

LAKEHEAD UNIVERSITY

PERCEIVED NUMEROSITY AND GESTALT PRINCIPLES  
OF PERCEPTUAL ORGANIZATION

by



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## ABSTRACT

Three experiments were conducted in an attempt to relate indirectly Gestalt principles of perceptual organization to the perception of numerosness<sup>1</sup>. Experiment 1 hypothesized that estimation of number is a function of the brightness contrast between focal (element) and contextual (background) variables. This hypothesis was not confirmed by the data, although indirect support from the present data and from previous research is discussed.

In Experiment 2 it was predicted that numerosness would be overestimated for homogeneous compared with heterogeneous stimulus arrangements. Two parallel experimental operations failed to yield data in support of the hypothesis.

Finally, in Experiment 3 stimulus patterns were quantified with the expectation that estimation of number would be an inverse function of informational content. Although an unexpected quadratic curve was obtained it is suggested that the data are in support of the experimental hypothesis. Tentatively, perceived number may be considered to be a function of the Gestalt notion of figural goodness.

The data do not provide a basis for the interpretation of the regular-random numerosity illusion (RRNI). However, they do not discredit the notion that the regular figures that have been used in the delineation of the illusion incorporate in their composition Gestalt principles of perceptual organization.

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1. The terms 'numerosness' and 'numerosity' will be used interchangeably throughout the text of this paper.

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Finally, whatever limitations and shortcomings of the study remain, they are my own responsibility.

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DEDICATION

Dedicated to my beloved wife  
Catharine Anne  
whose strength and courage  
I shall always admire.

## INTRODUCTION

### 1) Regular-random Numerosity Illusion: Data Review

Ginsburg (1976) described a perceptual illusion which is characterized by a marked tendency for an observer to judge a regularly arranged pattern of dots to be more numerous than its randomly arranged counterpart, while both stimuli are equal in physical number. Subsequently, this particular perceptual phenomenon has been referred to as the regular-random numerosity illusion (RRNI).

Using circular patterns Ginsburg (1978) and Cousins (1979) have replicated the illusion for various numbers of dots. It has been demonstrated consistently that estimations of number for regular arrangements exceed actual number while the converse is true for random arrangements.

Previous efforts to explain the illusion have focused largely on an expectancy-contrast model proposed by Birnbaum and Veit (1973). This model holds that through everyday experience the observer develops a subjective correlation between focal and contextual variables, and that expectancies, based on this correlation, are established. Where the subject is presented with stimulus information that is discrepant with respect to his expectancies, a process of contrast induces an illusion (see also Birnbaum, 1975).

The RRNI appeared to be consistent with an expectancy interpretation, particularly if one assumed that a negative subjective correlation between numerosity (focal variable) and regularity (contextual variable) had been established through everyday experience. When one considers the randomly arranged stimuli (eg. trees in the forests; stars in the sky) and the regularly arranged stimuli (eg. cars in a parking lot; buildings on a street) that are encountered on a daily basis, the assumption that random arrangements are expected to appear more numerous certainly seems plausible. If an expectancy interpretation were appropriate, based on this negative subjective correlation the observer would expect the random arrangement to be more numerous and by contrast with numerosity perceived, he would judge it to be less numerous.

Indirect evidence for an expectancy interpretation of the RRNI is provided by Ginsburg (1978) who elicited subject's expectations before and following estimations of number for dots in regular and random patterns. It was shown that before exposure to the stimuli subjects expected randomly arranged patterns to appear more numerous than objectively equal regular arrangements. Interestingly, another group of subjects tested following exposure to stimuli tended to have opposite expectations. Also Ginsburg and Deluco (1979) demonstrated a developmental trend in the strength of the illusion, implicating perceptual experience

as a contributing factor. However, in a study designed to examine the expectancy-contrast model more directly, less favourable results were reported.

By using a procedure similar to that employed by Birnbaum and Veit (1973), Cousins (1979) attempted to modify the RRNI by changing subjective correlations in an experimental situation. Different groups of subjects were exposed to conditions where item number was either positively correlated, negatively correlated or not correlated with regularity. It was expected that in each of the respective conditions the illusion would be reversed, enhanced and unaffected. The results did not yield a confirmation of the author's hypothesis as the illusion, apparently robust in nature, persisted in all conditions.

Subsequently, Ginsburg (1980) conducted a study that demonstrated the RRNI when rectangular regular and random arrangements are employed. In his discussion, Ginsburg relates the RRNI to a theoretical framework entirely different from and perhaps in opposition to that upon which the expectancy-contrast model is based: namely Gestalt theory. It is upon this theoretical framework and its implications for the RRNI and perceived numerosity in general that the present study focuses.

## 2) The Gestalt Thesis

On the basis of casual observation Frith and Frith (1972) proposed that elements in a single cluster would be

judged to be more numerous than the same number of elements displayed in several small clusters. In an experimental test their hypothesis was readily confirmed and the authors termed the phenomenon the "solitaire illusion". They related the illusion to Gestalt theory by suggesting that elements in a single cluster form a higher order "Gestalt" determined by the principles of continuity and spatial separation. However, these authors offer no explanation as to why such an arrangement might appear to be more numerous than several small clusters.

The early Gestalt psychologists discussed the notion of goodness of figure in delineating their position on perceptual organization. They conceived of a good figure as being one that is well organized and embraces any combination of qualities such as; similarity; regularity; symmetry; good continuation; common fate; simplicity, etc. As Ginsburg (1980) has pointed out it is reasonable to suggest that elements in a regular arrangement are inherently better figures than their random counterparts.

In the present study an attempt is made to relate certain Gestalt principles of perceptual organization to perceived numerosity. Interest in this particular relationship stems from Frith and Frith's assumption that elements that form a better Gestalt should appear more numerous, and the notion that items arranged in a regular fashion are intuitively better figures than their random counterparts. The demonstration of the existence of such a relationship would undoubtedly have implications for

the interpretation of the RRNI.

The aforementioned research problem is investigated by employing a set of experimental operations from three different orientations. The rationale underlying each experiment, and a description of each experimental hypothesis, will follow.

3) Experimental Operations

a) Experiment 1: Contrast

Since the early Gestalt psychologists discussed the notion of goodness of figure the concept has frequently been the subject of empirical research. Although difficulties have been encountered in operationally defining the term "good figure", empirical evidence has isolated several characteristics associated with the perception of such figures and other stimuli incorporating Gestalt principles of perceptual organization. A brief, and by no means exhaustive, survey of such research is described below.

French (1953) studied the discrimination of differences between dot patterns as a function of the number of dots in the pattern as well as the average separation of the dots. His findings clearly indicated that subjects best discriminate differences when both number of dots and average separation were relatively low. The principle of simplicity, characteristic of good figures, may have some bearing on these data. In a subsequent study, French (1954) examined the identification of dot patterns from memory as a function of complexity of the patterns. Although a relatively moderate number of dots (6-8)

was found to be optimal for pattern identification, ease of identification was clearly shown to be associated with patterns having dots arranged in either symmetrical or linear arrays.

Hochberg and McAlister (1953) measured frequency of responding to bidimensional and tridimensional levels of ambiguous figures (i.e. Kopferman cubes) as a function of figural goodness. Their results indicated that quantitatively defined good figures are perceived tridimensionally far more often than poor tridimensional figures. An information measure was employed as an index of figural goodness.

In discussing possible objective definitions of figural goodness, Hochberg and McAlister considered recognition thresholds as an appropriate index, but dismissed this alternative as being too limited and experimentally restrictive. However, in a subsequent study, Bitterman, Krauskopf and Hochberg (1954) found that foveal form threshold varies directly with the ratio of the form's perimeter to its area, which suggests that simple forms are recognized at lower threshold levels.

Beckwith and Restle (1966) studied the effect that object arrangement has on the speed and accuracy of enumeration. They found that in counting, a set of objects is grouped according to certain Gestalt principles (i.e. propinquity, good continuation, similarity). Their data showed that speed of counting increased with no loss in accuracy when their stimuli were characterized by some or all of these principles.

Koffka (1935), when discussing memory, suggested that a trace system, resulting from a well-organized field, is more stable and less subject to interference than that arising from a chaotic field. In an investigation of whether retention is a function of pattern goodness, Attneave (1955) found that, in three different memory tasks, symmetrical patterns were remembered more easily than asymmetrical patterns occupying the same number of cells in a matrix.

In a study concerning the estimation of number Saltzman and Garner (1948) found that reaction time (RT) was quicker for regular vs. random arrangements of dots throughout the entire range of dots used (1-10). Finally, in another RT task designed to examine whether response uncertainty is an inverse function of pattern goodness, Clement (1964) found that subjects responded more quickly and more uniformly in a verbal naming task with patterns judged to be higher in figural goodness.

The above evidence suggests that psychological processes including discrimination, recognition, identification, learning and memory are enhanced when good patterns, as opposed to poor, are employed as stimuli. Indirectly, this would suggest that good patterns are more readily perceptible and/or are more easily processed by human perceptual mechanisms.

In accordance with the present research problem, the experimental hypothesis for Experiment 1 states that estimation of number is a function of pattern perceptibility. That is to say,

it is predicted that the more perceptible the pattern of dots, the more numerous it will be judged. Horne and Turnbull (1977) provide evidence in support of such an assertion. They showed that longer stimulus duration leads to higher estimation. One interpretation of this finding may be that longer duration makes the pattern more perceptible and consequently produces higher estimation.

Perhaps another way in which a stimulus pattern may be made more perceptible is to increase the brightness contrast between focal (element) and contextual (background) variables. In Experiment 1 pattern perceptibility is operationally defined in these terms. Thus a random arrangement of black dots is predicted to be judged more numerous on a light gray background than a similar arrangement, equal in physical number, on a substantially darker background. The converse is predicted when white dots are employed.

b) Experiment 2: Homogeneity

Several Gestalt principles relate to the perception of form. The perceptual field is said to become organized, taking on form as parts become connected and groups of parts unite to form structure. Organization is said to be inevitable and natural where an organism is concerned, according to the Gestalt psychologists. Wertheimer (Woodworth & Schlosberg, 1954), in attempting to determine why some elements of the visual field form into figure while other units become part of ground, presented various patterns of dots, and observed which dots grouped themselves into figures most readily. Subsequently, he proposed certain principles of

perceptual organization. These principles include: nearness or proximity in the field of view; sameness or similarity; common fate of elements; and good continuation among others.

In an attempt to unify the principles of grouping, Musatti (Woodworth & Schlosberg, 1954) amalgamated them into one unitary law of homogeneity. He suggested that proximity, similarity, common fate, good continuation, in addition to certain environmental variables, are all mere instances of the underlying law of homogeneity and that it is this particular law that has a major contribution to perceptual organizing behavior.

In that adjacent units, and units of similar size, shape, and colour, tend to combine into better articulated wholes, and that a good form is a well-articulated one (Boring, 1950), it is logical to assert that Musatti's law of homogeneity is an underlying component of figural goodness.

The experimental hypothesis of Experiment 2 states that a perceptual field of homogeneous elements will be perceived as a better articulated whole than a pattern of heterogeneous elements. Thus, in accordance with the aforementioned research problem, it is predicted that homogeneous patterns of elements will be judged to be more numerous than their heterogeneous counterparts of equal physical number. This prediction follows Musatti's proposition that homogeneous patterns form a better Gestalt than heterogeneous patterns and the assumptions of Frith and Frith (1972) and Ginsburg (1980) that better Gestalten appear more numerous.

Element properties to be manipulated to achieve conditions of homogeneity and heterogeneity include brightness (i.e. all black or all white vs. black and white dot brightness) in Experiment 2a, and size (i.e. all small or all large vs. small and large dot sizes) in Experiment 2b.

c) Experiment 3: Information

The subjective and qualitative formulation of the Gestalt principles of perceptual organization have frequently made it difficult to test them. Koffka (1935) has stated that quality and quantity are not separate characteristics but rather two aspects of the same basic principles and that real organizations, units, and shapes must have a formula which will express them quantitatively.

Subsequently, there have been numerous attempts to quantify many of the qualitative Gestalt principles, including the principle of goodness of figure. Examples of such attempts will follow.

Hochberg and McAlister (1953) employed as a measure of figural goodness, the relative time devoted to each of the perceptual responses which may be elicited by the same ambiguous stimuli. These authors hypothesized that "...the probability of a given perceptual response to a stimuli is an inverse function of the amount of information required to define that pattern" (p. 364).

Attneave (1954, 1955, 1959) redefined the notion of good figure in terms of "redundancy" or "interdependencies among parts". He said that many of the Gestalt principles of perceptual organization

pertain to information distribution and that a good Gestalt is characterized by a high degree of "internal redundancy". Organization was said to be demonstrably measurable in informational terms and Attneave conceived of the principles of similarity, symmetry, good continuation, etc. to be examples of redundancy. Based on his information theoretic approach, Attneave hypothesized that various Gestalt laws might make good patterns easier to remember because their geometrical order is one way of reducing the uncertainty of these patterns.

In a characteristically different information theoretic approach, Garner (1962,1966) also examined informational properties of stimuli as opposed to their physical parts. He suggests that goodness of figure does not depend upon the characteristics of the individual stimulus, but on a set of alternatives from which that stimulus must be differentiated. That is to say, a pattern's goodness was said to vary inversely with a number of other equivalent patterns with which the subject classes that stimulus. Bear (1973) supports Garner's position by demonstrating that in good patterns the positions of missing elements are more readily predictable than the positions of missing elements in poor patterns. Bear suggests that these data demonstrate the Gestalt conception of a good figure as one that is well organized and Garner's notion that better figures have few alternatives.

The notion that the quantity of information of a pattern is an inverse function of figural goodness has been suggested

by various authors (eg. Hochberg & McAlister, 1953; Attneave, 1954, 1955, 1959; Garner, 1962, 1966) regardless of the measure of information employed. Thus it is reasonable to assume that regular arrangements contain less information than their randomly arranged counterparts. In Experiment 3 the informational content of stimulus arrangements, varying in degree of regularity, is determined. If it is assumed that good figures appear more numerous and contain less information than poor figures, it can be predicted that perceived numerosity is an inverse function of informational content. Thus, as degree of regularity increases, estimation of number is expected to increase.

#### 4) Summary

Some indication has been provided that perceived numerosity may depend upon certain underlying Gestalt principles of perceptual organization. Such consideration characterizes the research problem for the proposed study. A series of experimental operations, from different orientations, will be employed to investigate the problem.

Pattern perceptibility is believed to have a demonstrable effect upon number perceived. In the first experiment it is hypothesized that estimation of number is a direct function of the brightness contrast between elements of a pattern and its background. A set of parallel operations will be utilized to investigate whether homogeneity of elements has implications for perceived numerosity. In a second experiment it is hypothesized that an

array of homogeneous elements will be judged to be more numerous than a heterogeneous array of equal physical number. Finally, it is believed that perceived numerosity is an inverse function of the informational content of a pattern. In a third experiment it is hypothesized that, as degree of regularity increases and concomitantly informational content decreases, estimations of number will be greater.

It is evident that either the confirmation of some or all of the present experimental hypotheses will have implications for the RRNI. It should be noted that such confirmation will provide no indication as to why regular arrangements are typically judged to be more numerous than randomly arranged patterns of equal physical number. However, positive results of the present research will link perceived numerosity to Gestalt principles of perceptual organization and, consequently, establish a basis from which future research concerning the RRNI may be conducted.

## EXPERIMENT 1: CONTRAST

### Method

- 1) Design & Subjects Experiment 1 was characterized by a  $2 \times 3 \times 6$  single group repeated measures design. Thirty undergraduate students recruited from the Psychology Subject Pool at Lakehead University participated as subjects: 18 were females and 12 were males. These subjects ranged in age from 18 to 36 years with a mean age of 20.43 years.
- 2) Apparatus A Kodak carousel slide projector, complete with an 80 slide carousel and two timing mechanisms, was used to project stimuli onto a screen. One timing mechanism functioned to regulate the exposure duration of each stimulus slide. This duration was measured with a photo cell and timer and found to be 2.90 seconds. The other timer served to regulate the intertrial interval duration: photo-cell and timer indicated a 7.21 second duration.
- 3) Materials Thirty-six stimulus patterns were constructed by pasting 6 mm cardboard dots on 220 mm by 275 mm cardboard sheets. Each stimulus pattern was constructed by surrounding a central point with five evenly spaced rings: the radius of the outer ring being 76 mm. The five circles comprised 8, 16, 24, 32 and 40 equidistant potential dot positions for a total of 120. The positions to be taken by the prescribed number of dots were determined by using a table of random numbers. When all dot positions had been determined the rings were removed and the dots affixed. Finally photographs of each stimulus sheet were taken and a set of 35 mm stimulus slides was prepared. The six levels of number

that were employed ranged from 20 to 65 in increments of nine. Different conditions of perceptual contrast were created by employing three levels of gray background (dark, medium and light) and two levels of element brightness (black and white). This yielded 36 different stimuli. Appendix B-1 displays examples of these stimuli.

The levels of background brightness were varied by mixing different proportions of white and black water base paints (see Appendix A). Subsequently, a roller was used to paint the cardboard sheets. The medium gray ( $R = .29$ ) was selected since, according to pilot data, it gave no bias to either black or white elements in terms of estimation of number.

Table 1-1 provides an indication of the reflectance of each of the levels of background and element brightness. Reflectance was measured by using the Munsell Neutral Value Scale (1971 edition). One stimulus sheet representing each level of background brightness was selected for measurement.

Table 1-1  
Reflectance of Stimulus Sheets (% Reflectance)

	Background	White Elements	Black Elements
Dark	20	90	7
Medium	27	90	7
Light	36	90	7

Two observers made independent assessments and a consensus was taken as to the reflectance of the stimulus sheets and their elements.

Table 1-2 shows the luminance of the stimulus slides when projected upon the screen. Luminance was measured by using a Macbeth Illuminometer. Prior to measurement stimulus slides were projected upon a screen in the room used for experimentation. Again, one representative slide pertaining to each level of background was selected for measurement and a series of estimations was

Table 1-2  
Luminance of Stimulus Slides (ftL)

Background	White Elements	Black Elements
Dark	.86	3.87
Medium	1.24	8.35
Light	1.29	2.83

made by two independent observers. A consensus as to the luminance of each slide was subsequently taken.<sup>1</sup>

4) Procedure Subjects were run in one group of 30. Each subject was provided with a pencil and a response sheet (see Appendix D-1). Subsequently, the group was instructed to observe patterns of dots as they were briefly flashed on the screen and to estimate the number of dots in each

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<sup>1</sup>It may be noted that perfect correspondence between the reflectance of stimulus sheets and the luminance of stimulus slides was not attained. See Discussion (p. 64) for comments regarding this discrepancy.

pattern (see Appendix C-1). The projector was positioned approximately 10 m from the screen.

The group received 54 trials where stimulus slides were shown for 2.90 seconds and followed by a 7.21 second interval, within which time subjects recorded their estimations of number and prepared to observe the next slide. After every 10 trials subjects were informed as to which trial was next.

The first 18 trials served as training trials, while the remaining 36 were test trials. Test trials were characterized by one replication of each of the 36 stimulus patterns. For the training trials one-half of these patterns were randomly selected such that each level of element brightness, background brightness, and number were fairly represented.

The order of presentation was randomized such that three restrictions were met. The restrictions were:

- 1) no more than three patterns containing the same level of element brightness could be presented consecutively,
- 2) no more than two patterns characterized by the same level of background brightness could be presented consecutively, and
- 3) patterns containing the same level of number could not be presented consecutively.

Following testing the subjects were debriefed as to the nature and purpose of the study and reminded to indicate their age and sex.

## Results

The dependent variable<sup>1</sup> employed in the present experiment was each subject's estimation of number for each of the 36 test trials (see Appendix F-1).

An element brightness (EB: 2 levels) by background brightness (BB: 3 levels) by Number (No: 6 levels) factorial repeated measures Analysis of Variance was performed on the data (see Table 1-3).

In the light of the experimental hypothesis of the present experiment perhaps the most interesting finding was a statistically non-significant EB X BB interaction  $F(2,58)=0.80$ ,  $p>.05$ , accounting for virtually none of the total variability ( $\eta^2 = .0003$ ). This unexpected result is depicted in Figure 1-1 where it is readily apparent that estimation of number was relatively unaffected by level of background brightness regardless of the level of element brightness.

It is interesting to note that at all levels of background brightness patterns containing white elements were reported to be more numerous than those containing black elements. This finding is supported by a statistically significant main effect for EB,  $F(1,20) = 8.97$ ,  $p<.01$ ,

---

<sup>1</sup>See Discussion (p. 57) for comments regarding the criterion variable employed for data analysis.

Table 1-3

Summary of Repeated Measures Analysis of Variance:  
 Element Brightness X Background Brightness X Number

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	29		
Element Brightness (EB)	1	1456.03	8.97*
EB x S's (Error)	29	162.31	
Background Brightness (BB)	2	206.78	3.09
BB x S's (Error)	58	66.87	
Number (No)	5	41851.96	145.15**
No x S's (Error)	145	288.34	
EB x BB	2	72.08	0.80
EB x BB x S's (Error)	58	90.28	
EB x No	5	462.90	6.02**
EB x No x S's (Error)	145	76.94	
BB x No	10	277.26	3.15**
BB x No x S's (Error)	290	88.16	
EB x BB x No	10	155.07	1.33
EB x BB x No x S's (Error)	290	116.88	

\* p<.01

\*\* p<.001

Figure 1-1

Mean Estimation of Number Plotted Against  
Element and Background Brightness

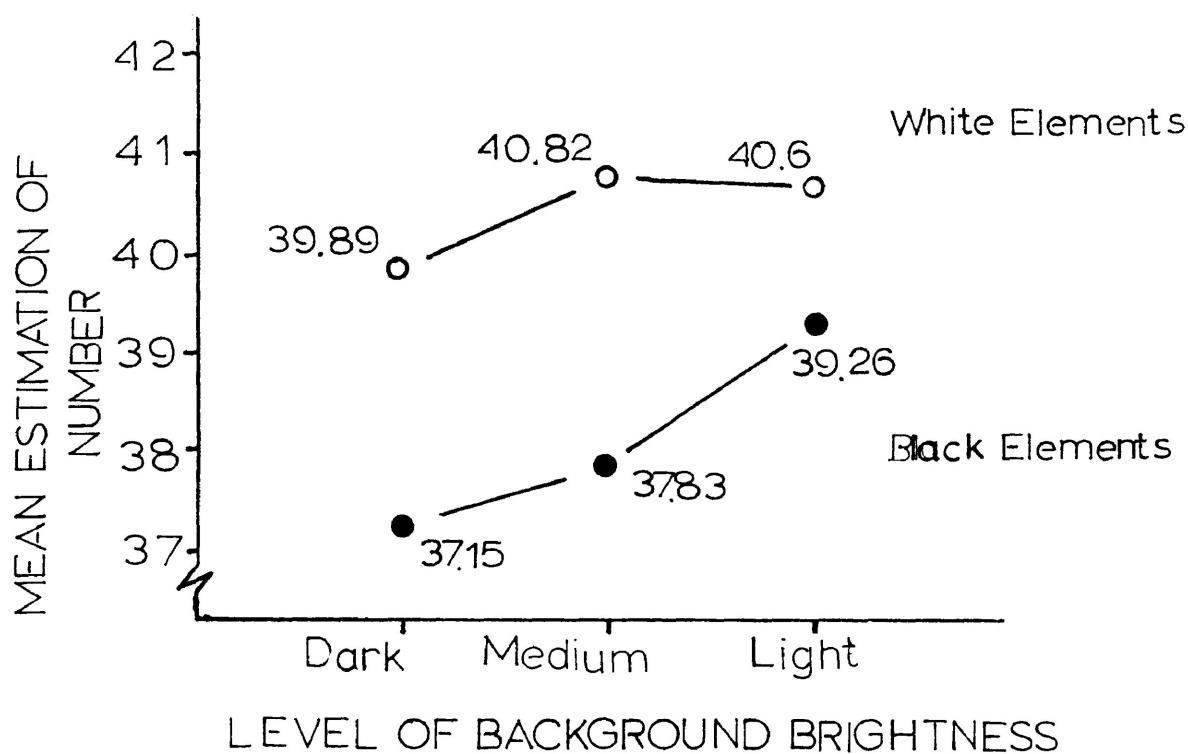
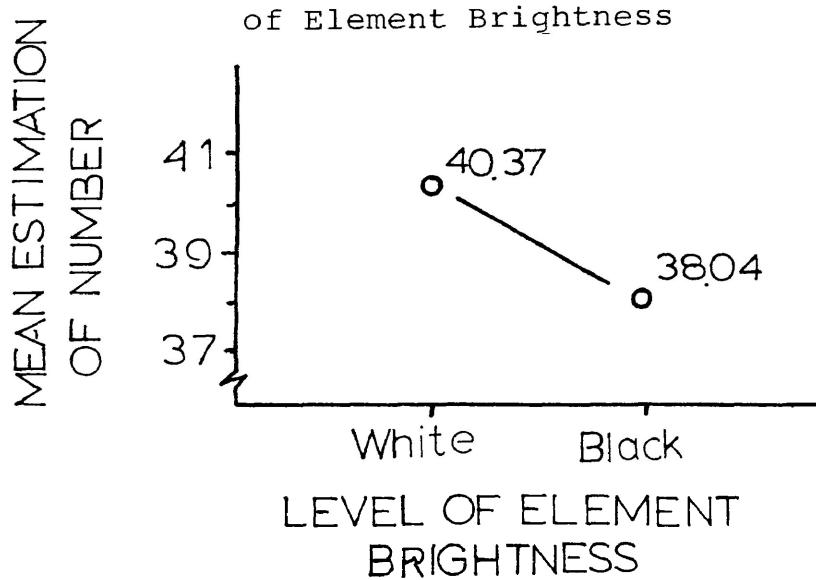


Figure 1-2

Mean Estimation of Number as a Function  
of Element Brightness



and is illustrated in Figure 1-2. Although patterns with white elements were judged to be more numerous it should be noted that this effect accounted for less than 1% of the total variability ( $\eta^2 = .003$ ).

As predicted, a statistically significant main effect for number was discerned,  $F(5,145) = 145.15$ ,  $p < .001$  (see Figure 1-3). This finding accounted for a large proportion of the total variability ( $\eta^2 = .475$ ) and is characterized by increases in the estimation of number as actual number increases. The predicted linear trend is confirmed by a statistically significant Trend Analysis for the linear component  $t(145) = 10.99$ ,  $p < .001$ , and a statistically non-significant quadratic trend.

The Analysis of Variance discerned two remaining and unexpected statistically significant effects: an EB X No interaction,  $F(5,145) = 6.02$ ,  $p < .001$ ; and a BB X No interaction  $F(10,290) = 3.15$ ,  $p < .001$ . These interactions each accounted for less than 1% of the total variability ( $\eta^2 = .005$  and  $.006$  respectively). Figures 1-4a and b provide an illustration of these interactions.

Figure 1-4a shows differential responding to levels of background brightness at the various levels of number. This finding is found not to be statistically significant when the Geisser-Greenhouse correction is employed,  $F(1,29) = 3.15$ ,  $p > .05$ . In Figure 1-4b it may be seen that at most, but not all levels of number, patterns containing white dots were estimated to be more numerous. This finding remains statistically significant when the Geisser-Greenhouse conservative

Figure 1-3

Mean Estimation of Number as a Function  
of Level of Number (Experiment 1)

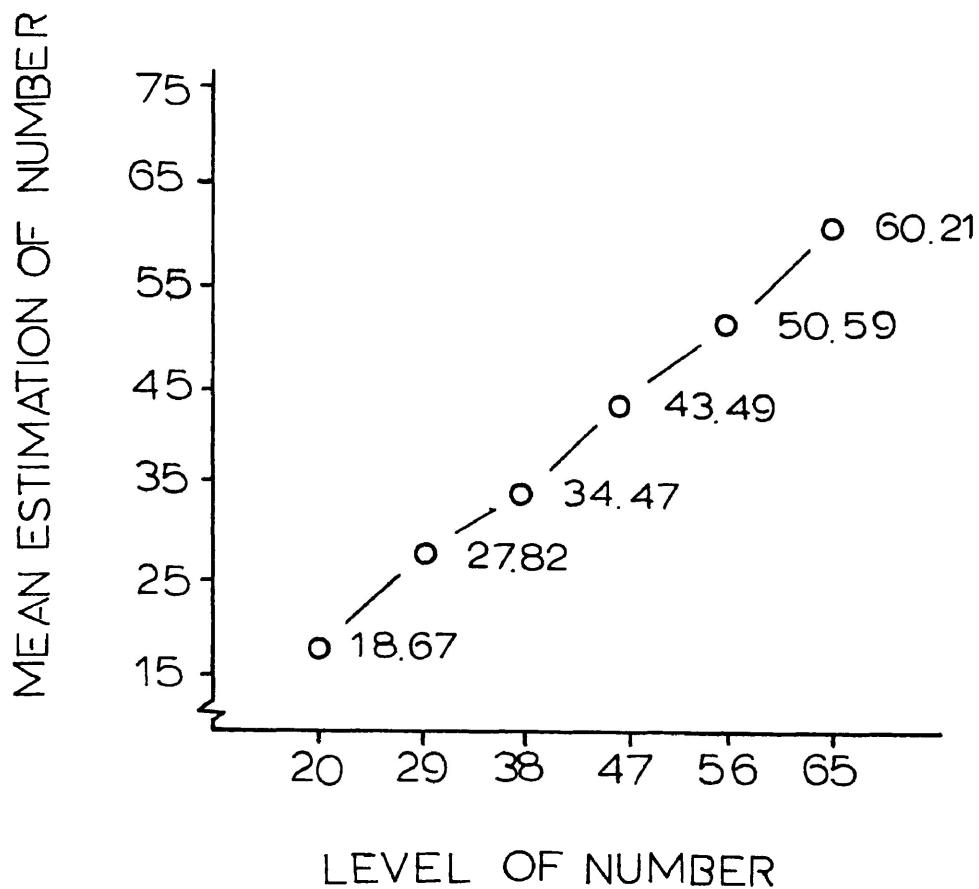


Figure 1-4a

Mean Estimation of Number as a Function  
of Background Brightness and  
Level of Number

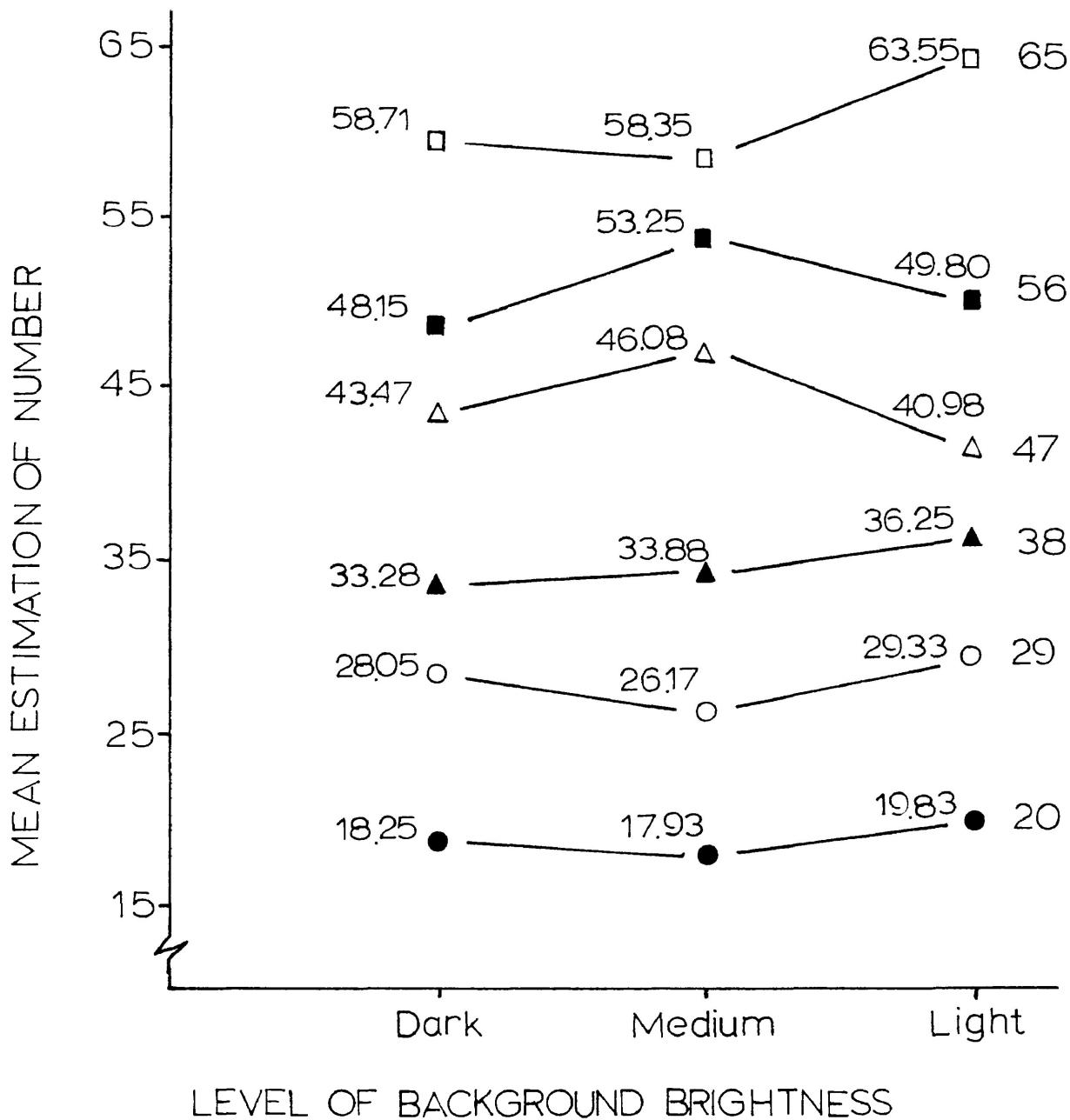
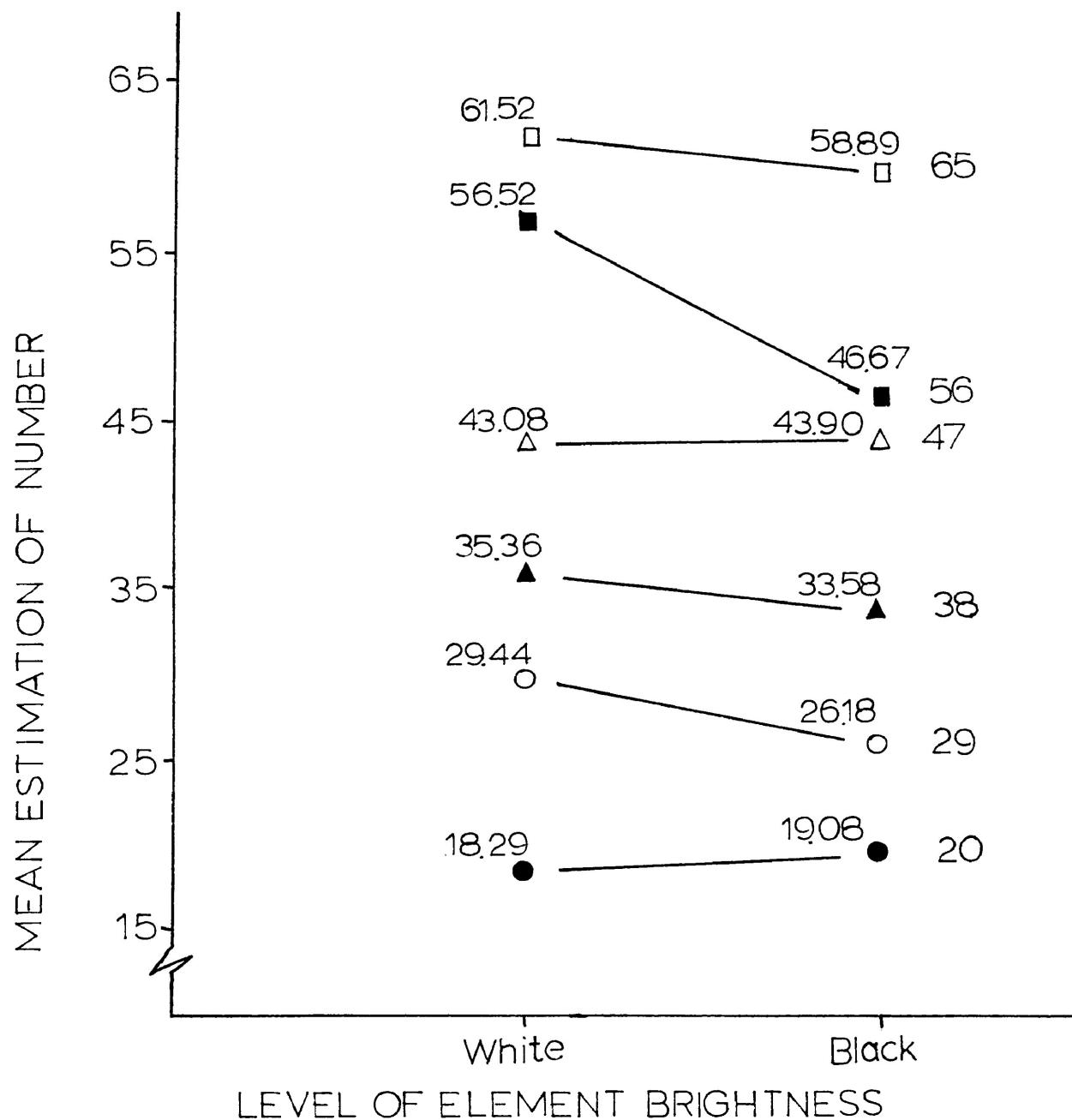


Figure 1-4b

Mean Estimation of Number as a Function  
of Background Brightness and  
Level of Number



correction is applied,  $F(1,20) = 6.02$ ,  $p < .001$ .

Unfortunately, neither of these illustrations offer any indication of systematic differences in responding, thus minimizing their interpretability. However, it is interesting to note that in all instances mean estimation of number was below the actual number of each stimulus pattern. This finding corroborates earlier research where both regular and random patterns were employed, in that random patterns tend to be underestimated (Ginsburg, 1979, 1978; Cousins, 1979).

## EXPERIMENT 2: HOMOGENEITY

### Method

1) Design & Subjects Experiment 2 was broken into two parallel experimental operations: Experiments 2a and 2b each characterized by single group  $3 \times 6$  repeated measures design. One group of 28 subjects participated in both experiments. Subjects were drawn from the Psychology Subject Pool at Lakehead University: 18 were females and 10 were males. These subjects ranged in age from 17 to 56 years with a mean age of 25.57 years.

2) Apparatus A Kodak carousel slide projector complete with an 80 slide carousel and a timing mechanism was used to project stimulus slides onto the screen.

The manually operated timing mechanism functioned to regulate the exposure duration of each stimulus slide.<sup>1</sup> This duration was 2.24 seconds. Intertrial intervals were timed manually by the experimenter with the aid of a wrist watch complete with second hand. Intertrial intervals were approximately 7 seconds.

---

<sup>1</sup>Technical difficulties prevented the continued use of the timing mechanism employed in Experiment 1.

3) Materials

a) Experiment 2a. Eighteen stimulus patterns were constructed by pasting 6 mm cardboard dots on 220 mm by 275 mm cardboard sheets. The rules governing pattern construction in Experiment 1 were employed in the present experiment. Photographs of each stimulus pattern were taken and a set of 35 mm stimulus slides was prepared prior to the experiment.

As in Experiment 1 the six levels of number that were employed ranged from 20 to 65 in increments of nine. Three different conditions of homogeneity were created by displaying all black, all white or  $\frac{1}{2}$  black -  $\frac{1}{2}$  white dots on a medium gray background. In the heterogeneous condition ( $\frac{1}{2}$  black -  $\frac{1}{2}$  white), after the dot positions had been determined, white and black dots were assigned by using a table of random numbers such that there were an equal number of these elements on patterns with even levels of number and a differential of one on patterns with odd levels of number.

The medium gray paint mixture employed in Experiment 1 was used to paint stimulus sheets prior to the application of the dots. Again, based on pilot data this level of gray ( $R = .29$ ) was found to be neutral with respect to white and black elements in terms of estimation of number. The reader is referred to Tables 1-1 and 1-2 (p. 15, 16) for an examination of the reflectance of the stimulus sheets and of the luminance of the stimulus slides for both background and pattern

elements. Appendix B-2 provides examples of the stimuli employed in Experiment 2a.

b) Experiment 2b. In the present experiment 18 stimulus patterns were constructed by pasting either 3 mm or 6 mm black dots on white 220 mm cardboard squares. Again the rules governing pattern construction in Experiment 1 were employed in the present experiment and a set of 35 mm stimulus slides was prepared prior to the experiment.

The same levels of number used in Experiments 1 and 2a were employed in the present experiment. Different conditions of homogeneity were created by displaying all small (3 mm), all large (6mm) or  $\frac{1}{2}$  small -  $\frac{1}{2}$  large dots on the stimulus sheets. The procedure for appropriating small and large dots in the heterogeneous condition is identical to that described in Experiment 2a. Appendix B-3 provides examples of the stimuli employed in Experiment 2b.

4) Procedure Subjects were run in one group of 28. Each subject was provided with a pencil and a response sheet (see Appendix D-2). Previous research has indicated that no detrimental effects of practice occur in a task of this nature over this number of trials. Therefore Experiment 2a was arbitrarily conducted first. Again subjects were instructed to observe patterns of dots as they were briefly flashed on the screen and to estimate the number of dots in each pattern (see Appendix C-1). The projector was positioned approximately 10 m from the screen.

The group received 36 trials where stimulus slides were shown for 2.24 sec. and followed by an interval of approximately 7 sec., within which time subjects recorded their estimations of number and prepared to observe the next slide. Following every 10 trials, subjects were informed as to which trial was next.

The first 18 trials served as training trials while the remaining 18 were test trials. Both training and test trials were characterized by one replication of each of the 18 stimulus patterns designed for Experiment 2a.

The order of presentation was randomized such that two restrictions were met. The restrictions were 1) no more than two patterns displaying the same condition of homogeneity could be presented consecutively, and 2) patterns displaying the same level of number could not be presented consecutively.

Following the completion of the 36 trials the subjects were informed that they had completed the first phase of the experiment. Response sheets were collected and new response sheets (identical to those used in Experiment 2a) were distributed.

The group was then presented with abbreviated instructions (see Appendix C-2) prior to the onset of Experiment 2b. The procedure for Experiment 2b was identical to that described for Experiment 2a, the only exceptions being that stimulus patterns designed for Experiment 2b were presented and a different random order of presentation, observing the same restrictions, was utilized.

Following testing, subjects were debriefed as to the purpose and nature of the study and reminded to indicate their age and sex.

Results

Experiment 2a The dependent variable<sup>1</sup> employed in the present experiment was each subject's estimation of number for each of the 18 test trials (see Appendix E-2).

A condition of homogeneity ( $H$ : 3 levels) by level of number ( $No$ : 6 levels) factorial repeated measures Analysis of Variance was performed on the data (see Table 2-1).

It was predicted that estimations of number for the heterogeneous condition would be lower than that for the combined homogeneous conditions. This hypothesis was not confirmed as a statistically non-significant main effect for  $H$ ,  $F(2,54)=2.25 \underline{p>.05}$ , accounted for less than 1% of the total variability ( $\eta^2=.001$ ). Although, as is apparent in Figure 2-1, estimations of number for the conditions of homogeneity were in the expected direction, orthogonal contrasts comparing the heterogeneous condition to the combined homogeneous conditions failed to reach statistical significance  $t(54) = -0.68, \underline{p > .05}$ .

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<sup>1</sup>See Discussion (p. 57) for comments regarding the criterion variable employed for data analysis.

Figure 2-1  
Mean Estimation of Number  
and Condition of Homogeneity  
(Experiment 2a)

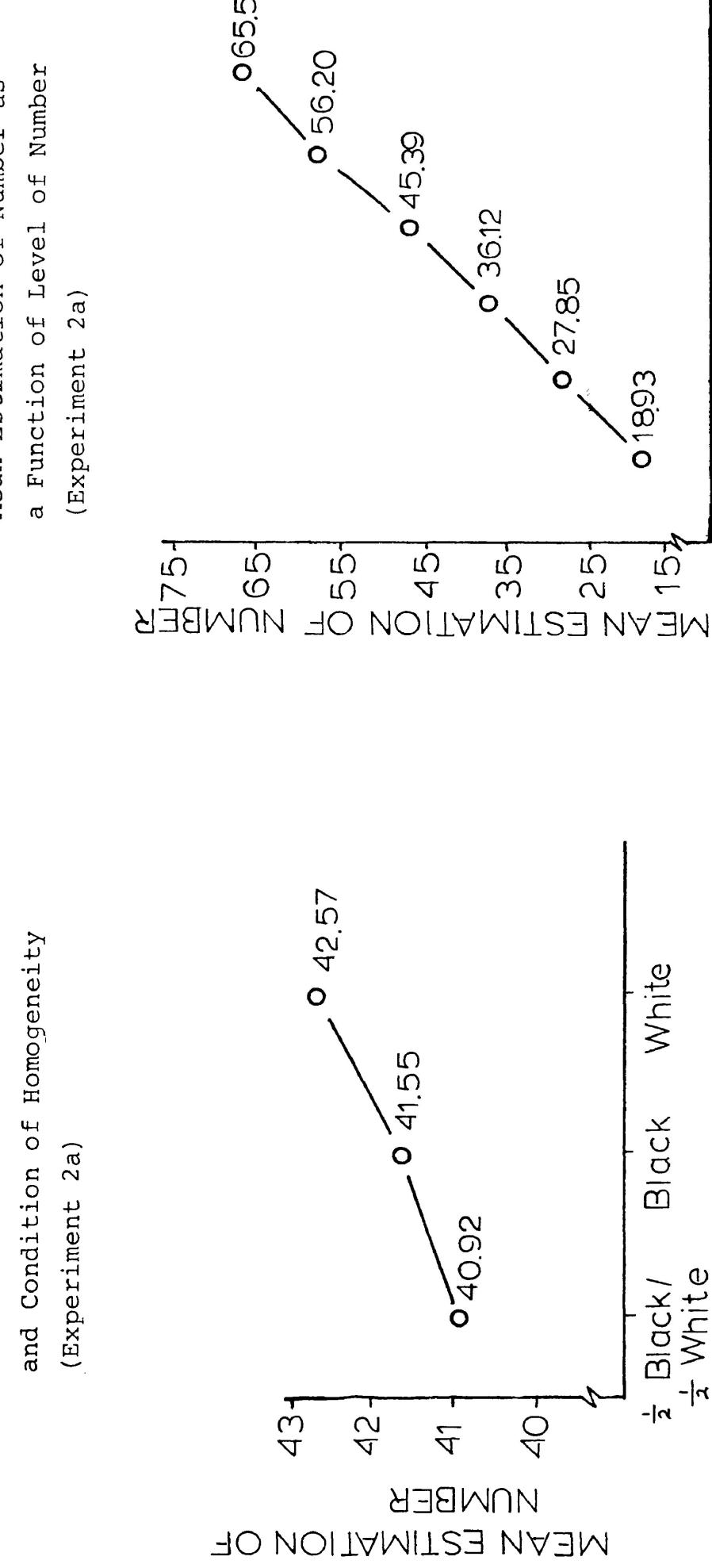


Figure 2-2  
Mean Estimation of Number as  
a Function of Level of Number  
(Experiment 2a)

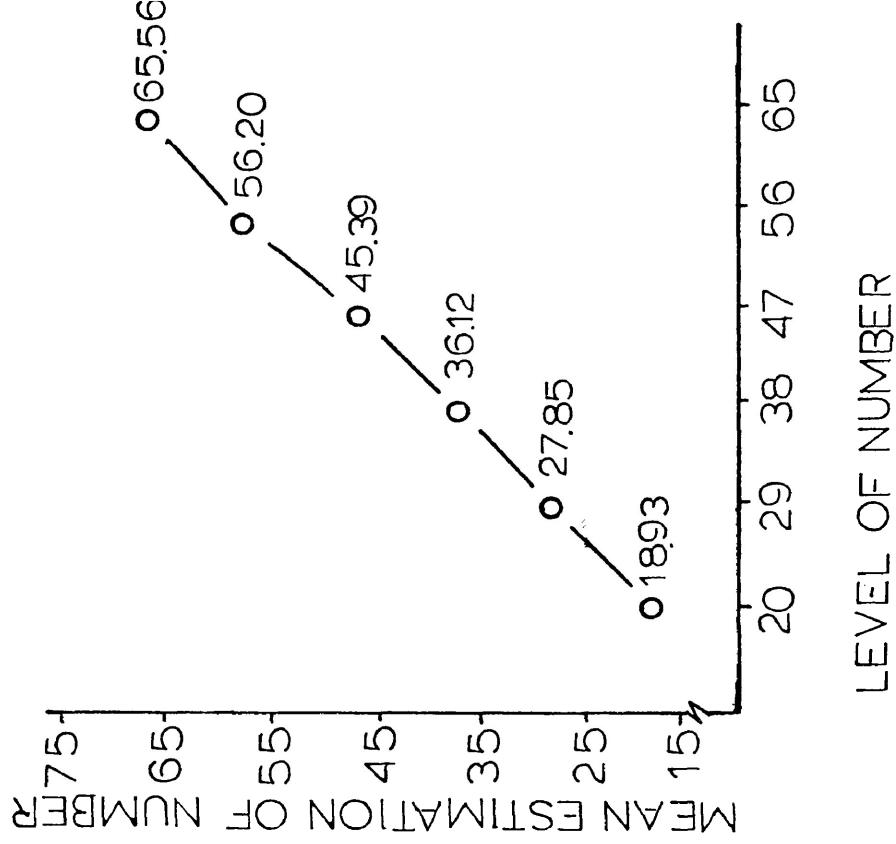


Table 2-1

## Summary of Repeated Measures Analysis of Variance:

Homogeneity x Number (Experiment 2a)

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	27		
Homogeneity (H)	2	117.04	2.25
H x S's (Error)	54	52.03	
Number (No)	5	25814.47	156.35**
No x S's (Error)	135	165.11	
H x No	10	137.93	2.05*
H x No x S's (Error)	270	66.97	

\*p&lt;.05

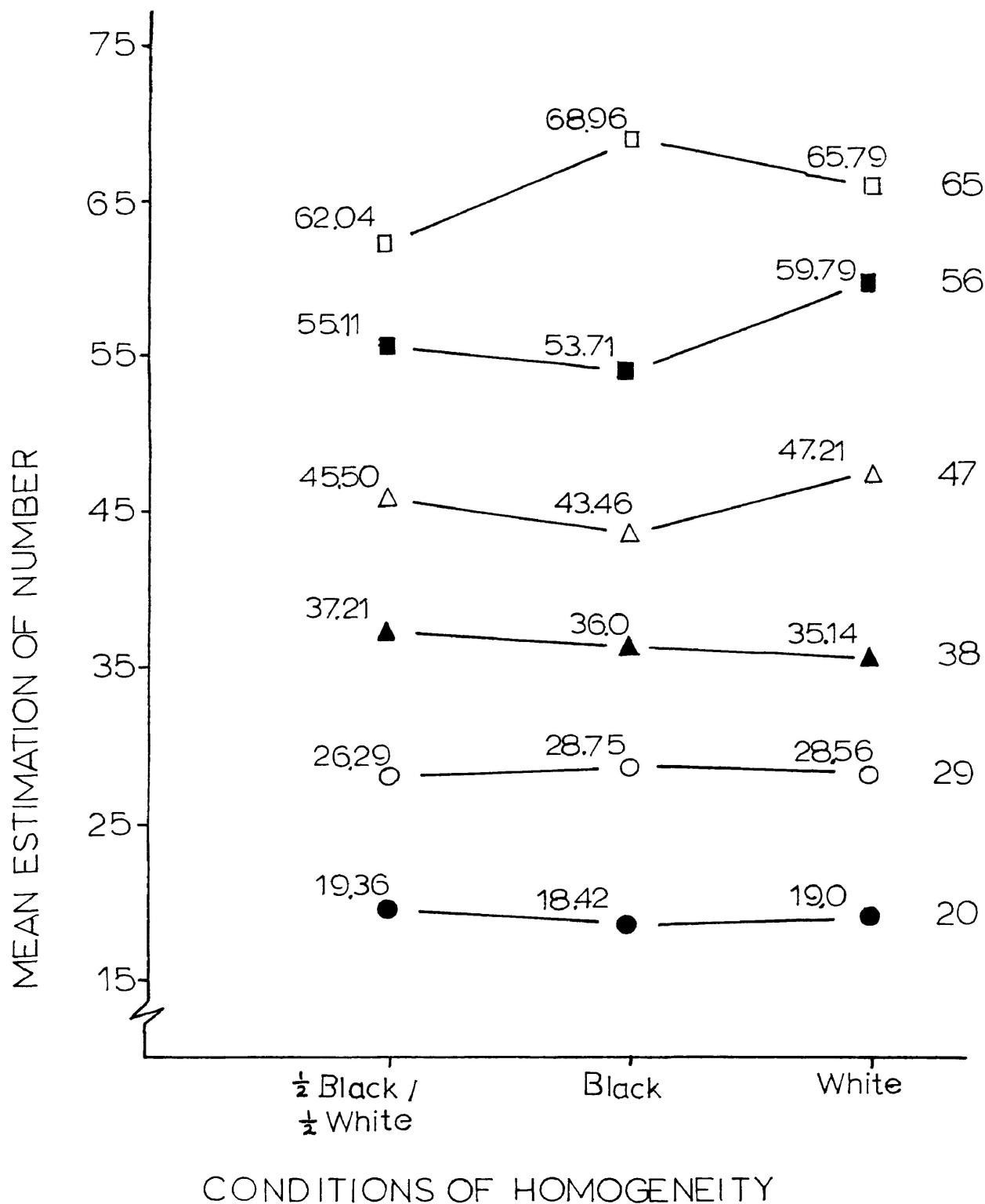
\*\*p&lt;.001

As predicted, a statistically significant main effect for No was discerned,  $F(5,135) = 156.35$ ,  $p < .001$ . Figure 2-2 reveals that subject's estimations of number increased linearly with the stimulus level of number. This linear trend was confirmed by a statistically significant Trend Analysis for the linear component  $t(135) = 16.12$ ,  $p < .001$ , and a statistically non-significant quadratic trend. The main effect for No accounted for well over 50% of the total variability ( $\eta^2 = .574$ ).

Finally an unexpected H x No interaction was found to achieve statistical significance  $F(10,270) = 2.05$ ,  $p < .05$ . This finding is characterized by a substantially different pattern of responding under conditions of homogeneity at the highest level of number (see Figure 2-3). In addition, where lower levels of number tend to be underestimated, as

Figure 2-3

Mean Estimation of Number as a Function  
of Condition of Homogeneity and Level of  
Number (Experiment 2a)



expected from previous findings where random arrangements are employed, this is not necessarily the case for higher levels of number in the present experiment. At any rate the H X No interaction accounts for less than 1% of the total variability ( $\eta^2 = .006$ ) and is found to be statistically non-significant when the Geisser-Greenhouse conservative correction is applied  $F(1,27) = 2.05$ ,  $p > .05$ .

Experiment 2b As in Experiment 2a the dependent variable for the present experiment was each subject's estimation of number for each of the 18 test trials (See Appendix E-3). Again a condition of homogeneity (H: 3 levels) by level of number (No: 6 levels) factorial repeated measures Analysis of Variance was used to analyze the data (see Table 2-2).

Table 2-2

## Summary of Repeated Measures Analysis of Variance:

## Homogeneity x Number (Experiment 2b)

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	27		
Homogeneity (H)	2	3002.63	24.24*
H x S's (Error)	54	123.88	
Number (No)	5	22513.75	152.21*
No x S's (Error)	135	147.91	
H x No	10	316.71	5.56*
H x No x S's (Error)	270	56.92	

\* $p < .001$

As in Experiment 2a it was predicted that estimations of number for the heterogeneous condition would be significantly lower than estimations for the combined homogeneous conditions. The Analysis of Variance discerned a significant main effect for H,  $F(2,54) = 24.24$ ,  $p < .001$ . However, as is readily apparent in Figure 2-4, this effect was not in the predicted direction. Although estimations of number for the "all small" condition were greater than for the " $\frac{1}{2}$  small/ $\frac{1}{2}$  large" condition, estimations of number for the "all large" condition were considerably lower. Orthogonal contrasts comparing the heterogeneous condition to the combined homogeneous conditions proved to be statistically non-significant  $t(54) = -0.08$ ,  $p > .05$ . The Scheffe multiple comparison procedure, comparing each mean with all others, revealed that the only means that differed significantly at the .05 level were the two homogeneous conditions. The main effect for H accounted for slightly over 1% of the total variability ( $\eta^2 = .014$ ).

Similar to Experiment 2a a predicted main effect for No was confirmed by the Analysis of Variance,  $F(5,135) = 152.21$ ,  $p < .001$ . This main effect accounts for over 25% of the total variability ( $\eta^2 = .262$ ). Again estimations of number were found to increase linearly with stimulus level of number, as can be seen in Figure 2-5. This notion is supported by a statistically significant Trend Analysis for the linear component,  $t(135) = 15.81$ ,  $p < .001$ , and a statistically non-significant quadratic trend.

Also similar to Experiment 2a, an unexpected H X No interaction was discerned,  $F(10,270) = 5.56$ ,  $p < .001$ . This

Figure 2-4  
Mean Estimation of Number as a  
Function of Condition of Homogeneity  
(Experiment 2b)

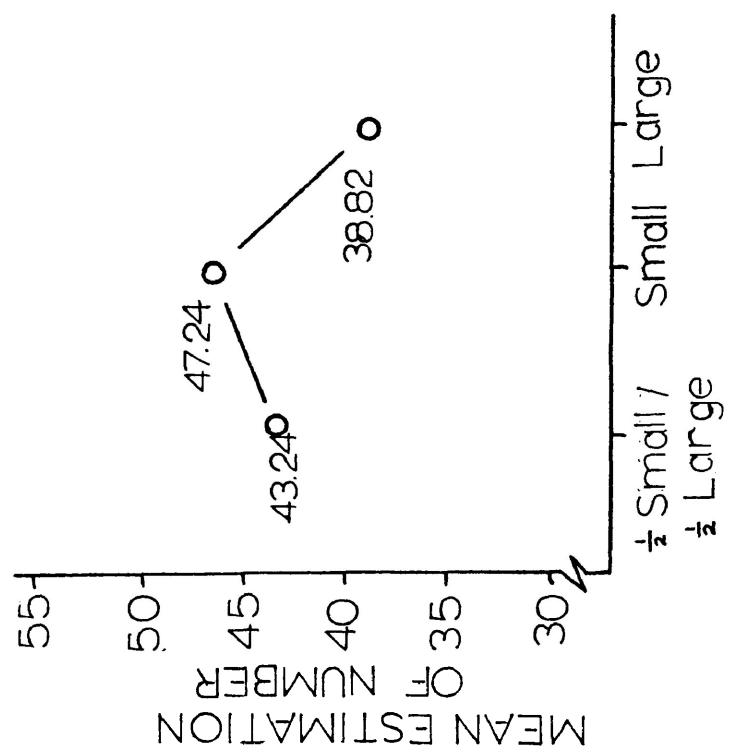
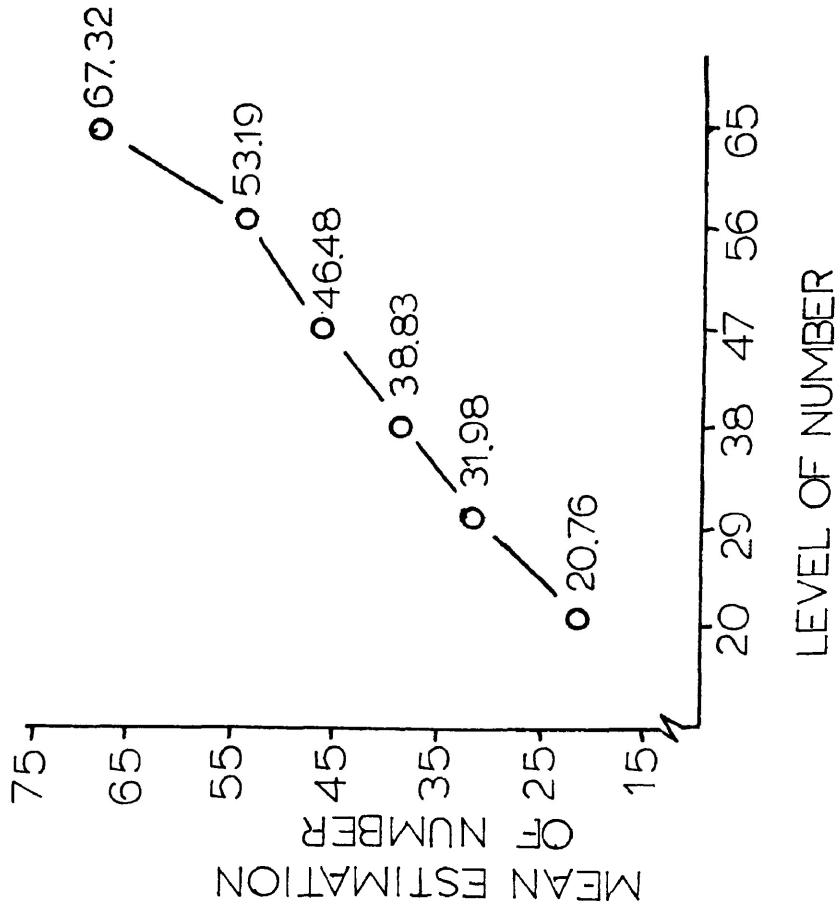


Figure 2-5  
Mean Estimation of Number as a  
Function of Level of Number  
(Experiment 2b)

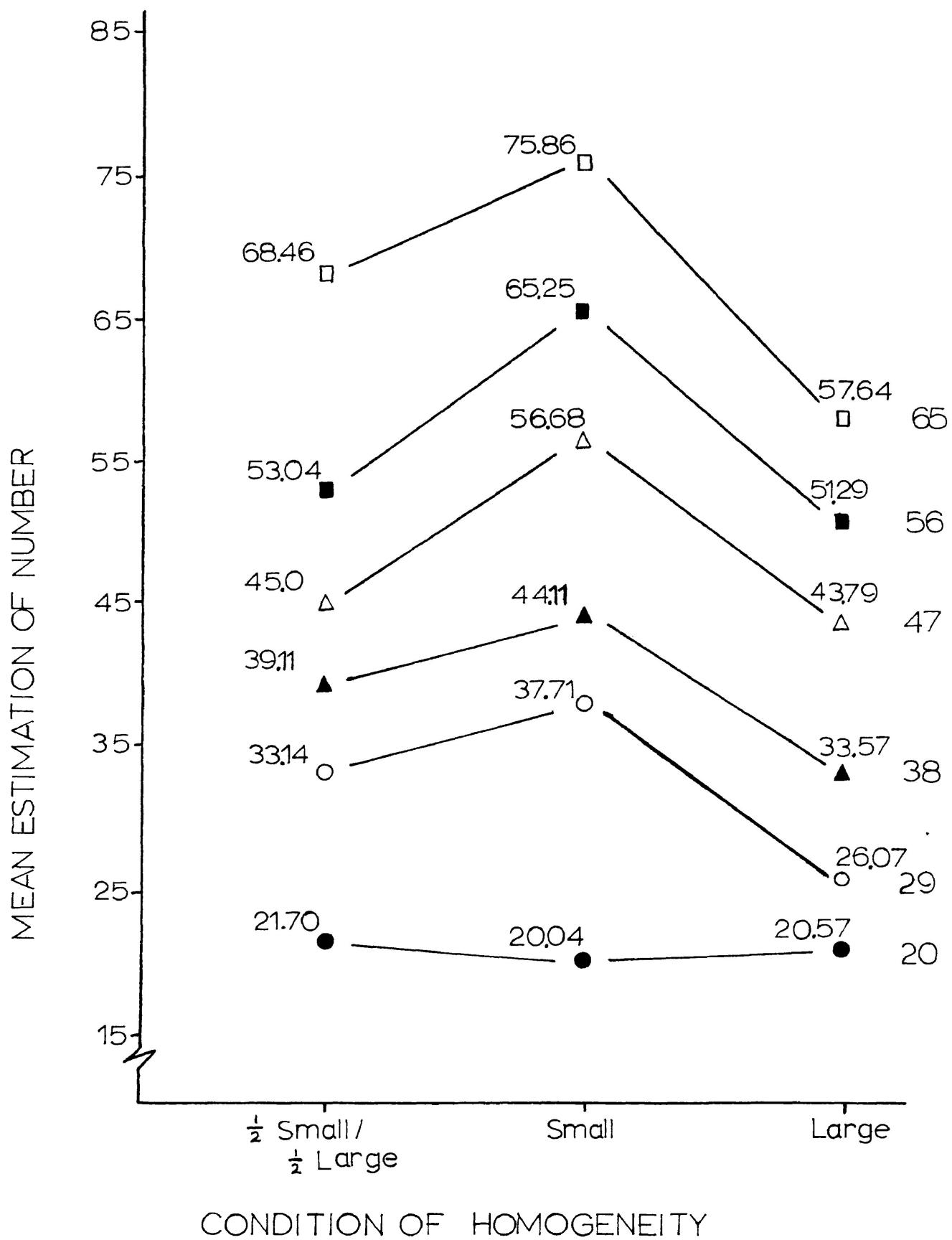


interaction is presented graphically in Figure 2-6. Again, similar to Experiment 2a, it is apparent that subjects tend to respond differently under different conditions of homogeneity at the highest level of number. Interestingly there appears to be no difference among conditions of homogeneity at the lowest level of number. Also the trend that estimations of number tend to be underestimated does not appear to be evident. Moreover, at almost all levels of number, estimations for the "½ small / ½ large" and the "all small" conditions are over-estimated.

The H X No interaction accounts for less than 1% of the total variability ( $\eta^2 = .007$ ) but remains statistically significant when the Geisser-Greenhouse conservative correction is applied  $F(1,27) = 316.71, p < .025$ .

Figure 2-6

Mean Estimation of Number as a Function  
of Condition of Homogeneity and Level of  
Number (Experiment 2b)



## EXPERIMENT 3: INFORMATION

### METHOD

1) Design & Subjects Experiment 3 was characterized by a  $3 \times 4$  single group repeated measures design. Twenty-one undergraduate students recruited from the Psychology Subject Pool at Lakehead University, participated as subjects: 11 were females and 10 were males. Ages ranged from 19 to 26 years and the mean age was 20.91 years.

2) Apparatus The apparatus described in Experiment 2 was employed in the present experiment (see p. 26).

3) Materials Twelve stimulus arrangements were constructed by pasting 6 mm black dots on 220 mm white cardboard squares.

The majority of the stimulus patterns were constructed utilizing a modified version of Attneave's (1955, p. 210 ff) technique which permits the informational measurement of both regular (symmetrical) and random (non-symmetrical) patterns. As depicted in Table 3-1 the stimuli were characterized by three levels of number (24, 40 and 60) and by four conditions of regularity (highly regular: REG3, regular: REG2, moderately regular: REG1, and random: RAND). Stimulus patterns are presented in Appendix B-4. A description of how the stimuli in each of the four conditions of regularity were constructed and an indication of how the informational content of each stimulus was determined will follow. The reader is referred to Table 3-1 for an elaboration of the characteristics associated with each stimulus pattern.

Highly regular stimuli (REG3) were constructed by using a matrix template which contained the same number of cells as the prescribed number of dots for a given level of number. The matrix template spanned the entire 220 mm square stimulus sheet. Dots were simply allocated to a position central to each cell in the matrix. Subsequently, the matrix was removed and the dots were fixed to the stimulus sheet. The resultant patterns were highly regular since each cell in the matrix was used and no other pattern could result. These patterns are symmetrical about their horizontal and vertical axes.

The method used to construct REG 3 patterns differs somewhat from that employed to construct the remaining nine patterns. All of the remaining patterns were constructed by employing a modified version of Attneave's (1955) technique.

Each pattern was constructed by randomly allocating a prescribed number of dots to positions in an initial matrix size. Dots were always allocated to a position central to matrix cells, each matrix cell had an equal opportunity of being assigned a dot, and there were always twice as many cells as the number of dots allocated. Final matrix size and concomitantly, number of dots, was dependent upon the number of "entire matrix" reflections required, such that the desired level of number was achieved.

To elaborate, consider stimulus condition level of number 24, REG2 (see Table 3-1). Here six dots were randomly assigned to a 3 x 4 matrix. To arrive at the level of number

Table 3-1: Information Content of Various REGs

---

Number of Dots Initially

---

Initial Matrix Size

---

Independent Variables:

Level of Number

24

60

Condition of Regularity <sup>1</sup>	REG3	REG1	RAND	REG3	REG2	REG1	RAND	REG2	REG1	RAND
Initial Matrix Size	4x6	3x4	4x6	3x5	4x5	5x5	6x10	5x6	6x10	10x12
Initial # of Cells	24	12	24	48	48	20	40	60	30	60
# Dots Initially	24	6	12	24	40	10	60	60	15	30
# Allocated	24	6	6x8	6x8	6x6	8x10	8x10	6x10	10x12	10x12
Final Matrix Size	4x6	6x5	6x8	48	48	80	80	60	120	120
Final # of Cells	24	48	48	48	48	80	80	60	120	120
# of Reflections Required	0	2	1	0	0	1	0	0	2	1
Combination Expression ( $nC_r$ )	$24C_{24}$	$12C_6$	$24C_{12}$	$4^9C_{24}$	$40C_{40}$	$20C_{10}$	$40C_{20}$	$80C_{40}$	$60C_{60}$	$30C_{15}$
Factorial Expansion <sup>2</sup>	$\frac{24!}{r!(n-r)!}$	$\frac{12!}{6!6!}$	$\frac{24!}{12!12!}$	$\frac{48!}{24!24!}$	$\frac{40!}{40!40!}$	$\frac{20!}{10!10!}$	$\frac{40!}{20!20!}$	$\frac{80!}{40!40!}$	$\frac{60!}{30!30!}$	$\frac{120!}{60!60!}$
Information (bits) <sup>3</sup>	0	9.83	21.30	44.80	0	18.47	36.06	76.51	0	27.18
										56.74
										116.25

- REG3: Highly Regular REG2: Regular REG1: Moderately Regular RAND: Random
- Under REG3 conditions  $(n-r)!$  is arbitrarily represented by 1! to avoid mathematical impossibility.
- See Figure 3-1 for computational process

required, the number of dot positions was doubled first by reflecting the initial matrix on its vertical axis and doubled once again by reflecting the newly constructed  $6 \times 4$  matrix on its horizontal axis. Thus both the initial number of matrix cells and the number of dots were increased fourfold. Subsequently, the final matrix template was removed and dots were affixed to positions central to designated matrix cells. A symmetrical arrangement based on two reflections was the result.

The construction of stimulus arrangement, level of number 24, REG 1 involved only one "entire matrix" reflection, thus increasing the initial matrix size ( $4 \times 6$ ) and the initial number of dots (12) twofold. The resultant stimulus arrangement of 24 dots was somewhat less regular (symmetrical) than condition REG 2. Finally, the stimulus pattern for level of number 24, RAND was constructed by randomly allocating 24 dots to a  $6 \times 8$  matrix. No reflections were required as this matrix spanned the 220 mm square stimulus sheet, and the resultant pattern was relatively irregular in nature. The remaining six stimulus arrangements were constructed in the same manner as those just described. Each of the stimulus patterns employed in Experiment 3 appears in Appendix B-4.

In the present experiment the major point of departure from Attneave's (1955) technique concerns informational measurement. The modified procedure is depicted in Figure 3-1, where the steps involved in computing the informational content of stimulus condition, level of number 24, REG 2 are outlined. The informational quantity in each of the remaining 11 stimuli was determined in the same fashion. The reader is

referred to Table 3-1 for an examination of the particulars associated with each of the stimulus arrangements.

It should be noted that at all levels of number, stimuli in condition of regularity REG3 are said to contain 0 bits of information because in each instance, the number of initial cells ( $n$ ) is equal to the number of dots initially allocated ( $r$ ). In each of the remaining conditions of regularity however, quantity of information is seen to increase with level of number. In terms of the experimental hypothesis for Experiment 3 it is important to notice that as degree of regularity increases, informational content decreases.

4) Procedure Subjects were run in one group of 21. Each subject was provided with a pencil and a response sheet (see Appendix D-3).

The experiment was broken into two phases. In phase 1 subjects were presented with the same instructions that were given to those participating in Experiments 1 and 2 (see Appendix C-1) and again their task was to record estimations of number. The projector was positioned approximately 10 m from the screen.

The group received 48 trials where stimulus slides were shown for 2.24 sec. and followed by an interval of approximately 7 sec. Following every 10 trials, subjects were informed as to which trial was next.

The first 12 trials were training trials and were characterized by one replication of each of the 12 stimulus

Figure 3-1

Computation of Informational Quantity (bits) of  
Stimuli for Experiment 3 (Example; Level of  
Number 24, REG2)

- 1) Derive combination expression ( $nC_r$ ) from initial number of cells (n) and initial number of dots allocated (r).

e.g.       $12 C_6$

- 2) Derive factorial expansion expression  $\frac{n!}{r!(n-r)!}$  from combination expression.

e.g.       $\frac{12!}{6! 6!}$

- 3) Convert to logarithms of factorials and conduct arithmetic operations.

e.g.       $8.68 - (2.86 + 2.86) = 2.96$

- 4) Convert from  $\log_{10}$  to  $\log_2$  by multiplying by the constant  $3.3223 \times 10^3$  (product is equal to information in bits).

e.g.       $2.96 \times 3.3223 = 9.83$  bits.

$*\log_{10} 10 = 1 (10^1)$

$\log_{10} 2 = 0.301 (10^{-3.01})$

Divide exponents (  1  ) to obtain the quotient 3.3223.  
                        .301

patterns. The remaining 36 trials were test trials and comprised three replications (in blocks of 12) of each stimulus pattern.

The order of presentation was randomized such that two restrictions were met. The restrictions were: 1) no more than two stimulus patterns with the same level of regularity could be presented consecutively, and 2) stimulus patterns with the same level of number could not be presented consecutively.

Following the completion of the 48 trials, subjects were informed that they had completed the first phase of the experiment. Response sheets were collected and new response sheets (see Appendix D-4) were distributed.

The group was then presented with the instructions for phase 2 (see Appendix C-3). The subject's task in this phase of the experiment was to examine each stimulus pattern and to rate it on a seven point scale in terms of perceived regularity: a rating of 7 indicating highly regular; a rating of 1 indicating highly irregular (see Appendix C-3).

Subjects were presented with 12 trials employed in phase 1. Each slide was presented for approximately 7 sec. and followed by an interval of about 10 sec. Both the exposure duration of each slide and intertrial intervals were manually regulated by the experimenter.

Following testing, subjects were debriefed as to the nature and purpose of the study and reminded to indicate their age and sex.

### Results

Phase 1 The dependent variable<sup>1</sup> employed in phase 1 was each subject's mean estimation of number for each of the 12 stimulus arrangements (see Appendix E-4). Thus 12 scores per subject were tabulated.

A condition of regularity (R: 4 levels) by level of number (No: 3 levels) factorial repeated measures Analysis of Variance was performed on the data (see Table 3-3).

Table 3-3

Summary of Repeated Measures Analysis of Variance:

Regularity x Number (phase 1)

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subject's (S's)	20		
Regularity (R)	3	2639.47	25.87*
R x S's (Error)	60	102.02	
Number (No)	2	18406.88	145.92*
No x S's (Error)	40	126.14	
R x No	6	472.36	9.03*
R x No x S's (Error)	120	52.32	

\*p<.001

---

<sup>1</sup>See Discussion (p. 57) for comments regarding the criterion variable employed for data analysis.

It was predicted that estimation of number would increase with degree of regularity. The Analysis of Variance was successful in yielding a statistically significant main effect for R,  $F(3,60) = 25.87$ ,  $p < .001$ , that accounted for approximately 10% of the total variability ( $\eta^2 = .105$ ). This main effect is graphically illustrated in Figure 3-2. It is readily apparent that stimulus condition REG3 (highly regular) was by far estimated to be most numerous. However differences appear to be minimal among all other conditions of regularity. A Trend Analysis revealed a statistically significant linear component  $t(60) = -40.0, p < .001$ , but also, a statistically significant quadratic component  $t(60) = 29.67, p < .001$ , was discerned. These observations weaken the support for the experimental hypothesis provided by the significant main effect for R.

Further investigation of these data using Newman-Keuls multiple comparison procedure revealed that: condition REG3 differs significantly from all other means; condition REG2 differs significantly from condition REG1; and no other differences were found to be statistically significant at the .05 level.

As predicted, a statistically significant main effect for No,  $F(2,40) = 145.92, p < .001$  was found. This main effect, presented in Figure 3-3, accounts for close to 50% of the total variability ( $\eta^2 = .489$ ) and is represented by a linear increase in estimation of number with stimulus level of number. The results of a Trend Analysis yielded

Figure 3-2

Mean Estimation of Number as a Function of Condition of Regularity

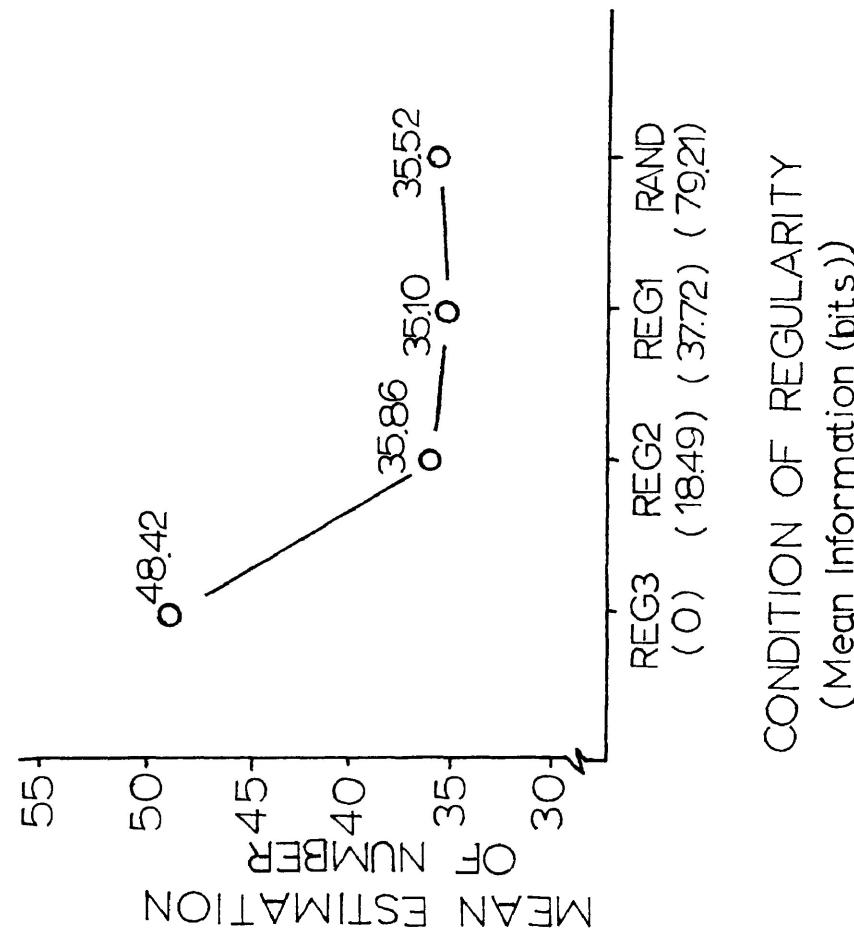
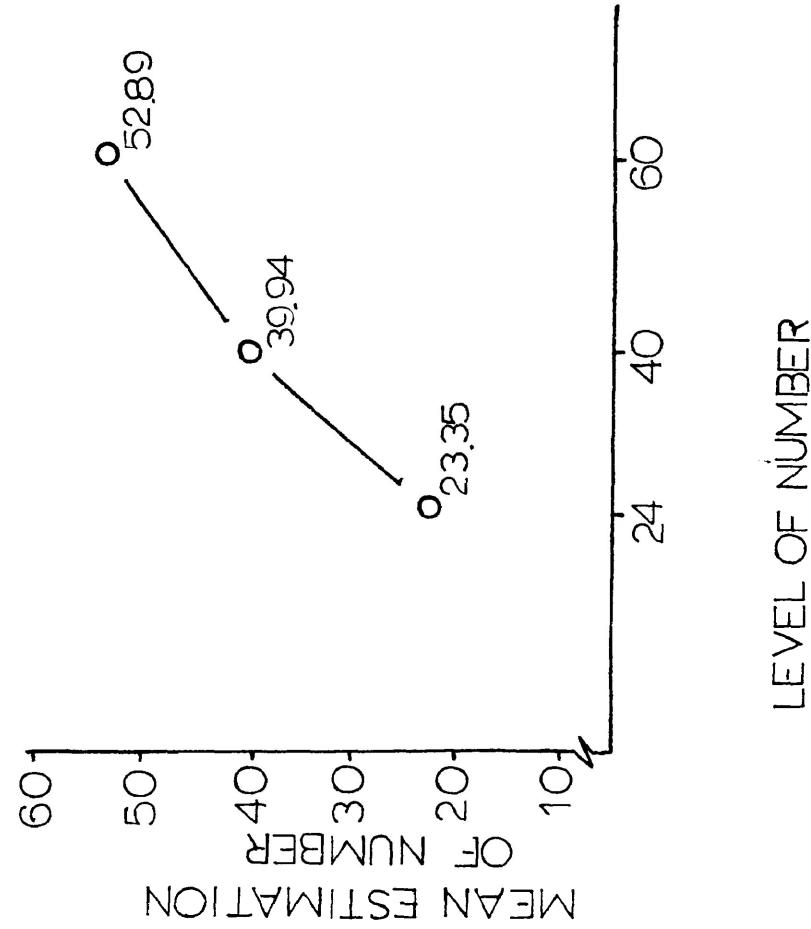


Figure 3-3

Mean Estimation of Number as a Function of Level of Number Regularity



a statistically significant linear component  $t(60) = 8.52, p < .001$ , and a statistically non-significant quadratic trend.

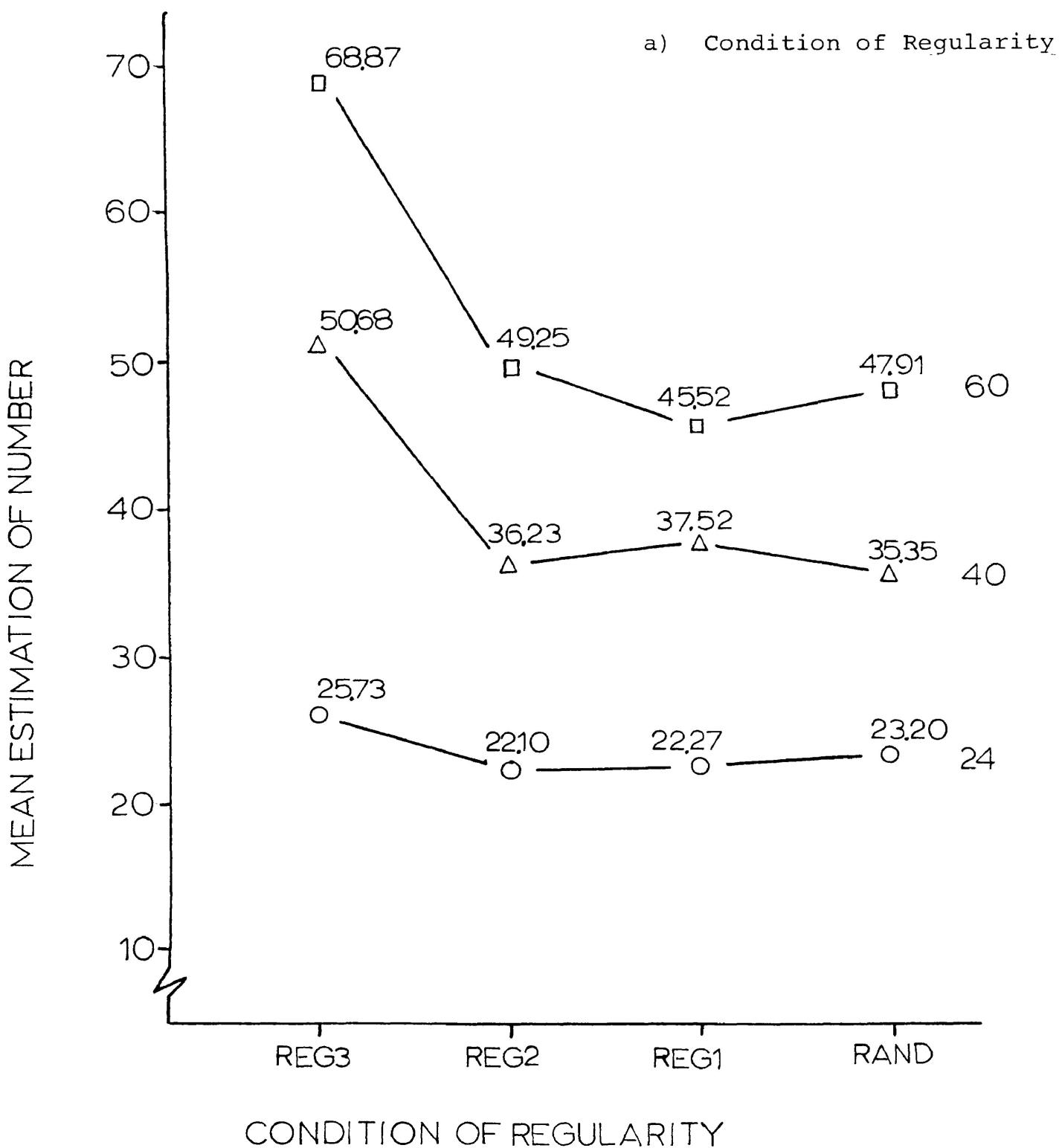
Finally an unexpected statistically significant R x No interaction was discerned by the Analysis of Variance,  $F(6,120) = 9.02, p < .001$ . This interaction, which accounts for approximately 4% of the total variability ( $\eta^2 = .038$ ) is represented graphically in Figure 3-4a. It is apparent that the quadratic component characteristic of responses at the various conditions of regularity is systematically exaggerated as level of number increases, and there is a marked discrepancy between responses at the highest as opposed to lowest level of number. This observation is also apparent in Figure 3-4b where level of number is plotted against the informational content of the stimulus arrangements.

Phase 2: Phase 2 was carried out so as to ascertain whether subjects did indeed discriminate among the conditions of regularity. The dependent variable employed in this phase of the experiment was each subject's rating of regularity for each of the 12 stimuli. These data were also subjected to a 3 x 3 factorial, condition of regularity (R) by level of number (No) factorial repeated measures Analysis of Variance (see Table 3-4).

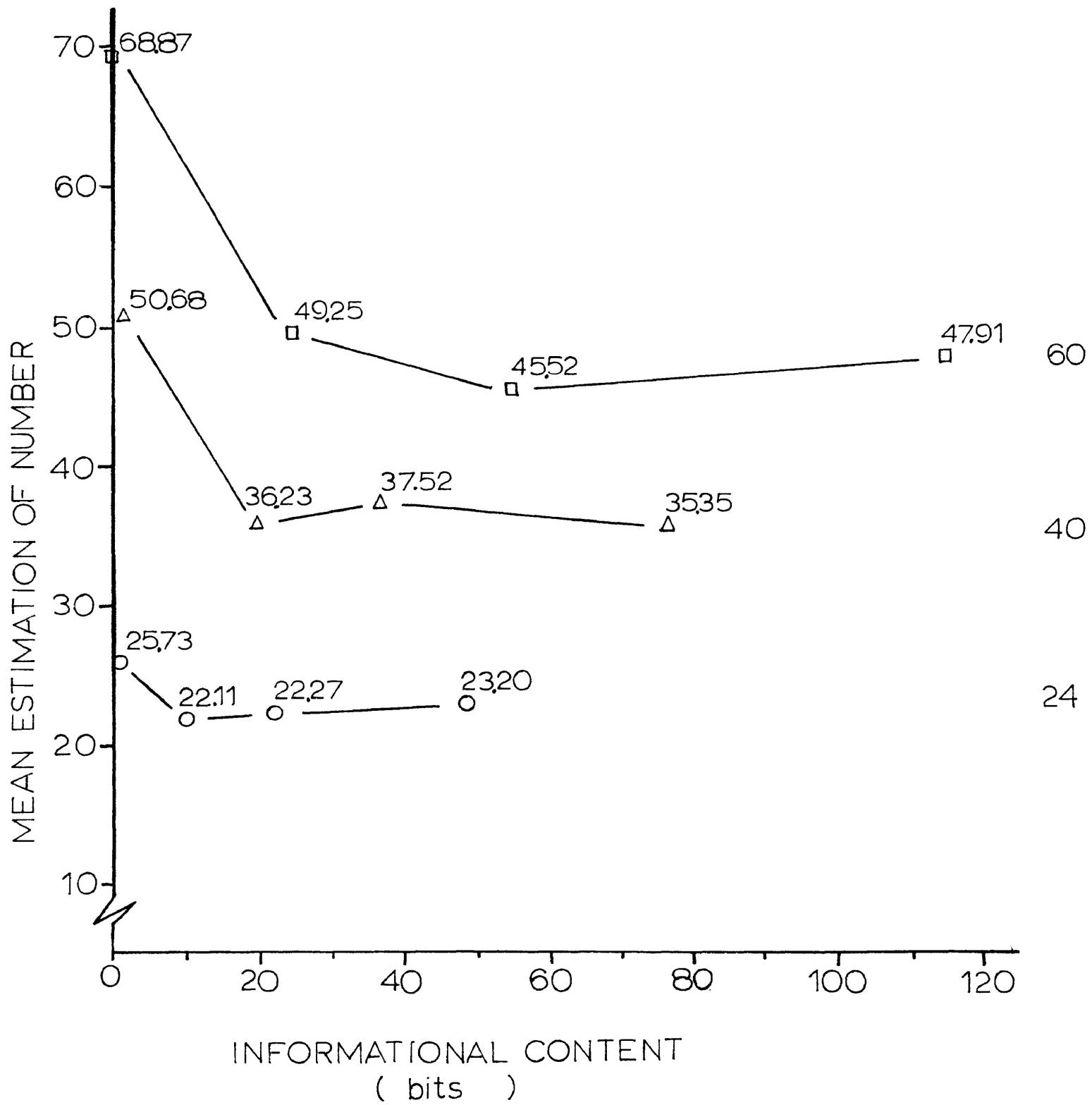
Figure 3-5 readily reveals that subjects perceived patterns assumed to be more regular to be so. This finding is supported by a statistically significant main effect for R,  $F(3,60) = 84.20, p < .001$ , that accounts for close to 60%

Figure 3-4

Mean Estimation of Number as a Function of Level of Number and a) Condition of Regularity b) Informational Content.



b) Informational Content



of the total variability ( $\eta^2 = .596$ ). This finding supports the notion that degree of regularity is an inverse function of informational content. It may be noted that the mean rating of regularity for the random arrangements (RAND) fall below the neutral point, indicating that subjects tended to perceive them as being irregular, whereas all other patterns, each containing some degree of symmetry, were rated as being regular.

Table 3-4

## Summary of Repeated Measures Analysis of Variance:

Regularity x Number (phase 2)

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	20		
Regularity (R)	3	215.17	84.20**
R x S's (Error)	60	2.56	
Number (NO)	2	12.21	13.53**
NO x S's (Error)	40	0.90	
R x NO	6	3.42	3.96*
R x NO x S's (Error)	120	0.86	

\* $p < .01$ \*\* $p < .001$ 

The Analysis of Variance also yielded an unexpected main effect for NO,  $F(2, 40) = 13.53$ ,  $p < .001$ . This effect accounts for less than 3% of the total variability ( $\eta^2 = .023$ ) and is characterized by a tendency for subjects to

Figure 3-5  
Mean Rating of Regularity as a Function of Condition of Regularity.

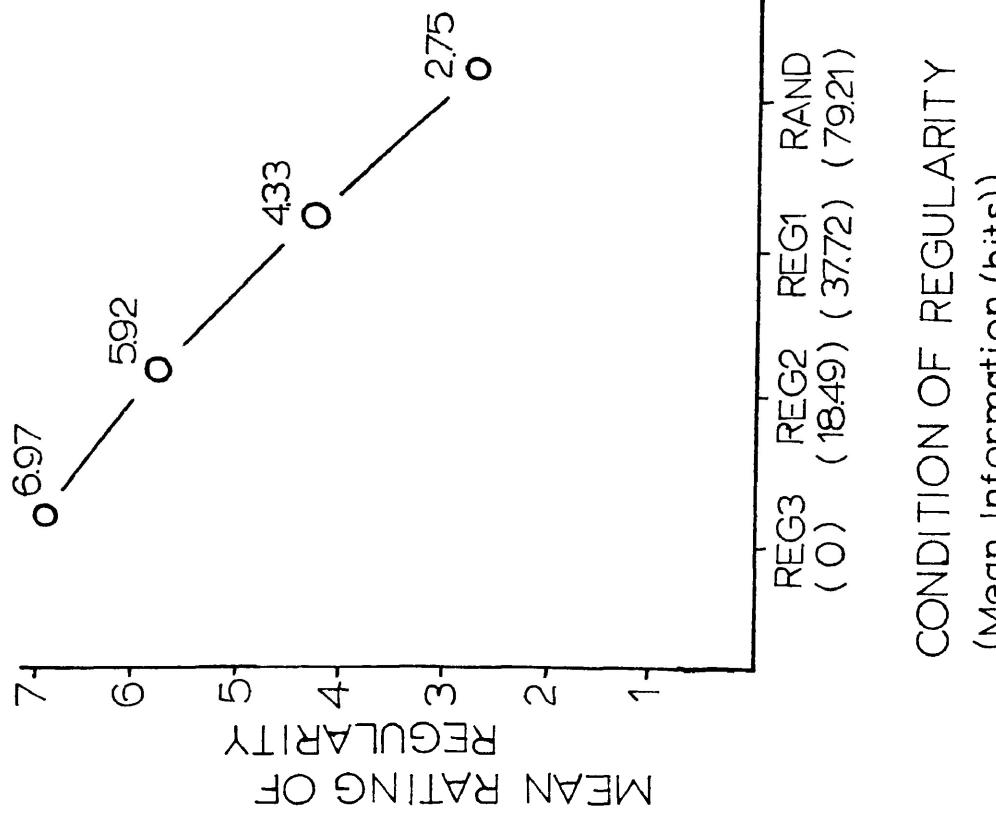
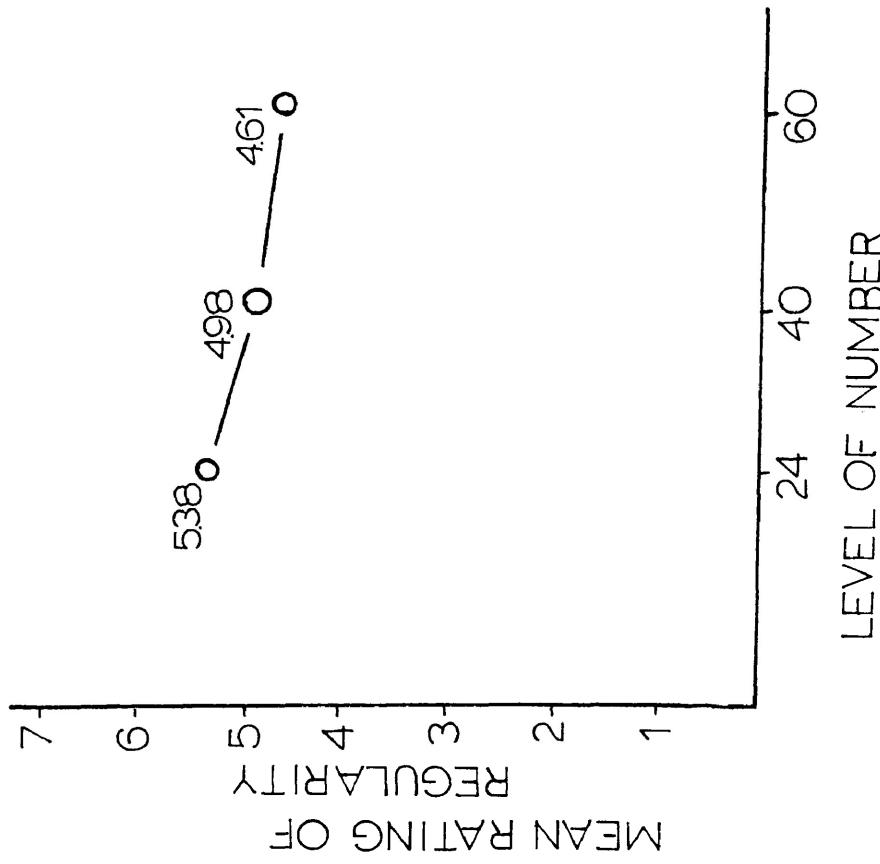


Figure 3-6  
Mean Rating of Regularity as a Function of Level of Number Larity.

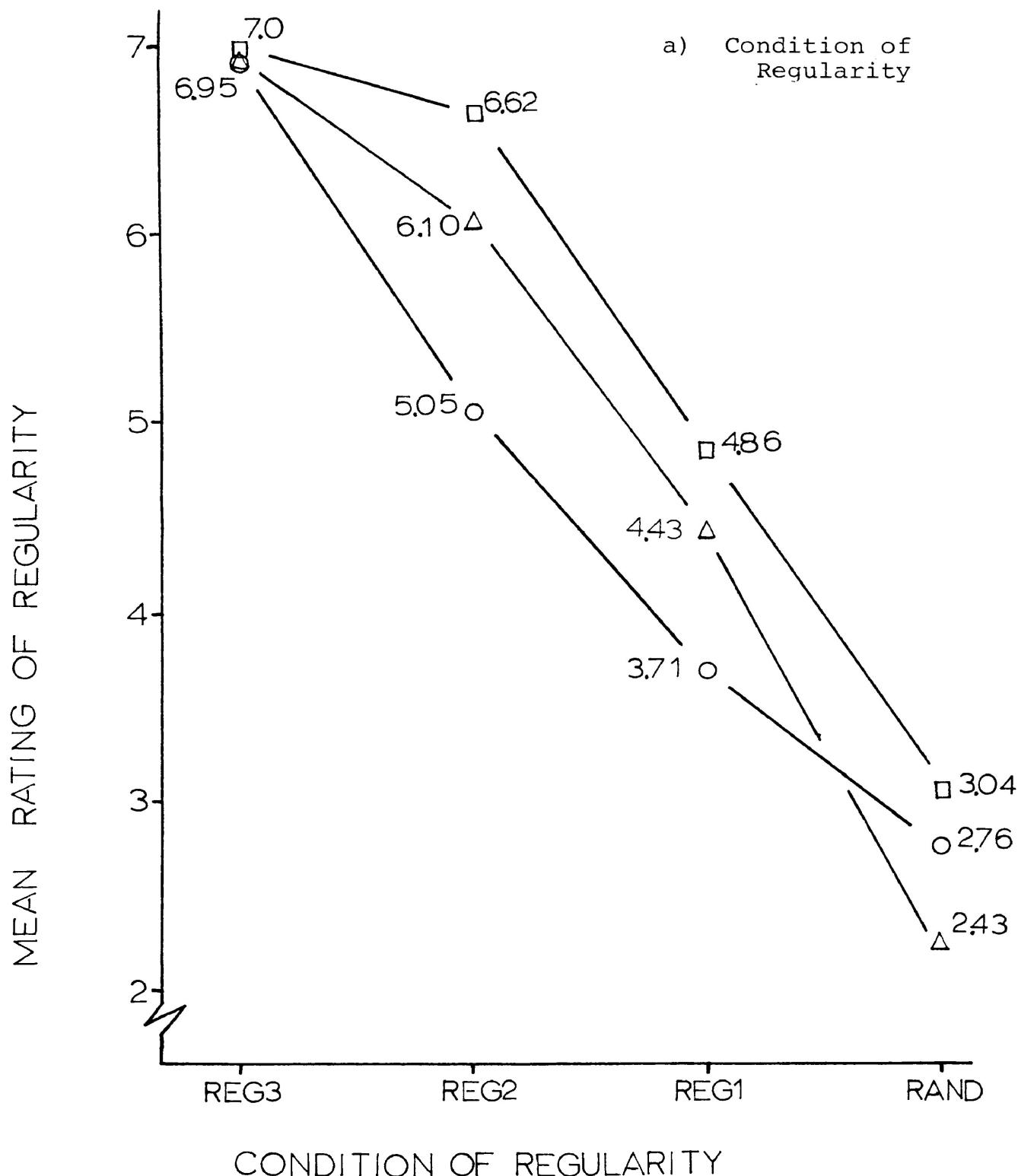


rate less numerous patterns as being more regular (see Figure 3-6). It is also found to remain statistically significant when the Geisser-Greenhouse conservative correction is applied,  $F(1,20) = 13.53$ ,  $p < .001$ . Newman-Keuls procedure revealed that the only means that differed significantly were the highest and lowest levels of number.

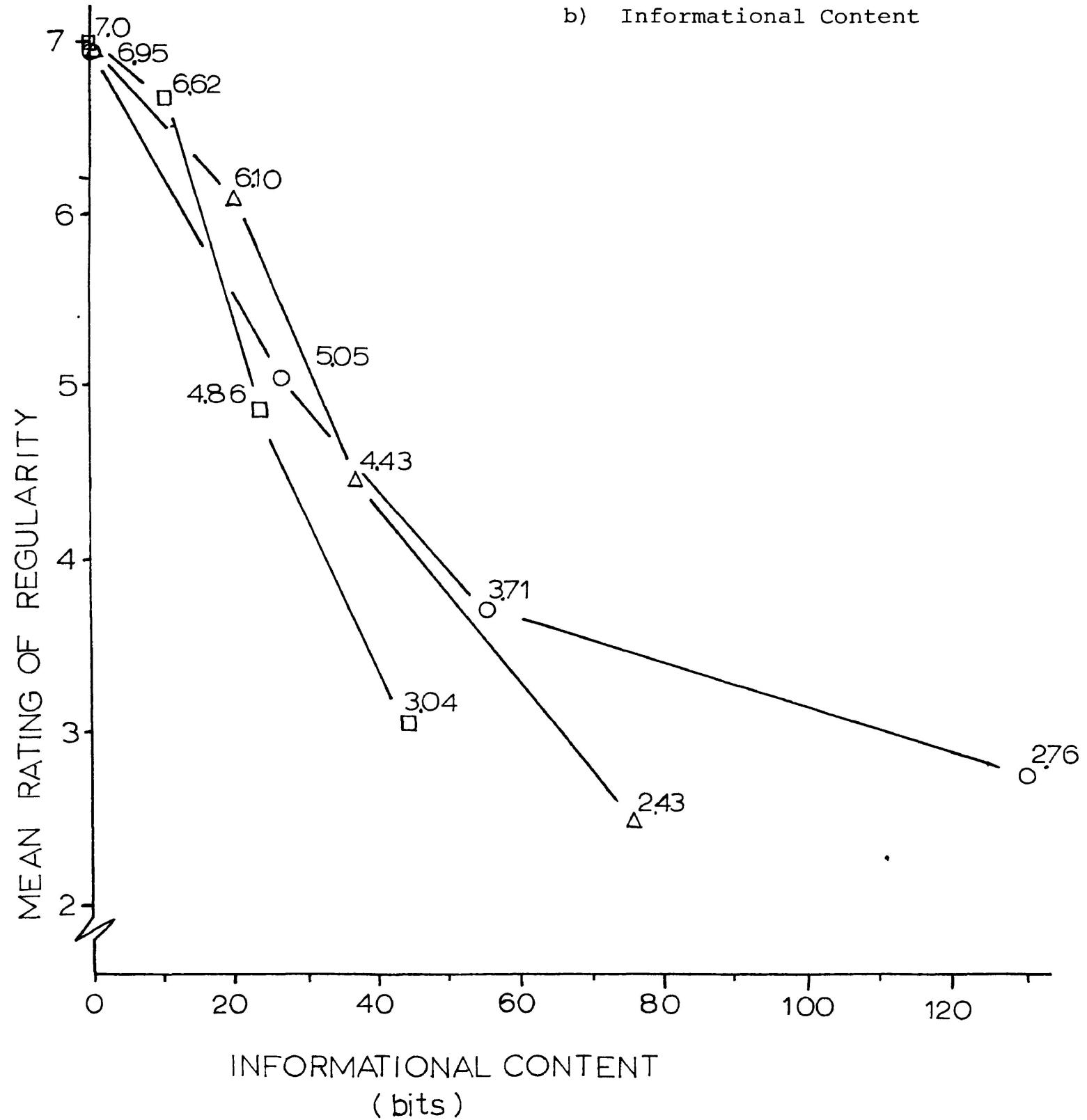
Finally the Analysis of Variance yielded a statistically significant  $R \times No$  interaction,  $F(6,120) = 3.96$ ,  $p < .001$ . This interaction (see Figure 3-7a) further elucidates the finding that subjects tended to judge less numerous patterns to be more regular by showing that this is particularly the case for patterns in the intermediate conditions of regularity. This observation is also apparent in Figure 3-7b where level of number is plotted against informational content of the stimulus arrangements. The interaction accounts for less than 2% of the total variability ( $\eta^2 = .019$ ) and becomes statistically non-significant when the Geisser-Greenhouse conservative correction is applied,  $F(1,20) = 3.96$ ,  $p > .05$ .

Figure 3-7

Mean Rating of Regularity as a Function of Level of Number and a) Condition of Regularity b) Informational Content



b) Informational Content



## DISCUSSION

### 1) Considerations Concerning Violations of Assumptions Underlying the Analyses of Variance

Whenever there is a relationship between means and variance there is also likely to be a tendency for a distribution to be skewed. Lindquist (1953) suggests it is sometimes desirable to transform criterion measures into derived measures whose variance is more homogeneous and whose distribution is more nearly normal.

Examination of the standard deviations presented in Appendix E reveals that the criterion measure used in all of the present experiments is undoubtedly positively correlated with the means. According to Bartlett (1947) "...when the variance is proportional to the means the square root may be considered..." an appropriate data transformation prior to an analysis of variance (p. 68).

The transformation of data to more closely align it with statistical assumptions has been the subject of debate for a considerable period of time. However in relatively recent times Glass, Peckham and Sanders (1972) have stated that the "...payoff of 'normalizing transformations' in terms of more valid probability statements is low and they are seldom considered to be worth the effort." (p. 273) and that "skewed populations have very little effect on either the level of significance or the power of the fixed effects model F test".

In the light of the present data the position of Glass et al. is supported. The results of within groups analyses of variance following square root transformations of the criterion measures for each experiment appear in Appendix G. When compared with analyses on the criterion measures themselves these results are virtually identical. Hence the non-transformed criterion measures were employed in all data analyses.

2) Effects of Element Number

In each of the three experiments the most pronounced effect on the dependent variable was attributable to the manipulation of level of number. That is to say, estimation of number was seen to increase linearly with objective level of number. In each of the experiments, significant main effects for level of number accounted for substantially more of the total variability than any other source of variability. These observations, however, come as no surprise as they have been demonstrated empirically and they are intuitively plausible.

Interestingly all of the interactions that were found to be statistically significant involved level of number as a contributing variable. It should be noted that most of these interactions account for less than 1% of the total variability, and some are effectively negated when the Geisser-Greenhouse conservative correction is employed to compensate for the liberal nature of within-groups designs. However, the fact remains that certain variables appear to

manifest different effects at different levels of number.

The literature provides a clear indication that more than one central mechanism may be involved in the estimation of numerosness and that level of number is a determinant of the nature of the mechanism that will be operative. For instance, Hunter and Sigler (1940) examined the effect of duration and intensity on the task of determining the correct number of dots. They found that the Bunsen-Roscoe law held where the task was a single discriminatory event (i.e. seven dots or less) but that this law did not apply when several such events were required (i.e. more than seven dots). In a study conducted by Taves (1941) the subject's task was to report the number of dots contained in rapidly presented stimuli and to rate the degree of confidence with which their report was made. Taves found that when more than six dots were presented reports became inaccurate and the subjects became less confident about the accuracy of their reports. Based on these data Taves advocated the existence of two central mechanisms for the judgment of numerosness.

Kaufman, Lord, Reese and Volkman (1949) confirmed the existence of two central mechanisms by demonstrating that subjects were less accurate, slower, and less confident when judgments of numerosness were made for patterns containing more than six dots. They suggested that subjects "subitize" patterns with less than six dots whereas they "estimate" when the number of dots exceeds six.

The term "subitize" is associated with the concept of "immediate apprehension" of number whereas estimation refers to a somewhat less rapid and accurate quantification process.

In a subsequent study, Jensen, Reese and Reese (1950) gave subjects time enough to count the elements in a given stimulus arrangement although they did not specifically instruct them to do so. Again, the existence of two mechanisms was demonstrated with the point of discontinuity at about six dots.

The above data clearly support the notion that two mechanisms are operative in the estimation of number. The first mechanism has been termed subitizing, and has been shown to operate up to about six stimulus elements. The second mechanism, termed estimating, was said to be operative for stimuli containing elements exceeding six. In the present experiment, since the lowest level of number employed was 20, it was not expected that evidence for two mechanisms would be found. Yet, in each of the three experiments, level of number was seen to contribute to the production of significant interactions. Furthermore, there appears to be a general tendency for responding at the lowest frequencies to be characteristically different from that at the highest frequencies.

As can be seen in Appendix F, variability in estimation of number clearly increases with level of number regardless of experimental condition. One possible

explanation for this finding might be that subjects are less confident in their estimations of number as the frequency of stimulus elements increases. Since a rather wide range of frequencies was employed in all of the present experiments, it is possible that subjects were responding to high frequency stimulus arrangements with substantially less confidence than when responding to stimuli with lower levels of number. If this is the case, this notion may be somewhat responsible for some, if not all, of the unexpected interactions where level of number is a contributing factor.

The data of Horne and Allee (1971) and of Horne and Turnbull (1977) may add credence to the aforementioned assertion since, in both studies, dot frequency was found to interact with other variables to produce significant effects. Although dot frequencies ranged only from 16 to 37 in both studies, an interesting statement by Horne and Allee may be germane to the present data.

"It is as if there were two processes operating, one in which information is perceived with slightly impaired efficiency and a second process at higher frequencies where there is less efficiency". (p. 92)

The data provided by Horne and Allee, Horne and Turnbull, and the present experiments, may suggest that it is inappropriate to think of estimation of number in terms of only two central mechanisms: subitizing and estimating. Although the first mechanism may be rather well defined and adequately characterized, the same may not be said for the second.

Klahr (1973) discusses several hypotheses in relation to quantification processes. He develops quantification models for the processes of subitizing and counting but because of a lack of data is forced to speculate about the process of estimation. Klahr suggests that the subject compares visual information in short term memory with a standard that is either derived from long term memory or from recently processed stimuli. He points out that in rapid presentations subjects may match the standard with only part of the stimulus thus leading to increased error with higher frequencies.

The examination of judged numerosness, associated variability and perhaps other correlative measures (e.g. the six point scale for confidence ratings employed by Taves (1944) and Kaufman et al. (1949)) may indeed reveal that the process of estimation is different for high, as opposed to intermediate (i.e. greater than than six elements), levels of number.

### 3) Effects of Element Size

The results of Experiment 2b reveal that small elements were estimated to be significantly more numerous than large, whereas estimations for stimuli in the heterogeneous condition fell somewhere in between.

Some data from previous studies are not consistent with this finding. Binet (Pollack and Brenner, 1969) found a strong tendency for his daughters (ages 2½ and 4) to overestimate the number of large, as opposed to small, objects. However, according to Daugherty (1964) Kastings

utilized a substantially larger subject pool and showed that children tend to respond differently than adults in that adults were found to overestimate smaller elements. Dougherty (1964) also found that adult subjects overestimate smaller elements, and suggested that they compensate for increasing size by decreasing estimates of numerosness.

More recent data may be consistent with a density interpretation of the phenomenon. Courtis (1970) also found that large dots were seen as fewer than an objectively equivalent quantity of smaller dots. In Courtis' study stimuli were constructed in such a way that element proximity was varied. That is to say, the small dots in the patterns were essentially a further distance from each of their neighbours than the large dots were in their respective arrangements. Thus, the patterns with large dots were essentially more dense than those with small dots. Horne and Allee (1971) manipulated the area covered by stimulus elements and found that estimates with the smaller area were significantly lower than estimates with the larger area. This was interpreted as a density effect. Finally, Krueger (1972) supported this notion by demonstrating that dots appear less numerous when bunched together than when spread apart.

In the present experiment stimuli were constructed in much the same way as those employed by Courtis (1970) in that elements were objectively further from their neighbours than the large elements. Thus stimuli with the small elements were less dense. With these considerations in mind the finding that patterns with small elements were judged to be more numerous than those with large elements appears to be consistent with previous research, and a density interpre-

tation seems appropriate. However, further empirical investigation is required if this interpretation is to be verified.

4) Evaluation of Hypotheses

a) Experiment 1: Contrast

In Experiment 1 it was predicted that estimation of number would increase as a direct function of the brightness contrast between focal (element) and contextual (background) variables. This hypothesis was not confirmed as evidenced by a statistically non-significant element brightness x background brightness interaction.

As can be seen in Tables 1-1 and 1-2 (pp. 15-16) there is a discrepancy between the reflectance of stimulus sheets and the luminance of stimulus slides. This inadvertent discrepancy is a direct result of the photographic technique employed to present stimuli on 35 mm slides. Apparently efforts to maintain the element : background brightness ratios, characteristic of the stimulus sheets, were not successful. As a result, the brightness contrast between focal and contextual variables has been "watered down" somewhat by the photographic process. This is particularly the case for stimuli characterized by the light gray background.

Although estimation of number was not influenced in the predicted direction, a statistically significant main effect revealed that stimulus arrangements containing white elements were perceived to be significantly more numerous

than those containing black elements. Beven and Turner (1964) found that element brightness (white and black) did not influence subjects' estimation of number when a neutral gray background was employed. This would imply that the backgrounds employed in the present experiment tended to provide a bias toward white elements and that brightness contrast may indeed influence estimation of number.

As part of an investigation to examine what variables might influence estimation of number, Horne and Allee (1971) employed three levels of background brightness (white, neutral gray, black) and two levels of element colour (yellow-red, yellow-green). A statistically non-significant colour-ground interaction was said to represent the absence of a contrast effect. In a subsequent study Horne and Turnbull (1977) employed only two levels of background brightness (white, black) but widened the spectrum of dot brightness by using three levels (yellow-red, yellow-green, and neutral gray). In this study a contrast effect reflected by a statistically significant colour-ground interaction was reported. Apparently, estimation was a direct function of contrast. The authors offer no explanation as to why these results differ from Horne and Allee. Possibly the use of a broader range of element brightness was a determining factor.

The data from the present experiment most certainly do not directly support the notion that estimation of number is a direct function of pattern perceptibility. However,

to suggest that one should unequivocally reject this notion would be premature, in that brightness contrast may have produced the main effect for element brightness. Furthermore, the results of Horne and Turnbull (1977) suggest that contrast deserves more attention as a variable that might influence perceived number.

The use of smaller groups and the manual presentation of stimulus sheets, thereby eliminating the photographic process, is recommended for further research.

Broadening the range of background brightness and concomitantly brightness contrast may also prove to be a worthwhile endeavour.

Pattern perceptibility may also be manipulated in other ways. For example, the illumination of a stimulus card can be easily manipulated on a tachistoscope. Also, patterns may be systematically blurred by manipulating the focusing apparatus of a slide projector. Further investigation along these, or other lines, may prove to be instrumental in reliably relating the perception of number to the perceptibility of the pattern.

b) Experiment 2: Homogeneity

In terms of the experimental hypothesis the data derived from both Experiments 2a and 2b did not turn out to be promising. In these experiments it was predicted that fields of homogeneous elements would be estimated to contain significantly more dots than their heterogeneous counterparts.

In Experiment 2a a statistically significant main effect for condition of homogeneity was not discerned. Moreover, the failure of the orthogonal contrast, comparing the homogeneous conditions (all white, all black) to the heterogeneous conditions ( $\frac{1}{2}$  white /  $\frac{1}{2}$  black), to reach statistical significance provides verification that the experimental hypothesis was not confirmed.

Experiment 2b succeeded in yielding a statistically significant main effect for condition homogeneity. However, as has been previously discussed, this main effect is attributable to the overestimation of small, as opposed to large, elements and may be interpreted as a density effect. When the two homogeneous conditions (all small, all large) are combined for comparison, the non-significant orthogonal contrast reveals that they do not differ substantially from the heterogeneous ( $\frac{1}{2}$  small,  $\frac{1}{2}$  large) condition. This particular result clearly indicates that the experimental hypothesis was not confirmed.

In attempting to offer possible explanations as to the results attained in Experiments 2a and 2b one is forced to speak in rather general and speculative terms. Beckwith and Restle (1966) have shown that in counting (a characteristically different task than number estimation), sets of objects are grouped according to Gestalt principles such as propinquity, good continuation, similarity, etc. These authors hypothesized that the subject counts within groups and somehow connects his sub totals (i.e. either by adding sub totals when all groups have been counted, or by adding to a cumulative total).

In the present experiment heterogeneous stimulus arrangements were constructed such that each of the two elements in the patterns were approximately equal in number. Whether or not this stimulus characteristic was readily apparent to the observer is debatable. However, assuming this to be the case, subjects may have been focusing on one element type only and subsequently doubling estimates to arrive at a final judgment. If this were true subjects would actually be responding to lower levels of number and perhaps level of number would influence judgment in an unanticipated way. As discussed previously, subjects may respond with less variability and more efficiency at lower frequencies, thus possibly negating any potential influences of the heterogeneous conditions.

Another possible explanation for the results may be that the operational definition underlying the heterogeneous condition may have been inadequate, and that subjects did not perceive this condition to be substantially different from its homogeneous counterparts. If this is the case one alternative might be to employ more than two levels of an element attribute, thus ensuring that subjects perceive the heterogeneous condition as being substantially different from an array of elements of one attribute level.

c) Experiment 3: Information

In Experiment 3 it was predicted that, as pattern regularity increased and informational content concomitantly decreased, estimation of number would increase.

The data provide partial support for this hypothesis.

The results of Experiment 3, Phase 2, clearly indicate that judged regularity is an inverse function of informational content. Thus subjects were able to discriminate among levels of regularity when instructed to do so. However, the results of Phase 1 show that estimation of number did not increase monotonically as quantity of information decreased, although estimation for the lowest level of information was certainly greater than that for the highest.

Estimations for the highly regular condition (zero bits of information) were quite different from responses to all other conditions. As expected, estimations for this condition were greater than objective levels of number, while estimations for the random conditions were not. This observation is consistent with Ginsburg (1976, 1978) and Cousins (1979) and provides further evidence of the RRNI.

However, it would appear that at each level of number the stimuli characterized by intermediate levels of regularity (information) are underestimated in a manner similar to the random condition. Possibly this observation may suggest that the degree of regularity characteristic of these stimuli is not sufficient to produce illusory effects. As mentioned above, the subjects successfully

discriminated among levels of regularity when instructed to do so. However, when engaged in the number estimation task, subjects may not have been sufficiently cognizant or appreciative of the regular nature of the intermediate patterns, such that estimation of number remained relatively unaffected. This explanation may seem somewhat unreasonable with respect to the REG2 condition where stimuli were symmetrical about both horizontal and vertical axes. However, post hoc comparisons showed this condition to be judged to be significantly more numerous than the REG1 condition.

Another factor that may have contributed to the results would be the highly regular nature of the stimuli characterized by zero bits of information. Possibly these stimuli contrasted with all other stimuli so dramatically that the regular features of the stimuli characterized by intermediate levels of information may have been suppressed during the number estimation task. Suppression of this nature would presumably be negated when the subjects were instructed to discriminate degree of regularity (i.e. Experiment 3, phase 2).

The aforementioned interpretation is highly speculative, and the reader is reminded that no data are available to support such assertions. However, if one were to ignore the fate of the intermediate levels of regularity (information), the experimental hypothesis for Experiment 3 is supported by the evidence that stimuli with the lowest informational content were judged to be substantially

more numerous than their counterparts with the highest informational content. Although this finding is not strengthened by the fate of the stimuli characterized by intermediate levels of regularity (information), it remains evident nonetheless. As such we have some further indication that good patterns of elements (highly symmetrical) are overestimated in terms of numerosness, relative to poor patterns (asymmetrical).

### 5) Conclusions

The research problem of the present study concerns the question of whether Gestalt principles of perceptual organization relate to the perception of number. Empirical studies have provided some indication that perceived numerosity may be a function of figural goodness, or, at least, some of the underlying principles that may contribute to figural goodness. The present study has attempted to investigate the effects that some factors, believed to be associated with good figures, have on the estimation of number.

The hypothesis for Experiment 1 was not confirmed, as estimation of number was not seen to be a function of the brightness contrast between focal and contextual variables. In that technical difficulties were inherent in this experiment, and that brightness contrast did appear to influence estimation of number, both in this study and in previous research, it is concluded that further investigation along these lines is required. Thus although the experimental

hypothesis was not confirmed in Experiment 1, it is believed that hastily abandoning this approach would be premature.

Experiment 2 was intended to show that homogeneous displays of elements would be estimated to be more numerous than their heterogeneous counterparts. Two experimental operations characterized by the same hypothesis, design, etc. were carried out. Neither confirmed the hypothesis. Furthermore, there was no real indication from the present data, nor from previous studies, that the modifications suggested would succeed in confirming the hypothesis. It is therefore concluded that this particular approach may be an inefficient, and perhaps inadequate, means of investigating the problem, although further empirical research could serve to verify such an assertion.

The hypothesis for Experiment 3 received partial support as patterns said to contain the lowest level of information were estimated to be much more numerous than their counterparts said to contain the highest informational content. Although estimation of number unexpectedly increased quadratically over levels of regularity (information), it is believed that these data are sufficient to tentatively provide an indication that estimation of number is a function of figural goodness.

The data, provided by the present experiments, certainly do not unequivocally, nor strongly, confirm the central question: i.e. whether perceived numerosity is related

to Gestalt principles of perceptual organization. However, they do leave us with some indication that this may be the case and no real evidence that this thesis is inappropriate. Only through further research can an adequate answer to this question be clarified.

The resolution of this research problem will possibly establish a meaningful basis from which an interpretation of the RRNI may emanate.

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## APPENDICES

- A. Proportions of White and Black Waterbase Paints Used to Create Levels of Background Brightness.
- B. Stimulus Samples.
- C. Instructions to Subjects.
- D. Response Sheets.
- E. Raw Data Summed Across Levels of Number.
- F. Standard Deviations Associated with Experimental Conditions.
- G. Analysis of Variance on Square Root Transformed Data.

## APPENDIX A

Appendix A.

Proportion of White and Black Waterbase Paints  
Used to Create Levels of Background Brightness.

Paint Ratio

White : Black

Light:                            3:1

Background  
Brightness:      Medium:                            4:3

Dark:                            3:4

## APPENDIX B

Appendix B-1

Representative Samples of Stimuli Employed

in Experiment 1: Contrast

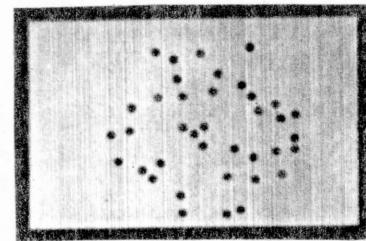
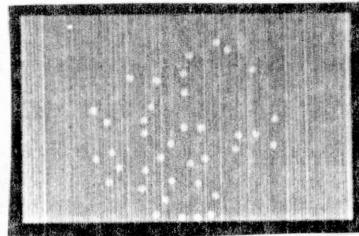
(Level of Number = 38)

Element Brightness

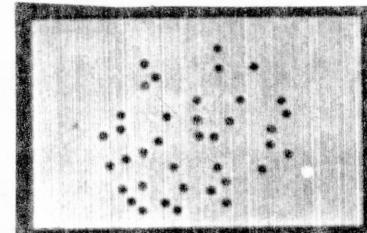
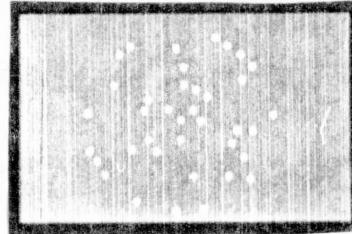
White

Black

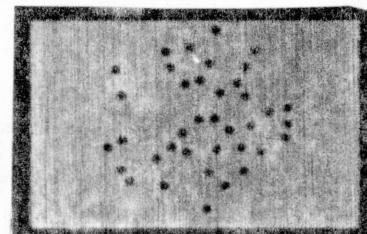
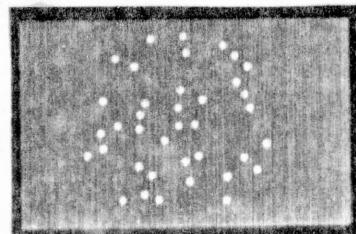
Light



Medium



Dark



B  
A  
C  
K  
G  
R  
O  
U  
N  
D

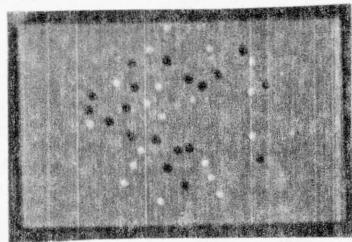
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R  
I  
G  
H  
T  
N  
E  
S  
S

Appendix B-2

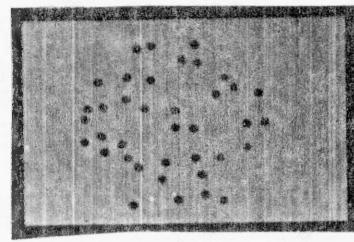
Representative Samples of Stimuli Employed  
in Experiment 2a: Homogeneity (Black and White Elements)  
(Level of Number = 38)

Condition of Homogeneity

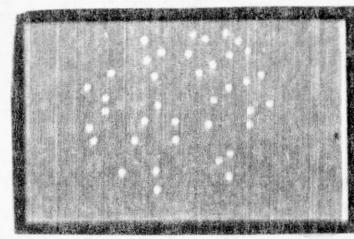
$\frac{1}{2}$  Black /  $\frac{1}{2}$  White



Black



White



Appendix B-3

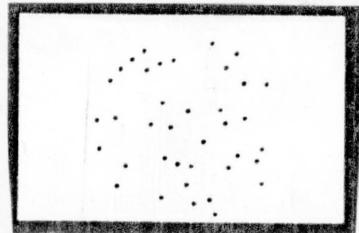
Representative Samples of Stimuli Employed  
in Experiment 2b: Homogeneity (Small and Large Elements)  
(Level of Number = 38)

Condition of Homogeneity

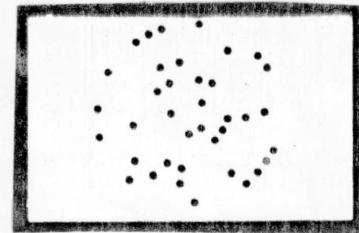
$\frac{1}{2}$  Small /  $\frac{1}{2}$  Large



Small

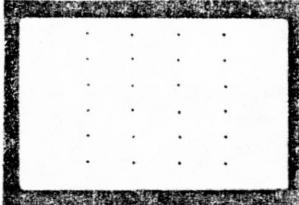
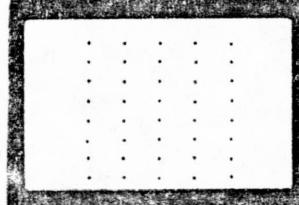
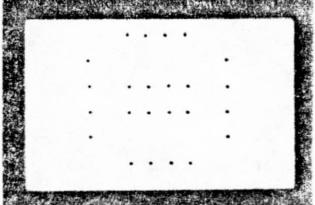
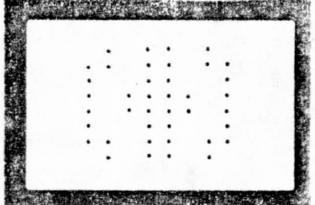
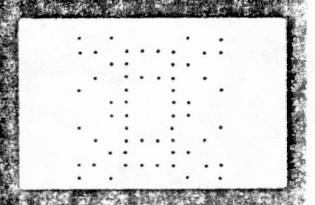
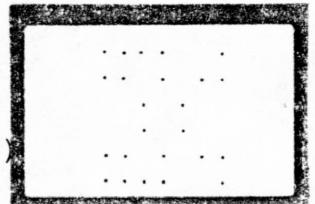
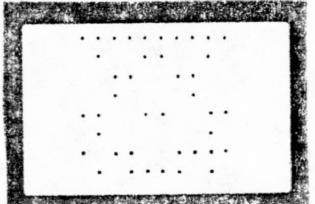
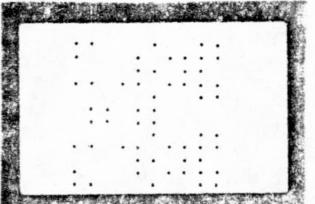
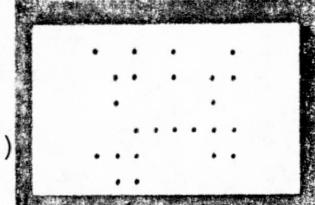
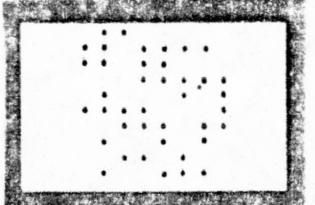
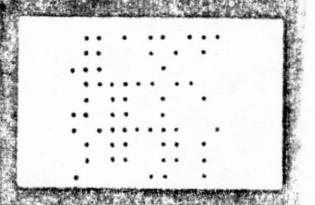


Large



Appendix B-4

Stimuli Employed in Experiment 3: Information

<u>Condition of Regu- larity</u>	<u>Level of Number</u>		
	20	40	60
REG3 (0)			
REG2 (18.49)			
( $\bar{x}$ infor- mation in bits)			
RAND. (79.21)			

## APPENDIX C

## Appendix C-1

### Instructions to Subjects for Experiment 1, Experiment 2a, and Experiment 3, phase 1.

In this experiment you will be shown a series of slides, each containing a particular number of dots. Each slide will be shown for a period of approximately three seconds and followed by an interval of about seven seconds. Your task is to estimate the number of dots on each slide. Please do not try to count, or in any way, calculate the number of dots. I am merely interested in your estimation of number. When you have estimated the number of dots, write it down on the paper provided, starting with the number one, and prepare for the next trial. Periodically I will indicate which trial is next. Should you discover a discrepancy, make a note and return to the appropriate space.

Are there any questions?

## Appendix C-2

### Abbreviated Instructions to Subjects in Experiment 2b

In the next experiment you will be shown a different series of slides, each containing a particular number of dots. The exposure duration of each slide and the duration of the interval between slides will be the same as in the previous experiment.

Again, your task is to estimate the number of dots on each slide. Remember not to try to count, or in any way, calculate the number of dots as I am merely interested in your estimation of number.

Write each estimate down in the appropriate space and make a note of any discrepancies.

Are there any questions?

Appendix C-3

Instructions to Subjects in Experiment 3, phase 2

In the next phase of the experiment you will be shown the first 12 slides once again. Each slide will be shown for approximately seven seconds and followed by an interval of about 10 seconds.

Your task is to rate each pattern in terms of how regular it appears to you. If you feel a pattern is highly regular rate it with the number 7. Conversely, if the pattern appears to be highly irregular rate it with the number 1. Please refer to, and familiarize yourself with, the rating scale at the top of your response sheet.

Are there any questions?

## APPENDIX D

Appendix D-1

Response Sheet for Experiment 1: Contrast

1. _____	19. _____	37. _____
2. _____	20. _____	38. _____
3. _____	21. _____	39. _____
4. _____	22. _____	40. _____
5. _____	23. _____	41. _____
6. _____	24. _____	42. _____
7. _____	25. _____	43. _____
8. _____	26. _____	44. _____
9. _____	27. _____	45. _____
10. _____	28. _____	46. _____
11. _____	29. _____	47. _____
12. _____	30. _____	48. _____
13. _____	31. _____	49. _____
14. _____	32. _____	50. _____
15. _____	33. _____	51. _____
16. _____	34. _____	52. _____
17. _____	35. _____	53. _____
18. _____	36. _____	54. _____

Name: \_\_\_\_\_

Age: \_\_\_\_\_

Sex: \_\_\_\_\_

Appendix D-2

Response Sheet for Experiment 2a and 2b; Homogeneity

1. _____	13. _____	25. _____
2. _____	14. _____	26. _____
3. _____	15. _____	27. _____
4. _____	16. _____	28. _____
5. _____	17. _____	29. _____
6. _____	18. _____	30. _____
7. _____	19. _____	31. _____
8. _____	20. _____	32. _____
9. _____	21. _____	33. _____
10. _____	22. _____	34. _____
11. _____	23. _____	35. _____
12. _____	24. _____	36. _____

Name: \_\_\_\_\_

Age: \_\_\_\_\_

Sex: \_\_\_\_\_

Appendix D-3

Response Sheet for Experiment 3, Phase 1: Information

1. _____	17. _____	33. _____
2. _____	18. _____	34. _____
3. _____	19. _____	35. _____
4. _____	20. _____	36. _____
5. _____	21. _____	37. _____
6. _____	22. _____	38. _____
7. _____	23. _____	39. _____
8. _____	24. _____	40. _____
9. _____	25. _____	41. _____
10. _____	26. _____	42. _____
11. _____	27. _____	43. _____
12. _____	28. _____	44. _____
13. _____	29. _____	45. _____
14. _____	30. _____	46. _____
15. _____	31. _____	47. _____
16. _____	32. _____	48. _____

Name: \_\_\_\_\_

Age: \_\_\_\_\_

Sex: \_\_\_\_\_

Appendix D-4

Response Sheet for Experiment 3, Phase 2: Information

1 Highly Irregular	2 Irregular	3 Moderately Irregular	4 Neutral	5 Moderately Regular	6 Regular	7 Highly Reugular
1. _____				7. _____		
2. _____				8. _____		
3. _____				9. _____		
4. _____				10. _____		
5. _____				11. _____		
6. _____				12. _____		

Name: \_\_\_\_\_

Age: \_\_\_\_\_

Sex: \_\_\_\_\_

## APPENDIX E

## Appendix E-1

Experiment 1: Raw Data Summed Across Levels of Number

Dark

Medium

Light

(Background Brightness)

White

Black

White

Black

White

Black

(Element Brightness)

1.	320	210	275	280	280	280	
2.	288	345	295	244	293	248	
3.	213	197	196	191	213	179	
4.	389	295	400	293	410	282	
5.	257	241	301	231	273	250	
5.	198	198	217	192	200	207	
7.	232	220	255	205	216	202	
8.	225	220	225	230	210	230	
S	9.	239	222	219	210	208	265
U	10.	221	195	222	224	204	230
B	11.	240	217	271	270	242	203
J	12.	187	239	263	235	307	247
E	13.	160	180	180	175	170	190
C	14.	222	176	214	192	176	187
T	15.	240	215	275	225	260	225
S	16.	176	158	168	167	163	164
	17.	252	256	244	261	230	258
	18.	179	176	203	174	210	190
	19.	197	179	182	184	172	194
	20.	166	137	138	156	163	139
	21.	245	221	224	224	233	227
	22.	269	232	254	262	267	242
	23.	330	303	289	310	315	389
	24.	378	462	476	419	458	439
	25.	225	180	187	183	205	230
	26.	179	200	165	185	185	185
	27.	210	210	220	235	240	235
	28.	222	200	236	194	196	211
	29.	202	241	275	194	354	261
	30.	283	262	279	247	255	278

## Appendix E-2

Experiment 2a: Raw Data Summed Across Levels of Number $\frac{1}{2}$  Black /  $\frac{1}{2}$  White

Black

White

(Condition of Homogeneity)

1.	231	246	223
2.	213	190	209
3.	242	217	231
4.	431	347	431
5.	349	337	315
6.	350	355	375
7.	209	189	176
8.	178	199	180
9.	173	193	185
S	10. 216	261	260
U	11. 139	133	138
B	12. 253	260	277
J	13. 290	290	275
E	14. 210	205	245
C	15. 201	244	232
T	16. 310	320	340
S	17. 230	260	260
	18. 283	278	282
	19. 227	213	233
	20. 280	340	325
	21. 220	210	253
	22. 280	310	290
	23. 220	320	285
	24. 280	212	240
	25. 160	140	154
	26. 210	204	210
	27. 230	246	264
	28. 249	262	264

Appendix E-3

Experiment 2b: Raw Data Summed Across Levels of Number

$\frac{1}{2}$  Small /  $\frac{1}{2}$  Large

Small

Large

(Condition of Homogeneity)

---

1.	235	270	210
2.	221	199	169
3.	240	253	225
4.	399	344	342
5.	378	445	355
6.	402	398	315
7.	221	283	203
8.	245	264	225
9.	203	320	101
S	10.	260	295
U	11.	132	112
B	12.	248	220
J	13.	278	279
E	14.	260	195
C	15.	265	255
T	16.	260	270
S	17.	275	253
	18.	259	198
	19.	254	208
	20.	235	245
	21.	277	234
	22.	235	248
	23.	300	205
	24.	258	225
	25.	189	159
	26.	243	240
	27.	222	215
	28.	270	231

## Appendix E-4

Experiment 3: Raw Data Summed Across Levels of Number

	REG3	REG2	REG1	RAND
(Condition of Regularity)				
(0)	(18.49)	(37.72)		(79.21)
			( $\bar{x}$ information in bits)	
1.	125	119	119	116
2.	133	112	123	109
3.	157	121	122	131
4.	176	80	77	78
5.	169	142	152	142
6.	95	90	73	77
S	7.	144	103	99
U	8.	192	120	118
B	9.	128	97	96
J	10.	168	92	117
E	11.	192	128	120
C	12.	197	92	89
T	13.	147	112	115
S	14.	124	70	70
	15.	114	71	63
	16.	143	128	122
	17.	132	83	100
	18.	106	78	145
	19.	134	161	127
	20.	151	93	87
	21.	124	118	118

## APPENDIX F

## Appendix F-1

Experiment 1: Standard Deviations Associated with  
Experimental Conditions

	Dark		Medium		Light		
	Black	White	Black	White	Black	White	
(Background Brightness)							
(Element Brightness)							
L E V	20	3.77	3.98	4.83	5.17	5.68	3.10
E L	29	6.81	6.93	6.77	14.53	12.41	6.18
O	38	7.32	13.96	7.33	9.90	13.99	13.17
F	47	13.37	18.27	14.49	14.62	14.32	14.91
N U	56	21.59	17.69	15.08	18.22	15.28	21.40
M B E R	65	19.18	19.03	17.47	26.49	18.81	27.45

Appendix F-2

Experiment 2: Standard Deviations Associated with  
Experimental Conditions

a) Experiment 2a

L E V E L  O F  N U M  B E R	$\frac{1}{2}$ White / $\frac{1}{2}$ Black (Condition of Homogeneity)	Black	White
20	4.12	3.25	5.08
29	7.75	9.49	7.38
38	11.65	12.41	10.38
47	11.96	12.11	11.97
56	15.95	17.50	21.92
65	23.64	20.65	19.39

b) Experiment 2b

L E V E L  O F  N U M  B E R	$\frac{1}{2}$ Small / $\frac{1}{2}$ Large (Condition of Homogeneity)	Small	Large
20	4.28	3.39	4.16
29	7.56	10.48	6.25
38	11.64	14.44	12.27
47	13.85	15.38	13.18
56	14.00	16.90	14.49
65	17.69	22.18	14.34

Appendix F-3

Experiment 3: Standard Deviations Associated with  
Experimental Conditions

---

	REG3	REG2	REG1	RAND
	(Condition of Regularity)			
0		(18.49)	(37.72)	(79.21)
	( $\bar{x}$ information in bits)			

---

L				
E	24	24	6.37	3.38
V				4.72
E	40	40	11.51	10.70
U				9.53
L				
M	60	60	16.00	12.86
B				13.31
O				12.89
E				
F				
R				

## APPENDIX G

## Appendix G-1

Summary of Repeated Measures Analysis of Variance Following Square Root Transformation: Element Brightness x Background Brightness x Number.

<u>SOURCE</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	20		
Element Brightness (EB)	1	6.50	8.77*
EB x S's (Error)	20	0.74	
Background Brightness (BB)	2	1.18	3.18
BB x S's (Error)	58	0.37	
Number (No)	5	271.42	250.38**
No x S's (Error)	145	1.08	
EB x BB	2	0.53	1.28
EB x BB x S's (Error)	58	0.41	
EB x No	5	2.67	6.82**
EB x No x S's (Error)	145	0.39	
BB x No	10	1.74	4.03**
BB x No x S's (Error)	290	0.43	
EB x BB x No	10	0.99	1.81
EB x BB x No x S's (Error)	290	0.54	

\* p<.01

\*\* p<.001

Appendix G-2

Summary of Repeated Measures Analysis of Variance Following Square Root Transformation: Homogeneity x Number (Experiment 2a)

<u>SOURCE</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subject's (S's)	27		
Homogeneity (H)	2	0.54	2.05
H x S's (Error)	27	0.27	
Number (No)	5	158.19	267.27**
No x S's (Error)	135	0.59	
H x No	10	0.67	2.14*
H x No x S's (Error)	270	0.31	

\* p<.05

\*\* p<.001

Appendix G-3

Summary of Repeated Measures Analysis of Variance Following Square Root Transformation: Homogeneity x Number (Experiment 2b).

<u>SOURCE</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subjects (S's)	27		
Homogeneity (H)	2	88.82	141.15*
H x S's (Error)	27	0.63	
Number (No)	5	263.40	372.41*
No x S's (Error)	135	0.71	
H x No	10	24.66	75.62*
H x No x S's (Error)	270	0.33	

\*p<.001

Appendix G-1

Summary of Repeated Measures Analysis of Variance Following  
Square Root Transformation: Regularity x Number (phase 1)

<u>Source</u>	<u>df</u>	<u>MS</u>	<u>F</u>
Subject's (S's)	20		
Regularity (R)	3	14.17	24.88 *
R x S's (Error)	60	0.56	
Number (NO)	2	122.92	230.33 *
No x S's (Error)	40	0.53	
R x No	6	1.75	6.18 *
R x No x S's (Error)	120	0.28	

\* p < .001