

**ECOLOGICALLY-BASED TAPER EQUATIONS
FOR MAJOR TREE SPECIES IN MANITOBA**

by

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in Partial Fulfillment of the Requirements for
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ABSTRACT

Kozak's variable exponent taper equation was fitted for balsam poplar (*Populus balsamifera* L.), trembling aspen (*Populus tremuloides* Michx.), white spruce (*Picea glauca* (Moench) Voss), black spruce (*Picea mariana* (Mill.) B.S.P.) and jack pine (*Pinus banksiana* Lamb.) in Manitoba. Stem analysis data were collected from a total of 37 balsam poplar, 71 trembling aspen, 97 white spruce, 300 black spruce and 298 jack pine trees. The data were collected within the Boreal Shield and Boreal Plain ecozones in Manitoba. Four ecoregions were sampled within the Boreal Shield ecozone including the Churchill River Upland, Hayes River Upland, Lac Seul Upland, and Lake of the Woods ecoregions. Three ecoregions were sampled within the Boreal Plain ecozone including the Mid Boreal Lowland, Mid Boreal Uplands and the Interlake Plain ecoregions. Stem taper variability between ecozones and ecoregions were tested using the F-test as the data permitted. Stem variability between site types were also tested for black spruce and jack pine. Ecozone-specific, ecoregion-specific, site type-specific and provincial taper equations were constructed corresponding to the results of the F-tests. A taper equation was developed for balsam poplar ecoregion 152-154. Ecozone-specific equations were derived for trembling aspen and white spruce. Provincial and site type-specific equations were derived for black spruce. Ecoregion-specific and site type-specific equations were derived for jack pine. Regional differences of stem taper were the result of geographic, climate and site differences. The results indicated that all the models performed quite well in predicting diameter inside bark. The residual plots of the balsam poplar and trembling aspen taper equations showed increasing variance and a non-normal distribution of residuals. However, the coefficients can still be considered appropriate and the predictive ability of the models was not affected. For each species, the diameter inside bark (dib) predictions of the Manitoba provincial models were plotted with the corresponding dib predictions of the Alberta provincial equation developed by Huang (1994). The plots indicated that the Manitoba equations performed well and were similar to the Alberta equations. For each species, dib predictions of the ecozone-specific, ecoregion-specific or site type-specific models were plotted with the dib predictions of the provincial model to display the differences indicated from the F-tests. Some of the equations were derived with less than the minimum 60 trees recommended by Kozak. Therefore, it is suggested that data be continually accumulated and used to update the equations developed in this study and to further determine if ecoregion-specific and site type-specific equations are required. Data should be collected outside the current range of diameter at breast height (dbh) and total tree height measurements to expand the application of the equations.

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1. INTRODUCTION

Increasing societal demands for fiber products has resulted in increasing industrial development. Therefore, ensuring sustainable forest management based on ecologically sound principles is necessary. One of the fundamental aspects of achieving an ecological understanding is being able to predict how forest ecosystems change over space and time and how they respond to natural disturbances, climate change and various forest management practices.

Jack pine (*Pinus banksiana* Lamb.), black spruce (*Picea mariana* (Mill.) B.S.P.), white spruce (*Picea glauca* (Moench) Voss), trembling aspen (*Populus tremuloides* Michx.) and balsam poplar (*Populus balsamifera* L.) are major tree species in Manitoba and are of utmost concern to the forest industry operating within the province. Forest ecosystems containing these tree species occur over a large geographical area. As a result, climate, site conditions and forest productivity vary greatly throughout the province. The ability to predict the growth and yield of forest stands located in various climate and site conditions is critical in the development of ecologically-based management plans and strategies. However, information regarding growth and yield and relationships between forest productivity and climate and site variables is currently lacking in Manitoba. Developing growth and yield models within an ecological framework and studying forest productivity in relation to climate and site variables is of utmost urgency. Accurate growth and yield predictions are pre-requisite not only for analyzing wood supply, calculating annual allowable cut (AAC) and calculating stumpage rates, but also for running ecologically-based management scenario models. Ecologically-based management models are essential for long term strategic planning.

One of the essential building blocks in forest growth and yield modeling is the equations/models for estimating individual tree volume of different species. The use of taper equations in estimating individual tree volume has recently become an increasingly popular trend. However, taper equations for major tree species have not been developed in Manitoba. The current individual tree volume equations used in Manitoba do not utilize taper equations and do not consider ecological differences in climate and site conditions. The current volume equations are based on mathematical regressions of total tree height and diameter at breast height. As well, previous testing of the current individual tree volume equations for black spruce has revealed significant bias and error (Wang 1997).

Taper equations have been shown to provide accurate diameter inside bark (dib) predictions in Canada (e.g. LeMay 1982, Kozak 1988, Gal and Bella 1994, Huang 1994, 1997, Huang *et al.* 1999, Huang *et al.* 2000). In particular, Kozak's taper equation (Kozak 1988) has been proven to fit well (Perez *et al.* 1990, Kozak 1991, LeMay *et al.* 1993, Kozak and Smith 1993, Muhairwe *et al.* 1994, Gal and Bella 1994). Huang (1994) used Kozak's model to develop ecologically-based taper equations for Alberta natural subregions. Regional differences of stem taper were the result of different biological, geographical and climate conditions (Huang *et al.* 2000). Huang *et al.* (2000) found that ecologically-based taper equations performed well at different portions of the stem and for various tree sizes.

The general objective of this thesis was to develop ecologically-based taper equations for major commercial tree species in Manitoba using Kozak's model. The species of interest include jack pine (*Pinus banksiana* Lamb.), black spruce (*Picea mariana* (Mill.) B.S.P.), white spruce (*Picea Gluaca* (Moench) Voss), trembling aspen

(*Populus Tremuloides* Michx.) and balsam poplar (*Populus balsamifera* L.). The specific goals of the thesis were 1) to develop ecoregion-specific and ecozone-specific taper equations to determine if a single provincial equation is sufficient for each species, and 2) to develop site-type-specific taper equations for jack pine and black spruce to test if a single equation is sufficient. The developed equations were compared to the Alberta equations developed by Huang (1994). Alberta's equations were used for the comparisons since they were developed using the same taper equation and are ecologically-based. To facilitate the application of the taper equations developed in the study, individual tree volume tables were derived from each taper equation.

2. LITERATURE REVIEW

2.1. Definition

Stem taper is the rate of narrowing in diameter in relation to the increase in total tree height for a given tree shape (Gray 1956). The stem profile of most conifers and some well formed hardwoods can be separated into three segments (Gray 1956, Assmann 1970, Husch *et al.* 1982, Newnham 1992). The basal, middle and top segments of the stem resemble the frustum of a neiloid, the frustum of a cubic or quadratic paraboloid, and a cone, respectively. Within the basal segment, the rate of taper decreases with increasing height above ground. Within the middle segment, above buttswell and below the live crown, the rate of taper increases with increasing height above ground. From the base of the crown to the top, the rate of taper slightly increases or remains constant with increasing height above ground (Valentine and Gregoire 2001). The overall shape of a tree is the same for all size classes within a species (Demaerschalk and Kozak 1977).

2.2. Factors affecting stem taper

Variations in taper are caused by differences in tree, stand and site characteristics as well as stand history (Larson 1963, 1965, Smith and Wilsie 1961). Tree characteristics that create variations in stem taper include changes in the size of the live crown, the distribution of the live crown and the length of the branch-free bole (Larson 1963). Site factors influence tree taper through their effects on crown development (Muhairwe *et al.* 1994). Differences of stem taper for different trees are the result of differences in diameter and height growth along the stem over time (Muhairwe 1999). Therefore, factors that affect tree growth in height and diameter (e.g. genetics, climatic

fluctuations, site quality, tree and stand age, crown size, canopy position, defoliation, species, stand density) also affect taper (Muhairwe 1994).

Newnham (1965) found that stem taper increases with age as long as the tree remains dominant, but as the tree becomes suppressed, the stem becomes more cylindrical in shape. Trees grown on good sites have large, long crowns while trees on poor sites have small, compact crowns. Smith and Wilsie (1961) found that during wet periods stem taper increased, while in dry periods stem taper decreased. Tall open grown trees have deep and vigorous live crowns and high taper. As stand density increases, lower branches die due to self-pruning. As a result, a longer branch-free bole is produced, taper decreases and the bole becomes more cylindrical in shape (Gray 1956, Muhairwe *et al.* 1994). However, trees with longer crown length have a larger diameter at breast height (dbh) and a greater rate of taper on the lower stem than trees with smaller crown length (Gray 1956, Fries 1965, Muhairwe 1994). Amateis and Burkhart (1987) examined stem taper of unthinned plantation, cutover plantation and natural origin stands of loblolly pine. They found that natural, open-grown trees had a greater rate of taper and suggested that competition may have a significant effect on tree form, taper and volume.

Silvicultural treatments that affect stand density also affect stem taper through changes of crown class and crown size (Muhairwe *et al.* 1994). Thinning decreases stand density and results in an increase in taper (Thomson and Barclay 1984). Tasissa *et al.* (1997) and Thomas and Parresol (1991) found that the coefficients of thinned and unthinned taper equations are significantly different while the coefficients of different thinning intensities are not significantly different. Pruning decreases crown size and

results in a decrease in taper (Larson 1963, 1965). Fertilization increases tree growth and vigor, which increases branch and crown size resulting in an increase in stem taper.

Trees with a larger crown ratio have greater taper than trees with a less crown ratio (Valenti and Cao 1986). Dell *et al.* (1979), Feduccia *et al.* (1979) and Baldwin and Polmer (1981) found that stem taper differed for different crown ratio classes, however, Burkhart and Walton (1985) found that crown ratio is related to stem taper but not strong enough to include in a taper equation.

Stem profile greatly affects merchantable volume (Cao *et al.* 1980). Trees with low taper provide more volume while trees with high taper provide less volume. Trees with less rapid taper for comparable sizes (total tree height and diameter at breast height) can have up to as much as 20% more volume (Heger 1965). Differences in volume may be attributed to differences in total height-dbh relationships, tree taper and form associated with different stand origins (Amateis and Burkhart 1987).

2.3. Taper equations, models or functions

Models which estimate radius, diameter, or cross-sectional area (inside or outside bark) at any point along the stem are considered taper models, profile models, taper equations, or taper functions (Valentine and Gregoire 2001). Taper equations provide i) predictions of diameter inside bark (dib) at any point along the stem, ii) estimates of total stem volume, iii) estimates of merchantable volume and merchantable height to any top diameter and from any stump height, and iv) estimates of individual log volumes (Kozak 1988). Taper equations can be used to predict individual tree volume, gross total volume (GTV), gross merchantable volume (GMV), merchantable length (ML) and number of trees per cubic meter of MV (Huang 1994).

The importance of studying taper equations is that no single theory, model or function has been able to explain all the variability in tree shape. Taper equations are a necessary and flexible tool for accurately estimating volumes, especially for specific market requirements (Newnham 1988, Muhairwe 1999). The weakness of many taper equations is that they exhibit a large degree of bias in diameter predictions over some portions of the stem, particularly underestimation of the lower portion and overestimation of the upper portion (Demaerschalk and Kozak 1977), and failure to account for differences in stem form between trees (Bi 2000). The accuracy and precision of volume estimates are dependent upon how well the taper equation fits the profile (Byrne and Reed 1986), therefore it is critical to carefully select a proper model.

Taper equations have become an increasingly popular trend (Kozak 1988, 1991, Perez *et al.* 1990, Newnham 1992, Flewelling 1993, Flewelling and Raynes 1993). They have been shown to provide accurate predictions in Canada (e.g. LeMay 1982, Kozak 1988, Gal and Bella 1994, Huang 1994, 1997, Huang *et al.* 1999, Huang *et al.* 2000). Taper equations are tools to provide accurate information on the current growing stock, which provides a sound basis for efficient forest management and mill utilization. They provide a description of the entire stem profile, which can be used for the computation of volume for any specified product and size class definition based on utilization standards (Avery and Burkhart 1994). Taper equations are better than volume equations, which directly predict volume, because individual log or sectional volumes can be estimated using various scaling rules (Williams and Reich 1997, Muhairwe 1999).

A good taper equation should include factors such as crown size, however, resources and conditions may limit variables to be measured (Muhairwe 1999). As

well, taper equations are based on forest inventory data, therefore, it is useful to develop equations based on data which are routinely measured (Muhairwe 1999). The use of total tree height in a taper equation is important since changes in tree shape are characterized by changes in tree height and diameter (Muhairwe 1994). Consequently, diameter at breast height outside bark (dbhob) and total tree height relationships explain a significant amount of tree-to-tree variation in form (Kozak 1988, Newnham 1992, Muhairwe 1994).

Taper equations can be categorized into four groups. 1) Simple equations which describe diameter changes from ground to top involving a single function of different forms (e.g. Behre 1923, Matte 1949, Gray 1956, Osumi 1959, Kozak *et al.* 1969, Demaerschalk 1973, Amidon 1984). These equations result in significant bias in estimating diameters close to the ground. However, they are easy to fit, easy to integrate for volume calculations and are easy to rearrange for the calculation of merchantable height (Kozak 1988). 2) Segmented equations which use different models for various parts of the stem and join these models in such a way that their first derivatives are equal at the point of intersection (e.g. Ormerod 1973, Max and Burkhart 1976, Demaerschalk and Kozak 1977, Cao *et al.* 1980, Matney and Sullivan 1980, Brink and von Gadow 1986). The parameters of these models are difficult to estimate and volume and merchantable height calculations are cumbersome. However, these equations provide less bias than simple equations. 3) An equation which uses one continuous function that describes the shape of the bole with a changing exponent from ground to top to compensate for the neiloid, paraboloid, and conic forms (e.g. Newberry and Burkhart 1986, Newnham 1988, Kozak 1988). These equations eliminate the necessity of using several functions to predict diameter inside bark (dib) at different points of the stem.

They are easy to develop and require less computing time (Kozak 1988). 4) Models which use approaches such as mixed linear models and polar coordinates (e.g. Sloboda 1977, Lappi 1986, Ojansuu 1987).

Since the 1920's, many have attempted to describe the shape of the stem. It is generally recognized that two equations describe stem profile better than one (e.g. Petterson 1927, Bennett and Swindel 1972, Ormerod 1973, Max and Burkhart 1976). Petterson (1927) suggested using a logarithmic function for the lower and middle portion of the stem and another logarithmic function with a different power index for the upper portion. Heijbel (1928) used a tangential function to describe the main stem and different equations for the top portion and the stem below 10% total height. Newnham (1958), Kozak and Smith (1966) and Kozak *et al.* (1969) used a simple quadratic parabola function to describe stem taper. Fries and Mattern (1965), Bruce *et al.* (1968), Bennet and Swindel (1972), Goulding and Murray (1976) developed taper equations based on polynomial functions. The weakness using polynomial functions to describe stem profile is the inability to describe the lower portion of the stem with significant butt swelling (Sterba 1980). To overcome this problem, the use of a higher order polynomial to represent swelling below dbh can be used (e.g. Fries and Matern 1965, Bruce *et al.* 1968). Demaerschalk (1971, 1972, 1973) and Munro and Demaerschalk (1974) proposed compatible equations but this reduced the accuracy of upper stem diameters (Cao *et al.* 1980). Compatible equations are equations from which both taper and volume functions provide identical results of total volume. Ormerod (1973) suggested using two simple power functions with a changing coefficient. Up to this point, every attempt to describe stem profile used one model to represent the entire length of the bole. All these attempts failed to solve the problem of bias since they were

too simple, systems were not conditioned properly to be continuous at points of intersection, not conditioned to give diameter of zero at the tip, and were based on dbhob, which is located at varying relative height and is highly affected by butt swell for some species (Demaerschalk and Kozak 1977). Max and Burkhart (1976) splined three polynomials together, one for each segment of the bole. Demaerschalk and Kozak (1977) developed a whole-bole system which linked two equations together. James and Kozak (1984) fitted the Demaerschalk and Kozak (1977) equation with standing tree data but it provided error due to the assumption that double bark thickness is a percentage of diameter and is consistent along the entire length of the stem. Alder (1978), Bitterlich (1979) and Thomas and Parresol (1991) used trigonometric functions to describe stem taper. Garay (1979) and Wensel and Krumland (1983) developed taper equations based on sigmoidal form. Kozak (1988) and Newnham (1988) introduced the variable-exponent and variable-form models, respectively. The value of the exponent determines the shape of the model and the value of the exponent varies with height to change the geometric shape. The use of a continuous model eliminates the necessity of using different models for different parts of the tree. When compared to the whole-bole system (Demaerschalk and Kozak 1977) and the segmented polynomial function (Max and Burkhart 1976), the variable-exponent model provides less local bias and greater precision in taper predictions (Newnham 1988, 1992, Kozak 1988, Perez *et al.* 1990, Kozak and Smith 1993, Muhairwe 1999). Flewelling and Raynes (1993) produced a variable-form model based on a system of three equations. Fang *et al.* (2000) developed a segmented variable-exponent equation where the values of the exponent vary among the three segments. Bi (2000) developed a trigonometric function which allows both the base and exponent of the function to vary with tree size, therefore the inflection point

varies. Bi (2000) found that the relative height of the inflection point varies with tree size. The size related changes in the relative height of the inflection point may explain why variations of the relative height of the inflection point in the base of the variable-exponent function had little effect on the prediction accuracy (Perez *et al.* 1990, LeMay *et al.* 1993). Trigonometric taper equations are more flexible in depicting changes in stem form within trees and between trees of different sizes (Bi and Long 2001). Valentine and Gregoire (2001) developed a "switching" model with both variable-form and variable-exponent where numerical switching functions are used to change the variable-exponent.

Kozak's taper equation (Eq. 1) is an allometric function with the general form $y = kx^c$, where y and x are dependent and independent variables, respectively, k is a constant, and c is an exponent that changes along the stem to describe stem form (Huang *et al.* 2000).

$$dib_i = a_0 D^{a_1} a_2^D X_i^{b_1 z_i^2 + b_2 \ln(z_i + 0.001) + b_3 \sqrt{z_i} + b_4 e^{z_i} + b_5 (D/H)} \quad \text{Eq. 1}$$

where:

dib_i = dib at point i along the stem (cm)

D = dbhob (cm)

$$X_i = (1 - \sqrt{h_i/H}) / (1 - \sqrt{p}) \quad \text{Eq. 2}$$

h_i = height above ground at point i along the stem (m)

H = total tree height (m)

$$p = (HI/H) * 100 (\%) \quad \text{Eq. 3}$$

HI = height of the inflection point (m)

$$z_i = \text{relative height } (h_i/H) \quad \text{Eq. 4}$$

$a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5$ = parameters to be estimated

The shape of any solid of revolution can be obtained by rotating an allometric curve around the x-axis which resembles the shapes of the bole, composite of neiloids,

cylinders, and cones (Husch *et al.* 1982). Kozak (1988) recommended that 60-100 trees, which cover a wide range of dbh and height measurements, be used when fitting the equation to provide optimum results. It was also suggested that a minimum of 6-15 diameter inside bark measurements, which are well spread out along the bole, be used for each tree to achieve the best results.

Equation 1 has been shown to provide an adequate fit (Perez *et al.* 1990, Kozak 1991, LeMay *et al.* 1993, Kozak and Smith 1993, Muhairwe *et al.* 1994, Gal and Bella 1994). Gal and Bella (1994) tested three equations including those developed by Demaerschalk and Kozak (1977), Kozak (1988) and Hilt 1980 (modified from Bruce *et al.* (1968)). The equations were tested using 12 timber species in Saskatchewan. They found that equation 1 achieves overall superior performance. Huang *et al.* (1999) demonstrated that the equation behaved well in predicting dib, gross total volume (GTV), merchantable height (MH) and merchantable volume (MV). The equation is flexible, easy to use, readily adaptable to any species, and is a good tool for obtaining accurate volume predictions (Huang *et al.* 2000). However, the equation does possess weaknesses: i) the equation can not be integrated directly to calculate total stem and log volumes, ii) volumes must be calculated by numerical integration from estimated dib predictions, and iii) MH can only be calculated using an iteration procedure (Kozak 1988, Kozak and Smith 1993, Muhairwe 1999). The equation also contains several polynomial terms and transformations of the same regressor, relative height, which results in multicollinearity (Kozak 1997).

Multicollinearity exists when there are high intercorrelations among the independent variables in multiple regression analysis and results in the following consequences: i) small changes in the data yield significant changes in the parameter

estimates, ii) coefficients have high standard errors, which affect their significance, and iii) coefficients may exhibit the wrong sign or unreasonable magnitude (Kleinbaum *et al.* 1988, Myers 1990, Fox 1991). Multicollinearity is commonly found in overcomplicated equations with several polynomial terms (Kozak 1997). The problem of autocorrelation arises from using multiple observations from the same tree, therefore, the assumption of independent error terms is violated (Kozak 1997). The presence of autocorrelation causes the following consequences: i) estimates are unbiased and consistent but no longer possess the minimum variance, ii) mean squared error (MSE) underestimates the real variance of the errors while standard errors of the coefficients may underestimate the true standard deviations, and iii) statistical tests using the t or F distributions and confidence intervals are no longer valid (Neter and Wasserman 1974, Kmenta 1986, Myers 1990). This invalidates standard regression hypothesis testing and interval estimation but can be solved using generalized nonlinear least squares (GNLS) and nonlinear mixed model techniques (Huang *et al.* 1997, Huang 1997). Accounting for autocorrelation and multicollinearity has little practical significance, since the estimates are still unbiased, therefore, these problems are usually ignored by practitioners whose primary objective is to achieve the best predictions (Huang 1994, Kozak 1997, Williams and Reich 1997, Huang *et al.* 1997).

The inflection point is a percentage of total tree height and is the location where stem form changes from neiloid at the butt to paraboloid (Demaerschalk and Kozak 1977, Newnham 1992). The relative height of the inflection point is relatively constant within a species, regardless of size (Kozak 1988), which contradicts the findings of Bi (2000) who found that the relative height of the inflection point varies with size. Perez *et al.* (1990) found that the value of p in equation 1 provided little impact on the

predictive properties of the model and therefore treated it as a parameter and estimated its value using nonlinear least squares (NLS). Kozak (1988) suggested using a constant value of 0.225. Studies by Demaerschalk and Kozak (1977), Perez *et al.* (1990) and Allen (1993) found the value of p to range between 0.2 and 0.25, between 0.15 and 0.35, and between 0.29 and 0.32, respectively. Muhairwe (1999) suggested using an average value for p of 0.25, since the effect caused by the actual averages for a species being different is expected to be minimal and not significant.

Improvement of equation 1 has been attempted, however, it may be difficult since the dbh-H relationship is strongly correlated with variables to be added (e.g. crown ratio, crown class, and site class) (Burkhart and Walton 1985, Muhairwe *et al.* 1994) and the dbh-H relationship already explains a significant amount of the variation (Kozak 1988, Newnham 1992, Muhairwe 1994). Muhairwe *et al.* (1994) expanded Kozak's original equation to include other stand variables (crown class, site class, breast height age, crown ratio, and quadratic mean diameter) but found that the improvement was small and the cost of measuring the extra variables to be unjustifiable. Muhairwe *et al.* (1994) suggested that improvements may be achieved by adding additional variables to the base of the function as opposed to the exponent. Kozak modified the equation and developed variations, however, equation 1 contained less multicollinearity and is still deemed superior (Kozak 1997). Kozak (1998) added upper stem diameters (outside bark) to improve equation 1 but found improvements to be small and not justifiable. However, if economically justifiable, a measurement between 40 and 50 percent above breast height is recommended. Tasissa *et al.* (1997) used a different equation but suggested that accounting for silvicultural treatments may improve the accuracy of taper equations.

2.4. Estimating volume using taper equations

The trends of bias in volume estimation using taper equations tend to underestimate small trees and overestimate large trees (Kozak 1997) and underestimate the lower portion of the stem and overestimate the upper portion of the stem (Demaerschalk and Kozak 1977). Two common sources of error in volume estimation are: i) model misspecification since the fit of the model depends on the shape of the stem and ii) diameter and length are not accurately measured (Biging 1988). Testing and validation of a taper model is usually accomplished by comparing volumes obtained using Smalian's or Newton's formula with volume estimates obtained using the taper equation. Kozak and Smith (1993) found volume estimates obtained using Smalian's formula to be consistently two to five percent greater than those obtained using the taper function. Smalian's formula usually overestimates tree volume (Husch *et al.* 1982). Therefore, volumes derived using the taper equation are considered to be more accurate (Huang 1994). However, this is a comparison between two predicted estimates of volume. The water displacement technique (Martin 1984) has been used to provide a better true estimate of volume, however, the procedure is expensive and specialized equipment is required (Huang 1994). The water displacement technique is still considered an estimate since the procedure has sources of errors (e.g. water sticks to the log, mechanical arms used to hold the log become submerged and affects water displacement). The proper procedure to validate a taper equation is to use an independent validation data set to observe the overall fit and prediction abilities of the taper equation (Muhairwe 1999). Fit statistics are used to observe how well the model fits the data set used in constructing the equation, while prediction statistics are used to indicate how well the model predicts diameter and volume (Muhairwe 1999). To observe

how well the equation performs at different locations along the stem, the prediction errors can be plotted against all four independent variables of the equation (Huang *et al.* 1999).

2.5. Ecologically-based taper equations

Ecologically-based forest management is important for providing sustainable forest operations. Ecologically-based taper equations provide more accurate volume predictions (Huang 1994) and assist in achieving sustainable management. Regional differences of stem taper are the result of different biological, geographical and climate conditions (Huang *et al.* 2000). Huang *et al.* (2000) found that ecologically-based taper equations performed well at different portions of the stem and for various tree sizes.

The ability of ecologically-based taper equations to accurately calculate the amount of volume of fiber on a land base improves the accuracy of merchandising, which results in an increase in utilization and profit of wood (Greber and Smith 1986). Therefore, individual tree volume tables developed through ecologically-based taper equations are an important asset to sound forest management. Applications of individual tree volume tables are: i) to predict total tree height (H), ii) to predict stump diameter outside bark (dob), iii) to predict dbhob, iv) to predict GTV from observed stump dob and H, v) to predict GTV from observed dbhob and predicted H, vi) to predict GTV from observed stump dob and H, vii) to predict GTV from observed stump dob and predicted H, and viii) to predict log volumes (Huang 1994).

Kozak's variable exponent taper equation (Eq. 1) has been shown to be a superior taper equation. Regional differences of stem taper have been observed. The differences are the result of different biological, geographical and climate conditions

(Huang *et al.* 2000). Stem taper equations have not been developed for major tree species in Manitoba. Therefore, equation 1 will be used to develop ecologically-based taper equations for major tree species in Manitoba. Ecologically-based taper equations will provide more accurate volume predictions, which will improve forest management in Manitoba.

3. STUDY AREA AND SPECIES

3.1. Study area

The study area covers two ecozones in Manitoba: Boreal Shield and Boreal Plain. A map of Manitoba's ecozones and ecoregions are provided in Figure 1 and Figure 2, respectively. The Boreal Shield ecozone includes the Churchill River Upland (88), the Hayes River Upland (89), the Lac Seul Upland (90), and the Lake of the Woods (91) ecoregions. The Boreal Plain ecozone includes the Mid Boreal Lowland (148), the Mid Boreal Uplands (152-154), and the Interlake Plain (155) ecoregions.

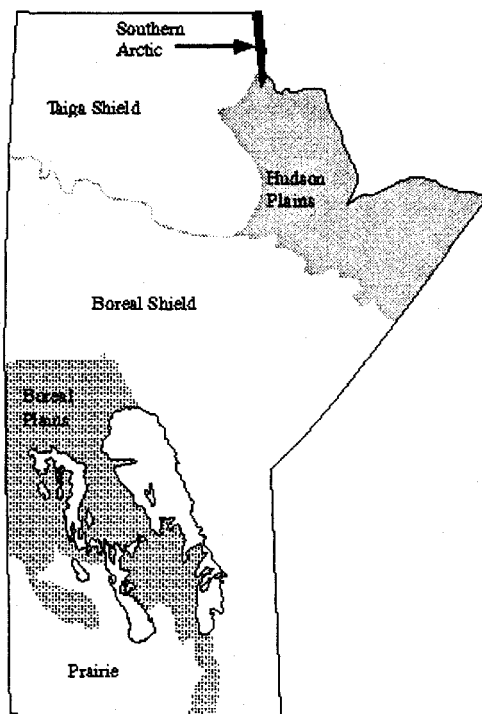


Figure 1. Map of Manitoba's ecozones (Manitoba Conservation 1995).

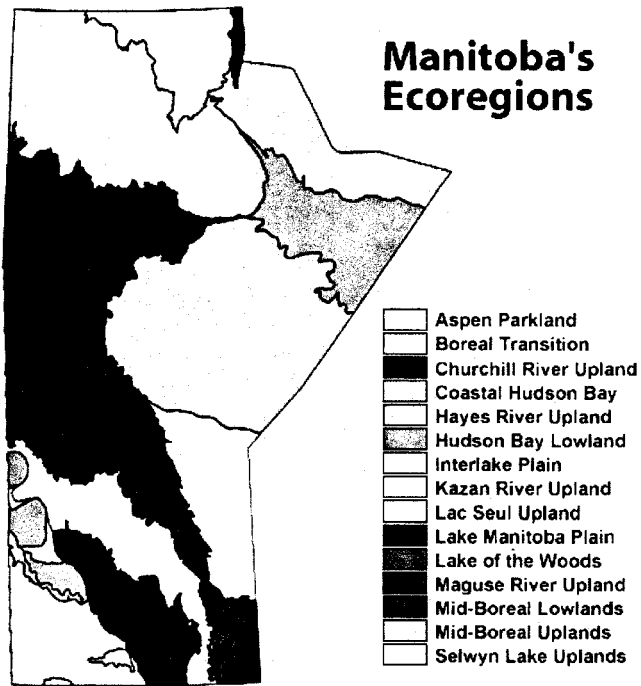


Figure 2. Map of Manitoba's ecoregions (Manitoba Conservation 2003).

3.1.1. Boreal Shield ecozone

The Boreal Shield ecozone is the largest ecozone within Canada. The ecozone is covered with many bedrock outcrops and small to medium lakes. It is characterized with long cold winters and short warm summers. The mean annual temperatures range between -4°C and 5.5°C . The mean summer temperatures range between 11°C and 15°C . The mean winter temperatures range between -20.5°C in the west and -1°C in the east. The mean annual precipitation ranges between 400 mm and 1000 mm. Forests cover over 80% of the Boreal Shield ecozone. Closed stands of conifer dominated by white spruce, black spruce, balsam fir, and tamarack populate the majority of the ecozone. However, the southern portion is attributed with a larger diversity of tree species including white birch, trembling aspen, balsam poplar, white pine, red pine, and jack pine. This ecozone is characterized with a mosaic of soils and exposed bedrock

(Environment Canada 2004). The ecozone is dominated with broadly rolling uplands and lowlands. The ecozone is composed of Precambrian granitic bedrock outcrops, moraines, glaciofluvial, and colluvial deposits. Luvisol soils are dominant in the southern portion while Brunisols are dominant in the northern portion (Zoladeski *et. al.* 1995).

Within the Boreal Shield ecozone, four ecoregions are recognized within Manitoba, which are pertinent to this study. These ecoregions are Churchill River Upland (88), Hayes River Upland (89), Lac Seul Upland (90), and Lake of the Woods (91).

The Churchill River Upland ecoregion experiences cool summers and very cold winters. The mean annual, summer, and winter temperatures are -2.5°C , 12.5°C , and -18.5°C , respectively. The mean annual precipitation ranges between 400 mm and 500 mm. The ecoregion is predominantly vegetated with closed stands of black spruce and jack pine, however, stands of trembling aspen, white birch, white spruce, and balsam fir cover significant areas, especially in the eastern portion (Environment Canada 2004).

The Hayes River Upland ecoregion experiences cool summers and very cold winters. The mean annual, summer, and winter temperatures are -4°C , 11.5°C , and -20°C , respectively. The mean annual precipitation ranges between 400 mm and 600 mm. Black spruce is the dominant species within the ecoregion. Stands predominantly consist of black spruce, jack pine, and small portions of white birch. Trembling aspen, white birch, white spruce, and balsam fir occupy significant areas, especially in the southern portion (Environment Canada 2004).

The Lac Seul Upland ecoregion experiences warm summers and very cold winters. The mean annual, summer, and winter temperatures are 0.5°C , 14°C , and

-14.5° C, respectively. The mean annual precipitation ranges between 450 mm and 700 mm. Stands dominating the ecoregion contain white spruce, balsam fir, and black spruce with small portions of trembling aspen and balsam poplar (Environment Canada 2004).

The Lake of the Woods ecoregion experiences warm summers and cold winters. The mean annual, summer, and winter temperatures are 1.5° C, 15° C, and -13° C, respectively. The mean annual precipitation ranges between 500 mm and 700 mm. This ecoregion is vegetated with stands succeeding from trembling aspen, white birch, and jack pine to white spruce, black spruce, and balsam fir (Environment Canada 2004).

3.1.2. Boreal Plain ecozone

The Boreal Plain ecozone contains considerably less bedrock outcrops and small lakes than the Boreal Shield ecozone. The ecozone is characterized as experiencing cold winters and moderately warm summers. The mean annual temperatures range between -2° C and 2° C. The mean summer temperatures range between 13° C and 15.5° C. The mean winter temperatures range between -17.5° C and -11° C. The mean annual precipitation ranges between 300 mm and 625 mm. Coniferous tree species dominating this ecozone include white spruce, black spruce, jack pine, and tamarack. Broadleaf trees dominate transitional areas close to the prairie grasslands and include white birch, trembling aspen, and balsam poplar (Environment Canada 2004). The ecozone is covered with a relatively flat to gently rolling landscape consisting of lacustrine deposits and largely hummocky to kettled glacial moraine. Luvisol soils are predominant across the ecozone, however, Black Chernozems are present in the southern portion and Brunisols and Organics are present in the northern portion (Zoladeski *et. al.* 1995).

Within the Boreal Plain ecozone, three ecoregions are within Manitoba and are pertinent to this study. These include Mid Boreal Lowland (148), Mid Boreal Uplands (152-154), and Interlake Plain (155).

The Mid Boreal Lowland ecoregion experiences short warm summers and cold winters. The mean annual, summer, and winter temperatures are -1°C , 13.5°C , and 17°C , respectively. The mean annual precipitation ranges between 375 mm and 625 mm. It is a relatively flat, low-lying ecoregion and wetlands occupy approximately 50% of the area. Stands of tamarack and black spruce occupy the poorly drained bogs and fens. Mixed deciduous and coniferous stands of trembling aspen and balsam poplar with white spruce, black spruce, and balsam fir in late successional stages predominate the ecoregion (Environment Canada 2004).

The Mid Boreal Upland ecoregions are represented by 10 separate ecoregions. Among them, only three are pertinent to this study including 152, 153, and 154. The ecoregion experiences short cool summers and cold winters. The mean annual temperatures range between -1°C and 1°C . The mean summer temperatures range between 13°C and 15.5°C . The mean winter temperatures range between -13.5°C and 16°C . The mean annual precipitation ranges between 400 mm and 550 mm. The ecoregion is characterized with mid-boreal mixed deciduous and coniferous forests containing closed stands of trembling aspen and balsam poplar with white spruce, black spruce, and balsam fir occurring in late successional stages. Poorly drained bogs and fens are dominated with tamarack and black spruce (Environment Canada 2004).

The Interlake Plain ecoregion experiences warm summers and cold winters. The mean annual, summer, and winter temperatures are 1°C , 15.5°C , and -14.5°C , respectively. The mean annual precipitation ranges between 425 mm and 575 mm. The

ecoregion is occupied with deciduous dominated boreal forest, consisting of stands of trembling aspen with small portions of balsam poplar. Open stands of jack pine occur on dry sandy sites (Environment Canada 2004).

3.2. Species

3.2.1. Balsam poplar

Balsam poplar (*Populus balsamifera* L.) is used for pulp, particle boards, and lumber. It is found from Newfoundland and Labrador across Canada to eastern British Columbia, north into Alaska and throughout the north eastern states (Burns and Honkala 1990b). Balsam poplar occurs on upland and flood plain sites which contain soils developed from lacustrine deposits, glacial till, outwash, and loess. It can be found on dry sandy soils to wet clay soils (Burns and Honkala 1990b). Best growth is achieved on moderately well drained sites. Balsam poplar can occur in pure stands, but is usually found in mixed stands where other species dominate. Species it is associated with in mixed stands include black spruce, tamarack, balsam fir, trembling aspen, and white birch (Burns and Honkala 1990b). Balsam poplar is usually associated with a heavy understory of shrubs.

3.2.2. Trembling aspen

Trembling aspen (*Populus tremuloides* Michx.) is the most widely distributed tree species in North America and is used for pulp, particle boards, and lumber. It is a pioneer species which quickly populates areas of bare soil caused by disturbances (Burns and Honkala 1990b). It grows on a wide variety of soils, but prefers sandy or gravelly sites. Trembling aspen is found from Newfoundland and Labrador across

Canada, through the north eastern states and south from Washington to California and Texas along the mountains (Burns and Honkala 1990b). Trembling aspen occurs on a wide variety of soils ranging from shallow and rocky to deep loamy sands and heavy clays. Chemical and physical properties of the soil strongly influence the growth and development of trembling aspen. Well drained loamy soils with high concentrations of organic matter and nutrients provide good sites for trembling aspen. Sandy soils, which contain low levels of moisture, provide poor sites. Trembling aspen usually occurs on north and east slopes where moisture conditions are more favorable (Burns and Honkala 1990b). Mature trees usually obtain a height between 20 m and 25 m and a dbhob between 18 cm and 30 cm. Trembling aspen is found in pure and mixed stands and is found with a wide variety of tree species (Burns and Honkala 1990b). Trembling aspen is usually associated with a heavy understory of shrubs.

3.2.3. White spruce

White spruce (*Picea glauca* (Moench) Voss) is used for pulpwood and constructional lumber. It is found from Newfoundland and Labrador west through Canada and south throughout the northern states (Burns and Honkala 1990a). White spruce grows in highly variable conditions of climates and soils. The wide variety of soils which white spruce is found upon include soils of glacial, lacustrine, marine, or alluvial origin and are composed of sand flats, clays, and organics. Productive sites are found on moderately well drained clay loams and well drained lacustrine soils (Burns and Honkala 1990a). Soil fertility, moisture, and physical properties all affect the growth of white spruce. White spruce grows best on sites with a dependable supply of well-aerated water. White spruce is commonly found in pure stands or mixed stands

where it is the major component. Species associated with white spruce in mixed stands include jack pine, black spruce, balsam fir, tamarack, trembling aspen, and white birch. White spruce trees of over 30 m tall with a dbhob of 60 to 90 cm are found on good sites (Burns and Honkala 1990a).

3.2.4. Black spruce

Black spruce (*Picea mariana* (Mill.) B.S.P.) is the most important pulpwood species in Canada. Black spruce ranges from northern Massachusetts to northern Labrador on the Atlantic coast, west across Canada to the west coast of Alaska (Burns and Honkala 1990a). Black spruce is commonly found on wet organic soils, however, a variety of soil types including deep humus, clays, loams, sands, coarse till, boulder pavements, and shallow soil mantles over bedrock do support more productive stands. Productive stands are usually found on dark peat soils with a high content of decayed woody material. The least productive stands are usually found on thick deposits of partially decomposed sphagnum peat. The most productive stands are found on upland sites, which provide better drainage (Burns and Honkala 1990a). Black spruce is commonly found in pure stands on organic soils and in mixed stands on mineral soils. Species black spruce is associated with in mixed stands include white spruce, jack pine, balsam fir, tamarack, trembling aspen, white birch, and balsam poplar. At maturity, black spruce usually has a height between 12 m and 20 m and a dbhob of 23 cm on good sites, and a height between 8 m and 12 m and a dbhob of 13 cm on poor sites. Regional differences in growth are associated to climatic factors, while differences within a region are associated with soil moisture and nutrients. Soil moisture is the major site factor which affects black spruce growth (Burns and Honkala 1990a).

3.2.5. Jack pine

Jack pine (*Pinus banksiana* Lamb.) is used for pulpwood, constructional lumber, and round timber. It is the most widely distributed pine species in Canada. It is a pioneer species which invades sites where the mineral soil has been exposed by major disturbances. It is usually found in even-aged pure or mixed stands on less fertile and dry soils (Burns and Honkala 1990a). Jack pine is found in Canada from the Northwest Territories east to Nova Scotia and south into the north eastern states (Burns and Honkala 1990a). Jack pine is usually found on sandy soils but also grows on loamy soils, thin soils over the granites and metamorphosed rocks of the Canadian Shield, over limestones, peats, and soils over permafrost. Jack pine grows best on well drained loamy sands. Jack pine is most commonly found on level to gently rolling sand plains, usually of glacial outwash, fluvial, or lacustrine origin (Burns and Honkala 1990a). Species associated with jack pine in mixed stands include trembling aspen, white birch, balsam fir, and black spruce. Jack pine trees grown in well stocked stands are usually short to medium-tall, with a narrow, open crown covering 30 to 45 percent of the stem. Trees grown in open areas exhibit a stocky stem of poor form and a wide, spreading crown with lots of branching commonly found to the ground (Burns and Honkala 1990a).

4. MATERIALS AND METHODS

4.1. Data collection

The stem analysis data used in this study were collected by Manitoba Conservation. Louisiana Pacific Canada Ltd., Swan River division, assisted with the collection of the trembling aspen and balsam poplar data within the Mid Boreal Upland ecoregion.

Stands were selected based on the following criteria: 1) pure as possible, 2) fully stocked, 3) even-aged, 4) minimal or no disturbances, and 5) mature. Plots were avoided near major roads since they affect drainage, which alters tree growth. Once a stand was located, either a 300 m² circular plot or a 625 m² square plot was established. There are two establishment methods since Manitoba Conservation changed their sampling procedure. Three trees within the dominant crown class were selected representing the largest, smallest, and average dbhob and height. These trees were felled and cookies were obtained from ground, 0.33 m, 0.67 m, 1.0 m, 1.3 m, and every 1.3 m interval after until a dob of 7.0 cm was reached. Cookies are cross-sectional slices of the stem. Cookies were then taken every 20 cm between 7.0 cm and 4.0 cm dob. Many variables were recorded for each tree, however, for the purpose of this study only dib, height above ground, dbhob, and total tree height were used.

The data was cleaned for the following scenarios: 1) cumulative height greater than total height, which was solved by deleting the top cookies, 2) dbh not equal to dob of cookie at 1.3 m height, which was solved by using the dob at 1.3 m since this was most likely more accurate, and 3) dib greater than dob, this was solved by recalculating dib by subtracting twice the bark thickness from dob.

4.2. Site type definition

Site types were created for black spruce and jack pine for the purposes of determining if site influences stem taper. The site type data were extracted from the original species data and whether ecoregion-specific, ecozone-specific, or provincial data were used depended on the results of the F-test.

The black spruce site types were categorized into two groups: lowland and upland, which were characterized by vegetation types defined in Zoladeski *et al.* (1995). Vegetation types were solely used since soil types are more susceptible to incorrect identification and were scarce throughout the data. The lowland site type was identified with the vegetation types V30, V31, V32, and V33. All other vegetation types were categorized as upland black spruce. The vegetation type V30 is Black Spruce/Labrador Tea/Feather Moss (Sphagnum) and is characterized as lowland black spruce stands occurring on wet, typically organic, poorly drained soils (Zoladeski *et al.* 1995). The vegetation type V31 is Black Spruce/Herb Rich/Sphagnum (Feathermoss) and is characterized as black spruce stands with small components of white cedar or tamarack occurring on wet, poorly drained organic soils with a carpet of sphagnum and feather moss and a well developed herb layer (Zoladeski *et al.* 1995). The vegetation type V32 is Black Spruce/Herb Poor/Sphagnum (Feather moss) and is characterized as lowland black spruce stands with the occasional tamarack occurring on wet, poorly drained organic soils with a carpet of sphagnum and feather moss and a sparse herb layer (Zoladeski *et al.* 1995). The vegetation type V33 is Black Spruce/Sphagnum and is characterized as poorly stocked stunted lowland black spruce stands occurring on wet peat deposits with a carpet of sphagnum and feather moss (Zoladeski *et al.* 1995).

The jack pine site types were categorized into two groups: rock and mineral soil using vegetation and soil types in Zoladeski *et al.* (1995). The rock site type was identified to be sites of rocky and shallow soil that supports vegetation types V25 and V26. The vegetation type V25 is Jack Pine/Feather Moss and is characterized as even-aged jack pine stands occurring on rapidly drained, fresh to moist, coarse-textured soils or very shallow soils over bedrock (Zoladeski *et al.* 1995). The vegetation type V26 is Jack Pine-Black Spruce/Lichen and is characterized as jack pine-black spruce stands occurring on dry to fresh mineral soil over rock outcrops (Zoladeski *et al.* 1995). The majority of soil types represented include SS1 (Discontinuous Organic Mat on Bedrock), SS2 (Extremely Shallow Soil on Bedrock), and SS3 (Very Shallow Soil on Bedrock) (Zoladeski *et al.* 1995). The mineral soil types support all other vegetation types.

4.3. Data analysis

The statistical analyses and individual tree volume tables were accomplished using SAS/STAT software (SAS institute Inc. 1990). The graphs were constructed using SYSTAT software (Wilkinson 1999).

Initial values of the parameters for equation 1 were estimated first using the linearized equation (Eq. 5) to reach fast convergence (Huang 1994). The nonlinear regression procedure (PROC NLIN) used with the Gauss-Newton iterative method (Gallant 1987) was used to estimate the parameters (Huang 1994). The linear model was fitted with provincial data for all species and the parameter estimates were used as initial values for equation 1, regardless of ecozone-specific, ecoregion-specific, or site type-specific. This was done since the estimates were used solely as initial values and it was believed that provincial initial values were satisfactory for each species.

$$\ln(\text{dib}_i) = \ln(a_0) + a_1 \ln(D) + \ln(a_2)D + b_1 \ln(X_i)z_i^2 + b_2 \ln(X_i) \ln(Z_i + 0.001) + b_3 \ln(X_i) \sqrt{z_i} + b_4 \ln(X_i) e^{z_i} + b_5 \ln(X_i)(D/H) \quad \text{Eq. 5}$$

where:

dib_i = dib at point i along the stem (cm)

D = dbhob (cm)

X_i = defined in equation 2

h_i = height above ground at point i along the stem (m)

H = total tree height (m)

p = defined in equation 3

HI = height of the inflection point (m)

z_i = defined in equation 4

$a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5$ = parameters to be estimated

The linear equation (Eq. 5) was not used further since examination proved that the equation in its nonlinear form with an additive error structure is more appropriate than the linear form since nonconstant error variance is evident in the linear equation, which causes the parameter estimates to be inefficient (Judge *et al.* 1988, Huang 1994). The error specification is correct in the nonlinear model (Eq. 1) (Huang 1994).

Nonlinear extra sum of squares procedure as demonstrated in Bates and Watts (1988) and Huang *et al.* (1994) was used to determine whether differences of taper between ecozones, ecoregions, and site types existed. Since the taper equation (Eq. 1) possesses eight parameters, using dummy variables/ indicator variables for all the parameters would increase the sensitivity of the equation. As a result, the F-test (Eq. 7) would show significant differences between all groups, as seen with preliminary work. Dummy variables are also known as indicator variables and are variables which account for categorical differences of parameters within regression equations. Dummy variables are frequently applied to models that allow for behavioral differences in geographic regions (Judge *et al.* 1988, Neter *et al.* 1990). The parameters a_1 and b_5 were the only parameters attributed dummy variables since they were highly correlated with dib and

were used by Huang (1994) and Huang *et al.* (2000). Using dummy variables for only the highly correlated variables is common for models with three or more parameters (Bates and Watts 1988). The full model can be written as equation 6 and the reduced model is represented as equation 1.

$$\text{dib}_i = a_0 D^{(a_1 + c_1 k_1 + \dots + c_t k_t)} a_2 X_i^{b_1 z_i^2 + b_2 \ln(z_i + 0.001) + b_3 \sqrt{z_i} + b_4 e^{z_i} + (b_5 + c_{t+1} k_1 + \dots + c_{2t} k_t)(D/H)} \quad \text{Eq. 6}$$

where:

dib_i = dib at point i along the stem (cm)

D = dbhob (cm)

X_i = defined in equation 2

h_i = height above ground at point i along the stem (m)

H = total tree height (m)

p = defined in equation 3

HI = height of the inflection point (m)

z_i = defined in equation 4

$a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5$ = parameters to be estimated

k = dummy variables for ecozones, ecoregions or site types

t = # of ecozones, ecoregions or site types

c = parameter estimates for ecozone, ecoregion or site type differences

Each combination of ecozones, ecoregions, and site types were tested using the null hypotheses ($H_0: c_1 = c_2 = \dots = c_t$) and the alternative hypothesis (H_1 : at least one of the equalities is not true). To determine these differences, the F-test was used and is illustrated in equation 7. An α -level of 0.05 was specified. If $F > F_{\text{critical}}(1-\alpha, df_R - df_F, df_F)$, reject H_0 .

$$F = \frac{(SSE_R - SSE_F)/(df_R - df_F)}{SSE_F / df_F} \quad \text{Eq. 7}$$

where:

F = F-statistic

SSE_R = error sum of squares associated with reduced model (Eq. 1)

SSE_F = error sum of squares associated with full model (Eq. 6)

df_R = degrees of freedom associated with reduced model (Eq. 1)

df_F = degrees of freedom associated with full model (Eq. 6)

Equation 1 was then fitted for each significant group and the mean square error (MSE) and coefficient of determination (R^2) for equation 1 were calculated using equation 8 and equation 9, respectively.

$$\text{MSE} = \frac{\sum_{i=1}^n (\text{dib}_i - \hat{\text{dib}}_i)^2}{n - m} \quad \text{Eq. 8}$$

where:

MSE = mean square error

dib_i = observed dib at point i along the stem (cm)

$\hat{\text{dib}}_i$ = predicted dib at point i along the stem (cm)

n = number of observations

m = number of parameters (m = 8)

$$R^2 = 1 - \frac{\sum_{i=1}^n (\text{dib}_i - \hat{\text{dib}}_i)^2}{\sum_{i=1}^n (\text{dib}_i - \bar{\text{dib}})^2} \quad \text{Eq. 9}$$

where:

R^2 = coefficient of determination

dib_i = observed dib at point i along the stem (cm)

$\hat{\text{dib}}_i$ = predicted dib at point i along the stem (cm)

$\bar{\text{dib}}$ = observed average dib (cm)

Differences of dib predictions were determined as percentages to compare relative differences. The percent differences were determined for comparison between Manitoba and Alberta provincial equations, ecozone-specific and provincial equations, ecoregion-specific and provincial equations, and site type-specific and provincial equations for each species as the results permitted. The percent differences were calculated using equation 10.

$$\text{diff} = \frac{\text{dib}_b - \text{dib}_a}{\text{dib}_a} * 100\% \quad \text{Eq. 10}$$

where:

- diff = percent difference (%)
- dib_a = dib prediction for Manitoba provincial equation (cm)
- dib_b = dib prediction for Alberta provincial, ecozone-specific, ecoregion-specific or site type-specific equations (cm)

Individual tree volume tables (Appendix IV) were constructed for each taper equation using the procedures illustrated in Appendix I.

5. RESULTS

5.1. Balsam poplar

A summary of the balsam poplar diameter at breast height outside bark (dbhob) and total tree height data is shown in Table 1. Data were only collected within the Mid Boreal Upland ecoregion (ecoregion 152-154) and consists of 37 trees. The mean, minimum, and maximum dbhob sampled were 24.3 cm, 17.9 cm, and 31.0 cm, respectively. The mean, minimum, and maximum total tree height sampled were 21.03 m, 16.90 m, and 24.50 m, respectively. The balsam poplar dib data are clustered together and no outlying data points are observed (Figure 3).

Table 1. Summary statistics of the balsam poplar data.

Ecoregion # and name	No. of trees	Variable	Mean	Min	Max	SD
152-154 - Mid Boreal Upland	37	D (cm)	24.3	17.9	31.0	3.10
		H (m)	21.03	16.90	24.50	2.23

D = dbhob, H = total tree height, SD = standard deviation

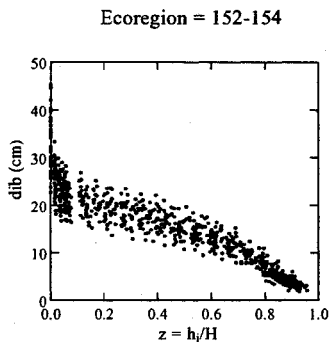


Figure 3. Balsam poplar dib data plotted by relative height (z) and fitted for ecoregion 152-154.

The fit statistics and the residual plot of the taper equation (Eq. 1) for balsam poplar are shown in Table 2 and Figure 4, respectively. The fit statistics and residual plots show that the taper equation provided a strong fit of the data. The balsam poplar

equation explained more than 98% of the total variation of dib. The residual plot shows the residuals clustered around zero indicating a good agreement between observed and predicted dib. However, the residuals show a wider scatter for larger dib measurements, primarily dib measurements below dbhob. This indicates that the residuals are not normally distributed and dib predictions around buttswell are less accurate.

Table 2. Fit statistics of the ecoregion 152-154 taper equation (Eq. 1) for balsam poplar.

Parameter	152-154
a_0	0.2569
a_1	1.5383
a_2	0.9751
b_1	0.7719
b_2	-0.1105
b_3	0.1448
b_4	-0.0811
b_5	0.1324
MSE	1.236146
R^2	0.983187
n	875

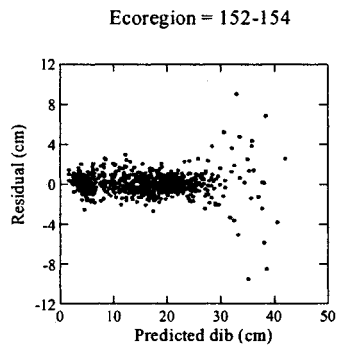


Figure 4. Residual plot of the ecoregion 152-154 taper equation (Eq. 1) for balsam poplar.

5.2. Trembling aspen

A summary of the trembling aspen dbhob and total tree height data are presented in Table 3. A total of 71 trees were sampled, 10 within the Boreal Shield ecozone and 61 within the Boreal Plain ecozone. Since there was a poor representation of the data within the ecozones, ecoregion-specific equations were not attainable and the data were observed at the ecozone level. The mean, minimum, and maximum dbhob sampled within the Boreal Shield ecozone were 19.9 cm, 9.7 cm, and 29.3 cm, respectively. The mean, minimum, and maximum total tree height sampled within the Boreal Shield ecozone were 19.61 m, 9.90 m, and 28.1 m, respectively. The mean, minimum, and maximum dbhob sampled within the Boreal Plain ecozone were 24.1 cm, 11.5 cm, and 35.1 cm, respectively. The mean, minimum, and maximum total tree height sampled within the Boreal Plain ecozone were 21.43 m, 11.70 m, and 26.40 m, respectively. The trembling aspen dib data are well clustered (Figure 5). The Boreal shield data appear to have a less tight cluster than the Boreal Plain ecozone, however, no outliers are visible.

Table 3. Summary statistics of the trembling aspen data.

Ecozone	No. of trees	Variable	Mean	Min	Max	SD
Boreal Shield Ecozone	10	D (cm)	19.9	9.7	29.3	6.03
		H (m)	19.61	9.90	28.10	5.13
Boreal Plain Ecozone	61	D (cm)	24.1	11.5	35.1	5.12
		H (m)	21.43	11.70	26.40	3.08
Provincial	71	D (cm)	23.5	9.7	35.1	5.42
		H (m)	21.20	9.90	28.10	3.47

D = dbhob, H = total tree height, SD = standard deviation

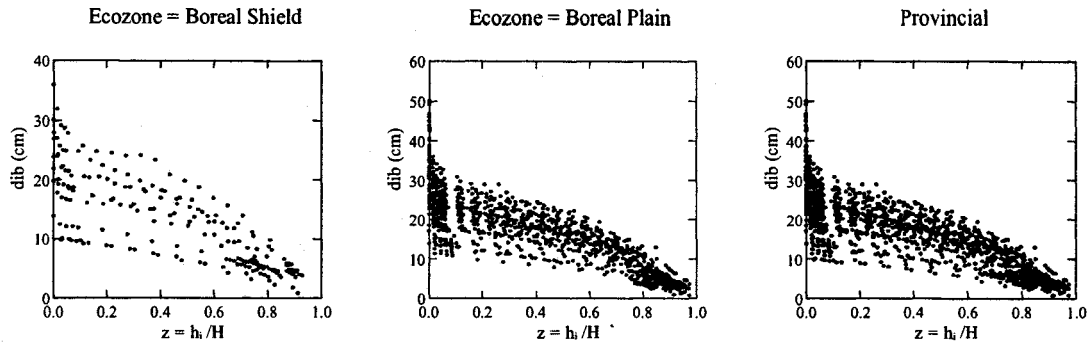


Figure 5. Trembling aspen dbh data plotted by relative height (z) and fitted by ecozone and provincial.

The F-test results determining if differences of trembling aspen stem taper exist between ecozones are shown in Table 4. The F-statistic calculated using equation 7 is 21.60, which is greater than the critical F-value of 3.00. This indicates that stem taper differed between these two ecozones and ecozone-specific taper equations are required for trembling aspen.

Table 4. F-test for ecozone differences of the taper equation for trembling aspen.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
Trembling aspen	2857.4	1616	2782.9	1614	21.60*	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plots of the trembling aspen taper equations (Eq.1) are shown in Table 5 and Figure 6, respectively. The fit statistics and residual plots show that the taper equation provided a strong fit of the data. The trembling aspen equations all explained more than 97% of the total variation of dbh. The residual plots show the residuals clustered around zero indicating a good agreement between observed and predicted dbh. However, the residuals show a wider scatter for larger dbh

measurements, primarily measurements below dbhob. This indicates that the residuals are not normally distributed and dib predictions around buttswell are less accurate.

Table 5. Fit statistics of the ecozone-specific and provincial taper equations (Eq. 1) for trembling aspen.

Parameter	Boreal Shield	Boreal Plain	Provincial
a_0	0.7855	0.4560	0.6549
a_1	1.0580	1.2991	1.1280
a_2	0.9950	0.9859	0.9930
b_1	0.1203	1.0188	0.9223
b_2	-0.0713	-0.1311	-0.1311
b_3	0.3202	0.5736	0.6458
b_4	0.2354	-0.3198	-0.2967
b_5	-0.2120	0.1047	0.0670
MSE	1.294318	1.691296	1.768216
R^2	0.977597	0.978322	0.976623
n	211	1413	1624

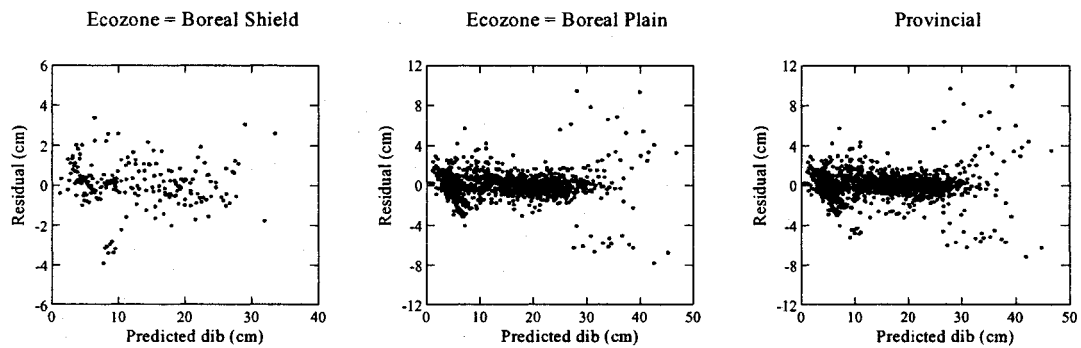


Figure 6. Residual plots of the ecozone-specific and provincial taper equations (Eq. 1) for trembling aspen.

5.3. White spruce

A summary of the white spruce dbhob and total tree height data is shown in Table 6. A total of 97 trees were sampled, 33 within the Boreal Shield ecozone and 64 within the Boreal Plain ecozone. Since there was a poor representation of the data within the ecozones, ecoregion-specific equations were not attainable and the data were

observed at the ecozone level. The mean, minimum, and maximum dbhob sampled within the Boreal Shield ecozone were 23.5 cm, 13.3 cm, and 37.1 cm, respectively. The mean, minimum, and maximum total tree height sampled within the Boreal Shield ecozone were 18.17 m, 11.56 m, and 25.65 m, respectively. The mean, minimum, and maximum dbhob sampled within the Boreal Plain ecozone were 27.9 cm, 14.7 cm, and 40.4 cm, respectively. The mean, minimum, and maximum total tree height sampled within the Boreal Plain ecozone were 21.23 m, 11.84 m, and 29.30 m, respectively. The white spruce dib data appear to be represented similarly for both ecozones and appear to be tightly clustered indicating there are no outlying data points (Figure 7).

Table 6. Summary statistics of the white spruce data.

Ecozone	No. of trees	Variable	Mean	Min	Max	SD
Boreal Shield Ecozone	33	D (cm)	23.5	13.3	37.1	6.68
		H (m)	18.17	11.56	25.65	3.68
Boreal Plain Ecozone	64	D (cm)	27.9	14.7	40.4	7.13
		H (m)	21.23	11.84	29.30	4.52
Provincial	97	D (cm)	26.4	13.3	40.4	7.27
		H (m)	20.22	11.56	29.30	4.50

D = dbhob, H = total tree height, SD = standard deviation

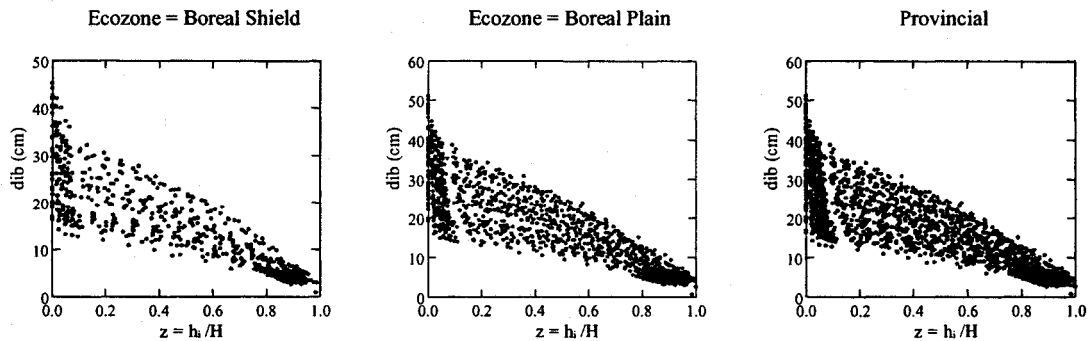


Figure 7. White spruce dib data plotted by relative height (z) and fitted by ecozone and provincial.

The F-test results for ecozone differences of the taper equation are shown in Table 7. The F-statistic calculated using equation 7 is 15.78, which is greater than the critical F-value of 3.00. This implies that stem taper differed between the ecozones and ecozone-specific taper equations are required for white spruce.

Table 7. F-test for ecozone differences of the taper equation for white spruce.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
White spruce	3808.8	2152	3753.7	2150	15.78*	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plots of the ecozone-specific and provincial taper equations (Eq. 1) for white spruce are shown in Table 8 and Figure 8, respectively. Based on the fit statistics and residual plots, all the models provided a good fit with more than 98% of the total variation of dib explained by the model and the residuals are all tightly clustered around zero indicating a random distribution.

Table 8. Fit statistics of the ecozone-specific and provincial taper equations (Eq. 1) for white spruce.

Parameter	Boreal Shield	Boreal Plain	Provincial
a_0	0.6554	0.7524	0.6977
a_1	1.1330	1.0797	1.1086
a_2	0.9923	0.9939	0.9931
b_1	0.1801	0.2469	0.2342
b_2	-0.0592	-0.0669	-0.0653
b_3	0.1254	0.1921	0.1814
b_4	0.0996	0.0374	0.0491
b_5	0.1230	0.1218	0.1233
MSE	1.415953	1.908303	1.769892
R^2	0.984431	0.982889	0.983337
n	708	1452	2160

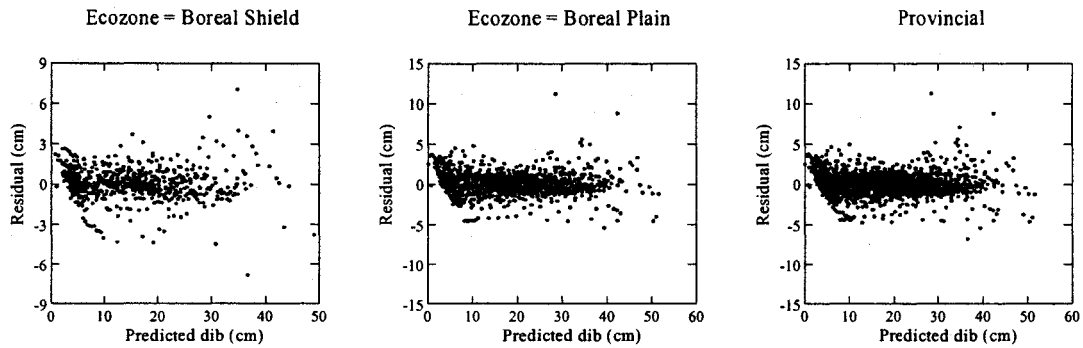


Figure 8. Residual plots of the ecozone-specific and provincial taper equations (Eq. 1) for white spruce.

5.4. Black spruce

5.4.1. Ecozone-specific and ecoregion-specific models

A summary of the black spruce dbhob and total tree height data is presented in Table 9. A total of 300 trees were sampled, 202 within the Boreal Shield ecozone and 98 within the Boreal Plain ecozone. The smallest and largest dbhob sampled were 5.3 cm within ecoregion 155 and 25.9 cm within ecoregion 91, respectively. The mean observed dbhob was 16.8 cm. The smallest and largest total tree heights sampled were 4.66 m within ecoregion 148 and 25.00 m within ecoregion 88, respectively. The mean total tree height sampled was 15.88 m. Some ecoregions were poorly represented. For example, ecoregions 90, 91, 152-154 and 155 were represented with 21, 21, 17 and 13 trees, respectively. However, the ecoregion-specific analysis was explored. The black spruce dib data appear to be lacking for some ecoregions, especially ecoregion 155, which contained only 13 trees (Figure 9). However, the ecozone-specific and provincial data are tightly clustered and show no outlying data points.

Table 9. Summary statistics of the black spruce data.

Ecoregion # and name	No. of trees	Variable	Mean	Min	Max	SD
Boreal Shield Ecozone						
88 - Churchill River Upland	34	D (cm)	14.9	5.5	25.7	5.66
		H (m)	13.31	5.96	25.00	5.16
89 - Hayes River Upland	126	D (cm)	17.4	7.7	25.3	3.08
		H (m)	16.29	6.65	22.7	2.68
90 - Lac Seul Upland	21	D (cm)	17.1	11.5	23.3	2.96
		H (m)	16.43	11.52	21.00	2.65
91 - Lake of the Woods	21	D (cm)	17.6	13.4	25.9	2.88
		H (m)	16.20	13.20	19.70	1.762
Boreal Plain Ecozone						
148 - Mid Boreal Lowland	68	D (cm)	16.0	5.5	25.8	3.82
		H (m)	16.04	4.66	23.7	3.66
152-154 - Mid Boreal Upland	17	D (cm)	18.9	15.7	25.0	2.35
		H (m)	17.63	14.10	21.60	2.07
155 - Interlake Plain	13	D (cm)	10.8	5.3	24.9	5.07
		H (m)	9.95	4.75	19.73	3.83
Provincial	300	D (cm)	16.8	5.3	25.9	3.83
		H (m)	15.88	4.66	25.0	3.44

D = dbhob, H = total tree height, SD = standard deviation

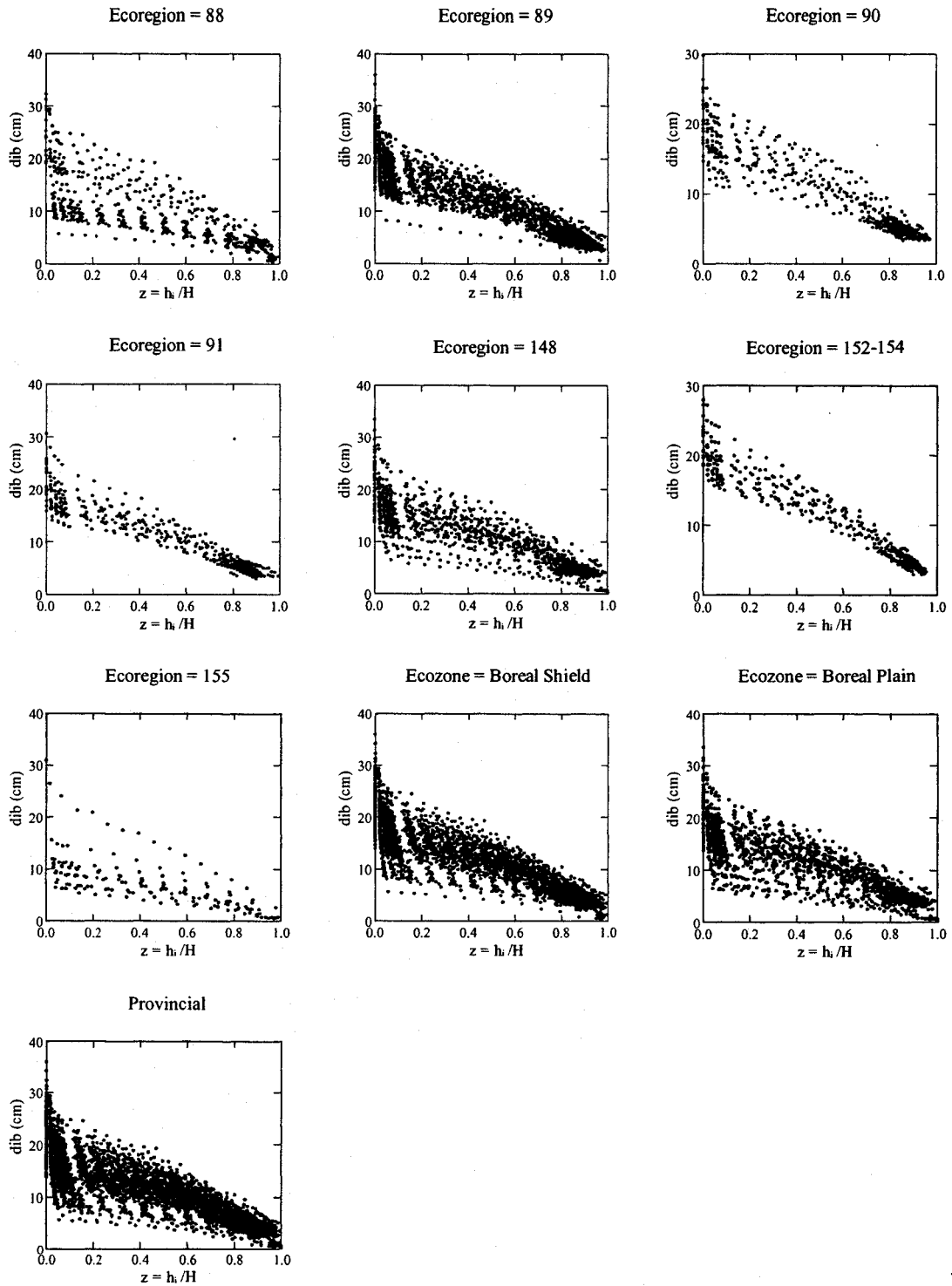


Figure 9. Black spruce dib data plotted by relative height (z) and fitted by ecoregion, ecozone, and provincial.

The F-test results for ecozone differences of the taper equation are shown in Table 10. The F-statistic calculated using equation 7 is 1.96, which is less than the critical F-value of 3.00. This implies that black spruce stem taper was not different between ecozones and a single provincial equation can be used. Therefore, ecoregion differences in stem taper were not explored.

Table 10. F-test for ecozone differences of the taper equation for black spruce.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
Black spruce	3936.7	5932	3934.1	5930	1.96	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plot of the provincial black spruce taper equation (Eq. 1) are shown in Table 11 and Figure 10, respectively. Based on the fit statistics and residual plot, the model provided a good fit with more than 98% of the total variation of dib explained by the model and the residuals are all tightly clustered around zero indicating a random distribution.

Table 11. Fit statistics of the provincial taper equation (Eq. 1) for black spruce.

Parameter	Provincial
a_0	0.8894
a_1	1.0163
a_2	0.9950
b_1	0.2866
b_2	-0.0853
b_3	0.6307
b_4	-0.1714
b_5	0.1491
MSE	0.663633
R^2	0.981481
n	5940

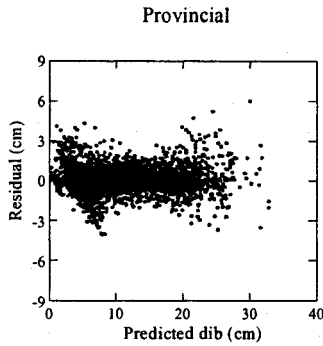


Figure 10. Residual plot of the provincial taper equation (Eq. 1) for black spruce.

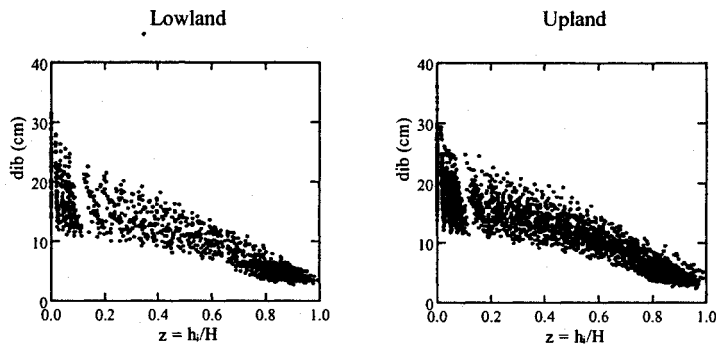
5.4.2. Ecosite-specific models

The black spruce site type data were extracted from the provincial data set since it was determined that black spruce stem taper did not differ between ecozones. The site type data contains fewer trees than the provincial data used for testing ecozone differences since vegetation types were not recorded for some trees. A summary of the black spruce site type dbhob and total tree height data is presented in Table 12. A total of 61 lowland and 167 upland trees were sampled. The smallest, largest, and mean dbhob sampled in the lowland site type were 11.5 cm, 25.9 cm, and 17.1 cm, respectively. The smallest, largest, and mean total tree heights sampled within the lowland site type were 9.22 m, 21.30 m, and 16.11 m, respectively. The smallest, largest, and mean dbhob sampled in the upland site type were 12.1 cm, 25.8 cm, and 17.7 cm, respectively. The smallest, largest, and mean total tree heights sampled within the upland site type were 9.61 m, 25.00 m, and 16.88 m, respectively. The black spruce site type dib data appear to be similarly distributed between the two site types and no outlying data points are observed (Figure 11).

Table 12. Summary statistics of the black spruce site type data (Provincial).

Site type	No. of trees	Variable	Mean	Min	Max	SD
Lowland	61	D (cm)	17.1	11.5	25.9	3.43
		H (m)	16.11	9.22	21.30	2.54
Upland	167	D (cm)	17.7	12.1	25.8	2.94
		H (m)	16.88	9.61	25.00	2.54

D = dbhob, H = total tree height, SD = standard deviation

Figure 11. Black spruce site type dib data plotted by relative height (z).

The F-test results for site type differences of black spruce are shown in Table 13. The F-statistic calculated using equation 7 is 5.92, which is greater than the critical F-value of 3.00. This indicates that stem taper differed between site types for black spruce and site type-specific equations are required.

Table 13. F-test for site type differences of the taper equation for black spruce.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
Black spruce	3299.5	4757	3291.3	4755	5.92*	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plots of the black spruce site type-specific taper equations (Eq. 1) are shown in Table 14 and Figure 12, respectively. Based on the fit statistics and residual plots, the model provided a good fit explaining more than 97% of the total variation of dib for both site types and the residuals are all tightly clustered around zero indicating a random distribution.

Table 14. Fit statistics of the site type-specific taper equations (Eq. 1) for black spruce.

Parameter	Lowland	Upland
a_0	0.5191	1.1427
a_1	1.3059	0.8876
a_2	0.9785	1.0017
b_1	0.4179	0.3417
b_2	-0.0841	-0.09
b_3	0.7781	0.6841
b_4	-0.3529	-0.2204
b_5	0.3231	0.1648
MSE	0.4844582	0.7565699
R^2	0.9862398	0.9792101
n	1289	3476

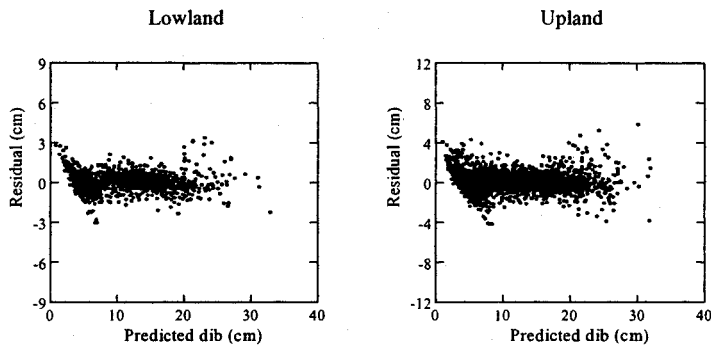


Figure 12. Residual plots of the site type-specific taper equations (Eq. 1) for black spruce.

5.5. Jack pine

5.5.1. Ecozone-specific and ecoregion-specific models

A summary of the jack pine dbhob and total tree height data is presented in Table 15. A total of 298 trees were sampled, 168 within the Boreal Shield ecozone and 130 within the Boreal Plain ecozone. The smallest, largest, and mean dbhob observed were 2.9 cm within ecoregion 88, 31.2 cm within ecoregion 91, and 18.6 cm, respectively. The smallest, largest, and mean total tree height sampled were 3.30 m within ecoregion 88, 24.60 m within ecoregion 91, and 15.55 m, respectively. The jack pine dib data are shown in Figure 13. The ecoregion-specific data appear to be well represented across the ecoregions. Ecoregions 89 and 90 contain a few trees outside the main cluster of data. These trees were not excluded since it was believed the difference of these trees from the main cluster of data was marginal and insignificant. As well, the quantity of data was lacking and it was believed that the deletion of trees may jeopardize the analyses. The ecozone-specific and provincial data appear to be more tightly clustered and contain no outlying data points.

Table 15. Summary statistics of the jack pine data.

Ecoregion # and name	No. of trees	Variable	Mean	Min	Max	SD
Boreal Shield Ecozone						
88 - Churchill River Upland	61	D (cm)	14.1	2.9	25.4	5.97
		H (m)	10.99	3.30	18.00	3.37
89 - Hayes River Upland	25	D (cm)	18.2	3.4	29.6	6.84
		H (m)	14.87	3.72	21.11	4.56
90 - Lac Seul Upland	47	D (cm)	19.3	6.9	28.8	3.80
		H (m)	17.17	7.70	22.00	2.81
91 - Lake of the Woods	35	D (cm)	22.4	16.6	31.2	3.44
		H (m)	18.52	12.75	24.60	2.45
Boreal Plain Ecozone						
148 - Mid Boreal Lowland	94	D (cm)	18.0	8.6	31.2	4.44
		H (m)	15.01	5.56	22.29	3.40
152-154 - Mid Boreal Upland	21	D (cm)	22.6	18.0	30.0	3.56
		H (m)	18.10	15.50	22.40	1.76
155 - Interlake Plain	15	D (cm)	15.91	4.7	25.1	7.23
		H (m)	13.39	5.50	21.10	5.77
Provincial	298	D (cm)	18.6	2.9	31.2	5.50
		H (m)	15.55	3.30	24.60	4.16

D = dbhob, H = total tree height, SD = standard deviation

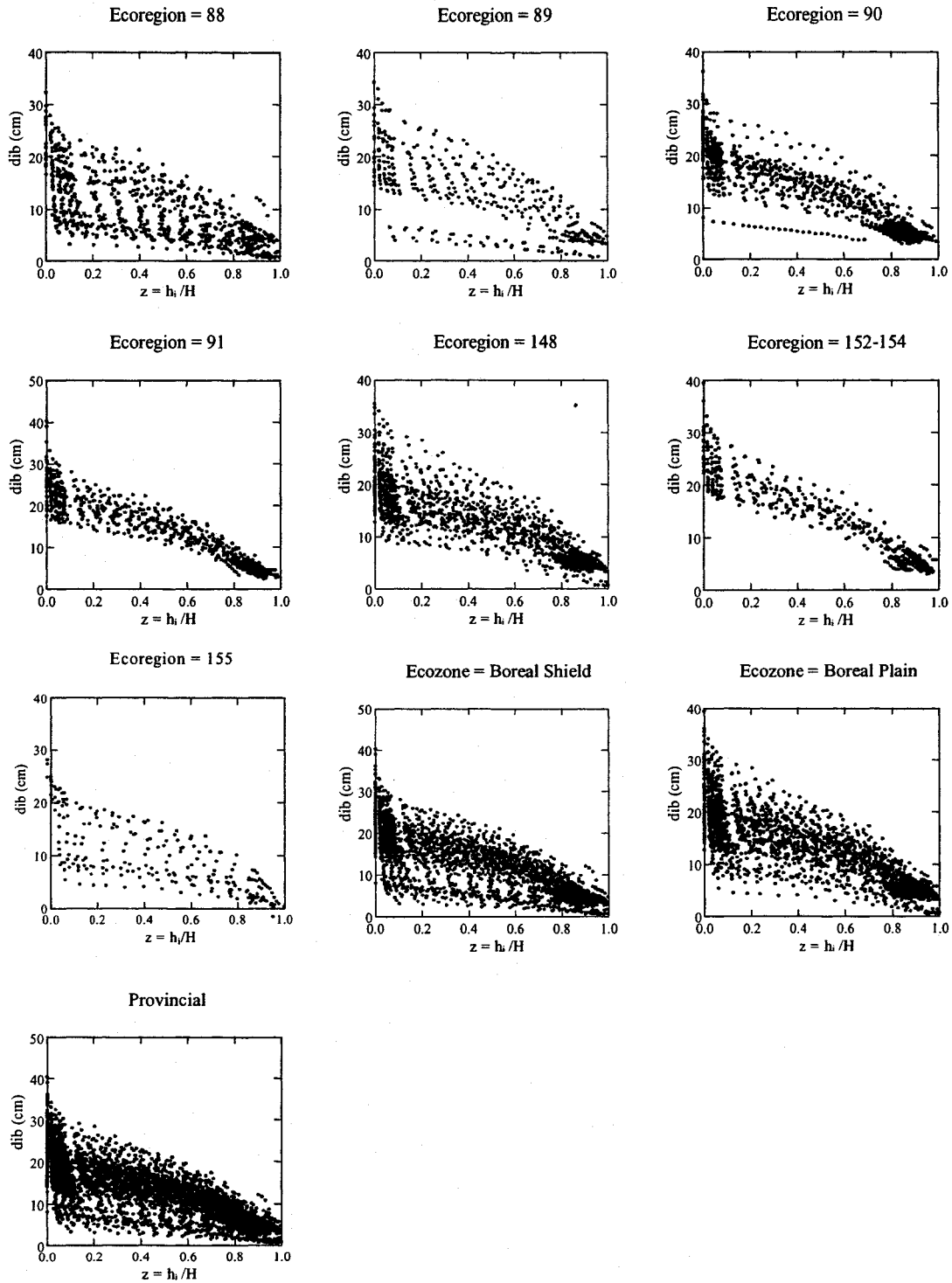


Figure 13. Jack pine dib data plotted by relative height (z) and fitted by ecoregion, ecozone, and provincial.

The F-test results for ecozone differences of the taper equation are shown in Table 16. The F-statistic calculated using equation 7 is 59.38, which is greater than the critical F-value of 3.00. This implies that stem taper differed between the ecozones and ecoregion differences need to be explored to determine if ecozone-specific or ecoregion-specific taper equations are required for jack pine.

Table 16. F-test for ecozone differences of the taper equation for jack pine.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
Jack pine	6748.5	5422	6603.8	5420	59.38*	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The ecoregion F-tests were conducted within ecozones since it was previously determined that ecozone differences existed. The F-test (Eq. 7) results for ecoregion differences of jack pine stem taper are shown in Table 17. All ecoregion combinations provided significant F-statistics, which are F-statistics less than the critical value of 3.00. This implies that jack pine stem taper differed between the ecoregions and ecoregion-specific equations are required.

Table 17. F-tests for ecoregion differences of the taper equation for jack pine.

Ecoregion	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
88-89	1472.2	1274	1455.9	1272	7.12*	3.00	<0.0001
88-90	1871.1	1860	1839.9	1858	15.75*	3.00	<0.0001
88-91	1837.9	1636	1706.3	1634	63.01*	3.00	<0.0001
89-90	1608	1418	1535.6	1416	33.38*	3.00	<0.0001
89-91	1540.1	1194	1358.3	1192	79.77*	3.00	<0.0001
90-91	1828.2	1780	1769.6	1778	29.44*	3.00	<0.0001
148-152-154	2663.9	2118	2354.5	2116	139.03*	3.00	<0.0001
148-155	1770.3	1865	1690.5	1863	43.97*	3.00	<0.0001
152-154-155	1044	713	1031.7	711	4.24*	3.00	<0.0001

a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plots for the jack pine ecoregion-specific and provincial taper equations (Eq. 1) are shown in Table 18 and Figure 14, respectively. Based on the fit statistics and residual plots, the models provided a good fit for all ecoregions with more than 97% of the total variation of dib predictions explained by the models. The residual plots show the residuals all tightly clustered around zero indicating a random distribution of residuals.

Table 18. Fit statistics of the ecoregion-specific and provincial taper equations (Eq. 1) for jack pine.

Parameter	88	89	90	91	148	152-154	155	Provincial
a_0	0.9690	0.8864	1.2446	0.9138	1.1236	1.6336	1.0950	0.9674
a_1	0.9568	1.0130	0.7872	0.9820	0.9006	0.7698	0.8971	0.9746
a_2	1.0012	0.9964	1.0122	0.9985	1.0018	1.0025	1.0018	0.9976
b_1	-0.0805	0.0695	0.1648	0.2919	0.1738	0.5719	0.5162	0.1459
b_2	-0.0659	-0.0178	-0.0351	-0.0414	-0.0251	-0.0924	-0.0564	-0.0402
b_3	0.4314	-0.1669	-0.2493	0.1462	-0.1237	0.9974	0.3560	0.0487
b_4	0.1196	0.2151	0.2737	0.0168	0.1434	-0.5121	-0.2258	0.1266
b_5	-0.0685	0.0784	0.0284	0.1665	0.1157	0.3402	0.2933	0.0753
MSE	1.019903	1.348637	0.923144	1.026653	0.904634	1.697812	0.918455	1.244656
R^2	0.973937	0.975319	0.979705	0.982809	0.977363	0.971280	0.981417	0.974126
n	862	420	1006	782	1639	487	234	5430

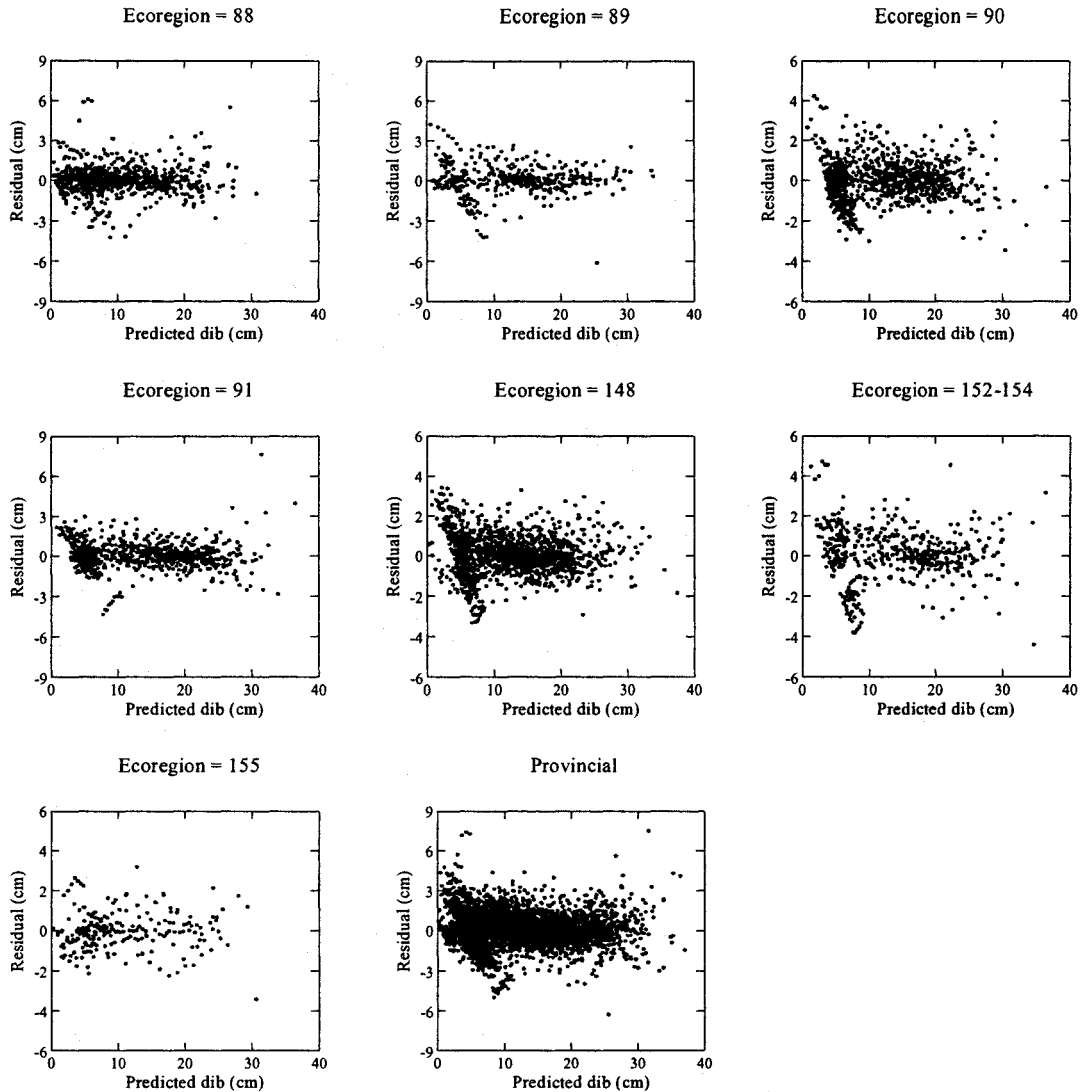


Figure 14. Residual plots of the ecoregion-specific and provincial taper equations (Eq. 1) for jack pine.

5.5.2. Ecosite-specific models

The ecoregion 90 jack pine data was used for site type testing since the ecozone and ecoregion analyses determined that ecoregion-specific equations are required for jack pine. The ecoregion 90 data contained the most trees with recorded vegetation and soil types. The site type data had fewer trees than the ecoregion data used for testing ecoregion differences because vegetation and soil types were not recorded for some

trees. A summary of the jack pine site type data is presented in Table 19. A total of 20 rock/shallow soil trees and 27 mineral soil trees were sampled within the Lac Seul Upland ecoregion. The smallest, largest, and mean dbhob sampled within the rock site type were 6.9 cm, 29.3 cm, and 17.3 cm, respectively. The smallest, largest, and mean total tree heights sampled within the rock site type were 7.70 m, 18.15 m, and 14.93 m, respectively. The smallest, largest, and mean dbhob sampled within the mineral site type were 15.1 cm, 28.8 cm, and 20.7 cm, respectively. The smallest, largest, and mean total tree heights sampled within the mineral site type were 15.05 m, 22.00 m, and 18.71 m, respectively. The jack pine site type dib data appear to be similarly distributed between the two site types (Figure 15). Both site types contain a few trees outside the main cluster of data. These trees were not excluded since the differences of these trees from the main cluster of data were marginal and insignificant. As well, the quantity of data was lacking and it was believed that the deletion of trees may jeopardize the analyses.

Table 19. Summary statistics of the jack pine site type data (Ecoregion 90).

Ecoregion # and name	Site type	No. of trees	Variable	Mean	Min	Max	SD
Boreal Shield Ecozone							
90 - Lac Seul Upland	Rock	20	D (cm)	17.3	6.9	29.3	3.94
			H (m)	14.93	7.70	18.15	2.66
90 - Lac Seul Upland	Mineral	27	D (cm)	20.7	15.1	28.8	3.03
			H (m)	18.71	15.05	22.00	1.63

D = dbhob, H = total tree height, SD = standard deviation

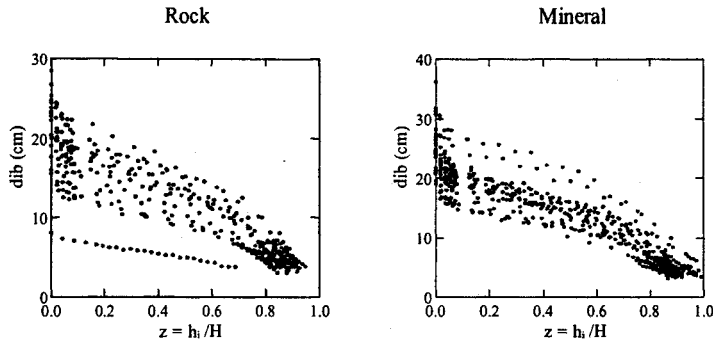


Figure 15. Jack pine site type dib data within ecoregion 90 plotted by relative height (z).

The F-test results for site type differences of jack pine stem taper are shown in Table 20. The F-statistic calculated using equation 7 is 11.22, which is greater than the critical F-value of 3.00. This implies that stem taper differed between the site types and site type-specific taper equations are required for jack pine.

Table 20. F-test for site type differences of the taper equation for jack pine.

Species	Reduced model		Full model		F-value ^a	F(critical)	P-value
	SSE	df	SSE	df			
Jack pine	921.3	998	901	996	11.22*	3.00	<0.0001

^a Asterisk (*) indicates a significant F-value at $\alpha=0.05$. SSE = error sum of squares, df = error degrees of freedom.

The fit statistics and residual plots of the site type-specific taper equations (Eq. 1) for jack pine are provided in Table 21 and Figure 16, respectively. The taper equation provided a good fit for both the site types with 98.5% and 97.8% of the total variation of dib explained for the rock and mineral site types, respectively. The residuals are all tightly clustered around zero indicating a good fit and a random distribution.

Table 21. Fit statistics of the site type-specific taper equations (Eq. 1) for jack pine.

Parameter	Rock	Mineral
a_0	1.1158	2.0531
a_1	0.8511	0.5446
a_2	1.0083	1.0235
b_1	0.3392	-0.0967
b_2	-0.0265	-0.0323
b_3	-0.2465	-0.4068
b_4	0.1713	0.487
b_5	0.1479	-0.1332
MSE	0.5418752	1.1033837
R^2	0.9854957	0.9777024
n	410	596

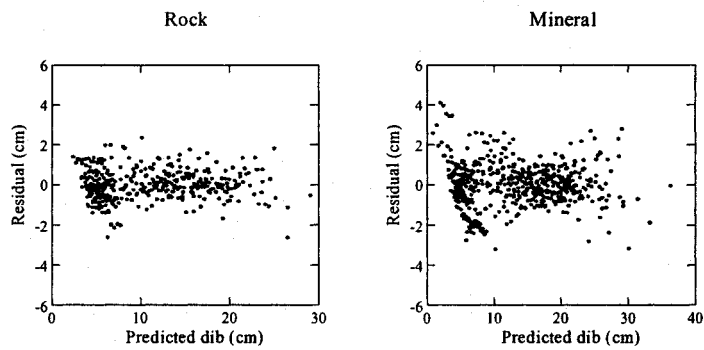


Figure 16. Residual plots of the site type-specific taper equations (Eq. 1) for jack pine.

6. DISCUSSION

Independent testing is the best procedure to validate regression equations. Therefore, taper equations developed in the study should be validated using an independent data set to observe how the equations will respond to other data. To do so, the provincial black spruce data were separated into two data sets containing 70 percent and 30 percent of the trees. The data set with 70 percent of the trees was used to fit the equation and produced a MSE of 0.6679. This fitted equation was applied to the data set containing 30 percent of the trees and produced a MSE of 0.6673, which is slightly smaller than the MSE produced by the data that was used to fit the model. It is believed that the developed equations for all the tree species would produce similar results and should be considered valid when using other data sets. The taper equations for other species were not validated in the same way as was done for black spruce because of limited sample sizes.

6.1. Balsam poplar

The balsam poplar taper equation provided a good fit explaining 98.3% of the total variation of dib. However, the residuals were not evenly distributed and exhibited a larger scatter for larger dib predictions. The predictions which were exhibiting the larger scatter were exclusively associated with diameter observations at the ground (0 cm. height) level. When these observations were excluded, the residuals were evenly distributed. In addition to irregular in shape, diameter at the ground level could be subjected to measurement error because these samples may have been cut into the soil. It should also be noted that observations at 0 cm. height are not typically used in fitting taper equations. Accurate estimations of dib and volume below 30 cm. height are not

important in practice because the stump height during harvesting is not typically less than 30 cm. The equation provided a slightly better fit than the Alberta provincial equation (Huang 1994), which explained 97.5% of the total variation. A graphical comparison of the dib predictions from the Alberta provincial equation and the Manitoba ecoregion 152-154 equation is shown in Figure 17. The equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows that predictions from these equations are quite similar. The Manitoba equation produced slightly larger dib predictions for all tree sizes. However, the difference is marginal. A graphical comparison of the percent differences between Manitoba and Alberta balsam poplar dib predictions is shown in Figure 18. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the Manitoba and Alberta equations. The Manitoba equation was standardized and the differences were calculated as a percent of the Manitoba equation for each tree size. The differences of predicted dib for small, medium, and large tree sizes range between -12.37% and 2.46%, -11.06% and 0.26%, and -7.25% and 1.44%, respectively. This indicates that the difference of balsam poplar stem taper between provinces is marginal and the Manitoba coefficients can be considered reasonable.

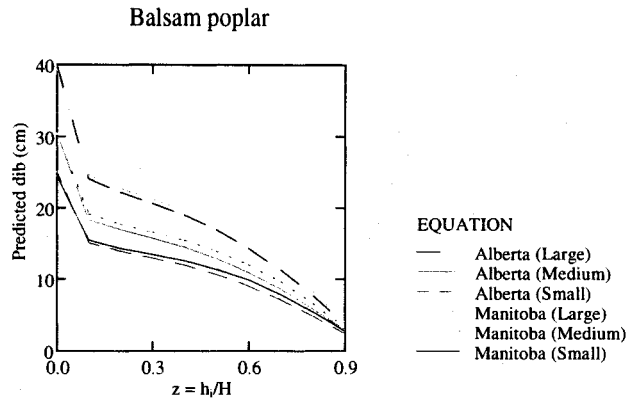


Figure 17. Comparison of Manitoba ecoregion 152-154 and Alberta provincial balsam poplar taper equations. Small: $D = 18$ cm, $H = 18$ m, Medium: $D = 22$ cm, $H = 20$ m, Large: $D = 29$ cm, $H = 24$ m.

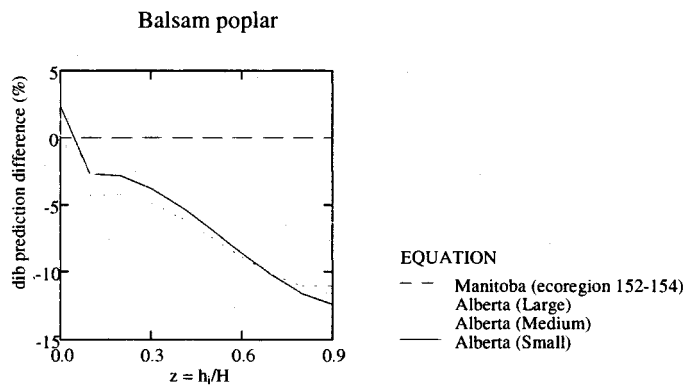


Figure 18. Comparison of differences between Manitoba ecoregion 152-154 and Alberta provincial balsam poplar dbh predictions. The differences are in relation to the Manitoba equations. Small: $D = 18$ cm, $H = 18$ m, Medium: $D = 22$ cm, $H = 20$ m, Large: $D = 29$ cm, $H = 24$ m.

Although, the Manitoba equation provided a good fit, the equation is only useful within ecoregion 152-154. The sample size was also small, based only on 37 trees, which is less than the 60 trees recommended by Kozak (1988). Additional balsam poplar data should be accumulated to explore ecozone and ecoregion differences. These additional data would also expand the applications of the taper equation. Huang (1994)

discovered differences of balsam poplar stem taper. The differences appear to be elevation related along the mountains in Alberta. Huang (1994) discovered two significant groups of natural subregions in Alberta. One group is located close to the mountains while the other is located east of the mountains in the central portion of the province. Although Manitoba does not contain drastic elevation differences compared to Alberta, ecozone-specific and ecoregion-specific differences should still be explored.

6.2. Trembling aspen

The provincial trembling aspen taper equation provided a good fit explaining 97.7% of the total variation of dib. Similar to balsam poplar, the residuals were not evenly distributed and exhibited a larger scatter for larger dib predictions. This large scatter was again caused by inaccurate predictions of dib at the ground level (0 cm. height). The equation provided a slightly poorer fit than the Alberta provincial equation (Huang 1994), which explained 98.0% of the total variation. A graphical comparison of the dib predictions from the Alberta and Manitoba provincial equations is shown in Figure 19. The equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows that predictions from these equations are quite similar. The Manitoba equation produced larger dib predictions for all tree sizes. However, the difference is marginal. A graphical comparison of the percent differences between Manitoba and Alberta trembling aspen dib predictions is shown in Figure 20. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the Manitoba and Alberta equations. The Manitoba equation was

standardized and the differences were calculated as a percent of the Manitoba equation for each tree size. The comparison shows a similar pattern for all tree sizes. The Manitoba equation produced significantly larger dib predictions within the lower portion of the stem at buttswell and continuous larger predictions throughout the remainder of the stem. The differences of predicted dib for small, medium, and large tree sizes range between -6.39% and 0.42%, -7.83% and -1.10%, and -7.67% and -0.83%, respectively. This indicates that the difference of trembling aspen stem taper between provinces is marginal and the Manitoba coefficients can be considered reasonable.

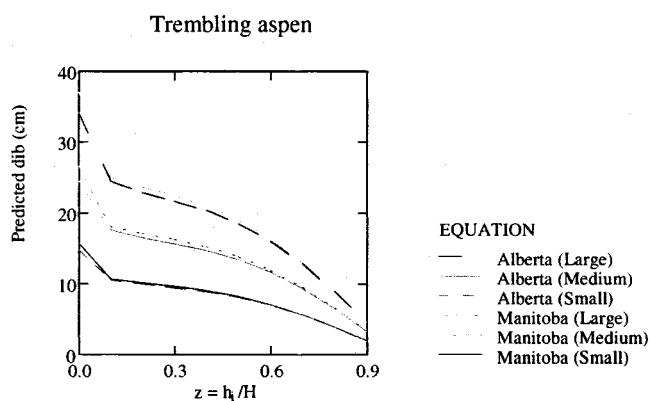


Figure 19. Comparison of Manitoba and Alberta provincial trembling aspen taper equations. Small: $D = 12$ cm, $H = 13$ m, Medium: $D = 20$ cm, $H = 21$ m, Large: $D = 28$ cm, $H = 26$ m.

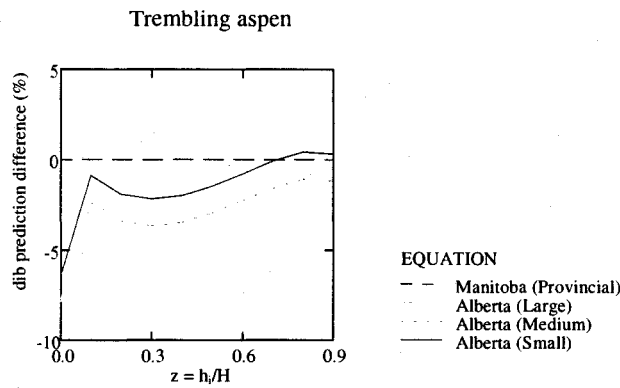


Figure 20. Comparison of differences between Manitoba and Alberta provincial trembling aspen dbh predictions. The differences are in relation to the Manitoba equations. Small: $D = 12$ cm, $H = 13$ m, Medium: $D = 20$ cm, $H = 21$ m, Large: $D = 28$ cm, $H = 26$ m.

The F-test concluded that ecozone-specific taper equations are required for trembling aspen. A graphical comparison of the Manitoba ecozone-specific and provincial trembling aspen taper equations is shown in Figure 21. The comparison shows the predictions obtained from these equations are quite similar for all three tree sizes. The difference is most evident for the small and large tree sizes. A graphical comparison of the percent differences between the ecozone-specific and provincial trembling aspen dbh predictions is shown in Figure 22. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the provincial and ecozone-specific equations. The provincial equation was standardized and the differences were calculated as a percent of the provincial equation for each tree size. The differences of predicted dbh for small, medium, and large tree sizes within the Boreal Shield ecozone range between -15.20% and 5.96%, -15.28% and 3.63%, and -11.38% and 1.74%, respectively. The differences of predicted dbh for small, medium, and large tree sizes within the Boreal Plain ecozone

range between -2.65% and 0.73%, 0.35% and 3.62%, and 0.51% and 2.89%, respectively. The Boreal Shield dib predictions appear to be overall less than the provincial predictions. This was expected since soils within the Boreal Shield ecozone are shallower and less productive, which result in slower growth. The Boreal Plain dib predictions appear to be consistently similar to the dib predictions obtained from the provincial equation. This is most likely due to the fact that the majority of the data used to derive the provincial equation were located within the Boreal Plain ecozone.

The ecozone-specific equations both provided a good fit explaining more than 97% of the variation of dib, which are similar to the Alberta equations which explain between 97.9% and 98.48% (Huang 1994). Huang (1994) determined that ecological differences influence trembling aspen stem taper in Alberta. Huang (1994) discovered four significant groups of natural subregions. The groups appear to be located in varying proximity to the mountains. Therefore, it was suspected that ecozone differences would exist in Manitoba. The differences within Alberta may be more elevation related, while the differences in Manitoba may be more climate and soils influenced. Additional trembling aspen data should be accumulated to explore ecoregion differences and to support the ecozone-specific equations with a broader range of dbh and total tree height observations. Since one of the two ecozones tested in the study had a small sample size (10 trees), re-testing ecozone differences is recommended when more data become available in the future.

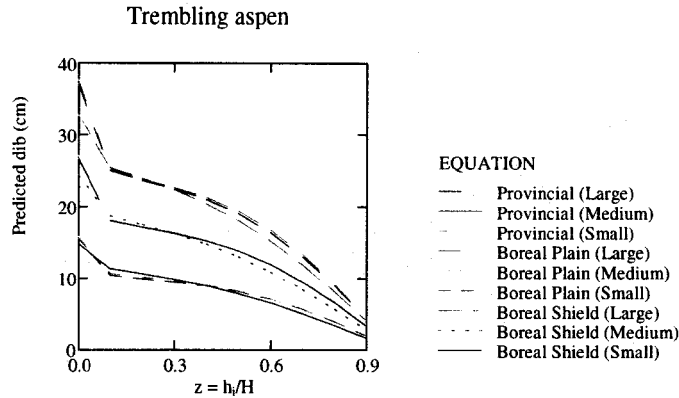


Figure 21. Comparison of Manitoba ecozone-specific and provincial trembling aspen taper equations. Small: $D = 12$ cm, $H = 13$ m, Medium: $D = 20$ cm, $H = 21$ m, Large: $D = 28$ cm, $H = 26$ m.

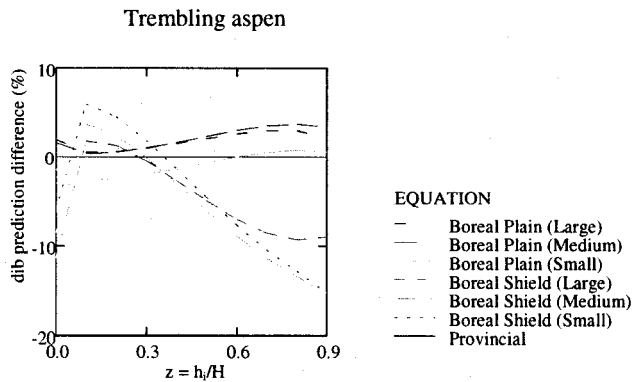


Figure 22. Comparison of differences between ecozone-specific and provincial trembling aspen dib predictions. The differences are in relation to the provincial equations. Small: $D = 12$ cm, $H = 13$ m, Medium: $D = 20$ cm, $H = 21$ m, Large: $D = 28$ cm, $H = 26$ m.

6.3. White spruce

The provincial white spruce taper equation provided a good fit explaining 98.3% of the total variation of dib. The Alberta provincial equation (Huang 1994) also explained 98.3% of the total variation. A graphical comparison of the dib predictions from the Alberta and Manitoba provincial white spruce equations is shown in Figure 23. Predictions from the two equations were compared using three tree sizes. The small,

medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows that the predictions from the two equations are quite similar. The Manitoba equation produced larger dib predictions for the medium and large tree sizes, however, the difference is marginal. A graphical comparison of the percent differences between the Manitoba and Alberta white spruce dib predictions is shown in Figure 24. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the Manitoba and Alberta equations. The Manitoba equation was standardized and the differences were calculated as a percent of the Manitoba equation for each tree size. The differences of predicted dib for small, medium, and large tree sizes range between -8.25% and 21.05%, -8.38% and 20.81%, and -6.56% and 22.50%, respectively. The Manitoba equations produced significantly smaller dib predictions around buttswell, however, the dib predictions throughout the remainder of the stem were quite similar to those obtained from the Alberta equation. This indicates that the difference of white spruce stem taper between provinces is marginal and the Manitoba coefficients can be considered reasonable.

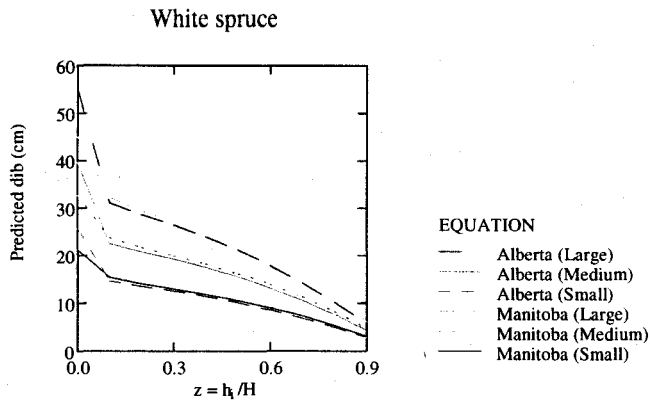


Figure 23. Comparison of Manitoba and Alberta provincial white spruce taper equations. Small: $D = 16$ cm, $H = 13$ m, Medium: $D = 25$ cm, $H = 20$ m, Large: $D = 35$ cm, $H = 25$ m.

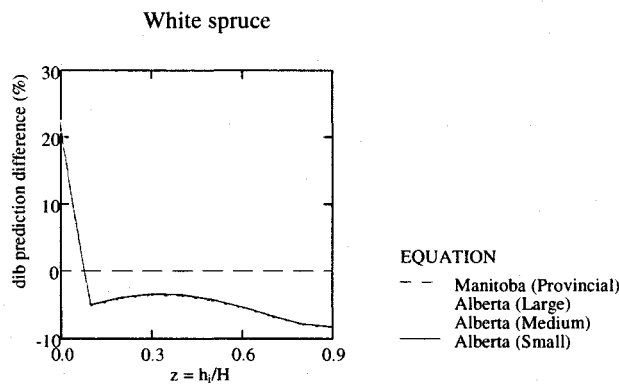


Figure 24. Comparison of differences between Manitoba and Alberta provincial white spruce dbh predictions. The differences are in relation to the Manitoba equations. Small: $D = 16$ cm, $H = 13$ m, Medium: $D = 25$ cm, $H = 20$ m, Large: $D = 35$ cm, $H = 25$ m.

A graphical comparison of the Manitoba ecozone-specific and provincial white spruce taper equations is shown in Figure 25. Despite the fact that the F-test indicated that ecozone differences exist for white spruce, the comparison shows an extremely small difference of the equations for all three tree sizes (Figure 25).

A graphical comparison of the percent differences between the ecozone-specific and provincial white spruce dib predictions is shown in Figure 26. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the provincial and ecozone-specific equations. The provincial equation was standardized and the differences were calculated as a percent of the provincial equation for each tree size. The Boreal Shield predictions are consistently less than the provincial predictions. However, the Boreal Plain predictions are consistently greater than those of the provincial equation. The differences of predicted dib for small, medium, and large tree sizes within the Boreal Shield ecozone range between -6.53% and -0.05%, -6.19% and 0.31%, and -6.17% and 0.33%, respectively. The differences of predicted dib for small, medium, and large tree sizes within the Boreal Plain ecozone range between 0.59% and 3.15%, 0.02% and 2.58%, and -0.15% and 2.46%, respectively. This further indicates that the differences of the white spruce ecozone-specific taper equations and the provincial equation are small and marginal.

The ecozone-specific equations both provide a good fit explaining more than 98% of the variation of dib, which are similar to the Alberta equations which explain between 97.7% and 98.5% (Huang 1994). Huang (1994) determined that ecological differences influence white spruce stem taper in Alberta. Huang (1994) discovered three significant groups of natural subregions. The groups appear to be located in varying proximity to the mountains. Therefore, it was suspected that ecozone differences would exist in Manitoba. The differences within Alberta may be more elevation related, while the differences in Manitoba may be more climate and soils influenced. Since white spruce growth is dependant upon soil fertility and a dependable supply of well-aerated water, it may be speculated that the Boreal Shield ecozone soils composed of bedrock

outcrops and rocky soils provide a less suitable growing condition than the deep mineral clay-loam lacustrine soils of the Boreal Plain ecozone.

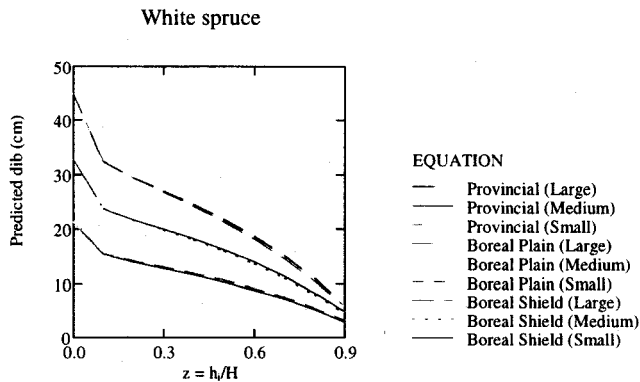


Figure 25. Comparison of Manitoba ecozone-specific and provincial white spruce taper equations. Small: $D = 16$ cm, $H = 13$ m, Medium: $D = 25$ cm, $H = 20$ m, Large: $D = 35$ cm, $H = 25$ m.

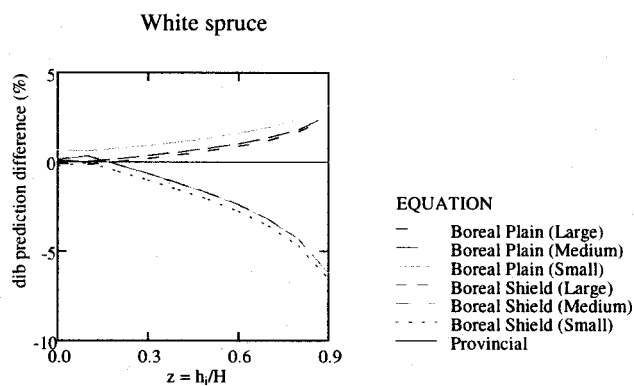


Figure 26. Comparison of differences between ecozone-specific and provincial white spruce dbh predictions. The differences are in relation to the provincial equations. Small: $D = 16$ cm, $H = 13$ m, Medium: $D = 25$ cm, $H = 20$ m, Large: $D = 35$ cm, $H = 25$ m.

Additional white spruce data should be accumulated to explore ecoregion differences and to support the ecozone-specific equations with a broader range of dbh

and total tree height observations. One of the ecozones tested in this study had a sample size of 33 trees, which is less than the minimum desired sample size of 60 trees. Re-testing is recommended when more data become available in the future.

6.4. Black spruce

6.4.1. The provincial model

The provincial black spruce taper equation provided a good fit explaining 98.1% of the total variation of dib. The Alberta provincial equation (Huang 1994) also explained 98.1% of the total variation. A graphical comparison of the dib predictions from the Alberta and Manitoba provincial black spruce equations is shown in Figure 27. The equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the coefficients were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows that the predictions from the two equations are quite similar. The Manitoba equation produced smaller dib predictions for the lower portion of the stem for all three sizes. A graphical comparison of the percent differences between the Manitoba and Alberta provincial black spruce dib predictions is shown in Figure 28. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the Manitoba and Alberta equations. The Manitoba equation was standardized and the differences were calculated as a percent of the Manitoba equation for each tree size. The differences of predicted dib for small, medium, and large tree sizes range between -8.72% and 35.29%, -8.05% and 36.26%, and -6.95% and 37.74%, respectively. The predictions near the lower portion of the stem were consistently greater for the Alberta equation. This indicates that black spruce trees located in Alberta possess a larger butt

swell than black spruce trees in Manitoba. The overall difference of black spruce stem taper between provinces is marginal, suggesting the Manitoba equations are reasonable.

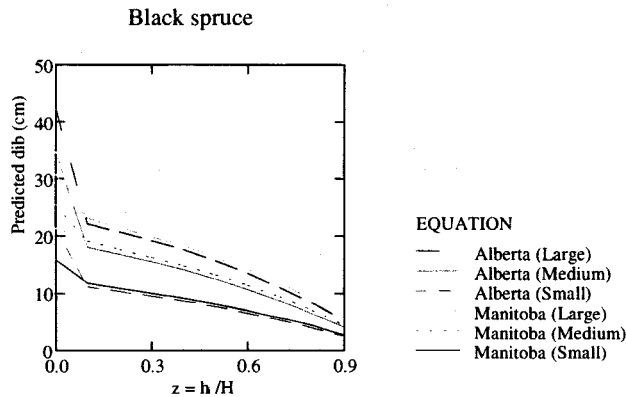


Figure 27. Comparison of Manitoba and Alberta provincial black spruce taper equations. Small: $D = 12$ cm, $H = 10$ m, Medium: $D = 20$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 24$ m.

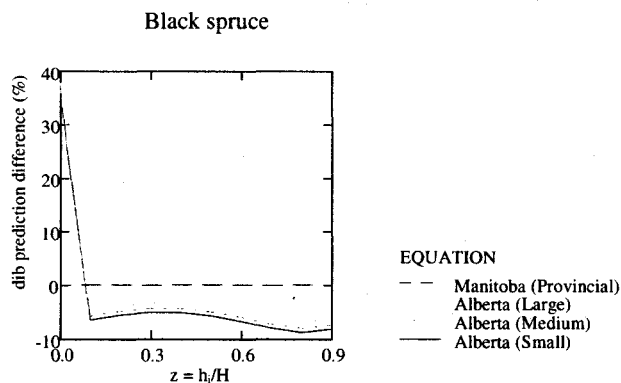


Figure 28. Comparison of differences between Manitoba and Alberta provincial black spruce dbh predictions. The differences are in relation to the Manitoba equations. Small: $D = 12$ cm, $H = 10$ m, Medium: $D = 20$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 24$ m.

Despite differences of black spruce stem taper between natural subregions were observed within Alberta, the F-test indicated that ecozone differences do not exist for black spruce in Manitoba. The differences experienced in Alberta may be more

elevation related. Huang (1994) determined that ecological differences influence black spruce stem taper in Alberta. Huang (1994) discovered two significant groups of natural subregions in Alberta. One group is located close to the mountains while the other is located east of the mountains in the central portion of the province. The lack of differences in Manitoba may be attributed to less elevation and climate differences.

6.4.2. Ecosite-specific models

The F-test indicated that site type differences of stem taper exist for black spruce. A graphical comparison of the site type-specific and provincial taper equations is shown in Figure 29. These equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows marginal differences for the small and medium tree sizes and greater differences for the large tree size. A graphical comparison of the percent differences between the site type-specific and provincial black spruce dib predictions is shown in Figure 30. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the provincial and site type-specific equations. The provincial equation was standardized and the differences were calculated as a percent of the provincial equation for each tree size. The lowland site type predictions are for the most part less than the provincial predictions. However, the upland site type predictions are consistently greater than those of the provincial equation. The differences of predicted dib for small, medium, and large tree sizes within the lowland site type range between -6.94% and 1.23%, -1.44% and 3.51%, and -5.51% and -0.81%, respectively. The differences of predicted dib for small, medium, and large tree sizes within the

upland site type range between 0.60% and 1.78%, -0.38% and 1.79%, and 0.18% and 1.80%, respectively.

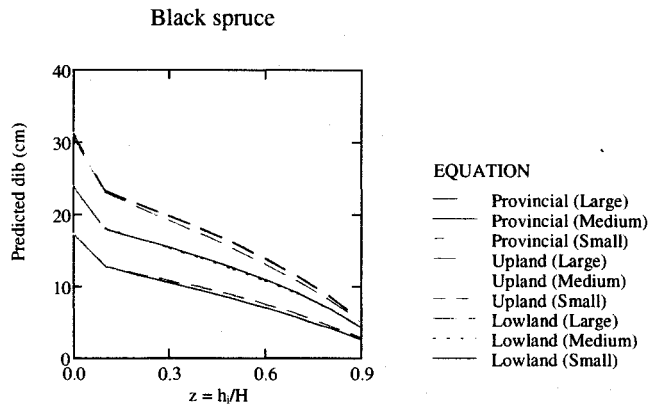


Figure 29. Comparison of Manitoba site type-specific and provincial black spruce taper equations. Small: $D = 13$ cm, $H = 10$ m, Medium: $D = 19$ cm, $H = 18$ m, Large: $D = 25$ cm, $H = 21$ m.

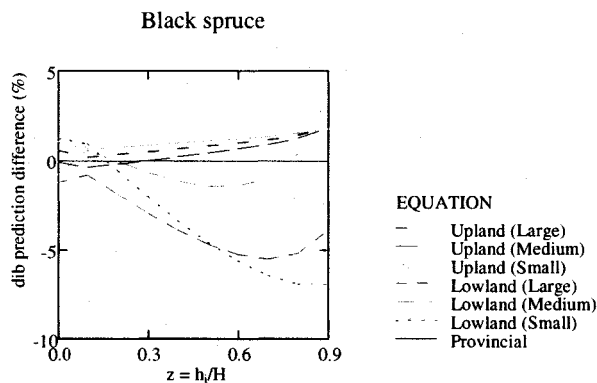


Figure 30. Comparison of differences between site type-specific and provincial black spruce dib predictions. The differences are in relation to the provincial equations. Small: $D = 13$ cm, $H = 10$ m, Medium: $D = 19$ cm, $H = 18$ m, Large: $D = 25$ cm, $H = 21$ m.

The equations provide a good fit explaining 98.6% and 97.9% of the total variation of dib for lowland and upland site types, respectively. Differences of black

spruce stem taper were expected since soil moisture significantly affects black spruce growth. The upland site type trees possess less taper than lowland site type trees. This is expected since black spruce growth is more productive on upland sites with good drainage. Upland sites contain more available nutrients and moderate moisture levels, which favor crown development and in turn, favor tree growth in height and diameter (Muhairwe 1994). On the other hand, lowland sites contain less available nutrients and high moisture levels, which inhibit crown development and in turn, inhibit tree growth in height and diameter (Muhairwe 1994).

6.5. Jack pine

6.5.1. Ecoregion-specific models

The provincial jack pine taper equation provided a good fit explaining 97.4% of the total variation of dib. The equation provided a slightly poorer fit than the Alberta provincial equation (Huang 1994), which explained 98.3% of the total variation. A graphical comparison of the Alberta and Manitoba provincial equations is shown in Figure 31. The equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows that the predictions from the two equations are quite similar. The Manitoba equation produced larger dib predictions for all three tree sizes, however, the difference is marginal. A graphical comparison of the percent differences between the Manitoba and Alberta jack pine dib predictions is shown in Figure 32. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the Manitoba and Alberta equations. The Manitoba equation was standardized

and the differences were calculated as a percent of the Manitoba equation for each tree size. The differences of predicted dbh for small, medium, and large tree sizes range between -7.00% and -1.75%, -6.75% and -1.59%, and -6.36% and -1.32%, respectively. The Manitoba jack pine trees probably occur on more nutrient rich sites and more favorable soil conditions, which produces trees with lower taper. However according to the percent differences, the differences of jack pine stem taper between provinces is marginal indicating the Manitoba coefficients can be considered reasonable.

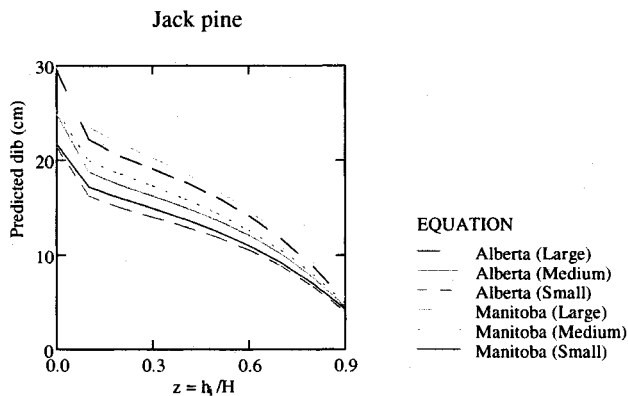


Figure 31. Comparison of Manitoba and Alberta provincial jack pine taper equations. Small: $D = 18$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

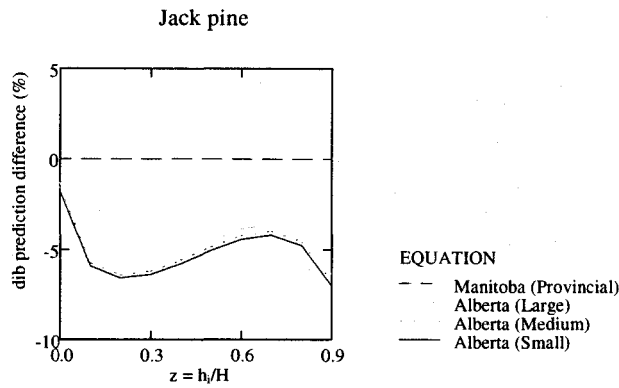


Figure 32. Comparison of differences between Manitoba and Alberta provincial jack pine dib predictions. The differences are in relation to the Manitoba equations. Small: $D = 18$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

The F-test indicated that ecoregion-specific taper equations are required for jack pine. A graphical comparison of the Manitoba ecoregion-specific and provincial jack pine taper equations are shown in Figure 33. The comparison shows differences of the equations for all three tree sizes. A graphical comparison of the percent differences between the ecoregion-specific and provincial jack pine dib predictions is shown in Figure 34. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the provincial and ecoregion-specific equations. The provincial equation was standardized and the differences were calculated as a percent of the provincial equation for each tree size. The differences range between -15.15% and 14.84%. The predictions are comparable for the first portion of the stem but tend to differ more in the latter portion of the stem. Ecoregions 88, 89, and 148 exhibited greater dib predictions than the provincial equation. Ecoregions 90 and 91 produced lower dib predictions in the latter portion of the stem than the provincial equation. Ecoregions 152-154 and 155 produced lower

predictions for medium and large trees and larger predictions for the small tree size. The ecozone-specific equations provided a good fit explaining more than 97% of the variation of dib, which are similar to the Alberta equation which explained 98.3%. Differences may be attributed to differences of climate and soil conditions between the ecoregions. Ecoregions 89 and 148 exhibited jack pine trees with less stem taper indicating faster growth in these two ecoregions. This is probably because these ecoregions possess more loamy soils, which provide more available nutrients and moisture. These favorable sites increase tree growth in height and diameter by increasing crown development (Muhairwe 1994). In contradiction, ecoregions 90 and 91 produced jack pine trees with higher stem taper indicating slower growth in these two ecoregions. This is probably because these ecoregions possess more sandy soils, which provide less available nutrients and moisture. These unfavorable sites inhibit tree growth in height and diameter by inhibiting crown development (Muhairwe 1994).

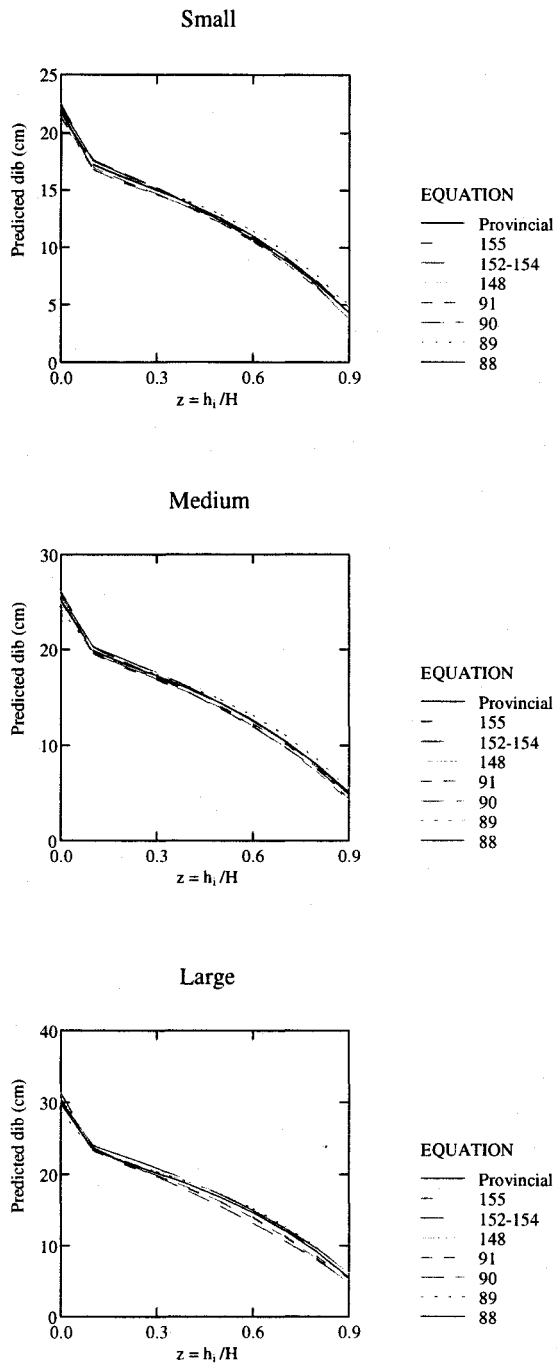


Figure 33. Comparison of Manitoba ecoregion-specific and provincial jack pine taper equations. Small: $D = 18$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

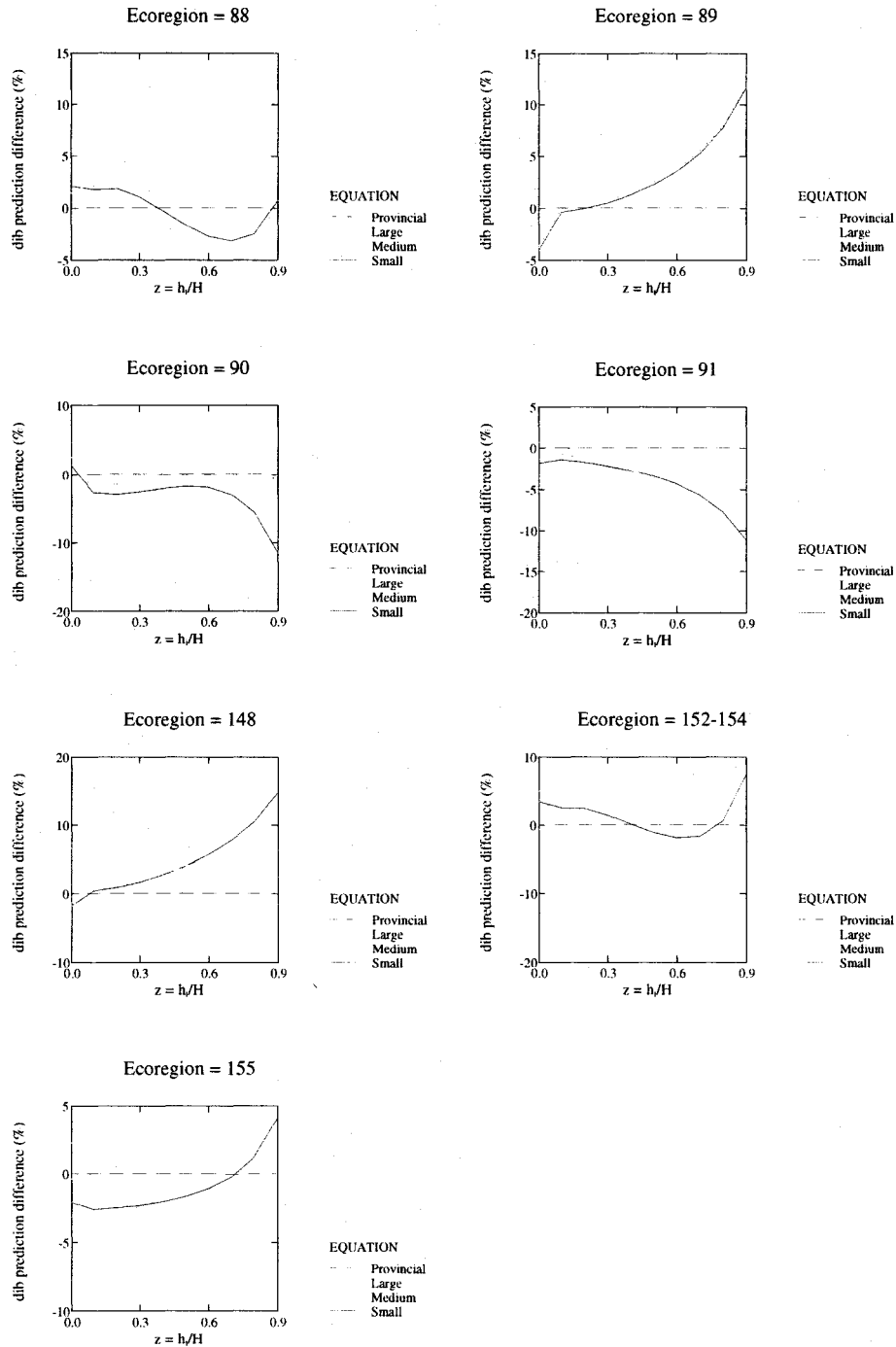


Figure 34. Comparison of differences between ecoregion-specific and provincial jack pine dib predictions. The differences are in relation to the provincial equations. Small: $D = 18$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

6.5.2. Ecosite-specific models

The F-test indicated a difference in stem taper between rock and mineral soil site types for jack pine. The equations provided a good fit explaining 98.5% and 97.8% of the total variation of dbh for rock and mineral site types, respectively. A graphical comparison of the site type-specific and provincial taper equations is shown in Figure 35. The equations were compared using three tree sizes. The small, medium, and large tree sizes used to compare the equations were attributed dbh and total tree height measurements within the bounds of the data. The comparison shows marginal differences for the small and medium tree sizes and greater differences for the large tree size. A graphical comparison of the percent difference between the site type-specific and provincial jack pine dbh predictions is shown in Figure 36. The percent differences were calculated using equation 10. They were estimated as an attempt to quantify the differences between the provincial and site type-specific equations. The provincial equation was standardized and the differences were calculated as a percent of the provincial equation for each tree size. The rock site type predictions are consistently less than the provincial predictions, while the mineral soil site type predictions are for the most part greater than those of the provincial equation with the exception of the small tree size. The mineral soil site type predictions are consistently larger than the rock site type predictions, with the exception of the small tree size. The differences of predicted dbh for small, medium, and large tree sizes within the rock site type range between -14.80% and -1.32%, -16.42% and 1.85%, and -16.77% and 4.83%, respectively. The differences of predicted dbh for small, medium, and large tree sizes within the mineral soil site type range between -13.06% and 3.86%, -2.55% and 2.05%, and -0.83% and 9.41%, respectively. The mineral soil site type trees experience lower

taper because they provide more favorable growing conditions for jack pine. The mineral soil sites provide more nutrients, moisture and growing space than rock sites. This provides greater crown development, which yields more vigorous tree growth in height and diameter.

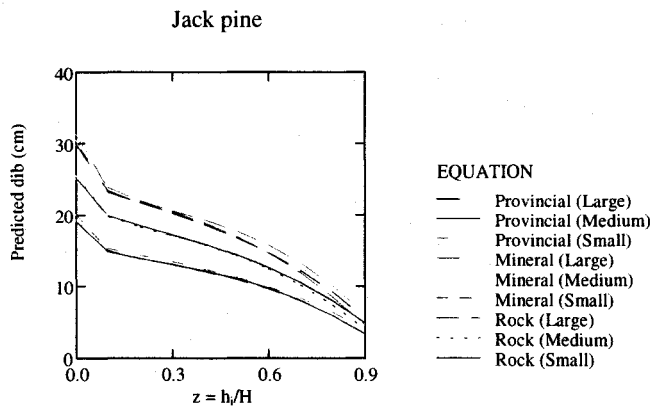


Figure 35. Comparison of Manitoba site type-specific and provincial jack pine taper equations. Small: $D = 16$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

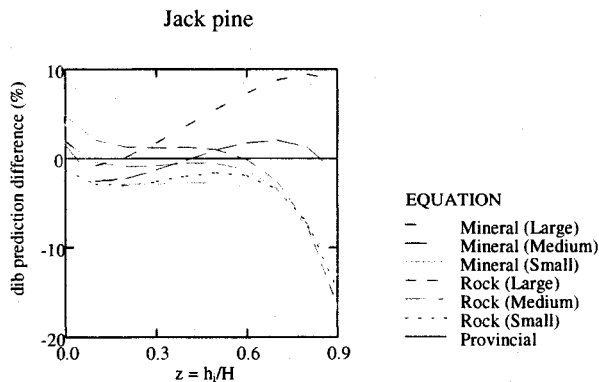


Figure 36. Comparison of differences between site type-specific and provincial jack pine dbh predictions. The differences are in relation to the provincial equations. Small: $D = 16$ cm, $H = 16$ m, Medium: $D = 21$ cm, $H = 17$ m, Large: $D = 25$ cm, $H = 18$ m.

7. CONCLUSIONS

Kozak's variable-exponent taper equation (Eq. 1) provided a good fit for all the species and models. All the models explained more than 97% of the total variation in dbh, indicating that Kozak's variable-exponent equation is an appropriate taper model for the major tree species of Manitoba. The increasing variance for large diameters exhibited in the residual plots for balsam poplar and trembling aspen were caused by inaccurate predictions of dbh at the ground level (0 cm. height).

Taper equations developed in this study generally agreed well with the taper equations developed by Huang (1994) in Alberta. The differences between the provincial equations were small and indicated that the Manitoba equations can be considered reasonable. The differences between the provincial equations may be the result of elevation differences between the provinces. Alberta contains more drastic elevation changes than Manitoba. As well, Alberta's climate and soils are more influenced by mountains, which affects stem taper.

Some of the equations were derived with less than the 60 trees recommended by Kozak. It is suggested that data be continually accumulated and used to update the equations developed in this study and to further determine if ecoregion-specific and site type-specific equations are required. Data should be collected outside the current range of dbh and total tree height measurements to expand the application of the equations. Vegetation and soil types should be recorded for all future black spruce and jack pine stem analysis data. Application of the taper equations and corresponding volume tables developed in this study should not proceed outside the range of dbh and total tree height data used in the study as results may be unreliable.

These findings support the ecological differences of stem taper and the specific equations should be used to achieve accurate dib and resulting volume predictions.

Using ecozone and ecoregion-specific equations may increase the complexity of forest management, however, using the ecologically-based equations will enhance the accuracy of dib and volume predictions.

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9. APPENDICES

9.1. APPENDIX I

Construction of Individual Tree Volume Tables

The developed taper equations (Eq. 1) along with the following submodels were used in the creation of the individual tree volume tables. The submodels include the diameter outside/ inside bark model (Eq. 11), the height-diameter model (Eq. 15), and the stump diameter and diameter at breast height model (Eq. 18). The diameter outside/ inside bark model predicts dob using dib measurements. This model was used to calculate the stump dob. The height-diameter model predicts total tree height using dbhob measurements. The stump diameter and dbhob model was used to predict stump dob using dbhob measurements.

Three tables were constructed for each taper equation including gross total volume (GTV), merchantable length (ML)/ gross merchantable volume (GMV), and trees/m³. They were constructed using a stump height of 0.3 m and a top dib of 7.0 cm. These scaling rules were defined using the industrial standards within Manitoba.

A diameter outside/inside bark model (Eq. 11) was fitted to examine the relationship between dib and dob. Equation 11 can be rearranged to predict dib from dob measurements (Eq. 12). The mean square error (MSE) and coefficient of determination (R^2) for equation 11 were calculated using equation 13 and equation 14, respectively.

$$dob_i = a + bdib_i \quad \text{Eq. 11}$$

$$dib_i = (dob_i - a)/b \quad \text{Eq. 12}$$

where:

dob_i = dob at point i along the stem (cm)

dib_i = dib at point i along the stem (cm)

a, b = parameters to be estimated

$$MSE = \frac{\sum_{i=1}^n (dob_i - \hat{dob}_i)^2}{n - m} \quad \text{Eq. 13}$$

where:

MSE = mean square error

dob_i = observed dib at point i along the stem (cm)

\hat{dob}_i = predicted dib at point i along the stem (cm)

n = number of observations

m = number of parameters (m = 2)

$$R^2 = 1 - \frac{\sum_{i=1}^n (dob_i - \hat{dob}_i)^2}{\sum_{i=1}^n (dob_i - \bar{dob})^2} \quad \text{Eq. 14}$$

where:

R^2 = coefficient of determination

dob_i = observed dib at point i along the stem (cm)

\hat{dob}_i = predicted dib at point i along the stem (cm)

\bar{dob} = observed average dib (cm)

A height-diameter model (Eq. 15) was fitted to predict total tree height as a function of dbhob. The mean square error (MSE) and coefficient of determination (R^2) for equation 15 were calculated using equation 16 and equation 17, respectively.

$$H_i = 1.3 + a(1 - e^{-bD_i})^c \quad \text{Eq. 15}$$

where:

H_i = total tree height for tree i (m)
 D_i = dbhob for tree i (cm)
 e = base of natural logarithm (≈ 2.71828)
 a, b, c = parameters to be estimated

$$\text{MSE} = \frac{\sum_{i=1}^n w_i (H_i - \hat{H}_i)^2}{n - m} \quad \text{Eq. 16}$$

where:

MSE = mean square error
 $w_i = 1/D_i$
 D_i = dbhob for tree i (cm)
 H_i = observed H for tree i (m)
 \hat{H}_i = observed average H (m)
 n = number of observations
 m = number of parameters ($m = 3$)

$$R^2 = 1 - \frac{\sum_{i=1}^n w_i (H_i - \hat{H}_i)^2}{\sum_{i=1}^n w_i (H_i - \bar{H})^2} \quad \text{Eq. 17}$$

where:

R^2 = coefficient of determination
 $w_i = 1/D_i$
 D_i = dbhob for tree i (cm)
 H_i = observed H for tree i (m)
 \hat{H}_i = predicted H for tree i (m)
 \bar{H} = observed average H (m)

A stump diameter and dbhob model (Eq. 18) was fitted to predict stump dob using dbhob measurements. A stump height of 0.3 m was used. The mean square error (MSE) and coefficient of determination (R^2) for equation 18 were calculated using equation 19 and equation 20, respectively.

$$\text{stpdob}_i = a + bD_i + cD_i^2 \quad \text{Eq. 18}$$

where:

stpdob_i = stump dob for tree i (cm)
 D_i = dbhob for tree i (cm)
 a, b, c = parameters to be estimated

$$\text{MSE} = \frac{\sum_{i=1}^n (\text{stpdob}_i - \hat{\text{stpdob}}_i)^2}{n - m} \quad \text{Eq. 19}$$

where:

MSE = mean square error
 stpdob_i = observed stump dob for tree i (cm)
 $\hat{\text{stpdob}}_i$ = predicted stump dob for tree i (cm)
 n = number of observations
 m = number of parameters (m = 3)

$$R^2 = 1 - \frac{\sum_{i=1}^n (\text{stpdob}_i - \hat{\text{stpdob}}_i)^2}{\sum_{i=1}^n (\text{stpdob}_i - \bar{\text{stpdob}})^2} \quad \text{Eq. 20}$$

where:

R² = coefficient of determination
 stpdob_i = observed stump dob for tree i (cm)
 $\hat{\text{stpdob}}_i$ = predicted stump dob for tree i (cm)
 $\bar{\text{stpdob}}$ = observed average stump dob (cm)

Merchantable length (ML) is defined as the portion of the bole that is located between stump height and a specified top diameter. ML was calculated by first specifying a top dib (7.0 cm) and rearranging the original taper equation (Eq. 1) to be expressed as equation 21.

$$z_i = h_i/H = (1 - (\text{dib}_i/K)^{1/c} (1 - \sqrt{p})) \quad \text{Eq. 21}$$

where:

z_i = defined in equation 4

h_i = height above ground to the specified top dib (m)

H = total tree height (m)

dib_i = top dib (=7.0 cm)

$$K = a_0 D^{a_1} a_2^D \quad \text{Eq. 22}$$

D = dbhob (cm)

$$c = b_1 (h_i/H)^2 + b_2 \ln(h_i/H + 0.001) + b_3 \sqrt{h_i/H} + b_4 e^{h_i/H} + b_5 (D/H) \quad \text{Eq. 23}$$

p = defined in equation 3

HI = height of inflection point (m)

$a_0, a_1, a_2, b_1, b_2, b_3, b_4, b_5$ = parameters to be estimated

A mathematical iteration routine was used to calculate MH. A top dib of 7.0 cm was used for industrial standards. An initial value of (h_i/H) for a specified top diameter was termed $(h_{\text{top}}/H)_0$. Huang (1994) suggested using an initial value of 0.9. This was used in equation 23 to calculate c , which was used in equation 21 to calculate $(h_{\text{top}}/H)_1$. The next step was to calculate $(h_{\text{top}}/H)_2$, which was accomplished using equation 24.

$$(h_{\text{top}}/H)_2 = \frac{(h_{\text{top}}/H)_1 + (h_{\text{top}}/H)_0}{2} \quad \text{Eq. 24}$$

where:

h_{top} = height above ground to the specified top dib (m)

H = total tree height (m)

This procedure was repeated until a desired precision was achieved in equation

25. Huang (1994) suggested using a precision level of 0.00000001.

$$\left| (h_{\text{top}}/H)_j - (h_{\text{top}}/H)_{j-1} \right| < 0.00000001 \quad \text{Eq. 25}$$

where:

h_{top} = height above ground to the specified top dib (m)

H = total tree height (m)

Merchantable height (MH) and merchantable length (ML) were calculated using equation 26 and equation 27, respectively. Merchantable height is the height to the specified top dib. Merchantable length is the length of the merchantable portion of the bole, which is the portion between the specified stump height and MH. A stump height of 0.3 m was used for industrial standards.

$$MH = (h_{top}/H)_j \times H \quad \text{Eq. 26}$$

where:

MH = merchantable height to the specified top dib (m)
 h_{top} = height above ground to the specified top dib (m)
 H = total tree height (m)

$$ML = MH - SH \quad \text{Eq. 27}$$

where:

ML = merchantable length between specified SH and MH (m)
 MH = merchantable height to the specified top dib (m)
 SH = stump height (0.3 m)

Gross merchantable volume (V_m) is the volume within the merchantable length inside bark. The first step in calculating V_m was dividing ML into 10 sections. The height above ground for the midpoint and top of each section was calculated using equation 28. Since there are 10 sections, 20 height above ground measurements were calculated.

$$h_i = i \times ML / 20 + SH \quad \text{Eq. 28}$$

where:

h_i = height above ground at point i along the stem (m)
 ML = merchantable length between specified SH and MH (m)
 SH = stump height (0.3 m)

The next step was to calculate the dib estimates for each corresponding height above ground using the original taper equation (Eq. 1). Merchantable volume was then calculated using Newton's formula (Eq. 29). Newton's formula is recommended for volume estimation since it is more accurate than Smalian's formula (Goulding 1979, Husch *et al.* 1982, Biging 1988, Figueiredo-Filho and Schaaf 1999).

$$V_m = \frac{ML/10}{6} (0.00007854)(dib_0^2 + 4dib_1^2 + dib_2^2) + \dots + \frac{ML/10}{6} (0.00007854)(dib_{18}^2 + 4dib_{19}^2 + dib_{20}^2) \quad \text{Eq.29}$$

where:

- V_m = merchantable volume (m^3)
- ML = merchantable length between specified SH and MH (m)
- dib_i = dib at point i along the stem (cm)

Gross total volume (V_{tot}) is the volume of wood contained in the entire stem inside bark and was calculated using equation 30.

$$V_{tot} = V_m + V_{tip} + V_{stp} \quad \text{Eq. 30}$$

where:

- V_{tot} = gross total volume (m^3)
- V_m = merchantable volume (m^3)
- V_{tip} = tip volume (m^3)
- V_{stp} = stump volume (m^3)

Tip volume (V_{tip}) was calculated using the equation for a cone (Eq. 31) and stump volume (V_{stp}) was calculated using the equation for a cylinder (Eq. 32).

$$V_{\text{tip}} = \pi(\text{dib}_{\text{top}} / 200)^2 (H - \text{MH}) / 3 \quad \text{Eq. 31}$$

where:

- V_{tip} = tip volume (m^3)
- dib_{top} = top dib (7.0 cm)
- H = total tree height (m)
- MH = merchantable height to the specified top dib (m)

$$V_{\text{stp}} = \pi(\text{dib}_0 / 200)^2 \text{SH} \quad \text{Eq. 32}$$

where:

- V_{stp} = stump volume (m^3)
- dib_0 = stump dib (cm) predicted from the taper equation (Eq. 1)
- SH = stump height (0.3 m)

The number of trees per cubic meter of merchantable volume was calculated using equation 33.

$$\text{Trees} / \text{m}^3 V_m = 1 / V_m \quad \text{Eq. 33}$$

where:

- $\text{trees} / \text{m}^3 V_m$ = trees per cubic meter of MV
- V_m = merchantable volume (m^3)

9.2. APPENDIX II

Fit Statistics and Residual Plots of Submodels.

9.2.1. Balsam poplar

The fit statistics of the ecoregion 152-154 submodels for balsam poplar are shown in Table 22. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit explaining more than 99% of the total variation of dob and stump dob, respectively. However, the height-diameter model (Eq. 15) provided a poorer fit with only 36.9 % of the total variation of total tree height explained. Although the model provided a poor fit, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Therefore, caution should be exercised when using the predicted total tree heights. The residual plots for the diameter inside/outside bark, height-diameter, and stump diameter-dbhob models are shown in Figure 37, Figure 38, and Figure 39, respectively. All the residual plots show a cluster around zero and no visible pattern, indicating a random distribution of residuals.

Table 22. Fit statistics of the ecoregion 152-154 submodels for balsam poplar.

152-154	
<u>Diameter inside/outside bark model (Eq. 11)</u>	
a	0.65856
b	1.06317
MSE	0.149684
R ²	0.998190
n	875
<u>Height-diameter model (Eq. 15)</u>	
a	28.7428
b	0.0466
c	0.9728
MSE	0.147706
R ²	0.369591
n	37
<u>Stump diameter-dbhob model (Eq. 18)</u>	
a	-7.3723
b	1.7746
c	-0.0131
MSE	0.000408
R ²	0.999970
n	37

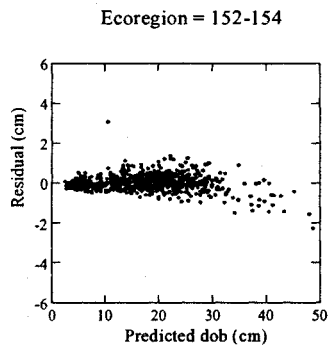


Figure 37. Residual plot of the ecoregion 152-154 diameter inside/outside bark model (Eq. 11) for balsam poplar.

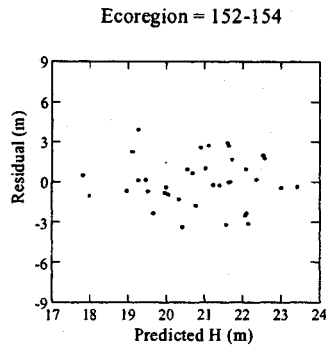


Figure 38. Residual plot of the ecoregion 152-154 height-diameter model (Eq. 15) for balsam poplar.

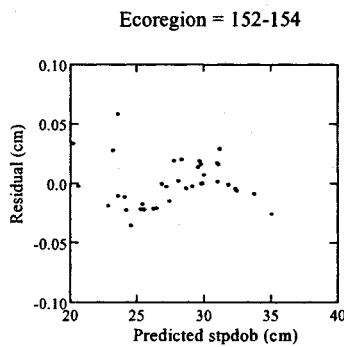


Figure 39. Residual plot of the ecoregion 152-154 stump diameter-dbhob model (Eq. 18) for balsam poplar.

9.2.2. Trembling aspen

The fit statistics of the submodels for trembling aspen are shown in Table 23. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit for all the models explaining more than 99% of the total variation of dob and stump dob, respectively. The height-diameter model (Eq. 15) provided a moderate fit for the ecozone-specific equations with 78.0% and 89.9% of the total variation of total tree height explained for the Boreal Plain and Boreal Shield ecozones, respectively. Although the model provided a moderate fit, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted

for application within the individual tree volume tables. Therefore, caution should be exercised when using the predicted total tree heights, especially for the Boreal Plain ecozone. The residual plots of the diameter inside/outside bark, height-diameter, and stump diameter-dbh models are shown in Figure 40, Figure 41, and Figure 42, respectively. All the residual plots show a cluster around zero and no visible pattern, indicating a random distribution of residuals.

Table 23. Fit statistics of the eczone-specific and provincial submodels for trembling aspen.

	Boreal Shield	Boreal Plain	Provincial
<u>Diameter inside/outside bark model</u>			
a	0.18208	0.43433	0.39958
b	1.06383	1.0519	1.05338
MSE	0.152981	0.122413	0.128244
R ²	0.997597	0.998578	0.998469
n	211	1413	1624
<u>Height-diameter model</u>			
a	42.0171	23.0375	23.9472
b	0.0337	0.1633	0.1283
c	1.1569	5.2256	3.1155
MSE	0.283248	0.172843	0.184107
R ²	0.899722	0.780331	0.824006
n	10	61	71
<u>Stump diameter-dbh model</u>			
a	-0.9291	-3.8616	-1.9176
b	1.2522	1.4469	1.2781
c	-0.00493	-0.00637	-0.0034
MSE	0.225146	0.002199	0.007263
R ²	0.996589	0.999948	0.999838
n	10	61	71

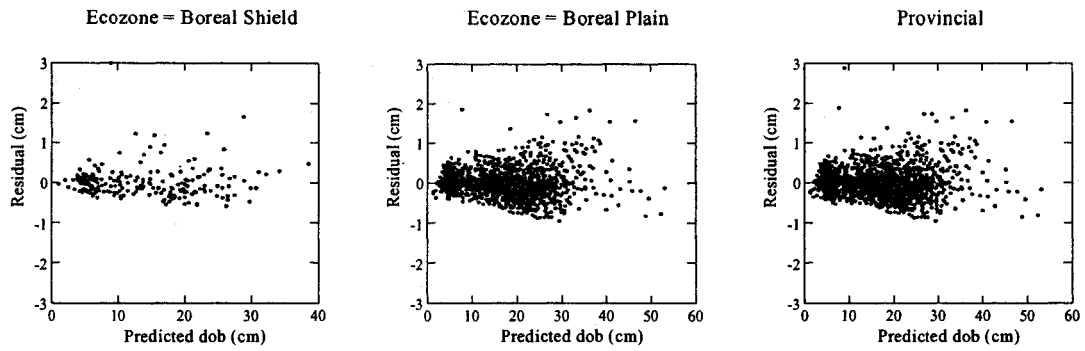


Figure 40. Residual plots of the ecozone-specific and provincial diameter inside/outside bark models (Eq. 11) for trembling aspen.

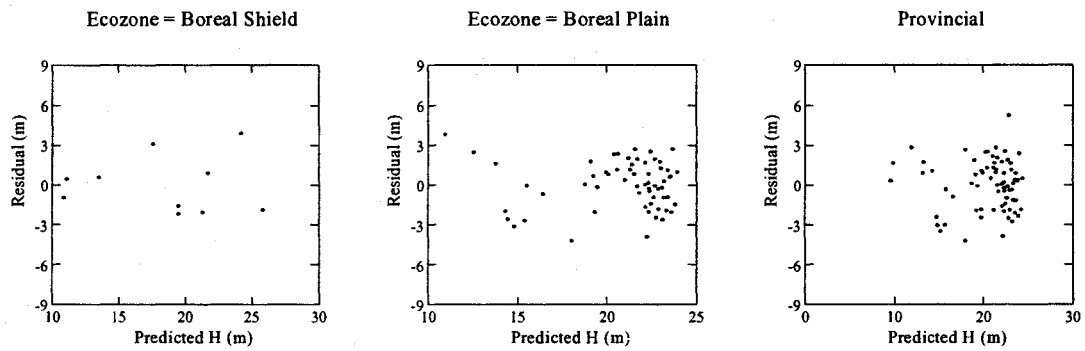


Figure 41. Residual plots of the ecozone-specific and provincial height-diameter models (Eq. 15) for trembling aspen.

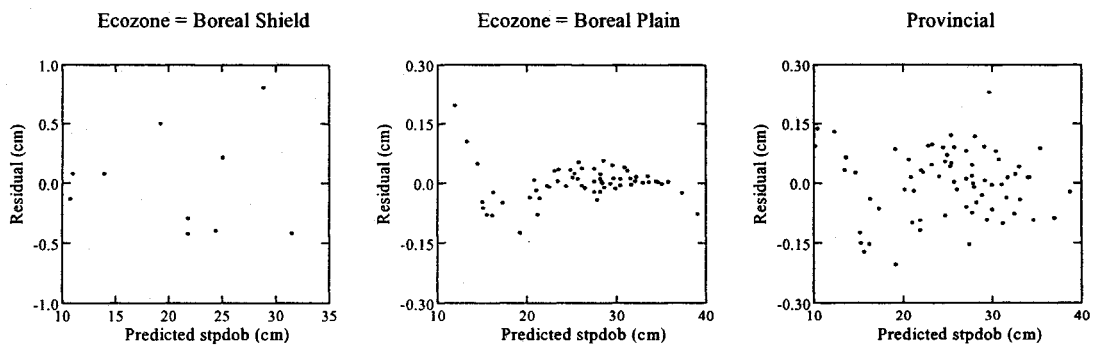


Figure 42. Residual plots of the ecozone-specific and provincial stump diameter-dbhob models (Eq. 18) for trembling aspen.

9.2.3. White spruce

The fit statistics of the ecozone-specific and provincial submodels for white spruce are shown in Table 24. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit for all the models accounting for more than 99% of the total variation of dob and stump dob, respectively. The height-diameter model (Eq. 15) provided a moderate fit for all the models with more than 70% of the total variation of total tree height explained. Although the models provided a moderate fit, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Therefore, caution should be exercised when using the predicted total tree heights. The residual plots of the diameter inside/outside bark, height-diameter, and stump diameter-dbhob models are shown in Figure 43, Figure 44, and Figure 45, respectively. The residual plots of the diameter inside/outside bark models and the height-diameter models show a cluster around zero and no visible pattern, indicating a random distribution of residuals. The residual plots of the stump diameter-dbhob models (Figure 45) show a pattern of increasing variance. This indicates a non-random distribution of residuals and indicates that the stump diameter-dbhob model may not be an appropriate model. Although the residuals show a non-random distribution, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Therefore, caution should be exercised when using the stump dob predictions, especially for larger predictions.

Table 24. Fit statistics of the ecozone-specific and provincial submodels for white spruce.

	Boreal Shield	Boreal Plain	Provincial
<u>Diameter inside/outside bark model (Eq. 11)</u>			
a	0.39517	0.48676	0.45216
b	1.02486	1.02212	1.0232
MSE	0.033366	0.040950	0.039138
R ²	0.999648	0.999647	0.999647
n	708	1452	2160
<u>Height-diameter model (Eq. 15)</u>			
a	23.0868	34.605	38.0155
b	0.0863	0.0408	0.0291
c	2.0248	1.3985	1.1104
MSE	0.197207	0.187331	0.194689
R ²	0.707113	0.799840	0.779316
n	33	64	97
<u>Stump diameter-dbhob model (Eq. 18)</u>			
a	-1.7114	-1.3506	-1.8149
b	1.2886	1.2791	1.3055
c	-0.00272	-0.00324	-0.0035
MSE	0.013431	0.017873	0.018999
R ²	0.999786	0.999734	0.999720
n	33	64	97

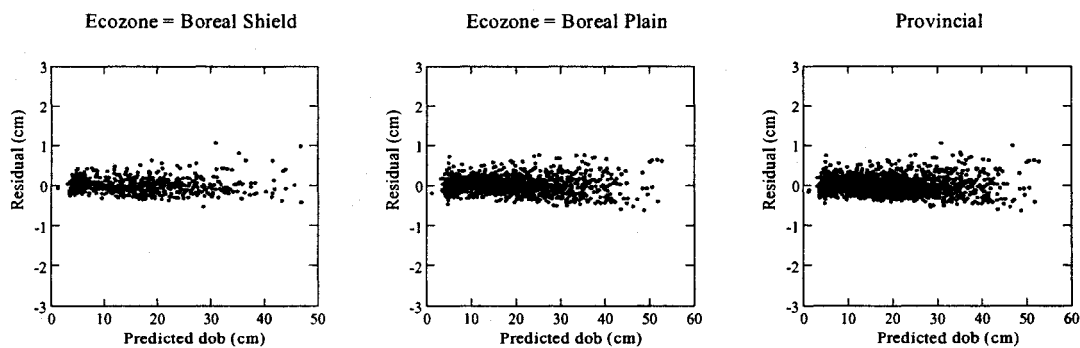


Figure 43. Residual plots of the ecozone-specific and provincial diameter inside/outside bark models (Eq. 11) for white spruce.

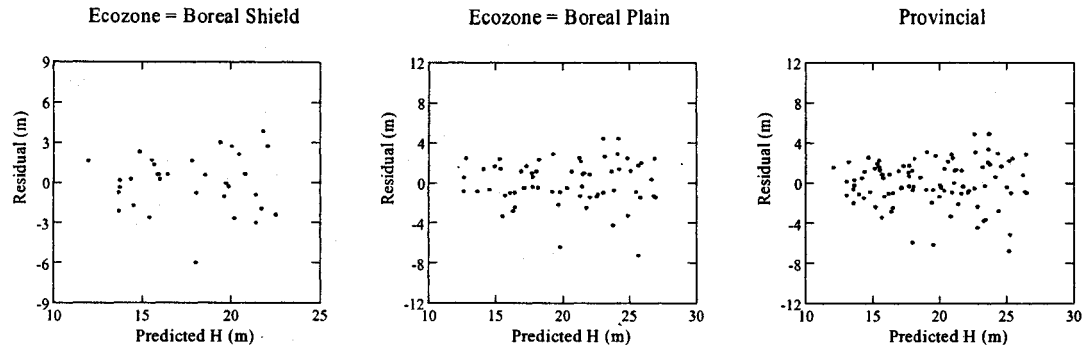


Figure 44. Residual plots of the ecozone-specific and provincial height-diameter models (Eq. 15) for white spruce.

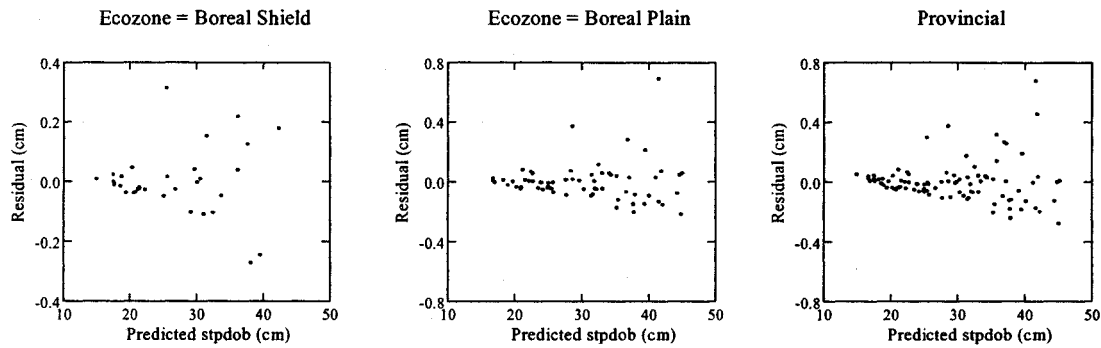


Figure 45. Residual plots of the ecozone-specific and provincial stump diameter-dbhob models (Eq. 18) for white spruce.

9.2.4. Black spruce

9.2.4.1. Provincial models

The fit statistics of the provincial submodels for black spruce are shown in Table 25. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit explaining more than 99% of the total variation of dob and stump dob, respectively. The height-diameter model (Eq. 15) provided a good fit explaining 87.5% of the total variation of total tree height. The residual plots of the diameter inside/outside bark, height-diameter, and stump diameter-dbhob models are

shown in Figure 46, Figure 47, and Figure 48, respectively. The residual plot of the diameter inside/outside bark model shows a cluster around zero and no visible pattern, indicating a random distribution of residuals. The residual plots of the height-diameter model (Figure 47) and the stump diameter-dbhob model (Figure 48) indicate the presence of increasing variance. This indicates a non-random distribution of residuals and indicates that the height-diameter model and the stump diameter-dbhob model may not be appropriate models. Although the residuals show a non-random distribution, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Therefore, caution should be exercised when using the total tree height and stump dob predictions, especially for larger predictions.

Table 25. Fit statistics of the provincial submodels for black spruce.

Provincial	
<u>Diameter inside/outside bark model (Eq. 11)</u>	
a	0.37747
b	1.02456
MSE	0.038568
R ²	0.998975
n	5940
<u>Height-diameter model (Eq. 15)</u>	
a	22.0772
b	0.1191
c	2.695
MSE	0.153136
R ²	0.875025
n	300
<u>Stump diameter-dbh model (Eq. 18)</u>	
a	0.0119
b	1.1432
c	-0.00169
MSE	0.005023
R ²	0.999743
n	300

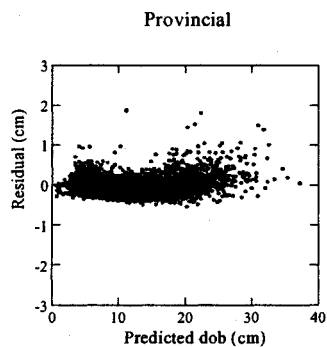


Figure 46. Residual plot of the provincial diameter inside/outside bark model (Eq. 11) for black spruce.

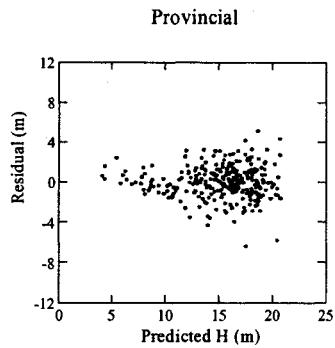


Figure 47. Residual plot of the provincial height-diameter model (Eq. 15) for black spruce.

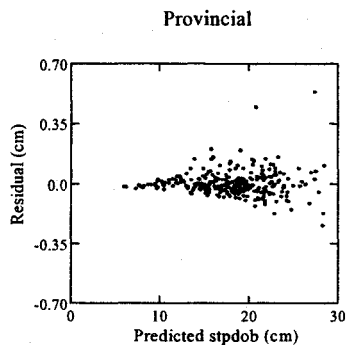


Figure 48. Residual plot of the provincial stump diameter-dbhob model (Eq. 18) for black spruce.

9.2.4.2. Ecosite-specific models

The fit statistics of the site type-specific submodels for black spruce are shown in Table 26. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit for both the site types explaining more than 99% of the total variation of dob and stump dob, respectively. The height-diameter model (Eq. 15) provided a moderate fit for the lowland site type and a poor fit for the upland site type explaining 74.8% and 57.7% of the total variation of total tree height, respectively. Although the model provided a poor fit, the submodels were specified to

be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Therefore, caution should be exercised when using the predicted total tree heights, especially for the upland site type. The residual plots of the diameter inside/outside bark, height-diameter, and stump diameter-dbhob models are shown in Figure 49, Figure 50, and Figure 51, respectively. All the residual plots show a cluster around zero and no visible pattern, indicating a random distribution of residuals.

Table 26. Fit statistics of the site type-specific submodels for black spruce.

	Lowland	Upland
<u>Diameter inside/outside bark model (Eq. 11)</u>		
a	0.3878	0.40792
b	1.02151	1.02364
MSE	0.032307	0.0433149
R ²	0.9991173	0.9988634
n	1289	3476
<u>Height-diameter model (Eq. 15)</u>		
a	20.8347	32.1195
b	0.135	0.0417
c	3.0383	1.1089
MSE	0.107521	0.1633656
R ²	0.7475618	0.5772468
n	61	167
<u>Stump diameter-dbhob model (Eq. 18)</u>		
a	-1.3889	0.9628
b	1.2807	1.0397
c	-0.00487	0.000885
MSE	0.0433943	0.0080557
R ²	0.9970843	0.9991892
n	61	167

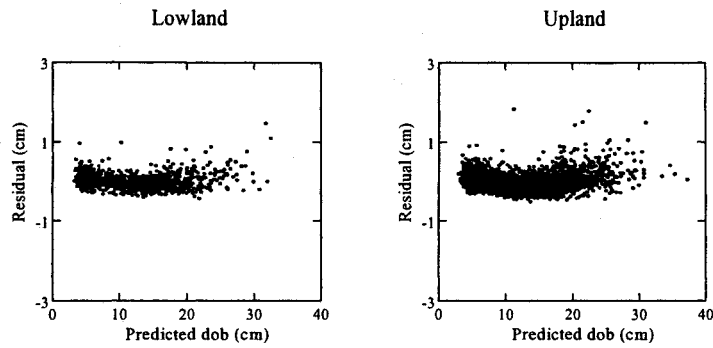


Figure 49. Residual plots of the site type-specific diameter inside/ outside bark models (Eq. 11) for black spruce.

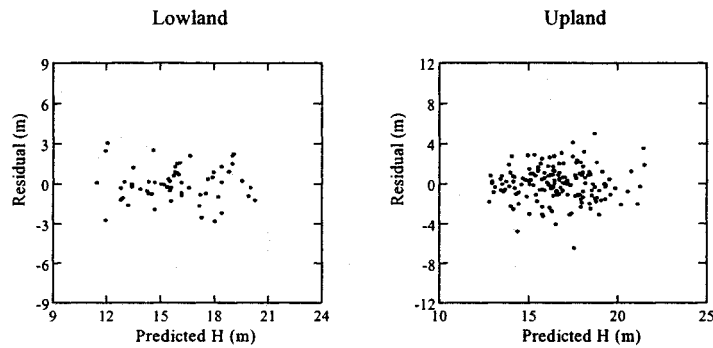


Figure 50. Residual plots of the site type-specific height-diameter models (Eq. 15) for black spruce.

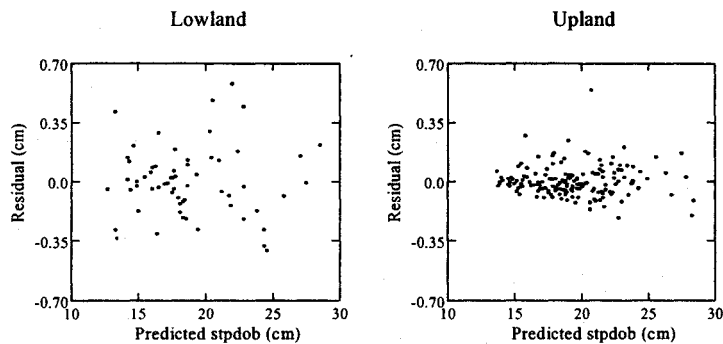


Figure 51. Residual plots of the site type-specific stump diameter-dbhob models (Eq. 18) for black spruce.

9.2.5. Jack pine

9.2.5.1. Ecoregion-specific models

The fit statistics of the ecoregion-specific and provincial submodels for jack pine are shown in Table 27. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit for all the models with more than 99% of the total variation explained for dob and stump dob, respectively. The height-diameter model (Eq. 15) provided a good fit for all the models explaining more than 84% of the total variation of total tree height. The residual plots of the diameter inside/outside bark, height-diameter and stump diameter-dbhob models are shown in Figure 52, Figure 53, and Figure 54, respectively. The residual plots of the height-diameter models show a cluster around zero and no visible pattern, indicating a random distribution of residuals. The residual plots of the stump diameter-dbhob models (Figure 54) show a cluster around zero and no visible pattern, with the exception of the provincial model. This indicates a non-random distribution of residuals for the provincial model and suggests that the model is not appropriate for the provincial data set. The residual plots of the diameter inside/outside bark models (Figure 52) indicate the presence of increasing variance. This indicates a non-random distribution of residuals and indicates that the diameter inside/outside bark model may not be an appropriate model. Although the residuals show a non-random distribution, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Since the diameter inside/outside bark model was used for stump dob predictions, caution should be exercised when using the stump dob predictions, especially for larger predictions.

Table 27. Fit statistics of the ecoregion-specific and provincial submodels for jack pine.

	88	89	90	91	148	152-154	155	Provincial
<u>Diameter inside/outside bark model (Eq. 11)</u>								
a	0.04797	0.10244	0.01824	0.07387	-0.03509	0.06821	0.0715	0.04326
b	1.05064	1.03214	1.05138	1.04256	1.04152	1.03323	1.04337	1.04194
MSE	0.094587	0.098577	0.14396	0.174087	0.104677	0.048417	0.119019	0.12359
R ²	0.9978	0.998285	0.997128	0.997305	0.997582	0.999224	0.997734	0.997636
n	862	420	1006	782	1639	487	234	5430
<u>Height-diameter model (Eq. 15)</u>								
a	31.3526	19.9378	26.6519	34.2235	18.4303	48.7248	41.0288	25.4006
b	0.0202	0.1099	0.0507	0.0141	0.1659	0.00585	0.0303	0.0634
c	0.8221	2.1513	1.0894	0.5311	4.9657	0.486	1.266	1.5326
MSE	0.150987	0.0764	0.178492	0.180817	0.205866	0.057856	0.231148	0.221091
R ²	0.88338	0.981832	0.715814	0.411925	0.768299	0.636017	0.929237	0.88493
n	61	25	47	35	94	21	15	298
<u>Stump diameter-dbh model (Eq. 18)</u>								
a	0.1397	-0.3065	1.3997	-1.884	0.758	1.069	0.3533	-0.1696
b	1.1016	1.1342	0.8899	1.2526	1.0251	1.031	1.0577	1.1482
c	-0.00046	-0.00173	0.00765	-0.00315	0.00194	0.000399	0.00071	-0.00187
MSE	0.03083	0.000864	0.008833	0.026898	0.009668	0.069707	0.112381	0.000521
R ²	0.99927	0.999986	0.999569	0.998244	0.999616	0.995736	0.998462	0.999987
n	61	25	47	35	94	21	15	298

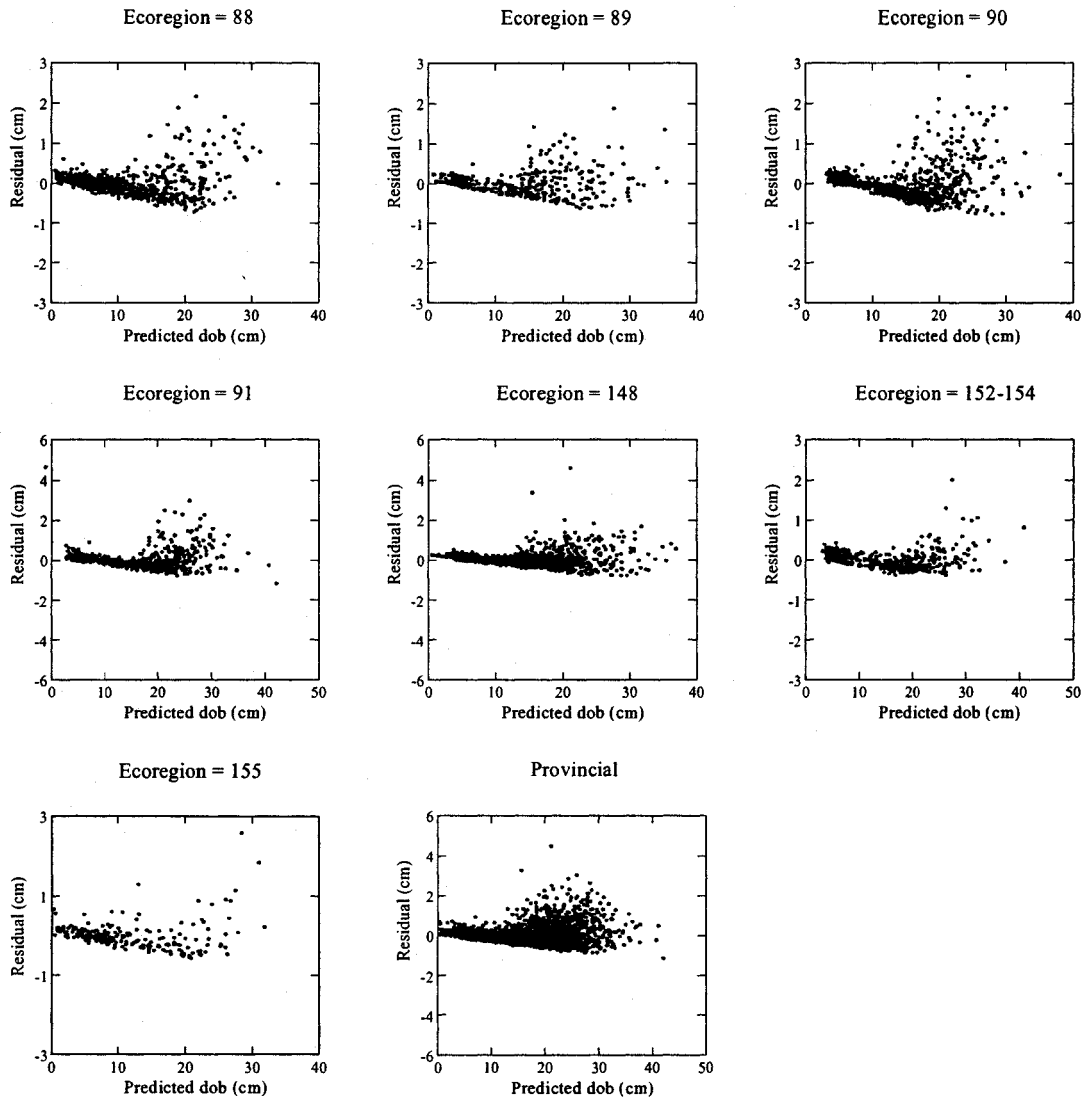


Figure 52. Residual plots of the ecoregion-specific and provincial diameter inside/outside bark models (Eq. 11) for jack pine.

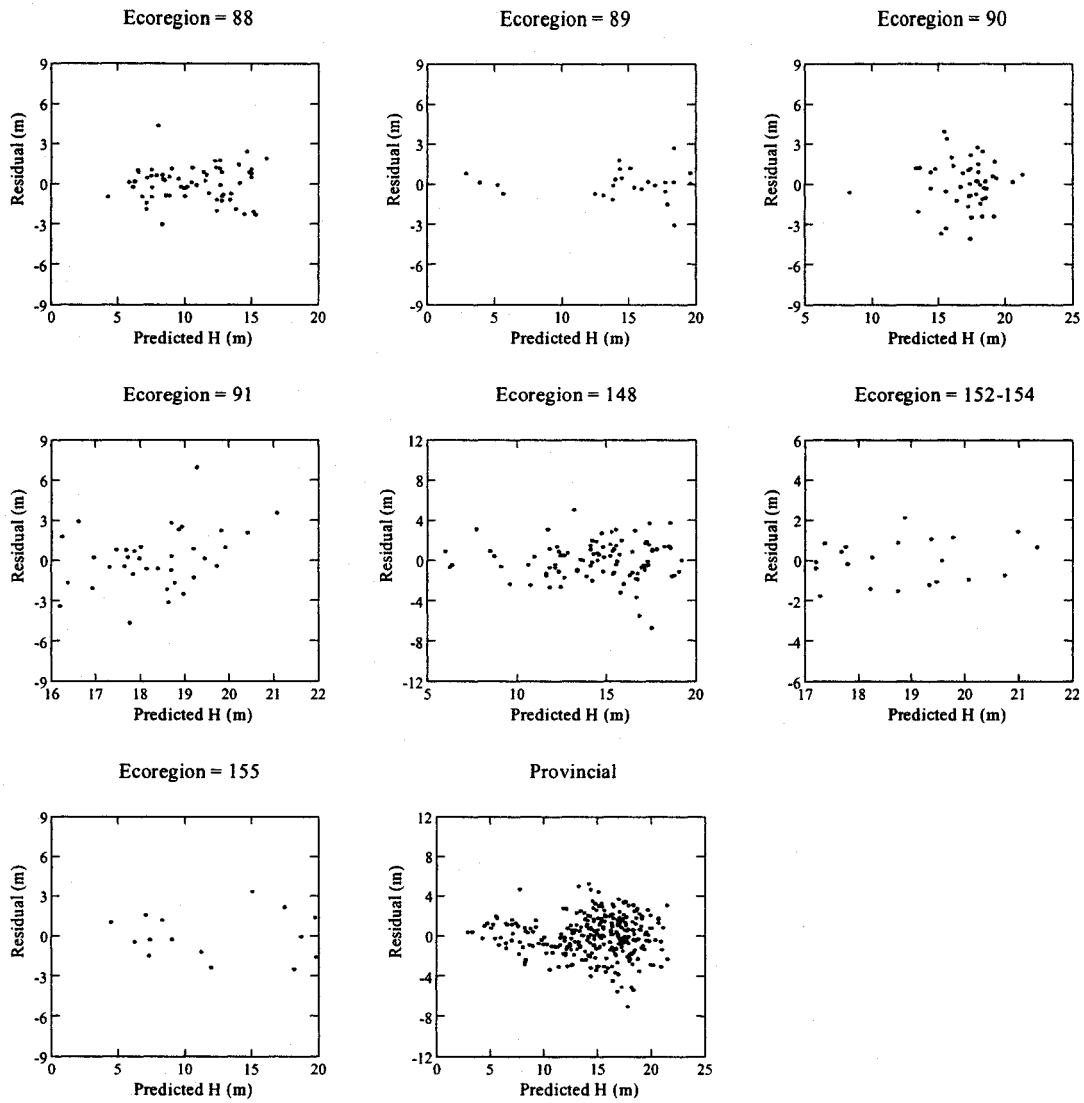


Figure 53. Residual plots of the ecoregion-specific and provincial height-diameter models (Eq. 15) for jack pine.

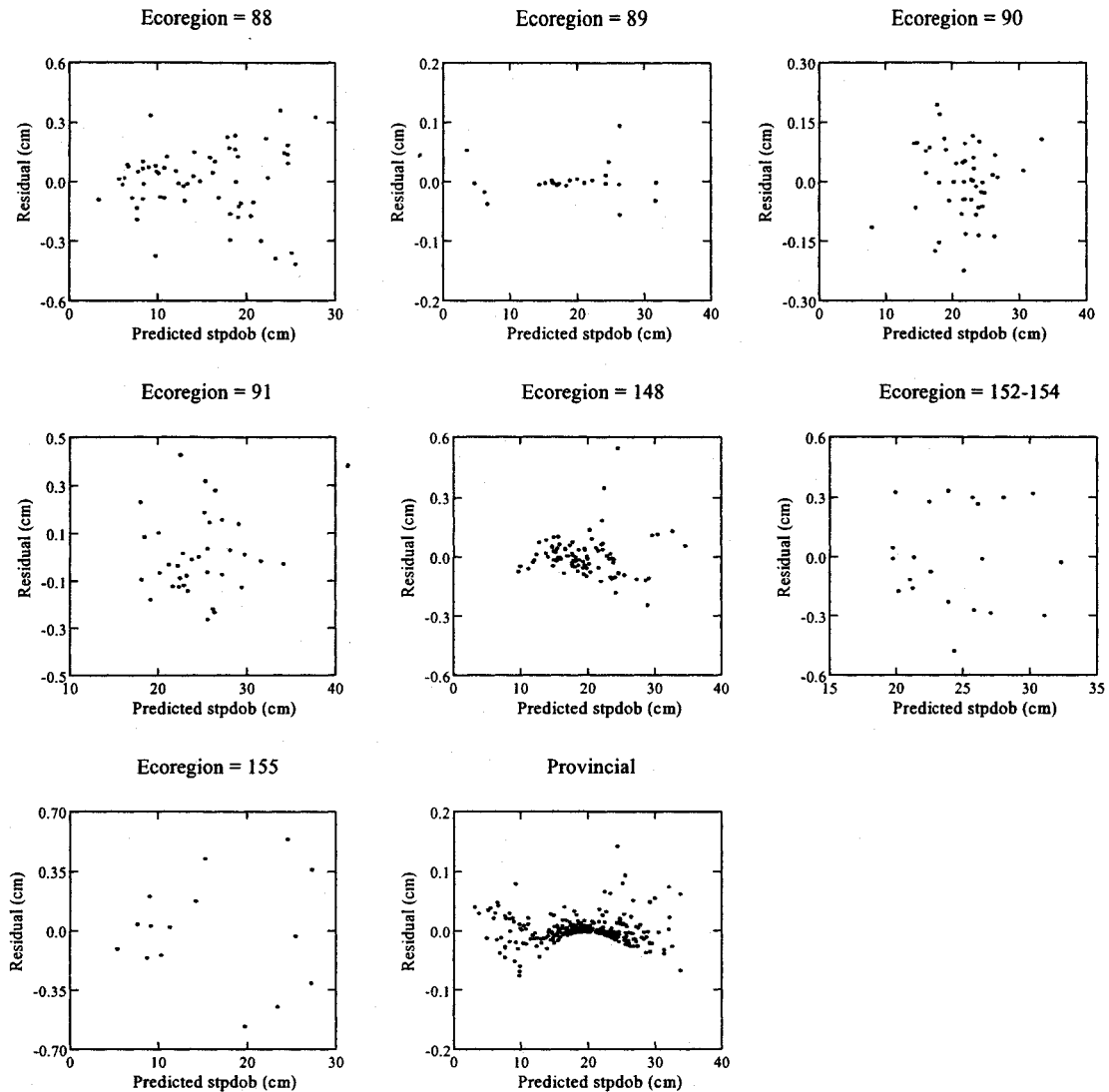


Figure 54. Residual plots of the ecoregion-specific and provincial stump diameter-dbhob models (Eq. 18) for jack pine.

9.2.5.2. Ecosite-specific models

The fit statistics of the site type-specific submodels for jack pine are presented in Table 28. The diameter inside/outside bark model (Eq. 11) and the stump diameter-dbhob model (Eq. 18) both provided a good fit for both the site types explaining more than 99% of the total variation of dob and stump dob. The height-diameter model (Eq.

15) provided a good fit for the rock site type explaining 75.5% of the total variation of total tree height, while it provided a poor fit for the mineral site type explaining only 36.3% of the total variation of total tree height. The residual plots of the diameter inside/outside bark, height-diameter, and stump diameter-dbhob models are shown in Figure 55, Figure 56, and Figure 57, respectively. The residual plots of the height-diameter models and the stump diameter-dbhob models show a cluster around zero and no visible pattern, indicating a random distribution of residuals. The residual plots of the diameter inside/outside bark models (Figure 55) indicate the presence of increasing variance. This indicates a non-random distribution of residuals and indicates that the diameter inside/outside bark model may not be an appropriate model. Although the residuals show a non-random distribution, the submodels were specified to be used by Manitoba Conservation. The submodels were solely fitted for application within the individual tree volume tables. Since the diameter inside/outside bark model was used for stump dob predictions, caution should be exercised when using the stump dob predictions, especially for larger predictions.

Table 28. Fit statistics of the site type-specific submodels for jack pine.

	Rock	Mineral
<u>Diameter inside/outside bark model (Eq. 11)</u>		
a	-0.03593	0.01981
b	1.06228	1.04786
MSE	0.156659	0.127125
R ²	0.996243	0.997642
n	410	596
<u>Height-diameter model (Eq. 15)</u>		
a	17.5955	-34.7539
b	0.1169	0.015
c	1.6449	0.5249
MSE	0.172251	0.094751
R ²	0.754658	0.363092
n	20	27
<u>Stump diameter-dbh model (Eq. 18)</u>		
a	0.7149	6.5592
b	0.9539	0.472
c	0.00704	0.0155
MSE	0.007824	0.047390
R ²	0.999664	0.996372
n	20	27

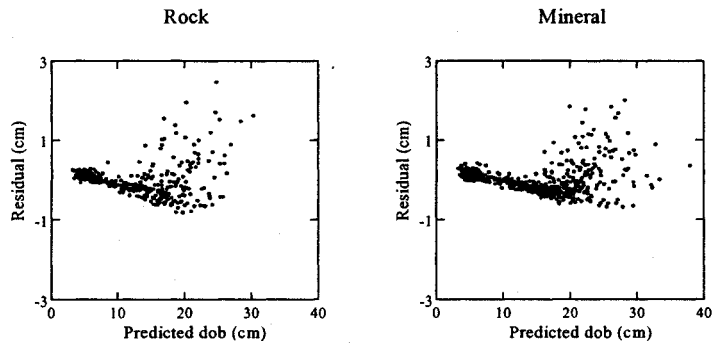


Figure 55. Residual plots of the site type-specific diameter inside/outside bark models (Eq. 11) for jack pine.

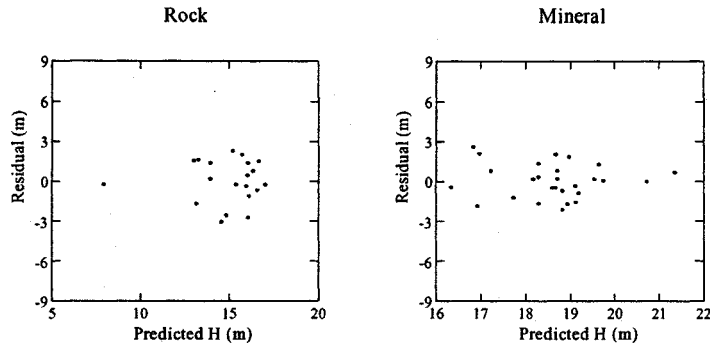


Figure 56. Residual plots of the site type-specific height-diameter models (Eq. 15) for jack pine.

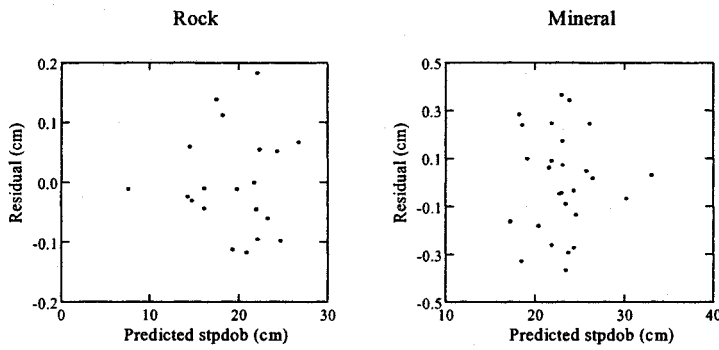


Figure 57. Residual plots of the site type-specific stump diameter-dbhob models (Eq. 18) for jack pine.

9.3. APPENDIX III

Applications of the Individual Tree Volume Tables

The individual tree volume tables (Appendix IV) were derived using the taper equations, the diameter outside/ inside bark models (Eq. 11), the height-diameter models (Eq. 15), and the stump diameter and dbhob (Eq. 18). Three tables were created for each taper equation. They were organized into gross total volume, merchantable length/ gross merchantable volume, and trees per cubic meter. The dbhob column at the left side of the tables was arranged in 2.0 cm classes. For example, the 2.0 cm dbhob class contains a range of 1.1 cm and 3.0 cm. The total tree heights at the top of the tables were arranged in 2.0 m classes. For example, the 12.0 m class contains a range of 11.1 m and 13.0 m. The stump dob column at the left side of the tables were predicted values for the corresponding dbhob classes. The stump dob provides a range with two boundary values.

The underlined values in the middle portion of the tables represent average height-diameter relationships, which were the dbhob and total tree height combination using the predicted height for each class. The tables can be used for the following applications.

1) Prediction of total tree height.

Using measurements of dbhob or stump dob, total tree height can be predicted using the column on the far right side of the tables. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), if a tree has a dbhob of 9.9 cm or has a stump dob of 9.0 cm, the predicted total tree height is 12.3 m.

2) Prediction of stump diameter outside bark.

Using measurements of dbhob, stump dob can be predicted. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), if a tree has a dbhob of 10.0, the stump dob is predicted to fall between 7.7 cm and 10.6 cm.

3) Prediction of diameter at breast height outside bark.

Using measurements of stump dob, dbhob can be predicted. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), if a tree has a stump dob of 9.0 cm, the dbhob is predicted to fall between 9.1 cm and 11.0 cm.

4) Prediction of gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter from observed dbhob and total tree height.

Using measurements of dbhob and total tree height, gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter can be predicted using the corresponding tables. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), if a tree has a dbhob of 10.0 cm and total tree height of 18.0 m, the tree contains 0.0363 m³ of gross total volume, 0.0164 m³ of gross merchantable volume, merchantable length of 3.65 m, and requires 60.837 trees to accumulate a cubic meter.

5) Prediction of gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter from observed dbhob and predicted total tree height.

Using measurements of dbhob, total tree height can be predicted and used to determine gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter using the corresponding tables. For example, using the balsam

poplar ecoregion 152-154 tables (Appendix IV), a tree with a dbhob of 10.0 cm has a predicted total tree height of 12.3 m which contains 0.0245 m³ of gross total volume, 0.0108 m³ of gross merchantable volume, merchantable length of 2.38 m, and requires 92.892 trees to accumulate a cubic meter.

6) Prediction of gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter from observed stump dob and total tree height.

Using measurements of stump dob and total tree height, gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter can be predicted using the corresponding tables. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), a tree with a stump dob of 10.1 cm and a height of 16.9 m contains 0.0324 m³ of gross total volume, 0.0146 m³ of gross merchantable volume, merchantable length of 3.23 m, and requires 68.716 trees to accumulate a cubic meter.

7) Prediction of gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter from observed stump dob and predicted total tree height.

Using measurements of stump dob, total tree height can be predicted and used to determine gross total volume, gross merchantable volume, merchantable length, and trees per cubic meter using the corresponding tables. For example, using the balsam poplar ecoregion 152-154 tables (Appendix IV), a tree with a stump dob of 9.7 cm has a predicted total tree height of 12.3 m which contains 0.0245 m³ of gross total volume, 0.0108 m³ of gross merchantable volume, merchantable length of 2.38 m, and requires 92.892 trees to accumulate a cubic meter.

8) Prediction of log volumes.

Using observed or predicted dbhob and total tree height measurements from the tables, individual log volumes can be calculated with varying top diameter and stump diameter measurements. This can be done using the same iteration procedure used to construct the tables (Appendix V).

9.4. APPENDIX IV

Individual Tree Volume Tables

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dbf

Species: Balsam poplar
Ecoregion: 152-154

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories from 3.1-5.0 to 39.1-41.0. Rows contain volume data for various tree heights and diameters.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: Trembling aspen
Ecoregion: 148, 152-154, 155 (Boreal Plain Ecozone)

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) ranging from 9.1-11.0 to 70.7-71.5. Rows provide volume data for various tree sizes.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dbh

Species: Trembling aspen
Ecoregion: 88, 89, 90, 91, 148, 152-154, 155 (Provincial)

Table with columns: DBHOB (cm), STUMP DOB (cm), Total Tree Height (m) (3.1-5.0 to 39.1-41.0), and Predicted HT. Rows represent different tree size classes. Underlined values in the middle portion of the table represent average height-diameter trees.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: White spruce
Ecoregion: 88, 89, 90, 91 (Boreal Shield Ecozone)

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) ranging from 3.1-5.0 to 39.1-41.0. Rows represent diameter classes and height classes. Values represent merchantable length and volume.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: White spruce
Ecoregion: 148, 152-154, 155 (Boreal Plain Ecozone)

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories (3.1-5.0 to 39.1-41.0). Rows represent diameter and height combinations. Values include merchantable length and volume. Underlined values indicate average height-diameter trees.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: White spruce
Ecoregion: 68, 69, 90, 91, 148, 152-154, 155 (Provincial)

Table with columns: DBHOB (cm), STUMP DOB (cm), and 20 diameter classes (3.1-5.0 to 39.1-41.0) plus Predicted HT. Rows represent combinations of diameter classes.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dbh

Species: Black spruce
Ecoregion: 86, 89, 90, 91, 148, 152-154, 155 (Provincial)

Table with columns for DBHOB (cm), STUMP DOB (cm), and diameter classes (3.1-5.0 to 39.1-41.0) and Predicted HT. The table contains numerical values for volume and length across various diameter and height bins.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: Jack pine
Ecoregion: 86

Table with columns: DBHOB (cm), STUMP DOB (cm), Total Tree Height (m) categories (3.1-5.0, 5.1-7.0, etc.), and Predicted HT. The table contains multiple rows of data for different tree sizes and heights.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dbh

Species: Jack pine
Ecoregion: 89

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories (3.1-5.0, 5.1-7.0, etc.). Rows represent tree height and diameter combinations. Underlined values in the table represent average height-diameter trees.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: Jack pine
Ecoregion: 90

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories from 3.1-5.0 to 39.1-41.0. Rows contain numerical data for each category, with underlined values representing average height-diameter trees.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: Jack pine
Ecoregion: 01

Table with columns: DBHOB (cm), STUMP DOB (cm), Total Tree Height (m) (3.1-5.0 to 39.1-41.0), and Predicted HT. Rows contain numerical values representing volume and length data for various tree heights.

Underscored values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dib

Species: Jack pine
Ecoregion: 152-154

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) ranging from 7.1-9.0 to 79.1-81.0. Rows contain volume data for various tree heights and diameters.

Underscored values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m³) from 0.3 m stump height to 7.0 cm top db

Species: Jack pine
Ecoregion: 155

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) bins (3.1-5.0 to 39.1-41.0). Rows represent diameter classes from 7.1-9.0 to 79.1-81.0 cm. Values represent merchantable length and volume. Underlined values indicate average height-diameter trees.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dbh

Species: Jack pine
Ecoregion: 88, 89, 90, 91, 148, 152-154, 155 (Provincial)

Table with columns: DBHOB (cm), STUMP DOB (cm), 3.1-5.0, 5.1-7.0, 7.1-9.0, 9.1-11.0, 11.1-13.0, 13.1-15.0, 15.1-17.0, 17.1-19.0, 19.1-21.0, 21.1-23.0, 23.1-25.0, 25.1-27.0, 27.1-29.0, 29.1-31.0, 31.1-33.0, 33.1-35.0, 35.1-37.0, 37.1-39.0, 39.1-41.0, Predicted HT. Rows contain volume and length data for various tree diameters and heights.

Underlined values in the middle portion of the table represent average height-diameter trees.

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top db

Species: Jack pine
Ecoregion: 90 Rock/ Shallow Soil

Table with columns for DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories from 3.1-5.0 to 39.1-41.0. Rows list various tree measurements and their corresponding merchantable lengths and volumes.

Underlined values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Merchantable length (m) / gross merchantable volume (m3) from 0.3 m stump height to 7.0 cm top dib

Species: Jack pine
Ecoregion: 90 Mineral Soil

Table with columns: DBHOB (cm), STUMP DOB (cm), and Total Tree Height (m) categories (3.1-5.0, 5.1-7.0, 7.1-9.0, 9.1-11.0, 11.1-13.0, 13.1-15.0, 15.1-17.0, 17.1-19.0, 19.1-21.0, 21.1-23.0, 23.1-25.0, 25.1-27.0, 27.1-29.0, 29.1-31.0, 31.1-33.0, 33.1-35.0, 35.1-37.0, 37.1-39.0, 39.1-41.0, Predicted HT). Rows list diameter and height ranges with corresponding volume values.

Undrilled values in the middle portion of the table represent average height-diameter trees

MANITOBA CONSERVATION - INDIVIDUAL TREE VOLUME TABLE (2004)
Trees / m³ gross merchantable volume from 0.3 m stump height to 7.0 cm top dib

Species: Jack pine
Ecoregion: 90 Mineral Soil

DBHOB (cm)	STUMP DOB (cm)	Total Tree Height (m)																			Predicted HT
		3.1-5.0	5.1-7.0	7.1-9.0	9.1-11.0	11.1-13.0	13.1-15.0	15.1-17.0	17.1-19.0	19.1-21.0	21.1-23.0	23.1-25.0	25.1-27.0	27.1-29.0	29.1-31.0	31.1-33.0	33.1-35.0	35.1-37.0	37.1-39.0	39.1-41.0	
7.1-9.0	10.7-12.1	147.625	99.449	73.164	57.465	47.177	39.956	34.624	30.533	27.297	24.676	22.510	20.692	19.143	17.809	16.648	15.629	14.726	13.922	13.200	12.4
9.1-11.0	12.1-13.6	71.847	50.903	38.611	30.896	25.678	21.936	19.130	16.953	15.215	13.797	12.619	11.625	10.775	10.041	9.399	8.834	8.333	7.886	7.484	13.6
11.1-13.0	13.7-15.3	46.132	33.499	25.700	20.700	17.276	14.802	12.936	11.482	10.318	9.366	8.574	7.904	7.330	6.834	6.400	6.018	5.679	5.375	5.103	14.8
13.1-15.0	15.4-17.1	32.032	23.962	18.587	15.054	12.605	10.823	9.473	8.418	7.571	6.877	6.299	5.809	5.390	5.026	4.708	4.428	4.179	3.957	3.756	15.8
15.1-17.0	17.2-19.1	23.022	17.856	14.022	11.424	9.598	8.258	7.239	6.439	5.796	5.268	4.827	4.453	4.133	3.855	3.612	3.398	3.208	3.037	2.884	16.7
17.1-19.0	19.2-21.1	16.836	13.629	10.854	8.901	7.505	6.472	5.682	5.059	4.557	4.145	3.799	3.507	3.256	3.038	2.847	2.678	2.529	2.395	2.274	17.6
19.1-21.0	21.2-23.3	12.421	10.565	8.548	7.060	5.977	5.167	4.543	4.050	3.651	3.322	3.047	2.814	2.613	2.439	2.286	2.151	2.031	1.924	1.827	18.4
21.1-23.0	23.4-25.6	9.197	8.274	6.816	5.674	4.824	4.181	3.683	3.287	2.966	2.701	2.478	2.289	2.127	1.985	1.862	1.752	1.655	1.567	1.489	19.2
23.1-25.0	25.7-28.0	-	6.526	5.484	4.606	3.934	3.420	3.018	2.697	2.436	2.219	2.038	1.883	1.750	1.634	1.533	1.443	1.363	1.291	1.227	19.8
25.1-27.0	28.2-30.6	-	5.171	4.443	3.767	3.235	2.820	2.493	2.231	2.017	1.840	1.690	1.563	1.453	1.357	1.273	1.199	1.132	1.073	1.019	20.5
27.1-29.0	30.7-33.3	-	4.109	3.618	3.101	2.677	2.341	2.075	1.859	1.683	1.536	1.412	1.306	1.215	1.135	1.065	1.003	0.948	0.898	0.853	21.1
29.1-31.0	33.4-36.1	-	3.270	2.958	2.564	2.227	1.955	1.736	1.559	1.412	1.290	1.187	1.098	1.022	0.955	0.896	0.844	0.798	0.756	0.719	21.7
31.1-33.0	36.2-39.0	-	2.603	2.426	2.129	1.861	1.640	1.460	1.313	1.191	1.089	1.002	0.928	0.864	0.808	0.758	0.714	0.675	0.640	0.609	22.2
33.1-35.0	39.2-42.1	-	-	1.994	1.774	1.561	1.381	1.233	1.111	1.009	0.923	0.850	0.788	0.734	0.686	0.644	0.607	0.574	0.545	0.518	22.8
35.1-37.0	42.2-45.2	-	-	1.641	1.482	1.314	1.167	1.045	0.943	0.858	0.786	0.724	0.671	0.625	0.585	0.550	0.518	0.490	0.465	0.442	23.3
37.1-39.0	45.4-48.5	-	-	1.352	1.240	1.109	0.990	0.888	0.803	0.731	0.671	0.619	0.574	0.535	0.501	0.471	0.444	0.420	0.398	0.379	23.7
39.1-41.0	48.7-52.0	-	-	1.114	1.040	0.937	0.841	0.757	0.686	0.626	0.574	0.530	0.492	0.459	0.430	0.404	0.381	0.361	0.342	0.326	24.2
41.1-43.0	52.1-55.5	-	-	0.918	0.873	0.794	0.716	0.647	0.587	0.537	0.493	0.456	0.423	0.395	0.370	0.348	0.328	0.311	0.295	0.281	24.6
43.1-45.0	55.7-59.2	-	-	0.756	0.733	0.674	0.611	0.554	0.504	0.461	0.424	0.393	0.365	0.341	0.319	0.300	0.284	0.268	0.255	0.242	25.0
45.1-47.0	59.4-63.0	-	-	-	0.617	0.572	0.522	0.475	0.433	0.397	0.366	0.339	0.315	0.294	0.276	0.260	0.245	0.232	0.221	0.210	25.4
47.1-49.0	63.2-66.9	-	-	-	0.519	0.487	0.447	0.408	0.373	0.343	0.316	0.293	0.273	0.255	0.239	0.225	0.213	0.202	0.191	0.182	25.8
49.1-51.0	67.1-70.9	-	-	-	0.436	0.414	0.383	0.351	0.322	0.296	0.274	0.254	0.237	0.221	0.208	0.196	0.185	0.175	0.166	0.158	26.1
51.1-53.0	71.2-75.1	-	-	-	0.367	0.353	0.328	0.302	0.278	0.256	0.237	0.220	0.205	0.192	0.181	0.170	0.161	0.152	0.145	0.138	26.5
53.1-55.0	75.3-79.4	-	-	-	0.309	0.301	0.282	0.261	0.241	0.222	0.206	0.191	0.179	0.167	0.157	0.148	0.140	0.133	0.126	0.120	26.8
55.1-57.0	79.6-83.8	-	-	-	-	0.257	0.242	0.225	0.208	0.193	0.179	0.167	0.156	0.146	0.137	0.129	0.122	0.116	0.110	0.105	27.1
57.1-59.0	84.0-88.4	-	-	-	-	0.219	0.208	0.195	0.181	0.168	0.156	0.145	0.136	0.127	0.120	0.113	0.107	0.101	0.096	0.092	27.4
59.1-61.0	88.6-93.0	-	-	-	-	0.187	0.179	0.168	0.157	0.146	0.136	0.127	0.118	0.111	0.105	0.099	0.094	0.089	0.084	0.081	27.7
61.1-63.0	93.3-97.8	-	-	-	-	0.159	0.154	0.145	0.136	0.127	0.118	0.111	0.104	0.097	0.092	0.087	0.082	0.078	0.074	0.071	28.0
63.1-65.0	98.1-103	-	-	-	-	0.136	0.133	0.126	0.118	0.110	0.103	0.097	0.091	0.085	0.080	0.076	0.072	0.068	0.065	0.062	28.3
65.1-67.0	103-108	-	-	-	-	0.116	0.114	0.109	0.103	0.096	0.090	0.084	0.079	0.075	0.070	0.067	0.063	0.060	0.057	0.054	28.5
67.1-69.0	108-113	-	-	-	-	-	0.098	0.095	0.089	0.084	0.079	0.074	0.069	0.065	0.062	0.058	0.055	0.053	0.050	0.048	28.8
69.1-71.0	113-118	-	-	-	-	-	0.085	0.082	0.078	0.073	0.069	0.065	0.061	0.057	0.054	0.051	0.049	0.046	0.044	0.042	29.0
71.1-73.0	118-124	-	-	-	-	-	0.073	0.071	0.068	0.064	0.060	0.057	0.053	0.050	0.048	0.045	0.043	0.041	0.039	0.037	29.3
73.1-75.0	124-129	-	-	-	-	-	0.063	0.062	0.059	0.056	0.053	0.050	0.047	0.044	0.042	0.040	0.038	0.036	0.034	0.033	29.5
75.1-77.0	129-135	-	-	-	-	-	0.054	0.053	0.051	0.049	0.046	0.044	0.041	0.039	0.037	0.035	0.033	0.032	0.030	0.029	29.7
77.1-79.0	135-141	-	-	-	-	-	-	0.046	0.045	0.043	0.041	0.038	0.036	0.034	0.033	0.031	0.029	0.028	0.027	0.026	29.9
79.1-81.0	141-146	-	-	-	-	-	-	0.040	0.039	0.037	0.036	0.034	0.032	0.030	0.029	0.027	0.026	0.025	0.024	0.023	30.1

Undertlined values in the middle portion of the table represent average height-diameter trees

9.5. APPENDIX V

Examples of SAS Script

The following are examples of the SAS script used in the completion of this thesis.

9.5.1. Linear model (example using jack pine)

```
data data1;
  infile "jp.csv" dlm = ",";

input ecoregion socode dbh h hag dib;

  lndib = log(dib);
  lndbh = log(dbh);
  x = (1 - sqrt(hag/h))/(1 - sqrt(0.25));
  z = hag/h;
  lnx = log(x);
  lnxz2 = lnx * z**2;
  lnz001 = log(z + 0.001);
  lnxlnz01 = lnx * lnz001;
  lnxsqrtz = lnx * sqrt(z);
  lnxez = lnx * exp(z);
  lnxdh = lnx * (dbh/h);

proc reg data = data1;
  model lndib = lndbh dbh lnxz2 lnxlnz01 lnxsqrtz lnxez lnxdh;
  output out = res1 p = pred r = resid;
proc plot data = res1;
  plot resid*pred;
run;
```

9.5.2. F-test (example testing jack pine within ecoregions 88 and 89)

```
data data1;
  infile "jp.csv" dlm = ",";

input ecoregion socode dbh h hag dib;

if ecoregion = 90 or ecoregion = 91 or ecoregion = 148 or ecoregion = 152
or ecoregion = 153 or ecoregion = 154 or ecoregion = 155 then delete;
run;
```

```

data data2;
set data1;

if ecoregion = 89 then k = 1;
else k= 0;

x = (1 - sqrt(hag/h))/(1 - sqrt(0.25));
z = hag/h;

proc nlin method = dud data = data2;
  parms a0 = 0.60177 a1 = 1.13290 c1 = 0 a2 = 0.96830 b1 = 0.54477
  b2 = -0.21367 b3 = 2.80306 b4 = -1.05037 b5 = 0.01891 c2 = 0;

  c = b1*z**2 + b2*log(z + 0.001) + b3*sqrt(z) + b4*exp(z) + (b5+c2*k)*(dbh/h);

  model dib = a0*dbh**(a1+c1*k)*a2**dbh*x**c;

output out = res1 p = pred r = resid;
proc plot data = res1;
  plot resid*pred;
run;

proc nlin method = dud data = data2;
  parms a0 = 0.60177 a1 = 1.13290 a2 = 0.96830 b1 = 0.54477 b2 = -0.21367
  b3 = 2.80306 b4 = -1.05037 b5 = 0.01891;

  c = b1*z**2 + b2*log(z + 0.001) + b3*sqrt(z) + b4*exp(z) + b5*(dbh/h);

  model dib = a0*dbh**a1*a2**dbh*x**c;

output out = res p = pred r = resid;
proc plot data = res1;
  plot resid*pred;
run;

```

9.5.3. Taper equation (example using jack pine ecoregion 88)

```

data data1;
  infile "jp.csv" dlm = ",";

input ecoregion socode dbh h hag dib;

if ecoregion = 89 or ecoregion = 90 or ecoregion = 91 or ecoregion = 148 or ecoregion =
152 or ecoregion = 153 or ecoregion = 154 or ecoregion = 155 then delete;

x = (1 - sqrt(hag/h))/(1 - sqrt(0.25));

```

```

z = hag/h;

proc nlin method = dud data = data1;
  parms a0 = 0.60177 a1 = 1.13290 a2 = 0.96830 b1 = 0.54477 b2 = -0.21367
  b3 = 2.80306 b4 = -1.05037 b5 = 0.01891;

  c = b1*z**2 + b2*log(z + 0.001) + b3*sqrt(z) + b4*exp(z) + b5*(dbh/h);

model dib = a0*dbh**a1*a2**dbh*x**c;
output out = res1 p = pred r = resid;
proc plot data = res1;
  plot resid*pred;
run;

```

9.5.4. Submodels (example using jack pine ecoregion 88)

```

data data1;
  infile "jp.csv" dlm = ",";

input ecoregion spcode dbh h hag dib dob cook1;

if ecoregion = 89 or ecoregion = 90 or ecoregion = 91 or ecoregion = 148 or ecoregion =
152 or ecoregion = 153 or ecoregion = 154 or ecoregion = 155 then delete;

proc reg data = data1;

model dob = dib;

output out = res1 p = pred r = resid;
proc plot data = res1;
  plot resid*pred;
run;

data data2;
  infile "jp.csv" dlm = ",";

input ecoregion spcode dbh h hag dib dob cook1;

if ecoregion = 89 or ecoregion = 90 or ecoregion = 91 or ecoregion = 148 or ecoregion =
152 or ecoregion = 153 or ecoregion = 154 or ecoregion = 155 then delete;

if cook1 = 0 then delete;

proc nlin method = dud data = data2;
  parms a = 20 b = 0.1 c = 1;
  bounds a>0, b>0, c>0;

```

```

model h = 1.3 + a*(1 - 2.71828**(-b*dbh))**c;

output out = res2 p = pred r = resid;
proc plot data = res2;
  plot resid*pred;
run;

data data3;
  infile "jp.csv" dlm = ",";

input ecoregion spcode dbh h hag dib dob cook1;

if ecoregion = 89 or ecoregion = 90 or ecoregion = 91 or ecoregion = 148 or ecoregion =
152 or ecoregion = 153 or ecoregion = 154 or ecoregion = 155 then delete;

if cook1 = 0 then delete;

z = 0.3/h;
x = (1 - sqrt(z))/(1 - sqrt(0.25));
dibstp = 0.9690*dbh**0.9568*1.0012**dbh*x**(-0.0805*z**2 + -0.0659*log(z +
0.001) + 0.4314*sqrt(z) + 0.1196*exp(z) + -0.0685*(dbh/h));
dobstp = 0.04797 + 1.05064*dibstp;

proc nlin method = dud data = data3;
  parms a = 1, b = 1, c = 0.001;

model dobstp = a + b*dbh + c*dbh**2;

output out = res3 p = pred r = resid;
proc plot data = res3;
  plot resid*pred;
run;

```

9.5.5. Individual tree volume tables (example using jack pine ecoregion 88)

```

data v1;

  do dbh = 8 to 80 by 2;
    do ht = 4 to 40 by 2; output;
    end;
  end;
run;

data v2;
set v1;

```

```

a0 = 0.9690;
a1 = 0.9568;
a2 = 1.0012;
b1 = -0.0805;
b2 = -0.0659;
b3 = 0.4314;
b4 = 0.1196;
b5 = -0.0685;

g0 = 0.9;

do until(abs(g0-g1)<0.00000001);
c = b1*(g0)**2 + b2*log(g0 + 0.001) + b3*sqrt(g0)+ b4*exp(g0) + b5*(dbh/ht);
g1 = (1 - (((a0*dbh**a1*a2**dbh)**(1/c))*(1 - sqrt(0.25))))**2;
g0 = (g0+g1)/2;
end;
run;

data v3;
  set v2;
  hi = g0*ht;
  mlen = hi - 0.3;

  mlen1 = 1*(hi - 0.3)/20 + 0.3;
  mlen2 = 2*(hi - 0.3)/20 + 0.3;
  mlen3 = 3*(hi - 0.3)/20 + 0.3;
  mlen4 = 4*(hi - 0.3)/20 + 0.3;
  mlen5 = 5*(hi - 0.3)/20 + 0.3;
  mlen6 = 6*(hi - 0.3)/20 + 0.3;
  mlen7 = 7*(hi - 0.3)/20 + 0.3;
  mlen8 = 8*(hi - 0.3)/20 + 0.3;
  mlen9 = 9*(hi - 0.3)/20 + 0.3;
  mlen10 = 10*(hi - 0.3)/20 + 0.3;
  mlen11 = 11*(hi - 0.3)/20 + 0.3;
  mlen12 = 12*(hi - 0.3)/20 + 0.3;
  mlen13 = 13*(hi - 0.3)/20 + 0.3;
  mlen14 = 14*(hi - 0.3)/20 + 0.3;
  mlen15 = 15*(hi - 0.3)/20 + 0.3;
  mlen16 = 16*(hi - 0.3)/20 + 0.3;
  mlen17 = 17*(hi - 0.3)/20 + 0.3;
  mlen18 = 18*(hi - 0.3)/20 + 0.3;
  mlen19 = 19*(hi - 0.3)/20 + 0.3;
  mlen20 = 20*(hi - 0.3)/20 + 0.3;

  z1 = mlen1/ht;
  z2 = mlen2/ht;
  z3 = mlen3/ht;

```

$z4 = mlen4/ht;$
 $z5 = mlen5/ht;$
 $z6 = mlen6/ht;$
 $z7 = mlen7/ht;$
 $z8 = mlen8/ht;$
 $z9 = mlen9/ht;$
 $z10 = mlen10/ht;$
 $z11 = mlen11/ht;$
 $z12 = mlen12/ht;$
 $z13 = mlen13/ht;$
 $z14 = mlen14/ht;$
 $z15 = mlen15/ht;$
 $z16 = mlen16/ht;$
 $z17 = mlen17/ht;$
 $z18 = mlen18/ht;$
 $z19 = mlen19/ht;$
 $z20 = mlen20/ht;$

$x1 = (1 - \sqrt{z1})/(1 - \sqrt{0.25});$
 $x2 = (1 - \sqrt{z2})/(1 - \sqrt{0.25});$
 $x3 = (1 - \sqrt{z3})/(1 - \sqrt{0.25});$
 $x4 = (1 - \sqrt{z4})/(1 - \sqrt{0.25});$
 $x5 = (1 - \sqrt{z5})/(1 - \sqrt{0.25});$
 $x6 = (1 - \sqrt{z6})/(1 - \sqrt{0.25});$
 $x7 = (1 - \sqrt{z7})/(1 - \sqrt{0.25});$
 $x8 = (1 - \sqrt{z8})/(1 - \sqrt{0.25});$
 $x9 = (1 - \sqrt{z9})/(1 - \sqrt{0.25});$
 $x10 = (1 - \sqrt{z10})/(1 - \sqrt{0.25});$
 $x11 = (1 - \sqrt{z11})/(1 - \sqrt{0.25});$
 $x12 = (1 - \sqrt{z12})/(1 - \sqrt{0.25});$
 $x13 = (1 - \sqrt{z13})/(1 - \sqrt{0.25});$
 $x14 = (1 - \sqrt{z14})/(1 - \sqrt{0.25});$
 $x15 = (1 - \sqrt{z15})/(1 - \sqrt{0.25});$
 $x16 = (1 - \sqrt{z16})/(1 - \sqrt{0.25});$
 $x17 = (1 - \sqrt{z17})/(1 - \sqrt{0.25});$
 $x18 = (1 - \sqrt{z18})/(1 - \sqrt{0.25});$
 $x19 = (1 - \sqrt{z19})/(1 - \sqrt{0.25});$
 $x20 = (1 - \sqrt{z20})/(1 - \sqrt{0.25});$

$dibm0 = (a0*dbh**a1)*(a2**dbh)*((1 - \sqrt{0.3/ht})/(1 - \sqrt{0.25}))**(b1*(0.3/ht)**2$
 $+ b2*\log(0.3/ht + 0.001)$
 $+ b3*\sqrt{0.3/ht} + b4*\exp(0.3/ht) + b5*dbh/ht);$

$dibm1 = (a0*dbh**a1)*(a2**dbh)*x1**(b1*z1**2 + b2*\log(z1 + 0.001) + b3*\sqrt{z1}$
 $+ b4*\exp(z1) + b5*dbh/ht);$
 $dibm2 = (a0*dbh**a1)*(a2**dbh)*x2**(b1*z2**2 + b2*\log(z2 + 0.001) + b3*\sqrt{z2}$
 $+ b4*\exp(z2) + b5*dbh/ht);$

$$\begin{aligned} \text{dibm3} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x3**(b1*z3**2 + b2*\log(z3 + 0.001) + b3*\text{sqrt}(z3) \\ &+ b4*\exp(z3) + b5*\text{dbh}/\text{ht}); \\ \text{dibm4} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x4**(b1*z4**2 + b2*\log(z4 + 0.001) + b3*\text{sqrt}(z4) \\ &+ b4*\exp(z4) + b5*\text{dbh}/\text{ht}); \\ \text{dibm5} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x5**(b1*z5**2 + b2*\log(z5 + 0.001) + b3*\text{sqrt}(z5) \\ &+ b4*\exp(z5) + b5*\text{dbh}/\text{ht}); \\ \text{dibm6} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x6**(b1*z6**2 + b2*\log(z6 + 0.001) + b3*\text{sqrt}(z6) \\ &+ b4*\exp(z6) + b5*\text{dbh}/\text{ht}); \\ \text{dibm7} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x7**(b1*z7**2 + b2*\log(z7 + 0.001) + b3*\text{sqrt}(z7) \\ &+ b4*\exp(z7) + b5*\text{dbh}/\text{ht}); \\ \text{dibm8} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x8**(b1*z8**2 + b2*\log(z8 + 0.001) + b3*\text{sqrt}(z8) \\ &+ b4*\exp(z8) + b5*\text{dbh}/\text{ht}); \\ \text{dibm9} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x9**(b1*z9**2 + b2*\log(z9 + 0.001) + b3*\text{sqrt}(z9) \\ &+ b4*\exp(z9) + b5*\text{dbh}/\text{ht}); \\ \text{dibm10} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x10**(b1*z10**2 + b2*\log(z10 + 0.001) + \\ &b3*\text{sqrt}(z10) + b4*\exp(z10) + b5*\text{dbh}/\text{ht}); \\ \text{dibm11} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x11**(b1*z11**2 + b2*\log(z11 + 0.001) + \\ &b3*\text{sqrt}(z11) + b4*\exp(z11) + b5*\text{dbh}/\text{ht}); \\ \text{dibm12} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x12**(b1*z12**2 + b2*\log(z12 + 0.001) + \\ &b3*\text{sqrt}(z12) + b4*\exp(z12) + b5*\text{dbh}/\text{ht}); \\ \text{dibm13} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x13**(b1*z13**2 + b2*\log(z13 + 0.001) + \\ &b3*\text{sqrt}(z13) + b4*\exp(z13) + b5*\text{dbh}/\text{ht}); \\ \text{dibm14} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x14**(b1*z14**2 + b2*\log(z14 + 0.001) + \\ &b3*\text{sqrt}(z14) + b4*\exp(z14) + b5*\text{dbh}/\text{ht}); \\ \text{dibm15} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x15**(b1*z15**2 + b2*\log(z15 + 0.001) + \\ &b3*\text{sqrt}(z15) + b4*\exp(z15) + b5*\text{dbh}/\text{ht}); \\ \text{dibm16} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x16**(b1*z16**2 + b2*\log(z16 + 0.001) + \\ &b3*\text{sqrt}(z16) + b4*\exp(z16) + b5*\text{dbh}/\text{ht}); \\ \text{dibm17} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x17**(b1*z17**2 + b2*\log(z17 + 0.001) + \\ &b3*\text{sqrt}(z17) + b4*\exp(z17) + b5*\text{dbh}/\text{ht}); \\ \text{dibm18} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x18**(b1*z18**2 + b2*\log(z18 + 0.001) + \\ &b3*\text{sqrt}(z18) + b4*\exp(z18) + b5*\text{dbh}/\text{ht}); \\ \text{dibm19} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x19**(b1*z19**2 + b2*\log(z19 + 0.001) + \\ &b3*\text{sqrt}(z19) + b4*\exp(z19) + b5*\text{dbh}/\text{ht}); \\ \text{dibm20} &= (a0*\text{dbh}**a1)*(a2**\text{dbh})*x20**(b1*z20**2 + b2*\log(z20 + 0.001) + \\ &b3*\text{sqrt}(z20) + b4*\exp(z20) + b5*\text{dbh}/\text{ht}); \end{aligned}$$

$$\begin{aligned} \text{mvol} &= 0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm0**2 + 4*dibm1**2 + dibm2**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm2**2 + 4*dibm3**2 + dibm4**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm4**2 + 4*dibm5**2 + dibm6**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm6**2 + 4*dibm7**2 + dibm8**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm8**2 + 4*dibm9**2 + dibm10**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm10**2 + 4*dibm11**2 + dibm12**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm12**2 + 4*dibm13**2 + dibm14**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm14**2 + 4*dibm15**2 + dibm16**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm16**2 + 4*dibm17**2 + dibm18**2) + \\ &0.00007854*(((\text{hi} - 0.3)/10)/6)*(dibm18**2 + 4*dibm19**2 + dibm20**2); \end{aligned}$$

```

trees = 1/mvol;
tipvol = 0.00007854*dibm20**2*(ht-hi)/3;
stpvol = 0.00007854*dibm0**2*0.3;
totvol = mvol + tipvol + stpvol;
run;

data v4;
  set v3;

tv1 = lag18(totvol);
tv2 = lag17(totvol);
tv3 = lag16(totvol);
tv4 = lag15(totvol);
tv5 = lag14(totvol);
tv6 = lag13(totvol);
tv7 = lag12(totvol);
tv8 = lag11(totvol);
tv9 = lag10(totvol);
tv10 = lag9(totvol);
tv11 = lag8(totvol);
tv12 = lag7(totvol);
tv13 = lag6(totvol);
tv14 = lag5(totvol);
tv15 = lag4(totvol);
tv16 = lag3(totvol);
tv17 = lag2(totvol);
tv18 = lag1(totvol);

if ht = 40;

  h = 1.3 + 31.3526 *(1 - exp(-0.0202*dbh))**0.8221;
  D1 = DBH - 0.9;
  D2 = DBH + 1.0;
  STUMP1 = 0.1397 + 1.1016*D1 + -0.00046*D1**2;
  STUMP2 = 0.1397 + 1.1016*D2 + -0.00046*D2**2;
  MX = '-';
run;

data p1;
  file "jp88totvol.csv" dlm = ",";
  set v4;
  PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
  STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
  tv1 21-28 .4   tv2 29-36 .4   tv3 37-44 .4 tv4 45-52 .4
  tv5 53-60 .4   tv6 61-68 .4   tv7 69-76 .4 tv8 77-84 .4
  tv9 85-92 .4   tv10 93-100 .4

```

```
tv11 101-108 .4 tv12 109-116 .4  
tv13 117-124 .4 tv14 125-132 .4  
tv15 133-140 .4 tv16 141-148 .4  
tv17 149-156 .4 tv18 157-164 .4  
totvol 165-172 .4 h 173-180 .1;
```

```
run;
```

```
data v5;
```

```
  set v3;
```

```
mv1 = lag18(mvol);  
mv2 = lag17(mvol);  
mv3 = lag16(mvol);  
mv4 = lag15(mvol);  
mv5 = lag14(mvol);  
mv6 = lag13(mvol);  
mv7 = lag12(mvol);  
mv8 = lag11(mvol);  
mv9 = lag10(mvol);  
mv10 = lag9(mvol);  
mv11 = lag8(mvol);  
mv12 = lag7(mvol);  
mv13 = lag6(mvol);  
mv14 = lag5(mvol);  
mv15 = lag4(mvol);  
mv16 = lag3(mvol);  
mv17 = lag2(mvol);  
mv18 = lag1(mvol);
```

```
m1 = lag18(mlen);  
m2 = lag17(mlen);  
m3 = lag16(mlen);  
m4 = lag15(mlen);  
m5 = lag14(mlen);  
m6 = lag13(mlen);  
m7 = lag12(mlen);  
m8 = lag11(mlen);  
m9 = lag10(mlen);  
m10 = lag9(mlen);  
m11 = lag8(mlen);  
m12 = lag7(mlen);  
m13 = lag6(mlen);  
m14 = lag5(mlen);  
m15 = lag4(mlen);  
m16 = lag3(mlen);  
m17 = lag2(mlen);  
m18 = lag1(mlen);
```

```

m = '/';

if ht = 40;

h = 1.3 + 31.3526 *(1 - exp(-0.0202*dbh))**0.8221;
  D1 = DBH - 0.9;
  D2 = DBH + 1.0;
  STUMP1 = 0.1397 + 1.1016*D1 + -0.00046*D1**2;
  STUMP2 = 0.1397 + 1.1016*D2 + -0.00046*D2**2;
  MX = '-';
run;

```

```

data p2;
  file "jp88mvol.csv" dlm = "," lrecl=275;
  set v5;
  PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
  STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
  m1 21-26 .2 m $ 27 mv1 28-33 .4
  m2 34-39 .2 m $ 40 mv2 41-46 .4
  m3 47-52 .2 m $ 53 mv3 54-59 .4
  m4 60-65 .2 m $ 66 mv4 67-72 .4
  m5 73-78 .2 m $ 79 mv5 80-85 .4
  m6 86-91 .2 m $ 92 mv6 93-98 .4
  m7 99-104 .2 m $ 105 mv7 106-111 .4
  m8 112-117 .2 m $ 118 mv8 119-124 .4
  m9 125-130 .2 m $ 131 mv9 132-137 .4
  m10 138-143 .2 m $ 144 mv10 145-150 .4
  m11 151-156 .2 m $ 157 mv11 158-163 .4
  m12 164-169 .2 m $ 170 mv12 171-176 .4
  m13 177-182 .2 m $ 183 mv13 184-189 .4
  m14 190-195 .2 m $ 196 mv14 197-202 .4
  m15 203-208 .2 m $ 209 mv15 210-215 .4
  m16 216-221 .2 m $ 222 mv16 223-228 .4
  m17 229-234 .2 m $ 235 mv17 236-241 .4
  m18 242-247 .2 m $ 248 mv18 249-254 .4
  mlen 255-260 .2 m $ 261 mvol 262-267 .4
  h 268-275 .1;
run;

```

```

data v6;
  set v3;
  n1 = lag18(trees);
  n2 = lag17(trees);
  n3 = lag16(trees);
  n4 = lag15(trees);
  n5 = lag14(trees);
  n6 = lag13(trees);

```

```

n7 = lag12(trees);
n8 = lag11(trees);
n9 = lag10(trees);
n10 = lag9(trees);
n11 = lag8(trees);
n12 = lag7(trees);
n13 = lag6(trees);
n14 = lag5(trees);
n15 = lag4(trees);
n16 = lag3(trees);
n17 = lag2(trees);
n18 = lag1(trees);

if ht = 40;

h = 1.3 + 31.3526 *(1 - exp(-0.0202*dbh))**0.8221;
D1 = DBH - 0.9;
D2 = DBH + 1.0;
STUMP1 = 0.1397 + 1.1016*D1 + -0.00046*D1**2;
STUMP2 = 0.1397 + 1.1016*D2 + -0.00046*D2**2;
MX = '-';

run;

data p3;
file "jp88trees.csv" dlm = ",";
set v6;
PUT D1 1-4 .1 MX $ 5 D2 6-9 .1
STUMP1 12-15 .1 MX $ 16 STUMP2 17-20 .1
n1 26-33 .3 n2 34-41 .3 n3 42-49 .3 n4 50-57 .3 n5 58-65 .3
n6 66-73 .3 n7 74-81 .3 n8 82-89 .3 n9 90-97 .3 n10 98-105 .3
n11 106-113 .3 n12 114-121 .3 n13 122-129 .3
n14 130-137 .3 n15 138-145 .3
n16 146-153 .3 n17 154-161 .3 n18 162-169 .3 trees 170-177 .3
h 178-185 .1;

run;

```