

**THE DEVELOPMENT OF JUNIOR INTERMEDIATE PRESERVICE
TEACHERS' MATHEMATICAL KNOWLEDGE AND VALUES**

by

Carlos Zerpa

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ABSTRACT

This research thesis is composed of two studies. The first study examined the validity and reliability of a locally developed instrument called the Perceptions of Mathematics (POM) questionnaire (Kajander, 2005). The POM questionnaire was used to measure mathematical knowledge and values on junior intermediate preservice teachers in the second study of this research thesis.

The second study investigated preservice teachers' change in mathematical knowledge and values via a mathematics methods course in education. This study included pretests and post-tests to examine preservice teachers' preliminary mathematical knowledge, beliefs, and changes, in these factors, after taking the mathematics methods course at the junior intermediate level.

The results in the first study show evidence that the POM questionnaire is a valid and reliable instrument. The results in the second study suggest that it is possible to change preservice teachers' conceptual and procedural mathematics knowledge via a mathematics methods course in education. In addition, the results of the second study suggest that preservice teachers' academic background does not appear to influence their change in conceptual mathematical knowledge and values via a mathematics methods course. On the other hand, preservice teachers' conceptual and procedural mathematical knowledge at the pretest, plus preservice teachers' courses taken in high school, appear to influence their change in conceptual mathematical knowledge the most. These findings have implications for mathematics educators of teachers, as well as school boards, to help them better assess professional development needs.

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CHAPTER 1 – INTRODUCTION

Mathematics is embedded in our lives. We begin the formal learning of mathematics, in elementary school and then throughout the rest of our secondary and, for some, post-secondary education. This mathematics journey, from elementary school to post-secondary education, helps us prepare for the scientific and technological changes taking place in today's world, which rely heavily on the understanding and application of mathematical concepts.

In order to meet the demands of the technological era, the Ontario Ministry of Education has reformed the mathematics curriculum in Ontario elementary and secondary schools. A goal of the new mathematics curriculum is to equip students with essential mathematical knowledge and skills to reason, solve problems and communicate (Ontario Curriculum Grade 1-8, 2005). Most importantly, students are meant to acquire the ability and motivation to continue learning on their own (Ontario Curriculum Grade 11, 2004).

Although the Ministry of Education has made a great effort to implement a mathematics curriculum that supports students' mathematics preparation, it is also important to consider how teachers' mathematical knowledge (i.e., "knowledge of mathematical concepts and procedures") and values (i.e., "mathematical conceptions and ideologies") influence students' mathematical knowledge and learning (Ambrose, 2004). In other words, how the quality of classroom teaching affects students' mathematical knowledge and performance (Expert Panel on Literacy and Numeracy Instruction, 2005) is crucially important. Indeed, teachers' knowledge about teaching and learning has been cited as the most important predictor of students' success (Greenwald, Hedges & Laine, 1996). Furthermore, teacher's ideologies influence student's mathematical values, which permit students to engage or not to engage in a mathematics course (Bishop,

Clarke, Corrigan & Gunstone, 2006).

The impact of the teacher as a factor in students' success found in previous research (Bishop et al, 2006; Greenwald et al, 1996), led to the investigation of preservice teachers' perceptions of conceptual and procedural knowledge and values at the junior intermediate level (Kajander, 2005). Developing preservice teachers' mathematical knowledge and values before they begin their classroom practice may enhance the mathematical knowledge and values that these teachers will bring to the classroom (Boyd, 1994; Kajander, 2005; Sowder, 2007). For that reason, this research thesis was aimed at examining preservice teachers' initial capacity (initial levels of conceptual and procedural mathematical knowledge and values) and their changes in mathematical knowledge and values after taking their Bachelor of Education degree, including a mathematics methods course.

The mathematics methods course included mathematical content related to patterning, numeracy, geometry and data management and my thesis advisor taught the entire course. The National Council of Teachers of Mathematics *Principles and Standards* (National Council of Teachers of Mathematics, 2000) guided the teaching strategies used in the mathematics methods course (NCTM, 2000). Detailed field notes were kept during each class of the course as part of the CRYSTAL research project conducted by my thesis advisor. These field notes were kept by an independent researcher (graduate student). As part of this CRYSTAL research project, I was able to examine these field notes, to determine the learning opportunities offered to these preservice teachers' candidates. For instance, in the mathematics methods course, teaching was focused on enhancing preservice teachers' conceptual understanding of the fundamental mathematics needed for teaching at the junior intermediate level by encouraging the preservice

teachers to make use of manipulatives and games to help them bridge the gap between the understanding of mathematical concepts and procedures. Furthermore, preservice teachers' mathematical learning was focused on building their knowledge by providing them with mathematical problems that allowed them to make use of their experiences and prior mathematical knowledge but with more emphasis on conceptual mathematical understanding. For example, preservice teachers were given problems like this:

- a) Use an area model with algebra tiles to show that $(X+2)(X+3)=X^2 + 5X + 6$. Label each area.
- b) Use a manipulative or model of your choice to illustrate and justify $2 - (-3)$. Show the answer.

In addition, the mathematical examples provided to the preservice teachers in the mathematics methods course were thoroughly discussed to allow all preservice teachers taking the mathematics methods course the opportunity to learn and build upon their existing knowledge regardless of their academic background. The curriculum delivered in the mathematics methods course was coherent in the sense that the mathematical problems and ideas were presented with the intention to better prepare preservice teachers to solve mathematical problems with more conceptual understanding at the junior intermediate level. Moreover, preservice teachers were allowed the opportunity to share their ideas with other members of the group, find other ways to solve the problems and build upon their existing knowledge.

The mathematics methods course instruction implemented some of the principles of reform in mathematics education by using the NCTM *Principles and Standards* (NCTM, 2000)

as a guide. The interpretation of what mathematics reform really is may be a dilemma (Hiebert, 1999), as there is no consistent image of what reform should look like in the classroom, and even less consensus about how it should be measured (Ross, Hogaboam-Gray & McDougal, 2002). The mathematics methods course examined in this study exemplified some of the characteristics of mathematics education reform as found in previous research (Ibid). For instance, many examples given in class to the preservice teachers were open-ended problems embedded in real-life contexts; and many of these problems had more than one possible solution method. Furthermore, the instruction was focused on the construction of mathematical ideas through preservice teachers' talk rather than the transmission through lectures and presentations.

The instructor's role in the course was more of a co-learner and creator of a mathematical community rather than sole knowledge expert. The mathematical problems presented to the class were undertaken with the aid of manipulatives and with access to other mathematical tools (calculators and computers) and the assessment of the class was integrated with every-day events. Hence, I believe that the mathematics methods course taken by the junior intermediate preservice teachers in the Bachelor of education program used a number of key characteristics of reform mathematics and the course was used as an intervention to potentially enhance preservice teachers' mathematical knowledge and values.

This research thesis also looked at the influence of preservice teachers' academic background on changes to their mathematical knowledge and values. The mathematical experiences that preservice teachers brought to their professional year Bachelor of Education program were compared to their growth. Changes in conceptual and procedural mathematical knowledge and values were compared to preservice teachers' backgrounds ("mathematics" or "non- mathematics," Ball, 1990). Preservice teachers categorised as having a mathematics

background were those with science, engineering and computer science backgrounds and are referred to as “mathematics teachers”. Preservice teachers referred to as “non-mathematics” teachers included those with arts, humanities and social science backgrounds. In addition, an attempt was made to predict gains in teacher knowledge based on preservice teachers' level of high school mathematics, mathematics courses taken in university as well as teachers' levels of conceptual and procedural mathematical knowledge and values at the beginning of the mathematics methods course (Ball, 1990; Ma, 1999; Boaler, 1999; Franz, 2000).

In summary, the work done on this research thesis was divided into two studies. The first study looked at the reliability and validity of the Perceptions of Mathematics (POM) questionnaire. The second study examined preservice teachers' change in conceptual and procedural mathematical knowledge and values after taking the mathematics methods course using the POM instrument to collect the data. Furthermore, this One-Group Pretest-Posttest Design study, which included a quantitative analysis of the data, looked at the influence of preservice teachers' academic background on changes to their mathematical knowledge and values as mentioned earlier. In addition, this study examined the possibility of creating a linear regression model to determine which factors may affect changes in preservice teachers' conceptual mathematical knowledge as a result of a professional development experience (the mathematics methods course) within their Bachelor of Education program.

CHAPTER 2 – RELIABILITY AND VALIDITY OF POM INSTRUMENT

2.1 Introduction

Measurements of Mathematical Knowledge

In mathematics education, knowledge is defined as a comprehension of mathematical topics, procedures and concepts and the relationship among these topics, procedures and concepts. Consequently, teachers must have a substantive knowledge of mathematics based on three criteria: correctness, meaning and connectedness (Ball, 1990). Ma (1999) states that Ball's (1990) vision of mathematical knowledge, however, has been limited by her data. Ma alludes that teachers with profound knowledge of mathematics are not only aware of the conceptual structure and procedures of mathematics, but are able to teach them to the students. Nevertheless, it has been claimed that teachers' mathematical knowledge can be enhanced by implementing changes in teachers' preparation and providing support to the teachers in order to attain profound understanding of fundamental mathematics (Hill & Ball, 2004).

In order to provide an appropriate preparation that improves teachers' mathematical knowledge, it is important to assess, and effectively measure, teachers' conceptual and procedural mathematical knowledge for teaching (Ibid). According to Hill, Schilling and Ball's (2005) research, assessing and measuring mathematical knowledge requires the implementation of valid and reliable instruments and therefore, the lack of validity and reliability of these measuring tools has stimulated a disagreement over how to properly assess teachers' mathematical knowledge needed for teaching. For instance, Hill et al (2005) state that some assessment tools measure teachers' abilities to solve middle-school mathematical problems, whereas others measure the ability to understand and apply mathematical content to teaching.

They advocate that appropriate assessment tools are needed to measure teachers' mathematical knowledge and capabilities in mathematics teaching.

Measurement Tools for Mathematical Content Knowledge (LMT Questionnaire)

In order to assess teachers' conceptual and procedural mathematical knowledge, it is essential to use valid and reliable instruments. With the utilization of valid and reliable instruments, it is possible to develop comprehensive theories of knowledge and apply the appropriate measurements (Ahn & Chang, 2004). For instance, it is possible to measure the knowledge that teachers have of mathematics with a focus on concepts, ideas and procedures and the knowledge that teachers have about how to teach mathematics (Ibid).

Research by Hill and Ball (2004) used assessment tools such as the Learning Mathematics for Teaching questionnaire (LMT) to measure teachers' mathematical knowledge. The LMT questionnaire is a paper and pencil instrument, which measures mathematical knowledge based on teachers' abilities to apply mathematical content to teaching-related situations. More specifically, the LMT questionnaire assesses content knowledge of mathematics with an emphasis on the following elementary curricular strands: number and operation, algebra, and geometry. The LMT questionnaire is not designed to examine individuals' mathematical knowledge. Instead, it is designed to compare groups of teachers' mathematical knowledge, or examine how a group of teachers' knowledge develops over time (Hill & Ball, 2005).

The validity of the LMT instrument was evaluated in a study conducted by Hill & Ball (2004) which assessed teachers' content knowledge of mathematics. This study looked for evidence of construct validity. The study hypothesized that teachers' knowledge of teaching elementary mathematics was multidimensional, in other words, it included knowledge of various

mathematical topics (e.g., number and operations, algebra) and domains (e.g., knowledge of content, knowledge of students and content). Hill and Ball (2004) used a response form questionnaire with 640 participants. The instrument included items in the following content areas (number concepts, operations, pattern and functions) and domains (knowledge of content, knowledge of students and content). Results of an exploratory factor analysis suggested that there were three underlying dimensions to teachers' knowledge of mathematics:

a) knowledge of content in number concepts and operations; b) knowledge of content in patterns, functions and algebra; and c) knowledge of students and content in number concepts and operations.

All of the knowledge of number concepts and operations items loaded on the first factor, and all the knowledge of patterns, functions and algebra items loaded on the second factor. The knowledge of students and content items (9 out of 14) loaded primarily on the third factor and a minority of the knowledge of students and content items loaded primarily on the first factor. In other words, mathematical ability and teaching ability was not just related to a general factor. In addition, Hill and Ball (2004) examined the reliability of the LMT instrument using Alpha coefficients. The reliabilities for patterns, functions and algebra measures, as well as the measures that combined number and operations items within each domain, range from 0.71 to 0.84. The lowest reliability of 0.71 occurred for the knowledge of students and content measures.

Measurements of Mathematical Values and Attitudes

Besides measuring teachers' mathematical content knowledge, it is also relevant to measure their mathematical values and attitudes (Stipek, Givvin, Salmon & MacGyvers, 2001). Ernest (1989) conducted a research study, which related to the knowledge, values and attitudes of the mathematics teacher. Ernest (1989) argued that besides mathematical knowledge, it is

important to consider teachers' values and attitudes. Ernest (1989) defined values as teachers' mathematical conceptions and ideologies and makes the argument that conceptions have a powerful impact on teaching. Therefore, based on their conceptions, teachers select their mathematical content, styles of teaching, and modes of learning for the students. In addition to measuring teachers' mathematics values, Ernest (1989) advocated that it is important to measure teachers' attitudes, which include liking, enjoyment, enthusiasm for the teaching of mathematics, and their confidence in their mathematics teaching abilities.

Nelson Attitudes and Practices for Teaching Mathematical Survey

One of the instruments utilized to measure teachers' beliefs about classroom mathematics teaching is the Nelson Attitudes and Practices for Teaching Mathematics Survey (NAPTMS). Hence, Ross, McDougal, & Hogaboam-Gray (2003) created the NAPTMS instrument with the intention of measuring elementary teachers' self-reported implementation of standards-based mathematics concepts in teaching. By measuring teachers' beliefs using the NAPTMS, it has been shown through the research findings that there is a relationship between teachers' beliefs and their practices (Ross et al., 2003; Stipek et al., 2001).

In the NAPTMS instrument, Ross et al (2003) included ten dimensions of elementary mathematics reform related to beliefs about classroom teaching practices. These dimensions were obtained from key NCTM documents and 154 empirical studies conducted from 1993 to 2000. All the items in each dimension were reviewed for face validity by a panel of experts in elementary mathematics and by teachers known to abide by the implementation of the NCTM *Standards*. In order to test the reliability of this instrument, Ross et al (2003) administered the survey twice to 517 Grade K- 8 teachers in two different districts. Using Cronbach's α as a measure of internal consistency, a coefficient $\alpha=0.81$ was obtained during the first administration

of the test in a single district and a coefficient $\alpha=0.81$ during the second administration of the test in a different district. The results indicated that the instrument was reliable.

The study also showed evidence of the instrument's validity. Predictive validity in which it was hypothesized by the authors that students taught by teachers who obtained high scores during the administration of the designed instrument survey (NAPTMS), would have higher academic performance in mathematics than those students taught by teachers who scored low. Teachers' scores correlated with students' achievement in schools $r=0.35$, $p<0.001$, $n=130$. The results show that mathematics achievement was higher in schools in which teachers had high scores on the standards-based teaching survey, which shows evidence of predictive validity.

The researchers also examined concurrent validity by linking the measures of the instrument to other measures taken at the same time. In this case, the researchers compared the survey responses of a small sample of teachers to observations of their teaching. Yet, the observations were not entirely consistent with the survey scores. The results do not show evidence of concurrent validity of the instrument; however, the researchers suggested that there is a continuing need for observational studies with more precise measurement to address concurrent validity.

The last form of validity was construct validity, in which the researchers conducted an interview process to describe the relationships between the observed and hypothesized standards-based survey scores in using a textbook to support standards-based teaching. The researchers hypothesized that teachers who were users of the textbook to support standards-based teaching, would score high on the survey, as supposed to those who were not. The results from the interview process showed that the two groups differed consistently in how they used the textbook in the classroom. Indeed, the teachers who scored high on the survey used the textbook to

support standards-based teaching, whereas teachers who scored low on the survey used the textbook to support traditional teaching. Thus, the results show evidence of construct validity.

Measurements of Mathematical Knowledge and Values (POM Questionnaire)

There is always a need to develop better tools to obtain a better understanding of how teachers' mathematical knowledge affects the quality of instruction (Ball et al., 2004). Furthermore, with better tools, it is possible to assess reform-based mathematical knowledge and values with a stress on conceptual understanding of mathematics (Ibid). For that reason, different tools have been developed to measure mathematical knowledge and beliefs such as the Learning Mathematics for Teaching (LMT) Survey (Hill et al., 2004) and the Nelson Beliefs Survey (Ross et al, 2003). These instruments; however, are not designed to measure mathematical knowledge and beliefs within one survey. These instruments only look at either measures of mathematical knowledge or beliefs, not both. Research indicates that while both knowledge and beliefs affect teaching, it is unclear exactly how they influence one another (Ambrose, 2004). A teacher with strong conceptual knowledge who highly values procedural fluency (a more traditional value) might still choose not to teach in a reform-based manner. Both mathematical understanding and deeply-held beliefs about what is important in mathematical knowing (Bishop et al, 2006) play a role in teacher education (Kajander, 2005). The Perceptions of Mathematics instrument (POM) (provided in Appendix A) was developed with the intention of assessing mathematical knowledge and values of preservice junior intermediate teachers. Specifically, the POM instrument was designed to measure the change in junior intermediate teachers' conceptual and procedural mathematical knowledge and conceptual and procedural mathematical values, as a result of teacher education programs (Kajander, Keene, Siddo & Zerpa, 2006).

The reliability of the POM instrument for measuring conceptual and procedural values was initially examined by Kajander et al (2006). In Kajander's study, it was reported that the resulting Cronbach's Alpha reliability coefficients for the POM instrument were 0.76 for procedural values and 0.79 for conceptual values for post-test administration of the survey. In addition, the study alluded that the face validity of the knowledge questions of the POM questionnaire was supported by drawing relationships to Ma's (1999) interview questions, and items from Hill and Ball's (2004) Learning Mathematics for Teaching (LMT) research study, as well as the feedback from mathematicians and practicing teachers. Nonetheless, there was a need to test the reliability of the conceptual and procedural knowledge questions of the POM instrument since the reliability measures reported in Kajander et al (2006) study only included reliability measures of conceptual and procedural values. Moreover, there was a need to show further evidence of the concurrent validity of the POM instrument by correlating the POM and LMT measures of mathematical knowledge. There was also a need to show further evidence of the concurrent validity of the POM instrument with respect to measures of conceptual and procedural mathematical values.

While an equivalent instrument to the POM measures of conceptual and procedural mathematical values (deeply held beliefs about what is important in mathematics) has not been found, examining correlation to the Nelson instrument (which examines beliefs about classroom teaching practices) may have potential. By showing stronger evidence of the reliability and validity of the assessment instrument (POM in this case), it may be possible to confidently evaluate preservice teachers' mathematical knowledge and values (Hill et al., 2004; Ball et al., 2004; Kajander et al., 2006). For that reason, this initial research study was conducted to show stronger evidence of the validity and reliability of the POM instrument when measuring

conceptual and procedural mathematical knowledge and conceptual and procedural mathematical values on preservice teachers.

2.2 Method

Instrument Reliability and Validity

Validity of the POM Instrument

Although the validity of the POM questionnaire was evaluated previously during the PRISM (Programming Remediation and Intervention for Students in Mathematics) study conducted by Kajander et al (2006) and by relating the POM questionnaire to Ma's (1999) interview questions, the purpose of this study was to provide further evidence of the concurrent validity of the POM instrument when measuring mathematical knowledge and values. For this study, in-service teachers' data collected in Kajander et al (2006) study, which included pretest and post-test data from three instruments (POM, LMT and NELSON) were used.

In order to further examine the concurrent validity of the POM instrument, it was hypothesized that two instruments – POM and LMT, would produce related measurements of mathematical knowledge during the pretest and post-test, since both instruments measure the same or similar constructs in terms of mathematical knowledge. In addition, it was also hypothesized that two instruments – POM and NELSON, would produce related measurements of mathematical beliefs during the pretest and post-test since both instruments measure aspects of mathematical beliefs. Therefore, the data collected from 30 in-service junior intermediate teachers in Kajander et al (2006) study who answered the three surveys (POM, LMT and NELSON) during the two administrations of the instruments (pretest and post-test) were used to test the hypotheses stated above.

The POM measures of mathematical knowledge (conceptual and procedural) were correlated to the LMT measures of mathematical content knowledge (number and operation, algebra and geometry) for the two administrations of the instruments (pretest and post-test). The magnitudes of the correlations were used to show evidence of concurrent validity of the POM instrument when measuring mathematical knowledge. Likewise, the POM measures of mathematical beliefs (conceptual and procedural values) were correlated to the NELSON measures of mathematical beliefs (manipulatives and attitudes) during the two administrations of the instruments (pretest and post-test). The strength of the correlations among the two instruments' variables was used to show evidence of the concurrent validity of the POM instrument when measuring mathematical values. It should be noted that the POM instrument was not initially developed to measure mathematical knowledge and values for junior intermediate in-service teachers; but rather, it was more specifically designed to measure mathematical knowledge and values for junior intermediate preservice teachers. Thus in order to strengthen the analysis, a new set of data was collected at the end of the winter semester in 2007.

During this data collection, 77 junior intermediate preservice teachers from the Bachelor of Education program at Lakehead University wrote the POM and a subset of the LMT questionnaire in the same administration (post-test). The new POM and LMT measures of mathematical knowledge were correlated and the strength of their relationship was used to show further evidence of the concurrent validity of the POM instrument when assessing junior intermediate preservice teachers' mathematical knowledge.

Reliability of the POM Instrument

Data collected from 111 junior intermediate preservice teachers from the Bachelor of Education program at Lakehead University in 2005-2006 were used to compute the Cronbach's alpha coefficients and determine the internal consistency of the POM instrument when measuring conceptual and procedural mathematical values. In addition, data collected from 77 junior intermediate preservice teachers from the Bachelor of Education program at Lakehead University in 2006-2007 were used to compute the Cronbach's alpha coefficients and show further evidence of the reliability of the POM instrument when measuring conceptual and procedural mathematical knowledge.

2.3 Results and Analysis

As stated in the literature, it is essential to use valid and reliable instruments to effectively assess teachers' mathematical knowledge, develop comprehensive theories of knowledge and apply the appropriate measurements (Ahn & Chang, 2004). Evidence for reliability and validity of the POM instrument are described below.

Evidence of Reliability of the POM

The reliability of the beliefs portion of the survey (POM) was examined by using a Cronbach's Alpha coefficient as a measure of internal consistency of the data. For this analysis, the data collected from 111 junior intermediate preservice teachers in 2005-2006 from the Bachelor of Education program at Lakehead University was used. The results show an alpha coefficient of 0.72 for procedural values and 0.72 for conceptual values for the pretest data. The results also show an alpha coefficient of 0.78 for procedural values and 0.82 for conceptual values for the post-test data.

In order to provide further evidence of the reliability of the POM instrument, the knowledge portion of the survey was examined. The post-test data collected from 77 junior intermediate preservice teachers in 2006-2007 from the Bachelor of Education program at Lakehead University was used to show evidence of internal consistency with respect to measures of mathematical knowledge. The results show an alpha coefficient of 0.83 for both conceptual and procedural mathematical knowledge.

Evidence of Concurrent Validity of the POM

Evidence of concurrent validity refers to a hypothesis linking survey scores to a relevant measure taken at the same time as the survey gets administered (Linn, 1989). In our case, the responses of the POM instrument conceptual and procedural knowledge were compared to the responses of the LMT instrument mathematical knowledge (number and operations, algebra and geometry). In addition, the responses of the POM instrument conceptual and procedural values were compared to the responses of the NELSON beliefs (manipulatives and attitudes). For this analysis, a small sample of 30 in-service teachers data collected in Kajander et al (2006) study who answered the two surveys (POM and LMT) of mathematical knowledge during two administrations of the instruments (pretest and post-test) was used to show evidence of concurrent validity of the POM instrument measures of mathematical knowledge. Furthermore, another small sample of 13 in-service teachers data collected in Kajander et al (2006) study who answered the two surveys (POM and NELSON) during two administrations of the instruments (pretest and post-test) was used to show evidence of current validity of the POM instrument measures of mathematical values.

During the first administration of the surveys, POM and LMT (pretest data), the results show positive significant correlations between the two instruments when measuring mathematical knowledge as depicted in Table 2.1.

Table 2.1.

Correlations Pretest Data LMT (NO, ALG, Geom) and POM (CK, PK)

		PK	CK	NO	ALG
CK	Correlation	0.63			
	Sig. (2-tailed)	0.00			
	N	30			
NO	Correlation	0.39	0.55		
	Sig. (2-tailed)	0.03	0.00		
	N	30	30		
ALG	Correlation	0.54	0.73	0.46	
	Sig. (2-tailed)	0.00	0.00	0.01	
	N	30	30	30	
GEOM	Correlation	0.57	0.61	0.60	0.64
	Sig. (2-tailed)	0.00	0.00	0.00	0.00
	N	30	30	30	30

Note. Correlations are significant at 0.05 level. CK = conceptual knowledge; PK = procedural knowledge; NO = number and operations; ALG = algebra; GEOM = geometry.

During the second administration of the surveys POM and LMT (post-test data), the results show significant positive correlations between the two instruments' measures of mathematical knowledge as depicted in Table 2.2. Thus, the results revealed significant positive correlations at both the pretest and post-test between both POM variables (conceptual and procedural knowledge), and all the three LMT variables (number and operations, algebra and geometry).

Table 2.2.

Correlations Post-test Data LMT (NO, ALG, GEOM) and POM (CK, PK)

		PK	CK	NO	ALG
CK	Correlation	0.80			
	Sig. (2-tailed)	0.00			
	N	30			
NO	Correlation	0.71	0.80		
	Sig. (2-tailed)	0.00	0.00		
	N	30	30		
ALG	Correlation	0.75	0.89	0.80	
	Sig. (2-tailed)	0.00	0.00	0.00	
	N	30	30	30	
GEO	Correlation	0.62	0.67	0.59	0.75
	Sig. (2-tailed)	0.00	0.00	0.00	0.00
	N	30	30	30	30

Note. Correlations are significant at 0.05 level. CK = conceptual knowledge; PK = procedural knowledge; NO = number and operations; ALG = algebra; GEOM = geometry.

The concurrent validity of the POM instrument conceptual and procedural values was examined by correlating the measures of the POM instrument mathematical values (conceptual and procedural values) to the NELSON instrument measures of beliefs (manipulatives and attitudes). The results show no significant correlations between the two instrument measures of mathematical values. Hence, no evidence of concurrent validity for the POM instrument measures of conceptual and procedural values were found during the two administrations of the surveys as depicted in Table 2.3 and Table 2.4. These results will be addressed further in the discussion.

Table 2.3.

Correlations using Pretest Data between NELSON (MANIP, ATT) and POM values (PV,VC)

		PV	CV	MANIP
CV	Correlation	-0.18		
	Sig. (2-tailed)	0.55		
	N	13		
MANIP	Correlation	0.23	-0.11	
	Sig. (2-tailed)	0.44	0.71	
	N	13	13	
ATT	Correlation	0.16	0.00	0.24
	Sig. (2-tailed)	0.59	0.99	0.42
	N	13	13	13

Note. Correlations are not significant at 0.05 level. CV = conceptual values; PV = procedural values; MANIP = manipulatives; ATT = attitudes.

Table 2.4.

Correlations using Post-test Data between NELSON (MANIP, ATT) and POM values (PV, CV)

		PV	CV	MANIP
CV	Correlation	-0.52		
	Sig. (2-tailed)	0.06		
	N	13		
MANIP	Correlation	-0.05	0.23	
	Sig. (2-tailed)	0.86	0.43	
	N	13	13	
ATT	Correlation	0.27	0.19	0.86
	Sig. (2-tailed)	0.36	0.52	0.00
	N	13	13	13

Note. Correlations are not significant at 0.05 level. CV = conceptual values; PV = procedural values; MANIP = manipulatives; ATT = attitudes.

Further Evidence of the Concurrent Validity of the POM Instrument

As stated in previous research, the POM instrument was designed to measure conceptual and procedural mathematical knowledge for junior intermediate preservice teachers (Kajander et al., 2006). The evidence of validity shown above was based on a small sample of 30 in-service teachers' data that was previously collected in Kajander et al (2006) study and used for this study. To provide further evidence of the validity of the POM when measuring conceptual and procedural mathematical knowledge on preservice teachers as stated in the methodology, a new set of data was collected from junior intermediate preservice teachers 2006-2007 from the Bachelor of Education at Lakehead University who answered the POM instrument and a subset of the LMT instrument at the same time (post-test). Scores were computed from both instruments and then correlated. The results once again revealed a positive significant correlation between both POM and LMT instruments when using just preservice teachers' data $r(77) = 0.58, p < 0.01$.

2.4 Discussion

Reliability and Validity of the POM

This study provides evidence of reliability and concurrent validity of the POM instrument (Kajander et al., 2006) in terms of measuring conceptual and procedural mathematical knowledge as well as conceptual and procedural mathematical values. The strong Alpha Coefficients ranging from 0.72 to 0.83 indicate that the POM instrument has internal consistency, when measuring junior intermediate preservice teachers' mathematical knowledge and values during the pretest and post-test. The lowest reliability of 0.72 occurred for the measures of conceptual and procedural values during the pretest. Hence, the reliability of the measures of conceptual and procedural mathematical knowledge of the POM instrument within the strands of number and operations, algebra and geometry is consistent with the reliability of the LMT measures with Cronbach's Alpha coefficients ranging from 0.71 to 0.84 as found by Hill and Ball (2004). Furthermore, the reliability of the measures of conceptual and procedural mathematical values of the POM is consistent with the reliability measures of the NELSON instrument with a Cronbach's Alpha coefficient of 0.81 (Ross et al., 2003).

The correlations between the POM and LMT instruments measures of conceptual and procedural mathematical knowledge within the strands of number and operations, algebra and geometry during the pretest and post-test as depicted in Table 2.1 and Table 2.2 show strong evidence of concurrent validity of the POM. Thus, evidence for validity of the POM measures of mathematical knowledge is consistent with the LMT measures of mathematical knowledge (Hill & Ball, 2004). Although the correlations between the two instruments were strong in terms measurements of mathematical knowledge within the strands of number and operation, algebra

and geometry, it was relevant to point out that the data used to show evidence of concurrent validity of the POM instrument for this study related to in-service teachers. Furthermore, the POM instrument was designed to measure mathematical knowledge of junior intermediate preservice teachers (Kajander et al, 2006) while the LMT instrument was designed to measure junior intermediate in-service teachers' mathematical knowledge (Hill & Ball, 2004). For that reason, a new subset of data from the LMT instrument was collected from preservice teachers and compared to preservice teachers' POM measures of mathematical knowledge taken at the same time. Significant correlations were found with the new set of data between the two instruments, POM and LMT; however, the correlation $r(77)=0.58, p<0.01$ was not as strong as the correlations found with the in-service teachers between the two instruments as shown in Table 2.1 and Table 2.2.

The correlations between the POM measures of conceptual and procedural mathematical values and Nelson measures of manipulatives and attitudes during the pretest and post-test as depicted in Table 2.3 and Table 2.4 are not significant and therefore, do not show evidence of concurrent validity of the POM in terms of measurements of conceptual and procedural mathematical values. Thus, the POM instrument measures of conceptual and procedural mathematical values in terms of validity are not consistent with the NELSON instrument measures of manipulatives and attitudes (Ross et al., 2003).

The results of this study may have implications in terms of assessing junior intermediate preservice teachers' conceptual and procedural mathematical knowledge and values. For instance, stronger correlations are needed between the POM and LMT measures of mathematical knowledge in order to show stronger evidence of the validity of the POM when assessing junior

intermediate preservice teachers' mathematical knowledge. Although strong correlations were found between the POM and LMT when measuring mathematical knowledge of junior intermediate in-service teachers, the results found from the junior intermediate preservice teachers' data suggest that new data collection pre and post with a larger sample size should be considered for both instruments. In the present study, the junior intermediate preservice teachers' sample size was limited by the number of students in the class and time constraints restricted data collection. This is why only a subset of the LMT that more closely related to the POM instruments was used to show further evidence of validity of the POM instrument when measuring junior intermediate preservice teachers' mathematical knowledge. Nonetheless, based on the strong evidence of validity of the POM instrument when measuring junior intermediate in-service teachers' mathematical knowledge as well the evidence found when measuring junior intermediate preservice teachers' mathematical knowledge, plus evidence of face validity from previous studies (Kajander, 2005) provided enough evidence to use the POM instrument to collect measures of mathematical knowledge from junior intermediate preservice teachers in the second study.

The results of the present study also may have implications in terms of measuring conceptual and procedural mathematical values since no evidence of concurrent validity was found when the POM measures of mathematical values were correlated to the NELSON measures of manipulatives and attitudes as shown in Table 2.3 and Table 2.4. Hence, a construct validity test should be considered, in which it should be hypothesized that preservice teachers' measures of mathematical values are composed of two dimensions. In other words, it should include measures of conceptual and procedural mathematical values. An exploratory factor

analysis should be conducted to determine if there are indeed two underlying dimensions when measuring preservice teachers' mathematical values. This construct validity test was not implemented in the present study because in order to conduct an exploratory factor analysis to test the hypothesis stated above a much larger data sample size is required. Based on the face validity of the POM instrument from previous studies (Kajander, 2005) and the literature, which states the importance of teachers' mathematical values in mathematics teaching to influence students' perceptions of mathematics learning (Ambrose, 2004), it was felt that there was enough evidence to proceed to use the POM instrument to collect preservice teachers' measures of conceptual and procedural mathematical values.

CHAPTER 3 – PRESERVICE TEACHERS' MATHEMATICAL DEVELOPMENT

3.1 Introduction

Mathematical Learning

Learning relates to growing new structures in the brain. These structures are composed of neurons and dendrites. The interconnectivity of dendrites and association of neurons create learning networks. These learning networks can be separated into conceptual and procedural knowledge (Smilkstein, 1993). Some cognitive theorists have used these learning networks to create a distinction between conceptual and procedural knowledge, where conceptual knowledge is defined as the core knowledge of concepts, “knowing that”, and procedural knowledge as the steps to solve a problem or acquire a goal, “knowing how” (Byrnes & Wasik, 1991). The distinction between conceptual and procedural knowledge has been applied to cognition such as memory and mathematical learning (Hiebert, 1987). In mathematical learning, conceptual knowledge precedes procedural knowledge and it forms the basis on which new procedures are acquired; but both conceptual and procedural knowledge cannot be mutually exclusive and must interact over time when solving problems (Byrnes & Wasik, 1991).

McCormick (1997) conducted a qualitative study which relates to conceptual and procedural knowledge in mathematics education. This study described how much contrast there is between computational procedures and the understanding of concepts when solving mathematical problems. More specifically, McCormick alludes that in countries like England and Wales, there has been a swing to design a curriculum with more emphasis on conceptual knowledge but using procedures as balance. McCormick also adds that the real world poses unpredictable challenges to students and quite often students find themselves in a situation

where they must inter-relate theory and practice to be able to solve problems. McCormick concluded that in order to perform and solve mathematical problems in our technological world, conceptual and procedural knowledge must be linked.

The link between conceptual and procedural mathematical knowledge has been studied in a more profound context by Mason and Spence (1999). In their study, both researchers linked conceptual and procedural knowledge through a new definition called "knowing-to". This definition of "knowing-to" refers to active knowledge or the knowledge present to solve problems in a fresh situation. The researchers also found that the traditional way of teaching mathematics refers to "knowing-about" in which students do not deviate from the examples taught in the classroom. More specifically, students lack the ability to draw paradigms to new situations. The researchers concluded that the absence of "knowing-to" might limit students' mathematical development. More explicitly, the absence of "knowing-to" is what prevents students and teachers from responding creatively in the moment to solve mathematical problems that are different from the ones experienced in the classroom.

Nonetheless, students' mathematical development can go through procedurally oriented phases before students can understand the meaning of the mathematical concepts. In other words, procedural knowledge can be integrated or assimilated into one's conceptual schema (Piaget, 1977). It is also important to realize that in other situations of mathematical development, conceptual and procedural knowledge interconnect to one another in mutually supportive and integrated ways. Consequently, students can improve their procedural knowledge by making use of written conceptual thoughts about mathematical notions (Rittle-Johnson, Siegler & Alabali, 2001).

The integration of conceptual and procedural knowledge has led to the development of alternative instructional mathematical strategies. These instructional mathematical approaches such as “iterative process” help highlight the importance of each mathematical lesson so that students can amalgamate conceptual and procedural mathematical knowledge leading to greater learning. The integration of conceptual and procedural knowledge reduces “overgeneralization (applying a concept or procedure in an inappropriate way) and under-generalization (failing to transfer to appropriate tasks)” (Rittle-Johnson & Koedinger, 2002, p. 974).

Sherin and Fuson (2005) conducted a study to demonstrate how children's multiplication strategies change because of the integration of their conceptual growth relating to number and computational procedures. The researchers found that although procedural knowledge plays a big role in multiplication, conceptual understanding cannot be isolated from the learning process. In other words, conceptual understanding should merge with practice in different ways to enhance students' comprehension of patterns and structures across computational resources, for example by teaching multiplication and division together. It is Sherin and Fuson's belief that this approach will help students improve their understanding of patterns while acquiring a rich network of concepts and multiplication strategies.

Mathematical Values

Besides merging conceptual and procedural mathematical knowledge to enhance students' comprehension of mathematics, it is also important to realize that teachers' beliefs about mathematics influence students' perceptions of mathematical concepts and procedures. In mathematics education values are defined as deeply held beliefs about what is important in mathematics and these values have a powerful impact on teaching (Ernest, 1989). In some cases, these values can discourage students from applying their mathematical knowledge to real life

situations or other situations outside the classroom structure (Boaler, 1999). Therefore, classroom experiences and teachers' mathematical values develop students' perceptions of mathematics (Ibid).

According to Boaler's (1999) research, some students can create their own mathematical perception and believe that mathematics is just made of numerous rules, formulas and equations that need to be memorized; but in other cases, students may believe that mathematics is about interacting with the problem, being creative and finding a solution without following a fixed classroom structure. These beliefs would have been influenced by environments created by teachers they have had, resulting in deeply held values about what is important in mathematics. Hence, it is essential for teachers to develop a profound understanding of fundamental mathematics, for instance, how mathematical procedures work as well as the understanding of mathematical concepts, in order to change their values (Ma, 1999).

Developing a deep understanding of mathematics not only influences teachers' mathematical values, but also changes the way they teach in the classroom (Ernest, 1989; Stipek, Givvin, Salmon & MacGyvers, 2001). Recent studies have shown that teachers' mathematical conceptions, ideologies and development influence students' mathematical values and efforts in learning mathematics (Schommer-Atkins, Duell & Hutter, 2005). This is why in mathematics education research, values are considered an essential part of the educational process. In mathematics education, teachers' values are a crucial influence in the ways students choose to engage or not engage in a mathematics course (Bishop et al., 2006). Values can also affect mathematical performance based on the feelings that mathematics evokes in many adults and children. For instance, after grade three many children have already developed their opinion about mathematics (Franz, 2000). In high school and university, the situation becomes worse,

with many students avoiding mathematical courses because they believe that mathematics is dull and senseless (Ibid).

Teachers' Mathematical Knowledge

In addition to the classroom influence of teachers' beliefs, teachers' knowledge of mathematics has become an area of concern in the last two decades. There has been an implicit disagreement over the knowledge of mathematics that teachers need to know in order to teach. Some researchers argue that teachers' capabilities in higher level mathematics are the most important attributes (Hill & Ball, 2004). Others believe that higher level mathematics ability is not sufficient to teach, and believe that teachers must have knowledge about how to teach mathematics to students (Ma, 1999; Ambrose, 2004; Schommer-Aikins et al, 2005). Hence, teaching mathematics to students should be treated as a system of interacting features to minimize the gap between teaching and students' learning (Hiebert et al., 2005). This system of interacting features such as the knowledge that teachers and students bring to the lesson, tasks presented in the classroom, students' discourse and participation, the assessments and the physical materials available for teaching is what defines the learning conditions for the students (Ibid). Once these learning conditions are defined then what matters is how these features together are enacted with students to help them achieve their goals (Ibid).

After all, teaching mathematics is not simply knowing in front of the students. Teaching mathematics entails making the content accessible, interpreting students' questions and ideas, and being able to explain concepts and procedures in different ways (Hill, Sleep, Lewis & Ball, 2007). Therefore, teachers must be able to understand and explain to their students why mathematical algorithms work and how these algorithms may be used to solve problems in real life situations (Ibid). Hence, the skills required for teaching mathematics are multidimensional;

this means that this capacity does not relate to one general factor such as mathematical ability or teaching ability but rather, it relates to a system of features that interact with one another to help teachers transfer mathematical knowledge to their students (Ibid). In this system of interacting features for teaching mathematics to students, teachers may opt to use a constructivist model for teaching mathematics, in which students may actively contribute to the construction of their mathematical knowledge rather than being passive recipients of information (Johnson & Munakata, 2005). Furthermore, in this constructivist model approach for teaching mathematics, the teacher may be a facilitator or coach, who assists his or her students to construct their own conceptualizations and solutions to mathematical problems (Piaget, 1997). Hence, students' mental mathematical abilities may develop through various paths of discovery, which may have been created by the teacher (Clark, 1999).

It is important for teachers to keep in mind, however, that implementing a constructivist approach in mathematical learning is not an easy task; the process involves modifying aspects of established knowledge, methods of reasoning and technical vocabulary to construct new mathematical knowledge (Shechter, 2001). Moreover, in the constructivist model approach, students are supposed to reject the idea of acquiring knowledge by being told or lectured about it; but rather build their own mathematical knowledge by working together, exploring patterns, testing their own hypothesis, reflecting on concepts and applications and justifying their reasoning (Francisco, 2005). Previous research has shown that instruction based on constructivist principles encourages students to construct the necessary mathematical knowledge to solve a problem (Kroesbergen & Van Luit, 2002). For instance, Steele (1994) conducted a study on helping preservice teachers confront their conceptions about mathematics and mathematics teaching and learning. In Steele's study, the researcher wanted prospective mathematics teachers

to learn to think mathematically and understand the nature of mathematics through problem solving. The study found that through a constructivist approach, teachers can create an environment that allows students to construct their own knowledge by linking mathematical concepts to procedures through the use of physical material in the classroom such as manipulatives.

Manipulatives and Constructivism

The appropriate use of manipulatives in mathematics teaching can help teachers initiate students' mathematical thinking, and elicit students' creativity and problem-solving skills (Steel, 1994). Even students who have not been engaged by formal mathematical teaching methodologies often find productive ways through the use of manipulatives toward a mathematical solution (Marshall, 2004). Furthermore, the manipulatives can be used to link students' concrete experiences to mathematical concepts and generalizations, in order to give them meaning, but of course this process must be supported and encouraged by the teacher as shown in previous research (Kamii, Rummelsburg & Kari, 2005). Kamii, Rummelsburg & Kari (2005) conducted a study on teaching arithmetic to an experimental group composed of low-performing first graders. Through the use of arithmetic manipulatives and word problems, the researchers stimulated students' exchange of viewpoints. At the end of the year, children in the experimental group were compared with low-performing first graders who received traditional instruction. The researchers' findings indicate that children in the experimental who did not have traditional instruction in arithmetic, but used manipulatives to stimulate their mathematical thinking did considerably better than those who received traditional mathematical instruction without manipulatives during the entire year.

The point is that features of mathematics teaching such as the use of manipulatives, a de-emphasis on the use of paper and pencil skills, and a focus on students' active construction of mathematical knowledge and communication about solutions to challenging mathematical problems are common to standards-based curricula (Hiebert et al., 2005) and have a common set of goals aligned with the Standards (National Council of Teachers of Mathematics, 1989; 1990; 2000).

Mathematics Reform and Constructivism

The recommendations for the new mathematics standards-based curricula began with a shift toward deep conceptual understanding of mathematics along with procedural fluency developed by utilizing constructivist principles (Ross, Hogaboam-Gray & McDougal, 2002). These constructivist principles as stated in other research studies encourage students to interact with each other and the environment to construct and discover new knowledge (Kroesbergen & Van Luit, 2002). Hence, the constructivist approach has inspired the mathematics reform by providing the principles and theoretical foundations of the reformed elementary and high school curriculum in Ontario. Such principles and theoretical foundations are also described more fully in the National Council of Teachers of Mathematics (NCTM) *Principles and Standards*, which is typically used as a guide for defining mathematics reform by those who make decisions about mathematics education of students from prekindergarten to grade 12 (Hickey, Moore & Pellegrino, 2001; NCTM, 1989;1990; 2000; Ross et al, 2002). The NCTM *Principles* include: equity, curriculum, teaching, learning, assessment and technology. These principles describe high quality mathematics education by creating a coherent curriculum, which effectively organizes and integrates important mathematical ideas; by describing the implementation of effective teaching, which requires knowing and understanding mathematics, students as learners

and pedagogical strategies; by supporting student learning with mathematical understanding; by integrating assessment and instruction, in which assessment provides the information teachers need to make appropriate instructional decisions; and finally by implementing technology as a way of engaging students in mathematical learning as well as facilitating their understanding of mathematical concepts when solving mathematical problems (NCTM, 2000).

In addition, the content strands of the NCTM *Standards* include: number and operations, algebra, geometry, measurements, and data analysis and probability. Number and operations are essential in the NCTM *Standards* because historically speaking, number and operations have been a cornerstone of the mathematics curriculum; algebra because it emphasizes relationships among quantities, functions and analysis of change; geometry because of spatial visualization and reasoning; measurement because of practicality and pervasiveness of measurement in so many aspects of every day life; data analysis and probability because they will help students make decisions in businesses, politics, research and other aspects of every day life in which students can formulate and answer questions using methods for data analysis and make inferences and conclusions (Ibid).

Students' Reform-based Mathematical Learning

The NCTM *Principles and Standards* have provided some of the chief characteristics of the mathematics education reform curriculum in Ontario (Ross et al., 2002). Although the implementation of the reformed mathematics curriculum is not consistent across all the elementary and secondary schools in Ontario (Ibid), the literature shows that students taught in a reform-based approach have more opportunities to enjoy mathematical learning without memorizing formulas. Rather, by exploring concepts, and that such an approach minimizes students' fears and concerns about mathematical performance and encourages students to learn in

a classroom climate in which risk-taking is encouraged and supported by the teacher and other students in the classroom (Hiebert, 1999). Furthermore, students taught using a reform-based approach are able to acquire greater skills in using mathematical tools to improve their prior knowledge and construct new knowledge than those taught with the traditional mathematics approach, in which the emphasis is more in mathematical procedures (Romberg, 1997). For example, Fennema, Franke and Carpenter (1993) tracked a teacher over four years as the teacher implemented a program that focused on helping students construct deep understanding of mathematical concepts and strategies for solving problems embedded in their everyday experiences. The researchers found that this teacher had a profound effect on her students. Her students solved more complex mathematical problems than other grade 1 pupils and adapted their mathematical procedures in response to problem requirements. Villasenor and Kepner (1993) found that children who were in a classroom that fully implemented mathematics reform were also more successful in traditional mathematics tasks. Heibert (1999) found that reform-based teaching programs promote students' deep understanding of mathematics. Cardelle-Elawar (1995) found that providing students with reform-based instruction and including mathematical tasks embedded in real-life experiences contributed to superior grades 3-8 student performance on mathematical problem-solving. Stein, Remillard and Smith (2007) found that the learning environment is a critical factor in students' mathematical learning and that the curriculum implemented in the classroom is more effective when the normative practices in the classroom promote a reform-based learning environment associated with students' mathematical understanding in problem-solving. The researchers also found that students' mathematical achievement was highest among students who experienced a standards-based curriculum in a reform-based learning environment over two consecutive years.

The implementation of reform mathematics, however, is a difficult process (Senger, 1998). Even teachers chosen as exemplars of reform mathematical practices regress from reform methods to traditional methods (Ibid). Indeed, some research studies show that the most challenging in the implementation of reform mathematics is the management of students' talk about mathematical reasoning, including finding the right balance between encouraging student construction of knowledge without leaving them floundering (Ball, 1993; Ross, Haimes, & Hogaboam-Gray, 1996; Smith, 2000). For example, Bosse (1998) studied the recommendations of the National Council of Teachers of Mathematics (NCTM) *Standards* (NCTM 1989; 2000) in light of a historical perspective in the United States. Bosse's paper focuses on the educational high school reform movement that took place in the United States in the mid-1990's. In this study, Bosse emphasizes that the NCTM *Standards* expect K-12 teachers to grasp and develop new curricula philosophically consistent with these *Standards* and ideas of mathematical reform. Bosse's findings indicate that teachers and the public perceived the new curricular suggestions to be quite extensive and beyond the expertise of the K-12 teachers. In addition, insufficient teacher training did not adequately prepare teachers to continue the reform effort. Earl and Southerland (2003) conducted a similar study but with an emphasis on the perception of students on the impact of reform education in Ontario secondary schools. The researchers found that while some students were very accepting of the new curriculum, others found it to be very condensed and difficult. The researchers concluded that the reform had a profound affect on students, both personally and academically, and that students' perspectives should be considered in providing valuable information for educators and policy makers.

Despite the concerns related to the reform movement, teachers and policy makers felt that with the traditional curriculum, students were far from understanding concepts in mathematical

learning (Kenney & Silver, 1997). The reasons for this mathematical deficiency were many; in some situations, students did not have the opportunity to learn important mathematics (NCTM, 2000). In other instances, the curriculum did not engage them (Ibid). In addition, the traditional curriculum often did not prepare students to enter university with satisfactory mathematical understanding to think conceptually at the university level (Kajander & Lovric, 2005). The reform process is intended to offer an engaging curriculum based on constructivist principles, which allows students to develop mathematical understanding and proficiency (McCormick, 1997; Steele, 1994). Mathematical understanding and proficiency open the doors to productive futures; whereas, a lack of mathematical competence keeps those doors closed (NCTM, 2000). Everyone needs to understand mathematics and all students should have the opportunity and support necessary to learn mathematics with a profound understanding (Expert Panel on Literacy and Numeracy, 2005; NCTM, 2000).

In order for students to have a more profound understanding of mathematics, it is important to develop teachers' mathematical content knowledge and values so that teachers can change the way they teach in the classroom to influence students' mathematical learning using a more reform-based approach (Ball, 1990; Ma, 1999; Stipek et al, 2001). Indeed, the most powerful mechanism for overcoming the barriers to mathematics reform teaching is professional development (Hill, Schilling & Ball, 2005). It has been shown that teachers can complement their mathematical knowledge by additional professional development (Ibid). Since teachers' mathematical development contributes to students' mathematical success (Greenwald, Hedges, & Laine, 1996) such professional development is of crucial importance.

Teachers' Mathematical Development

One way to facilitate teachers' mathematical development is by deepening their mathematical understanding and changing their epistemological beliefs via professional development experiences (Hill & Ball, 2004; Kajander, Keene, Siddo & Zerpa, 2006). Kajander et al (2006) conducted a study of 40 in-service grade 7 teachers from urban and rural areas. Teachers were tested before and after an eight-month intervention. Professional development experiences were provided for the volunteering teachers with an emphasis on conceptual understanding of fundamental mathematics, appropriate use of manipulatives, use of representations and differentiated instruction. This included three days of professionally delivered in-service training on number and operation, as well as online courses for some of the participants. The researchers found that changes in mathematical knowledge and values were possible even in such a short time. Also, teachers' beliefs about the need to focus on procedural learning decreased, which was argued as indicative of a shift towards a more reformed based conception as shown in previous studies (Kajander, 2005). Furthermore, it should be explained here that in reform literature, it is argued that conceptual aspects of learning also promote procedural learning without specific focus on procedural skills (NCTM, 2007). This is why a diminished emphasis on procedural values may be an indicative of a shift to a more reform-based conception (Kajander, 2005).

Ball (1996) also found that the use of professional development experiences can change teachers' traditional ways of mathematical thinking. It can shape teachers' understanding of mathematical concepts and help them be more flexible when listening to students' new ideas and innovations. Learning about this type of teaching however, requires more than knowledge and skills, it entails patience, curiosity, generosity, confidence, trust and imagination. According to

Ball, some of these qualities can certainly be acquired and enhanced through professional development. The professional development must convey a learning process based on critical discussions in which teachers interact and exchange ideas. Such a learning process is different from traditional professional development in which teachers just collect handouts and reproducible worksheets and eagerly file them. Instead, a professional development experience should provide teachers with information, tips, guidance and ideas complemented with critical discussions. It should also include a deep conceptual re-examination of the mathematics itself. For that reason, teachers need experience with linking concrete ideas and mathematical models to the generalizations, which may be embedded in the procedures. Hence, this type of mathematical practice often goes far beyond teachers' prior experiences. Ball concludes that the lack of critical discussion and reflection during professional development experiences may cause teachers to formulate their own interpretation and implementations, which leads to individualism and isolation of teaching. This individualism makes it difficult to develop common standards for teaching mathematics in which teachers will have the opportunity to debate, improve, and change their understanding of mathematical knowledge and values.

For that reason, the professional development experience should include a vision that requires teachers to shift their mathematical thinking and values so that they have different ideas about what they should be trying to accomplish in the classroom to engage and improve students' mathematical knowledge (Sowder, 2007). Indeed, this shift in teachers' mathematical thinking and values should be initiated during their preservice training experiences because the demands on teachers are more intense during their in-service teaching career and therefore, shifting teachers' mathematical knowledge and values at the in-service level is more difficult (Ibid). Furthermore, reform teaching preparation must involve interaction between preservice

and in-service teachers since preservice teachers will be observing and assisting mathematical classes delivered by in-service teachers who may or may not be mathematics reformers; therefore, preservice teachers must learn about the sources and obstacles to curriculum reform (Boyd, 1994).

Hence, universities need to prepare preservice teachers for reform-based teaching in innovative classrooms, so that preservice teachers can gain more experience in how to implement students' reform-based learning into their classroom practices. Furthermore, universities need to develop partnerships with schools to address the need to prepare preservice teachers to meet the demands of the new reform-based mathematics curricula (Ibid). Support for effective teaching needs to begin in preservice teachers' education. For that reason, this study has focused on the evolving knowledge and values of preservice teachers based on a mathematics methods course in education.

3.2 Method

Purpose

There were three goals that guided this study. The first goal was to examine preservice teachers' change in mathematical knowledge and values during a one year teacher certification program, which includes a mathematics methods course. The second goal of the study was to investigate the relationship between preservice teachers' previous academic background to changes in their mathematical knowledge and values. The final goal of the study was to investigate if a regression model could be utilized to predict change in conceptual knowledge by identifying the variables that were significantly related to change in conceptual mathematical knowledge such as preservice teachers' mathematical background, preservice teachers'

conceptual and procedural mathematical knowledge and values at the pretest, and mathematics courses taken in high school and university.

Research Questions

1. To what extent do preservice teachers change their conceptual and procedural mathematical knowledge and values during a Bachelor of Education program which includes a mathematics methods course?
2. How does academic background influence preservice teachers' conceptual and procedural mathematical knowledge and values?
3. Can a regression model be used to predict change in conceptual mathematical knowledge?

Design

The design used for this study was a One-Group Pretest-Posttest design. In this design a single group is measured not only after being exposed to an intervention but also before. One of the advantages of this design is that it does not require a control group. One of the disadvantages of this design, however, is that there are some uncontrolled variables (history, maturation, instrument decay, statistical regression and attitude of subjects) that may influence the outcome of the study and therefore, are considered threats to the internal validity of the data (Linn, 1989). Nonetheless, this design has been used in other educational research studies (Ibid). For this study, though, the One-Group Pretest-Posttest design was implemented because the mathematics methods course is a compulsory course in the professional year for junior intermediate preservice teachers in the Bachelor of Education program at Lakehead University; therefore, it was not

possible to create a control group. Hence, the One-Group Pretest-Posttest design although not the strongest design, was appropriate for this study.

Participants

Data collected from 111 junior intermediate preservice teachers were used for this One-Group Pretest-Posttest design study to examine preservice teachers' change in conceptual and procedural mathematical knowledge and values based on a mathematics methods course in education. All participants were Bachelor of Education students from the junior intermediate professional year program at Lakehead University in 2005-2006. The participants were recruited from the EDUC 4151 (Curriculum Instruction in Mathematics) classes in the Faculty of Education. All participants signed a consent letter outlining the participants' rights, including their right to withdraw at any time from the study, their voluntary participation in the project and their right to know the purpose of the study (See Appendix B).

Instrument and Measurements

The POM questionnaire was administered at the beginning of the EDUC 4151 mathematics methods course (Curriculum and Instruction in Mathematics) to 111 junior intermediate preservice teachers to collect the pretest data and after six months, the POM was administered again to collect the post-test data. The junior intermediate preservice teachers commenced the EDUC 4151 mathematics methods course (Curriculum and Instruction in Mathematics) in September, 2005 and at this time, a pretest was administered to the preservice teachers using the Perceptions of Mathematics questionnaire (POM) (Kajander, 2005). The EDUC 4151 course included a focus on conceptual understanding of mathematics using a reform-based approach as explained in the introduction of this research thesis. The course

emphasized how concepts relate to procedures and how these procedures can be used once the mathematical concept is understood. The EDUC 4151 mathematics methods course ended in February, 2006, and the Perceptions of Mathematics questionnaire (POM) was administered to the preservice teachers again to collect the post-test data.

The Perceptions of Mathematics questionnaire (POM) (Kajander, 2005; Kajander, Keene, Siddo & Zerpa, 2006) was used to measure relative levels of preservice teachers' conceptual and procedural mathematical values (perceptions, assumptions, ideologies and beliefs about what is important in mathematics learning) and relative levels of conceptual (meaning of concepts) and procedural (knowledge of method or skill) mathematical knowledge. The strand measurements for conceptual and procedural mathematical knowledge included number and operations, algebra and geometry. Hence, four dependent variables were measured in this study – conceptual and procedural knowledge, and conceptual and procedural values. All the dependent variables were scaled out of 10 and provided information on preservice teachers' levels of mathematical knowledge and values. This information was used to assess changes in preservice teachers' mathematical understanding during the year.

The importance of understanding how mathematical procedures work and how these procedures interlink with concepts when connecting ideas or solving mathematical problems, as addressed by the measures of conceptual values and conceptual knowledge respectively, were of interest in this study (Mason & Spence, 1999; Ma, 1999). Furthermore, some of the mathematical knowledge questions administered on the POM instrument to measure mathematical knowledge were based on Ma's (1999) interview questions for elementary teachers. These questions had an emphasis on profound understanding of fundamental mathematics (Ma, 1999). The questions entailed calculations with whole numbers, decimals,

fractions and integers, and included operations of subtraction, multiplication and division as well as linear relations, area and perimeter as posed by Ma's (1999) interview questions.

The demographic variables such as preservice teachers' mathematics courses taken in high school, mathematics courses taken at university, and academic background were also measured with the POM questionnaire and the answers were used to compare the level of mathematical knowledge and values between mathematically (mathematics, computer science and engineering majors) and non-mathematically (arts, humanities or social science majors) oriented preservice teachers. In addition, the relationship between each of these variables and change in conceptual mathematical knowledge (ΔCK) was examined to determine which variables influenced preservice teachers' change in conceptual knowledge the most.

Procedures and Analyses

During each administration of the POM questionnaire (pretest and post-test), preservice teachers were asked to answer the POM instrument mathematical knowledge questions using written mathematics procedures and the result of this work was scored as procedural knowledge. The preservice teachers were also asked to explain in writing the method used to answer each question by providing diagrams, models or using another example to support their explanation. The result of this work was scored as conceptual knowledge. For the POM values questions, preservice teachers were asked to answer each procedural and conceptual values question using an interval scale from 0 to 3, zero meaning low and three meaning high. For each preservice teacher, all the procedural values item scores were added together for the procedural values score, and all the conceptual values item scores were added together for the conceptual values score, giving two separate scores out of 30. Each result was divided by 3 to scale the results out of 10.

Scoring of the knowledge questions was done by two researchers who double marked papers and compared scores until consistency was attained. The procedural knowledge questions were added together and scaled out of 10. Similarly, the conceptual knowledge questions were added together and scaled out of 10. The results of the POM instrument mathematical knowledge and values were scaled out of 10 to choose a common scale in order to explore significant differences and effect sizes of the POM instrument with respect to other instruments measures of mathematical knowledge and values for future research.

Statistical Analysis of the Change in Mathematical Knowledge and Values

The data obtained from the POM questionnaire was statistically analyzed to determine the junior intermediate preservice teachers' change in conceptual and procedural knowledge, and change in conceptual and procedural mathematical values. The POM questionnaire also provided information to statistically appraise the effect of preservice teachers' mathematical background on their mathematical knowledge and values and investigate the best predictor of change in conceptual mathematical knowledge. Descriptive statistics was used to summarize, organize and better understand the data. The descriptive statistics provided a representation of the intervention effect (presumed to be mainly the EDUC 4151 course) in changing 111 preservice teachers' mathematical knowledge and values. T-tests for repeated measures were used to analyze the intervention effect between the pre and post-test for each dependent variable, namely conceptual knowledge (CK), procedural knowledge (PK), conceptual values (CV) and procedural values (PV). Cohen's effect sizes (Cohen, 1998) for repeated measures t-tests were computed for each dependent variable used in this analysis. Because multiple t-test comparisons were performed, a Bonferroni correction (Shaffer, 1995) was implemented to keep the type I error rate at 0.05. A Levene's test was implemented to check for homogeneity of variance. Since homogeneity of

variance was violated for measures of conceptual mathematical knowledge and measures of procedural mathematical values, nonparametric Wilcoxon's signed rank tests were conducted (Freund, 1999).

Statistical Analysis Based on the Influence of Preservice Teachers' Mathematical Background

From the data, 82 cases of non-mathematics preservice teachers with arts, humanities or social science background were compared to 29 cases of mathematics preservice teachers with engineering, computer science or technology backgrounds. Descriptive statistics was used to compare and provide a representation of the data in terms of means and standard deviations between the two groups of preservice teachers for the pre and post-test data. Four factorial ANOVAS with two independent variables each, time (pre and post) and mathematical background (non-mathematics, mathematics) were used to examine main effects and interaction effects of the independent variables on conceptual knowledge, procedural knowledge, conceptual values and procedural values. Cohen's effect sizes for analysis of variance (Cohen, 1998) measures were computed for each dependent variable used in the analysis. Since four 2-way ANOVAS were performed, a Bonferroni correction (Shaffer, 1995) was used to account for the entire variance in the analysis. A Kruskal-Wallis nonparametric test was conducted since there was a violation of the assumption of homogeneity of variance for conceptual mathematical knowledge measures.

Predicting Change in Conceptual Knowledge

Pearson's Product Moment Correlations were explored between change in conceptual knowledge (ΔCK) and each of the following pretest variables: conceptual knowledge (CK), procedural knowledge (PK), procedural values (PV), conceptual values (CV), academic background, mathematics courses taken in university and mathematics courses taken in high

school using the pretest data collected from 111 junior intermediate preservice teachers. The strength of the correlations was examined to identify which factors at the pretest (conceptual knowledge, procedural knowledge, procedural values, conceptual values, academic background, university mathematics courses and high school mathematics courses) significantly related to change in conceptual knowledge and potentially could be used as predictors of change in conceptual knowledge. A regression analysis was performed to create a linear mathematical model to predict change in conceptual knowledge as shown in Table 3.1. The beta standardized coefficients from the regression model were used to identify the variables or factors that had the highest impact on change in conceptual mathematical knowledge.

Table 3.1.

Predicting Model for Preservice Teachers' Change in Conceptual Knowledge

Dependent Variable	Independent Variable
y: change in conceptual knowledge	X ₁ : procedural mathematical knowledge
	X ₂ : mathematics courses taken in high school
	X ₃ : mathematics courses taken in university
	X ₄ : procedural mathematical values
	X ₅ : conceptual mathematical values
	X ₆ : academic background
Predicting Equation	
$y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + C$	
where: ($\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6$) are the unknown weights of the independent variables (C) constant value	

3.3 Results and Analysis

Changes in Conceptual and Procedural Mathematical Knowledge

An analysis of the data shows significant changes in junior intermediate preservice teachers' conceptual and procedural knowledge between the pretest and post-test. Descriptive statistics as shown in Table 3.2 indicated that the mean conceptual knowledge increased from the pretest (M=0.97, SD=1.41) to the post-test (M=4.78, SD=2.53). In addition, the mean procedural knowledge increased from the pretest (M=6.97, SD=2.09) to the post-test (M=8.47, SD=2.12). For conceptual knowledge the standard deviation is considerably smaller at the pretest; there was more score variability at the post-test.

Table 3.2.

Descriptive Statistics Measures of Conceptual and Procedural Knowledge

	N	Minimum	Maximum	Mean	Std. Deviation
PREPK	111	0.00	10.00	6.97	2.09
PRECK	111	0.00	8.00	0.97	1.41
POSTPK	111	2.00	9.00	8.47	2.12
POSTCK	111	0.00	10.00	4.78	2.53

Note. Scale used for the scores is from 0 to 10. PRECK = conceptual knowledge at the pretest; PREPK = procedural knowledge at the pretest; POSTCK = conceptual knowledge at the post-test; POSTPK = procedural knowledge at the post-test

In order to determine the significance of these differences between the pre and post-test data with respect to conceptual and procedural mathematical knowledge, two repeated measures t-test were performed. Because two repeated t-test measures were performed, a Bonferroni correction was implemented to keep the Type I errors rates at 0.05. The repeated measures t-test suggest that there was a significant improvement in preservice teachers' conceptual knowledge $t(110) = -15.04, p < 0.025, d = 1.43$ (large effect) and there was also a significant improvement in

preservice teachers' procedural knowledge $t(110) = -6.83, p < 0.025, d = 0.64$ (moderate effect). Since there was a discrepancy of variance between the pretest and post-test conceptual knowledge, a Levene's test for equality of variance was performed. The Levene's test revealed a significant difference between the pre and post-test variance, $F(1,220) = 38.6, p < 0.05$ for conceptual knowledge. Since homogeneity of variance was violated for pre and post-test conceptual knowledge measures, a nonparametric repeated measures Wilcoxon's signed ranks test was conducted. The results of the Wilcoxon's signed ranks test also revealed significant difference between the pre and post-test for conceptual knowledge measures, $Z = -8.5, p < 0.05$. In summary, the results of this analysis show that preservice teachers' conceptual and procedural mathematical knowledge increased significantly from the pretest to the post-test.

Changes in Conceptual and Procedural Mathematical Values

The results of this study also show significant changes in junior intermediate preservice teachers' conceptual and procedural values between the pretest and post-test. Descriptive statistics as shown in Table 3.3 indicate that the mean conceptual values increased from the pretest ($M = 7.83, SD = 1.22$) to post-test ($M = 8.45, SD = 1.36$). Note that the mean procedural values decreased from the pretest ($M = 7.89, SD = 1.22$) to the post-test ($M = 6.16, SD = 1.58$).

Table 3.3.

Descriptive Statistics Conceptual and Procedural Values

	N	Minimum	Maximum	Mean	Std. Deviation
PREPV	111	4.00	10.00	7.89	1.22
PRECV	111	4.30	10.00	7.83	1.22
POSTPV	111	1.30	9.30	6.16	1.58
POSTCV	111	3.00	10.00	8.45	1.36

Note. Scale used for the scores is from 0 to 10. PREPV = procedural values at the pretest; PRECV = conceptual values at the pretest; POSTPV = procedural values at the post-test; POSTCV = conceptual values at the post-test.

It was also important to assess how significant these differences were between the pre and post-test data with respect to procedural and conceptual mathematical values. In order to assess these differences, two repeated measures t-test were performed. Because two repeated t-test measures were performed, a Bonferroni correction was implemented to keep the type I errors rates at .05. The repeated measures t-test suggest that there is a significant improvement in preservice teachers' conceptual values $t(110) = -4.38, p < 0.025, d = 0.41$ (small effect), and a significant decrease in preservice teachers' procedural values $t(110) = 12.32, p < 0.025, d = 1.17$ (large effect). Since there was a discrepancy of variance between the pretest and post-test procedural values, a Levene's test for equality of variance was performed. The Levene's test revealed a significant difference between the pre and post-test variance, $F(1,220) = 7.48, p < 0.05$ for procedural values. Since homogeneity of variance was violated for pre and post-test procedural values, a nonparametric repeated measures Wilcoxon's signed ranks test was conducted. The results of the Wilcoxon's signed ranks test also revealed significant difference between the pre and post-test for procedural values, $Z = -8.22, p < 0.05$. In summary, the results of this analysis show that preservice teachers' conceptual mathematical values increased

significantly from the pretest to the post-test and their procedural mathematical values decreased significantly from the pretest to the post-test.

Influence of Preservice Teachers' Academic Background on their Mathematical Knowledge and Values

In this study, the 82 junior intermediate preservice teachers with arts and humanities backgrounds were compared to the 29 junior intermediate preservice teachers with mathematical backgrounds in order to find out if preservice teachers' backgrounds influence their mathematical knowledge and values. Descriptive statistics in Table 3.4 indicate that the mean conceptual knowledge score for preservice teachers with backgrounds in arts, humanities and social sciences ($M=0.74$, $SD=1.15$) was lower than the mean conceptual knowledge score for preservice teachers with backgrounds in science, engineering and computer science ($M=1.62$, $SD=1.84$) at the pretest. Moreover, the mean conceptual knowledge score for preservice teachers with background in arts, humanities and social sciences ($M=4.47$, $SD=2.60$) was lower than the mean conceptual knowledge score for preservice teachers with backgrounds in science, engineering and computer science ($M=5.65$, $SD=2.14$) at the post-test. The descriptive statistics also indicate that the mean conceptual knowledge score for preservice teachers with arts, humanities and social science backgrounds at the pretest ($M=.74$, $SD=1.15$) was lower than their mean conceptual knowledge score at the post-test ($M=4.47$, $SD=2.60$). Similarly, the mean conceptual knowledge score for preservice teachers with science, engineering and computer science backgrounds at the pretest ($M=1.62$, $SD=1.84$) was lower than their mean conceptual knowledge score at the post-test ($M=5.65$, $SD=2.14$).

Table 3.4.

Conceptual Knowledge Means and Standard Deviations Pre and Post-test Data by Preservice Teachers' Background

Test	Background	Mean	Std. Deviation	N
pretest	non-mathematics	0.74	1.15	82
	Mathematics	1.62	1.84	29
	Total	0.97	1.41	111
post-test	non-mathematics	4.47	2.60	82
	mathematics	5.65	2.14	29
	Total	4.78	2.53	111

Note. The scale used for the conceptual knowledge scores is from 0 to 10. Non-mathematics preservice teachers with arts, humanities and social science backgrounds; mathematics = preservice teachers with mathematics, computer science and engineering backgrounds.

A 2-way ANOVA (pre-post and background) was used to examine the main effects and interaction effects for these two factors on conceptual knowledge. Levene's test was conducted to examine equality of variance and the test revealed that the variance is heterogeneous, $F(3, 218)=16.29, p<0.05$. The analysis of variance revealed no significant interaction between the two independent variables (pre-post and background) on conceptual knowledge, $F(1,218)=0.310, p>0.0125$. There was however, a main effect of time (pre-post), which means that conceptual knowledge increased significantly from the pre to post-test for both groups (mathematics and non-mathematics preservice teachers), $F(1,218)=160, p<0.0125, f=1.46$ (large effect size) as depicted in Figure 3.1. Indeed, a review of Figure 3.1 suggests that the conceptual knowledge scores were significantly higher for mathematical preservice teachers at the pretest and post-test when compared to non-mathematical preservice teachers, $F(1,218)=11.22, p<0.0125, f=0.22$ (small effect size). Nonetheless, it is important to point out that there was a violation of the

assumption of homogeneity of variance for conceptual knowledge; for that reason, a nonparametric Kruskal-Wallis test was conducted and the results are similar to those found with the parametric test (ANOVA). Conceptual knowledge increased significantly for both groups of preservice teachers from the pretest to the post-test $\lambda^2=109$, $p<0.0125$ and there was a significant difference between both groups of preservice teachers $\lambda^2=6.58$, $p<0.0125$.

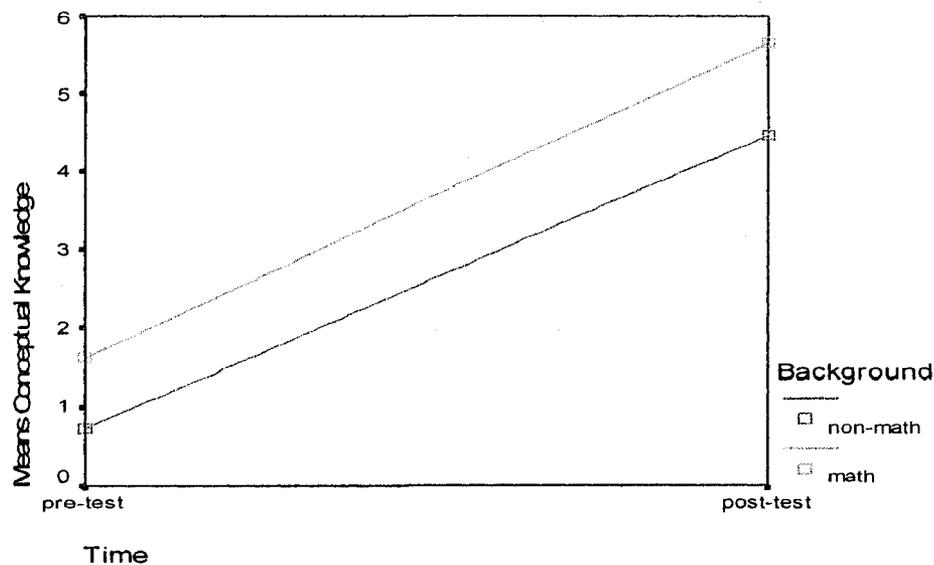


Figure 3.1. Means of Conceptual Knowledge Pre and Post-test Based on Preservice Teachers' Background.

Descriptive statistics in Table 3.5 indicate that the mean procedural knowledge score for preservice teachers with backgrounds in arts, humanities and social sciences ($M=6.73$, $SD=2.13$) was lower than the mean procedural knowledge score for preservice teachers with backgrounds in science, engineering and computer science ($M=7.65$, $SD=1.83$) at the pretest. Moreover, the mean procedural knowledge score for preservice teachers with backgrounds in arts, humanities and social sciences ($M=8.32$, $SD=2.32$) was lower than the mean procedural knowledge score for

preservice teachers with backgrounds in science, engineering and computer science (M=8.89, SD=1.39) at the post-test. The descriptive statistics also indicate that the procedural knowledge scores for preservice teachers with arts, humanities and social science backgrounds were lower at the pretest (M=6.73, SD=2.13) than their procedural knowledge scores at the post-test (M=7.53, SD=2.36). Similarly, the procedural knowledge scores for preservice teachers with science, engineering and computer science backgrounds were lower at the pretest (M=8.08, SD=1.19) than their procedural knowledge scores at the post-test (M=8.35, SD=1.16).

Table 3.5.

Procedural Knowledge Means and Standard Deviations Pre and Post-test Data by Preservice Teachers' Background

Test	Background	Mean	Std. Deviation	N
pretest	non-mathematics	6.73	2.13	82
	mathematics	7.65	1.83	29
	Total	6.97	2.09	111
post-test	non-mathematics	8.32	2.32	82
	mathematics	8.89	1.39	29
	Total	8.47	2.12	111

Note. The scale used for the procedural knowledge scores is from 0 to 10. Non-mathematics = preservice teachers with arts, humanities and social science backgrounds; mathematics = preservice teachers with mathematics, computer science and engineering backgrounds.

A 2-way ANOVA (pre-post and background) was used to examine the main effects and interaction effects for these two factors on procedural knowledge. Levene's test to examine equality of variance was conducted and the test revealed that homogeneity of variance was preserved, $F(3, 218)=1.27, p>0.05$. The analysis of variance revealed no significant interaction between the two independent variables (pre-post and background) on procedural knowledge,

$F(1,218)=.310, p>0.0125$. There was however, a main effect of time (pre-post), which means that procedural knowledge increased significantly from the pre to post-test for both groups (mathematics and non-mathematics preservice teachers), $F(1,218)=19.69, p<0.0125, f=0.30$ (medium effect size) as depicted in Figure 3.2. Indeed, a review of Figure 3.2 suggests that procedural knowledge scores were higher for mathematics preservice teachers at the pretest and post-test when compared to non-mathematics preservice teachers; however, the 2-way ANOVA revealed no significant differences between the two groups of preservice teachers, $F(1,218)=5.43, p>0.0125$.

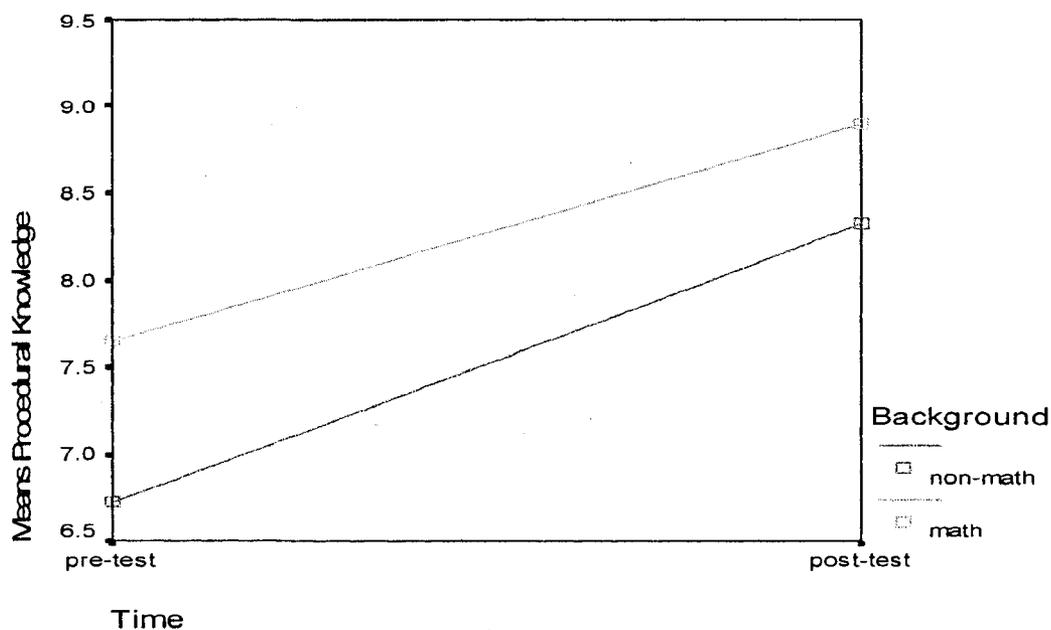


Figure 3.2. Means of Procedural Knowledge Pre and Post-test Based on Preservice Teachers' Background.

Descriptive statistics in Table 3.6 indicate that the mean conceptual values for preservice teachers with backgrounds in arts, humanities and social sciences ($M=7.74, SD=1.22$) was lower than the mean conceptual values for preservice teachers with backgrounds in science,

engineering and computer science (M=8.08, SD=1.19) at the pretest. Moreover, the mean conceptual values for preservice teachers with backgrounds in arts, humanities and social sciences (M=8.40, SD=1.45) was slightly lower than the mean conceptual values for preservice teachers with backgrounds in science, engineering and computer science (M=8.61, SD=1.08) at the post-test. The descriptive statistics also indicate that the conceptual values for preservice teachers with arts, humanities and social science backgrounds were lower at the pretest (M=7.74, SD=1.22) than their conceptual values at the post-test (M=8.07, SD=1.38). Similarly, the conceptual values for preservice teachers with science, engineering and computer science backgrounds were lower at the pretest (M=8.08, SD=1.19) than their conceptual values at the post-test (M=8.35, SD=1.16).

Table 3.6.

Conceptual Values Means and Standard Deviations Pre and Post-test Data by Preservice Teachers' Background

Test	Background	Mean	Std. Deviation	N
pretest	non-mathematics	7.74	1.22	82
	mathematics	8.08	1.19	29
	Total	7.83	1.22	111
post-test	non-mathematics	8.40	1.45	82
	mathematics	8.61	1.09	29
	Total	8.45	1.36	111

Note. The scale used for the conceptual values scores is from 0 to 10. Non-mathematics = preservice teachers with arts, humanities and social science backgrounds; mathematics = preservice teachers with mathematics, computer science and engineering backgrounds.

A 2-way ANOVA (pre-post and background) was used to examine the main effects and interaction effects for these two factors on conceptual values. Levene's test to examine equality of variance was conducted and the test revealed that homogeneity of variance was preserved,

$F(3, 218)=0.519, p>0.05$. The analysis of variance revealed no significant interaction between the two independent variables (pre-post and background) on conceptual values, $F(1,218)=0.108, p>0.0125$. There was however, a main effect of time (pre-post), which means that conceptual values increased significantly from the pre to post-test for both groups (mathematics and non-mathematics preservice teachers), $F(1,218)=9.05, p<0.0125, f=0.20$ (small effect size) as depicted in Figure 3.3. Indeed, a review of Figure 3.3 suggests that conceptual values were higher for preservice teachers with mathematics related backgrounds at the pretest and post-test when compared to non-mathematics preservice teachers; however, the 2-way ANOVA revealed no significant differences between the two groups of preservice teachers, $F(1,218)=1.97, p>0.0125$.

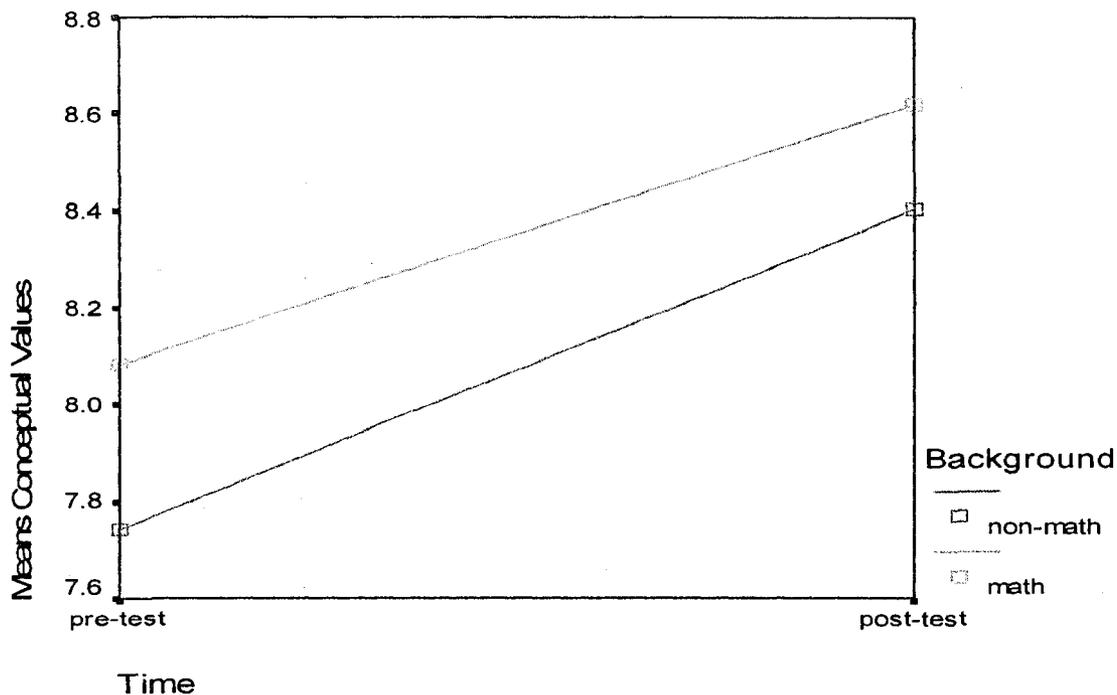


Figure 3.3. Means of Conceptual Values Pre and Post-test Based on Preservice Teachers' Background.

Descriptive statistics as shown in Table 3.7 indicate that the mean procedural values for preservice teachers with backgrounds in arts, humanities and social sciences ($M=7.92$, $SD=1.24$) were slightly higher than the mean procedural values for preservice teachers in science, engineering and computer science ($M=7.80$, $SD=1.20$) at the pretest. Moreover, the mean procedural values for preservice teachers with backgrounds in arts, humanities and social sciences ($M=7.07$, $SD=1.65$) were slightly higher than the mean procedural values for preservice teachers in science, engineering and computer science ($M=6.89$, $SD=1.68$) at the post-test. The descriptive statistics also indicate that the procedural values for preservice teachers with arts, humanities and social science backgrounds at the pretest ($M=7.92$, $SD=1.24$) were higher than their procedural values at the post-test ($M=7.07$, $SD=1.65$). Similarly, the procedural values for preservice teachers with science, engineering and computer science backgrounds at the pretest ($M=7.80$, $SD=1.20$) were higher than their procedural values at the post-test ($M=6.89$, $SD=1.68$).

Table 3.7.

Procedural Values Means and Standard Deviations Pre and Post-test Data by Preservice Teachers' Background

Test	Background	Mean	Std. Deviation	N
pretest	non-mathematics	7.92	1.24	82
	mathematics	7.80	1.20	29
	Total	7.89	1.22	111
post-test	non-mathematics	6.22	1.57	82
	mathematics	5.98	1.61	29
	Total	6.16	1.58	111

Note. The scale used for the procedural values scores is from 0 to 10. Non-mathematics = preservice teachers with arts, humanities and social science backgrounds; mathematics = preservice teachers with mathematics, computer science and engineering backgrounds.

A 2-way ANOVA (pre-post and background) was used to examine the main effects and interaction effects for these two factors on procedural values. Levene's test to examine equality of variance was conducted and the test revealed that homogeneity of variance was preserved, $F(3, 218)=2.43, p>0.05$. The analysis of variance revealed no significant interaction between the two independent variables (pre-post and background) on procedural values, $F(1,218)=0.093, p>0.0125$. There was however, a main effect of time (pre-post), which means that procedural values decreased significantly from the pre to post- test for both groups (mathematics and non-mathematics preservice teachers), $F(1,218)=66, p<0.0125, f=0.54$ (large effect size) as depicted in Figure 3.4. Indeed, a review of Figure 3.4 suggests that procedural values were higher for non-mathematics preservice teachers at the pretest and post-test when compared to mathematics preservice teachers; however, the 2-way ANOVA revealed no significant differences between the two groups of preservice teachers, $F(1,218)=0.69, p>0.0125$. Since a series of 2-way ANOVAS were used, a Bonferroni correction was implemented to account for the entire variance and keep the Type I error rates to 0.05.

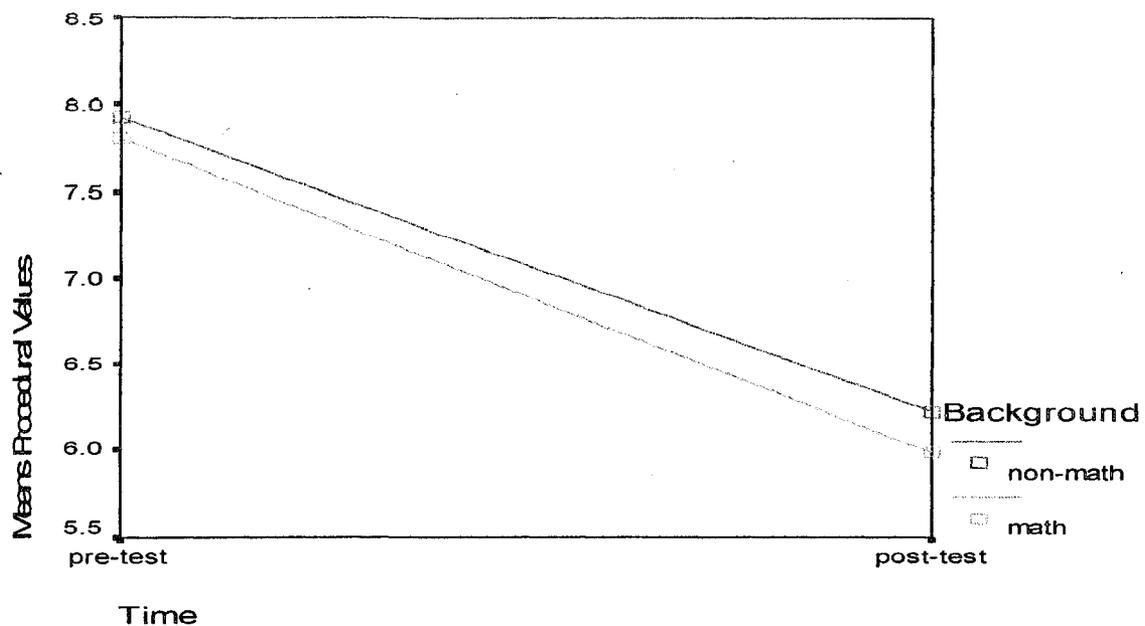


Figure 3.4. Means of Procedural Values Pre and Post-test Based on Preservice Teachers' Background.

In summary, the statistical analysis shows that both groups of preservice teachers (mathematics and non-mathematics) increased their conceptual and procedural mathematical knowledge from the pretest to the post-test. Furthermore, mathematics preservice teachers had higher conceptual and procedural mathematical knowledge than non-mathematics preservice teachers at the pre and post-test. In addition, both groups of preservice teachers increased their conceptual mathematical values from the pretest to the post-test; however, both groups of preservice teachers decreased their procedural values from the pretest to the post-test.

Predicting Change in Conceptual Mathematical Knowledge

This study also examined the relationship of change in conceptual mathematical knowledge with other factors such as mathematical background, conceptual and procedural values, conceptual and procedural knowledge, high school mathematical level and university mathematical level to explore the possibilities of creating a regression model to predict change in conceptual knowledge using the pretest data. In addition, this model will help shed light on the factors that may affect changes in conceptual mathematical knowledge before taking a methods course in mathematics education. For this analysis, descriptive statistics as shown in Table 3.8 as well as Pearson Product Moment Correlations as shown in Table 3.9 were computed using the pretest data for junior intermediate preservice teachers and change in their conceptual knowledge from the pretest to the post-test. The results revealed that change in conceptual mathematical knowledge was significantly correlated to procedural knowledge, $r=-0.27$, $n=111$; conceptual knowledge, $r=-0.36$, $n=111$; high school mathematics level, $r=0.24$, $n=111$.

Table 3.8.

Descriptive Statistics Pretest Data for Change in Conceptual Knowledge and Factors

	Mean	Std. Deviation	N
Δ CK	3.81	2.66	111
High School Mathematics	1.45	0.50	111
University Mathematics	2.06	3.07	111
Background	1.26	0.44	111
PV	7.89	1.22	111
CV	7.83	1.22	111
PK	6.97	2.09	111
CK	0.97	1.41	111

Note. Δ CK = change in conceptual knowledge from the pretest to the post-test; High School Mathematics = mathematical level gained from high school; University Mathematics = level of mathematics taken at university; Background = mathematics or non-mathematics major; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest.

Table 3.9.

Pretest Data Correlations between Change in Conceptual Knowledge and other Factors

		Δ CK	HIGHM	UNIVM	BACKM	PV	CV	PK
HIGHM	Correl	0.24						
	Sig	0.01						
	N	111						
UNIVM	Correl	0.16	0.28					
	Sig	0.10	0.00					
	N	111	111					
BACKM	Correl	0.05	0.20	0.45				
	Sig	0.60	0.03	0.00				
	N	111	111	111				
PV	Correl	-0.01	0.10	-0.07	-0.04			
	Sig	0.31	0.28	0.46	0.67			
	N	111	111	111	111			
CV	Correl	-0.01	0.06	0.09	0.12	0.29		
	Sig	.923	0.54	0.33	0.11	0.00		
	N	111	111	111	111	111		
PK	Correl	0.27	0.31	0.36	0.11	0.04	0.14	
	Sig	0.00	.001	0.00	0.04	0.65	0.14	
	N	111	111	111	111	111	111	
CK	Correl	-0.36	0.25	0.18	0.27	0.07	0.17	0.25
	Sig	0.00	0.00	0.06	0.00	0.44	0.07	0.00
	N	111	111	111	111	111	111	111

Note. Δ CK = change in conceptual knowledge from the pre to the post-test; HIGHM = level of high school mathematics; UNIVM = level of mathematics taken at university; BACKM = mathematics or non-mathematics majors; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest. Correlation is significant at the 0.05 level (2-tailed).

Based on the magnitude of the correlations found, a regression analysis as shown in Table 3.10 was performed to assess, if and how, change in conceptual mathematical knowledge can be predicted by conceptual and procedural knowledge at the pretest, academic background (mathematics and non-mathematics majors), high school mathematics, university mathematics

and conceptual and procedural values. For this regression analysis, collinearity statistics were implemented by computing the variance inflation factor (VIF) as depicted in Table 3.10. The variance inflation factor was found to be less than 10, which indicates that the independent variables are not linearly related. The results from this regression analysis support the results of the correlations, suggesting that change in conceptual knowledge may be predicted from the high school mathematics level ($\beta=0.26$, $p<0.05$), procedural knowledge ($\beta=0.30$, $p<0.05$) and conceptual knowledge ($\beta=-0.52$, $p<0.05$) pretest data. Nonetheless, the low value for R^2 (0.35) as shown in Table 3.11, indicates that this prediction model, although significant, leaves 65 percent of the variance in change in conceptual mathematical knowledge scores unexplained.

Table 3.10.

Results of the Regression Analysis Beta Coefficients with Change in Conceptual Knowledge as the Dependent Variable Using the Pretest Data

Model		Unstandar Coeff		Standar Coeff	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Const)	0.70	1.92		0.36	0.71		
	HIGHM	1.37	0.46	0.26	2.93	0.00	0.83	1.206
	UNIVM	0.03	0.08	0.03	0.33	0.73	0.70	1.429
	BACKM	0.35	0.55	0.06	0.62	0.53	0.75	1.329
	PV	-0.23	0.18	-0.10	-1.26	0.21	0.88	1.125
	CV	0.09	0.18	0.04	0.50	0.61	0.87	1.144
	PK	0.38	0.11	0.30	3.35	0.00	0.79	1.263
	CK	-0.98	0.16	-0.52	-5.99	0.00	0.84	1.182

Note. Dependent Variable – ΔCK = change in conceptual knowledge from the pre to the post-test. Independent Variables – HIGHM = level of high school mathematics; UNIVM = level of mathematics taken at university; BACKM = mathematics or non-mathematics majors; PV = procedural values at the pretest; CV = conceptual values at the pretest; PK = procedural knowledge at the pretest; CK = conceptual knowledge at the pretest; VIF = variance inflation factor less than 10.

Table 3.11.

Regression Analysis Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.58	0.35	0.301	2.23

Based on the independent variables that were significant (high school mathematics, procedural knowledge and conceptual knowledge), a trimmed model was created as shown in Table 3.12. With the trimmed model, change in conceptual knowledge may be predicted from the high school mathematics level ($\beta=0.26$, $p<0.05$), procedural knowledge ($\beta=0.32$, $p<0.05$) and conceptual knowledge ($\beta=-0.50$, $p<0.05$) pretest data. The variance inflation factor was less than 10, which indicates that the variables are linearly independent. Therefore, a model was created to predict change in conceptual knowledge, and the standardized coefficients for the equation below were obtained from Table 3.12.

Prediction Equation from Regression Model

$$\Delta CK = .26(HM) + .32(PK) - .5(CK)$$

where

ΔCK change in conceptual mathematical knowledge

HM level of high school mathematics

PK procedural mathematical knowledge at the pretest

CK conceptual mathematical knowledge at the pretest

This trimmed model, although significant, has low value for R^2 (0.35), which leaves 65 percent of the variance unaccounted for on change in conceptual mathematical knowledge scores

as shown in Table 3.13. Hence this model may provide a useful starting point, but it must be remembered that there may be other factors not addressed by this study, which are needed to account for the rest of the variance in the model.

Table 3.12.

Trimmed Model Regression Analysis Beta Coefficients with Change in Conceptual Knowledge as the Dependent Variable Using the Pretest Data

Model		Unstandard Coeff		Standard Coeff	t	Sig	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Const)	-0.11	0.85		-0.13	.89		
	HM	1.38	0.45	0.26	3.05	.00	.87	1.14
	PK	0.40	0.10	0.32	3.76	.00	.86	1.15
	CK	-.95	0.15	-0.50	-6.03	.00	.90	1.10

Note. Dependent Variable – Δ CK = change in conceptual knowledge from the pre to the post-test. Independent Variables – HM = level of high school mathematics; PK = procedural mathematical knowledge at the pretest. CK = conceptual mathematical knowledge at the pretest.

Table 3.13.

Trimmed Regression Analysis Model Summary Using High School Mathematical, Procedural and Conceptual Knowledge as Independent Variables

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	0.57	0.33	0.309	2.21

3.4 Discussion

This study was conducted to answer three questions: to what extent do preservice teachers change their conceptual and procedural mathematical knowledge and values during a Bachelor of Education program which includes a mathematics methods course?; how does academic background influence preservice teachers' conceptual and procedural mathematical knowledge and values?; and, can a regression model be used to predict change in conceptual knowledge?

Previous research studies have shown that teachers can improve their mathematical knowledge and change their deeply held beliefs about mathematics, referred here as values, to better develop students' mathematical knowledge as a result of professional development (Hill & Ball, 2004; Kajander, 2005; Kajander et al., 2006; Sowder, 2007). In this study, significant changes in preservice teachers' levels of mathematical knowledge and values were found between the pretest and post-test data after taking the mathematics methods course in education. Hence, this study further emphasizes that a preservice teacher education experience can change preservice teachers' traditional ways of mathematical thinking to what appears to be a more reform-based conception of teaching at the junior intermediate level.

Changes in Conceptual and Procedural Mathematical Knowledge

The literature shows that in traditional mathematical teaching and learning, the emphasis is more on procedural mathematical fluency (Hiebert, 1999; Martin, 1995; McCormick, 1997). In this study, the results at the pretest revealed that preservice teachers' conceptual understanding of basic mathematical quantities and operations was extremely low; however, their procedural mathematical abilities were relatively high. In addition, the same results were found when the preservice teachers were analyzed in two separate groups (mathematics and non-mathematics

background). This may imply that these preservice teachers typically came from a more traditional mathematical learning approach.

The literature shows that teachers can improve their conceptual and procedural mathematical knowledge through reform-based teacher education programs as well as professional development experiences (Ball, 1996; Boyd, 1994; Kajander et al., 2006). In this study, after the intervention, preservice teachers' conceptual and procedural mathematical knowledge increased significantly. These results suggest that the mathematics methods course seems to have offered an opportunity to deepen content specific mathematics understanding for preservice teachers as well as support the improvement of procedural mathematical skills. More specifically, the mathematics methods course appeared to offer an avenue to deepen preservice teachers' conceptual levels of fundamental mathematical knowledge, even though some evidence argues that having high levels of procedural mathematical knowledge makes it harder for teachers to switch to a more conceptual mathematical approach (Hiebert, 1999). Nevertheless, the results of this study show that preservice teachers' conceptual knowledge seems to have increased significantly and with a large effect size from pretest to the post-test. Hence, preservice teachers appeared to have deepened their conceptual mathematical knowledge of fundamental mathematics to a type of knowledge "knowing why," which includes more emphasis on mathematical understanding. Such understanding may be more applicable to a reform-based teaching environment in which students improve their conceptual mathematical knowledge along with procedural mathematical fluency and build upon their mathematical understanding in order to construct new mathematical knowledge (Hiebert et al., 2005; Kamii et al., 2005).

The literature also shows that teachers can improve their procedural knowledge by making use of deepened conceptual knowledge and written conceptual thoughts about

mathematics (NCTM, 2000; Rittle-Johnson et al., 2002). Furthermore, the literature shows that teachers' construction of conceptual knowledge allows them to create generalizations and these generalizations of mathematical concepts also help teachers to improve their procedural knowledge (Sherin & Fuson, 2005). In this study, preservice teachers had relatively high levels of procedural knowledge before the intervention. After the intervention, preservice teachers' procedural knowledge appeared to have increased significantly with a moderate effect size from the pretest to the post-test, even though the methods course focused on conceptual learning. Hence, this result seems to support an important premise of mathematics education reform, namely that a conceptually based learning environment also supports procedural skill development (NCTM, 2000).

These findings suggest that it may be potentially possible to improve preservice teachers' conceptual mathematical knowledge as well as their procedural knowledge as a result of a mathematics intervention, as found in previous studies (Ball, 1996; Kajander et al., 2006). Indeed, the findings suggest the possibility that through a mathematics methods course, it may be feasible to facilitate mathematical learning for preservice teachers to improve their conceptual understanding of fundamental mathematics, as has been found elsewhere (Hill & Ball, 2004).

In addition, the literature shows that mathematics methods courses may permit teachers to become better facilitators of knowledge, and ultimately help students build their own mathematical knowledge through group work and class interaction and therefore, solve mathematical problems with conceptual understanding (Kazemi & Franke, 2004). Results of the present study suggest that the intervention appeared to be effective in improving preservice teachers' conceptual mathematical knowledge, which is needed for teaching with a more reform-based approach.

The data analysis of this study illustrates an example of preservice teachers' possible growth in conceptual and procedural knowledge. For instance, before the intervention, preservice teachers had high levels of procedural knowledge, which means that perhaps these preservice teachers had more traditional training in mathematics with more emphasis on procedural knowledge. After the intervention, preservice teachers' conceptual and procedural mathematical knowledge both appeared to have improved to a higher level, although the mathematics intervention was focused on improving preservice teachers' conceptual mathematical knowledge.

The significant change of preservice teachers' conceptual and procedural knowledge from the pretest to the post-test suggests that conceptual knowledge and procedural knowledge are not mutually exclusive, but seem to interact over time when solving mathematical problems; therefore, changes in conceptual knowledge also impact changes in procedural knowledge and such an interaction between conceptual and procedural knowledge facilitates the link between theory and practice when solving mathematical problems in different contexts as found in other research studies (Byrnes & Wasik, 1991; Mason & Spence, 1999; McCormick, 1997).

It is important for teachers to guide student learning in mathematics to include deep conceptual mathematical understanding along with procedural mathematical fluency (Ball, 1990; Ma, 1999; Stein et al, 2007). This study shows that preservice teachers seemed to have improved their conceptual and procedural mathematical knowledge, and such an improvement subsequently may support preservice teachers to teach mathematics to their students with deeper understanding of mathematical concepts and procedures during their classroom practices.

Changes in Conceptual and Procedural Mathematical Values

Previous studies showed that teachers' conceptual mathematical development is multidimensional; in other words, teachers' conceptual mathematical development may include

mathematical values as well as knowledge of various mathematical topics and domains such as knowledge of content and knowledge of students (Ambrose, 2004; Hill, Schilling & Ball, 2005).

In this study, preservice teachers' conceptual and procedural values appeared to have changed from the pre to the post-test as a result of the mathematics intervention. For instance, conceptual values were high before the intervention and rose significantly throughout the experience with a small effect size. A potential reason for these high levels of conceptual values at the pretest may be that these preservice teachers were already shifting their beliefs to a more conceptual mathematical approach for teaching based on the way they previewed the mathematics methods course and their future participation in it as well as the influence of other mathematical experiences during their previous studies.

The literature also shows that developing deep conceptual mathematical knowledge influences teachers' conceptual and procedural mathematical values (Ernest, 1989; Hiebert, 1999; Stipek et al., 2001).

In this study, preservice teachers' procedural values dropped significantly with a large effect size while conceptual values rose with a small effect size over the duration of the mathematics methods course. Indeed, similar results have been found in previous research (Kajander, 2005). The decrease in preservice teachers' procedural values and the increase in conceptual values may indicate that preservice teachers shifted their mathematical values towards believing that conceptual mathematical knowledge precedes procedural mathematical knowledge and also believing that once the mathematical concept is understood, procedural knowledge follows by creating generalizations. Hence, the mathematics methods course and perhaps the experiences acquired by these preservice teachers throughout the Bachelor of

Education program may have supported changes in preservice teachers' mathematical values to what I conjecture to be a more reform-based conception.

The literature shows that reform-based teaching programs promote students' deep understanding of mathematical concepts and that mathematical procedural knowledge develops along with conceptual understanding by creating generalizations (Boaler, 1999; Hiebert, 1999). Furthermore, the literature also shows that teachers' mathematical values have a powerful impact on the teaching approach that gets implemented in the classroom as well as on students' mathematical development (Ernest, 1989). Indeed, students taught in a reform-based approach are able to acquire greater skills in using mathematical tools to improve their mathematical knowledge and construct new knowledge than those taught with a traditional approach in which the emphasis is more in mathematical procedures (Romberg, 1997; Stipek et al., 2001). In this study, the data show that the mathematics methods course in education studied in this research, which I argued earlier followed a reform-based approach, appeared to have influenced the conceptual and procedural mathematical knowledge and values, which preservice teachers may subsequently bring to the classroom to influence their students' perceptions of mathematics. Thus, a reform-based conception may help teachers better prepare their students to engage and assimilate the scientific and technological changes taking place in the information age as described by the NCTM *Standards* (2000) and the changes in many mathematics curricula (Sowder, 2007).

The literature shows that professional development experiences facilitate teachers' mathematical development by improving their mathematical knowledge and changing their mathematical values toward a more reform-based conception (Hill et al., 2004; Kajander et al., 2006). Moreover, the literature shows that teacher preparation for reform-based teaching should

begin at the preservice phase of a teacher's career (Boyd, 1994). In this study, it is important to note that preservice teachers appeared to have changed their mathematical knowledge and values as a result of the intervention and these results seem to support the evidence stated in the literature. Hence, the mathematics methods course appeared to offer an opportunity for preservice teachers to experience reform-based learning and significantly deepen their conceptual understanding of fundamental mathematics. Furthermore, the mathematics methods course appeared to offer an avenue to possibly shift preservice teachers' beliefs about mathematics toward a more reform-oriented conception. Thus, the results found in this study suggest that preservice teachers' preparation in all strands of the elementary mathematics curriculum may potentially enhance their understanding and shift beliefs. Such growth may be an important factor in implementing effective reform mathematics education at the classroom level (Sowder, 2007).

Influence of Academic Background on Preservice Teachers' Knowledge and Values

The literature shows that teachers can change their conceptual and procedural mathematical knowledge and values toward a more reform-based teaching conception as a result of a professional development experience at the preservice or in-service phase of their teaching career (Ball, 1996; Boyd, 1994; Hill et al, 2004; Kajander, 2005; Kajander et al, 2006; Sowder, 2007). In this study, the mathematics methods course appeared to be effective in helping both groups of preservice teachers (mathematics and non-mathematics backgrounds) improve their conceptual and procedural mathematical knowledge and shift their beliefs toward what I conjecture to be a reform-based conception. Furthermore, the results of this study suggest that previous levels of formal mathematics background may not always result in high levels of

conceptual understanding of mathematics. For instance, in this study, both groups of preservice teachers (mathematics and non-mathematics) had very low conceptual knowledge at the pretest.

In addition, the statistical results suggested that conceptual knowledge was significantly different between the two groups of preservice teachers at the pretest and post-test, although with a small effect size. Nonetheless, both groups of preservice teachers seemed to have significantly improved their levels of conceptual and procedural mathematical knowledge and values from the pretest to the post-test in ways that support mathematics reform.

The literature shows that helping teachers develop deep conceptual mathematical knowledge influences their conceptual and procedural mathematical values, which are important in implementing reform-based learning in the classroom (Ernest, 1989; Hiebert, 1999; Stipek et al., 2001). The literature also shows that preservice teachers' reform-based experiences must begin at the preservice phase of a teacher's career (Boyd, 1994). In this study, I explored whether those preservice teachers with mathematics related background were harder to shift toward a more reform-based approach. The descriptive statistical analysis of the data however, suggests that preservice teachers with a mathematical background already had slightly higher conceptual values than those with a non-mathematical background at the pretest. Furthermore, the analysis suggests that there was a significant change in mathematically experienced preservice teachers' conceptual values from the pretest to the post-test with a small effect size, which means that this group of preservice teachers may have improved their conceptual values, even though these conceptual values were already at a high level. Arts and humanities majors also appeared to have improved their conceptual values in a similar way from the pretest to the post-test to the same extent as the mathematics majors.

In addition, both groups of preservice teachers seemed to have decreased their procedural values significantly with a moderate effect size from the pretest to the post-test. Decreasing their procedural values from the pretest to the post-test suggests that both groups of preservice teachers (mathematics and non-mathematics) may have come to believe more strongly that in mathematical learning conceptual knowledge precedes procedural knowledge and it forms the basis on which new procedures are acquired by creating generalizations (Boaler, 1999; Byrnes & Wasik, 1991). Hence, both groups of preservice teachers seemed to have shifted to believe in a more reform-oriented approach. Such changes may increase the possibility that these preservice teachers will bring this new mathematical reform-oriented approach into their classrooms, by placing more emphasis on mathematical concepts and letting the mathematical procedures develop through generalizations as opposed to the traditional way of teaching mathematics, in which the emphasis tends to be more on mathematical procedures (Hiebert et al., 2005).

Finally, it should be noted that regardless of preservice teachers' mathematical background, both groups of preservice teachers appeared to have improved their mathematical knowledge and shifted their values to include more emphasis on the importance of conceptual understanding of elementary mathematics. Hence, the data show that a junior intermediate mathematics methods course in education may offer an avenue to potentially change preservice teachers' mathematical knowledge and values to a more reform-based conception regardless of their academic background.

Predicting Change in Conceptual Knowledge

The literature shows that the number of university mathematics courses taken by preservice teachers during their undergraduate majors does not increase their conceptual understanding of fundamental mathematics needed for teaching mathematics to students in a

reform-based approach (Ball, 2004; Foss, 2000). Hence, preservice teachers may need specialized training experiences such as a mathematics methods course in education in order to learn how to teach mathematics to students with a reform-based approach (Ball, 2004; Kajander et al, 2006; Ma, 1999; Sowder, 2007).

In this study, the final goal was to develop a regression model to predict preservice teachers' change in conceptual mathematical knowledge (how much conceptual mathematical knowledge was gained from the pretest to the post-test) based on their conceptual and procedural mathematical knowledge and values at the pretest, high school mathematics courses, university mathematics courses and academic background (mathematics or non-mathematics majors). The data showed significant correlations between change in conceptual mathematical knowledge and high school mathematics, and change in conceptual mathematical knowledge and preservice teachers' levels of conceptual and procedural mathematical knowledge at the pretest. The mathematical courses taken at university, however, did not correlate to change in conceptual mathematical knowledge.

Similarly, academic background (mathematics or non-mathematics majors) did not correlate to change in mathematical knowledge. These results suggest that academic background and mathematics courses taken at university do not seem to play a role in changes to junior intermediate preservice teachers' conceptual mathematical knowledge. On the other hand, the results suggest that preservice teachers' knowledge of fundamental mathematics as gained from the elementary or high school as well as preservice teachers' levels of conceptual and procedural mathematical knowledge at the pretest may be relevant in determining how much mathematical training may be necessary to improve preservice teachers' conceptual mathematical knowledge along with procedural mathematical fluency. Hence, this mathematical training is important for

teachers in order to teach mathematics to their students with a reform-based approach (Hill, Rowan & Ball, 2005).

The literature shows that developing a deep understanding of mathematics influences teachers' mathematical values and the way teachers instruct in the classroom (Boaler, 1999; Stipek et al, 2001). In this study however, preservice teachers' conceptual and procedural mathematical values did not correlate to change in their mathematical knowledge. One of the reasons for this lack of correlation may be the influence of other uncontrolled variables such as the way the preservice teachers viewed the mathematics methods course in education and their participation in it, on changes to their conceptual mathematical knowledge since there was no comparison group.

The literature shows that high levels of conceptual understanding of fundamental mathematics are important to teach mathematics to others with profound understanding (Ball, 1996; Hill & Ball, 2004; Ma, 1999). In this study, I needed to find the weight of each independent variable in the mathematical model in order to predict change in conceptual mathematical knowledge. Since the correlations only indicated the strength of the relationship between the dependent variable and each independent variable, I decided to conduct a regression analysis to explore the impact of these independent variables (preservice teachers' levels of conceptual and procedural mathematical knowledge and values at the pretest, high school mathematics courses, university mathematics courses and academic background) on change in conceptual mathematical knowledge.

Based on the results of the regression analysis, the level of high school mathematics attained and the levels of conceptual and procedural mathematical knowledge at the pretest were the best predictors of change in conceptual knowledge. Furthermore, the beta standardized

coefficients (values obtained by standardizing all variables to unit variance before the regression was run) within the model indicated that the preservice teachers' conceptual mathematical knowledge at the pretest had the highest weight. This means that each value of the coefficient of preservice teacher' level of conceptual mathematical knowledge at the pretest is the expected increase on change in conceptual knowledge with a 1-unit increase in preservice teachers' level of conceptual mathematical knowledge at the pretest when other regressors are held constant. For instance, with preservice teachers' levels of procedural knowledge at the pretest and the level of high school mathematics variables held constant, each increase from preservice teachers' level of conceptual mathematical knowledge at the pretest is associated with a decrease of -0.50 unit on change in conceptual knowledge. In other words, preservice teachers with high levels of conceptual mathematical knowledge at the pretest may tend to change less in conceptual mathematical knowledge according to this regression model. Conversely, the conceptually weaker student seemed to have grown the most in conceptual mathematical understanding over the intervention.

In addition, the regression model in this study shows that with initial levels of conceptual knowledge and the level of high school mathematics variables held constant, each increase from preservice teachers' levels of procedural mathematical knowledge at the pretest is associated with an increase of 0.32 unit on change in conceptual knowledge, which means that preservice teachers with high levels of procedural mathematical knowledge at the pretest may tend to change more in conceptual mathematical knowledge. Finally, the results of the regression analysis show that with preservice teachers' pretest levels of procedural and conceptual mathematical knowledge variables held constant, each increase from the level of high school mathematics is associated with an increase of 0.26 unit on change in conceptual knowledge.

Hence, preservice teachers with more high school mathematics courses may change more in terms of conceptual mathematical knowledge.

This combination of attributes paints a picture of students who, knowingly weak in conceptual understanding, nevertheless persevere and take more high school mathematics courses, which they survive by using procedural skills rather than by ever managing to develop conceptual understanding. Such a combination of factors appears to be typical for students who grow most in conceptual knowledge over the methods course. In addition, the regression model shows that although high school mathematics and preservice teachers' levels of conceptual and procedural mathematical knowledge at the pretest were the best predictors of change in conceptual knowledge, the low value for R^2 indicated that 65 percent of the variance was unaccounted for in terms of predicting change in conceptual knowledge.

Therefore in order to account for a higher percentage of the variance, other factors may be taken in consideration in future models. Moreover, a larger sample may be needed to create a stronger linear model to predict change in conceptual mathematical knowledge. In summary, the findings from this regression model may indicate that preservice teachers with high levels of procedural mathematical knowledge and high school mathematics benefited the most from the intervention.

This information may be useful for future teacher educators to help them assess preservice teachers' mathematical knowledge and provide these preservice teachers with more appropriate mathematical training in order to meet the expectations of the reform-oriented mathematics curriculum before these preservice teachers enter the classroom environment. This information may also be useful for preservice teachers to help them identify their weaknesses and strengths and develop the necessary mathematical knowledge to be able to teach and engage

their students in solving mathematical problems to increase students' understanding of mathematical concepts. Finally this information may be useful for school boards to address the need for teacher training courses or programs to facilitate the transition of mathematics teaching from the old curriculum to the new reform-based curriculum.

CHAPTER 4 – CONCLUSIONS AND RECOMMENDATIONS

This thesis was divided into two studies. The first study shows evidence for the validity and reliability of the POM instrument when measuring the conceptual and procedural mathematical knowledge of junior intermediate preservice teachers. Furthermore, the study shows evidence of the reliability of the POM instrument when measuring conceptual and procedural mathematical values. The study, however, does not show evidence of validity of the POM instrument when measuring conceptual and procedural values. Nonetheless, based on the evidence of validity and reliability found on this study and the face validity of the POM instrument from previous studies (Kajander, 2005), I felt that there was enough evidence to use the POM as a valid and reliable instrument to collect the data and answer the research questions stated in the second study.

The second study was conducted to answer three questions related to the development of junior intermediate preservice teachers' mathematical knowledge and values as a result of a mathematics methods course within their Bachelor of Education program.

The first question was: *To what extent do preservice teachers change their conceptual and procedural mathematical knowledge and values during a Bachelor of Education program which includes a mathematics methods course?*

The results of this study suggest that significant changes in preservice teachers' mathematical knowledge and values are potentially possible via a Bachelor of Education one year program, which includes a reform-based mathematics methods course. The study shows that although preservice teachers' conceptual mathematical knowledge was still low after the intervention (less than 50% according to the instrument used), preservice teachers appeared to have significantly improved their conceptual knowledge after completing the mathematics

methods course. Furthermore, preservice teachers' procedural mathematical knowledge also appeared to have improved significantly from the pretest to the post-test even though the intervention was based on conceptual understanding of fundamental mathematics. This means that a well designed mathematics methods course may potentially enhance preservice teachers' understanding of mathematical concepts as well as procedures by making emphasis on conceptual mathematical development. Thus, a well designed mathematics methods course might offer an avenue to help preservice teachers become better facilitators of knowledge by providing them with new classroom teaching techniques based on a reform mathematics approach. Hence, the results of this study suggest that the intervention may have offered a new opportunity for preservice teachers to improve their understanding of mathematical concepts, create generalizations and construct new mathematical knowledge.

This study also shows that preservice teachers' values about mathematical learning appeared to have changed from the pretest to the post-test. Although there was not a control group in this study, it appears that the mathematics methods course may have offered an avenue to shift preservice teachers' mathematical values to a more reform-based conception. As discussed previously, conceptual values increased and procedural values decreased after the mathematics intervention, which means that preservice teachers seemed to have shifted their beliefs from traditional teaching to a reform-based approach which places more emphasis on mathematical understanding, and less emphasis on procedural practice and fluency. Before the intervention, the data suggest that the majority of these preservice teachers had high procedural values, which may indicate that these preservice teachers experienced a more traditional mathematical learning approach. Thus, this study conjectures that changing preservice teachers' beliefs to value a more reform-based approach through a mathematics methods course in

education, may influence the type of mathematical knowledge and preferred teaching approach that these teachers bring into their classroom practices. Moreover, it may increase students' opportunities to learn mathematics with engagement and conceptual understanding by conceptualizing ideas before generating and applying mathematical procedures to solve problems (Franz, 2000). Furthermore, influencing teachers' beliefs may increase students' opportunities to enjoy mathematical learning without memorizing formulas and without fear; but rather, exploring concepts and encouraging students to learn in a classroom climate in which risk-taking is encouraged and supported by the teacher and other students in the classroom (Hiebert, 1999). This is in contrast to the traditional way of teaching mathematics, in which students' mathematical learning is more based on manipulating mathematical procedures and memorizing formulas with less emphasis in mathematical concepts (Hiebert et al., 2005; Romberg, 1997).

It is also relevant to look at the study from a different perspective. What if these preservice teachers revert to traditional teaching after taking the mathematics methods course? If this is the case, there is a need to follow up on these teachers' classroom practices. Hence, a longitudinal study may be necessary to examine classroom practices of in-service teachers who had taken the mathematics methods course within the Bachelor of Education program at the junior intermediate level. This longitudinal study might provide evidence of how well these preservice teachers had implemented the mathematical knowledge and values acquired through the mathematics methods course in their classroom practice.

In summary, this study shows that it is possible to shift preservice teachers' mathematical knowledge and values to a more reform-based conception through a mathematics methods course in education before these preservice teachers enter their classroom practices.

Such a shift may increase students' opportunities to learn mathematics with engagement and conceptual understanding.

The second question was: *How does academic background influence preservice teachers' conceptual and procedural mathematical knowledge and values?*

This study also provides evidence that regardless of preservice teachers' academic background (mathematics and non-mathematics), it may be potentially possible to shift their conceptual and procedural mathematical knowledge and values to improve their capacity to teach using a more reform-based approach. The mathematics methods course studied appears to be instrumental in shifting both groups (mathematics and non-mathematics) of preservice teachers' mathematical knowledge and values to a different level from where these teachers started. For instance, the data shows that at the pretest both groups of preservice teachers had extremely low conceptual knowledge and high procedural knowledge and for some preservice teachers no scoreable evidence of conceptual knowledge was demonstrated on the instrument items, yet via the intervention, which included more emphasis in mathematical understanding, both group of preservice teachers moved to a higher level of conceptual and procedural mathematical knowledge.

Hence, regardless of preservice teachers' mathematical background, the intervention appears to have been effective in helping preservice teachers link their conceptual and procedural mathematical knowledge when solving problems. This finding is very interesting and it supports the literature, which states that in mathematical learning conceptual knowledge precedes procedural knowledge and ultimately forms the basis for creating new procedures; however, both conceptual and procedural knowledge cannot be mutually exclusive and must interact overtime through learning network mechanisms (Byrnes & Wasik, 1991; Smilkstein, 1993).

In addition, the data shows that regardless of teachers' mathematical background, it is potentially possible to shift their conceptual and procedural values to a more reform-based conception for teaching mathematics at the junior intermediate level. For example, at the pretest, the data shows that these preservice teachers (mathematics and non-mathematics) had high procedural mathematical values, which implies that both groups of preservice teachers may have associated a more traditional mathematical approach for teaching with more emphasis on procedures as an important aspect of mathematical learning.

Via the intervention, both groups of preservice teachers seemed to have balanced their mathematical values by giving equal importance to both (concepts and procedures) in mathematical learning. Furthermore, the data shows that regardless of preservice teachers' mathematical background, their conceptual knowledge was still below the 50% mark after the intervention. This finding further emphasizes the need to better prepare preservice teachers with a deeper understanding of mathematical concepts regardless of their mathematical background before these teachers begin their classroom practices at the junior intermediate level. This finding seems to highlight the importance of professional development at all levels in education to deepen teachers' mathematical knowledge and shift their values. Teachers need to be able to teach mathematics to their students in an environment in which students can improve their procedural knowledge by making use of their conceptual thoughts about mathematical notions; an environment in which students will be able to integrate concepts and procedures to develop better mathematical strategies when solving problems (Rittle-Johnson et al, 2001).

In summary, regardless of preservice teachers' mathematical background, an effective mathematics methods course in education may offer an excellent starting point for preservice teachers' conceptual and procedural mathematical growth as shown in this study. Hence, these

findings again underscore the importance of mathematical learning experiences, which shape preservice teachers' mathematical knowledge and values to a more reform-based orientation that may ultimately help them better prepare their students to adapt, perform and succeed mathematically.

The third question was: *Can a regression model be used to predict change in conceptual mathematical knowledge?*

Teachers' conceptual mathematical understanding is considered an important element in mathematics reform (Hiebert, 1999); therefore, teachers need to have a profound understanding of the mathematical concepts that they will be teaching to their students in the classroom (Ma, 1999; Sowder, 2007). Hence, in order to better improve teacher's conceptual understanding of mathematical concepts as an important element of mathematics reform, it is essential to determine which factors impact preservice teachers' change in conceptual mathematical knowledge after taking a mathematics methods course in education (Boyd, 1994; Ross et al, 2002).

For that reason, in this study, a regression mathematical model was created to predict preservice teachers' change in conceptual knowledge from the pretest to the post-test and further determine the factors that may impact their change in conceptual mathematical knowledge after taking a mathematics methods course in education. The findings as stated in the discussion revealed that the number of high school mathematics courses taken and the levels of conceptual and procedural mathematical knowledge at the pretest seemed to have impacted preservice teachers' conceptual mathematical growth the most. This means that, according this model, preservice teachers with high levels of conceptual mathematical knowledge at the pretest did not change as much in terms of increasing their conceptual mathematical knowledge as a result of

the intervention, compared with preservice teachers with high levels of procedural mathematical knowledge at the pretest and a larger number of high school courses taken, who changed more in terms of improving their conceptual mathematical knowledge.

These findings may have implications for mathematics educators of preservice teachers in terms of helping them assess and better prepare preservice teachers. Indeed, these findings further underscore the importance of preservice teachers' reform-based mathematical preparation in a Bachelor of Education program before these teachers commence their classroom practices. In addition, the mathematical model as depicted in Table 3.12 suggests that having high levels of procedural knowledge in place, possibly related to having taken more high school mathematics courses, seems to relate to stronger growth in mathematical understanding; it is conjectured that these preservice teachers might also be more committed to learning mathematics. These preservice teachers however, will need more conceptual mathematical development based on the data provided.

It is also important to point out that although these factors were significant in predicting preservice teachers' change in conceptual knowledge, the factors did not account for the entire variance in the model and other factors need to be explored for future research. In summary, regardless of which factors are missing, the model highlights the importance of assessing preservice teachers' initial levels of conceptual and procedural mathematical knowledge as well as the number of high school mathematics courses taken in order to possibly impact preservice teachers' conceptual mathematical growth via a mathematics methods course in education.

4.1 Limitations

The first study could have been improved by collecting a larger sample size and showing further evidence of construct validity of the POM instrument. However, the sample size was limited by the number of students in the classroom. Moreover, this first study could have been improved by implementing a mixed methods approach to show evidence of credibility and dependability of the POM instrument from a qualitative point of view along with the present evidence of validity and reliability from the quantitative point of view. For instance, dependability could have been accomplished by implementing open-ended interview questions during the pretest and post-test data collection and comparing the results from two groups of preservice teachers in the junior intermediate one year Bachelor of Education program, who could have written the POM questionnaire at two different times. Credibility could have been accomplished by triangulating the results obtained from field notes, open-ended questions and member checks during the pretest and post-test data of this study.

The second study could have been improved by selecting random samples from different classes to avoid the possibilities of collecting data from a biased sample. There is a need to use a larger sample size to minimize standard error and make the mean comparisons more robust with a higher power of criterion. In addition, a control group is needed to minimize threats to the internal validity of the data and therefore, diminish the possibilities of committing a type I or type II error. Since there was no control group and randomization of data, the threats to the internal validity of the results in this study due to uncontrolled variables may include history, testing, instrument decay, statistical regression and attitude of subject. History may be a threat because other events (mathematical knowledge gained from other courses, tutoring, workshops or seminars) outside of the research study could have altered or affected participants'

performance. Testing may be a threat to the results of this study because the design was One-Group Pretest-Posttest and therefore, there is a possibility that preservice teachers could have performed better the second time due to practice. Instrument decay may be a threat to this study due to the possible fatigue of the person correcting the surveys. Statistical regression may be a threat because extremely low scoring individuals would have been more likely to show more improvement. Attitude of participants may be a threat because of the way the participants may have viewed the study and their participation in it. It was difficult however, to get a control group for this study since the mathematics methods course is part of the Bachelor of Education curriculum and therefore, it is a compulsory course for all junior intermediate preservice teachers. Nonetheless, in order to ameliorate these threats (history, testing, instrument decay, statistical regression and attitude of subject) to the internal validity of the data, it would be advisable for future research in this area to test an experimental and control group before the intervention and after the intervention and then implement an analysis of variance.

In the second study a multivariate analysis (MANOVA) instead of a series of ANOVAS for each dependent variable could have been implemented to analyze the effect of the mathematics intervention with respect to conceptual and procedural knowledge as well as conceptual and procedural values (CK, PK, CV, PV) from the pretest to the post-test between the two groups of preservice teachers (mathematics and non-mathematics). Hence, implementing a series of individual ANOVAS may produce error rates such as Type I or Type II error. These error rates may affect correlated dependent variables in the analysis. A MANOVA however, will reduce these number errors since the MANOVA creates discriminant factors which are independent from each other and uncorrelated. In addition, a MANOVA is more efficient because it eliminates error rates and reduces the number of statistical test in the analysis.

Although a MANOVA would have been more appropriate for the data analysis of this study, there are assumptions with a MANOVA that makes it hard to interpret the data. In these assumptions discriminant functions are considered to be normally distributed, discriminant functions are assumed to have equal variance, and correlation patterns of variables are assumed to be equal for each discriminant factor or function. Thus, in order to overcome these violations and assumptions, it is important to have a large sample size for each group and although with a MANOVA it is easy enough to form a linear combination of dependent variables to create a discriminant factor, it is not always easy to determine what this linear combination measures. Based on these concerns, it was decided for this study to use individual ANOVAS for each dependent variable and a Bonferroni correction was implemented to keep the error rates at 0.05 level.

Finally, there is certainly a need to provide further evidence of the validity of the POM instrument in the first study when measuring conceptual and procedural mathematical values. Hence, other approaches must be taken into consideration such as construct validity or mixed methods approach to authenticate the POM instrument measures of conceptual and procedural values.

4.2 Future Research Ideas

A longitudinal study would be advisable to examine teachers' mathematical knowledge and values during the preservice and in-service phases of their career, and relate this to their classroom practices over time. This will allow researchers to have a better understanding of whether teachers have reverted from a more reform-based conception in their training to a more traditional approach in their teaching.

A study to statistically compare in-service teachers' mathematical knowledge and beliefs to the mathematical knowledge and beliefs of their students would be valuable to shed light on how teachers' reform-based training influences their students. Furthermore, since it is difficult to get a control group because the mathematics methods course is a compulsory course within the Bachelor of Education program, a mixed methods approach study may be prudent to examine the effect of the intervention (mathematics methods course) from two points of views (quantitative and qualitative) and therefore provide more strength to the findings.

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APPENDICES

APPENDIX A

Perceptions of Mathematics Questionnaire (POM) (Kajander, 2005)

Information for Participating Teachers

Research Project Title: *Teachers' Evolving Mathematical Understandings*

Researcher(s): Ann Kajander, Ralph Mason

Sponsor (if applicable): NSERC (CRYSTAL), University of Manitoba

This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

Research Study by Dr. Ann Kajander Faculty of Education, Lakehead University, email ann.kajander@lakeheadu.ca, phone (807) 343-8127

The purpose of this research is to examine mathematics beliefs and knowledge of teachers, and to help you study your own abilities, values, and growth areas. Participation may give you a better idea of your own level of mathematical understanding at the conceptual level, as well as a better understanding of your values in the teaching and learning of mathematics.

Participation in this study is strictly voluntary, and individual results will not be communicated to Lakehead Public Schools. Submissions will be numbered, and confidentiality maintained - at no time will your name be used in reporting any research results.

Teachers who volunteer to participate will be asked to complete the Perceptions of Mathematics Survey which contains mathematical questions as well as questions about your beliefs about mathematics. Completing the survey should take less than an hour and all answers are acceptable. You may also be asked for comments about how well you feel the Survey characterizes your values and understanding in mathematics, and how the Survey might be improved. You may also be asked if you wish to voluntarily participate in several brief interviews. Any data collected will be recorded by participant number and will be kept completely confidential. At no point will names of participants be made public.

Final analysis of results will be made public and participants will be made aware of how they can see the results. Participation is voluntary and participants may withdraw at any time from the Study with no repercussions. Data will be securely stored at Lakehead University for seven years.

If you are willing to participate, please sign the attached Consent Form and submit it with your Survey. Thank you for your interest in this project!

Ann Kajander

Consent Form

Your signature on this form indicates that you have understood to your satisfaction the information regarding participation in the research project and agree to participate as a subject. In no way does this waive your legal rights nor release the researchers, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from the study at any time, and /or refrain from answering any questions you prefer to omit, without prejudice or consequence. Your continued participation should be as informed as your initial consent, so you should feel free to ask for clarification or new information throughout your participation. Feel free to contact

Dr. Ann Kajander ann.kajander@lakeheadu.ca (807)343-8127

This research has been approved by the University of Manitoba Research Ethics Board as well as Lakehead University Research Ethics Board. If you have any concerns or complaints about this project you may contact any of the above-named persons or the Human Ethics Secretariat at 204-474-7122. A copy of this consent form has been given to you to keep for your records and reference.

My signature on this sheet indicates I agree to participate in a study by Dr. Ann Kajander, of Lakehead University on *Teachers' Evolving Mathematical Understandings* and it also indicates that I understand the following:

1. I am a volunteer and can withdraw at any time from the study.
2. There is no apparent risk of physical or psychological harm.
3. The data I provide will be confidential and data will be securely stored at Lakehead University for 7 years.
4. I will receive a summary of the project, upon request, following the completion of the project.

I have received explanations about the nature of the study, its purpose, and procedures. I am willing to answer a written survey and I am also aware I may be asked to participate in related interviews, from which I may also withdraw at any time.

Participant's Signature _____ Printed Name: _____

Researcher Signature _____ Date _____

Code: _____

Date: _____

Perceptions of Mathematics "POM" Survey
A. Kajander, Lakehead University

Research has shown that the prior ideas and understandings about mathematics that are brought to classrooms by teachers are very important in terms of how teachers will decide to teach mathematics. It is important to honestly assess what your current understanding is, in order to move forward as a teacher. This survey will have no bearing whatsoever on any course grades or evaluations, but rather will help you make some decisions about how to best focus your learning. You will have an opportunity to reassess yourself at the end of the year. You may find the survey 'hard' in places at this point. This is to be expected – don't be alarmed!

Completing the survey will allow you to create your own personal mathematical 'Profile', which will give you an idea of *how* you understand mathematics, and how you value different types of mathematical learning opportunities. There is no 'right' answer – everyone will be different.

Professional development affords the chance to think about what kind of teacher of mathematics you are and want to become, and to move towards that goal. This survey is designed as an important first step in determining and achieving your goals.

"Perceptions of Mathematics" Survey
Procedural and Conceptual Content and Values Scale for Mathematics

Part 1.

Please answer the following as well as you can remember (check off the box or fill in as required).

My teaching experience in years (counting the current year) is

- 1 to 4
- 5 to 9
- 10 or more

1. The highest level of mathematics I passed in high school was

- grade 10
- grade 11
- grade 12
- one or more OAC courses (or equivalent)

2. Mathematics courses I have taken at the university level (leave blank if none taken)

- introductory course in mathematics for future teachers
- first year algebra or calculus
- one or more statistics courses
- first year course(s) plus other second or third year mathematical courses

3. My gender is

- female
- male

4. Which category most closely describes your undergraduate major?

- arts, humanities or social sciences
- science, engineering, computer science or technology
- mathematics
- other _____

2. Values Questions

Please answer these questions by circling the response, where 0 is low or poor or disagree, and 3 is high or positive or agree. Please do not add other responses such as "not sure" – choose the closest response to your feeling.

- 1) It is important to me to be able to get the correct answer to mathematical questions. 0 1 2 3
- 2) It is important to me to really understand how and why mathematical procedures work. 0 1 2 3
- 3) It is important for everyone to be able to accurately do basic mathematical calculations such as addition or multiplication, without a calculator. 0 1 2 3
- 4) Everyone needs to deeply understand how and why mathematical procedures work if they are going to make effective use of them. 0 1 2 3
- 5) It is important to be able to recall mathematical facts such as addition facts or times tables quickly and accurately. 0 1 2 3
- 6) It is important to have to think through and understand a variety of different approaches to problems. 0 1 2 3
- 7) It is the teacher's job to teach the steps in each new mathematical method to the students before they have to use it. 0 1 2 3
- 8) There are often several correct ways to get a right answer. 0 1 2 3

- 9) Accurate and efficient calculation skills are highly important in mathematics. 0 1 2 3
- 10) It enriches student understanding to have to think about different ways to solve the same problem. 0 1 2 3
- 11) It is important to practice on many familiar shorter mathematical questions in school. 0 1 2 3
- 12) It is important to develop connections between related ideas and models in mathematics. 0 1 2 3
- 13) Most people learn mathematical best if they are taught the methods step by step. 0 1 2 3
- 14) When I'm learning mathematical I really want to know "how" and "why" the methods and ideas work. 0 1 2 3
- 15) Calculators shouldn't be used too much in school because they can lessen opportunities to practice computational skills. 0 1 2 3
- 16) Children learn deeply by investigating new types of problems different from ones they've seen before. 0 1 2 3
- 17) There is usually one best way to write the steps in a solution to a mathematical question. 0 1 2 3
- 18) Most people learn mathematical best if they explore problems in small groups to discuss and compare different approaches. 0 1 2 3
- 19) Learning to follow "the steps" to generate correct answers is very important. 0 1 2 3
- 20) It is important to develop connections between ideas by working on multi step problems. 0 1 2 3

3. Mathematics Questions

for questions 1 to 3 below on this page:

PART a): **Answer** the questions, showing your steps as needed to illustrate the method you used.

PART b): **Explain** what you can about why and how the method you used in a) works, using explanations, diagrams, models, and examples as appropriate. If possible, do the question another way.

1. 1.6×3

a) b)

2. $5 - (-3)$

a) b)

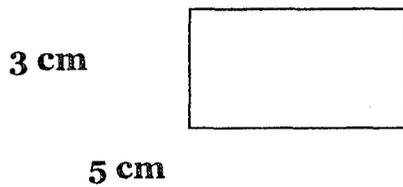
3. $1\frac{3}{4} \div \frac{1}{2}$

a) b)

4. Find and state the pattern rule that relates n and the result.

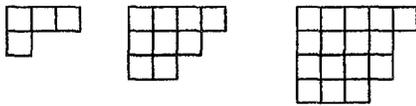
n	result
1	4
2	9
3	16
4	25

5. For the rectangle below, calculate



- a) the perimeter
- b) the area

5. State up to 3 different forms of an algebraic pattern rule for the number of tiles in each diagram below (depending on the frame number), which you would mark as correct if they were submitted by a student. Use n as the frame number. (The diagrams show the first 3 terms of the pattern).



1 2 3

- a)
b)
c)

Is it true that as the perimeter of a rectangle increases, so does the area? Explain.

APPENDIX B

Ethical Clearance

Lakehead

UNIVERSITY

Office of Research

Tel (807) 343-8283
Fax (807) 345-7749

August 15, 2005

Dr. Ann Kajander
Faculty of Education
Lakehead University
955 Oliver Road
Thunder Bay, Ontario P7B 5E1

Dear Dr. Kajander:

Re: REB Project #: 124 04-05
Granting Agency name: University of Manitoba CRYSTAL grant (NSERC)
Granting Agency Project #: 19389

Based on the recommendation of the Research Ethics Board, I am pleased to grant ethical approval to your research project entitled, "Teachers' Evolving Mathematical Understandings".

The Research Ethics Board requests an annual progress report and a final report for your study in order to be in compliance with Tri-Council Guidelines. This annual review will help ensure that the highest ethical and scientific standards are applied to studies being undertaken at Lakehead University.

Completed reports may be forwarded to:

Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
FAX: 807-346-7749

Best wishes for a successful research project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/len

cc: B.L. Crutchley, University of Manitoba
Margot Ross, Finance, Lakehead University
Research Office, Lakehead University





UNIVERSITY
OF MANITOBA

EDUCATION RESEARCH
CENTRE
100 UNIVERSITY AVENUE
WINNIPEG, MANITOBA R2S 1S6

EDUCATION RESEARCH
CENTRE
100 UNIVERSITY AVENUE
WINNIPEG, MANITOBA R2S 1S6
TEL: 204-945-5000
FAX: 204-945-5001

APPROVAL CERTIFICATE

02 September 2005

TO: Ann Kajander
Principal Investigator

FROM: Stan Straw, Chair
Education/Nursing Research Ethics Board (ENREB)

Re: Protocol #E2005:084
"Teachers' Evolving Mathematical Understandings"

Please be advised that your above-referenced protocol has received human ethics approval by the **Education/Nursing Research Ethics Board**, which is organized and operates according to the Tri-Council Policy Statement. This approval is valid for one year only.

Any significant changes of the protocol and/or informed consent form should be reported to the Human Ethics Secretariat in advance of implementation of such changes.

Please note that, if you have received multi-year funding for this research, responsibility lies with you to apply for and obtain Renewal Approval at the expiry of the initial one-year approval; otherwise the account will be locked.



Office of Research

(807) 343-8283
(807) 346-7749

MEMORANDUM

Date: February 8, 2006

To: Dr. Ann Kajander

From: Dr. Richard Maundrell

Subject: REB Project # 124 04-05

Thank you for your correspondence dated January 31, 2006 requesting an amendment to your approved ethics protocol entitled "Teachers' Evolving Mathematical Understandings".

The changes you have proposed have been reviewed, and they are acceptable to the Research Ethics Board.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board



UNIVERSITY
OF MANITOBA

OFFICE OF RESEARCH
SERVICES
Office of the Vice-President (Research)

244 Engineering Bldg.
Winnipeg, MB R3T 5V6
Telephone: (204) 474-84
Fax: (204) 261-0325
www.umanitoba.ca/resea

RENEWAL APPROVAL

04 May 2006

TO: Ann Kajander
Principal Investigator

FROM: Stan Straw, Chair
Education/Nursing Research Ethics Board (ENREB)

Re: Protocol #E2005:084
"Teachers' Evolving Mathematical Understandings"

Please be advised that your above-referenced protocol has received approval for renewal by the **Education/Nursing Research Ethics Board**. This approval is valid for one year only.

Any significant changes of the protocol and/or informed consent form should be reported to the Human Ethics Secretariat in advance of implementation of such changes.

Lakehead

UNIVERSITY

Office of Research

August 15, 2007

Tel (807) 343-8283
Fax (807) 346-7749

Dr. Ann Kajander
Faculty of Education
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1

Dear Dr. Kajander:

Re: REB Project #: 124 04-05
Granting Agency name: NSERC (Sub-grant from University of Manitoba)
Granting Agency Project #: N/A

On the recommendation of the Research Ethics Board, I am pleased to grant renewal of ethical approval to your research project entitled, "Teachers' Evolving Mathematical Understandings". This approval includes the amendment noted on page 2 of your Request for Renewal form.

Ethics approval is valid until **August 15, 2008**. Please submit a Request for Renewal form to the Office of Research by July 15, 2008 if your research involving human subjects will continue for longer than one year. A Final Report must be submitted promptly upon completion of the project. Research Ethics Board forms are available at:

<http://boit.lakeheadu.ca/~research/www/internalforms.htm>

During the course of the study, any modifications to the protocol or forms must not be initiated without prior written approval from the REB. You must promptly notify the REB of any adverse events that may occur.

Completed reports and correspondence may be directed to:

Research Ethics Board
c/o Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
Fax: (807) 346-7749

Best wishes for a successful research project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/len

cc: Office of Research
Margot Ross, Office of Financial Services

