Abstract

Research has investigated the use of locally valued activities to contextualize mathematics for First Nations students; for example, Beatty and Blair (2015), Lipka, Sharp, Adams, and Sharp (2007), Nicol, Archibald, and Baker (2013), and Wagner and Lunney Borden (2010). In this multidisciplinary case study, I have explored the mathematical thinking that resulted from a contextualized mathematics unit collaboratively implemented in a small Ontario First Nation elementary school. Although not considered decolonizing research, this project was influenced by culturally responsive methods and pedagogy (Battiste, 2002; Doige, 2010; Lipka, 2007; Lunney Borden & Wiseman, 2016; Nicol, Archibald & Baker, 2010). The Education Manager (EM), a local resident and member of the First Nation who represented the community in matters of education, collaborated on this project. She shared information gleaned from community surveys that expressed a desire for more outdoor and hands-on activities for elementary school students. Along with the teacher, the EM and I choose to use a school garden to contextualize the mathematics. Through collaboration with the teacher, a variety of mathematics problems were created that connected to the garden. Some of the problems were inquiry based, which is more closely related to Indigenous Ways of Knowing than traditional school mathematics (Battiste, 2005; Doige, 2010; Lipka, 2007; Lunney Borden & Wiseman, 2016; Nicol, Archibald & Baker, 2010). Lessons were implemented by the researcher over three weeks in a Grade 3/4/5 classroom. The mathematical thinking that resulted from the problems was organized and analyzed. The effectiveness of inquiry and contextualized mathematics was compared to more teacher-led methods. The findings of this study suggest that contextualized inquiry-based mathematics, connected to locally valued activities, elicits rich mathematical thinking.
Acknowledgements

To the families, students, community, and school that participated in this study, thank you for welcoming me into your school and sharing your time with me. It was a great experience to watch these students grow and learn and to be part of such a positive atmosphere.

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You are the best.
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Chapter 1: Introduction

1.1 Personal Introduction

My first twenty years as a teacher were spent on Haida Gwaii, an archipelago off the northern coast of British Columbia, Canada. The population of the islands is about 5,000. Approximately half of the residents are members of the Haida Nation, the First Nation whose ancestral home is Haida Gwaii. The Haida people are in the process of regaining much that has been lost: political power, cultural ceremonies, and language. The name of the islands was officially changed from the “Queen Charlotte Islands” to “Haida Gwaii” in 2009 (CBC News, December 11, 2009).

While living on Haida Gwaii I taught elementary and secondary school in the public-school system, as well as Adult Education at the college. My students reflected the population of the communities; about 50% were Haida students. Living in such a beautiful place, with a tumultuous and painful history, affected me deeply. Although I was aware of the devastating effects of European contact and residential schools for First Nation communities, I had not previously been immersed in a community struggling with the devastating consequences of losing their culture and so many of their people. Families were trying to build their way back from a painful past. My students and their families were recovering and rebuilding while living with devastating loss. As a teacher, one thing I could do was ensure that the truth of the past and support for the community were priorities in the classroom.

I saw the power that education could have for students if their lives were valued by being reflected in the classroom. I experienced firsthand the engagement and confidence that came when students saw themselves, and their history, respectfully brought into school.
As a teacher, I began to remove the wall between the classroom and the rest of the community. Parents were encouraged to visit, and students were taken out into the community more frequently. To include students’ own ideas and concerns in the school day I began using a project based approach to teaching. I learned how to address curriculum within content that was valued by the students in my class by incorporating contextualized and holistic methods into my teaching. I connected the curriculum to the culture and community of my students.

We spent time outside in the forest and at the beach. We read traditional stories and had family and community members come into class. I participated in community events and feasts so that I could learn about my students’ culture and get to know parents, aunties, uncles, and grandparents. I felt lucky to live where family and community were valued. I am not of First Nations descent, but I have had the privilege to be surrounded by Haida culture.

I started my Master of Education degree on Haida Gwaii with Cynthia Nicol who was researching culturally responsive mathematics education from a critical pedagogy perspective. She helped teachers contextualize mathematics in local culture using inquiry. This helped me to address many of the issues I had struggled with while teaching mathematics including inappropriate and unengaging material and a focus on memorizing instead of understanding mathematics. Implementing inquiry contextualized by locally valued activities helped all students to feel valued at school. After completing two master’s courses, I had to move to Ontario for personal reasons. We left our beloved island and I put my education aside for a few years.

In Ontario, I took a position as a Job Development Officer (JDO) for a local Tribal Council. The First Nation communities that I worked in as a JDO were all implementing some local cultural programming in their schools. After a few years, I met Ruth Beatty who was working on culturally responsive mathematics education at Lakehead University and I
applied to finish my education at Lakehead. One of the Education Managers (EM) was interested in collaboration. The EM, who lived in the community and was a member of the First Nation, was a member of the Band Administration which is the political body in the community. She represented the wishes of the community in matters of education. One of her responsibilities was to help implement relevant programming in the local elementary school.

She and I had become friends. We had worked together for three years through the Tribal Council and had many conversations about how to bring more relevant content into the small community-run elementary school. She knew me and my philosophy of education.

The EM and I spoke off and on until I was accepted into Lakehead to complete my MEd degree. From then on, we planned together in collaboration with the teachers at the school. I spent a year volunteering, helping to plant gardens, and doing math with students prior to starting to document learning.

1.2 Context of the Study

Current research in culturally responsive education for First Nations students suggests several guidelines. First Nations communities are all unique. Establishing a respectful relationship with members of the community is integral to work that will be of benefit to its residents (Donald, Glanfield, & Sterenberg, 2013; Nicol, Archibald, Baker, 2013; Lipka et al., 2007; Munroe et al., 2013). Inviting elders and community members to guide and contribute to the project is commonly practiced. Community members are invited into the school to spend time with students. Culturally responsive research often connects mathematics to decolonizing activities such as embracing local language and supporting the revitalization of local traditional practices (Donald et al., 2013; Nicol & Archibald 2013; Lipka et al., 2007; Munroe et al., 2013). Mathematical thinking unique to the community’s history can be
particularly empowering for students (Beatty & Blair, 2015; Lipka, 2005; Wagner & Lunney Borden, 2010).

This research project, although influenced by culturally responsive research and pedagogy, focuses on the mathematical thinking of students as a result of contextualized mathematics. A locally valued activity, gardening, was chosen to contextualize the mathematics. Although gardening does not revitalize First Nations culture, this community had chosen to focus on food security and the school garden was considered the flagship of this project (EM, personal communication, September 2017). The community had a long history of farming with many community members continuing in this profession. Gardening was a way to be self sufficient and nurture the community (EM, personal communication, September 2017). The EM, whose children attended the school, felt that a school garden could reflect the goals and values of the community. In addition, she took me around to visit the parents of the children in the school, so we could seek feedback concerning this project. Parents were very supportive.

Gardens would not normally be considered culturally responsive, in fact they may even represent colonization to many First Nations communities. Yet, the members of this community saw gardening and farming as a way to be self sufficient, be connected to nature, and have a source of healthy food (EM, personal communication, 2015). As part of the school garden project the harvest was shared with the community. This reflects the important community value of taking care of others (EM, personal communication, 2015).

Although this project does not reflect all the currently accepted practices of culturally responsive research, it is influenced by Indigenous methodologies. Some decolonizing ideas are included such as contextualizing curriculum in locally valued activities, using holistic methods, collaborating with community members and teachers, and using inquiry learning
A context which focused on the revitalization of culture, and the inclusion of elders and more local community members, would have more closely reflected what is currently accepted practice for culturally responsive academic research in Canada (Beatty & Blair 2015; Donald et al., 2013; Nicol, Archibald & Baker, 2010; Lipka et al., 2007). With a focus on the effects of contextualizing mathematics and the effects of using inquiry on mathematical understanding in children, this research can support other work that seeks to contextualize mathematics with Indigenous Knowledges to embrace and reflect the culture of Indigenous students.

The Province of Ontario has committed to “improve achievement among First Nation, Métis, and Inuit students” (Ontario Ministry of Education: Aboriginal Education Office, 2007, p. 5). Among the scholarly recommendations for a culturally responsive math program for First Nations students are that the program be holistic and contextualized in local activities, culture, people, and places (Doige, 2003; Lipka et al., 2005; Munroe et al., 2013). To meet the expectations of the Ontario Mathematics Curriculum, a mathematics program should be contextualized and inquiry- or problem-based (Ontario Ministry of Education, 2005). Inquiry mathematics, which is holistic and contextualized, aligns well with both the pedagogical recommendations of the Ontario Ministry of Education for mathematics and some of the pedagogical recommendations for First Nations students.

A school garden was chosen to contextualize the mathematics for this research. Growing food is valued in the First Nation community that hosted this research project. It is a very small community (population 150) that currently, and in the past, used farming and gardening as a source of income and food. The Coldwater-Narrows Agreement, ratified in
2012 (Bell) was based in the removal of members of this First Nation from land which they had turned into successful farmland in 1836. (EM, personal communication, Sept 2017).

According to the Government of Canada Fact sheet,

Over the next six years, the First Nations constructed a road (which ultimately came to be Ontario Highway No. 12) over the old portage route between the two villages and cleared the land along the road for farming.

Schools, houses, barns and mills were also built at the two villages (2010).

Today many community members are involved in farming and the students at the school are familiar with growing food and cooking with garden crops. In addition, the Education Manager shared the results from an extensive survey process that the community had just undergone. The results showed a strong desire among community members to have students from the school participate in hands-on learning and spend more time outside. The EM, teacher, and I felt this could be accomplished by connecting math and gardening.

Expectations from the Ontario curriculum were also integrated into the mathematics. Students in this community attended school on the First Nation until Grade 5 when they were bussed out of the community and went to a local public school. This transition could be difficult, and the community requested that students follow the Ontario curriculum to ease the transition and support their success after Grade 5. Although following the curriculum could be seen as using colonizing practices, the community requested the inclusion of curriculum and so this request was followed.

This instrumental case study was designed to explore the mathematical thinking of students that resulted from lessons contextualized in gardening and cooking, and connected to the expectations for teaching fractions in the Ontario Mathematics Curriculum. The collaborative nature of this project resulted in the creation and delivery of a variety of types of
lessons, some were inquiry-based, most were contextualized by the garden, and others were more teacher driven and not inquiry based. This created an opportunity for comparison of the mathematical thinking of students resulting from a variety of pedagogical styles of teaching and learning mathematics. This project considers the effects of inquiry and real-life context, as compared to more traditional teacher led mathematics, on the mathematical thinking of students. The results may contribute to the discussion of context-based mathematics, garden education, and First Nations education.

1.3 Purpose of the Study

The purpose of this study was to contribute to a body of research that is focused on the improvement of mathematics education for First Nations students by:

1) Collaboratively designing a contextualized mathematics unit for students in a First Nation community that considers the requests of the community to be hands-on, outside, and include the expectations of *Ontario Mathematics Curriculum*.

2) Collaborating with the teacher and EM to implement a math unit contextualized by a school garden in a small First Nation elementary school.

3) Documenting and analyzing the mathematical thinking of students which resulted from delivering mathematics problems with a variety of pedagogies contextualized by gardening.

1.3.1 Research Questions.

1) What types of mathematical thinking are supported by a variety of mathematics problems connected to the fraction expectations of the Number Sense and Numeration strand of the *Ontario Mathematics Curriculum*?

2) How does using the garden to contextualize mathematics affect the
mathematical thinking in elementary students in a small First Nation community?

3) Are there other benefits to using contextualized learning for First Nations students?

1.4 Significance of the Study

Meeting the needs of First Nations students requires curriculum that is culturally responsive (Ontario First Nation, Metis and Inuit Education Policy Framework, 2007; Gay, 2002; Nicol et al., 2010; Lipka et al., 2007). In Ontario, students in provincially run or associated schools are also expected to meet the curriculum requirements of the Ministry of Education. One aspect of implementing culturally responsive curriculum is that it should reflect local values and activities (Doige, 2003; Lipka et al., 2005; Munroe, Lunney Borden, Murray Orr, Toney, & Meader, 2013). Connecting locally valued activities to classroom projects and curriculum is more straightforward in areas such as Language Arts and the Fine Arts than in Mathematics. Mathematics textbooks are rarely connected to local activities.

The teacher who participated in this project, Ms. T, expressed that using inquiry mathematics and integrating local activities were practices that she wished to explore. She wanted to increase the cultural responsiveness of her mathematics program. Collaborating in contextualizing mathematics in real life activities was one step in that direction.

Increasing the cultural responsiveness of Mathematics programs in Canada is needed. According to graduation and employment rates, the original peoples of this country, including First Nations (FN) people, are not, in general, receiving an effective or successful education (Statistics Canada, 2015).

As can be seen in Table 1, there are low graduation rates and consequently high unemployment rates, among FN individuals (Aboriginal Peoples Fact Sheet for Canada).
This indicates that FN students’ needs are not being met in school. Table 1 uses statistics from the *Aboriginal Peoples Fact Sheet for Canada*, which gathered statistics from the “National Household Survey” (2011) and the “Aboriginal Peoples Survey” (2012), (Statistics Canada, 2015).

![Table 1](image)

<table>
<thead>
<tr>
<th>Comparison of Level of Education</th>
<th>On Reservation</th>
<th>Off Reservation</th>
<th>Non-Aboriginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Certificate/Diploma</td>
<td>47.2</td>
<td>25.6</td>
<td>12.1</td>
</tr>
<tr>
<td>High School Graduate</td>
<td>18.0</td>
<td>24.4</td>
<td>23.2</td>
</tr>
<tr>
<td>College Graduate</td>
<td>14.6</td>
<td>21.9</td>
<td>21.3</td>
</tr>
<tr>
<td>University Bachelor’s Degree or Above</td>
<td>4.6</td>
<td>10.8</td>
<td>26.5</td>
</tr>
</tbody>
</table>

These school statistics result in the employment rates shown in Table 2.

![Table 2](image)

<table>
<thead>
<tr>
<th>Comparison of Employment Rate</th>
<th>On reservation</th>
<th>Off reservation</th>
<th>Non-Aboriginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment rate</td>
<td>47.0</td>
<td>62.7</td>
<td>75.8</td>
</tr>
</tbody>
</table>

Note that the percentage of the non-Aboriginal population that has not graduated from High School is 12.1% whereas the on-reserve FN population without a high school diploma is almost four times higher at 47.2%. Almost half of the on-reserve population of FN students do not complete high school.

There are many issues that must be addressed to begin to change these statistics. In the
document, *Beyond Shadows: First Nations, Métis and Inuit Student Success*, published by the Canadian Teacher’s Federation, Toulouse stated that, “Residential schools, federal day schools, stereotypes, racism, oppression and poverty are only some of the shadows cast on Indigenous peoples” (2013, p. 5). Along with loss of culture, language, and land, the effects of these events on First Nations people can be seen in results such as, “high mortality, low graduation rates, increased diabetes, and youth suicide among others” (Toulouse, 2013, p. 5).

The challenge of changing these statistics and improving the quality of life for First Nations people must be met in Canadian society and therefore, in Canadian schools.

From the *Ontario First Nation, Metis, and Inuit Education Policy Framework: Delivering Quality Education to Aboriginal Students in Ontario’s Provincially Funded Schools* (2007):

Factors that contribute to student success include teaching strategies that are appropriate to Aboriginal learner needs, curriculum that reflects First Nation, Métis, and Inuit cultures and perspectives, effective counseling and outreach, and a school environment that encourages Aboriginal student and parent engagement. (p. 6)

In December of 2015 The Truth and Reconciliation Commission of Canada recognized that education in Canada was not addressing the needs of FN students. The Commission made several calls to action pertinent to education. In the section titled “Education for Reconciliation” the following calls to action were made:

62) We call upon the federal, provincial, and territorial governments, in consultation and collaboration with Survivors, Aboriginal peoples, and educators, to:

i. Provide the necessary funding to post-secondary institutions to educate teachers on how to integrate Indigenous knowledge and teaching
methods into classrooms.

ii. Provide the necessary funding to Aboriginal schools to utilize Indigenous knowledge and teaching methods in classrooms. (p. 121)

As well as in the section titled “Education”:

7) We call upon the federal government to develop with Aboriginal groups a joint strategy to eliminate educational and employment gaps between Aboriginal and non-Aboriginal Canadians. (p. 224)

10) We call on the federal government to draft new Aboriginal education legislation with the full participation and informed consent of Aboriginal peoples. The new legislation would include a commitment to sufficient funding and would incorporate the following principles:

i. Providing sufficient funding to close identified educational achievement gaps within one generation.

ii. Improving education attainment levels and success rates.

iii. Developing culturally appropriate curricula.

iv. Protecting the right to Aboriginal languages, including the teaching of Aboriginal languages as credit courses.

v. Enabling parental and community responsibility, control, and accountability, similar to what parents enjoy in public school systems.

vi. Enabling parents to fully participate in the education of their children.

vii. Respecting and honouring Treaty relationships. (p. 224)

These calls to action should contribute to FN communities being supported, and First Nations students being successful in this country. To implement these suggestions teachers
will need to find ways to integrate Indigenous Ways of Knowing into classrooms while also meeting the needs of the curriculum. One way to do this, as pointed out by research in culturally responsive education, is to encourage teachers to address the sharp contrast between traditional Western pedagogy and those of Indigenous Ways of Knowing, which are more holistic and subjective when compared to mainstream thinking (Battiste, 2005; Doige, 2003; Munroe et al., 2013; Oskineegish, 2013). Contextualizing mathematics lends itself to a more holistic way of teaching mathematics.

This project considers integrating the wishes of a First Nation community by using a more holistic pedagogy of contextualized mathematics to improve the classroom environment for students. Considering the mathematical thinking of students resulting from mathematics connected to their school garden may give insight into this part of culturally responsive math education. In turn this may help to create a more successful and positive experience for First Nations students in Canadian elementary schools.
Chapter 2: Literature Review

2.1 Introduction

As stated in the *Ontario First Nation, Metis, and Inuit Framework* from the Ontario Ministry of Education, the Province is committed to improving achievement among First Nations students (2007, p. 5). The Framework also states that:

There are several issues that impact on Aboriginal student achievement, including a lack of awareness among teachers of the learning styles of Aboriginal students and a lack of understanding within schools and school boards of First Nation, Métis, and Inuit cultures, histories, and perspectives.


As an alternative to traditional Western educational practices, using holistic pedagogy grounded in local values and beliefs, is recommended by scholars for First Nations students (Abrams, 2013; Doige, 2003; Lipka et al., 2005; Munroe et al., 2013). Culturally responsive mathematics, contextualized in local activities, has shown favorable results with increased engagement among students and increased support from the community (Civil, 2002; Howard & Perry, 2007; Lewthwaite, 2014; Lipka et al., 2005; Munroe et al., 2013). A culturally responsive program should reflect the community in which students live by using both culturally responsive content and pedagogy in the classroom.

2.2 Culturally Responsive Education

A teacher is practicing culturally responsive education when she or he is concerned with equity in the way that lessons are developed and delivered (Gay, 2002). Teachers “incorporate important aspects of the family and community culture of their students.” (Banks & Ambrosio, 2002, p. 1705). Culturally responsive education, which contextualizes learning
in students lived experiences, is used to engage culturally diverse learners (Gay, 2002, p. 106). Research has found that student achievement increases because of school experiences connected to students’ own “cultural and experiential filters” (Gay, 2002, p. 106). More than just including content, “experiential filters” refers to an understanding and respect for the ways in which different cultures interact with the world. For First Nations students, as Battiste points out,

Such rethinking of education from the perspective of Indigenous knowledge and learning styles is of crucial value to both Indigenous and non-Indigenous educators who seek to understand the failures, dilemmas, and contradictions inherent in past and current educational policy and practice for First Nations students. The immediate challenge is how to balance colonial legitimacy, authority, and disciplinary capacity with Indigenous knowledge and pedagogies. (2005, p. 4)

In Canada, current work in culturally responsive education seeks to decolonize classrooms to support the learning of Indigenous students (Donald et al., 2013; Munroe et al., 2013; Nicol et al., 2013). This includes contextualizing learning in culturally revitalizing activities such as community story-telling, language, and other historically important cultural activities. In this research, the context was a school garden. Although not a decolonizing activity it was valued in the community and allowed for the contextualization of the math in an outdoor and hands-on activity.

Learning through the care for a school garden was an accessible way for the teacher and students to be outdoors. In the garden, students could learn about nature while contributing to the well-being of the community through the provision of healthy food. Also, this context supported the use of more holistic and inquiry based mathematics, both
pedagogies encouraged for use in culturally responsive programs for First Nation students (Donald et al., 2013; Nicol et al., 2010; Lipka et al., 2007).

It is important to consider the cultural relevancy of both the content and the way in which it is delivered to students. Traditional school mathematics are often delivered by the teacher or from a textbook (Van de Wall, 2015). To deliver mathematics in a more holistic and less compartmentalized way, which is more reflective of Indigenous Ways of Knowing, an inquiry or problem-solving method can be used (Lipka et al., 2005; Nicol et al., 2013; Wagner & Lunney Borden, 2010).

2.2.1 Culturally responsive pedagogy. Pedagogy, or the ways in which teachers plan and deliver curriculum, can vary greatly. Students can learn in a variety of ways such as via lectures, textbooks, guest speakers, or by investigating real life problems. In a traditional Western style of classroom pedagogy students are seated at desks receiving curriculum from their teacher, often using a commercially prepared textbook. Culturally responsive education requires culturally responsive pedagogy. The Report of the Royal Commission on Aboriginal Peoples, states that a supportive environment for First Nations students, “must be holistic”, meaning that schools must address the intellectual, spiritual, emotional, and physical development of participants (1996, p. 414). As a non-First Nation person raised in a Westernized culture and educated in Westernized schools it is difficult to imagine what Indigenous knowledges would look like in a school setting. There is a paradigm shift required; “The Aboriginal approaches to learning are spiritual, holistic, experiential/subjective, and transformative. In contrast, mainstream approaches to learning are secular, fragmented, neutral/objective, and seek to discover definitive truth” (Doige, 2003, p. 147).

In the expansive literature review, Battiste states, “Education for wholeness, which
strives for a level of harmony between individuals and their world, is an ancient foundation for the educational processes of all heritages. In its most natural dimension, all true education is transformative, and Nature centered” (2002, p. 30) and that, “Educational reforms must end the fragmentation Eurocentric educational systems impose on First Nations students and facilitate the goal of wholeness to which Indigenous knowledge aspires” (2002, p. 30).

An experiential method of teaching can deliver holistic pedagogy by involving students intellectually, physically, spiritually, and emotionally. Experiential learning means that students participate in learning through real life, hands-on, activities as opposed to the print based learning often found in schools (Dewey, 1968). To do this, the curriculum can be contextualized by activities in which students will participate. School lessons can be built from activities rather than from textbooks or curriculum. This allows for a more holistic experience of learning, a pedagogy more closely tied to Indigenous Ways of Knowing.

Since all First Nations communities have unique cultures, languages, and practices, it is not appropriate to try a one-size-fits-all solution but, as pointed out by scholars, there are some commonalities that can be reflected in the classroom to better meet the needs of First Nations students (Nicol et al., 2013; Oskineegish, 2013; Wagner & Lunney Borden, 2010). For instance; “teachers can be aware of the widespread values of community involvement, and the benefits of holistic teaching” (Oskineegish, 2013, p. 7).

Instead of learning experiences being structured around a textbook or a curriculum concept, learning can be structured around an activity valued in a community. This allows mathematics to “emerge in the course of teaching and learning, rather than framing the manner in which teaching and learning proceed” (Wagner & Lunney Borden, 2010). Holistic methodology contextualized by place can contributes to a more culturally responsive pedagogy.
2.2.2 Holistic education. Holistic learning engages the whole child and students are active participants in their learning (Dewey, 1968). In a research study from Nicol et al., (2010) a group of teachers working together to implement culturally responsive education in their classrooms were guided by Archibald’s (2008) storywork. This work frames Indigenous learning in the ideas of respect, relevance, reciprocity, and responsibility; “This perspective represented a different way of viewing knowledge and knowing. A mechanistic, compartmentalized view of teaching was replaced by a view that recognized the interconnected, wholistic, and synergistic intricacies of teaching.” (Nicol et al., 2010, p. 53).

Students can learn holistically through experiential education, or active involvement in their learning. Teaching math connected to a garden where students plant, weed, water, and harvest, uses a holistic and experiential pedagogy.

A garden may also support emotional and spiritual engagement of students. Many First Nation communities, including the one participating in this project, embrace caring for the people and places in the community as an important value. Students can share the bounty of the garden with family and friends. The garden can support intellectual engagement through learning about nature and growing plants. Also, because gardening is a social activity, the students’ social engagement is also supported.

The word holistic can apply to the student experience as well as the way curriculum is delivered. A holistic program often integrates different areas of curriculum usually around a project or theme. Holistic can refer to integrating a variety of parts of the school curriculum such as Language Arts, Physical Education, and Music or integrating the various parts within a curriculum area such as addition, multiplication, and geometry in the math curriculum.

In mathematics, the different conceptual areas are often treated in isolation. Holistic mathematics instruction combines different conceptual areas within one lesson (Van de
Walle, 2015). Using real life activities for education allows students to be engaged holistically while also providing a theme around which one can integrate learning. A holistic math activity would attempt to engage students at different personal levels (physical, emotional, spiritual, and intellectual) while also connecting various parts of the curriculum. Inquiry mathematics, or using problem solving to teach mathematics, can be contextualized by real life activities. Inquiry mathematics can be connected to Indigenous Ways of Knowing when it is holistically delivered and contextualized in locally valued activities.

2.2.3 Culturally responsive math education. Research has been done in the area of culturally responsive mathematics programs for First Nations students. Indigenous culture has been brought into the classroom and connected to mathematical concepts. Although the number of studies that have been done in this area is small, researchers have found that engagement in mathematics improved (Lewthwaite, 2014; Wagener & Lunney Borden, 2010) and curriculum could be effectively delivered (Beatty and Blair, 2015; Lipka et al., 2007; Munroe et al., 2013) when students’ own cultures were represented in their learning. In an Australian study, Howard and Perry (2007) suggested that sensitivity to and understanding of Indigenous students, and appropriate culturally connected curriculum, can enhance learning. In Alaska, Lipka et al. (2005), have found positive results academically for both Indigenous and non-Indigenous students through the implementation of *Math in a Cultural Context*.

Problem solving, or inquiry is often used in these culturally responsive projects because it reflects a holistic style of learning and a respect for students’ cultural or traditional ways of learning. Nicol et al. (2013), pointed out that inquiry is integral to culturally responsive mathematics, “A focus on inquiry seemed to shift views from cultural and mathematical deficiency to those that are respectful of students’ ways of understanding and cultural diversity” (p. 85). A pedagogy of inquiry contextualized by locally important
activities is also about engagement. As Lunney Borden and Wiseman state, “integration is not so much about specific content and control but rather about pedagogy and how engagement in teaching and learning allows for growth in mind, body, spirit, and heart.” (2016, p. 144).

The Indigenous community that participated in this study requested that the culturally responsive mathematics unit include expectations from the *Ontario Mathematics Curriculum*. As will be discussed in the next section, inquiry learning is recommended in the Ontario curriculum.

2.3 Mathematics Education

2.3.1 *Ontario mathematics curriculum.* In Ontario, the mathematics curriculum delineates the expectations for content in schools. Specific mathematics concepts must be introduced at different grade levels. The concepts build upon each other in a spiral nature, each conceptual area becoming more complex as students move up the grades in school. The expectations for fractions from the Number Sense and Numeration strand of the mathematics curriculum were integrated into the development of the problems for this unit (Ontario Ministry of Education, 2005). They can be seen in Appendix A.

In addition to the concepts that students are required to learn, the *Ontario Mathematics Curriculum* recommends an inquiry-based program, which means that a problem-solving approach is recommended. “Problem solving forms the basis of effective mathematics programs and should be the mainstay of mathematical instruction” (Ontario Ministry of Education, 2005, p. 11). These ideas are well supported by the research into the ways in which children’s mathematical understanding develops (Bobis, Anderson, Martin, & Way, 2011; Lamon, 2007; Verschaffel, Greer, & Torbeyns, 2006).

From the introduction to the document *Teaching and Learning Mathematics: The*
Report of the Expert Panel on Mathematics in Grades 4-6 in Ontario, (2006) the characteristics of “Effective Mathematics Instruction” include the following:

- is focused on having students make sense of mathematics
- is based on problem solving and investigation of important mathematical concepts
- includes students as active rather than passive participants in their learning
- has students communicate and investigate their thinking through ongoing discussion (p. 8).

These ideas are a result of the development of research into children’s learning. There have been many influences on their development. One influence is constructivist theory, which posits that students learn by building knowledge rather than being filled like an empty vessel. Students need to use mathematics rather than just receiving it:

In fields other than mathematics, we’ve understood this constructive nature of learning. We teach students to become good writers by involving them in the process of writing. In science, we engage learners in actively inquiring, in formulating hypotheses, and in designing experiments. We teach art by allowing learners to create it. Have we traditionally been teaching mathematics in our classrooms or only the “history” of mathematics some past mathematicians’ constructions and their applications? Is there any connection at all between “school mathematics” and “real mathematics”? (Fosnot & Dolk, 2001, p. 19)

For students to participate in their learning they need to be given more than the “history of mathematics”. To use mathematics to work on problems requires that students use
mathematical thinking, rather than memorizing procedures, to solve problems. This is possible if students learn and work at a conceptual level rather than a procedural level.

Students are using mathematics, or mathematizing, when they are using their mathematical understanding to solve a problem rather than using a memorized procedure (Lamon, 2007). For example, students can learn to add fractions by finding a common denominator through the process of multiplying the two denominators. This is a procedure. Students can also learn to add fractions by using fraction strips that allow them to manipulate concrete objects which represent fractions. Manipulating the strips and exploring ideas can lead to the understanding that two quarters are the same as one half. This illustrates understanding the use of a common denominator because it makes sense rather than because it is a rule. Problem solving facilitates mathematizing because it allows students to explore underlying concepts and connect them to their current understanding.

2.3.2 Inquiry or problem solving. Math problems elicit mathematizing and rich mathematical thinking when they have multiple ways of being solved (Lamon, 2007; Utley & Reeder, 2012). This can be done if problems are open ended and students are given sufficient time to work on them (Stigler & Hiebert, 2004). Rather than “2+2=?” we can be more open ended and ask, “Show me two ways to represent 4”. This moves the focus to the work rather than the answer and from the procedure of calculation to the concept of addition.

An analysis of a large-scale study conducted in 1995, the Third International Mathematics and Science Study (TIMMS) video study, (Stigler & Hiebert, 2004) compared teaching methods and performance by students on standardized mathematics testing in several countries around the world. The authors found that students in the United States were under performing compared to countries such as Japan and the Netherlands. Upon close examination of the teaching methods used it was found that teachers were using problem
solving with students for as much time in the US as in other countries but that the teachers in
the US turned problem solving lessons into procedural lessons. Problems that were worded to
lead students to a conceptual investigation were taught in such a way that they stopped being
exploratory, stopped having more than one answer, and became the practice of skills already
learned. Practicing skills is the same as using a procedure. Conceptual learning includes
mathematizing and encourages connections between various mathematical concepts (Lamon
2007; Stigler & Hiebert, 2004). The difference in the way problems were used by teachers
was seen by the researchers as the core issue contributing to lower scores in American
students. Students were not getting any time in mathematics to consider mathematical
concepts and use mathematical thinking.

2.3.2.1 Contextualized problem solving. To help children make sense of mathematics it has
been found that not only problem solving, but contextualized problem solving, is very helpful
(Bonoto, 2005; Verschaffel et al., 2006). It has been noted that students confronted with math
problems at school will use less common sense compared to when they are out of school. In
school, as opposed to out of school, students will often use standard algorithms in an,
inefficient and meaningless way, suggesting a limited understanding of the
required mathematical concepts and procedure. In contrast, when offered
the same type of problem in out of school contexts, children are able to
efficiently answer these problems using diverse self-invented mental
computation. (Verschaffel et al., 2006, p. 57)

Bonoto (2005), used cultural artifacts familiar to students and an interactive teaching
method to counteract the tendency of students to suspend sense making and exclude reality
from their mathematical problem solving in school. The positive benefits to this method were
likened to those seen in the Realistic Mathematics Education program developed in the
Netherlands. The key components of Realistic Mathematics Education are similar to the characteristics of “Effective Mathematics Instruction” from the *Ontario Mathematics Curriculum* and consist of the following:

- Learning math is a constructive activity, learning math in “the course of social activity” that has a purpose
- Use of realistic or meaningful context problems for development of math knowledge
- Use carefully chosen mathematical models to bridge between students’ intuitive knowledge and formal math
- Learning through social interaction and cooperation for reflection and enhancement of understanding
- Interconnecting the various learning strands.

(Verschaffel et al., 2006, p. 55)

Connecting problems to real contexts facilitates understanding of deeper, more complex, underlying concepts (Bonoto, 2005; Lamon, 2007). Contextualized mathematics is also encouraged by mathematics researchers because of its ability to engage students in relevant and meaningful learning (Bonoto, 2005; Calder & Brough, 2013).

Contextualized mathematics can be delivered through holistic education. Contextualizing inquiry in locally valued activities is one step in meeting the needs of First Nations communities and their students.

### 2.3.3 Connecting inquiry learning, holistic learning, and contextualized instruction

Inquiry learning, holistic learning, and contextualized instruction align well with each other. Using inquiry to teach mathematics requires teachers to use problem solving. Math problems can be connected to real life activities or contextualized within valued
activities. A focus on local community values and activities narrows the contextualization to the local community. Inquiry learning and contextualized instruction both support a holistic and experiential way of learning. Holistic learning is one component recommended for Indigenous education (Doige, 2003; Lipka et al., 2005; Munroe et al., 2013).

The open-ended problems used in inquiry mathematics allow a holistic approach to the content. Mathematics can be approached as a connected group of concepts rather than isolated topics. Understanding the connections between concepts encourages the development of mathematical understanding (Bonoto, 2005; Lamon, 2007; Fosnot & Dolk, 2001). Table 3 helps to illustrate the connections between inquiry mathematics, contextualized instruction, and holistic learning.

<table>
<thead>
<tr>
<th>Area</th>
<th>Inquiry Mathematics</th>
<th>Contextualized Instruction</th>
<th>Holistic Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holistic</td>
<td>Can be</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Experiential</td>
<td>Can be</td>
<td>Can be</td>
<td>Can be</td>
</tr>
<tr>
<td>Contextualized by real life</td>
<td>Can be</td>
<td>Can be</td>
<td>Yes</td>
</tr>
<tr>
<td>Problem solving</td>
<td>Yes</td>
<td>Can be</td>
<td>Can be</td>
</tr>
<tr>
<td>Student centered</td>
<td>Yes</td>
<td>Can be</td>
<td>Yes</td>
</tr>
</tbody>
</table>

In this project these conceptual areas are connected in the form of a garden-based mathematics unit.

2.3.4 Meeting in the Garden. School gardens have become popular in the last few years as they bring a plethora of benefits to children. In the academic article; “Impact of
Garden-Based Learning on Academic Outcomes in Schools: Synthesis of Research Between 1990 and 2010”, Williams and Dixon (2013), considered 152 articles. Choosing 48 studies that met selection criteria based around clear and relevant methodologies with measured outcomes they reported that:

The results of the studies indicate strong and frequent positive impacts in all areas studied. The results of the studies show overwhelmingly that garden-based learning had a positive impact on student’s grades, knowledge, attitudes, and behavior. (p. 225)

These findings are encouraging because they demonstrate increased academic achievement as well as behavioral, social, and health benefits. Yet, the authors concluded the article by noting that garden-based learning research has been less than rigorous and that teachers find the lack of resources a challenge to using a garden for teaching purposes. These conclusions are echoed in recent studies, which also show positive effects on students (Blair, 2009; Christopher-Ipaktchian, 2014; Pittman, 2011) with teachers often struggling to incorporate this learning tool into the curriculum (Blair, 2009; Christopher-Ipaktchian, 2014).

Hands-on activities like gardening are partly effective in education because they are appealing to children. Being outside, seeing plants grow, and eating food they tended themselves, are very engaging activities.

2.3.5 Engagement. Research has pointed out that engagement is critical for learning mathematics (Bobis et al., 2011; Gettinger & Walter, 2012; Lein et al., 2016). In fact, when it comes to problem solving, a study from Lein et al. (2016) found that, “Engagement was a stronger predictor than prior mathematics achievement and accounted for 26.9% of the unique variance in mathematics problem solving, whereas prior mathematics achievement accounted for 18.4% of the variance” (p. 119). Engagement can be defined as “on task behavior” (Lein
et al., 2016, p. 117) or displaying a “productive disposition” (Bobis et al., 2011, p. 35). Bobis et al. (2011), have made recommendations for ways of increasing student engagement, which include contextualizing mathematics with student interests and using open ended problems.

Garden-based learning contextualizes mathematics in real world examples, which gives students concrete activities within which they can use their mathematics skills. This real-world context may serve to engage students and the engagement serves to enable a deeper understanding of mathematics. Students appreciate using math to solve real problems as opposed to abstract problems and, as they work, they can process and practice their math skills in meaningful ways (Bobis et al., 2011; Bonoto, 2005).

For all students, contextualizing mathematics in activities that are familiar and enjoyable should encourage engagement and facilitate learning. For First Nations students, this means using activities that are familiar and enjoyable to the individuals that are being taught, as opposed to culturally inaccessible content. Students need to see themselves in the mathematics activities. To do this, problems for inquiry math can be connected to chosen activities and designed for students to explore mathematical concepts.

2.4 Mathematical Concepts

The teacher involved in this study and the Education Manager requested that the math unit address the expectations of the fraction section of the Number Sense and Numeration strand of the Ontario Mathematics Curriculum. These expectations can be reviewed in Appendix A. The curriculum includes the development of a facility with numbers known as “number sense” a subset of which is “fraction sense”.

2.4.1 Number sense. Fraction sense and number sense refer to the ability to work with fractions and whole numbers in a holistic manner, relating addition, subtraction,
multiplication, and division to working with each other and to fractions (Aksu, 1997; Bonoto, 2005; Fosnot & Dolk, 2001, Lamon 2007, Sowder et al., 1998; Verschaffel et al., 2006). In textbooks, these concepts are usually dealt with separately, even in problem solving.

Research has suggested that the development of number sense occurs when students have sufficient time to work with a variety of mathematical concepts and understand how they are connected (Aksu, 1997; Bonoto, 2005; Lamon, 2007; Sowder et al., 1998; Verschaffel et al., 2006). For example, addition is connected to multiplication because one type of multiplication is repeated addition. Another example is that fractions, or parts of a whole, are connected to division because the numerator is divided by the denominator.

Development of mathematical thinking was explored in an Australian study of number sense and problem solving (Louange & Bana, 2010). These researchers defined number sense as “estimation” and problem solving as “any question where the answers and the procedure are not obvious.” In this year long investigation of Year 7 students, the authors found that there was a very strong correlation between the ability to estimate and the ability to solve a problem. In the Ontario curriculum, estimation is part of the Number Sense and Numeration strand. Thus, this strand of the mathematics curriculum is important for problem solving in general.

Facility with numbers is increased by being able to deconstruct numbers into their parts. This can lead to using friendly numbers such as 5 and 10. For example, one can add 7 and 6 by breaking the 6 into 3 and 3 and then adding one 3 to the 7 to make 10. With one 3 left over the answer is 10 plus 3 or 13. In an experimental study focused on teaching young children decomposition strategies, Cheng (2012), found that a computational strategy of decomposing numbers is highly advantageous in later grades if taught to children when they
are younger. Cheng points out that using the alternative “counting on” strategy for addition and subtraction becomes unwieldy as students move to larger numbers. Having a deep understanding of these concepts facilitates the move from concrete to abstract thinking, or working with real objects to using symbolic representations such as numerals. Also, working with numbers using the decomposition strategy leads to deeper understanding of the concepts of addition and subtraction, which would contribute to the development of number sense (Cheng, 2012).

Conceptually based, contextualized, problems support students in developing flexibility in number sense because students explore the connections between concepts (Bonoto, 2005). This same flexibility with concepts applies to fraction concepts and is called fraction sense (Asku, 1997; Lamon 2007). Number sense involves an understanding of additive and multiplicative thinking. Multiplicative thinking involves comprehension of the ideas within the areas of multiplication, division, and fractions. Part of the challenge of elementary mathematics is a transition from additive to multiplicative thinking.

2.4.2 Additive to multiplicative thinking. The transition from additive to multiplicative thinking is challenging and takes a long time to develop (Lamon, 2007; Sowder et al., 1998). Part of the reason this transition is difficult is that the definition of the unit changes between the two conceptual areas (Lamon, 1999; Sowder et al., 1998).

In multiplication, a unit becomes a complex entity that has a quantity greater than one. Students must keep track of both the number of groups and the number in each group. In multiplication and division two quantities are combined to make a third quantity that is unlike the original two quantities. This is different from additive operations where the new quantity is directly related to the original quantities (Milligan, 1998). As Lamon (1999) points out, students who are comfortable with multiplicative thinking are students who easily build and
use composite units when the situation calls for it. In her review of the literature from the
Second Handbook of Research on Mathematics Teaching and Learning entitled “Rational
Numbers and Proportional Reasoning: Toward a theoretical framework for research” she
discusses research in this area and concludes that being able to create units “may be an
important mechanism in accounting for the development of increasingly sophisticated
mathematical ideas” (2007, p. 643).

The unitizing that happens in multiplicative thinking has a part/whole construct that
allows students to think about both the complex unit and the individual parts at the same time.
This is the flexibility in “one’s reasoning that is needed in the field of rational numbers”
(Lamon 2007, p. 643). This research also states that students may benefit from extended time
with this concept as well as an opportunity to connect the concepts of addition, subtraction,
multiplication, and division.

Problem solving is an excellent way to connect these concepts, giving students time to
work with them at their own pace before they begin to learn formal algorithms. Problem
solving, as compared to the more procedural mathematics of practicing an algorithm, allows
students some flexibility and creativity in their thinking. This often means that they will use a
variety of mathematical concepts while working on one problem (Bonoto, 2005; Lamon,
2007). This allows them to see the connections between concepts in mathematics. Working
with fractions, a subset of multiplicative thinking, was the focus of the mathematics in this
project.

2.4.3 Fractions and rational numbers. Fractions are conceptually rich, being in and
of themselves both a multiplication and division “problem”; \( \frac{3}{4} \) means 3 divided by 4, or
3/4 of something, which represents multiplication because we divide the whole into four equal parts and then consider three of these parts (3 times 1/4 is 3/4). The conceptual area of fractions is considered a pivotal concept in elementary mathematics because of its complexity and subsequent difficulty for students to grasp (Cooper et al., 2012; Lamon, 2007).

Students often use whole number concepts, that are not applicable, to work with fractions (McNamara & Shaughnessy, 2010, p. 4). For example, when multiplying whole numbers, the product of two numbers is larger than both factors. Whereas with fractions, the product is smaller than at least one of the factors. For example, in the question 10 x ½ =5 the product is smaller than the multiplicand, 10. Another idea that is confusing for students is that as the numeral in the denominator of a fraction gets larger the value of the fraction gets smaller. This is the opposite of the case with whole numbers. For example, 1/5 is smaller than 1/2 even though 5 is greater than 2.

The transition from working with whole numbers to working with fractions can be supported by the development of fraction sense. Fraction sense involves a fluidity and flexibility with fractions and rational number concepts (Lamon, 2007). Encouraging students to think mathematically through problem solving before introducing formal algorithms should assist in the development of fraction sense. As Aksu (1997) has related:

A common type of error in teaching fractions is to have students begin computations before they have an adequate background to profit from such operations. Students must understand the meanings of fractions before performing operations with them. (p. 375)
One way to encourage fraction sense is by using estimation. Comparison using benchmarks such as 0, 1, and 1/2 helps students to get an idea of the size of fractions in a concrete way (Van de Walle, 2015).

2.5 A Framework for Problems and Investigating Student Thinking

Learning about and understanding fractions can be addressed in a variety of ways. The traditional way, used in most textbooks, breaks down the skills and procedures involved in learning to work with fractions into small steps. Students then learn to use and practice the steps. This method teaches students the procedure but does not develop conceptual understanding. Using an inquiry or problem-solving approach that emphasizes conceptual understanding has been shown to increase mathematical competency with fractions over the long term (Cramer, Post, & del Mas, 2002; Lamon, 2007; Pitta-Pantani & Nicolaou, 2016).

Fraction instruction is often subdivided into subcon structs or subsets as areas of instruction. Historically these subsets are:

- part-whole
- part-part
- fraction as quotient
- fractions as operator
- measurement

(Fosnot & Dolk, 2002; Lamon, 2007; Ontario Ministry of Education, 2014)

More recently a variety of authors have proposed additional conceptual areas for teaching and learning fractions. These ideas build on the subcon structs and are intended to support holistic teaching methods. Fosnot and Dolk (2002), who have encouraged a conceptually based constructivist program where students use problem solving to grapple
with the big ideas of mathematical concepts, have suggested the following areas to address when studying fractions with elementary students;

1) **Part whole relation**: a fraction is the relationship between the part (numerator) and the whole (denominator).

2) **Equivalency vs congruency**: different shaped fractions can have the same area or contain the same fraction of the whole.

3) **Connecting multiplication and division to fractions**: fair sharing is a fraction, the numerator and denominator are related through division, a fraction of an object is a multiplicative idea.

4) **The whole matters**: when comparing two fractions the whole is imperative because fractions are relations.

5) **Relations on relations**: when dividing and multiplying fractions students must consider two different wholes at the same time.

Lamon (2007), in a discussion of rational numbers and proportional reasoning, pointed out that rational numbers are a type of proportional reasoning and that this complex area of mathematical understanding is difficult to separate into parts. It is easier to see it as a web of interconnected ideas under the umbrella of multiplicative reasoning. She suggested several related areas for focus when teaching rational number concepts in the elementary classroom. These are:

1) **Measurement**: conceptual ideas of measurement such as the inverse relationship between the size of a unit and the number of times you can measure it out of a quantity, or the existence of infinite subunits between measures.

2) **Rational number interpretations**: part-whole, measure, operator, quotient, ratio.
3) **Reasoning up and down**: simple proportional reasoning without using an algorithm. For example, if half a cup of sugar is $1.00 then one whole cup of sugar will be one half plus one half or $1.00 plus $1.00 which is $2.00.

4) **Sharing and comparing**: partitioning or fair sharing strategy often a first experience with fractions. For example, dividing up a chocolate bar with friends (sharing) and making sure everyone gets an equal amount (comparing).

5) **Unitizing and norming**: partitioning into manageable pieces or chunking. These are common pre-instructional strategies that students have. These strategies involve being able to see a ratio and use it without calculations. Students use groups to create a unit that can be applied to solving a rational number problem.

6) **Relative thinking**: comparative thinking where separate numbers change relative to each other.

7) **Quantities and covariation**: transforming quantities while maintaining the relationships of the numbers.

There is some overlap between the subdivisions of Fosnot & Dolk (2002) and those of Lamon (1999, 2007). When taken together these concepts provided a comprehensive framework for developing problems for this project, analyzing the data, and considering the results.

### 2.6 Conclusion

My goal in this multidisciplinary research project was to investigate the mathematical thinking of students from contextualized mathematics problems to move towards a more culturally responsive mathematics program. In this action research project, a variety of types of lessons, including some contextualized by the school garden and using an inquiry method,
were implemented. The children’s mathematical thinking resulting from these lessons was analyzed and compared.

Inquiry mathematics and Indigenous Ways of Knowing have some pedagogical similarities including holistic and context-based learning. Inquiry mathematics is recommended by scholars for the development of number and fraction sense (Lamon, 2007; Bonoto, 2005) and is a more holistic way of learning mathematics. Contextualizing inquiry-based mathematics with gardening can teach mathematics conceptually while engaging students in authentic, holistic, outdoor experiences.
Chapter 3: Methods

3.1 Design

3.1.1 Research in and with Indigenous communities. This research project is influenced by the practices of culturally responsive research. Although there are aspects included in this project that reflect current expectations for research in and with Indigenous communities, this project does not reflect the full spectrum of those expectations.

According to Donald, Glanfield, and Sterenberg (2012), researchers in Indigenous education have a responsibility to “disrupt colonial logics” (p. 55). This disruption should take place when considering the mathematics itself, the way it is taught, and the way the research is conducted (Donald et al., 2012). As can be seen from Table 4, created from the work of Donald et al., (2012), this project is influenced by, but does not fully represent, culturally responsive research in Canada.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Indigenous Education Research</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematics</td>
</tr>
<tr>
<td>Colonial influence</td>
<td>Mathematics follows the European idea that math is a pristine body of knowledge to be delivered.</td>
</tr>
<tr>
<td>Contextualized Garden Math project</td>
<td>Math ideas from curriculum but students had flexibility and use of invented and alternative algorithms.</td>
</tr>
</tbody>
</table>
This research project addresses a part of what is needed to creating curriculum that may be more responsive to the needs of First Nations students, that of contextualizing mathematics and using inquiry based problems. There are many other important aspects, such as integrating language and connecting cultural activities, that should be considered when creating culturally responsive content (Donald et al., 2012, Munroe et al., 2013; Nicol et al., 2013).

3.1.2 Participatory action research. This research project used a participatory action approach to collaborate with a teacher and the EM in a small First Nation community. This research method was chosen because it is democratic, socially responsive, and takes place in context (Miles, 2000). The relationship between myself, the students, the teacher, and the community was integral to this project. By choosing this method of research I was choosing to focus on the needs and expectations of the community.

As noted by Park (1999), participatory research, “most clearly distinguishes itself from other forms of action-related research by the fact that it issues from the felt needs of a community” (p. 143). The wish of the community to integrate more outdoor education and hands-on learning were the motivating factors for this project. I was made aware of this wish by the Education Manager of the community whom I had met several years before this project. Parents contributed to this school on a regular basis. Parents’ and children’s’ ideas
and collaboration were encouraged and supported by myself and the school. I was invited to work with the teacher and the students to implement a garden-based mathematics unit at the school in May of 2014, so we met to discuss this opportunity.

At the beginning of the project I met with the teacher and the Education Manager. I went into the school over the course of a one year to spend time with the students, help start a garden, and get to know the teacher. I brought in ideas for problems and Ms. T, the teacher, would look at them or try them with the students and then we would discuss them. Sometimes they were too complicated, or they did not align with the values of the school community. For example, we discussed having a sale of garden produce and the teacher felt that this would go against the idea that students were caring for the community. She did not want money to be part of the project, so we took those problems out. As well, the content of the problems was changed to support the classroom teacher and the program she had planned for students. Originally the problems were to be based on Number Sense, but this was changed to Fraction Sense to align with the unit that was being taught.

During the recorded portion of the project the teacher and I met after each class to plan the next one. We both came up with ideas. These often evolved from the previous class. This project changed many times to try to accommodate the teacher and students. The purpose of the project was to help the teacher and students move closer to a math program that more closely reflected the community. This project attempted to support the teacher who was learning how to contextualize mathematics and connect it to the students lives. Although some of the classes ended up being more teacher focussed and less inquiry driven, this was a result of this collaborative relationship. Many adjustments were made to lessons and problems as a result of our discussions.

3.1.2.1 Meetings with the Education Manager. The initial idea of having more time outside
engaged in experiential activities was generated within the First Nation community through surveys (EM, personal communication, March 2014). This idea was communicated to me by the Education Manager who works for the Band Administration, a political body that works for the community. Meetings among the teacher, myself, and the Education Manager were held to decide how more time outside engaged in experiential learning could be accomplished. It was felt that a school garden would provide many opportunities for mathematical thinking and would easily connect to families and community, connections which were important to the school. The Education Manager also supported the idea of gardening with students because the recent history of the community was one of farming. The residents of the community had supported themselves with farming since the early 1800’s (EM, personal communication, 2017; Government of Canada, 2010). As well, the community was beginning a new initiative around food security and the small school would be the flagship project (EM, personal communication, 2017).

3.1.2.2 Meetings with teacher. The teacher at the school was involved in creating the problems, as well as implementing and assessing the garden math unit. As this project evolved, plans were changed to accommodate the teacher’s and the school’s schedule. The lessons evolved through observation of the students and ongoing consultation with the teacher. As Greenwood (2007) stated:

Action research is social research carried out by a team that encompasses a professional action researcher and the members of an organization, community, or network (“stakeholders”) who are seeking to improve the participants’ situation. AR promotes broad participation in the research process and supports action leading to a more just, sustainable, or satisfying situation for the stakeholders. (p. 3)
The involvement of the community through surveys and the involvement of the teacher with planning helped to shape the project. Although not the focus of the research there was an open-door policy in the classroom. Several adults from the community visited or helped in some way. One community member offered to bless the garden with the students, conducting a tobacco ceremony. This event connected the students more closely to their cultural traditions. Using action research allowed the project to evolve in the best interest of the community and the students. This also allowed the teacher to see a variety of benefits of contextualized problem based mathematics and holistic education.

3.1.2.3 Critical action research. This project uses a critical action research method. The word critical comes from the postmodernist view that, “Instead of claiming the incontrovertibility of fact, postmodernists argue that truth is relative, conditional and situational and that knowledge is always an outgrowth of prior experience” (Miles, 2000). Critical action research is socially responsive and focused on improvement (Miles, 2000). The goal of this project was to contribute to the improvement of mathematics education in this community and for First Nations students in general. The pedagogy of critical action research matches the pedagogy of this math unit, one that moves closer to embracing the values and culture of the community and its students.
3.1.2.4. School visits. Once the teacher agreed to collaborate on this research in October of 2014, I started to visit the school. I spent time getting to know students and teacher before beginning to document the research project. We created several gardens including the one shown in Figure 1. I spent many hours digging and weeding with students. This gave me time to develop relationships with the students. As pointed out by Lunney Borden and Wiseman, “We have learned from Aboriginal colleagues that teaching and learning are fundamentally about relationships”, and relationships take time to develop (2016, p. 143).

![Figure 1. Garden created at the school.](image)

Over the course of seven months I made eight visits to the school. Each time I stayed for the day and we worked in the garden and on math problems. The teacher and I created open-ended math problems for students to work on that were connected to what we were doing in the garden that day. Although these were not documented for the research project, it gave us time to adjust to this way of working and doing mathematics. Examples of the problems students worked on included estimating and then using non-standard units to measure the length and width of their gardens and calculating the number of bulbs of garlic
they needed to plant based on the number of cloves in a bulb. By working with the students during this time I developed a trusting relationship with them, which is important to the success of a culturally responsive mathematics program (Bonner, 2014; Lunney Borden & Wiseman, 2016). A summary of the visits that I made to the school can be seen in Table 5.

<table>
<thead>
<tr>
<th>Date</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 2014</td>
<td>Meetings with the Education Manager</td>
</tr>
<tr>
<td>October 2014</td>
<td>Research proposal presentation to staff at education forum</td>
</tr>
<tr>
<td>Jan 2015-April 2015</td>
<td>Meetings with teachers regarding topics, methods, and activities</td>
</tr>
<tr>
<td>Feb 4th, 2015-Oct 22, 2015</td>
<td>Eight volunteer visits at the school to help prepare and plant garden and help with math class prior to starting recording math lessons</td>
</tr>
<tr>
<td>October -November 2015</td>
<td>Implemented lessons and documentation of project</td>
</tr>
</tbody>
</table>

3.1.3 Case study. This qualitative study took an in-depth look at students’ responses to five mathematics lessons. Creswell has stated that qualitative research can be considered a case study if it is the study of an issue, “within a bounded system” (2007, p. 73). Furthermore, if the case is used to understand a broader concept, then it is defined as an instrumental case study (Stake, 1995, p. 3). As this research may inform contextualized mathematics, inquiry mathematics, and various ideas such as the use of holistic and experiential education, it may be defined as an instrumental case study.

The use of a case study methodology included the development of codes, analyzing the results qualitatively with development of themes, and an in-depth discussion of the results
3.1.4 The community. This project was conducted in a small central Ontario First Nation community. The residents have lived in this area since the early 1800s when they were forcibly moved by the Canadian Government. The residents were moved from a previous forced settlement, or reservation, where they had become accomplished farmers. Farming became part of the culture of the community and continues to be to this day. Residents also fish and gather wild foods to supplement their diet. The outdoors is an important part of life in this small Indigenous community (EM, personal communication, March 2014).

Currently approximately 150 people live in the community with others living close by and traveling to the community for work or to see family. This area is somewhat isolated by geographic features. Although the teachers live a short drive away they are not residents of the community. A political body, the Band Administration, runs the day to day affairs of the community.

3.1.5 The site. The small school in the community houses two classrooms, two teachers, two assistants, an administrative assistant, and 15 students. At the time of the implementation of this project, one class had students from Kindergarten to Grade 2 (Primary), the other had students from Grades 3 to 5 (Junior). The school is on the reserve, but the teachers are from the local provincial school board. The community had a tuition agreement with the local school board and the school followed the Ontario Provincial Curriculum. Students write the provincial standardized test, the Education Quality and Accountability Outcomes (EQAO) tests, in Grade 3.

After Grade 5 students are bussed out of the community to a local public school. Participating in provincial standardized tests and bussing students to a public school for Grade
6 contributed to community members and teachers being concerned about academic achievement. This added a layer of expectation to the mathematics unit that required the inclusion of curriculum expectations in planning.

I worked with both classes but only documented the mathematics instruction in the Junior class. The Junior teacher, Ms. T., taught three grades in one room. She is not of First Nations descent but her husband, and therefore her five children, are. She has worked in First Nations education for many years and was interested in exploring contextualized mathematics. She felt that this would help her to bring more culture from the community into the classroom.

Although Ms. T. taught other content areas of the class in a holistic manner, her math program was mostly traditional “school math” and used textbooks and worksheets. Ms. T expressed that she felt it was challenging to meet the curriculum expectations for three grade levels. Using the textbook made her feel like she was better able to meet curriculum goals as well as track student learning. Students were introduced to, and had access to, manipulatives but their mathematics program was not problem or inquiry-based.

Ms. T wanted to increase the amount of time spent on inquiry in the mathematics program. In other subject areas inquiry was being implemented. Students had choices and did group work. They were comfortable having responsibility for their learning and working within an unstructured environment. Once students got to know me, they adapted easily to the mechanics of inquiry-based mathematics, working well independently and with their peers. The students’ names have been changed to protect their anonymity. Their ages increase in order; two Grade 3 students (Bonnie and Daniel), four Grade 4 students (Alberta, Rose, Joe, and Noah), and one Grade 5 student (Amber).

The school yard contained a jungle gym, a grassy area for play, and three small
gardens. These gardens had been created in the fall of 2014 when I first started connecting with the teachers at the school. I helped to plant them in the spring of 2015. One was planted with a three sisters garden consisting of beans, squash, and corn, a traditional planting by First Nations people who lived in this area in the past. “The Three Sisters, maize (Zea mays ssp. mays), bean (Phaseolus vulgaris), and squash (Cucurbita pepo), were the dominant crops in many Northeast Native American agricultural systems during the late prehistoric and historic periods” (Hart, p. 87, 2008). The Iroquois are historically known to have used this system of planting, but the Ojibwe people who now live in this area of Ontario are also known to have used Three Sisters gardens (Ojibwe Garden Program, 2013).

Another garden was planted with an assortment of vegetables including beets, kale, and radishes. These were chosen for their hardiness and use in salads, an easy dish that we could make with students. In the fall, we planted garlic and some berry bushes. The berry bushes were planted to start some more permanent garden plants from which students could harvest on their own. The school also had a kitchen downstairs that was used for snacks and lunches. We had access to this kitchen for food preparation.

3.2 Procedure

3.2.1 Permission. The Research Ethics Board of Lakehead University approved this research which was conducted with a vulnerable population, school aged children in a First Nation community. Historically, First Nations communities have not been treated ethically or with respect. This community was confined to a reservation in the early 1800’s. After the extensive work of building roads and farms, they were forcibly removed with no compensation until recently (Government of Canada, 2010). This was in addition to the Government removing children and placing them in Residential schools. The reality of this
mistreatment was taken into consideration at all times. The research ideas were approved by the Band Education Manager who consulted with the board of the Band Administration. Once the Band Administration approved the project further planning could proceed. All parents and students received explanatory letters and permission slips and were made aware verbally that they could decline to participate in the research at any time with no consequences. Permission to video record and take pictures of students was received.

As this was a small group of children, many of whom had siblings in the school, the Education Manager took me to meet with the parents in person to discuss the project and go over the permission slips (see Appendix B). This gave me time to explain the project and answer any questions. We discussed the connection of the mathematics to the garden allowing time for parents to air any issues they may have with the project. The parents were very supportive of the project.

Teachers (Appendix D) and students (Appendix C) also gave consent by signing permission slips. By this time the teachers were well versed in the project. The students were read the Assent Letter (Appendix C) during class time. It was made clear to them that although they were required to participate in mathematics class, any recording of their work was completely up to them and would not affect their marks.

As a follow up, I attended a year-end event hosted by the school where I had an opportunity to discuss the research with parents. A report outlining the major findings of the research will be sent to parents and the completed thesis will be made available to the community.

3.2.2 Data sources.

3.2.2.1 In-class lessons. Data was triangulated by using field notes, video recording, photos,
artifacts of student work, ongoing student interviews, and discussions with the teacher. Each lesson was video recorded as were student explanations of their work. Student work was photographed. Student artifacts, including chart paper diagrams, were collected. After each lesson, I wrote detailed field notes to describe and critique the lesson and to document student mathematical thinking. Most of the problems were open-ended with no prescribed solution strategy, I was interested in documenting the various strategies students used in their problem solving and observing any additional benefits to doing contextualized mathematics.

3.2.2.2 Interviews. The teacher, Ms. T., was interviewed before and after the project and I took notes on our conversations throughout the project. Some of our conversations after class were also videotaped. Students were asked questions throughout the project to encourage them to explain their mathematical thinking. All paper artifacts and other data were kept in a locked cabinet to which only my supervisor and I had access. Data files were kept on a password protected device, to which only I had access.

3.2.3 Unit development. After discussion with the teachers and at least eight visits to the school to volunteer in the classroom, mathematics problems were created and discussed. The resulting lessons are outlined below. The Junior teacher requested that students explore fraction concepts. In addition to relevant research and curriculum materials, a variety of texts were consulted for examples of inquiry learning and mathematical problem solving (Hajra, 2015; Lyon & Bragg, 2011; McNama & Shaughnessy, 2010; Ontario Ministry of Education, 2014; Pittman, 2011; Schuster & Anderson, 2005; Small, 2009). The following ideas, which were explored in the Literature Review, were included when developing the problems:

From the research on inquiry (e.g., Lamon, 2007; Stigler & Hiebert, 2004; Utley & Reeder, 2012).
• Group work
• Problems that can be approached in different ways

From number sense and fraction sense (e.g., Aksu, 1997; Bonoto, 2005; Lamon, 2007)

• Use and understand estimation
• Connections between multiplication, addition, subtraction, and division
• Extra time to explore concepts before using traditional algorithms

From culturally responsive education research (Lipka et al., 2005; Nicol et al., 2013; Wagner & Lunney Borden, 2010)

• Community requested outdoor activities
• Community requested hands-on activities
• Holistic, contextualized in place

To meet teacher requests for content, some of the lessons created were teacher-led and less inquiry-based. In addition, this project was documented in the month of October when most of the gardening for the season was complete. Although many undocumented math classes were spent outside using the garden as a concrete context, this was not possible for the lessons that fell within the timeframe of the documentation. Connections were made to the garden by using the harvest to cook, and by making outlines of small gardens in the classroom for one lesson.

As some lessons were less contextualized and more traditional teacher led lessons, this allowed for comparison of the inquiry lessons with the teacher led lessons. The lessons that were directly connected to the garden were compared to those that were not contextualized this way. Although the math was done inside the class, it was usually connected to work done outside in the garden. Most of the problems were social in nature and somewhat open ended. Students often worked in partners and could approach a problem in their own way with
chosen materials or manipulatives.

The teacher and I collaborated on creating the problems. We both proposed ideas and then we would discuss them. We discussed content and methods of teaching. After each lesson, we discussed the pros and cons of the completed lessons concerning student engagement, difficulty level, and student conceptual understanding. These conversations informed the next lesson. When we found students could work independently more capably than we were expecting, we were comfortable giving them more independence. We also noted that students were choosing to work in partners that were very effective.

3.3 Research Lesson Overview

Table 6 outlines the sequence of the lessons and describes their mathematical and culturally responsive connections. A more detailed description of each lesson is included in the next section.
<table>
<thead>
<tr>
<th>Lesson</th>
<th>Mathematical Connections</th>
<th>Cultural Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>29/10/2015 Lesson 1:</td>
<td>Understanding and using estimation, introduction to fractions, introduction to multiplying fractions.</td>
<td>Connecting learning to community and family as this soup was shared. Experiential and holistic-cooking soup together.</td>
</tr>
<tr>
<td>Fraction Lesson with Recipe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and Cooking Soup</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03/11/2015 Lesson 2:</td>
<td>Looking at and comparing different sized fractions by making fraction strips. Introduction to vocabulary numerator and denominator, introduction of concept of relative size of fractions being smaller as the denominator gets bigger.</td>
<td>Experiential (hands-on using manipulatives) and social math activity allows for discussion, group work, and learning from one another. One “whole” connected to the garden.</td>
</tr>
<tr>
<td>Making Fraction Strips,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order by Size</td>
<td></td>
<td></td>
</tr>
<tr>
<td>06/11/2015 Lesson 3:</td>
<td>Exploration of concept of one half; exploration of measurement and concepts connected to fractions/multiplicative thinking, being exposed to alternate ways of mathematizing the problem.</td>
<td>Connection to planting, taking care of the garden; experiential math (social and physical), inquiry-based problem.</td>
</tr>
<tr>
<td>Find Half of a Garden</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/11/2015 Lesson 4:</td>
<td>Oral student presentations, exposure to thinking of other students and a variety of methods that could be used, multiple concepts connected to finding one half.</td>
<td>Experiential, discussion, sharing, learning from other students.</td>
</tr>
<tr>
<td>Student Explanations of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Half of a Garden Problem</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/11/2015 Lesson 5:</td>
<td>Being exposed to the idea that the same sized part of a whole can be named in different ways for example 1/2 is equal to 2/4’s</td>
<td>Social, learning from peers, working with manipulatives is experiential. One “whole” is one garden.</td>
</tr>
<tr>
<td>Culminating Activity with</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction Strips</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.4 Diagnostic Assessment

Some of the students were assessed using the PRIME (Professional Resources and Instruction for Mathematics Educators) Number and Operations Strand Kit (Small, 2005) prior to implementation of the lessons. PRIME is a Canadian developed researched tool that uses problem solving questions to gauge students’ understanding of mathematics. Students’ answers can be analyzed for levels of understanding. The purpose of this assessment was to have a general idea of students’ understanding of number and fraction sense to help to create appropriate lessons for the unit. As well, this information would help later with the analysis of students’ mathematical thinking.

These assessments were conducted in a small room outside of the classroom, which was quiet and private. A couple of the students did not want to be assessed, and two students only did half of the assessment. I read the questions to students and helped them with some of the questions, making notes of any help that was given. The assessments were scored using the associated scoring sheet. Results were given to the teacher and discussed. No post assessments were done using the PRIME assessment tool. Given the short time frame and the fact that we used the assessments to plan the unit, the results of a post assessment would not be valid.

3.5 Data Analysis

Initially all the videos were transcribed, artifacts were gathered and organized, and field notes were collated. This data was reviewed, and notes were made in the margins pertaining to mathematical understanding, surprising or unexpected information, and connections between the various types of data (Miles & Huberman, 1994).

The data was hand coded. An initial coding for fraction and number sense was made.
These codes can be seen in Tables 7 and 8 respectively. After this, more codes were created to note mathematical thinking that did not fall under the first set of codes (Table 9). All the coded data was reviewed again to note patterns. This comparison led to the development of themes and the analysis of the data.

3.5.1 Codes. The a priori codes were developed to identify fluidity and flexibility of mathematical thinking. These codes came from the research on the conceptual development of number and fraction sense in children. These ideas were reviewed in the Literature review and are summarized here.

To develop the codes for fraction sense, the work of Fosnot and Dolk (2002) and Lamon (2007) were combined when possible to create one code. Table 7 summarizes the connections between these ideas and their codes.

<table>
<thead>
<tr>
<th>Table 7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fraction Sense Codes</strong></td>
</tr>
<tr>
<td><strong>Fosnot and Dolk (2002)</strong></td>
</tr>
<tr>
<td>Equivalency vs congruency</td>
</tr>
<tr>
<td>Connecting multiplication and division to fractions</td>
</tr>
<tr>
<td>The whole matters</td>
</tr>
</tbody>
</table>
The information from the research on number sense was used to make Table 8. As outlined in the Literature Review from the work of Fosnot & Dolk, 2002, and that of Lamon, 2007, as well as Asku, 1997; Bonoto, 2005; Fosnot & Dolk, 2001; Lamon 2007; Sowder 1992; Sowder et al., 1998; and Verschaffel, Greer, & Torbeyns, 2006, the ability to connect the various concepts in mathematics to each other increases general number sense.

<table>
<thead>
<tr>
<th>Category</th>
<th>Code</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding and subtracting to solve problems</td>
<td>NS-AD</td>
<td>Using addition and subtraction to solve fraction problems.</td>
</tr>
<tr>
<td>Multiplying and dividing to solve problems</td>
<td>NS-MD</td>
<td>Using multiplication and/or division while solving problems other than that which is connected to using fractions.</td>
</tr>
<tr>
<td>Deconstructing numbers</td>
<td>NS-DN</td>
<td>Breaking numbers apart to more easily make calculations.</td>
</tr>
</tbody>
</table>
Lastly, codes emerged during coding that were connected to number and fraction sense, or to problem solving. These generally came from the first and second reviews of the data, from notes in the margins, during the hand coding. They are summarized in Table 9.

<table>
<thead>
<tr>
<th>Category</th>
<th>Code</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connecting adding and subtracting to multiplying and/or dividing</td>
<td>NS-AM</td>
<td>Using or showing an understanding of the conceptual connection between addition and multiplication or subtraction and division.</td>
</tr>
<tr>
<td>Rounding numbers to Friendly numbers</td>
<td>NS-RN</td>
<td>Changing a number to a multiple of 5 or 10 to make it easier to work with.</td>
</tr>
<tr>
<td>Using benchmarks</td>
<td>NS-BE</td>
<td>Comparing fractions to 0, ½ or 1 to talk about their relative size.</td>
</tr>
<tr>
<td>Estimating</td>
<td>NS-ES</td>
<td>Being able to make an educated guess regarding calculations to help with fact checking.</td>
</tr>
<tr>
<td><strong>Table 9</strong></td>
<td></td>
<td><strong>Additional Codes</strong></td>
</tr>
<tr>
<td>Category</td>
<td>Code</td>
<td>Explanation</td>
</tr>
<tr>
<td>Using compensation</td>
<td>DA-COM</td>
<td>Taking an amount from one number and adding it to another number as a strategy for adding or subtracting e.g. if 20+40=60 then 18+42 also =60.</td>
</tr>
<tr>
<td>Counting</td>
<td>DA-CO</td>
<td>Using counting as a strategy to solve a problem.</td>
</tr>
<tr>
<td>Skip counting</td>
<td>DA-SK</td>
<td>Using counting by groups such as 30, 60, 90 to add or multiply numbers.</td>
</tr>
<tr>
<td>Adding fractions</td>
<td>DA-AD</td>
<td>Combining unit or composite fractions to create a new total.</td>
</tr>
<tr>
<td>Connect fractions and decimals</td>
<td>DA-FD</td>
<td>Seeing that a decimal is another way to represent a fractional part of a whole number.</td>
</tr>
<tr>
<td>Measuring-Using a Ruler</td>
<td>DA-MER</td>
<td>Using a ruler to measure length.</td>
</tr>
</tbody>
</table>
Measuring—Using Non-standard units  DA-MNS  Using tools other than rulers to measure length.
Connect the concrete and the abstract  DA-CCA  Seeing the connection between concrete activities and their abstract representations.
Manipulatives  DA-MAN  Using manipulatives to help solve problems.
Drawing  DA-DRA  Using drawing to help solve problems.
Geometric thinking  DA-GEO  Engaging in geometric thinking to work on problems.

Table 10 summarizes the coding of the data from each lesson.

<table>
<thead>
<tr>
<th>Student</th>
<th>Grade</th>
<th>Lesson 2 Fraction Strips</th>
<th>Lesson 2 Student led exploration</th>
<th>Lesson 3 and 4 Find Half of a Garden and presentations</th>
</tr>
</thead>
</table>
| Bonnie   | 3     | FS-PR                     | FS-PR                            | FS-MO
|          |       |                           |                                  | NS-DN
|          |       |                           |                                  | DA-MER
|          |       |                           |                                  | DA-CO
| Daniel   | 3     | Worked with Joe           | FS-ER                            | FS-SC
|          |       |                           | DA-CCA                          | NS-AM
|          |       |                           | DA-AD                           | DA-COM
| Alberta  | 4     | Absent                    | Absent                           | Worked with Joe
| Joe      | 4     | FS-PR                     | Worked with Daniel               | FS-ER
|          |       |                           |                                  | FS-TC
|          |       |                           |                                  | FS-ER
|          |       |                           |                                  | NS-MD
|          |       |                           |                                  | FS-ER
|          |       |                           |                                  | DA-MER
| Rose     | 4     | FS-PR, FS-UN              | FS-ER, FS-PR                     | NS-ES
|          |       |                           | DA-MNS                          | FS-ER
|          |       |                           | DA-MER                          | DA-GEO

Table 10
*Codes for Individual Students in Lessons 2, 3, and 4*
<table>
<thead>
<tr>
<th>Name</th>
<th>Num</th>
<th>Worked with</th>
<th>Worked with</th>
<th>FS-PR</th>
<th>FS-UN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noah</td>
<td>4</td>
<td>Worked with Amber</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amber</td>
<td>5</td>
<td>Worked with Noah</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Worked with Amber:
  - FS-SC
  - DA-SK
  - FS-MO
  - DA-FD
  - FS-MO
  - DA-MER
  - FS-MO
  - NS-AM
  - NS-AD
  - NS-ES
  - NS-ES
  - NS-DN

- Worked with Noah:
  - FS-PR
  - DA-DRA
  - NS-BE
Chapter 4: Results

4.1 Action Research

This research project used an action research method. The teacher, Ms. T, and I worked together to create and implement the unit and I spent time getting to know the students so that we worked well together by the time I was recording this project. Using a collaborative approach had many advantages. For example, the research was flexible and attuned to the needs of the students. The teacher and I consulted after every lesson to make the next lesson more effective. For example, we discussed ways to support student understanding without giving them too much help, using effective questioning. Also, we discussed issues that arose with students that surprised us such as unusual ways of solving problems. Our conversations often revolved around informal formative assessment. Were students ready to move on?

We planted a Three Sisters garden in the spring of 2015, a type of garden traditionally planted by First Nations people in this area (Hart, p. 87, 2008) using corn, beans, and squash. It can be seen in Figure 2. The design of this garden was discussed with students. The vegetables complemented each other. The squash covered the ground to minimize the need for water and keep weeds away, the corn acted as stakes giving the beans something to grow up. This garden requires minimal maintenance and could be planted and then left unattended for long periods.

A community member who was quite involved in the garden and the school came and blessed the new garden with a tobacco ceremony that involved all the students. This was not planned ahead of time but was very much appreciated. She said a prayer to the Creator,
sprinkled tobacco on the garden, and asked for it to be abundant to help bring healthy food to the community. The students were present and participated in the ceremony. It was a beautiful fall afternoon and we all spent some time appreciating the outdoors and being able to be there in the garden. Although this research did not use a fully culturally responsive methodology, there were moments like this, unplanned but impactful for students, partly because of the holistic nature of the project.

![Image of a garden]

*Figure 2. Three Sisters Garden planted at the school.*

In the fall of 2015 we harvested about 10 butternut squashes from the Three Sisters Garden. These were cooked and frozen. The teacher and I decided to make soup with the students, so I looked at creating fraction problems contextualized by making soup. After consultation with the teacher, this produced Lesson 1. This lesson turned out to be very well received by students and was a rich source of mathematical thinking. Contextualizing with cooking appeared to engage students. They were very happy to take their soup home to their families, being able to say that they grew, and harvested the squash, and helped to make the soup.

4.2 Diagnostic Assessment Results
The PRIME (Professional Resources and Instruction for Mathematics Educators) Number and Operations Strand Kit (Small, 2007) was used to assess some of the students. A few students were unwilling to participate or did not complete the assessment. These students expressed that they found the assessment too difficult or too long. The results from completed and incomplete assessments were used to help to create the lessons for the garden math unit, a summary of which can be found in Table 11. Assessment results are considered here to help describe the variety of developmental levels of mathematical understanding that existed in the class.

The PRIME assessment score gives a broad range for results. For example, a score between 8 and 17 on Operations Diagnostic Tool C means that a student is in Phase 2, which is defined as approximately grades 1-3. A score of 18-23 on the same test would place the student in Phase 3 or approximately Grades 3-5. Thus, one can say that a student is closer to the beginning or the end of a phase and therefore give approximations of grade level. The students who were tested performed within the range expected for their grade level. In general, students' understanding of addition, subtraction, and multiplication was in the average range for their age.

During the assessments, students were offered clarification and assistance if it was requested. Of the students who were tested, several expressed that the assessment was too long, and I noted that their answers were less enthusiastic near the end of the assessment. From my experience volunteering in the class, there were several instances where I felt that the student could do the math, but they did not understand the question. I helped students with some of the questions when I thought the issue was vocabulary or test writing skills. Any assistance I gave was noted and taken into consideration for scoring. Given students' issues with test writing, and the amount of help I gave them, the scores cannot be considered
accurate, however, the information was useful and helped us to plan the lessons.

The results of the assessment were given to the teacher. She and I discussed the results, which gave us an idea of what the students’ strengths and challenges were regarding number sense and numeration concepts. Most of the students struggled with decimals, fractions, rounding, and estimation. They had strengths in addition, subtraction, and multiplication. Those who attempted the assessment could successfully complete a multiplication word problem, at their grade level, but all the students had trouble with a division word problem.

As a result of this assessment I planned a division problem connected to fraction sense, Lesson 3, Find Half of a Garden. Also, with considerations to the academic research and the Ontario curriculum, I created problems that dealt with fractions and estimation while also being connected to the garden. The lessons were discussed with the teacher and adapted further to meet her needs and those of her students.

A summary of the results of the assessments can be seen in Table 11.

<table>
<thead>
<tr>
<th>Student</th>
<th>Grade</th>
<th>Assessment</th>
</tr>
</thead>
</table>
| Bonnie  | 3     | Did not complete  
Observed to struggle with grade level math |
| Joe     | 3     | Not assessed-observed to be confident in grade level math |
| Daniel  | 4     | Did not complete-observed to have some gaps in math understanding |
| Alberta | 4     | Not assessed  
Observed to enjoy mathematics and be comfortable with grade level work |
4.3 Lessons Results

In general, the lessons in the math unit were designed to help students develop fraction sense. Since this project was collaborative, and to accommodate the wishes of the teacher, a variety of types of lessons were delivered. This variety made it possible to compare the mathematical thinking resulting from different teaching methodologies. Lesson 3, Find Half of a Garden, was the most open-ended inquiry-based lesson and the most clearly contextualized by the garden and food. Lesson 1, Fraction Lesson with Recipe and Cooking Soup, was conceptual but teacher delivered rather than student driven. Lessons 2 and 5 used fraction strips, which are concrete manipulatives, but relate to the mathematics in a more abstract way than lessons connected more directly to work in the garden or to cooking. Lessons 2 and 5 were also less open ended than Lesson 3 although students did find several different ways to approach them.

Results are reported chronologically as the lessons were presented to students. For each lesson the problem that students were given is stated, the procedure is presented, and students’ conceptual understanding is considered. Conceptual understanding is presented with attention to the development of fraction sense and number sense. Students’ work for each lesson, where appropriate, is grouped by category of conceptual thinking that appeared while students worked on the problems. Each lesson generated different categories of
mathematizing.

4.3.1 Lesson Procedures. Overviews of lessons and directions were given at the front of the classroom while students sat at their desks, or while sitting on the carpet with the whole group. Instructions were kept to a minimum, taking about five minutes. Lesson 1 used a traditional delivery of information that included student participation and culminated in the group cooking soup together. All the other lessons were primarily composed of open-ended problem solving, meaning there was more than one way to work on the problem, so there were a variety of solutions. Students were encouraged to work wherever they wanted in the class, in groups or alone.

Before they worked on the problems, students were given a piece of chart paper and markers to record their thinking. This allowed space for more than one person to show their work and it allowed them to see each other’s work easily. Also, students could not erase their work, which meant we could see the development of their thinking and any changes they made during their problem solving.

4.3.2 Lesson 1: Fraction Lesson with Recipe and Cooking Soup.

October 29, 2015

The Butternut Squash Soup recipe was increased to make enough soup to feed the families of the students in the class. This was a teacher led lesson where we worked as a class to multiply the recipe. After the math lesson, we went downstairs and made the soup. I had organized stations, so all the students could participate. The soup was sent home with students to share with their families.

During this lesson, several different concepts were introduced to students. The first concept was rounding numbers and using friendly numbers. Using the recipe, I had written
out on a piece of chart paper, I went through the following:

- We needed enough soup for 48 people (calculated previously by class)
  - I asked students which friendly number was close, and we chose 50
- recipe was for 4-6 people
  - We chose to use the friendly number 5 as a multiplier

Students knew that 5×10=50 because they could count by 10s. They could also see 50 as multiplying two friendly numbers, 5 and 10. We then looked at how we could feed 50 people and they were able to deduce that we could multiply the recipe by 10. This calculation was easier than using the exact answer of 48 and dividing by 4, 5, or 6. Also in cooking it is okay to have extra. I demonstrated how to round numbers to the closest 10 and discussed why it was appropriate, and even advantageous, to use estimation in this situation. Students were shown that estimation, multiplication, and friendly numbers are mathematical concepts used in real life and that they are useful when doing calculations inside as well as outside of school.

Multiplying by 10, a concept introduced in the early grades, made the lesson accessible to all the students. Counting by tens is introduced in Grade 2 with two-digit multiplication introduced in Grade 4 (*Ontario Mathematics Curriculum*, 2005). I showed students the recipe, which I had written out on chart paper (Figure 3), and asked them which numbers they thought would be the easiest numbers to multiply by 10. We multiplied the whole numbers 1, 2, and 3 first to get 10, 20, and 30. Then we went on to the fractions.
Introducing fractions with this recipe allowed the exploration of the concept of multiplication within the confines of two simple fractions, 1/4 and 1/2, making this lesson accessible to the three grade levels (3, 4, 5) in this class. Traditional algorithms regarding multiplication or addition of fractions were not used. This gave students time to consider the concepts involved in multiplying fractions before learning any formal algorithms (Aksu, 1997; Lamon, 2007).
Next, we considered multiplying the one-half fraction by ten. As we were planning to cook the soup that day and did not have much time, a group lesson was necessary. With students directing me, I drew halves of an orange on the board until we had 10 halves, drawing one half circle on top of the other (to make a whole) and then wrote the number of halves (below) and the number of whole oranges above the diagram. Figure 4 represents the diagram that was on the whiteboard for students to see.

| Number of oranges needed (two halves make a whole) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1               | 2               | 3               | 4               | 5               |                 |
|                 |                 |                 |                 |                 |                 |
|                 |                 |                 |                 |                 |                 |
| 2               | 4               | 6               | 8               | 10              |                 |

**Figure 4. Pictorial representation of halves of an orange in the soup recipe.**

Students could see that there were two sets of numbers. The number at the top of the picture represented the number of whole oranges and the number at the bottom represented the number of half oranges. The students called out the rest of the totals as I drew the 10 halves necessary for our big batch of soup.

I did the same thing with the next fraction, (1/4) the quarters of a cup of juice, drawing a square cut into four and having students count out 10 of these. We looked at the patterns in
the numbers. Students participated by counting the quarters, pointing out the creation of wholes, asking questions, and answering my questions such as “What should I do next?”, “Do you see a pattern?”, and “How many 1/4s do we have now? How many wholes?”.

![Figure 5. Pictorial representations of quarters of a cup of juice in the soup recipe.](image)

Although this was not an inquiry-based problem, it was holistic because it was connected to cooking. The squash used in the recipe was grown in the students’ garden and students knew that they were bringing soup home to their families.

Although this was a teacher led lesson, it was more conceptual than procedural. The holistic nature of the problem, using a recipe and making soup for others, anchored the mathematics in an activity rather than a mathematical concept (Figure 6). In contrast, mathematics learned in a textbook is anchored to a singular concept which does not nurture
the development of fraction sense. Multiplying the whole recipe allowed the comparison of multiplying whole numbers and fractions, which supported the inclusion of a variety of mathematical concepts.

During this lesson, students were exposed to the concepts of unitizing, reasoning up, and covariation as discussed by Lamon (2007). Unitizing, for example, occurred when considering \( \frac{1}{2} \) an orange plus \( \frac{1}{2} \) an orange becoming a whole orange, two quantities (\( \frac{1}{2} \) and \( \frac{1}{2} \)) are combined to make a different quantity, one whole. The number of wholes can be counted by understanding this unit. Reasoning up is considered by the act of finding the number of wholes in 10 halves or 10 quarters because we increased the number of wholes in a multiplicative manner with the corresponding number of parts. When we explored the idea that increasing the number of halves also changes the number of wholes, this is the concept of covariation. From the work of Fosnot & Dolk (2002) this problem presented the idea of part-whole relations because one half an orange means one part of two pieces that exist when you cut an orange in half. One quarter of a cup of juice is one part of the four equal parts that exist when you divide it into four parts. Lastly, the concept that the whole matters is considered in the idea that half of an orange is different from half of a cup of orange juice, the relative size of one half changes depending on the whole that it is related to.

Contextualizing the lesson allowed the instruction to focus on solving a problem rather than teaching a concept. This allowed several concepts to be considered and connected within the lesson. Students were exposed to the connection between addition and multiplication because we used repeated addition to multiply the ingredients in the recipe. They saw that division and fractions were related because we were dividing wholes into parts to make fractions and multiplying fractions (parts) to make wholes.

In addition, connecting this lesson to concrete objects that students would be cutting
themselves, when we went downstairs to cook, allowed students to connect the concrete and the abstract. The concrete actions of preparing ingredients and making soup were connected to the abstract ideas involved in naming and multiplying fractions.

![Image of a child preparing food](image)

*Figure 6. Preparing butternut squash soup.*

### 4.3.3 Lesson 2: Making Fraction Strips, Order by Size.

November 3rd, 2015

Students were provided with a set of blank fractions strips (not labelled with corresponding fraction), such as Figure 7, that was commercially prepared and photocopied on to paper.
I started this lesson by asking students to identify the “one half” fraction strip. I asked them how they knew what to look for and we discussed the idea that one half has two equal parts. We proceeded to fold the fraction strip representing 1/2 in half and consider what fractional part this would be called if there were now four equal parts. Students could then find the 1/4 strips by comparing the size to one half of the one-half strip. We proceeded in this manner, considering the size of the next set of strips, the number of pieces of the whole they represented, and other fraction strips to which they were related. The class filled in the names of the fraction on the strips and cut them out.

This lesson was intended to allow students to work with concrete objects that embody the mathematical idea that a fractional piece of a whole gets smaller as the corresponding number in the denominator gets bigger. For example, 1/3 is a larger piece of a whole than 1/6, although the denominator is smaller, 3 is smaller than 6. This is a challenging concept for students because it is the inverse of the relationship they are used to where whole numbers become larger as quantities grow (McNamara & Shaughnessy, 2010). This concept is part of
moving from additive to multiplicative reasoning (Lamon, 2007).

The group was given two fraction questions. These were written on the chart paper at the front of the class and then read to them to ensure clarity. The questions both asked students to arrange fractions in order from largest to smallest part of the whole. The first question asked students to work only with unit fractions. The second question, which was a bit more challenging, asked students to order various fractions including one that represented a whole number (3/3). Students were encouraged to use their fraction strips to work on this question. The first question would require them to consider only one fraction strip at a time whereas the second question required that they put strips together to create fractions such as 2/5 or 4/6 and then compare the sizes of the resulting parts of the whole.

Question #1: Put these in order (rewrite) from smallest to largest part of the whole
1/2, 1/6, 1/4, 1/3, 1/5

Question #2: Put these in order (rewrite) from smallest to largest part of the whole
1/2, 3/3, 2/5, 4/6

While working on Question #1 most of the students initially ordered the fractions by increasing size of the numeral in the denominator, which resulted in the opposite ordering than requested. As we went around the room we asked students questions to help them explain their thinking. The students enjoyed using the fraction strips and after they worked on the first question most of them worked on the second question. Some of the students started on their own investigations such as writing out the fraction strips in symbolic form on paper. After students worked on the problems for half of an hour, we gathered together to go over the questions and look at their answers.

Bonnie initially did not use her fraction strips to work on the problem. She drew
circles reminiscent of a Venn diagram (Figure 8), with what she believed to be true about the relative size of the fractions. She put fractions with denominators 5 and less, 1/1, 1/2, 1/3, 1/4, 1/5, in one circle labelled “little” and fractions with denominators 6 and greater 1/6, 1/7, 1/8, 1/9, 1/10, in a circle labelled “big”. These can be seen in Figure 8.

![Figure 8. Bonnie's diagram for Lesson 2.](image)

You can also see from Figure 8 that Bonnie’s idea of 1/3 is still developing. She does not understand that 1/3 means that the whole is divided into three equal pieces. She does show that she knows there are four quarters in a whole but does not indicate in this picture that the quarters would be equal in size.

Gathering the fractions into two groups indicates that she may not even see them as individual entities. Also, she has misinterpreted the sizes, seeing the fractions with smaller numbers in the denominators as smaller pieces. She was not yet aware of the concept of part whole relations.

Noah and Amber, two of the older students, also ordered the fractions by increased size of the numeral in the denominator rather than by increased size of the actual fraction. In
Figure 9 Amber is pointing to the diagram of a “whole” labelled 1/1.

Figure 9. Noah and Amber working on Lesson 2.

I had just asked “Which is the smallest part of the whole?” and Amber pointed to the “whole” itself. Realizing that she was pointing to the smallest number in the numerator, 1, rather than the smallest piece, she and Noah stated that they would have to rework the problem.

Noah realized they had reversed the order of the pieces because he made a connection between the symbol and the physical size of the fraction strip. He understood that the “one whole” piece is not the smallest part, but had the smallest number in the denominator (1/1) among the fractions. He understood that they had ordered the pieces by increasing size of the numeral in the denominator rather than increasing size of the fraction strip, or part of the whole. These students proceeded to re-order their pieces which they did correctly.

Another group recognized that they were also confused by the question. They could
see that there were two different issues, the size of the piece and the size of the numeral in the denominator, but they were unsure which to use to sort their fraction strips. When I approached the group, we had the following conversation:

J: We are not sure if the little number has to go first or the big number has to go first.

L: Ok, let’s read that question again on the chart paper. What does it say?

Which ones are the smallest parts on the paper that you can see right now?

Okay, let’s do it this way, which one’s smaller, this (1/2) or this (1/8)?

J: *initially points to 1/2.*

L: Smaller piece of paper?

Both boys point to the piece labelled 1/8.

(Transcripts, video, Day 3)

It is clear from these examples that many of the students found the inverse relationship between the size of the numeral in the denominator and the size of the actual fraction strip or fractional piece to be confusing. Using the fraction strips allowed them to work with concrete objects but the concept was challenging. After close investigation and discussion, the students could see the inverse relationship between the size of the numeral in the denominator and that of the fractional piece.

The second question was a bit more complex, asking students to consider the size of fractions that were composites of their fraction strips rather than unit fractions. Most of the students attempted this question after completing Question #1.

4.3.3.1 Question #2. Put the fractions 1/2, 3/3, 2/5, 4/6, in order from smallest to largest part of the whole.

This was an enrichment question meant for students if they completed question
number one. Several students worked on this question, however most students focussed on creating composite fractions without ordering them. This exploration helped expose students to different types of fractions other than just the unit fractions in Question #1 and allowed them to consider connections between the abstract fraction notations in the question and the more concrete representations of the fractions with the fraction strips.

Rose used the fraction strips to help her work on Question #2 (Figure 10) and then also rewrote the numerals on her chart paper.

![Figure 10. Rose writes out the fractions on her chart paper.](image)

She put the appropriate fraction strips together to create the composite fractions for the question and then lined up the various strips to look at their relative sizes. After this she used a more abstract way of looking at the mathematics by writing out the unit fractions that created the composite fractions in Question #2. For example, there are three 1/3 fraction strips, then three 1/3 fractions in written form to make the 3/3 fraction from the question. Using the symbolic form in her work is a step this student took on her own. You can also see from Figure 10 that Rose answered the second problem by lining up the fractions 1/2, 4/6, and
3/3 and looking at their relative sizes.

Noah and Amber also completed Question #2. Their diagram can be seen in Figure 11.

*Figure 11. Amber and Noah use diagrams to compare fractions.*

Figure 11 includes one large diagram showing a variety of fractional parts as compared to the whole as well as several smaller diagrams representing different fractions. The diagrams divide the parts into the appropriate number of pieces, such as 10 pieces for tenths, but the pieces are sometimes different sizes or do not take up the whole space in the “whole”. Noah and Amber have designated the answer to Question #2 by ordering smaller
diagrams for the fractions 1/2, 3/3, 2/5, 4/6 from largest to smallest with the labels #1, #2, #3, etc. This is the reverse of what was requested but I went over it with them and they were aware that they had reversed the question, by accident, and had ordered the fractions this way. They were also aware that the whole was the largest part and so had not misunderstood the concept but rather had transposed their work in the reverse order from the question. They explained their thinking this way;

A: So first we drew out all of the fractions. See this one is one out of two, four out of six, three out of three. Three (out of three) is, of course, the first one because it is one whole.

N: Four out of six we discovered it because it covers in more than half (referring to the small diagrams). One out of two was the second because it covers in half and then two out of five covers in less than that because it only covers in two spaces.

(Transcripts, video, Day 3).

These students considered the relative sizes of different fractions through drawings using direct modeling. They compared each fraction to the benchmark of “1/2” to help them order the composite fractions. Realizing that the amount of the whole that was covered (the size of the fraction) could be ranked, they could order the fractions without finding a common denominator. These students clearly understood that fractions are parts of a whole.

Although there were some issues with what these students were doing—their fraction drawings show fractional pieces of different sizes meaning they are not yet fully understanding the concept that fractions are equal shares, they used their drawings effectively to reason out their answer. They understood that by comparing the amount of coverage of the whole to the benchmark of one half they could order the pieces without having to be precise. Using comparisons between the fraction pairs was sufficient.
Although only Lily, Noah, and Amber worked on Question #2, the other students explored the fraction strips after completing Question #1. One pair of students engaged in their own inquiry and wrote out the symbolic representations for the fraction strips on the chart paper. They made groupings to represent a “whole” from different unit fractions. For example, they wrote out ten of the one tenth unit fractions together and four of the one fourth fractions together (Figure 12). This activity helped to reinforce the idea that the number of pieces or parts, when making fractions, corresponds to the numeral in the denominator and that fractions are parts of a whole. The students used a symbolic, and therefore more abstract and more challenging, representation of the concrete fraction strips.

*Figure 12. Daniel and Joe exploring fractions.*

4.3.4 Lesson 3 and 4: Find Half of a Garden and Student Explanations of Half of a Garden Problem.

November 6th and 10th, 2015

Lessons 3 and 4 have been put together because Lesson 4 consisted of presentations of student work from Lesson 3. Ruth Beatty, my supervisor, attended this lesson and videotaped
students working.

The problem in this lesson was, “If we were going to grow two crops we would need to find the middle of the garden to divide it in half. Please show me where this line would go in your garden. You can use any method you would like.” Students experienced this when planting the Three Sisters garden as we had planted two rows of corn first. Then we planted beans surrounding the corn and inter-plantings of squash.

It happened to be raining the day we did this activity, which had been planned for outdoors. We did this lesson inside instead. I used masking tape to mark out “gardens” on the floor. Students were asked to add a line of tape delineating half of their rectangle. They were permitted to work alone or in pairs and encouraged to use tools and manipulatives from the classroom.

If students found one half of their garden, then they were asked to find one third of their garden. This is a more difficult task because partitioning a rectangle into equal thirds is more difficult than dividing a space into two halves. Students worked on this activity for 45 minutes. Two days later they presented their thinking to the rest of the group. The following analysis of student work combines the two classes to best portray students’ thinking during the problem solving.

The youngest student, Bonnie, worked with the teacher on this problem. The older students asked a few questions but generally worked on their own. All students used some form of measurement. The following analysis is divided into two categories, measuring using standard and non-standard units. After the analysis is a short narrative concerning the group that attempted the second part of the question, finding thirds of a garden.

4.3.4.1 Measuring using non-standard units. Two students used nonstandard units to find the middle of their garden, Bonnie, in Grade 3, and Rose, in Grade 4.
Rose understood the problem to mean that she needed to find the center of her rectangle because of the word “middle”. She began to jump from the outside of the garden to the inside, using the movement of her body to locate the center of the rectangle. She jumped many times from the same spot outside the garden to the center before she started to make a small rectangle out of tape on the inside. This student’s use of jumping came from her favorite activity, dancing. Her knowledge of dance and her desire to use movement to explore mathematics helped her to connect to the problem. She added tape to the center rectangle one side at a time by jumping in and out of the rectangle from the edge to the center (Figure 13). Rose jumped from two different sides into the center.

![Figure 13. Rose jumps into her rectangle.](image)

Next, she proceeded to estimate the center of the small rectangle where she taped down an “x”. After this Rose made a model of her garden on her chart paper. In the model,
she showed the original outline of the garden, the outline of the small rectangle in the middle, the location of the “x”, and the lines showing where half of the garden would be. She made the line delineating one half of her garden from corner to corner of the large rectangle, through the center of the small rectangle, and the “x”.

Rose then proceeded to measure the line segments in her diagram (Figure 14). This allowed her to see many mathematical relationships. She had a rectangle with two opposite and equal sets of sides. The x that she drew was in the center of both the rectangles.

![Rose working on her diagram of the garden.](image)

Rose was the only student who divided her rectangle in half from corner to corner. The other students drew a straight line down the center of the rectangle that represented their garden. By sharing her work with the class, the rest of the students could see a visual example of the idea that there are different ways to divide a rectangle in half demonstrating the concept that equivalency (one half) does not equal congruency (same shape) from Fosnot and Dolk (2002). Rose began this problem using non-standard units, jumping, and ended with standard units. One other student, Bonnie, used non-standard units in her work on this problem.

Bonnie worked with the teacher. Ms. T helped Bonnie to pick manipulatives that
suited her and asked Bonnie questions to help her think about how to solve the problem. To measure the length of one side of the “garden” they used a set of 100 Unifix cubes that were strung on a rope in colored groups of five, alternating red and yellow (Figure 15). Two colored groups of five make ten, so Bonnie could count by 5’s or 10’s while seeing that 10 is made of two 5’s.

![Bonnie demonstrating the use of the blocks for measuring.](image)

*Figure 15. Bonnie demonstrates the use of the blocks for measuring.*

Since she was the youngest, and sometimes lost her way when counting large numbers, this manipulative helped support her counting. A ruler was used for a small section (16 cm) of the garden that was longer than the Unifix cube string. A ruler is more abstract than the string of cubes but serves a similar purpose as a number line, where the student can see the numbers written in order. Using these two different manipulatives, ruler and string of cubes, allowed for a rich experience of measuring. Bonnie experienced both abstract (ruler) and concrete (Unifix cubes), standard (ruler) and non-standard (Unifix cubes) tools to work on this problem. Additionally, these manipulatives represented the difference in size between 100 and 16 while concretely deconstructing the number 116 into 100 (a friendly number), with the Unifix cubes, and 16, with the ruler.
To find half of the total, 116, Bonnie and Ms. T found half of each of the deconstructed numbers (100 and 16), 50 and 8, and then added them together. Deconstructing the numbers connects adding and dividing showing that numbers can be broken down into parts, dealt with separately, and then added back together. The rest of the students used a variety of methods revolving around measurement with standard units.

4.3.4.2 Measuring using standard units. Three groups of two students used a ruler to measure one side of their garden and then divided the result in half to find the middle. They used a variety of mathematical concepts to work out their answers.

Daniel, in Grade 3, used several different methods to find the middle of his rectangle. He had used the perimeter of his chart paper as the outline of his “garden” rather than using a rectangle made of tape on the classroom floor. Initially he guessed where the center would be and drew a line down the page using this estimate as a reference. He was not yet able to measure across the top and divide this large number in half using a traditional algorithm, but he did know that the measurements on either side of the center line had to be equal. Using this information, he proceeded to measure the resulting lengths on either side of the line.

In Figure 16, you can see that there is a small diagram at the top left corner of the page that shows the numbers 27 and 31 (which add to 58) on either side of the center line and then 52 (numerals reversed, should be 25) and 36 beneath these (add to 61).
Daniel understood that his line had moved as it went down the page, so he decided to choose a point in his line that he estimated to most accurately show the midline of the page. He drew an “X” at this point and used this spot for further calculations. After measuring either side of the “X” he found 26 cm and 31 cm.

Daniel found an answer to the problem, Find Half of a Garden by using compensation to create two equal groups. He started by taking one centimeter away from the larger number, 31 cm, and added this to the smaller number, 26 cm, to get 27 and 30. He did this again to get 28 and 29. Daniel stopped here unsure what to do next as he did not know how to share a whole number. He had balanced the numbers out to create almost equal numbers on either side of the center line by subtracting from the larger number and giving to the smaller number. This shows that he understands that one half means the two sides need to be equal.
He also related adding and subtracting to division and fractions.

Connecting adding, subtracting, and dividing is one of the ways students transition from additive to multiplicative reasoning. This student showed the class that subtracting from the larger number and adding this quantity to the smaller number, with the goal of having two equal numbers, is the same as dividing a quantity by two. In this work, partitioning the numbers by sharing and comparing is concrete (visually the line is in the middle), and abstract (finding equal quantities or numbers). Daniel was also using sharing and comparing from the ideas of Lamon (2007).

Like Daniel, there were other students who struggled to divide odd numbers in half. They dealt with this either by using two very close numbers (Alberta and Joe) or by changing the number to an even one and then dividing it (Noah and Amber). Half of a garden is a concrete idea, which is likely more accessible to students than half of a number, which is abstract.

Noah and Amber began working on this problem by measuring the length of a side of the garden. They knew they had to divide this number somehow but had to reason through the process to connect the fraction “one half” with the process of dividing by two. Initially they were unaware that the denominator of a fraction tells you the number with which you divide the whole. For example, if you are finding a quarter you divide your whole by four. In this problem, finding one half of a garden, they would have had to divide their total by two. This transcript is from the beginning of the work period;

N: Well you divide a hundred and seven in half. A hundred and seven divided by zero (several second pause-thinking). Oh, the equal, I know where the equal part is probably around 65 cm so let me see.

(Transcripts, video, Day 4)
At first this student equated divide in half with divide by zero. Then he realized that he needed to find two equal parts, the same amount on each side. He is not yet aware that he needs to divide by two even though he understands that the two numbers that make up 107 should be equal. His conjecture is that 65 and 65 will give him 107.

Noah and Amber tried several methods other than the traditional algorithm of division to find two numbers that would add to 107. They were struggling because the number was odd, 107 cm. They deconstructed the number into the partial quotients 100 and 7 and they knew that 50 plus 50 gave them 100. They did not know what to do with the 7 that was left over.

R: So, what would half of a hundred and seven be?

N: We were thinking 50?

A: Half of 107 would be like 52.

(Transcripts, video, Day 4)

They know half of 107 was more than 50 and so offer a conjecture of “like 52.” The first response of 50 indicates that they have realized that dividing into halves means dividing by 2 (into 2 equal pieces) but have difficulty using that understanding to divide 107 by 2. Ruth asked them a question to support their thinking by asking them to check their conjecture of 52.

R: What’s 52 and 52?

A: Wait, wait.

N: 57

A: It’s gonna be, oh it’s so hard. You can’t find 7. It’s not an even number. Maybe 5 but 10. It might be 3+3, 53 plus 53 is 56 and 4+4 is 8. You can’t get 7, it’s impossible. (transcripts, video, Day 4)
Their answer indicates that they understand the operations they are attempting to carry out, that they need to divide 107 into two parts, but they also realize that 7, since it is an odd number, cannot be evenly divided into equal halves (into equal whole numbers). Instead, as seen in the transcript, they try successive whole numbers (double 3 of 53 to get 106 and double 4 of 54 to get 108). Since they are only dealing with whole numbers as units to divide, they are unable to find a solution for dividing the 7 in 107, and so declare the task impossible.

Even though they are unsuccessful at dividing into two equal halves, their conversation indicates that they have developed some understanding of the relationship between dividing by 2, and dividing a quantity in half. In the end, they measured their garden again and found it to be 110cm, which they could easily deconstruct into 100 and 10 and then divide by two to find 50 and 5.

Over the course of working on this problem these two students used several mathematical concepts. From the ideas of Fosnot and Dolk (2002), Amber and Noah connected division to fractions and considered the part-whole relation. These concepts can be seen when the students divided by two to find one half and when they knew to change the part based on the size of the whole. They used estimation to predict, or guess and check, while problem solving. Also, this pair of students used partitioning to create manageable pieces by deconstructing the number to divide. Lastly, they were using relative thinking when they separated numbers and changed them relative to each other. The 100 and the 7, once chunked into two units, had to change in the same way, both being divided by two. Using these concepts in the work for one problem helps students move from additive thinking to multiplicative thinking because they are using additive concepts (subtraction and addition) within multiplicative concepts (division and relative thinking). Students see that finding one half is also division, that division means equal parts, and that it helps them when they can
deconstruct numbers using subtraction to make the division more accessible.

Joe and Alberta also used standard measurement to work on this problem. They connected a variety of mathematical concepts while finding their answer. First, they measured across one side of their garden using a ruler and found it to be a total of 43 inches. The rulers they used had inches and centimeters on them, these students used inches to measure as they were not told otherwise. They drew a model on their chart paper that was 12 inches across, as this length of side fit on the chart paper, whereas 43 inches would not have fit.

Students were not asked to make a scale model, nor were they expected to measure the picture that represented their work, but by doing so this group considered mathematical concepts that were not considered by other students while working on this problem. By creating a scale model on the chart paper, and choosing to use standard measures (inches) to show their work, these students ended up creating a ratio. There were two gardens, the one taped to the floor and the model representing it on the chart paper. The model on the chart paper was smaller than the floor garden, giving two sets of numbers that were different but multiplicatively related.

Joe explained to the class that he had to find half of both his “garden” and his model. Students were presented with the idea that if you divide a larger number in half (43) and a smaller number in half (12) you get answers relative to the number you started with, relative in a multiplicative rather than additive way. Although this was not explicitly stated, being exposed to the concept even implicitly may support students to think about the differences between additive and multiplicative thinking. Here the student is explaining how he worked on the problem and made his model:

J: I measured the square and I got a 43-inch square. I turned it into a 12 inch so that it could fit on the paper.
J: To find the middle I sliced my number in half a.k.a divided 12 by two and got 6. I went to the 6-inch mark and I drew a line.

(Transcripts, video, Day 5)

His language was accessible for other students (sliced, a.k.a.) while helping to describe the process of working on this problem. He showed the class that half of 12 was 6 and half of 43 was 21. Although half of 43 is 21.5, 21 is as close as you can get while using whole numbers. Joe had two sets of numbers on his chart paper; 12 and half of it being 6, 43 and half of it being 21 generating ratios of 6:12 and 21:43 and which are both variations on 1:2.

This presentation connected to the concept of “the whole matters” from Fosnot and Dolk (2002) because half of 43 is a different quantity than half of 12. Students can see that a half can be different sizes while also showing that the size of the half is dependent on the size of the whole. This is known as the idea of covariation from Lamon (2007). The numbers 43, 12, and 2 vary proportionately with 21 (.5) 6, and 1.

Lessons 3 and 4, Find Half of a Garden, and the presentations, allowed students to expand their number sense by connecting many different concepts within the same problem. This lesson also seemed to encourage the use of estimation to check answers, perhaps because the fraction one half is a familiar one. Students looked at their lines to see if the two halves were even before, during, and after calculations. As well, in this problem there is a relationship between the concrete and the abstract. Students were physically dividing the garden in half while dividing the abstract quantity, that of the numeral for the measured length, in half. This led to comments that connected concrete and abstract concepts such as:
N: No that’s 10cm. *(Counts five back from 60 to 55)* 5cm on each side, put the tape here.

(Transcripts, video, Day 4)

This comment shows that this student understands that there is a concrete, physical, middle, with equal quantities on either side and that he can apply this understanding to the abstract number 10 with the abstract amounts of 5 being equal and on either side of the middle in the number 10.

**4.3.4.3 Question #2.** Students who completed the first problem were given another problem to explore, “Find One Third of Your Garden”. Noah and Amber were the only group to attempt Question #2. Their response to the problem showed the understanding that developed through the course of their work on the first problem. They discovered that dividing in half meant to divide by two and they used this information to understand that finding thirds meant dividing by three. In this transcript they had begun working on Question #2, Find One Third of a Garden.

N: It’s a hundred and ten this way *(without measuring, just remembers).*

A: But now we need to try to divide it into threes.

N: 110 is the width.

A: Oh, we need to divide 110 by threes.

N: Three, where is the calculator? 36.66666

A: Can I see this? One hundred and ten divided by three… aah, this is going to be harder than I thought it was going to be.

(Transcripts, video, Day 4)

These students learned that a fraction is equal to dividing the whole into the number of parts indicated by the denominator. They had also learned that dividing is sometimes complex
and requires an understanding of rational numbers that they did not yet have. When dividing 110 by three on the calculator they found a repeating decimal, similar to dividing an odd number by two during Question #1. It was evident that they did not know how to translate this into terms that they understood. They did not have a way to explain a repeating decimal with their understanding of fractions.

4.3.5 Lesson 5: Culminating Activity with Fraction Strips.

November 12, 2015

The last lesson brought back the fraction strips to finish the unit with some general fraction concepts. Students were asked to use the fraction strips that they made to consider the question: “If the whole represents a garden, how many ways can you fill it up?”

I prepared a class fraction strip set out of large strips of paper so that all the students could see, and we could work as a group. We sat on the carpet together and, using the large set of strips, I demonstrated the activity. We used a variety of different fractions to fill up the “one whole” strip. For example, we started with two half gardens. Then we tried a half and two quarter gardens. Students were then asked to go and experiment the same way with their own fraction strip sets.

The student-made fraction strips were not quite equal sized pieces because of their homemade nature. This led to fractions fitting into the whole that should not have fit. Thus, students were finding relationships that were not true. I called the students back to the center of the room and showed them what was happening, explaining why it was a problem. We did not proceed further with this activity. To finish the unit, I reviewed some of the concepts we had covered and the vocabulary they had learned including numerator and denominator.

4.4 Summary of Results
To consider the mathematical thinking that resulted from these lessons as well as the pedagogical information for each lesson Table 12 was constructed. The table is followed by a summary that is considered in more detail in the Discussion.
<table>
<thead>
<tr>
<th></th>
<th>Lesson 1</th>
<th>Lesson 2</th>
<th>Lesson 2 part 2</th>
<th>Lesson 3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Fractions with Recipe and Cooking</td>
<td>Making Fraction Strips, Order by Size</td>
<td>Student Led Exploration with Fraction Strips</td>
<td>Find Half of a Garden</td>
</tr>
<tr>
<td><strong>Meaningful context?</strong></td>
<td>Yes</td>
<td>Connected to garden very abstractly</td>
<td>Connected to garden very abstractly</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Open ended?</strong></td>
<td>No</td>
<td>Somewhat, could choose method</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Teacher led, or student led learning?</strong></td>
<td>Teacher led</td>
<td>Student led</td>
<td>Student led</td>
<td>Student led</td>
</tr>
<tr>
<td><strong>Could work in partners?</strong></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Types of mathematical thinking</td>
<td>Lesson 1</td>
<td>Lesson 2</td>
<td>Lesson 2 part 2</td>
<td>Lesson 3 and 4</td>
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<tr>
<td></td>
<td>Unitizing</td>
<td>Fractions are parts of a whole</td>
<td>Part Whole relations</td>
<td>Equivalency is not congruency</td>
</tr>
<tr>
<td></td>
<td>FS-UN</td>
<td>FS-TC</td>
<td>FS-PR</td>
<td>FS-ER</td>
</tr>
<tr>
<td></td>
<td>Part/whole relations</td>
<td>Fractions are relations between the numerator and the denominator</td>
<td>Equivalency vs congruency</td>
<td>Connecting compensation and division</td>
</tr>
<tr>
<td></td>
<td>FS-PR</td>
<td>FS-PR</td>
<td>FS-TC</td>
<td>FS-MO</td>
</tr>
<tr>
<td></td>
<td>Covariation</td>
<td>Relations are between the numerator and the denominator</td>
<td>Using benchmarks</td>
<td>Connecting fractions to dividing, multiplying, subtracting, adding</td>
</tr>
<tr>
<td></td>
<td>FS-TC</td>
<td>FS-TC</td>
<td>FS-ER</td>
<td>FS-MO</td>
</tr>
<tr>
<td></td>
<td>Reasoning up</td>
<td>The whole matters</td>
<td>Estimating</td>
<td>Part whole relations</td>
</tr>
<tr>
<td></td>
<td>FS-PR</td>
<td>FS-ER</td>
<td>NS-ES</td>
<td>FS-PR</td>
</tr>
<tr>
<td></td>
<td>The whole matters</td>
<td>Adding fractions</td>
<td>Drawing</td>
<td>Using benchmarks</td>
</tr>
<tr>
<td></td>
<td>FS-UN</td>
<td>FS-TC</td>
<td>DA-DRA</td>
<td>NS-BE</td>
</tr>
<tr>
<td></td>
<td>Adding fractions</td>
<td>Connecting division and fractions</td>
<td>Connect concrete and abstract</td>
<td>Estimating</td>
</tr>
<tr>
<td></td>
<td>FS-TC</td>
<td>FS-MO</td>
<td>DA-CCA</td>
<td>NS-ES</td>
</tr>
<tr>
<td></td>
<td>Connecting trigonometry and multiplication</td>
<td>Connecting addition and multiplication</td>
<td></td>
<td>Estimating NS-ES</td>
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<td></td>
<td>NS-AM</td>
<td>DA-AD</td>
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<td></td>
<td>Relative thinking</td>
<td>Skip counting</td>
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<td>DA-AD</td>
<td>DA-SK</td>
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<tr>
<td>Connecting dividing to multiplying, subtracting, adding&lt;br&gt;NS-AM&lt;br&gt;Skip counting with 2-digit numbers&lt;br&gt;DA-SK&lt;br&gt;Drawing&lt;br&gt;DA-DRA&lt;br&gt;Geometry&lt;br&gt;DA-GEO&lt;br&gt;Measuring, using a ruler&lt;br&gt;DA-MER</td>
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</table>
The results of this project indicate that using inquiry and contextualized garden mathematics contributed positively to the development of students’ conceptual understanding of fractions. The lesson that appears to have nurtured the most varied conceptual thinking was Lesson 3, Find Half of a Garden. It was contextualized in gardening and was also the most inquiry based problem.

Lesson 1, Fraction Lesson with Recipe and Cooking Soup, a teacher led lesson, also provided a rich variety of mathematical concepts perhaps because it was connected well to the garden project and anchored in a real-life activity rather than a concept. Lesson 2, Making Fraction Strips, Order by Size, was effective in supporting students’ understanding of some fraction concepts but seemed to be limited in the number of concepts that students considered. Interestingly, most students embarked on their own investigations with the fraction strips, which added to their conceptual understanding.

In this project, contextualization in real activities, gardening or cooking, seemed to offer lessons with the richest source of mathematics. Open ended problems such as Lesson 3, Find Half of a Garden, that offered students a variety of entry points and methods of finding solutions, encouraged mathematizing with a variety of concepts. These concepts can then be connected to each other which helps students create a deeper understanding.

The lessons in this unit contributed to students’ development of fraction sense by encouraging connections between a variety of mathematical concepts. Giving students inquiry based problems that could be approached at a variety of levels of understanding and with a variety of learning styles allowed them to connect their own thinking with new mathematical concepts.

After considering the coding and Table 12 the following themes emerged:

1) Breadth of thinking: variety of mathematical concepts used.
2) Depth of thinking: connections between mathematical concepts.

3) Differentiation: problems accessible to different learning styles.

4) Differentiation: different levels of understanding within the same problem.

5) Using common sense and estimation.

6) Perseverance and engagement

These points will be connected to this research and previous research in the areas of culturally responsive education, mathematical understanding, and inquiry methods of teaching in the next section.
Chapter 5: Discussion

5.1 Introduction

This research project was designed to analyze the mathematical thinking which resulted from the implementation of lessons contextualized by the school garden. The unit connected hands-on activities such as gardening and cooking to the *Ontario Mathematics Curriculum* through problem solving activities. The goal was to link mathematics to locally valued activities using an inquiry method as one step towards a more culturally responsive program for the First Nations students in a small central Ontario First Nation school and to consider the richness of the mathematical thinking that resulted.

Videos of student work and explanations regarding their work were collected. Artifacts of student work were collected, and field notes were taken. This data was coded and analyzed. Results from the coding were collected into a table to consider connections to the variety of types of lesson that were taught in this project (Table 12). Analysis of this table facilitated the development of themes. The themes will be elaborated upon and discussed within the framework of the research questions for this project in the following manner:

1) What types of mathematical thinking are supported by a variety of math problems connected to the fraction expectations of the Number Sense and Numeration strand of the *Ontario Mathematics Curriculum*?
   a) Depth of thinking: raising complex mathematical ideas, using complex methods to calculate and mathematize.
   b) Breadth of thinking: variety of mathematical concepts used.

2) How does using the garden to contextualize mathematics affect the mathematical thinking in elementary students in a small First Nation community?
a) Differentiation: problems accessible to different learning styles.

b) Differentiation: different levels of understanding within the same problem.

c) Using common sense and estimation.

3) Are there other benefits to using contextualized learning for First Nations students?

a) Perseverance and engagement.

5.2 Research Question #1

What types of mathematical thinking are supported by a variety of mathematics problems connected to the fraction expectations of the Number Sense and Numeration strand of the Ontario Mathematics Curriculum?

The use of inquiry and contextualization appear to encourage a rich mathematical environment for students. This research project suggests that contextualizing learning in a school garden and using inquiry-based mathematics could encourage the development of fraction sense in elementary age students because these pedagogies create a learning environment which supports the use of multiple mathematics concepts in one lesson. When students share ideas in class, this can lead to further connections between a larger number of concepts. Variety of concepts and connections made between them can support the development of a conceptual understanding of mathematics and thus the development of fraction sense (Bonoto, 2005; Lamon, 2007; Sowder, 1992).

Prior to a discussion comparing the lessons, Table 13 provides a summary for reference.

Table 13
Summary of Lessons
Lessons 1 and 3, which were explicitly connected to gardening or cooking, appear to have generated the greatest variety of mathematical concepts (See table 12). During these lessons, students chose different approaches, materials, and manipulatives to use while working on the problems. These lessons, 1 and 3, were more reflective of Indigenous Ways of Knowing than the other lessons in the unit because they were both holistic and directly connected to the lives and communities of the students.

Lessons 2 and 5 were more aligned with “school math” instruction. These lessons were devised to help students learn specific concepts about fractions. They were less holistic and less experiential than 1 and 3, and did not generate the same depth or breadth of thinking as those problems that were contextualized by real life. Although I attempted to make Lessons 2 and 5 open ended and conceptually based, they had the clearest right and wrong answers and used more abstract manipulatives (fraction strips) compared to the concrete materials in the other lessons such as ingredients, the soup, and the garden.

A table was created to contrast and compare the most relevant attributes of each lesson and the mathematical thinking that resulted from its implementation (Table 12). In this table, ideas from research on inquiry-based mathematics are considered in the following questions:

- Is there meaningful context?
- Is it open ended?
- Teacher led, or student led learning?
• Could students work in partners?

Some attributes were included in all the questions and are therefore not included in the table. For example, all questions were conceptually, rather than procedurally, based as recommended by scholars, so this attribute was not considered in the chart (Utley & Reeder, 2012; Fosnot & Dolk, 2001; Lamon, 2007). Even the problems that were teacher led (Lesson 1), focused on understanding the concepts involved rather than teaching a procedure. The two problems that were the least contextualized (the ones that used fraction strips, Lessons 2 and 5), were still designed to help students understand fraction concepts by manipulating concrete materials.

Several researchers have pointed out that to deeply understand mathematics students need to make connections between different concepts (Lamon, 2007; Sowder, 1998). In this study there were examples of rich mathematical thinking and signs of development in understanding. In most of the problems students considered a variety of mathematical concepts and often made connections between various concepts.

5.2.1 Breadth of thinking: Variety of mathematical concepts used. Student work during the lessons was coded for types of mathematical thinking. Some of the lessons generated thinking about more mathematical concepts than others. The greatest amount of mathematical thinking that was observed occurred during Lesson 3, Find Half of a Garden. The data showed that students discussed or used the following mathematical concepts during this math lesson, while working on Question #1:

• Measuring, using a ruler

• Measuring using non-standard units

• Adding 2-digit numbers

• Deconstructing numbers
• Dividing numbers to find one half

• Skip counting with 2-digit numbers

• Unitizing

• Connecting addition, subtraction, and division

• Estimating

• Making a model

• Geometric ideas around rectangles

• The whole matters (Fosnot & Dolk, 2002)

• Part whole relations (Fosnot & Dolk, 2002)

• Covariation (Lamon, 2007)

As compared to Question #1 in Lesson 2:

• Fractions are parts of a whole

• Fractions are relations between the numerator and the denominator

• Unitizing

In Lesson 3, Find Half of a Garden, students chose different materials to work with (manipulatives) including connecting cubes, rulers, calculators, jumping, and drawing. They chose different strategies such as finding the center with estimation and then checking or deconstructing numbers to divide them in half. Students connected different concepts to their calculations. For example, fractions were connected to division (one half means divide by two) and division was connected to subtraction, when trying to find two equal numbers on either side of a center line. Some students shared their work with a partner or other students in the class.

Lesson 1, Fraction Lesson with Recipe and Cooking Soup, although teacher led, was tied to the activity of cooking. The variety of mathematical concepts that surfaced in this
lesson is in between the variety for Lessons 2, Making Fraction Strips, Order by Size and 3, Find Half of a Garden, and include:

- Adding whole numbers and fractions
- Skip counting and multiplying whole numbers and fractions
- The whole matters (Fosnot & Dolk, 2002)
- Part whole relations (Fosnot & Dolk, 2002)
- Reasoning up (Lamon, 2007)
- Relations on relations (different quantities, number of wholes changes with the number of halves) (Lamon, 2007)
- Unitizing (Lamon, 2007)
- Covariation (Lamon, 2007)

Although Lesson 1, Fraction Lesson with Recipe and Cooking Soup, was not an inquiry problem, it was contextualized by the garden and cooking. Anchoring the lesson to a real-life activity, rather than being anchored to a single concept the way mathematics is organized in a textbook, allowed the mixing and connection of a variety of mathematical ideas.

In a school math textbook, for example, the class moves from one concept to another looking at multiplying in one chapter and division in another. Mathematical concepts such as addition, geometry, and fractions are dealt with separately. In Lesson 1, the connecting idea was the activity of cooking. This allowed a variety of concepts to be considered within one lesson. Having a variety of mathematics concepts within the same problem allowed the teacher to show the connections between concepts in mathematics. These connections encourage the development of fraction sense and support students in the transition from additive to multiplicative thinking (Lamon, 2007; Sowder et al., 1998).
5.2.2 Depth of thinking: Connections between mathematical concepts. Students often found connections between different mathematical concepts while working on the problems. For example, in Lesson 3, Find Half of a Garden, Daniel, in Grade 3, was unsure how to divide a two-digit number. Rather than measure across the top of the page and divide by two to find the center, this student guessed at the approximate middle and then measured the two resulting sides. He realized that he could subtract and add his way to finding two equal numbers, balancing them out using compensation until they were equal. This was presented to the class. The other students saw that adding, subtracting, and dividing were related and connected to fractions. The concept of, “a half means to divide by two” was related to other concepts such as the part whole relationship, the whole matters, and division (Fosnot & Dolk, 2002). This student showed that to divide by two means to find two equal parts (multiplicative thinking and partitioning from Lamon, (2007)), and finding two equal parts can be done with adding and subtracting (compensation). Considering the concepts that this student used, and the connections that he made, one can see that this student's presentation to the class was mathematically very rich.

Working on the same problem, Noah and Amber connected fractions, decimals, adding, and multiplying. While they were working on the Find Half of a Garden problem this pair of students were having trouble finding half of 107, an odd number. Noah tried using a calculator but did not understand how to work with the decimal. Amber decided to add two equal numbers together, guessing and checking, until she got to 107, but the odd number confused them. They knew that adding two equal numbers to find the total was the same thing as dividing the total by two. They worked on this problem using what they knew.

Noah understood that he was looking for part of a number and that decimals were another way to represent part of a number. By trying to find half of 7 he was connecting
decimals, fractions, addition, subtraction, and division at a level that he understood.

Another student, Bonnie, worked with the teacher on Lesson 3. They used a ruler, meter stick, and a rope with 100 Unifix cubes. Using these tools allowed Bonnie to work with the physical idea of the relationship of the numbers 8 to 16 and 50 to 100. She deconstructed numbers using concrete tools. She also worked with the connections between addition, subtraction, division, and fractions in a concrete and meaningful way.

She worked with the concept of division and its connection to fractions (1/2 of 100 means divide 100 by two). This is what Lamon (2007) calls reasoning down. With help from the teacher Bonnie found half of each of 100 and 16, the deconstructed quantities from 116. Finding half of the two parts of a deconstructed number shows the student that the parts change in a similar fashion, in a multiplicative way. For example, 116 divided by two is the same as 100 divided by two (50) plus 16 divided by two (8). As well, the relationship of 50 to 100 is the same as 8 to 16. These numbers can be looked at as ratios: 50:100, 8:16, and 1:2, all of which are equivalent. For a young student in Grade 3 this was a rich mathematical experience.

Combining an inquiry method with group work, discussion, and class presentations deepens the mathematics that students are exposed to which likely allows them to make more connections beyond those made when they were working on the problem themselves. In addition, when students have already worked with the problem at their own level of understanding they may be more likely to understand other students’ explanations. Each student approaches a question in their own way, often making different connections. Students working in groups were exposed to each other's strategies and, during presentations, to the strategies used by others in the class. This exposure to the same problem, with which they are familiar, being solved in a variety of ways, creates many mathematical connections for
students.

Connecting concepts may have influenced the rate and depth at which students were learning. Although this was a short period of study, only three weeks, there was some evidence of learning during this project.

5.2.3 Changes in understanding over time. This project was not long enough to expect to see significant change in student learning. Yet, there were instances of development in student understanding.

Over the course of working on Lesson 3, Noah learned that a fraction meant to divide by the number in the denominator. At first, he did not understand that finding half meant to divide the garden in two equal pieces as he indicated when he said, “Well you divide a hundred and seven in half. A hundred and seven divided by zero.” When something is divided in half the number two, the divisor, comes from the denominator in the fraction. This student understood that one half meant two equal parts, which he showed by looking for two equal numbers that added up to his total. He could not yet translate this into the operation of division. Once he realized that he could divide the length of the garden by the number two he connected adding two equal parts to the idea of division. Later, during the lesson he was asked to look at the second question, “Find a Third of a Garden”. While working on this part he showed that he had come to understand that the numerator is related to the number of parts and that he needed to divide the length of his garden by three to find one third. This student went through the process of learning that a fraction is a division question, or an operator. One third of something means show one of three equal portions which can be accomplished through the operation of division, specifically by three, the denominator of the fraction.

In Lesson 2, Daniel, Joe, and Rose, all connected the concrete fraction strips to the abstract symbolic representation by writing out the fractions in sets on their chart paper. This
was done after they completed the problem that they had been asked to work on. They were not asked to do this extra work but were exploring the concepts on their own. As this lesson, Lesson 2, generated fewer mathematical concepts than some of the other more contextualized problems, it is interesting that students directed their own learning in a way that would indicate growth of understanding, moving from concrete to abstract representations.

5.3 Research Question #2

How does using the garden to contextualize mathematics affect the mathematical thinking in elementary students in a small First Nation community?

The contextualized inquiry problems seemed to support a wide variety of learning styles and levels of development. This allowed students to do mathematics in ways which suited their ability and their preferred methods of building understanding. The large variety of ways of interacting with the problems may have contributed to students’ individual understanding of the mathematics and may, through discussion, have created deeper understanding for their peers.

Connecting a variety of mathematical concepts occurred as a result of several aspects. Contextualizing mathematics is a more holistic way of learning. A variety of concepts can come together because the focus is on the activity, like the garden, rather than on a mathematics concept. Also, the use of inquiry, allowed students to approach problems in a variety of ways. This variety of approaches, paired with a social learning environment, enriched the mathematical thinking that students were exposed to. Teaching holistically, which is recommended by scholars (Battiste, 2002; Doige 2003; Nicol et al 2010) for First Nations students helped to create an environment rich in mathematical ideas and discussion.

5.3.1 Differentiation: Different levels of understanding within the same problem.
Students were given choices as to how they solved the problems and whether they worked in a group or by themselves. They consistently chose methods and materials that were appropriate for their level of ability and understanding. Few students picked the same strategy and all strategies were appropriate for students’ mathematical levels as evidenced by their ability to use the materials to help them work on the problems.

The inquiry nature of the problems allowed students who had a strong facility with numbers to use that understanding. Rather than work at a level below that which they were capable, they could work at their own level making them less at risk of boredom and disengagement. They also exposed the other students in the class to this level of development and more advanced ways of understanding the mathematics.

In Lesson 2, students explored fractions in their own ways after working on the assigned problem. Allowing students to work on their own investigations led to students trying a variety of mathematics at their own level of understanding. Some students wrote out the fractions on their paper, making wholes out of the parts. Others connected the more complicated fractions to the fraction strips and then to the abstraction of symbols on paper.

In Lesson 3, Find Half of a Garden, students chose a variety of methods and materials that suited their levels of understanding. Bonnie used mostly non-standard measuring tools, other students used rulers, where others even drew scale models. This variety of materials was possible because of the inquiry approach to the mathematics. This problem even supported one student using a kinesthetic method to do her work, finding the center of her garden by jumping in and out.

5.3.2 Differentiation: Problems Accessible to Different Learning Styles. Rose used movement to work on the problem from Lesson 3, Find Half of a Garden. She enjoyed dance and being able to jump in and out of her rectangle taped to the floor allowed her to work on
this problem kinaesthetically. Accommodating her learning style increased the likelihood that she would be engaged and learning. Textbooks and worksheets are not generally designed for students who would like to use jumping to do their mathematics.

Rose also introduced an important concept to the class that was not addressed by other students. She was the only student that divided her garden in half by creating two triangles. The other students created two rectangles for the two halves of their garden. Rose, with the line delineating half going from corner to corner, showed the class that congruency is not the same as equivalency, one of the key concepts suggested by Fosnot and Dolk (2002). Cutting her rectangle in half from corner to corner showed the other students that “half” can come in different shapes.

This student had a kinesthetic learning style. She expressed to me at the outset of the lesson that she thought she may be doing the question wrong because she did not approach it by measuring with a ruler but by measuring with jumping. Being able to be different and insightful at the same time and present this thinking to the class is invaluable to building her confidence in mathematics and in school in general. It may also encourage creative thinking in her classmates. Rose could bring herself and her way of learning to math class (Figure 17). She could work on her problem in a way that suited her personality and embraced her love of dance.
5.3.3 **Using common sense and estimation.** Common sense and estimation are connected in mathematics. One way to encourage students to use common sense is to teach them to estimate. Using common sense is an important part of problem solving (Bonoto, 2005; Verschaffel et al., 2006). For example, students who use common sense might deconstruct numbers to add them up because it is faster and makes it easier to make mental computations. Students use common sense to judge the appropriateness of their answers, which is related to using estimation. This helps to guide decision making to determine whether an answer has been found or if something else must be done. It is imperative that students use common sense when doing mathematics yet, they often do not.

As noted in the research, students often fly in the face of common sense when solving mathematics problems in school (Bonoto, 2005; Verschaffel et al., 2006). Using contextualized problems has been found in other studies (Bonoto, 2005; Verschaffel et al., 2006) to increase the use of common sense during mathematics. This research supports those
findings.

During this project, students used common sense regularly. It appears that the lessons encouraged this because they were contextualized with real situations and the work was often inquiry based. There was more than one way to solve the problems, which may have helped students to become more self reliant. Prior to this unit, when assessing students using the PRIME assessment tool, it was found that many of the students were confused about estimation. It was encouraging to watch students use estimation while working on the problems.

In the, Find Half of a Garden problem of Lesson 3, there were many instances of using common sense. Students used their ability to estimate half of their garden. Amber and Noah checked their calculations by judging the relative sizes of the two sides of the garden after they drew their halfway point. They decided they were correct because the line they had drawn appeared to be in the middle of their rectangle. Perhaps, in contrast to traditional school math with many paper pencil questions, having one problem and a large rectangle taped to the floor was more concrete and therefore encouraged students to use common sense and estimation.

In the PRIME assessment that was done for the group, estimating was an area of mathematics that students were unfamiliar with. Students did not know how to use rounding numbers to estimate. I modeled one way of using rounding and estimating during Lesson 1 by finding the closest friendly number to 48 (50) and using 5 instead of 4 or 6 when working with the soup recipe. This example connected rounding and estimating to a concrete real-life activity. It showed students that in some situations you do not have to be exact. There are times that you can use friendly numbers to find an answer that simplifies the mathematics.

Students also used estimation with the fraction strips. In Lesson 2, Question #2, which
challenged students with more complex fractional representations, one group created diagrams and compared fractions using a greater than, less than, technique which is a type of estimation.

Using a meaningful context pulled students in and encouraged them to try different strategies until they found something that worked. The contextualized nature of the problems, reflective of one aspect of Indigenous Ways of Knowing, seems to have encouraged estimation and helped students to see its value. Working on the problems in this unit had other positive outcomes that influenced the richness of the mathematics.

5.4 Research Question #3

Are there other benefits to using contextualized learning for First Nations students?

The benefits of using contextualized inquiry included high levels of perseverance and engagement as well as increased connections to people and places in the community.

5.4.1 Perseverance and engagement. Engagement is integral to a successful mathematics experience (Bobis et al., 2011; Gettinger & Walter, 2012; Lein et al., 2016). Students that do not pay attention during class, who would rather be doing something else, do not learn mathematics. Engagement can be defined as “on task behavior” (Lein et al., 2016, p. 117) or displaying a “productive disposition” (Bobis et al., 2011, p. 35). Recommended ways of increasing engagement in students include contextualizing mathematics with student interests and using inquiry based problems (Bobis et al., 2011). Contextualizing math in place gives students real problems to work on such as deciding how to multiply a recipe before cooking. This real-world context serves to engage students partly because they can process and practice their math skills in meaningful ways (Bobis et al., 2011; Bonoto, 2005). To engage First Nations students and move in the direction of decolonizing education,
mathematics could be contextualized in local history and local use of mathematics (Nicol et al., 2013; Wagner & Lunney Borden, 2010). Although this was not done here, the results appear to support the use of contextualization in real life activities to increase student engagement.

I did not know what to expect regarding student engagement when we started working on the problems in this unit. Giving students one or two problems to work on when they are used to a page full of school math questions and desk work can turn chaotic. In my experience as a teacher, switching from seat work to group work with a class sometimes leads to a minimum of work or an inability to be self-directed. We did some very simple things such as finding the midline of a rectangle. Students could have worked for five minutes and declared that they were done. The level of engagement was surprising to me, as was the minimal amount of encouragement and support students needed. This was a small group with several adults, three including myself, but students were self-directed and worked on the problems for extended periods of time. I think there were a few reasons for this; students had prior experience with inquiry learning in other parts of the curriculum, students could make choices, the garden was an engaging context, and the math itself was intellectually challenging.

This class had not previously done a lot of inquiry math, but Language Arts was often taught holistically. Students had experience working independently, with chosen materials, on big questions, in and out of the classroom. They were experienced with this type of learning and could adapt to using inquiry for mathematics. Being student led, inquiry puts choice in the hands of students.

Most of the problems in this unit could be solved in many ways. Students could choose the way they worked on problems, at their own level and learning styles, while using a
variety of manipulatives. For example, one student had some difficulty focusing on learning, or being on task, during whole class work. He often chose to work with one other student. In this small group, he could learn from a peer. It is less intimidating to ask questions and perhaps easier to be engaged in small groups. Also, if a student had chosen the partner it was usually someone they wanted to work with and to whom they would listen.

Most of the problems were connected to a fun activity such as planning to make soup or planning to plant vegetables. Although students did not work in the garden during the math classes, I had worked in the garden with them from April to October planting, weeding, and harvesting. We had been talking about feeding their families for several months and they were very excited to make soup and bring it home. Students ate from their garden and this was motivating for them. We often had greens to try and they, most for the first time, ate baked garlic on crackers and baked radishes. Having planted, cared for, and harvested the vegetables seemed to create interest and increase their courage so they would try new things.

As well as the context being engaging, the mathematics also seemed to engage students. Perhaps the number of concepts involved in one problem, which makes the math challenging, also made the math interesting. Students worked hard, beyond my expectations, and often worked past the parameters of the problems. Students played with the mathematics. Daniel and Joe wrote out fractions to see the wholes symbolically on paper in Lesson 2 and Rose measured all the line segments in her garden in Lesson 3, both activities they chose to do on their own, out of curiosity. Students appeared to be engaged with the mathematics and they often persevered with difficult questions. Noah and Amber tried to find half of 107 in many ways and students did not hesitate to change their thinking and start over when working with the fraction strips.

After watching students work on the problems, seeing the variety of ways they
approached them and the variety of materials they used, it appears that the ease with which differentiation occurred may also have had an influence on the levels of engagement and perseverance that occurred during the implementation of this unit.

5.4.2 Time. Contextualized inquiry appears to increase the amount of time students spend engaged in mathematics. Increasing the amount of time students are engaged in problem solving that encourages the use of multiple mathematical concepts means that students are spending more time thinking about how concepts are connected. In her study Lamon (2007), pointed out that students who were in the experimental group took two years to catch up and overtake students in the control group. The students, Lamon stated, needed time to understand the concepts related to understanding fractions and to see how they were connected. Time allowed them to develop a conceptual understanding of rational numbers. This benefitted them in the long run as seen in the eventual rise of the experimental group’s test results above those of the control group, who had been taught using traditional algorithms. Teaching students traditional algorithms allows them to perform complex operations on fractions such as adding, subtracting, multiplying, and dividing without understanding the mathematics underneath these operations. Although allowing students the time necessary to understand these complex relations initially put them behind in test scores, they eventually caught up and superseded the control group.

If time is an important element in learning mathematics conceptually then we should maximize the amount of time students are actively working on mathematics. If students are disengaged, or working at the wrong level during math class, then no amount of increase in time spent on mathematics will improve their understanding. Maximizing the amount of time students are engaged in mathematizing during their mathematics lessons is imperative.

The teacher, Ms. T, expressed that she found the garden math “inspiring”, and chose
to implement more inquiry and more place based contextualized mathematics in her program after the completion of this project. In a follow-up interview, she informed me that she had decided to complete a math teaching course to enrich her understanding and increase her ability to use inquiry mathematics with her class. She had told me that she loved inquiry, its holistic nature and organic learning method, but had always struggled to implement it in mathematics lessons. Participating in this project, seeing her students using inquiry math, and being immersed in an inquiry math unit, was a powerful experience for her. This is a very strong testament to the quality of the learning that took place and the effect of collaboration through action research.

The holistic nature of the math also allowed for a variety of connections to the community and to topics that were important to the students. They were very happy to take the soup home. They participated in a ceremony blessing the garden. We had salads and baked garlic on crackers. One day we found a nest of baby mice in the hay that was being used to mulch the garden. These things added to the engagement of students and their connection to the mathematics.

5.5 Limitations of the Study

This project reflects some aspects of what is considered accepted practice in Indigenous education. To be more closely aligned with research in this area several more steps could have been taken. As part of the project, the community could have been consulted more directly (Donald et al., 2013; Nicol et al., 2010). Although community surveys were connected to this project, meetings with community members could have directed the contextualization of the mathematics in a different direction. Ongoing consultation with community members would have created a more legitimate action research cycle (Nicol et al.,
2013). Also, students would likely have benefited from following their own mathematical inquiries (Wagner & Lunney Borden, 2010). This would have given them even more agency, and likely more engagement.

This research project is a snapshot. The children from one class, in one small community, are a small sample size. More study on this topic, with larger numbers of students, is needed to contribute to the strength of these findings. Other limitations to consider include the length of the study, the type of assessment used, and the effects of other factors that contribute to student learning.

There are many factors that influence the success of learning in a classroom, the content and quality of the lessons are only part of the picture. The students themselves, the class size, the way they are taught outside of the project, their relationship or connections with their teacher, and their attitudes towards school, are all factors that contribute to the effectiveness of a program. Although they were not considered here, many factors may have contributed to the depth and breadth of mathematical thinking that was observed.

Another limitation is the length of this study. Although I was connected to the school over the course of a year, the recorded portion of this case study was only six classes over three weeks. This amount of time, although significant for this study, is quite short. Long term research, with a wider focus, would be invaluable to contribute to this discussion.

Lastly, student assessment was ongoing and qualitative during this project. A further long-term study that implemented quantitative pre- and post- assessments would be beneficial. Observation of a control group for comparison would also contribute greatly to understanding the effects of using contextualized inquiry-based education.
Chapter 6: Conclusion

This research project was designed to consider the mathematical thinking that resulted from the implementation of a mathematics unit with meaningful context in a First Nations community. Lessons were connected to the Ontario curriculum and were created with current research in mind. Some of the lessons were inquiry-based and garden-based, most were holistic in an attempt to reflect parts of the pedagogy of Indigenous Ways of Knowing, whereas others were less contextualized and less inquiry-based. Using different pedagogies allowed for comparison of various types of problems and the mathematical thinking that they generated. The lesson that was the most reflective of holistic teaching connected to Indigenous Ways of Knowing, Lesson 3, Find Half of a Garden, also generated the greatest number of mathematical concepts.

A school garden was used to anchor the mathematics unit. The academic research concerning garden-based learning shows that gardens often increase positive behavior in school as well as increasing academic performance (Blair, 2009; Christopher-Ipaktchian, 2014; Pittman, 2011). Contextualized learning, in the form of activities around the school garden, can be culturally responsive if these activities are valued by the First Nation community to which students belong or identify.

Garden-based learning could be used in any school, and would benefit the First Nation students attending, if the values of caring for community and nature were reflective of their community values. All First Nations communities must be treated individually but it has been found that caring for people and land is often a deeply held value in many First Nations communities (Abrams, 2013; Oskineegish, 2013; Scully, 2012). Garden-based learning reflects these values, has been shown to improve school performance, connects well with
holistic learning and inquiry mathematics, and is accessible to teachers and schools.

A collaborative approach was used to create this contextualized mathematics unit. A context important to the community, gardening and feeding others, was integrated with the expectations of the math curriculum. The results show positive outcomes and lend support to other research using contextualized learning as an option for culturally responsive mathematics (Beatty & Blair, 2015; Lipka et al., 2005; Munroe et al., 2013). Also, there seem to be benefits to contextualized and inquiry-based mathematics for elementary students in general. Student problem solving led to rich mathematical thinking. Those problems that were most closely contextualized by the garden and cooking, and left the greatest room for different ways of thinking, generated the greatest number of mathematical concepts. This meant that students made the greatest number of connections between concepts during the inquiry and contextualized problems.

Students seemed to be engaged in the problems created for this project, often working outside of the parameters of the questions. Differentiation occurred as students used various methods and materials to work on the problems. This added to the variety of concepts that were considered and the connections that were made between them. Students working in a social setting, as well as formally sharing their work, added to the richness of mathematical thinking. The holistic nature of the mathematics, related to Indigenous Ways of Knowing, added to the variety of mathematical thinking by allowing students to approach problems in their own ways. Holistic learning, recommended by scholars for Indigenous education (Donald et al., 2013; Nicol et al., 2010; Lipka et al., 2007), increased opportunities to connect to community members and activities that were important to students. This encouraged engagement and perseverance while working on the mathematics.

As pointed out in the Literature Review, inquiry-based learning shows the best results
over the long term. More long-term research into the benefits of contextualized inquiry-based mathematics for First Nations students would be an important next step. Research that has culturally appropriate assessment and can therefore report quantitative as well as qualitative data would also be helpful.

First Nations students require a culturally responsive program to close the gap that exists in education, especially in graduation rates (Ontario First Nation, Metis and Inuit Education Policy Framework, 2007; Gay, 2002; Nicol et al., 2010; Lipka et al., 2007). A program that grounds learning in locally valued activities using a holistic methodology would increase the number of mathematical concepts that students can consider and interconnect. This mathematical environment would benefit all students. As Toulouse (2013), points out, “pedagogy and practices that honor Indigenous learners” such as experiential opportunities, holistic approaches to teaching, differentiated instruction, community engagement, and hands-on activities “are also reflected in the literature regarding factors that contribute to overall student success in equitable school systems” (p. 17).

Inquiry mathematics, contextualized education, and holistic learning are similar in their pedagogies. They should all be ingredients in a culturally responsive program (Lipka et al., 2005; Nicol et al., 2013; Wagner & Lumney Borden, 2010), and they work well together to deliver content. From the results of this study, it appears that a program that connects locally valued activities and mathematics can deliver a rich mathematics program. As inquiry is considered, by scholars (Bonoto, 2005; Calder & Brough, 2013; Lamon 2007) and in the Ontario curriculum (Ontario Ministry of Education, 2005) to be the most recommended way of approaching mathematics with children, contextualizing math in activities valued by local First Nation communities would allow teachers to take a step closer to a culturally responsive program for First Nations students.
Consultation with First Nations communities regarding important values and activities is the first step. Embracing those values, bringing them into the school, and bringing them into children’s mathematics is the next step. Using an engaging contextualized inquiry-based program can provide a math program that enriches students' understanding of mathematics and possibly contributes to more positive school outcomes for First Nations students.
References


Mazmanian, L. (2013). *Evaluation of a garden project curriculum for 1st and 2nd*


http://www.edu.gov.on.ca/eng/literacynumeracy/LNSAttentionFractions.pdf


The Final Report of the Truth and Reconciliation Commission of Canada: Canada’s Residential Schools Volume 6


https://www.ctf-fee.ca/Research-Library/BeyondShadows_EN_Web.pdf


### Appendix A: Fraction Expectations Condensed from the Ontario Mathematics Curriculum

(Ontario Ministry of Education, 2005)

#### Fraction Expectations from the Ontario Mathematics Curriculum

<table>
<thead>
<tr>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
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<tbody>
<tr>
<td>divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation (p. 55)</td>
<td>represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered; – compare and order fractions (i.e., halves, thirds, fourths, fifths, tenths) by considering the size and the number of fractional (p. 66)</td>
<td>represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, number lines) and using standard fractional notation; (p. 78)</td>
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<tr>
<td></td>
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<td>demonstrate and explain the concept of equivalent fractions, using concrete materials (p. 78)</td>
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<tr>
<td></td>
<td></td>
<td>demonstrate and explain the relationship between equivalent fractions, using concrete materials (e.g., fraction circles, fraction strips, pattern blocks) and drawing (p. 67)</td>
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</table>
Appendix B: Consent letter and form for Parents

Lakehead Letterhead

Date here

Dear Parent or Guardian of ____________________,

My name is Linda Grant and I am a graduate student at Lakehead University. I would like to work with the teachers and students at the school to connect math and gardening. The teachers would like to use the garden more often as part of the school day and connecting it to math allows them to cover the math curriculum while being outside.

As a research project for my Master of Education degree I would like to gather information concerning student’s exposure to this type of teaching and their attitudes about mathematics in the garden. During this project, titled “Contextualized Garden-Based Mathematics” I would like to take photographs of students doing math and tape record lessons and conversations with students so that I can transcribe and consider this information afterwards. I would also like to ask students to write and draw about their experiences learning math in the garden. Any responses that I do gather I would like to use to understand student’s attitudes towards learning and how this connects to their mathematical understanding. As well, I would like to assess students learning using the PRIME assessment, which is a Canadian made and researched tool to gauge students’ understanding of mathematics.

Your child’s participation in any of these activities would be completely voluntary. If
you and your child agree to participate any data connected to a particular child will be kept completely confidential. No children’s names will be used in my report or any other public use of the information gathered. The results will be kept in a locked filing cabinet or my computer until the report is written. The report will be part of my Master of Education Thesis and will be published through the University of Lakehead.

There is some risk involved in taking children outside and working in the garden. These risks are minimal. The children and teachers are used to being outside as a result of the Coyote Learning program, which helps to make this project even safer.

This project aims to increase students mathematical understanding, increase the amount of time spent on outdoor education, and to help teachers to create their own math problems that are connected to outdoor projects. As well, results, and the math unit, may help students and teachers in other schools to benefit from garden math.

I would be happy to give you a copy of the report once it is finished. My email address is included below if you wish to contact me.

If you have any further questions about this project, please contact me at:

Linda Grant
(705) 345-8954
L.grant3@lakeheadu.ca

or my supervisor Ruth Beatty at:
(705) 330-4008 ext. 2619
rbeatty@lakeheadu.ca

Sincerely,

Linda Grant
Parental Consent Form

I have read and understand the information letter for the “Contextualized Garden-Based Mathematics” project and:

(Please check the appropriate box and sign below)

1. My child can be in photographs that will be taken to document this project.

2. My child can respond in writing and pictures to the math project.

3. My child can be recorded for the purposes of this research.

4. I do not want my child to participate in any documentation of this project.

__________________________
Child’s Name

__________________________
Date

__________________________
Parent/Guardian Name

__________________________
Parent/Guardian signature
Appendix C: Assent letter for Students

This letter was read to students as a group and then they were allowed privacy to sign or not sign the letter. This letter was included with their parental consent form. The parental consent takes precedence and determines student participation in the project.

Students;

I am a student at Lakehead University. I have been working with your teachers to make math more fun at school. We have decided to do math in the garden. As part of my school project at the University I would like to see how doing math out in the garden works for you as a student. Do you learn well outside, are there problems with it, or good things about it? I am doing this so that other teachers and students can learn from our project. I would like to videotape your math lessons in the garden and interview you about what you like or don’t like about this project.

Your teachers are going to expect everyone to do the math because it is part of your math class, but you can ask me not to videotape or interview you. It is no problem if you say no, it will not affect your mark and I will still be happy to work with you on your garden math.

I will be happy to answer any questions you have about this project! If you do not wish to be videotaped or do not want to be interviewed then you can tell me, or your teacher or parents and I will honor your wishes. Also, you may change your mind at any time and decide not to be videotaped, interviewed, or answer any of my interview questions. If you are comfortable with participating in this project you can sign your name on this form and with your parents’ permission I will record your participation in this project.

Thankyou,
Linda Grant
Appendix D: Consent Letter for Teachers

Lakehead letterhead

Date

Dear (teacher’s name or EM),

As you know, I am a graduate student at Lakehead University and I would like permission to work with you to connect math and gardening at the school. The intent of the project titled “Contextualized Garden-Based Mathematics” is to be able to use the garden more often as part of the school day.

As a research project for my Master of Education degree I will develop and help to implement a unit of number sense based problems. If you agree, we will co-teach the garden unit and I will record your input as to the strengths and weaknesses of the unit. I would like to gather information concerning teachers and student’s exposure to this type of teaching and their attitudes about mathematics in the garden. I would also like to record lessons and conversations with students and teachers so that I can transcribe and consider this information afterwards. I hope to use the responses that I gather to understand student’s attitudes towards learning and how this connects to their mathematical understanding. With parental permission, I will assess students learning using the PRIME assessment, which is a Canadian made and researched tool to gauge student’s understanding of mathematics.

Here is a tentative schedule that can be adapted:

May 18-Pre-Assessment (using PRIME)

May 19-June 12-Implementation (two visits with a one-hour lesson per week for four weeks)
June 15-19- Post Assessment (using PRIME)

Your participation in any of these activities would be completely voluntary. If you agree to participate, any data connected to you will be kept confidential. No names will be used in my report or any other public use of the information gathered. The results will be kept in a locked filing cabinet or on my computer until the report is written. The report will be part of my Master of Education Thesis and will be published through the University of Lakehead.

There is minimal risk involved in taking children outside and working in the garden. You and the children are used to being outside because of the Coyote Learning program, which helps to make this even safer.

This project aims to increase students mathematical understanding and increase the amount of time spent on outdoor education. As well, results, and the math unit, may help students and teachers in other schools to benefit from garden-based math.

I would be happy to give you a copy of the report and the unit once they are finished.

If you have any further questions about this project, please contact me at:

Linda Grant
(705) 345-8954
Lgrant3@lakeheadu.ca

or my supervisor Ruth Beatty at:
(705) 330-4008 ext. 2619
rbeatty@lakeheadu.ca

Thank you for considering my request,

Linda Grant