

LAKEHEAD UNIVERSITY

Quantile-based Reliability Analysis and Design in Slope Stability

by

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Declaration of Authorship

I, SUKHDEEP SINGH, declare that this thesis titled, 'Quantile-based Reliability Analysis and Design in Slope Stability' and the work presented in it are my own. I confirm that:

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Abstract

The study and analysis of slopes are essential for understanding their performance and, in particular, their stability, reliability, and deformations. Traditional slope stability analysis involves predicting the location of the critical slip surface for a given slope and computing a safety factor at that location, which belongs to the deterministic frame. It is found that multiple sources of uncertainties often exist in the evaluation of slope stability. When assessing the stability of slopes in the face of risks, it is desirable, and sometimes necessary, to adopt reliability-based approaches that consider these uncertainties explicitly.

The thesis develops an efficient methodology of soil modeling using maximum entropy based quantile distribution constrained by probability weighted moments, conducts field vane shear soil testing in the Nipigon river area and establishes the soil strength models. The research proposes a new reliability-based method to study the stability of the Nipigon river slope and carries out a reliability-based design of slopes by combining quantile-based reliability and multi-objective optimization.

In general, the probability distribution describes the randomness of soil parameters collected empirically or tested by the few numbers of collected soil samples. However, the substantial effect of sample size on the estimation of random properties of the soil strength requires an extensive data to explore uncertainties, which is uneconomical and sometimes impossible to obtain. This study aims to consolidate recent advancement in probabilistic characterization and develops an inverse cumulative distribution function (ICDF) or quantile distribution, for direct quantification of the actual variability of various soil samples. Based on the analysis, a framework is developed that streamlines the formulation of probability weighted moments (PWM), and maximum entropy (MaxEnt) based distribution function for various soil properties when estimated using different field or laboratory tests, leading to a reliable procedure for applications of the proposed framework to different site characterization problems. Examples are provided to illustrate the implementation and step-by-step procedures of the proposed framework.

This research further extends the reliability approach for slope stability problems and utilizes the first-order reliability method (FORM) with quantiles for improving the efficiency of the FORM with relatively small samples. Reliability analysis is combined with deterministic slope stability analysis and implemented using an efficient algorithm. The analysis is validated through comparison with other reliability methods and used to explore the effect of variability of the soil properties on slope system. It is found that, when variability of soil properties is defined by assuming a conventional distribution, the variance of factor of safety is overestimated or underestimated. The approach not only provides sufficiently accurate reliability estimates of slope stability but also significantly improves the computational efficiency of soil slope design in comparison with conventional design methods.

Keywords: slope stability, landslide, quantile function, first-order reliability method, response surface method, reliability-based design.

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Contents

Declaration of Authorship	ii
Abstract	iii
Acknowledgements	v
List of Figures	viii
List of Tables	ix
Abbreviations	x
1 Introduction	1
1.1 Motivations	2
1.2 Research objectives	3
1.3 Thesis outline	3
2 Literature review	5
2.1 Deterministic analysis in slope engineering	5
2.1.1 Determining slope stability with limit equilibrium concept	6
2.1.2 Stability analysis with finite element method	11
2.2 Advancement in slope engineering	12
2.2.1 Uncertainties in geotechnical engineering	13
2.2.2 Probabilistic modeling for soil variability	14
2.2.3 Reliability evaluation in geotechnical engineering	19
2.2.4 Non-probabilistic approach	21
2.3 Summary	22
3 Characterizing soil variability using quantile functions	24
3.1 Quantile-based modeling of geotechnical parameters	24
3.1.1 Probability weighted moments	25
3.1.2 Entropy-based quantile distribution with PWMs as constraints	29
3.1.3 Efficiency of PWM-based quantile function	31
3.2 Verification of PWM-based distribution for geotechnical parameters	33
3.2.1 Example 1	33
3.2.2 Example 2	36

3.3	Quantile-based soil characterization of Nipigon river landslide	37
3.3.1	Soil investigation of Nipigon river site	37
3.3.2	Modeling of soil variability from vane shear test samples	40
3.4	Summary	41
4	Quantile-based reliability analysis of Nipigon river landslide	43
4.1	Reliability in geotechnical engineering	43
4.1.1	Measures of reliability for slope failure system	44
4.1.2	Performance function for slope analysis	46
4.2	Computation of failure probabilities with quantile-based method	49
4.2.1	Calculations example	52
4.3	Case studies using quantile-based method	53
4.3.1	Homogeneous soil slope	54
4.3.2	Reliability analysis of Nipigon river landslide	56
4.3.3	Influence of water table and river currents	61
4.4	Summary	62
5	Remedial measures and slope design for Nipigon river landslide	63
5.1	Remedial measures for soil failure	63
5.2	Probabilistic design for Nipigon river landslide	68
5.2.1	Probabilistic design framework of soil slopes	68
5.2.2	Problem description, design parameters, design space	71
5.3	Summary	72
6	Conclusion and further recommendation	73
6.1	Contributions	73
6.2	Further research	76
	Appendix A	78
A.1	Probability weighted moments: a basic definition and statistical background	78
	Appendix B	83
B.1	Measurement data of vane shear test	83
	Appendix C	85
C.1	Response surface method and sensitivity analysis	85
	References	87

List of Figures

1.1	Landslide at Saint-Jude in Canada, May 2010 (Locat et al., 2017)	1
2.1	Theoretical model for the safety factor of slope.	6
2.2	Non-deterministic analysis in slope Stability	22
3.1	Normalized bias error for PWM and conventional moments	28
3.2	Algorithm for MaxEnt (QF) estimates	31
3.3	Normalized bias Pareto quantile with shape parameter ($POE = 10^{-2}, n = 10$).	32
3.4	Lognormal quantile with different COVs ($POE = 10^{-2}, n = 10$).	33
3.5	MaxEnt approximation compressive strength using different PWMs	34
3.6	Comparative ICDF of rock compressive strength using different sample sizes	35
3.7	MaxEnt approximation of c' with ICDF and PDF	36
3.8	Evidences of slope instabilities on the Nipigon river banks	38
3.9	Geotechnical investigations in study by Dodds et al. (1993)	39
3.10	Location map of landslide and a photo from actual landslide	39
3.11	Input variable: shear strength	41
4.1	Relationship of reliability and probability	44
4.2	Limit state function and design point (Holick, 2009)	47
4.3	The process of response surface method	49
4.4	Transformation of non-normal random variable (Holick, 2009).	50
4.5	Algorithm of first order reliability method with quantiles	53
4.6	Geometry slope stability problem case 1	54
4.7	Probability density function for homogeneous soil layer problem	56
4.8	Geometry slope stability problem: Nipigon river landslide	57
4.9	Sensitivity analysis with various variables	59
4.10	Probability of failure for various COVs with comparison of MC simulations	60
5.1	Reliability-based design for slope	70
5.2	Probability failure of each design	72
B.1	A shear vane tester equipped with a 33mm diameter vane.	83

List of Tables

3.1	Summary of estimated probability weighted moments for different sample size	34
3.2	Area and relative error(RE) of various sizes using ICDF and PDF	35
3.3	Summary of estimated PWMs and entropy's parameter λ s.	36
3.4	Summary of laboratory testing	40
3.5	Vane shear strength data results (kPa)	40
4.1	Calculated results of reliability index for case 1	52
4.2	Soil properties for homogeneous soil slope	55
4.3	Calculated results of reliability index for case 1	55
4.4	Reliability comparison between the three probabilistic methods for case 1 . . .	56
4.5	Soil parameters	58
4.6	Reliability comparison between the three probabilistic methods	60
4.7	Comparison between the three probabilistic methods for Nipigon river landslide	61
4.8	Results of slope stability analysis	62
5.1	The increase of factor of safety with vegetation	64
5.2	Summary of slope remedies	67
A.1	Moment Expression	78
A.2	Various distributions for a random variable	79
A.3	Approximate guidelines for coefficients of variation of soil parameter from Phoon & Kulhawy (1999b)	80
A.4	Sample observations of uniaxial compressive strength(MPa) example 1(data from Deng et al. (2004))	81
A.5	Data pairs of c' and ϕ' used in case study chapter 3 (data from Di Matteo et al. (2013))	82
C.1	Different random variables design points adopted for developing the response surface	85

Abbreviations

Latin Uppercase letters

E[x]	the expected value of x
FOS	factor of safety
FORM	first order reliability method
FOSM	first order second moment method
ICDF	Inverse Cumulative Distribution Function
FV	field vane shear test
LRFD	load and resistance factor design
ML	maximum likelihood
MOM	method of moments
MaxEnt	maximum entropy distribution
MCS	Monte Carlo Simulation
N	normal distribution
PDF	probability distribution function
PEM	point estimate method
POE	probability of exceedance
P_f	probability of failure
PWM	probability weighted moments
R	specified allowable stress or resistance
RBA	reliability based analysis
RSM	response surface method
RBD	reliability based design

Lowercase letters

c_u	undrained shear strength
$Var(x)$	variance of x
W_i	vertical stress acting at the base of slice i
X	measured quantity; variable

Y	population of a random variable
Z	limit state parameter
b_k	probability weighted moments
c	cohesion
$f(x)$	PDF of x
$f'(x)$	a priori PDF of x
g	performance function
h	horizontal direction
k	ratio S_u/σ'_v empirical transformation factor; or hydraulic conductivity
l	location
l_i	base length of slice i
z	depth

Greek letters

β	safety index
σ'_p	preconsolidation pressure
ϕ	friction angle
τ_{FV}	shear strength value obtained from FV test

I dedicate this to my parents, and my sister, for the support they have given me, especially over the last two years as a student in Thunder Bay.

Chapter 1

Introduction

A slope failure is the movement of a mass of rock or earth falling down from a slope, under the influence of gravity in different forms (Nemčok et al., 1972). There are various factors affecting slope failure phenomena like earthquakes, heavy rainfall, rapid snow melting and construction activities. Slope failures contribute to a significant loss of lives and money. The collapsed soil or rocks affect a large amount of area. A lot of studies have been performed on large landslides, and most of them considered as catastrophic landslides. A giant landslide occurred in Las Colinas, Central America due to the earthquake in 2001 affected 100,000 people (Baum et al., 2002). The occurrence of landslides is not just a natural process, but also a result of the increased vulnerability of communities and infrastructure resulting from excessive urban development, poor quality control and incomplete understanding of hazards.



FIGURE 1.1: Landslide at Saint-Jude in Canada, May 2010 (Locat et al., 2017)

In Canada, there are numerous cases of landslide due to rapid snow melting and earthquake. These are affecting the resources and lives of Canada. The landslide occurred at Saint-Jude in Canada is a recent example as shown in [Figure 1.1](#). The climate change and global warming are affecting the hilly regions, and coastal areas as well as infrastructure and other structures related to slopes ([Locat et al., 2017](#)). The studies claim that variability in soil parameters and fluctuations in rainfall will increase the chances of landslides in future. It is a challenging task to understand, and to respond the effects of climate change and preparing strategies to deal with it. Therefore, it is a rising concern for engineers to mitigate and prevent the slope failure.

1.1 Motivations

The reliability assessment of structures is considered as a formidable task in civil engineering. A civil engineer is responsible for developing reliable and effective systems for the society by mitigating risk and reducing failure. The geotechnical parameters and geological conditions are random variables due to the nature of origins. In general, engineers choose a *single value* or average in traditional geotechnical analysis rather than accounting the variability and quantifying the risk associated with projects. The conventional methods generally calculate the factor of safety (FOS), and it is assumed that the same value can be used to varying degree of uncertainties ([Duncan, 2000](#)). The FOS itself can be overly conservative in some cases. The traditional procedure may affect the risk assessment and contribute to failure. The reliability and probability-based concepts quantify the randomness of soil properties and help in designing safe and economic structure.

Reliability concepts provide a brief description of uncertainties and evaluate the combined effect of variability in different parameters on a structure. However, reliability theory has not been widely adopted for geotechnical problems, because this approach requires more laboratory/field investigations, effort and time than deterministic approaches. Although it is a challenging problem in slope engineering to obtain statistical data from a project site, the advantages of reliability theory are obvious. It can increase the efficiency in the design process and enhance the safety of structure which is valuable. Therefore, an advanced probabilistic

approach with the traditional approach is necessary for evaluating the safety of geotechnical structures.

1.2 Research objectives

The research is focusing on the evaluation of the effects of soil variability and uncertainty on slope stability analysis within the framework of probabilistic and reliability methods. The main intention is to apply a new probabilistic approach on the variability of soil parameters and to discover the failure probability in soil slope with soil shear strength parameters. To achieve this aim, soil slope stability, probability and reliability methods in civil engineering are studied. In addition to this, some case studies are elaborated to demonstrate the application of the proposed method.

1.3 Thesis outline

The present thesis focuses on non-deterministic methodology for evaluation of slope stability analysis. The main contents are:

Chapter 2 reviews the existing methodology in the slope stability analysis. The deterministic methods are discussed with advantages, disadvantages, and applicability to structure according to different assumptions. The reliability theory and its applications in slope stability are reviewed, and the impact of probabilistic methods on slope stability is also presented.

Chapter 3 starts by considering uncertainties as the main challenge in slope engineering and underlying the importance of using probabilistic methods as advancement to the deterministic analysis. The description of statistical and probabilistic terms related to quantile-based distribution is introduced with a detailed explanation. The chapter introduces the verification of proposed analysis which shows the difference between ordinary and probability weighted moments for modeling soil variability, and how the quantile function generates an efficient distribution with maximum entropy constrained by PWMs. The advantages of dealing with various sample sizes by the applied method are presented.

Chapter 4 represents the application of probabilistic and reliability analysis on three examples including homogeneous and non-homogeneous soil layer with correlated variables. With the

development of computer, quantile-based reliability approach is applied to analyze the slope stability and compared with most widely popular probabilistic methods.

Chapter 5 represents the reliability-based design in geotechnical engineering. The importance of calculating the probability of failure seeks an optimal design, which is insensitive to the variation in the uncertain input parameters.

Chapter 6 summarizes the essential findings of the research along with the conclusions and recommendations for further research.

Chapter 2

Literature review

The chapter reviews the concepts of slope failure and evaluation of safety factor or performance indicator with several approaches. Section 2.1 describes the deterministic approaches in slope stability analysis. The advancement in slope analysis is discussed in Section 2.2 with probabilistic and non-probabilistic approaches. The discussions on previous researches and need for an efficient methodology for modeling of soil variability is presented in Section 2.3.

2.1 Deterministic analysis in slope engineering

The quantitative deterministic solution can be generated by assuming some assumptions with uniquely defined parameters. However, it is an iterative process which includes calculations for a number of the trails or assumed slip surfaces to find the most critical slip surface. A solution or factor of safety equal to unity is defined as a limit state; a value less or more than unity is defined as a failure or stable, respectively.

The natural properties of soil are mostly recognized as complex and some assumptions are necessary for analysis of a particular slope mechanism. There are different kinds of soils and rocks with a different texture, nature, and properties. For example, sand and clay are entirely different in shape, structure, and permeability. Sand is generally considered as cohesion less soil, and clay is cohesive soil. Both cohesive and cohesion less soils require a different mechanism for calculation of shear strength and principle effective stress.

The performance of slope stability can be defined by its performance indicator. In a deterministic framework, there are two concepts, **Limit Equilibrium Method (LEM)** and **Finite Element methods (FEM)**, resulting in different performance indicators. For example, a *factor of safety* is the indicator of limit equilibrium, and the *critical seismic coefficient* is used for evaluation of stress deformation approaches in seismic conditions. The decision for choosing performance indicator is dependent on the area and type of structure.

2.1.1 Determining slope stability with limit equilibrium concept

The central concept of limit equilibrium is to analyze the stability of any soil mass or rock assuming incipient failure along a potential slip surface. In general, a *critical slip surface* is assumed, and the resisting, distributing forces are estimated enabling the formulation of limit equilibrium method. The safety indicator can be expressed by using Coulomb's equation for shear strength in terms of effective stress (Chowdhury et al., 2009):

$$F = \frac{s}{m} = \frac{c + \sigma' \tan \varphi}{c_m + \sigma' \tan \varphi_m}, \quad (2.1)$$

where s is shear strength, m is mobilized shear strength, σ' is the normal stress at failure surfaces which can be calculated from unit weight (γ) and height (h_i); c and φ are cohesion and friction angle respectively in the original state, known as soil shear strength parameters and c_m, φ_m are the required mobilized shear strength (m) parameters.

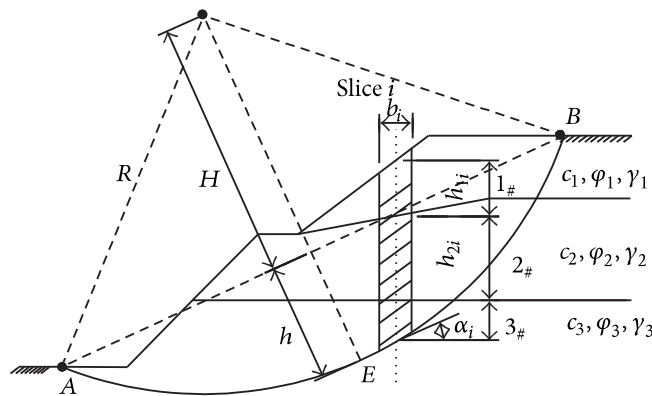


FIGURE 2.1: Theoretical model for the safety factor of slope.

A theoretical model of slope failure is presented in [Figure 2.1](#). The circle shows the slip failure area in a slope with different layers composing different properties. The slip surface is defined with a radius (H) and an angle θ .

The first technique based on the method of slices was introduced by [Fellenius \(1936\)](#), and then researchers develop this approach with the different assumptions ([Bishop, 1952](#); [Janbu, 1975](#); [Morgenstern & Price, 1965](#)). Numerous slice based methods were introduced in the 1980s and 1990s ([Vanmarcke, 1980](#)). These methods are widely used and available in commercial software.

In the limit equilibrium method, many slip surfaces can be considered for analysis. The results will be different depending on the slices and assumed slip surface. The slip surface with minimum value is considered as a *critical slip surface*. The factor of safety from the critical slip surface has to be optimized to obtain location and shape of failure. The stability analysis is based on determining the factor of safety in one slice and then repeating the procedure with another slice.

The shape of slip surface is assumed as circular or non-circular (arbitrary shape). The two-dimensional and three-dimensional analysis of slope stability problems are generally considered. In the two-dimensional analysis, the factor of safety is obtained as a ratio of resisting and disturbing moments taken about the center of slip circle as:

$$F = \frac{sR^2\theta}{Wx}, \quad (2.2)$$

where s is an average shear strength along the slip surface, R is the radius, θ is the central angle of the circular arc, and the weight W of the potential sliding mass acting on horizontal at x distance.

In clay, frictional angle φ and shearing strength c can be estimated from the triaxial results. It is more desirable to conduct *in-situ tests* like vane shear test and cone penetration test to estimate the undrained shear strength because it is challenging to obtain undisturbed samples from the field for triaxial test. The value of cohesion increases with the depth, and it is proportional to the effective overburden pressure and water table. It is very common getting

different cohesion values from the same clay layer.

Bishop (1952) demonstrated the analysis solution in an iterative procedure by assuming values of the inter-slice or slide wall forces. The forces acting on the base of the slice are the mobilized shear strength s/F , the total normal force $P = pl$, total pore pressure ul . Resolving these forces in the direction of the weight W it is easy to get an expression for p . By substituting the p in the Equation (2.1),(2.2) and taking moments of all forces (resisting and disturbing) about the centre of the critical slip surface:

$$F = \frac{\sum c_b + (W - u_b) \tan \varphi/m_\alpha}{\sum W \sin \alpha}. \quad (2.3)$$

Bishop's approach is applicable for circular slip surfaces, but it may not be circular in cross-section. Morgenstern & Price (1965) introduced a new approach by satisfying both force and moment equilibrium with Newton Raphson iteration technique. The factor of safety depends on the assumed side force function, and line of thrust in this method can be obtained in terms of effective stresses. The method is considered as best approach by some researchers.

Janbu (1975) proposed an approach based on the force and moment equilibrium of a typical vertical slice and force equilibrium of the sliding mass as a whole. The stability analysis using Janbu's approach with overall horizontal equilibrium as a stability criterion can be expressed as:

$$F = \frac{\sum b_s \sec^2 \alpha}{\sum (W + dT) \tan \alpha}, \quad (2.4)$$

where dT is the difference of tangential or shear forces on two successive slices; b is the width of the slice. The initial calculations can be made by assuming $dT = 0$.

Sometimes the convergence problems are encountered when applying the Janbu generalized method to individual slope stability studies mostly when the pore pressures are high. This problem was modified with a new innovative method, extended Janbu method involving iterative computations. The modeling considers the potential sliding mass into two parts, an upper part and a lower part, separated by an internal vertical surface. The computer-based solution with optimization procedure allows the critical slip surface of any shape and a minimum factor

of safety to be estimated. It can be used as a powerful and versatile tool for slope stability analysis based on limit equilibrium approach.

There is a little change in old methods by assuming non-vertical slices, interslice functions, changing of a factor of safety with different locations. [Sarma \(1987\)](#) proposed a different approach by determining the critical horizontal acceleration that is required to bring the state of critical equilibrium in the soil mass. He used the pseudostatic approach in which an appropriate horizontal force is applied to the center of gravity of sliding mass.

Many optimization methods are also applied in searching the failure zone including the numerical analysis and non-numerical analysis methods.(e.g., Genetic algorithm, Neural network method, Bionic Algorithm) ([Baker, 1980](#); [Greco, 1996](#); [Chen et al., 2008](#); [Li et al., 2015](#)). The development of slope stability analysis software like *Plaxis* and *Geo Studio* is a significant achievement in this field. This software is more convenient and can generate results with different situations like earthquake, heavy rainfall and seepage analysis in slopes.

The accuracy and feasibility of different approaches are discussed by [Duncan \(2000\)](#):

- The Morgenstern Price method is more rigorous but more accurate for stability analysis.
- The safety factor calculated for Simplified Bishop Method is greater than Fellenius Method by 6%-7%, and it produces almost same results with a comparison of Morgenstern & Price Method.
- Simplified Bishop Method is simple and produces more accurate results for circular slide face. It is most widely used in slope analysis problem.
- The Extended Janbu approach can be used for arbitrary shapes, and it is a powerful tool with a combination of computers.

Sometimes, it is difficult to choose an appropriate method for a site. It depends on the project and type of soil. There are two types of stress analysis. The total stress analysis, also known as short-term analysis, is acceptable for saturated clays and sand. The accuracy of analysis is generally focused on the undrained shear strength of soil. The effective stress or long-term analysis is useful for large projects. The failures corresponding to drained conditions may affect

the structure and literature proved that sometimes landslides occur frequently on the same site with high pore water pressures. The drained conditions provide the history and future of soil. It requires more tests and undisturbed samples which is more time and cost affecting procedure.

Various soil strength models: The strength of a soil is a unique value but depends on many factors such as the stress situation at breaking point, stress history, pore water and drainage conditions, loading or shearing rate. It is a crucial task in any slope stability analysis is to asses all of these parameters and evaluate as a real problem. This is done through geotechnical investigations and a study of the geological history. The modeling of the slope can begin on the basis of the established parameters.

1. **Drained analysis:** For a drained, effective stress, slope stability analysis, one has to assign the different regions Mohr-Coulomb models that simply define the strength according to the classic Mohr-Coulomb equation:

$$s = c' + \sigma' \tan \varphi'. \quad (2.5)$$

The factor of safety in a drained analysis is denoted $F_{\varphi'}$

2. **Undrained analysis** For an undrained analysis one can simply apply the Mohr-Coulomb model or use a predefined material called *Undrained strength* where the material strength is described by the value and the pore pressure have no effect on the materials shear strength.

$$s_u = c_u = \tau_{fu}. \quad (2.6)$$

The factor of safety in an undrained analysis is denoted F_c

3. **Combined analysis** This analysis includes both drained and undrained. It is method to calculate the safety factor which analyze each slice for both drained and undrained analyses. It is used for clay or silt deposits. The value of c' and ϕ' differ from case to case, but empirical relations can be used in this case:

$$c' = 0.1 \times \tau_{fu}, \quad \varphi' = 30. \quad (2.7)$$

The combined factor of safety is denoted F_k .

The limit equilibrium approach is convenient for complex soils and provides a reliable solution. Different methods can be used for different site conditions and project requirements. To assess the improvement in slope reliability within a probabilistic framework, it is necessary to use these concepts and methods of analysis.

2.1.2 Stability analysis with finite element method

The Finite Element Method (FEM) is the most widely accessible approach in conducting strain analysis or failure due to the complex mechanism (e.g., creeps, liquefaction of soils or internal deformation). It may be difficult to assume that slope failure can occur in only a particular area. The progressive failure may occur in over-consolidated or fissured clay and finite element method can determine this type of failure. The numerical simulation methods are adopted in several well-known *geotechnical finite element* (Griffiths & Fenton, 2004; Matsui & San, 1992). The other advanced numerical methods include the Boundary Element Method (BEM), the Explicit Finite Difference Method, Discrete Element Methods such as the Distinct Element Method (DEM) and Discontinuous Deformation Analysis (DDA) (Jiang, 2013).

The numerical simulation method mainly considers the relationship between stress and strain of slope material and the approach is not only dependent on geometry, shape and material in homogeneity. The two main finite element techniques are slip Surface Stress Analysis (SSA) and the other one is the Strength Reduction Method (SRM), developed by Matsui & San (1992). In SSA method, the potential slip surface is defined in advance and then analyzes the stress distribution on this surface after numerical simulation converged, ultimately calculates the safety factor based on the principle of weighted average. Giam & Donald (1988) invented the pattern search method to get the critical slide face and minimum safety factor based on stress level. Zou investigated the initial and potential range of slide face through the stress distribution, and then searched the most critical surface and its corresponding factor of safety.

The strength reduction method (Hamdy et al., 2003) is more popular method than slip surface stress analysis due to its simplicity and can be conducted in available software such

as FLAC, Ansys, etc. In this approach, the original shear strength parameters are reduced to bring the slope to fail. The surface under consideration is discretized, and the equivalent body forces are applied to the system. The Mohr-Coulomb criterion is adopted, and the finite element analysis can be performed under different conditions.

In numerical simulation methods, there are mainly three kinds of slope failure criterion:

- there is always a change in the rate of displacement in the system (Er-Xiang, 1997).
- a failure mechanism has developed (Matsui & San, 1992).
- the most commonly used criterion is a non-linear equation solver with a pre-set maximum number of iterations (Dawson et al., 1999; Griffiths & Fenton, 2004).

The study illustrated that factor of safety analyses using limit equilibrium and finite difference methods can be expected to produce very similar results for both simple and complex slope cases. An important limitation of the conventional methods is that it requires an arbitrary selection of the search areas and shape of the potential failure surfaces prior. This is an inconsistent measure of performance of soil slopes, which need to develop with more reliable tools to incorporate soil heterogeneity in a quantitative scheme amenable to engineering design.

2.2 Advancement in slope engineering

The deterministic approaches never assume uncertainties in design, but it should be considered. The failure occurs even when the factor of safety is higher than required. The probabilistic method is a technique to analyze the uncertainty and failure probability in a structure. It requires the assessment of *failure probability*, can be calculated from the treatment of performance function based on the geotechnical model. The probabilities calculated from observational data and historical data is enough to determine the performance indicator. The *reliability index* is the performance indicator of a probabilistic approach. The calculation of relative probabilities is also important, but it may difficult due to the complexity of geotechnical problems. The approach has been receiving more acceptance in geotechnical engineering due to its efficiency.

2.2.1 Uncertainties in geotechnical engineering

Soil is a geological material formed by weathering, erosion, and sedimentation processes and, save for residual soils, transported by physical means to their present locations. They have been subjected to various stress and physical and chemical changes. There may be randomness in data due to spatial variability and errors in testing. Uncertainty in geotechnical engineering can be classified into three types (Phoon & Kulhawy, 1999a; Van Gelder, 2000):

- **Aleatoric uncertainty:** Physical or aleatoric uncertainty is the natural randomness of a quantity such as the variability of shear strength from point to point within soil volume, the randomness of boundary conditions. Measurements and statistical estimations can quantify these kinds of uncertainties. It requires more experimental data and certain laboratory results.
- **Epistemic or statistical uncertainty:** Statistical uncertainty can be defined as lack of data or information about slope failure conditions. In slope probabilistic, data is available in very limited, insufficient and it is challenging to fit small data in probability distributions. It results from the data exploration uncertainties, data handling, and transcription error. It is evident that soil properties will be different when the samples and sample sizes are different. Statistical uncertainty is further explained in chapter 3.
- **Decision model uncertainty:** This type of uncertainty is related to time management that includes objectives, time preferences, and budget. Slope stability design and analysis is a process which includes different random variables in a relationship through some mathematical models, and these models are based on mechanical abstracts about the real methods. The model uncertainty leads to simplification postulates and unknown boundary conditions, also by other variables which are not contained in the models for the unknown effects.

Since the performance of geotechnical structure depends on soil properties of a profile, it is important to characterize the soil profile probabilistically. The probabilistic characterization of soil profiles provides more geotechnical information regarding the soil conditions at a

particular site, a basis for predicting the stability of slopes and for quantifying the probability of failure, and enables a geotechnical engineer to assess critically and compare various site investigation and testing programs (Jaksa et al., 1999).

2.2.2 Probabilistic modeling for soil variability

In a probabilistic analysis, the parameter which affect the performance with variability are considered as *random variables* or *noise parameters*. In slope stability, soil parameters are random variables and present variability in the system. Random variables are a range of values obtained from various experiments like in-situ and lab results.

Descriptors of randomness

The variability in parameters is represented by statistical and probabilistic approaches (Duncan et al., 2014). *Statistical moments* and *Probability distributions* are the basic descriptors of a random variable. These descriptors can be used to estimate the variability of geotechnical problems.

Statistical moments

Basically, first two moments (Mean, Variance) of a random variable are considered as statistical parameters representing variation of data. The procedure generally involves defining the material properties by their first and second moments: *Mean*, \bar{x} , and *Variance*, s , which define the probability density function and the coefficient of variation, *COV*. The mean of a data set is the sum of the data points in the data set divided by the total number of data points in the data set. The variance of a random variable is the mean value of the square of the deviation of that variable from its expected value or mean. The mean is the most common measure for the center of a data set. The variance is a measure of dispersion about the mean value of a data set. High and low values of dispersion indicates higher and lower uncertainty respectively.

Statistical analysis of geotechnical engineering parameters have been published by researchers (Phoon & Kulhawy, 1999a; Youssef et al., 2016). These *second moment* statistics are useful for reference purposes but they are largely generic with a wide range of dispersion and hence may not represent the most economical or cost effective case. They should not be used for design for the following reasons:

- The statistics of most geotechnical engineering parameters are dependent on in-situ state.
- The testing methods and/or procedures used in measuring parameters are not stated in most of publications. The same soil parameter can be estimated using different methods and/or procedures which results in huge difference.
- It is difficult to evaluate homogeneity of soil from the calculated statistics.

Proper knowledge of uncertainties is required for these statistics to be applied correctly in different cases, as it is difficult to apply same procedure for all situations. When the sample is small, the parametric estimation of a distributions are mostly inaccurate. Values of the parameters of the probability distribution functions estimated using the available sample series should be unbiased and close to their population values. Generally, Method of Moments (MOM), Maximum Likelihood (ML) and Probability Weighted Moments (PWM) are used for parameter estimations of probability distribution functions.

The MOM and MI are popular approaches and most widely used in geotechnical engineering (Christian & Baecher, 2002; Phoon et al., 2003). The *Probability weighted moments*, which has been investigated by many researchers, was originally proposed by Greenwood et al. (1979) and widely used by researchers. Hosking & Wallis (2005) investigated the properties of parameters estimated by the PWM method for the Generalized Extreme Value (GEV) distribution using fairly long observed series, and they gave a good summary of the PWM method. He showed that the PWM method is superior to the Maximum-Likelihood (ML) method in parameter estimations. However, its use in engineering analysis shows its efficiency (Deng & Pandey, 2008, 2009; Yu, 2008) and it can be used for geotechnical problems.

Probability distribution

Probability distribution ($f(x)$) refer to continuous random variables and represent the characteristics of a random variable. The reliability analysis is mainly focused on first two moments of random variable but sometimes it is recommended to evaluate the skewness of distribution using n moments. A probability density function of variable X defines the probability

of occurrence of the particular value x .

$$P[X = x] = f(x) \quad (2.8)$$

probability that the value lies between two values x_1 and x_2 is

$$p[a \leq x \leq b] = \int_a^b f(x) dx. \quad (2.9)$$

The cumulative distribution function CDF or $F(x)$ measures the integral of the probability density function from minus infinity to plus infinity

$$F(x) = \int_{-\infty}^{\infty} f(x) dx = 1, \quad (2.10)$$

it must be a continuous non-decreasing function with the values in the interval $[0, 1]$.

Quantile function or inverse cumulative distribution

The quantile function [or inverse cumulative distribution function (ICDF)] is a probabilistic measure that is widely employed in both statistical and engineering applications, mathematically expressed as (Kendall & Stuart, 1977):

$$F_{(r)}(x) = \sum_{k=r}^n \binom{n}{k} F^k(x) [1 - F(x)]^{(n-k)}, \quad (2.11)$$

$F_{(r)}(x)$ is the r th order statistics, the $F(x)$ can be written as u or $x = F^{-1}(u)$, $0 \leq u \leq 1$ and by substituting in Equation (2.11) the expected value of r th order statistics can be obtained as:

$$E(X_{r:n}) = r \binom{n}{r} \int_0^1 x(u) u^{r-1} (1-u)^{n-r} du, \quad (2.12)$$

note that $x(u)$ denotes the quantile function of a random variable. The expected minimum and maximum of a sample of size n can be obtained as:

$$E(X_{n:n}) = n \int_0^1 x(u) u^{n-1} du \text{ and } E(X_{1:n}) = n \int_0^1 x(u) (1-u)^{n-1} du. \quad (2.13)$$

In probabilistic analysis, the density functions of all random variables must be determined accurately to minimize the errors. There are a numerous distribution types used in mathematics and statistics. However, only a few distributions are commonly used in geotechnical engineering like normal, lognormal, gamma etc explained in Appendix A. The process of selecting and fitting a probability distribution that approximates a dataset best can be accomplished using many approaches and techniques.

Two techniques commonly used are plotting a histogram of the data and choosing a distribution that appears to best-fit the data (histogram) or the Pearson's moment-based system. Laboratory test results indicate that most soils can be considered as random variables having a normal or lognormal distribution (Harr, 1989; Christian et al., 1994; Duncan, 2000). However, best-fit probability distributions for geotechnical parameters are dependent on data set, largely dependent on soil type and in-situ state. Hence, it is impossible to select best-fit distributions for soil parameters.

Numerically defined maximum entropy distribution

The numerically defined distributions are also a substitute for parametric analysis. The numerical defined distributions are obtained by classical algorithms and mostly fit for a different range of data but it requires at least 40 to 50 samples for generation of smooth distribution curves (Siddall & Diab, 1975). Maximum entropy distribution defined by Jaynes is highly unbiased distribution with moments as constraints and optimization algorithm. The entropy of a random variable can be written as:

$$H [f(x)] = - \int_R [f(x) \ln f(x)] dx, \quad (2.14)$$

which is further maximized as:

$$\bar{H} = - \int [f(x) \ln f(x)] dx = \text{maximum}, \quad (2.15)$$

subjected to some known moment constraints or equations of moments:

$$\int x^n f(x) dx = \mu_n, \quad (n = 0, 1, 2, \dots, N) \quad (2.16)$$

where x is a sample estimate of population, μ_x are sample moments considered in the analysis and entropy function can be represented as H . Using Lagrange's method to solve the entropy density function, solution takes the form:

$$f(x) = \exp\left(\lambda_0 + \sum_{n=1}^N \lambda_n x^n\right), \quad (2.17)$$

where λ_k denotes unknown Lagrangian multiplier. $\lambda_0 - 1$ is used as the first multiplier as a matter of convenience.

The maximum entropy method has achieved good performances in structural reliability analysis (Deng et al., 2012), geotechnical engineering analysis (Zhang et al., 2013; Most, 2009), and rock mechanics parameters (Deng et al., 2004). The main problem in choosing maximum entropy is selecting the constraints as ordinary moments are highly biased for variety of data. Consequently, there is a need to develop probabilistic approach to analyze the soil properties.

Correlation in slope stability parameters

In slope engineering practice, the use of correlations or relationships in soil parameters provide a fast, cost-effective means of predicting the value of some parameters based on the value of some other parameters. In probabilistic approach, the quantification of the correlation between two or more soil properties provides a more realistic assessment of uncertainty in design parameters (Uzielli et al., 2005).

The random variables may be correlated or independent. If correlated, the likelihood of a certain value of random variable may depend on the other random variable. For example, the depth may be correlated with the properties of soil. The correlation between two or more soil properties has been found to be dependent in varying degrees on soil type, testing method used to obtain the numerical value of the parameter itself, and the homogeneity of the soil (Phoon et al., 2003).

Modeling and quantifying uncertainties in random variables are the initial and essential steps in *reliability-based analysis* and *design*. The primary task of planning and design is to ensure satisfactory performance. The insurance of safety can be given in the form of probability of success in satisfying the performance criterion. This probabilistic assurance of performance

is referred as *reliability*. Traditional approaches simplify the problem by considering the uncertain parameters to be deterministic and accounting for the uncertainties through the use of empirical safety factors. Deterministic safety factors do not provide adequate information and compromise with the goal of safety levels and minimizing cost. The use of probabilistic analysis in design is expected to provide more information about system behavior, the influence of different uncertain variables on system performance.

2.2.3 Reliability evaluation in geotechnical engineering

The reliability-based or probabilistic analysis was introduced to recognize the importance of uncertainties in structural engineering. Variability is present in different forms, but researchers justify it as a mathematical problem and generate a reliability index. The reliability index is the maximum distance between the original point and limit space in Gauss space (Hasofer & Lind, 1974). The reliability theory for slope engineering was proposed by Matsuoka & Kuroda (1974) with the design of embankments based on probabilistic approach. Then numerous researchers contribute to reliability-based design and analysis (Ang et al., 2007; Chowdhury et al., 2009). The structure is considered safe or unsafe depending on the distance between the limit state surface and design point.

Cornell (1969) introduced a simple two-variable approach to produce a linearization of performance function. The method is also known as Mean Value First Order Second Moment Method (MVFOSM). The result is dependent on mean value and partial derivatives of the safety margin. These methods are accurate for linear performance function but produce an error with non-linear or implicit performance function (Duncan, 2000; Griffiths & Fenton, 2004). Then researchers updated this method with First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) to optimize and linearize the implicit function about critical points (Breitung, 1984; Nowak et al., 1994). The transformation of non-normal variables to normal variables was another problem in reliability methods. Rackwitz & Flessler (1978) proposed a method to transform the non-normal variables to equivalent normal random variables and this method is known as JC method. . It has drawbacks such as the degradation of accuracy resulted from the multiple most probable points, the non-linearity of the performance function or the non-normality of random variables. The first order approximation assumed in

FORM could lead to an underestimate of the probability of failure if the actual limit state function curves towards the mean values. [Bin & Songhong \(2004\)](#) introduced reliability approach with quantile method. The direct iteration calculation of a nonlinear function under the condition of correlated variables was discussed by using coordinate transformation and matrix operation. The calculation formulas based on the quantile value method was derived and this algorithm is simple in calculation and high in precision.

In most of the geotechnical problems, the performance function is defined as an implicit function, and the reliability methods need the gradients of performance function. Response surface method (RSM) was developed to overcome and simplify the calculation problems. The main function of response surface method is to transform the implicit form into an explicit performance function by evaluating the impact of the input parameters on the system response. It generates a relationship between the significant variables and system response to reduce the complexity of analysis procedure. [Wong \(1985\)](#) used response surface in slope analysis and researchers used this approach with various developments ([Cho, 2009](#); [Li et al., 2015](#)). The probability assessment of multiple layered slopes is more convenient with RSM. The reliability methods with RSM can be used for uncertainty analysis after approximating the performance function.

Another popular method to evaluate the statistical moments of performance function is the Point Estimate Method (PEM). [Rosenblueth \(1975\)](#) introduced the point estimate method and then further developed by other researchers ([Harr, 1989](#); [Hong, 1996](#); [Christian & Baecher, 2002](#)). This approach is based on replacing the probability distribution with discrete points having the same mean value, standard deviation and skewness of performance function. The shape of any PDF used for any random variable is presented by mean and two hypothetical point masses located at plus and minus one standard deviation from the mean.

Simulation methods like Monte Carlo method is also a well-adopted approach in probabilistic analysis. It requires a little knowledge, and an efficient result can be obtained. The main deficiency of this method is that it is a time-consuming process. Researchers adopted this approach and identified the uniqueness of this method with comparison of other approaches ([Malkawi et al., 2000](#); [Low & Tang, 2007](#)).

There are numerous methods to calculate the reliability index and probability of failure with deterministic methods. The main idea is to identify the risk and uncertainty in structures and designing the safe and reliable design for society. The approaches are quite popular and efficient but there is still a need for improvement for risk analysis.

Slope design based on reliability analysis

In the past few decades, reliability-based design (RBD) has been gaining increasing attentions in geotechnical engineering. For practical purpose, RBD methods are usually adopted in current geotechnical designs, such as load and resistance factor design (Phoon et al., 2003). Theoretically, all existing reliability analysis methods can be directly used in full probabilistic design with trial-and-error procedure. There are some simulation-based methods, subset simulations and monte carlo simulations, have been applied to developing more specialized full probabilistic design approaches, such as robust design approach (Low et al., 2011). To ensure a desired accuracy of reliability estimates, a considerable number of samples are usually needed in simulation-based methods. Moreover, the required number of samples increases with the number of possible designs, leading to a significant increase in computational efforts. The improvement in reliability design is required to simplify and overcome the the shortcoming of available methods.

2.2.4 Non-probabilistic approach

There are some non-probabilistic analysis approaches used in slope stability analysis. These are used when it is challenging to fit distributions to same input data obtained from laboratory tests. There are other approaches to solve this problem such as Interval analysis (Moore & Lodwick, 2003), Evidence Theory (Dempster, 1967), Fuzzy Set Theory (Klir & Folger, 1988), Random Set Theory (Kendall & Stuart, 1977), Grey number Theory (Julong, 1989).

Interval analysis is used to describe the parameter uncertainties in the system. An interval number can be used as a random variable whose distribution function is unknown but assuming non-zero in the range of interval (Dietzel et al., 2011). This concept is a basic approach for defining uncertainties without probabilistic distributions. The worst and best set

of random variables can be obtained by using interval method. Random set theory also applied in geotechnical engineering mainly with designing of tunnels. Many researchers used and developed this theory with finite element method (Peschl & Schweiger, 2003).

The Fuzzy set approach (Zadeh, 1996) is another popular approach for the analysis of uncertainty data. Shrestha & Duckstein (1997) introduced fuzzy reliability index to measure the reliability and probability of failure in civil engineering problem. The theory is convenient for landslide susceptibility and slope design (Kavzoglu et al., 2014).

The non-deterministic approach is a good research topic, but there is less efficiency in using these approaches. Sometimes, it may difficult to use with practical application and collaborating with available reliability theories.

2.3 Summary

The topic of slope analysis with deterministic approaches was introduced with knowledge of uncertainties present in process. The main uncertainties are related to properties of soil and modeling of soil data. The main limitation of conventional approach is neglecting the uncertainties and preferring a single value, which increase the chances of slope failures.

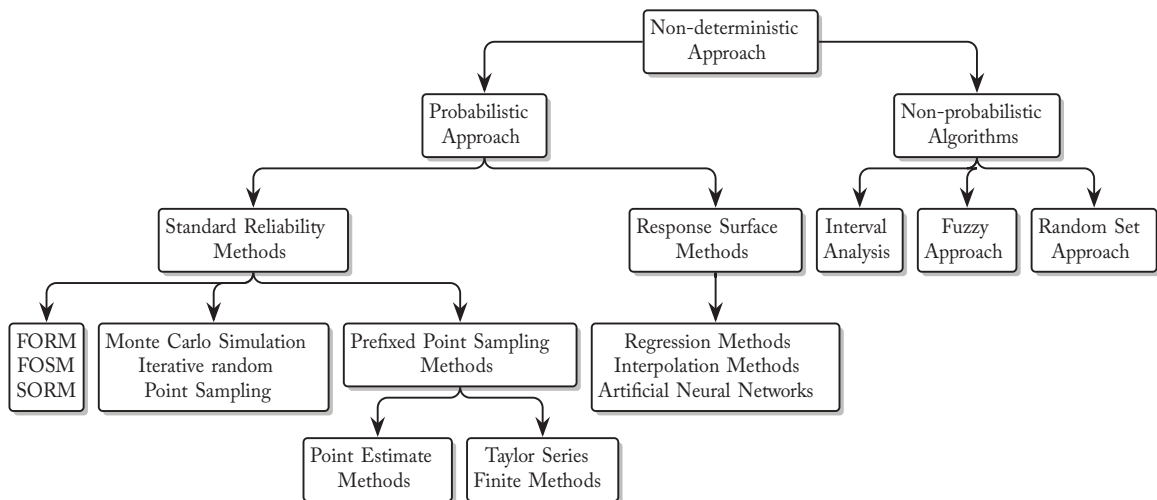


FIGURE 2.2: Non-deterministic analysis in slope stability

The probabilistic approach is an advantageous process by considering the soil parameters as random variable and defining in terms of distributions and statistical moments. The reliability index helps in enhancing the performance evaluation of slopes. The use of deterministic

analysis by incorporating uncertainties associated with the performance of the geotechnical structure is the simplest and most obvious advantage of a probabilistic approach or reliability analysis.

Figure 2.2 presents a summary of all methods discussed above. The non-probabilistic approach shows less efficiency as sometimes, it may be difficult to use with practical application and collaborating with available reliability theories. In conclusion, the estimation of the adequacy of a slope found by using a probabilistic analysis compared to the calculated traditional methods remains questionable. It shows a need of more progress in the field of geotechnical engineering and reliability-based design.

Chapter 3

Characterizing soil variability using quantile functions

This chapter proposes a numerical method for determining inverse cumulative distribution function (ICDF) of soil properties containing various uncertainties. Section 3.1 gives the reasons why a new distribution method for soil slope systems is needed and proposes maximum entropy based ICDF. Section 3.2 includes the verification of the proposed method and section 3.3 describe the implementation of a quantile-based approach in quantifying the uncertainties present in vane shear test data from the Nipigon river landslide. The discussion is made in Section 3.4.

3.1 Quantile-based modeling of geotechnical parameters

As pointed out above, soils are geological materials formed by weathering processes and transported by physical means to their present locations. They have been subjected to various stresses, pore fluids, physical or chemical changes and treated as random variables. The randomness present in the soil makes it difficult to design a reliable structure. The randomness can be observed mathematically using statistical moments and distributions. Moments can be estimated directly from a sample of observed values. In the ensuing, two categories of moments are employed.

Conventional moments are referred as moments about the origin or central moments. Suppose a sample contains n observations : x_1, x_2, \dots, x_n . The conventional moments μ_n of a distribution are defined by

$$\mu_n = \frac{1}{m} \sum_{i=1}^m x_i^n, \quad (3.1)$$

or

$$c_n = \frac{1}{m} \sum_{i=1}^m (x_i - \mu_1)^n, \quad (3.2)$$

where m is the sample size, n is the highest order of moments. μ_n is the n th moment about zero, c_n is the n th moment about central. x_i is i th value of m random variables. μ_n and c_n of the same variable can be transformed from each other using the binomial theorem.

First two moments are considered as parameters of the normal distribution. Non-zero values of c_3 is an indication of asymmetry or positive or negative skewness (depending on the sign of c_3), while non zero values of c_4 are an indication of non normal kurtosis (Siddall & Diab, 1975). However, the estimates of higher order ordinary moments (order>2) from a small sample (size<30) tend to be highly biased. In geotechnical engineering, the soil sample size is sometimes less than 30. At this time, direct use of ordinary moments would lead to inaccuracy.

3.1.1 Probability weighted moments

The problem of estimating a distribution for specifying a finite number p from a random variable can be solved by probability weighted moments. The probability weighted moments are the representation of corresponding population quantities. PWMs can be estimated by linear combinations of an ordered data set.

The probability-weighted moment of a random variable was formally defined by Greenwood et al. (1979) as:

$$M_{i,j,k} = E[X^i u^j (1 - u)^k] = \int_0^1 x(u)^i (u)^j (1 - u)^k du, \quad (3.3)$$

where i, j, k are real numbers. The two forms of PWMs are useful:

$$\alpha_k = M_{1,0,k} = \int_0^1 x(u) (1 - u)^k du, \quad (k = 0, 1, \dots, m), \quad (3.4)$$

and

$$\beta_k = M_{1,k,0} = \int_0^1 x(u) (u)^k du, \quad (k = 0, 1, 2, \dots, m), \quad (3.5)$$

PWMs are the normalized expectations of minimum or maximum of k random observations.

Special cases of these estimators include the sample mean:

$$\bar{x} = n^{-1} \sum x_i = a_0 = b_0, \quad (3.6)$$

and the extreme data values:

$$x_1 = na_{n-1} \quad (3.7)$$

$$x_n = nb_{n-1}.$$

From an order random sample of size n , unbiased estimates b_k and a_k of α_k, β_k can be calculated as:

$$a_k = \frac{1}{n} \sum_{i=1}^n \binom{n-i}{k} x_i / \binom{n-1}{k}, \quad (k = 0, 1, \dots, n-1), \quad (3.8)$$

and

$$b_k = \frac{1}{n} \sum_{i=1}^n \binom{i-1}{k} x_i / \binom{n-1}{k}, \quad (k = 0, 1, \dots, n-1), \quad (3.9)$$

where $\binom{i}{k} = \frac{i!}{(i-k)!k!}$, $k = 0, 1, \dots, n-1$.

The a_k and b_k are related in same way as α_k and β_k respectively, and general equation is:

$$a_k = \sum_{k=1}^n (-1)^k \binom{i}{k} b_k, \quad (3.10)$$

$$b_k = \sum_{k=1}^n (-1)^k \binom{i}{k} a_k, \quad (3.11)$$

$$\begin{array}{ll}
a_0 = b_0 & b_0 = a_0 \\
a_1 = b_0 - b_1 & b_1 = a_0 - a_1 \\
a_2 = b_0 - 2b_1 + b_2 & b_2 = a_0 - 2a_1 + a_2 \\
a_3 = b_0 - 3b_1 + 3b_2 - b_3 & b_3 = a_0 - 3a_1 + 3a_2.
\end{array}$$

The above representation of a_r and b_r as order-statistics show the clear relationship between statistics and population quantiles which they estimate, and a_r, b_r are refer as sample PWMs. Sample PWM moments may be used similarly to ordinary moments for generating ICDF, but sometimes, it is difficult to generate a conventional distribution even with PWMs due to their properties. Hence, numerically defined distributions offer a better result with the combination of PWMs.

Comparison of PWMs with conventional moments

The method of PWMs is computationally more tractable than the method of maximum-likelihood or method of moments. The asymptotic standard errors of the PWM estimators, compared to maximum likelihood estimators, usually show PWMs to be more reasonably efficient. The method of moments involves the higher power of the data and sample PWMs are the linear function of the data. The PWMs are more robust than the conventional moments and less affected by the sample variability or the presence of outliers.

The biasness of probability weighted moments are calculated from small sample data involving the conventional moments. The conventional moments are computed from the Appendix A to assess the normalized bias of sample estimates. Let D_k denote the difference between the k th sample estimate of a moment (or quantile) and the exact value obtained from the parent distribution. Then the bias is defined as the average of D_k , i.e. $\sum D_k/M$, M being the number of simulation samples.

The normalized bias error in Figure 3.1 shows that the PWMs are least biased as compared to conventional moments (order = 4). Conventional moments can be used for large data, but it shows an inaccurate estimation of a sample with small size. Comparing the practicality of two methods, it concluded that some distributions for which explicit expression for the parameters

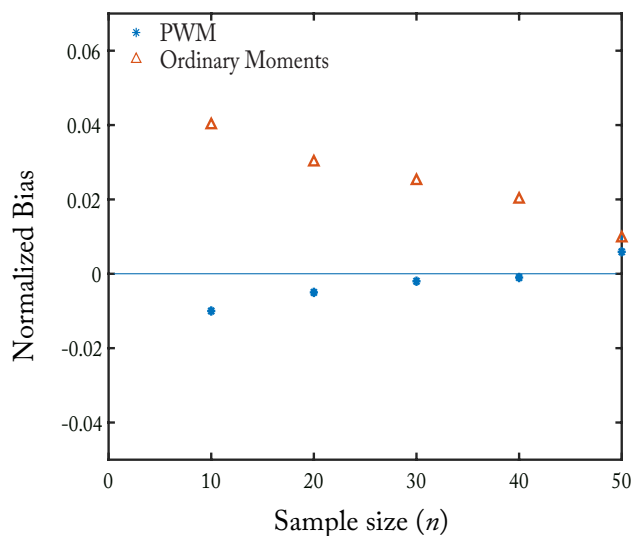


FIGURE 3.1: Normalized bias error for PWM and conventional moments

in terms of the PWMs cannot be found, either simple approximations exist which are adequate for all the practical purposes.

PWMs as moment of inverse cumulative distribution function

The PWMs can be used to generate quantile function. The main advantage of quantile function is that it can be used for continuous or discrete random variables as other distributions are mainly focused on one of the random variables. Quantiles show excellent performance for estimating the extreme tails with finite sample sizes. The quantile function of a non-negative random variable X in terms of the PWMs can be described as

$$x(F) = \sum_{r=1}^{\infty} (2r - 1)b_k P_{r-1}(F), \quad 0 < F < 1, \quad (3.12)$$

is convergent in mean square, i.e.

$$R_s(F) = x(F) - \sum_{r=1}^s (2r - 1)b_k P_{r-1}(F), \quad 0 < F < 1, \quad (3.13)$$

the remainder R_s after stopping the infinite sum after s terms, satisfies

$$\int_0^1 (R_s(F))^2 dF = 0 \text{ as } s = \infty, \quad (3.14)$$

where $dF(x)$ is a probability measure, which is a monotonic, continuous and non-negative function with the variance of X exists as

$$Var(X) = \sum_{r=2}^{\infty} (2r-1)b_k^2. \quad (3.15)$$

As the quantile functions are convenient to understand the trend of data and can be used for any sample size, these are appropriate for geotechnical problems.

3.1.2 Entropy-based quantile distribution with PWMs as constraints

The maximum entropy approach offers a definite procedure for the construction of probability distribution. The entropy is maximized under the constraints of moments and by introducing appropriate Lagrange multipliers, one seeks maximization of the functional entropy. The numerically defined maximum entropy can achieve more robustness in geotechnical parameter estimations. The maximum entropy distribution is a non-parametric approach, which means that no assumptions are needed about the shape of a random variable; there is only need to define the statistical moments as constraints. The PWM as moment estimator is consistent and asymptotically unbiased. Therefore, it is a desirable method for geotechnical problems.

The entropy of a quantile function can be written as:

$$H = - \int_0^1 [x(u) \ln x(u)] du, \quad (3.16)$$

and the available information is presented in terms of PWMs (Pandey, 2000),

$$\int_0^1 u^k x(u) du = b_k \quad (k = 0, 1, \dots, N), \quad (3.17)$$

where b_k is a sample estimate of population PWM, entropy function can be augmented as \bar{H}

$$\max \quad \bar{H} = - \int_0^1 x(u) \ln x(u) du - (\lambda_0 - 1) \left[\int_0^1 x(u) du - b_0 \right] + \sum_{i=1}^N \lambda_k x(u), \quad (3.18)$$

these equations can be represented in a simple way using Lagrangian multipliers and Newton Raphson algorithm:

$$\int_0^1 \exp(\lambda_0 + \sum_{i=1}^N \lambda_k x(u)) du = b_0, \quad (3.19)$$

Then we can find

$$\lambda_0 = -\ln \left[\int_0^1 \exp\left(\sum_{i=1}^N \lambda_k x(u)\right) du \right] + \ln b_0. \quad (3.20)$$

to derive the quantile function, entropy is maximized using the usual condition:

$$\frac{\partial \bar{H}}{\partial x(u)} = 0, \quad (3.21)$$

by using partial derivatives , we obtain:

$$\frac{\partial \lambda_0}{\partial \lambda_k} = -\frac{1}{b_0} \int_0^1 u^k \exp \left(\lambda_0 + \sum_{i=1}^N \lambda_i x(u) \right) dx, \quad (3.22)$$

since the last integral is equal to moments:

$$\frac{\partial \lambda_0}{\partial \lambda_k} = -\frac{b_k}{b_0}, \quad (3.23)$$

This is equivalent to:

$$\frac{\partial \lambda_0}{\partial \lambda_k} = -\frac{\int_0^1 u^k \exp \left(\sum_{i=1}^N \lambda_i x(u) \right) du}{\int_0^1 \exp \left(\sum_{i=1}^N \lambda_i x(u) \right) du}, \quad (3.24)$$

Substitution from equation (3.24) into (3.21) and subsequent simplification leads to the following solution (Pandey, 2000);

$$x(u) = \exp \left[-\sum_{k=0}^N \lambda_k u^k \right]. \quad (3.25)$$

Equation (3.25) has no analytic solution. The steps are programmed in commonly available commercial software packages with non-linear programming techniques.

The steps involved in the simulation experiment, shown in [Figure 3.2](#), are briefly described as follows:

1. Obtain n values from test results and arrange them in increasing order.
2. Estimate Probability-weighted moments of the data sample. Mostly four sample moments are considered for estimating the quantile function.
3. Generate probability of each quantile using Maximum Entropy method with probability weighted moment.

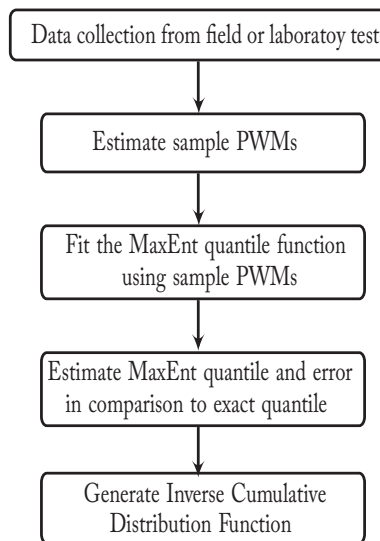


FIGURE 3.2: Algorithm for MaxEnt (QF) estimates

3.1.3 Efficiency of PWM-based quantile function

A simulation experiment was designed to estimate the bias and RMSE of quantile estimates obtained from MaxEnt quantile function against some benchmark estimates. A random sample of size n was simulated from a known distribution, e.g., Generalized Pareto and lognormal, with pre-selected parameters. From the sample, PWMs of order N were estimated and MaxEnt QF was fitted from the procedure described in previous section. The required quantile value was computed from the MaxEnt QF and benchmark distribution. The simulation was repeated M times to estimate the quantile bias and RMSE.

Consider the estimation of a Pareto quantile ($POE = 10^{-2}$) from sample size $n = 10$. In the simulation, the generalized Pareto distribution (GPD) is taken as the parent distribution with a fixed scale parameter ($d = 1.0$) and varying values of the shape parameter c , ranging from -0.4 to $+0.4$. The simulation consisted of $M = 10,000$ cycles. Four sample PWMs of order 0 to 3 ($N = 3$) were considered in generating the MaxEnt QF. Using the first two PWMs, the GPD parameters were estimated to calculate the benchmark quantile value. The variation of normalized bias with the shape parameter is compared in Figure 3.3. In general, MaxEnt QF results in slight underestimation, less than 5%, except when $c = -0.4$. For $c > 0.2$, the normalized bias of MaxEnt estimates is very close to that of the benchmark results. As expected, the MaxEnt estimates approach benchmark values as the tail heaviness of GPD decreases. The tail heaviness, in the present notation, is inversely proportional to the shape parameter, for example, a GPD with $c = -0.4$ has much heavier tail than $c = 0.1$.

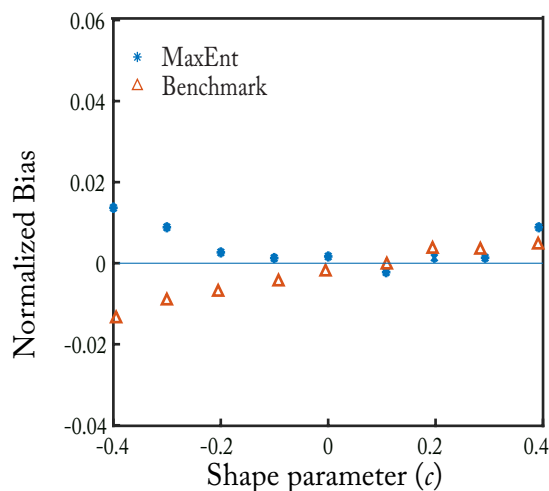


FIGURE 3.3: Normalized bias Pareto quantile with shape parameter ($POE = 10^{-2}$, $n = 10$).

In the second example, lognormal distribution is considered as parent distribution in the simulation. The objective is to estimate the lognormal quantile with probability of exceedence of ($POE = 10^{-2}$) from a sample of size 10 using four sample PWMs in the MaxEnt approach. To compute the benchmark quantile estimate, a lognormal distribution was generated using the sample mean and variance. The simulation involved 10000 cycles, and it was repeated for several COV (coefficient of variation) values of the lognormal distribution, ranging from 0.1 to 1.0. It is interesting to note from Figure 3.4 that the MaxEnt quantile RSME is within 3% for range of COV values from 0 to 0.4. However, the RMSE tends to be higher than the

benchmark estimates, especially for $COV > 0.6$. The nominal values of design loads ($COV < 0.6$) and soil properties ($COV < 0.3$) correspond to the POE of order (10^{-2}). Therefore, the proposed MaxEnt approach can provide reliable estimates of such nominal values from a very small sample that may belong to a fairly general distribution.

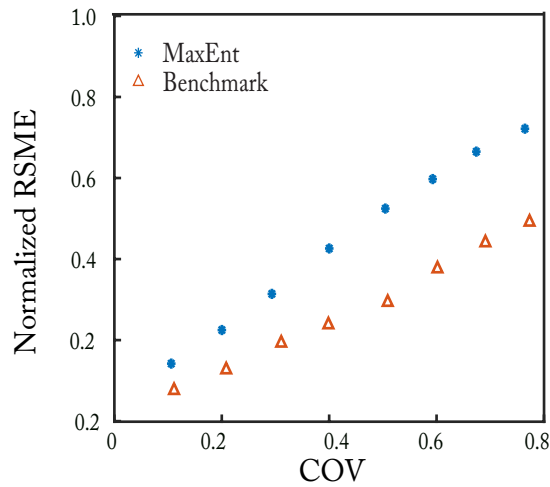


FIGURE 3.4: Lognormal quantile with different COVs ($POE = 10^{-2}$, $n = 10$).

3.2 Verification of PWM-based distribution for geotechnical parameters

The section provides the efficiency of PWM-based distribution in geotechnical engineering with comparison to other popular moments by applying them to benchmark examples. The approach is used to analyze data fulfilling the basic assumptions of stationary. The possible outcomes and limitations are discussed while applying them to a set of data. The results of these examples are compared to different approaches and literature database.

3.2.1 Example 1

The uniaxial compressive strength of rock from an open-pit slope of China (Deng et al., 2004) is used to characterize and generate probability curves. The data presented in Appendix A is analyzed with conventional and probability-weighted moments and data sample size is considered as 10 to 50.

The result of PWMs calculated from various samples is shown in Table 3.1. The sample estimation method can also be compared in terms of the efficiency, asymptotic or finite-sample, of the estimators.

TABLE 3.1: Summary of estimated probability weighted moments for different sample size

Sample	b_0	b_1	b_2	b_3	b_4	b_5	b_6
(N=10)	28.580	14.8178	10.0656	7.6433	6.1713	5.1804	4.9327
(N=20)	28.3950	14.6642	9.9294	7.5219	6.0616	5.0804	5.0103
(N=40)	28.960	14.975	10.145	7.6857	6.1913	5.1862	5.1566
(N=50)	28.942	14.9533	10.1267	7.6699	6.1781	5.1749	5.1521

When sample size is 50, the ICDF using MaxEnt and first five PWMs was derived as:

$$x(u) = \exp(-3.2022u - 0.6898u^2 + 1.260u^3 - 1.0096u^4 + 0.1749u^5). \quad (3.26)$$

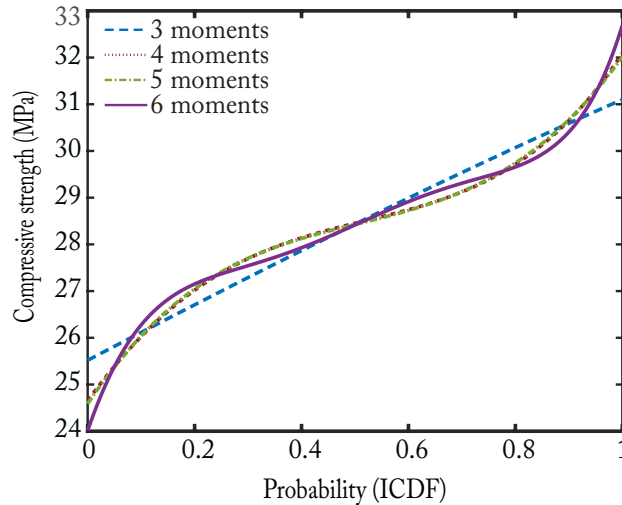


FIGURE 3.5: MaxEnt approximation compressive strength using different PWMs

Figure 3.5 represents various curves with different PWMs of 20 sample data. The estimation of ICDF from four weighted moments is showing a reliable curve. The estimation of higher order PWMs (order >8) from small samples (size < 20) is problematic and shows some errors. Hence, the four moments are appropriate with maximum entropy in estimation of geotechnical data.

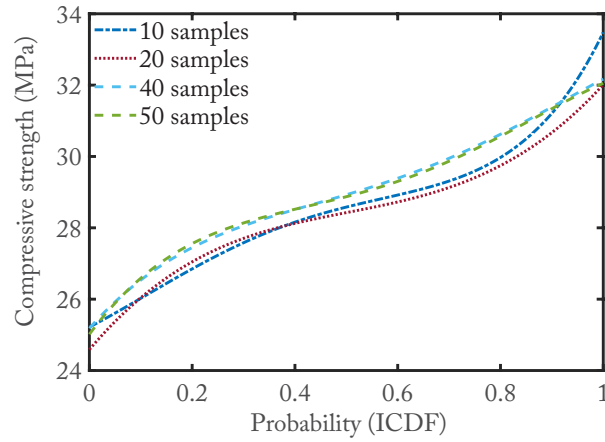


FIGURE 3.6: Comparative ICDF of rock compressive strength using different sample sizes

The measure efficiency is the area under these curves and its percentage relative error (RE) is defined as as percentage ratio of an absolute error to the specified true value of the area:

$$RE(\%) = \frac{|A_{size} - A_{K50}|}{A_{K50}} \times 100, \quad (3.27)$$

where A_{size} is the calculated area using MaxEnt method of a sample size, A_{K50} is the calculated area using Kolmogorov test of sample size 50. For convenience, A_{K50} is specified as the true area. The test is also conducted for various sample sizes for measurement of accuracy.

TABLE 3.2: Area and relative error(RE) of various sizes using ICDF and PDF

Sample size	ICDF using		PDF using	
	PWMs moments Area	RE (%)	conventional moments Area	RE (%)
50	28.9391	9.87E-03	0.9305	0.43
40	28.9421	4.32E-04	0.9324	0.64
20	28.4772	1.61	0.9445	1.93
10	28.5771	1.26	2.1416	131.14

The variation of different sample sizes are compared in Figure 3.6. Table 3.2 shows the area and RE under various sample sizes. It can be concluded that moments and sample size have significant influence on the estimation accuracy of probability curves of variability of rock properties. Probability curves derived by PWMs share more common area with Kolmogorov test than those by conventional moments. The RE by PWMs is much smaller than RE by conventional moments. The MaxEnt quantile function is very close showing the accuracy of

estimation. This shows that approach is useful in estimation of extreme quantiles like loads and material properties. The MaxEnt approach can provide a reliable result of such nominal values from a very small sample that is impossible for general distributions.

3.2.2 Example 2

The approach developed in this study is applied to characterize the site data of c' and ϕ' of fine-grained alluvial soils at the Paglia River alluvial plain in Central Italy (Di Matteo et al., 2013). The database is presented in Appendix A. In this example, three data sets with a limited number of c' and ϕ' data pairs are randomly selected from the 200 data pairs to perform probabilistic characterization based on quantile function method.

TABLE 3.3: Summary of estimated PWMs and entropy's parameter λ s.

Sample	b_0	b_1	b_2	b_3	λ_1	λ_2	λ_2	λ_4
cohesion (N=10)	19.5	11.14944	7.76122	5.944045	-1.8742	-5.3678	7.7657	-3.8037
cohesion (N=20)	19.2764	11.0187	7.77195	6.01853	-2.2323	-2.8363	3.5234	-1.7893
cohesion (N=30)	19.7839	11.1031	7.77910	6.00140	-2.4169	-2.0566	2.3161	-1.146

Quantile function of 10 data values with 4 PWMs :

$$x(u) = \exp(-1.8742u - 5.3678u^2 + 7.7657u^3 - 3.8037u^4). \quad (3.28)$$

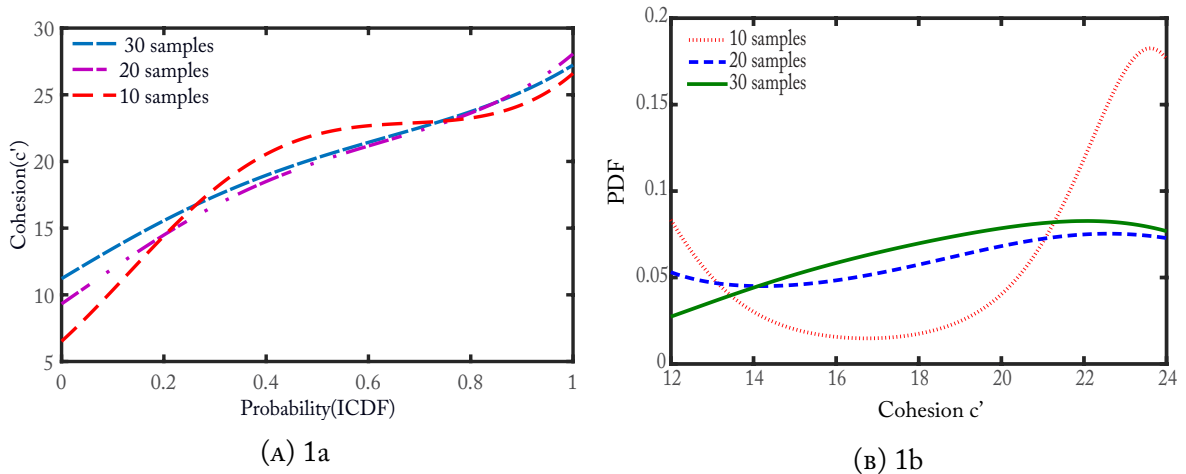


FIGURE 3.7: MaxEnt approximation of c' with ICDF and PDF

As from [Figure 3.7](#), it can be determined that even 10 sample values can generate a smooth distribution curve as compared to probability distribution using maximum entropy constrained as conventional moments. Different samples are fitted to the maximum entropy QF using the values of probability weighted moments of data. The values of moments (b_k) and entropy distribution (λ) parameter of maximum entropy quantile functions are demonstrated in [Table 3.3](#). The descriptive statistics of each data pair subgroup are very similar demonstrating that even small values can generate accurate and reliable results. The benefit of this non-parametric approach is obvious: one can evaluate the consequences of the outliers in a parametric way; moreover, it is also possible to estimate the variability of the data using the Maximum Entropy approach.

3.3 Quantile-based soil characterization of Nipigon river landslide

The process of site characterization allows the uncertainties in the determination of a geotechnical parameter or behavior of that parameter on the slope stability. After characterizing the sub-soil, actual behavior of parameter on the structure can be controlled during the construction phase. The methodology of performing measurements before construction is linked with probabilistic design to increase the knowledge and hence reduce the uncertainties. The site investigations include two parts, in-situ testing and laboratory investigation. The evaluation of shear strength in soft soils using in-situ methods (field vane shear test) is based on empirical relationships. The in-situ vane shear test is considered more convenient and economical.

3.3.1 Soil investigation of Nipigon river site

Nipigon is a municipality in Ontario, Canada. It is mostly covered with forests. In April 1990, a massive landslide occurred on the Nipigon River, north of Nipigon and encompassed an area of 101,500 square meters, estimated flow of 300,000 cubic meters of soil.

The test site is located about 9 km north of the municipality of Nipigon. The site situated near the Alexander Dam, and it is property of TransCanada Pipeline and Ontario Hydro.



FIGURE 3.8: Evidences of slope instabilities on the Nipigon river banks

The landslide affected the development of city and due to progressive failure in this area, the rehabilitation plans are becoming a challenge. A description of the Quaternary deposits along the landslide is found from the report of Trow Consulting Engineers and Lakehead University investigations. The land in this area is a glaciolacustrine plain and delta consisting of sands and silts. The sides of river are formed of fine-grained deposits with silt and sand may be found embedded in clay. The higher and steeper slopes formed at the river bank is main cause of small failures. The river banks are strongly marked by scars from erosion and landslides.

At the site, [Dodds et al. \(1993\)](#) carried out geotechnical investigations in one section at three different locations: Borehole H-1 was positioned 8 m east of the bank edge and close to Trow's Borehole H-1. Borehole H-2 was located about 165 m east of Borehole H-1 and Borehole H-3 was positioned just east of the landslide limit and TransCanada Pipelines right-of-way. At all points the following tests were carried out: field vane tests, undisturbed sampling with piston sampler, pore pressure measurements with open system piezometers and piezocone. The investigation locations are shown in [Figure 3.9](#).

In the present study, investigations were carried out at previous locations, one point close to the river (BH-1) and at one point above the crest (BH-3). From the top of slope and river bottom, the height is about 15m. The inclination of slope is about 40° . The actual landslide site is shown in [Figure 3.10](#).

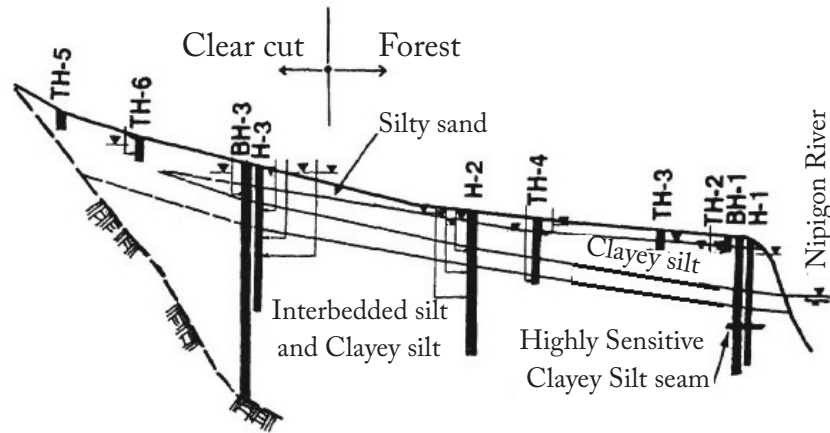


FIGURE 3.9: Geotechnical investigations in study by [Dodds et al. \(1993\)](#)

The geotechnical ground investigation methods and the laboratory tests are used to obtain the soil parameters. In this study, vane shear test and auger boring are used for field investigations with the help of civil engineering graduate students (Dhawan Joshi, Sankalp Yerra, Navjot Kanwar) under supervision of Dr. Jian Deng. The aim was to determine the shear strength and classify the soil for analysis the stability. Site investigations were carried out at three locations on the slope, borehole B1 close to the crest, borehole B2 near the crest and borehole B3 near the river. At all locations vane shear test was performed at depth of 0.22m, 0.55m and 0.85m. The soil samples were collected and used for calculation of Atterberg's limits ([Table 3.4](#)).



FIGURE 3.10: Location map of landslide and a photo from actual landslide

TABLE 3.4: Summary of laboratory testing

Sample	W_L	W_P	PI	LI	W	sand %	silt (%)	clay (%)
BH-1 0.5-1.0m	45.5	24.7	20.8	0.33	31.5	11	71	18
BH-1 1.0-1.6 m	39	18.9	20.1	0.97	38.4			

Note: W_L is liquid limit ; W_P is plastic limit ; PI is plasticity index ; LI is liquidity index ; W is water content.

3.3.2 Modeling of soil variability from vane shear test samples

The measurements from the field vane tests have been corrected with the plasticity index. An evaluation of the undrained shear strength from the field vane test has been made using the relationship described in Appendix B. The measurement data shows the need of statis-

TABLE 3.5: Vane shear strength data results (kPa)

No.	c_u (kPa)	No.	c_u (kPa)	No.	c_u (kPa)
1	55	18	70	35	42
2	35	19	72	36	71
3	47	20	35	37	102
4	39	21	44	38	100
5	38	22	59	39	55
6	56	23	72	40	42
7	57	24	85	41	32
8	52	25	70		
9	69	26	47		
10	68	27	75		
11	52	28	82		
12	35	29	69		
13	42	30	72		
14	40	31	68		
15	40	32	40		
16	62	33	42		
17	60	34	35		

tical analysis. In the presence of a significant trend of the data, proposed approach is used to demonstrate the variation in data. The distributions are generated with 10, 20, 30 and 40 values.

Soils samples are collected for the analysis of soil properties such as grain size distribution and water content are measured (shown in Table 3.4) .

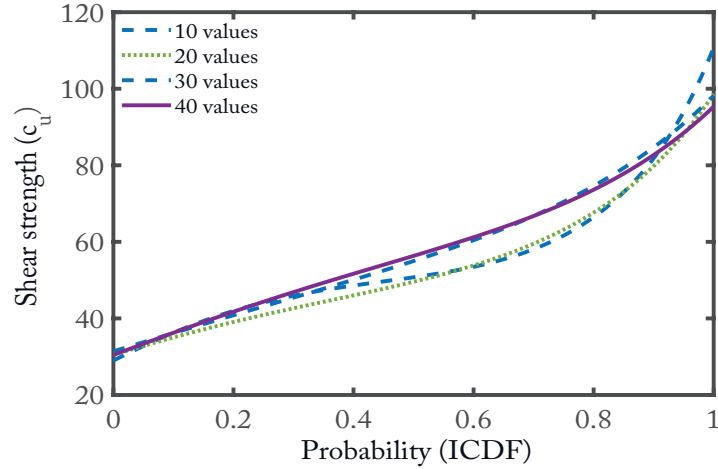


FIGURE 3.11: Input variable: shear strength

Using four probability weighted moments, the results of maximum entropy distribution for field vane shear data:

$$x(u) = \exp(-3.4160u - 1.9333u^2 + 2.0188u^3 - 1.2270u^4). \quad (3.29)$$

The efficiency of PWMs with various sample sizes can be seen in [Figure 3.11](#). It can be concluded that moments and sample size have significant influence on the estimation accuracy of probability curves of soil parameters. Apart from this case the MaxEnt quantile distribution allows a very flexible representation of shear strength parameters as random variable.

The ICDF generated with PWMs is showing high accuracy, which is influencing the reliability of this approach.

3.4 Summary

A numerical method for determining distribution free curve from both the probability weighted moments and maximum entropy approach, which governs the almost unbiased estimation, has been developed.

It is distribution-free because no classical theoretical distributions were assumed in advance. The inference result provides a universal form of probability curve. Probability curves derived by MaxEnt and PWMs are inverse cumulative density functions [ICDF] and can be accurately derived by MaxEnt and sample moments. It is concluded that proposed method

enable more secure inferences to be made from small samples about an underlying probability curve, especially when sample size is 40 or smaller.

The PWMs are compared with conventional moments and it is proved that PWM moments are more unbiased and efficient. The maximum entropy constrained with PWM makes the approach more convenient and appropriate for geotechnical data. The soil modeling of real values collected from Nipigon river bank with maximum entropy QF shows the effectiveness of the proposed algorithm. The slope stability analysis is performed on shear strength parameters evaluated from Nipigon river landslide in *Chapter 4*.

Chapter 4

Quantile-based reliability analysis of Nipigon river landslide

Section 4.1 describes the evaluation of performance function of soil slopes using response surface method. Quantile-based reliability method for complex soil slope system is developed in Section 4.2. The ordinary first-order reliability method is extended to quantile-based reliability method and applied to the slope stability analysis of the Nipigon river landslide in Section 4.3. In Section 4.4, conclusions and results are discussed.

4.1 Reliability in geotechnical engineering

The risk and safety analysis is dependent on Supply and Demand of structure. The structure can be considered as safe if Supply is more than Demand. The concept may be clear from the relationship:

$$Z = R - S \quad (4.1)$$

where Z is performance function, R is Resistance and S is Load applying on the structure.

R and S are considered as random variables in engineering design. The relationship between Load and Resistance and the probability of failure can be defined as a distribution function. The mean value is defined as the distance of the means of random variables, and the probability of failure is indicated by the overlap of the distribution function of resistance and

load. Distributions of the variables R and S further depend on appropriate parameters, for example on moment parameters μ_R, μ_S and σ_R, σ_S are mean and variance of random variables, respectively. The essential objective of reliability theory is to assess the probability of failure p_f and to find the necessary conditions for its limited magnitude.

Figure 4.1 shows an example of probability density functions of both the variables E and R and their respective location. In particular, the moment parameters (the means and standard deviations) may be considered as relative values related to the resistance mean μ_R .

$$P_f = f\left(\frac{\mu_r}{\mu_s}, \sigma_r, \sigma_s\right) \quad (4.2)$$

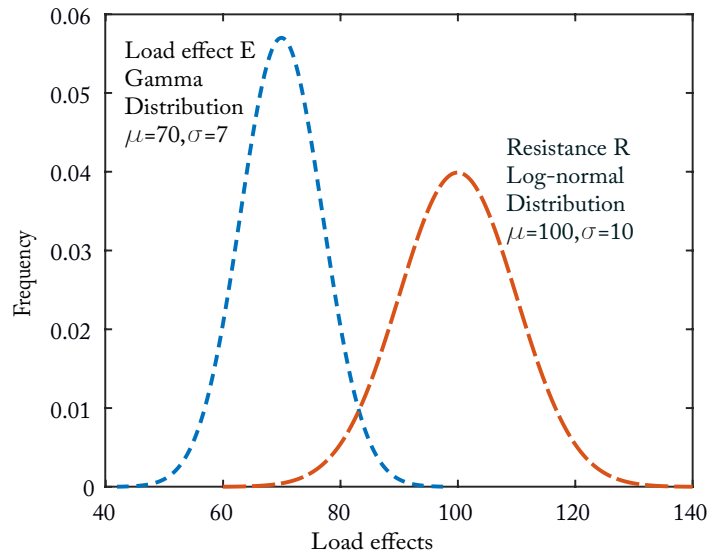


FIGURE 4.1: Relationship of reliability and probability

4.1.1 Measures of reliability for slope failure system

“Reliability is the probability of an object (variable or system) performing its required function adequately for the specified period of time under stated conditions” (Harr, 1977). The probability that slope will remain stable under specified design conditions is the reliability of slope. The design conditions include the load, water level conditions, soil properties, etc. If all the variables are assumed as normal and limit state equation is $Z = R - S$, then (Cornell,

1969):

$$\mu_Z = \mu_R - \mu_S, \quad (4.3)$$

$$\sigma_Z^2 = \sigma_R^2 + \sigma_S^2. \quad (4.4)$$

The probability of $Z < 0$ can be determined as:

$$P_f = P(Z = R - S < 0), \quad (4.5)$$

$$P_f = 1 - \phi\left(\frac{\mu_R}{\mu_S}\right), \quad (4.6)$$

and reliability index β can be represented as:

$$\beta = \left(\frac{\mu_R}{\mu_S}\right), \quad (4.7)$$

and using reliability index (β) with a probability of failure can be represented as:

$$P_f = 1 - \phi(\beta), \quad (4.8)$$

If the random variables are not normal, they should be transformed into independent normal distributed variables. If the probability density function of safety factor is normally distributed, the corresponding reliability index β is defined as:

$$\beta = (\mu_{FS} - 1)/\sigma_{FS} \quad (4.9)$$

where μ_{FS} is the mean of safety factor and σ_{FS} is the standard deviation of safety factor. If the probability density function is log-normally distributed, the reliability index of slope can be given as.

$$\beta = \ln\left(\frac{\mu_{FS}}{\sqrt{1 + \left(\frac{\sigma_{FS}}{\mu_{FS}}\right)^2}}\right) / \sqrt{\ln(1 + (\sigma_{FS}/\mu_{FS})^2) - 1} \quad (4.10)$$

The system is considered as safe if probability of failure is less than 10^{-2} . This is the foundation of *risk-based concept*. With this approach, the information of density function of each data is

usually difficult to obtain, but this methodology is more efficient than deterministic analysis.

4.1.2 Performance function for slope analysis

The safety of slopes is dependent on specific performance criteria, the relevant random variables like load and resistance parameters and the functional relationships among them corresponding to each performance criterion (Haldar & Mahadevan, 2000). The performance function can be described as

$$Z = g(X_1, X_2, X_3 \dots X_n). \quad (4.11)$$

The limit state function can be defined $Z = 0$. The area above the limit state is considered as safe and below the limit state is unsafe. The limit state equation plays an important role in structural reliability. The performance function may be explicit or implicit, complicated or straightforward. The reliability analysis methods have been developed corresponding to different types of complexities in performance function. The probability of failure, p_f in the terms of the performance function can be described as:

$$p_f = \int_{g(X) < 0} \dots \int f_X (X_1 X_2 X_3 \dots X_n), dx_1 dx_2 \dots dx_n \quad (4.12)$$

in which $f_X (X_1, X_2 X_3 \dots X_n)$ is a joint probability density function for the basic random variables $X_1, X_2, X_3 \dots X_n$ and the integration is performed over the failure region, that is $g() < 0$.

In slope probabilistic analysis, Chowdhury et al. (2009) defined the limit state of performance function as following:

$$F(x_1, x_2, \dots, x_n) - 1 = 0 \quad (4.13)$$

or

$$\ln F(x_1, x_2, \dots, x_n) = 0 \quad (4.14)$$

where, $F(x_1, x_2, \dots, x_n)$ is the safety factor function about $x_i (i = 1, 2, 3 \dots n)$. The performance function of slope stability may be in different form, depending on method or parameters and it is mostly implicit function. Figure 4.2 shows the graphical representation of reliability

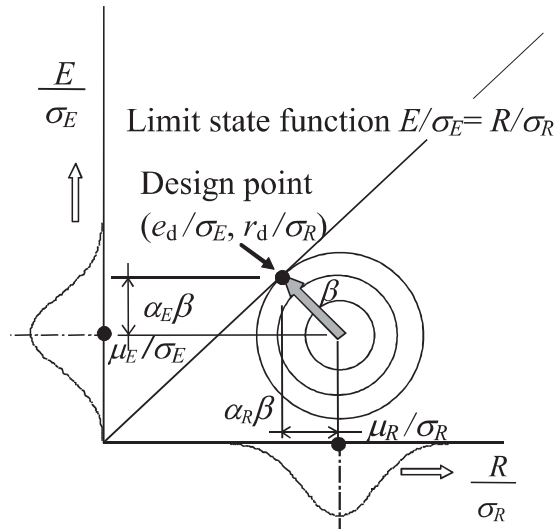


FIGURE 4.2: Limit state function and design point (Holick, 2009)

analysis with effects of variations and partial derivatives. It depends on the first two statistical moments of distribution of parameters. The safe and unsafe region boundaries are defined as the distance of design points from the limit state function. A linear limit state function with a few random variables can be solved with *Mean Value First Order Second Moment (MVFOSM)* or *First Order Reliability Method (FORM)*, but a nonlinear limit state function requires more complex reliability analysis. A nonlinear implicit function can be analyzed by response surface. As slope stability analysis is a complex problem and contain various random variables, it shows a need of additional framework to understand the system behaviour.

Response surface for evaluation of performance function

Response surface method (RSM) is used in the study to improve the results by converting implicit performance function into explicit. The response surface method consists of experimental results and response surface analysis. The multivariate polynomial models arise in the calculations of experimental results.

The process includes the generation of an explicit function from series of experiments, usually called runs, by changing input variables and considering the effect of output response. It improves the quality of information and eliminates the information of unused data. The primary goal of this method is to analyze the performance function with sufficient information to precisely estimate model parameter. The first order model is sufficient in our study to evaluate the reliability.

A simple model of a performance indicator with two controlled factors cohesion and friction angle can be represented as (Li et al., 2015):

$$FOS_j(X) = a_j + \sum_{i=1}^{N_c} (b_{i,j} * c'_i) + \sum_{i=1}^{N_c} (c_{i,j} * \varphi'_i), \quad (4.15)$$

where $FOS_j(X)$, $j = 1, 2, \dots, N_c$, is the safety factor for the j th potential slip surface; $X = (x_1, x_2, \dots, x_n)^T$ is the vector of input random variables in the physical space, in which n is the number of input random variables; $a_j = (a_{1,j}, b_{1,j}, \dots, b_{n,j}, c_{1,j}, \dots, c_{n,j})^T$ is the vector of unknown coefficients with a size of $N_c = 2n + 1$. Response surface analysis aims to interpolate the available data in order to predict the correlation locally or globally between variables and objectives. The term c'_i and φ'_i are considered as input random variables. If there is a curvature in the data, a first-order model would show a significant error. Polynomial models are generalized to any number of predictor variables x_i ($i = 1, 2, 3, \dots, N_c$). This design is used to fit first order response surface method.

The factor of safety for the i th potential slip surface is evaluated from N_c as $(\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_n})$, $(\mu_{x_1} \pm k\sigma_{x_n}, \mu_{x_2}, \dots, \mu_{x_n})$, $(\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_n} \pm k\sigma_{x_n})$ where k is a coefficient for generating the sampling points, and $k = 1.65$ is used. μ_{x_n} and σ_{x_n} are the mean and standard deviation of the i th random variable as shown in Appendix C. The response surfaces are constructed as procedure explained in Figure 4.3 and used to generate explicit functions with regression analysis.

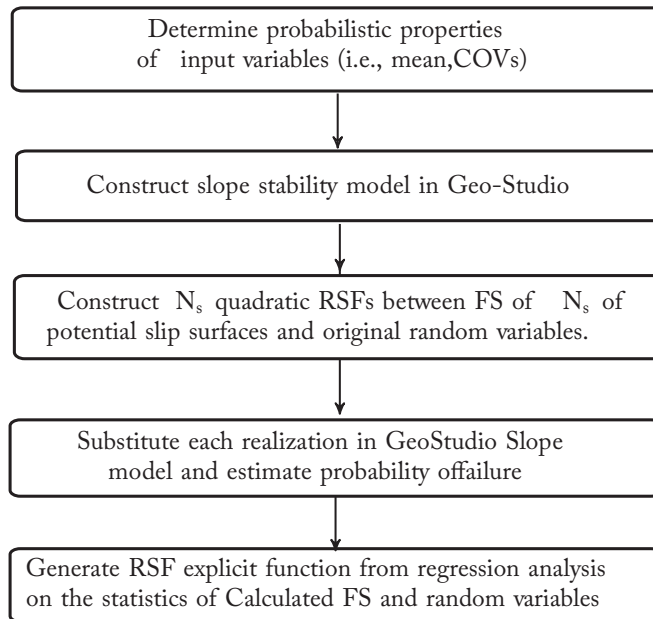


FIGURE 4.3: The process of response surface method

4.2 Computation of failure probabilities with quantile-based method

The FORM method is a great approach to reliability analysis. It is based on the Taylor Series expansion of the safety factor or the performance function at the critical points on the failure surface. This method provides analytical approximations for the mean and standard deviations of the probability of failure based on the variables.

The iteration procedure of the FORM method can be described in following steps (Holick, 2009):

1. Consider a multivariate case of slope stability system when basic variables are described by a vector $X[X_1, X_2, \dots, X_n]$,

$$G(X) = g(X_1, X_2, \dots, X_n) = 0 \quad (4.16)$$

- the basic variables X are transformed into a space of standardized normal variables U , and the performance function $G(X) = 0$ transformed into $G'(U) = 0$;

$$U = \frac{(X - \mu_x^e)}{\sigma_x^e} \quad (4.17)$$

in which μ and σ are respectively mean and standard deviation of random variable X . According to Rosenblatt's transformation, equivalent mean and standard deviation for non-normal variables are calculable as follows:

$$\sigma_x^e = \frac{1}{f_x(x)} \phi[\Phi^{-1} F_X(x)] \quad (4.18)$$

$$\mu_x^e = x - \sigma_x^e \Phi^{-1}[F_X(x)] \quad (4.19)$$

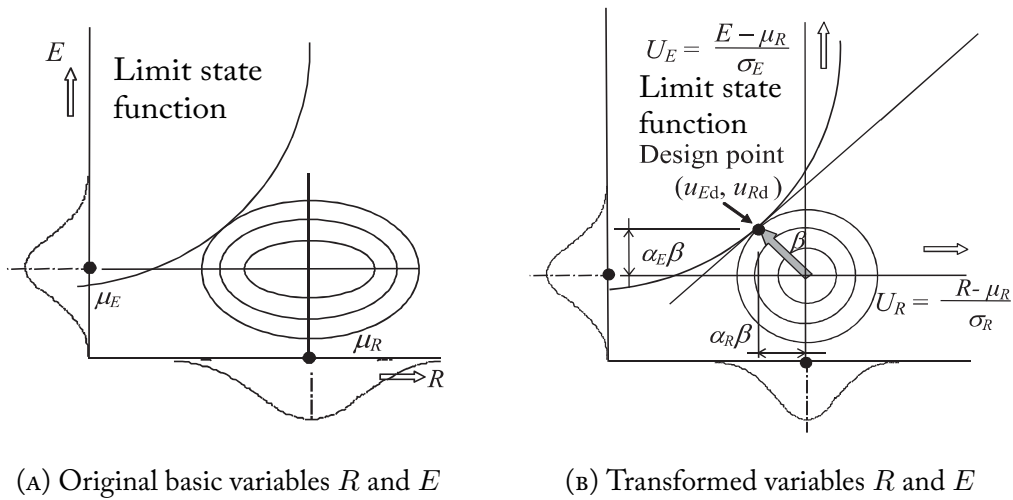


FIGURE 4.4: Transformation of non-normal random variable (Holick, 2009).

In plan view, the probability density function can be visualized as a contour plot involving a series of ellipses, and the limit state function can be seen as a line separating the failure and safe regions, see Figure 4.4.

- the failure surface $G'(U) = 0$ is approximated at a chosen given point by a tangent hyperplane (using Taylor expansion);

4. the reliability index β is determined as the distance of the design point from the origin and then the failure probability, P_f , is given as $P_f = \phi(-\beta)$;

First order reliability method with quantile function

A quantile-based reliability method for the direct iterative calculation of nonlinear function under the condition of correlated variables is more convenient and time-saving method.

Let n basic random variables affecting structural reliability be x_1, x_2, \dots, x_n obey the general distribution and distribution function is denoted as $F_{x_i}(x_i)$, the correlation coefficient between x_i and x_j is ρ_{ij} . The limit state equation for slope system is still Equation 4.16. The basic variable reduced by maximum entropy quantile estimation:

$$\left. \begin{aligned} \phi(\beta_{x_i}) &= F_{x_i}(x_i) \\ x_i &= F_x^{-1}(\phi(\beta_{x_i})) \end{aligned} \right\}, \quad (4.20)$$

where $\phi(\cdot)$ is the standard normal distribution function and F_x^{-1} is the inverse cumulative distribution function. In this way, limit state equation becomes

$$G = g(F_{x_1}^{-1}(\phi(\beta_{x_1})), F_{x_2}^{-1}(\phi(\beta_{x_2})), \dots, F_{x_n}^{-1}(\phi(\beta_{x_n}))) = 0, \quad (4.21)$$

iterative formula for reliability index β obtained as:

$$\beta^* = \frac{-\sum_{i=1}^n \left. \frac{\partial g}{\partial \beta_{x_i}} \right|_{P^*} \beta_{x_i}^* + g(F_{x_1}^{-1}(\phi(\beta_{x_1})), F_{x_2}^{-1}(\phi(\beta_{x_2})), \dots, F_{x_n}^{-1}(\phi(\beta_{x_n})))}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \left(\left. \frac{\partial g}{\partial \beta_i} \frac{\partial g}{\partial \beta_j} \right) \Big|_P^*}}, \quad (4.22)$$

$$\beta_{x_i}^* = \frac{-\sum_{j=1}^n \rho_{ij} \left(\left. \frac{\partial g}{\partial \beta_i} \right) \Big|_{P^*}}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} \left(\left. \frac{\partial g}{\partial \beta_i} \frac{\partial g}{\partial \beta_j} \right) \Big|_P^*}} \beta^* \quad (4.23)$$

available from limit state equation (4.21)

$$\left. \frac{\partial g}{\partial \beta_i} \right|_{P^*} = \left(\left. \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial \beta_i} \right) \Big|_{P^*} = \left(\left. \frac{\partial g}{\partial x_i} \right) \Big|_{P^*} x_i'^*, \quad (4.24)$$

where x_i^{*} is the basic random variable in the calculation of quantile derivative.

The quantile method for reliability index β is iteratively calculated using equation (4.25) and (4.26):

$$\beta^* = \frac{-\sum_{i=1}^n \left. \frac{\partial g_1}{\partial \beta_i} \right|_{P^*} x_i^{*} \beta_{x_i}^* + g(x_1^*, x_2^*, x_3^*, \dots, x_n^*)}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} x_i^{*} x_j^{*} \left(\left. \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} \right) \right|_P^*}}, \quad (4.25)$$

$$\beta_{x_i}^* = \frac{-\sum_{i=1}^n \left. \frac{\partial g_1}{\partial \beta_i} \right|_{P^*} \rho_{ij} x_j^{*}}{\sqrt{\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} x_i^{*} x_j^{*} \left(\left. \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} \right) \right|_P^*}} \beta^*, \quad (4.26)$$

$$x_i^* = F_{x_i}^{-1}(\Phi(\beta_{x_i}^*)), \quad (4.27)$$

The *partial reliability index* of the fundamental variables in the ultimate state equation of the structure can be obtained by inverse transformation of the *reduced gaussian variable* of each basic variable, and the corresponding *design value* is obtained.

$$x_i^* = \frac{dF_{x_i}^{-1}(\beta_{x_i}^*)}{d\beta_{x_i}^*}, \quad (4.28)$$

The equations are used to calculate the basic operational formulae of the basic variables *partial reliability index* and *design value* in the design of the limit state of the structure, and the iterative method is often applied in calculating the above formulas. The steps are described in Figure 4.5.

4.2.1 Calculations example

The limit state equation is $g(x_1, x_2) = 1 + x_1 x_2 - x_2 = 0$, the random variable $x_1 = LN(2, 0.4)$, $x_2 = LN(4, 0.8)$, correlation coefficient $\rho_{12} = -0.1$ (Bin & Songhong, 2004).

The solution results and comparison with the FORM method are shown in Table 4.1

TABLE 4.1: Calculated results of reliability index for case 1

Method	x_1^*	x_2^*	β_{x_1}	β_{x_2}	Reliability index β
FORM	0.809	5.245	-	-	4.585
Quantile-FORM	0.810	5.264	-4.471	1.641	4.585

The calculation can be performed directly in the original space, avoiding the complicated process of finding the feature vector and the matrix operation (especially in the case of many

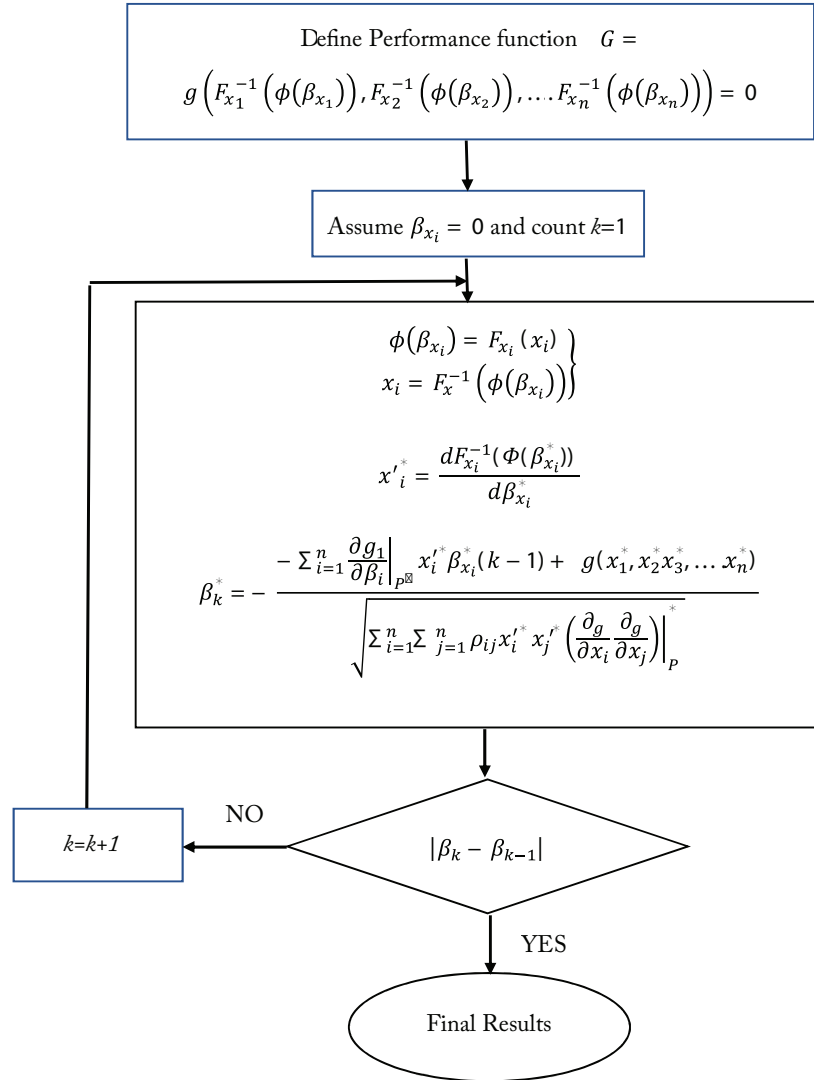


FIGURE 4.5: Algorithm of first order reliability method with quantiles

variables). It is very difficult to find the eigenvalues in simple FORM, which makes this calculation process simple.

4.3 Case studies using quantile-based method

The research provided an efficient reliability-based method to derive the reliability of structures, quantile-FORM, which may be applicable in engineering practice to evaluate the reliability of geotechnical structures. Although quantile-FORM is a proven reliability-based method, applicability of this method with deterministic slope analysis must be verified. The application of quantile FORM, which represent the variability of soil properties in slope system, is shown in various problems of geotechnical engineering. In order to be confident it

delivers correct results, it is checked against Monte Carlo simulations (MCS) and Point Estimate Method (PEM).

4.3.1 Homogeneous soil slope

The first case with the study of stability analysis of a homogeneous soil slope is analyzed in GeoSlope 2007 with limit equilibrium method. The basic geometry of this benchmark is shown in Figure 4.6. A slope with a height of $H = 10m$ and a slope angle of 2 : 1 is considered. The soil strength parameters cohesion c , friction angle φ and unit weight γ are considered as independent random variables and it is assumed that mean values (μ_x) of input parameters are known as shown in Table 4.2. The soil type is assumed as uniform in the whole region and water table is considered as negligible. To verify the model performs as expected, initially a

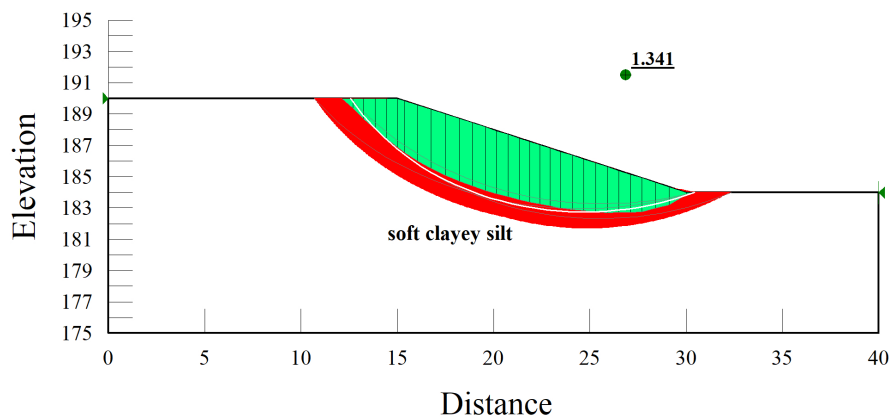


FIGURE 4.6: Geometry slope stability problem case 1

deterministic calculation with mean values is performed. Then, input parameters are assumed to be normally distributed in the first case for reliability analysis. Subsequently, the parameter variation feature is utilized to perform the proposed approach with the random variables. Typical coefficients of variation (COV) can be obtained from literature, soil investigation or design standards. The standard deviation can be calculated with the use of the coefficient of variation provided in Appendix (A).

The response surface method is used to make the implicit performance function to explicit based on the assumed variables. The regression analysis was performed based on least square error approach. The parameters are considered as uncorrelated. The lower limit ($\mu + 1.65\sigma$)

TABLE 4.2: Soil properties for homogeneous soil slope

Property	Symbol	Unit	Mean(μ_X)	COV (X)
Unit Weight	γ	$[kN/m^3]$	19	0.2
Friction Angle	φ	$[^\circ]$	12	0.25
cohesion	c	$[kPa]$	9	0.2

and upper limit ($\mu + 1.65\sigma$) of the variables are considered to quantify each point in design sets (see Appendix C).

The performance function can be defined as

$$G(x) = FOS - 1, \quad (4.29)$$

and the approximations of performance function evaluated from response surface is

$$G(x) = 0.617615 - 0.03285 * \gamma + 0.062458 * c + 0.059697 * \varphi - 1. \quad (4.30)$$

The reliability index (β) of the slope is evaluated for a constant COV value with proposed approach. The procedure will be elaborated in detail for constant values. The structures are considered as safe if the factor of safety is more than unity. The factor of safety is evaluated by quantile-based FORM method and compared with other methods. It is possible to generate the distribution function of probability of failure from statistical values, presented in Table 4.3.

TABLE 4.3: Calculated results of reliability index for case 1

Method	γ	c	φ	β_γ	β_c	β_φ	β
Quantile-FORM	21.139	8.087	9.576	0.563	-0.807	-0.507	1.107

The results of quantile-based reliability analysis were obtained in three iterations and the design points of random variables are too near as compared to other methods. The partial factors of limit state function helps in calculation of design points of FORM.

Considering the results of quantile-FORM method, one can see that accuracy and feasibility of quantile-FORM is more than other methods. It also overcomes the convergence problem by decreasing the number of iteration. The reliability index from PEM, FORM and

TABLE 4.4: Reliability comparison between the three probabilistic methods for case 1

Method	μ_{FS}	σ_{FS}	β	$P_f(\%)$
Quantile-FORM	1.258	0.308	1.1077	13.4036
FORM	1.147	0.334	1.107512	10.79
MCS (20000)	1.3658	0.261	1.437	5.4
PEM	1.211	0.286	1.2375	10.79

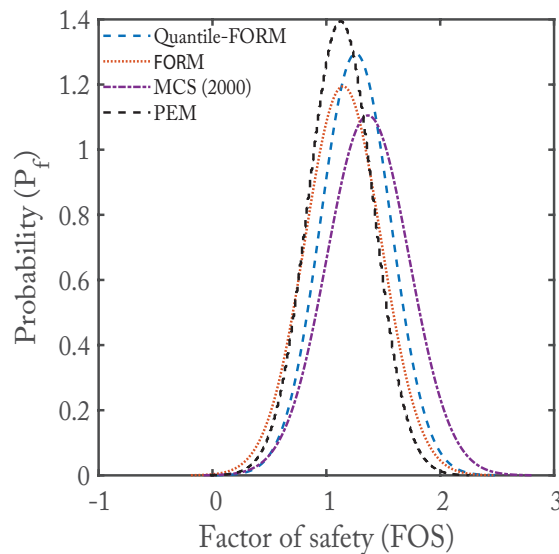


FIGURE 4.7: Probability density function for homogeneous soil layer problem

Monte Carlo simulation are compared in Table 4.4 and Figure 4.7. Significant difference in reliability values from the different methods could be explained by the different underlying approach of each method. In all methods, reliability index is calculated as the distance between mean and the threshold of standard deviations. Monte Carlo performs simulations with different combination of mean random variables; FORM works in a physical space of input stochastic variables, where it iteratively searches the design point. The efficiency and accuracy of FORM is dependent on optimization algorithm. The quantile-based FORM is efficient and easy algorithm for geotechnical problems.

4.3.2 Reliability analysis of Nipigon river landslide

The study of slope stability is based on a combination of limit equilibrium method and quantile-based reliability approach. The first situation, Initial state, is the original slope failure. It considers the variation in soil parameters like shear strength and slope geometry. The second situation, Influence of water table, is the computation of factor of safety with the fluctuation of

the water table. When it freeze or rain, the water is absorbed by the upper layer leading to the loss of soil strength and affecting the failure criteria. The increase of water content decreases the shear strength of cohesive soils. The third situation, Influence of river currents, is effective with the increase in pore water pressure due to the rising of water or downstream water flow.

Initial conditions

The purpose of analyzing the original slope is to determine the shear strength and failure of slope considering the data available from site investigations. The deterministic model is prepared in Geo-studio 2007 with the shear strength values explained in Chapter 3. The slope is considered in its natural and undisturbed state. The stability analysis is based on the influence of soil variability on various layers. The variation of shear strength parameters in layer II to IV is assumed from the literature. The calculations are compared with different approaches including different correlation functions describing the variability of slope reliability. The sensitivity analysis is also conducted for this problem to understand the contributions of the various sources of variability to the failure probability of slope system.

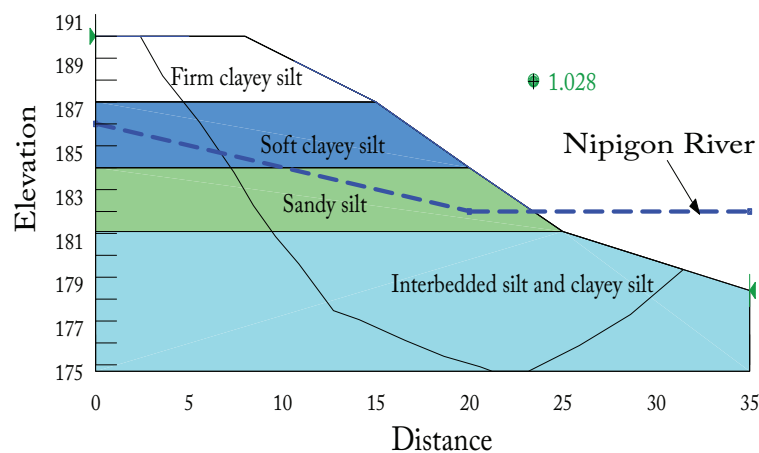


FIGURE 4.8: Geometry slope stability problem: Nipigon river landslide

Parametric studies

The aim of this section is to investigate the effects of the coefficient of variation and spatial variability of soil properties on the estimated probability of failure p_f of a slope. The information was obtained from three boreholes, 40 vane shear test results and Trow's investigation to

model their layer compositions in a geotechnical model. The model has been used as the base for statistical simulations. Four main soil types were derived from this data set summarized in Table 4.5.

TABLE 4.5: Soil parameters

	Layer	c' in kPa (μ)	φ' in degree (μ)	γ (μ)
I	Firm clayey silt	54	-	19
II	soft clayey silt	36	-	19
III	sandy silt	-	32	19
IV	interbedded silt and clayey silt	28	-	19

Soil type I consists of firm clayey silt, Soil type II consists of soft clayey silt, soil type III consists of sandy silt, soil type IV consists of interbedded silt and clayey silt. The measurement data is combined with literature review to describe variability of subsoil by combination of measurements and expert judgment.

Sensitivity analysis of initial situation

Since many input parameters are involved in this problem, influential parameters must be identified. To ensure no unnecessary parameters are included in reliability analysis, a sensitivity analysis is performed to identify which parameters have a major influence on the results. The global sensitivity analysis is investigated by using response surface approach. By using the response surface approach, it is possible to derive the global sensitivity measures analytically. The results of sensitivity are obtained with properties of different layers and water table. Furthermore it is mentioned that the unit weight of all layers is kept constant. All other soil properties are included in the sensitivity analysis. The lower and upper bound of analysis is defined as values below and above standard deviation. In order to determine the standard deviation, coefficients of variation ($COV = \sigma/\mu$) is assumed from the Appendix A. One can see the effect of increasing and decreasing parameters. Combining these results with parametric studies, it is possible to analyze the system behaviour of slope stability by considering main parameters.

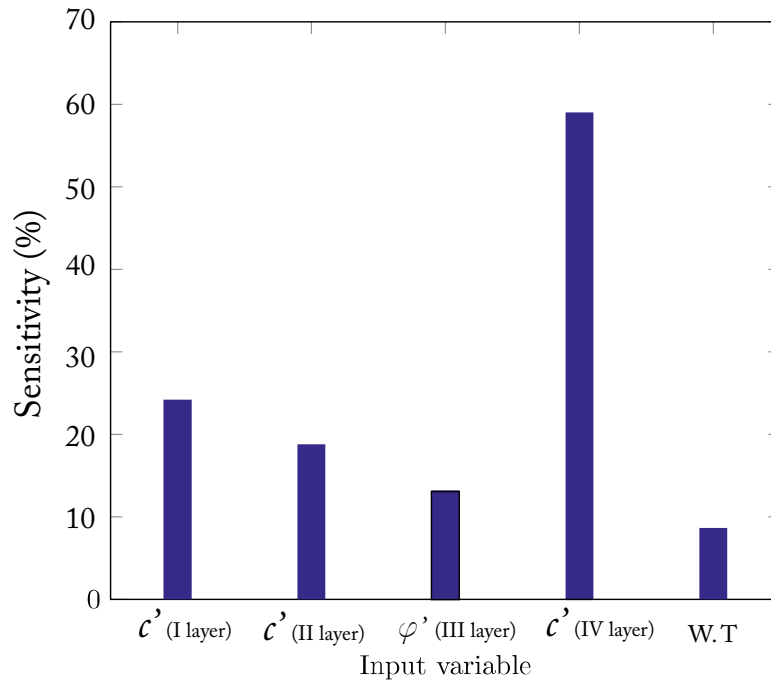


FIGURE 4.9: Sensitivity analysis with various variables

In order to determine the sensitivity scores all possible combinations are calculated, while changing only one variable at a time. One soil property may have a major influence when a certain criterion is considered, and a negligible influence at the other. The results shows the effect of weighting parameters on slope stability and factor of safety. The sensitivity of soil properties on the factor of safety computed by changing five properties. The results of this analysis are shown in Figure 4.9. It can be seen that most sensitive variables are the soil cohesion (c), soil friction angle(φ) and water table or piezometric line. From this analysis, it is concluded that these variables should be assumed stochastic when the factor of safety is evaluated. Here, the response surface is a problem-specific model that replaces the numerical model (i.e., software package) for computing the sensitivities. The response surface method is further used to create an explicit performance function for reliability analysis.

Reliability analysis using various random variables

First of all, shear strength parameters are considered as main uncertainty. In addition to this, the un-drained shear strength of top and bottom layer with friction angle of sandy layer is considered as ICDF. The reliability index (β) of this slope stability problem is evaluated for

a range of COV values. The procedure will be elaborated in detail for a constant $COV = 0.30$ here. The COV is defined as 0.1 to 0.3 for reliability analysis.

Assuming the factor of safety follows a normal distribution, the reliability index (β_{normal}) is calculated and compared with different COV s and probabilistic approaches. Figure 4.10

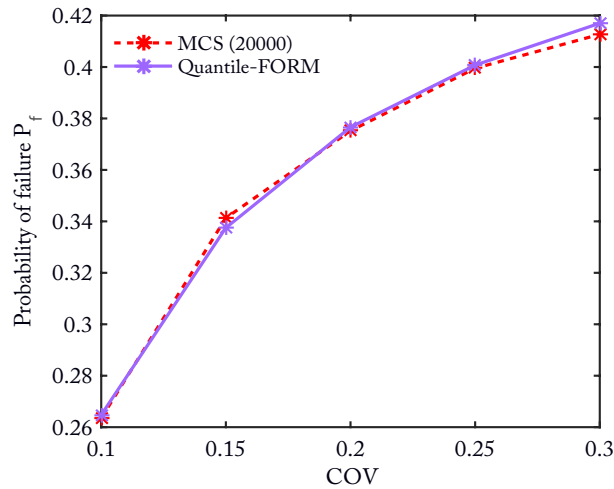


FIGURE 4.10: Probability of failure for various COV s with comparison of MC simulations

shows the result of a range of COV values. As the COV increases the probability of failure increases and reliability index decreases. It can be seen that the reliability indices obtained with the quantile-FORM are in good agreement with Monte Carlo simulations with a small and acceptable error. The main advantage of quantile-based FORM on MCS is that, it is applicable for 40-50 samples but MCS requires minimum 200 samples for accurate results. In order to prevent the overestimation of the overall safety, it is important to choose the distribution of probability of failure appropriately.

For quantile based FORM method, considering now the safety factor of slope as a function of correlated soil parameters with different layers. This illustrates that there is a significant difference. The results are shown in Table 4.6, the correlation coefficient is assumed from 0.95–

TABLE 4.6: Reliability comparison between the three probabilistic methods

ρ	μ_{FS}	σ_{FS}	β	P_f
0.95	1.0552	0.0806	0.6813	0.2478
0.90	1.0487	0.0789	0.6961	0.2432
0.85	1.0449	0.0772	0.711	0.2385
0.80	1.0408	0.0756	0.711	0.2338

0.8. The standard deviation of safety factor decreases up to around 0.19 and the probability of failure decreases from 24% to 22%.

Comparison and discussion

The influence of the soil properties on the calculated reliability index using three probabilistic methods is shown in Table 4.7. One can see that generally the results of quantile-FORM is closer to MCS and FORM. It means that accuracy of FORM increases with quantile function and it is suitable for complex geotechnical problems with more input variables and correlations.

TABLE 4.7: Comparison between the three probabilistic methods for Nipigon river landslide

Methods	Quantile function				Normal distributed				Correlated quantile functions			
	μ_{FS}	σ_{FS}	β	P_f	μ_{FS}	σ_{FS}	β	P_f	μ_{FS}	σ_{FS}	β	P_f
Quantile	1.02	0.25	0.39	0.34	1.10	0.21	0.40	0.34	1.04	0.07	0.71	0.23
FORM	1.10	0.14	0.68	0.24	1.09	0.11	0.62	0.26	1.10	0.10	0.92	0.17
MCS	1.05	0.14	0.34	0.36	1.05	0.15	0.33	0.36	1.08	0.11	0.69	0.24

4.3.3 Influence of water table and river currents

The influence of water content and pore water pressure is a problem concerning the undrained shear strength of soils. Thus, the change in pore water pressure changes the principal stresses and results in variation in consolidation and compressibility of soils. The pore water pressure from the running water between layers can affect the slope stability analysis. This section includes the effect of variation in water content and pore water pressure

As the water content increases in soil due to rain or snow, it profoundly affects the slope stability. Initially, the slope's water content is 30%, but it may increase to 40 – 45%. The undrained shear strength decreases with an increase of water content. Shear parameters are assumed for analysis based on variation of water content and pore pressure parameters. The water level of the river is also assumed as a random variable to estimate the effect of river currents. Table 4.8 shows the variation of safety factor as a function of water content and river currents.

TABLE 4.8: Results of slope stability analysis

Ground water conditions	Factor of safety	Probability of failure
water draw down by 2m	1.227	0.34
water draw down by 4 m	1.15	0.21
water at the level of river currents	1.029	0.13

When the water table was at 2m depth, the safety factor obtained 1.227 with a failure probability of 34% and 13.4% when the water table was at the same level of river currents. The objective of this scenario was to identify the influence of water table which results in probability of failure more than 5% given the uncertainties in soil thicknesses and properties.

4.4 Summary

In this chapter, a quantile-based method is introduced to apply in the slope reliability analysis problem. Its stability and efficiency are compared with the MCS and FORM method to evaluate the performance function of the Nipigon river slope.

The first case study is showing the example of a single layer slope stability. An analytical solution is adapted to quantify the effects of soil variability with global and sensitivity analyses of input parameters. It is concluded that Monte Carlo and quantile-based method show similar results, which proves that the proposed method performs well for slope stability analysis. The disadvantage of Monte Carlo is that the output distribution must be known with 200 samples, in contrast to quantile-based can be used with 30 or 40 samples.

The second case study presents reliability analysis for Nipigon river landslides. The uncertainty of the complex geotechnical conditions is quantified. By using Geo-studio with stochastic approaches, these results are calculated, which offers another insight into the effect of soil heterogeneity and the resulting risk. This study quantifies the influence of water as the probability of failure increases about 10% when water currents and water table effects are included. As from the results, it indicates that the area of Nipigon is prone to landslides and requires more attention.

Chapter 5

Remedial measures and slope design for Nipigon river landslide

By taking the failure probability of soil slopes as main concern, remedial methods for slope stability are discussed in Section 5.1. Slope design effected by soil parameters and system reliability index are investigated in Section 5.2. Failure probability and slope design parameters are considered, a new height and slope angle of Nipigon river landslide is calculated from combination of quantile-based method and optimization.

5.1 Remedial measures for soil failure

There are several considerations in case of landslides. First of all, the variation in soil properties and geological data make each design a different scenario. Second, the slope stability mechanism is same for different type of slopes. Third, the most reliable stability analysis is main factor affecting the slope design. Judgment, experience, and intuition, combined with the best data-gathering and analytical techniques, all contribute to optimal solution.

The landslides are most difficult to detect and costly in construction. It is clearly shown that erosion will be a cause for concern in the coming century, when it comes to slope stability along river banks. Practically this means that preventive measures will likely have to be undertaken to prevent this type of failure from undermining the slope stability. These measures consisted of placing geo-synthetics, rocks and other reinforcements along the bottom of river stretching

hundreds of meters in both north and south direction. The possibility and approach to bottom erosion protection might be to sort of the effects of main landslide. There are some remedies for Nipigon river banks.

1. **Vegetation** slope stability through hydro logical and mechanical effects of vegetation is the most economical in this case. Hydrological effects involve the removal of soil water by evapotranspiration through vegetation thus reducing the soil weight, which can lead to an increase in soil suction or reduction in pore water pressure, hence an increase in the shear strength (Ali et al., 2012). The plant root matrix also effect the shear strength of root as deep roots compact the the soil layers contributing the ability of soil to resist the shear stress.

TABLE 5.1: The increase of factor of safety with vegetation

Depth of roots (m)	cohesion $cR(kPa)$	Factor of Safety
0.5	60	1.301
	70	1.454
	80	1.532
1	60	1.308
	70	1.487
	80	1.521
1.5	60	1.43
	70	1.59
	80	1.616
2	60	1.589
	70	1.650
	80	1.692

The effects of vegetation can be computed with conventional slope stability analysis. In the limit equilibrium method, the shear strength of soil by incorporating the impact of the root matrix, the Mohr-Columb equation becomes:

$$\tau = (c' + cR) + (\sigma - \mu)\tan\phi', \quad (5.1)$$

where cR is apparent cohesion. The apparent soil cohesion caused by the plant root matrix system is added to initial cohesion. The method can be applied by increasing the cohesion of the first layer as with a variation of $0 - 20kPa$. The effect of the vegetation

layer to Nipigon bank is applied by increasing the cohesion and calculating the safety factor as shown in [Table 5.1](#). The maximum depth of root zone is assumed as $2m$. The results show relatively increment of safety factor.

2. **Surface drainage** is one of the possible remedies for correction of existing landslides. Drainage helps in both reducing the weight of mass and increasing the strength of soil material. The design of slopes also considers the movement of water on the surface. Surface water rises the soil erosion and creates the chances for failure of slopes as surface water flow across the face of the cut slope and will seep into the soil at the head of the cut ([Abramson et al., 2002](#)).

There are numerous slope treatments to promote rapid runoff and improve slope stability. Some of the measures are (i) using concrete slope paving or rock fills, (ii) providing trenches or drainage ditches. The asphalt paving is also useful in highway embankments. Surface drainage measures require minimal design and offer more protection to slopes.

3. **Subsurface drainage** is considered as an expensive treatment but sometimes beneficial. Since seepage forces act to increase the driving force on a landslide, the control of subsurface water is of vital importance. Sometimes, groundwater constitutes the most essential single contributory cause for the majority of landslides. The design of slopes is also dependent on pore water pressure, and it varies according to the climatic and geological conditions.
4. **Buttress or counterweight fills** is an external load applied to slopes to resist the pressure of soils. The ability of any restraining structure to perform as a designed stabilizing mass is a function of the resistance of the structure to (a) overturning, (b) sliding at or below its base, and (c) shearing internally. An overturning analysis is performed by treating the buttress as a gravity structure and resolving the force system to ensure the proper location of the resultant. Potential sliding at or below the base requires a similar analysis, and care must be taken in both the design and the construction phases to ensure adequate depth for founding the buttress and prescribed quality for the layer on which the buttress is placed. These designs are also suitable for high river currents.

5. **Chemical treatment** is used sometimes for treating the soil with different methodologies. The use of chemicals for treating the clay minerals along the plane of potential movements is a useful technique. Chemicals help in ion-exchange and increase the shear strength of soil. The treatment depends on the soil type, mineralogy and prevailing groundwater conditions in the slide mass. Most of the chemical treatments include lime, chemical grout, and potassium injections. A large volume of lime and cement grout are mostly expensive ([Indraratna et al., 2015](#)).
6. **Electrokinetic applications** involves electrolysis reactions, electroosmotic flow, electrophoresis can be used to change the properties of soil. The process is mainly dependent on the electric energy, and this energy helps in the treatment of soil by exchanging ions. This technique causes migration of pore water between previously placed electrodes; the loss of pore water, in turn, causes consolidation of the soil and a subsequent increase in shear strength.
7. **Combination of treatments** is also a practical approach in the stabilization of soil slopes, mainly for landslide prone area. The long-term stability of this treatment requires that the horizontal drains function correctly for the life of the structure. A slow-moving landslide can be treated chemically. The geotechnical design may not consider entirely safe if all the problems, (soil strength, groundwater level, slope geometrics) are not handled properly. Thus, a combination of several methods will generally be required.

TABLE 5.2: Summary of slope remedies

Procedure	Best Application	Limitation	Remarks
Drain Surface	In any design scheme; also be part of any remedial design	Will only correct surface infiltration or seepage due to surface infiltration	Slope vegetation should be considered in all cases
Drain subsurface	On any slope where lowering of groundwater table will effect or aid slope stability	Cannot be used effectively when sliding mass is impervious	Stability analysis should include consideration of seepage forces
Reduce weight	At any existing or potential slide	Requires lightweight materials that are costly	Stability analysis must be performed to ensure proper use and placement area of lightweight materials
Use buttress and counterweight fills	At an existing slide, in combination with other methods	May not be effective on deep-seated slide	Stability analysis is required to determine soil-pile force system for safe design
Install anchors	A good option for highway design	Involves depth control based on ability of foundation soils to resist shear forces from anchor tension	Study must be made of in-situ soil shear strength
Treat chemically	Where sliding surface is well defined and soil reacts positively to treatment	has not had long-term effectiveness evaluated	Laboratory study of soil chemical treatment must precede field installation
Use electroosmosis	To relieve excess pore pressures at desirable construction rate	Requires constant direct current power supply and maintenance	Methods are experimental and costly

5.2 Probabilistic design for Nipigon river landslide

The uncertainties in soil parameters and adopted models can lead to uncertainties in decision making for design. In geotechnical engineering, the least cost and reliable design are acceptable and known as optimal design. In this approach, safety requirements are usually accepted concerning the target factor of safety or reliability index. In this situation, a theory of fully probabilistic design applies to cope with the uncertainties. The simplified reliability-based design is dependent on the accuracy of statistical data. However, it is not possible in case of soil parameters due to its randomness.

The RBD approach in geotechnical engineering has been gaining attention due to its simplicity and high accuracy. This simplified method is usually adopted for piles, foundations and retaining walls (Phoon et al., 2003; Salgado & Kim, 2013). The simplified approach can be directly used with various reliability methods. The reliability-based robust design approach is originated from the structural engineering as an alternative to conventional designs. This design seeks an optimal design that is robust against the parameter uncertainties and satisfies the safety and economic requirements.

5.2.1 Probabilistic design framework of soil slopes

In the existing design approach, the robustness is increased by decreasing the standard deviation of a probability of failure. After the computation of system failure probability using the quantile-based FORM, the optimization algorithm is used to minimize the failure and locate the optimal design considering the safety requirements and cost-effectiveness. The focus of this research is to introduce the concept of robustness and an application to the existing example of the Nipigon River landslide.

Optimization theory

The optimization is introduced to find values of the design variables corresponding to a minimum in the costs. Such optimization is done if the shape and properties of a slope have been defined, but the dimensions still have to be designed. Here, optimization is used with reliability approaches to find the cost-effectiveness and input variables. This simplified

approach seeks an optimal design, represented by a set of design parameters (d), such that the design robustness $R(d, \theta)$ and cost $C(d)$ are optimized simultaneously, while the design (safety) constraint based on the system response $g(d, \theta)$ is satisfied. The performance function of a slope system is used in this theory.

Find d to optimize: $[C(d), R(d, \theta)]$

Subject to: Safety constraint as a function

where d – design parameters; θ – uncertain parameter; C – cost; R – robustness measure; g – system response.

In general, uncertainties in a design process can be classified in *epistemic uncertainties*. A probability distribution describes this uncertainty and can be reduced by more information. Examples of epistemic uncertainties, model error, and errors due to numerical approaches to find a solution. When a probabilistic approach is used to find a solution to a design problem, the values of all uncertain parameters are described by a probability distribution. The overall assumption in this method is that it is accurate to describe every uncertain parameter by a probability distribution if the uncertainty in this distribution is taken into account.

Optimization theory with reliability constraint

The optimization with a constraint as probability of failure of slope system can be defined as:

Find	x	
Minizing	$f(x)$	
such that	$P[g_i(x, \theta) \leq 0] \leq P_i$	$i = 1, 2, \dots, p$
in which:		
x	is the vector of design parameters	
θ	is the vector of uncertain parameters	
$f(x)$	is the objective function	
g_i	is the set of inequality constraints	
P_i	is the allowed probability of failure of constraint g_i	

In this optimization task, the constraint can also be expressed by a target reliability β_{target} or P_{target} so that the constraint becomes:

$$\beta_i(x) \geq i_{target} \quad i = 1, 2, \dots, p \quad (5.2)$$

in which the reliability index can be determined as described below for every limit state function $g_i(x, \theta)$

1. Perform a single-objective optimization with respect to each objective function of concern, $f(d)$, using the quantile-based FORM. This optimization will provide an optimal design with respect to safety requirements. By repeating this single-objective optimization for each design with different values of parameter, a number of designs $[f_1(d), f_2(d), f_3(d)]$ can be identified in the design pool.

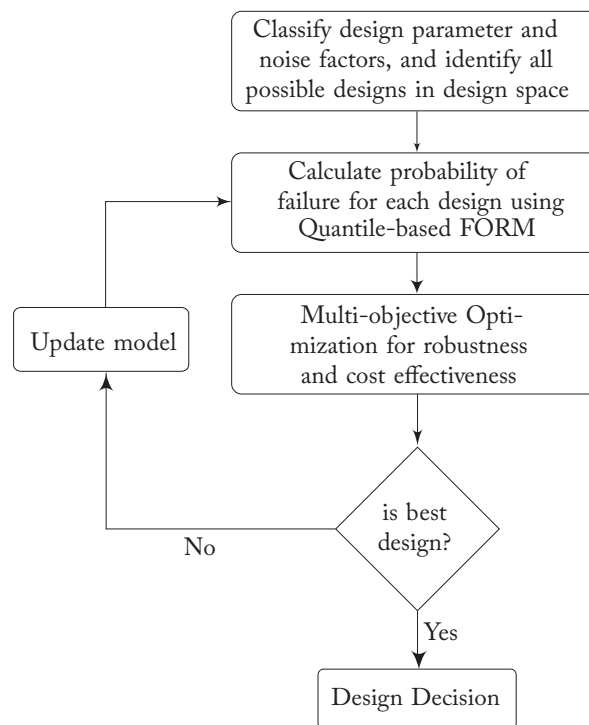


FIGURE 5.1: Reliability-based design for slope

2. Determine the corresponding maximum value of each objective function among all designs
3. Normalize the objective functions into values ranging from 0 to 1 using transformation.
4. Compute the distance from the normalized point to the normalized objective functions for each design in the design pool. The design that meets the safety requirements and cost effectiveness is an acceptable design.

The algorithm is represented in [Figure 5.1](#) and further implemented in slope design problem.

5.2.2 Problem description, design parameters, design space

The design for the massive landslide of Nipigon river is a challenging task. The possibility to design a slope with a robust height will be determined using RBD approach. This approach is based on the optimization of cost and reliability analysis. The focus is on finding the advantages of robust design with the quantile-based approach. The total cost is the function of investment costs and risk, can be represented as:

$$C = I_o + I'(h - h_0) + \frac{P_f D}{r' - g}, \quad (5.3)$$

$$I = I_0 + I'(h - h_0),$$

where I_o is fixed cost; I' is cost per meter heightening; h is the height; h_0 is the initial height; P_f is the probability of failure; D is the damage given failure; r' is the annual growth; This expression for C can be used to find an optimum in the cost. The cost is mainly the soil excavated or filled.

The proposed approach is applied to evaluate the system failure probability $P_{f,sys}$ of the design scenario. The results are calculated by converting P_f into β . In this study, the target reliability index are taken as 4 to 5 with failure probabilities of, $P_f = 10^{-3}$ and 10^{-4} . The slope height H and angle θ is taken as design points. The soil is considered as homogeneous in design and parameters are ranging from 23-28 for θ and 9 – 11m for height.

The number of potential slip surfaces varies from 6000- 8000, due to variation of design parameters for different height and angle. The soil is assumed as single layer with undrained shear strength of 40 *kPa* and *COV* is 0.2. Based on the mean values the deterministic FOS calculated as 1.104, the possible θ values vary from 26° to 28° and the possible height H is ranging from 10m to 11m.

The approach provides 45 designs from which 12 designs are accepted as most safe and economic. As in the [Figure 5.2](#), the design with blue dots are providing $P_f = 10^{-5}$ with high construction cost but the green and red dots are optimal points for design and cost effectiveness. The height of slope is showing more effect on reliability index than slope angle. The optimal design is demonstrated with $\beta = 4$ or $P_f = 10^{-3}$ as $H = 10m$. The reduction in statistical

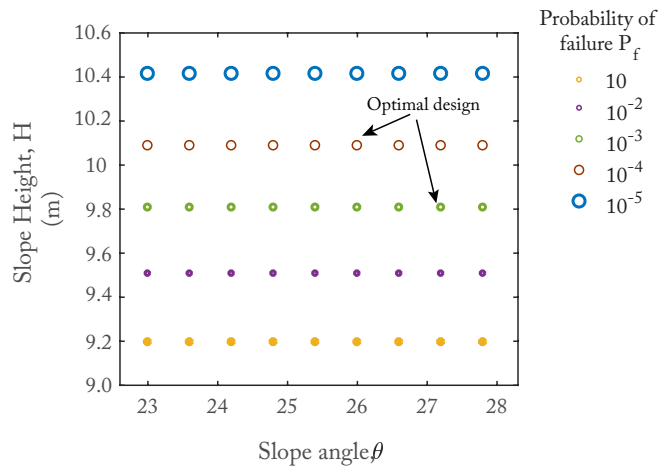


FIGURE 5.2: Probability failure of each design

uncertainty using the developed approach is important from an engineering point of view since more economical geotechnical designs can be achieved by reducing uncertainties in soil parameters

5.3 Summary

The chapter introduced a probabilistic design with remedial measures. A careful attention must be given to the protection of earth slopes. Protection may be in the form of retaining wall or gravity wall with designed hydraulic features to ensure dissipation of the destructive forces of the anticipated flow. One should never assume that the soil slope adjacent to river currents is adequate until slope is protected for the long term effects of water.

The probabilistic design based on combination of quantile-based FORM and multi-objective optimization is more convenient and does not require more sophisticated models. The design allows deterministic model and uncertainty analysis in a parallel fashion. The Nipigon river slope is designed with analysis of large number of varying parameters. The high computational efficiency provided by the reliability analysis method satisfies engineering requirements in practice and can significantly enhance the application of probabilistic design in soil slopes.

Chapter 6

Conclusion and further recommendation

In this thesis the quantile-based distribution is generated for soil variables and used for reliability-based design of Nipigon river landslide using field vane shear results. Conclusions and recommendations based on the research done in this thesis are listed below.

6.1 Contributions

Quantile-based distribution in geotechnical engineering

This section summarizes the conclusions of chapter 3.

- The proposed framework of uncertainty quantification offers an unbiased mathematical approach for quantifying the variation in shear strength parameters of soil from a limited number of site-specific data. It provides a logical route to determine characteristic values when extensive testing cannot be performed, which is a difficulty for majority of geotechnical projects. Since these moments can be estimated correctly for small-sample observations and a perfect distribution can be generated. The proposed approach effectively tackles the difficulty in obtaining meaningful information from a variety of soil samples collected from in-situ or laboratory tests.
- In the study of probabilistic soil modeling, different samples are well presented with the proposed approach. Its unique distribution appears appropriate for various sample sizes and variance. Examining the probability weighted moments with normalized

bias, we can see that PWMs are most unbiased and error is nearly about 5% which is negotiable. One would anticipate that the accuracy of maximum entropy distribution would increase with the PWMs as constraints. It is confirmed in most examples, which presumably represents the feasible region for the maximum entropy based quantile distribution constrained by the probability weighted moments.

- Probabilistic assessments are made to study the relative influence of variability on soil parameters. Soil properties are evaluated using the proposed probabilistic approach with vane shear data results. ICDF is generated to characterize soil variability of soil parameters. Lastly, the potential of the framework of uncertainty quantification, a case study on the Nipigon river landslide shows the effect of soil variability at different scales for the random input parameters.

Quantile-based FORM reliability analysis for slope stability

The quantification of the effects of soil variability is probably one of the most important issues in geotechnical design. By using proposed probabilistic method in geotechnical problem, the recommendation for the most reliable way to calculate the probability of failure is an easy task.

- From the results presented in chapter 4, it can be concluded that the methodology provides a rational and robust way of assessing the reliability of slopes from small sample data. Sensitivity analyses quantify the contribution of each uncertain variable and help the engineer simulate effects of random variables, which allows simplifying the calculation. The performance function is available in an implicit form, and it is replaced with response surface approach.
- Quantile-FORM uses gradients to estimate the variance of the factor of safety at a selected expansion point, and closed form solution of the partial derivatives is used to calculate the performance indicator of the slope. The method performed well in comparison to FORM and Monte Carlo simulations when estimating the reliability of slope failures.

- The first example is the homogeneous slope and the second is the Nipigon river slope with multiple layers. The finding of this research warrant the following conclusions: (a) the sample size does not affect the model and the analysis can be performed efficiently with small samples where MCM approach need minimum 200 samples for accurate results. (b) For the homogeneous slope, a good agreement is observed in the calculated reliability index (β) for FORM and quantile-FORM but little bit variation with MCM. (c) For the non-homogeneous slope, the calculated reliability index (β) based on the two used methods is in good agreement for non-correlated analysis, but it shows some difference in the correlated analysis.

Reliability-based design for slope stability

Probabilistic-design based on combination of quantile-based FORM and multi-objective optimization is more convenient and does not require more sophisticated models. The design allows deterministic model and uncertainty analysis in a parallel fashion. The Nipigon river slope is designed with analysis of large number of varying parameters. The high computational efficiency provided by the reliability analysis method satisfies engineering requirements in practice and can significantly enhance the application of probabilistic design in soil slopes.

The following conclusions are drawn from the results of the study on the efficient geotechnical design:

- The developed design is demonstrated to be effective and intuitive. Higher variation of the performance function signals lower design efficiency, which implies a higher degree of uncertainty.
- The evaluation of design and system performance requirement, share common computational steps, as both can be analyzed using quantile-based method. Thus, the computational efficiency is greatly improved over other existing reliability-based design approaches.
- the design using proposed approach has been shown effective, which allows for consideration of some reduction in the variation of random variables within the framework of

the reliability-based design, can yield more cost-efficient designs while improving the design robustness. It must be mentioned that further research is required for a generalized conclusion.

6.2 Further research

Many issues could be recommended, based on the outcomes of this research, and considering the stage of development of slope stability analysis research. The main recommendations are presented in this section.

As an extension, the slope stability analysis should be extended with the probabilistic approach proposed in this research to increase the feasibility and efficiency. This approach can be used for any type of slope may be man-made or natural. Additional studies should be conducted to gain more knowledge and experience in the field of stochastic quantification of soil properties. Moreover, the quality of the probability weighted moments within the framework of uncertainty quantification and reliability based design is an important task and it can be extended by fractional probability weighted moments in modeling of soil data.

Nipigon river banks are most critical for a prone landslide and require more research. In this research only shear strength parameters are considered due to time limitation. There are some other factors affecting the area such as snow, permeability, temperature and time effect. In future studies, the uncertainties of these factors can be considered in slope probabilistic analysis based on quantile function.

An interesting part in the context of economic design is the optimization of a geotechnical structure including uncertainties. Reliability-based design optimization should include different failure modes of complex structures incorporating the variability of loads and resistance forces properly.

This probabilistic approach can be used to carried out the whole landslide risk management, while vulnerability can be conveniently expressed as a conditional probability. The research require all input parameters as random variable to achieve a better result.

The quantile-based approach may be applied to calibration of load and resistance factors in the Load and Resistance Factor Design (LRFD) method. In this problem, the objective

would be to reduce the effect of the variability of geotechnical parameters on the calibrated resistance factors. However, more efforts on calibrating the model uncertainty in such cases and incorporating it into reliability-based design of soil slopes are still need of hour.

Appendix A

A.1 Probability weighted moments: a basic definition and statistical background

The comparison of probability weighted moments with other methods is presented in Table (A.1).

TABLE A.1: Moment Expression

Distributions	Conventional Moment ($M_{l,o,o}(integer\ l > 0)$)	Probability Weighted Moments ($M_{l,0,n}(real\ j, k > 0)$)
Normal	$E[(Y - E(Y))^n]$	$b_k = \frac{1}{m} \sum \binom{i-1}{k} x_i / \binom{m-1}{k},$ ($k = 0, 1, \dots, m$)
Weibull	$\sum_{n=0}^{\infty} \frac{t^n \lambda^n}{n!} \Gamma(1 + n/k)$	$b_k = \frac{m}{1+k} + \frac{a\Gamma(1+1/b)}{(1+k)^{1+1/b}}$
Gumbel	$d^x [e^{\theta m} \Lambda(1 - a\theta)] / d\theta^I$	$b_k = \frac{m}{1+j} + \frac{a \ln(1+1/j) + \epsilon}{(1+k)^{1+1/b}}$

TABLE A.2: Various distributions for a random variable

Distribution	Probability distribution Function (PDF)	Cumulative Distribution Function (CDF)	Inverse Cumulative Distribution Function (ICDF)
Normal	$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$F(x) = \int_{-\infty}^x \frac{e^{-x^2/2}}{\sqrt{2\pi}}$	$h(x) = \frac{\phi(x)}{\Phi(-x)}$
Log-normal Distribution	$f(x) = \frac{e^{-\frac{(\ln((x-\theta)/m))^2/(2\sigma^2)}{(x-\theta)\sigma\sqrt{2\pi}}}}{x} \quad x > \theta; m, \sigma > 0$	$F(x) = \Phi\left(\frac{\ln(x)}{\sigma}\right) \quad x \geq 0; \sigma > 0$	$G(p) = \exp(\sigma\Phi^{-1}(p)) \quad 0 \leq p < 1; \sigma > 0$
Weibull	$f(x) = \frac{\gamma}{\alpha} \left(\frac{x-\mu}{\alpha}\right)^{\gamma-1} \exp\left(-\left(\frac{x-\mu}{\alpha}\right)^\gamma\right) \quad x \geq \mu; \gamma, \alpha > 0$	$F(x) = 1 - e^{-(x^\gamma)} \quad x \geq 0; \gamma > 0$	$Z(p) = (-\ln(p))^{1/\gamma} \quad 0 \leq p < 1; \gamma > 0$
Gamma	$f(x) = \frac{(\frac{x-\mu}{\beta})^{\gamma-1} \exp(-\frac{x-\mu}{\beta})}{\beta\Gamma(\gamma)} \quad x \geq \mu; \gamma, \beta > 0$	$F(x) = \frac{\Gamma_x(\gamma)}{\Gamma(\gamma)} \quad x \geq 0; \gamma > 0$	$S(x) = 1 - \frac{\Gamma_x(\gamma)}{\Gamma(\gamma)} \quad x \geq 0; \gamma > 0$
Extreme Value Type I	$f(x) = \frac{1}{\beta} e^{-\frac{x-\mu}{\beta}} e^{-e^{-\frac{x-\mu}{\beta}}}$	$F(x) = 1 - e^{-e^{-x}}$	$Z(p) = \ln(\ln(\frac{1}{p}))$

TABLE A.3: Approximate guidelines for coefficients of variation of soil parameter from [Phoon & Kulhawy \(1999b\)](#)

Design Property	Test	Soil Type	Point COV[%]	spatial avg. COV[%]
$s_u(UC)$	Direct(lab)	Clay	20-55	10-40
$s_u(UU)$	Direct(lab)	Clay	10-35	7-25
$s_u(field)$	VST	Clay	15-50	15-50
$s_u(field)$	VST	Clay	15-50	15-50
ψ	Direct (Lab)	Clay, Sand	7-20	6-20
$\psi(CV)$	PI	Clay	15-20	15-20
K_O	Direct(SBPMT)	Clay	20-45	15-45
K_O	Direct(SBPMT)	Sand	20-45	35-50
K_O	KD	clay	35-45	35-50
K_O	N	clay	40-75	-
E_D	N	Silt	40-60	35-55
E_D	Direct(DMT)	Sand	15-70	10-70

Note: s_u = undrained shear strength

UU = unconsolidated -undrained triaxial compression test

CIUC = consolidated isotropic undrained triaxial compression test

$s_u(field)$ = corrected shear strength from vane shear test

K_o = in-situ horizontal stress coefficient

N = blow counts in a standard penetration test

KD = dilatometer horizontal stress index

PI = plasticity index

E_D = dilatometer modulus

VST = Vane shear test

q_T = correlated cone trip resistance

TABLE A.4: Sample observations of uniaxial compressive strength(MPa) example 1(data from [Deng et al. \(2004\)](#))

No.	Data	No.	Data	No.	Data
1	29	18	28	35	29.6
2	30.4	19	29.8	36	27.3
3	31.7	20	28	37	28.2
4	26	21	30.1	38	29.5
5	28.2	22	27.1	39	31.3
6	27.9	23	28.3	40	30.6
7	26.5	24	32.4	41	29.4
8	27.4	25	29.6	42	31
9	29.9	26	30.9	43	27.6
10	28.8	27	30.6	44	28.2
11	29.1	28	29.2	45	25.7
12	30.2	29	30.7	46	28.5
13	25.2	30	28.3	47	28.7
14	28.4	31	31.5	48	31.3
15	26.8	32	26.4	49	29.4
16	29.3	33	31.1	50	28.9
17	27.3	34	27.8		

TABLE A.5: Data pairs of c' and ϕ' used in case study chapter 3 (data from [Di Matteo et al. \(2013\)](#))

No.	Case I (10 data)		Case II (20 data)		Case III (30 data)	
	c' (kPa)	ϕ'	c' (kPa)	ϕ'	c' (kPa)	ϕ'
1	23.56	25.99	15.02	28.53	16.3	27.2
2	22.82	26.41	24.51	25.6	17.5	26.59
3	18.9	27.3	27.03	25.4	15.73	28.09
4	21.75	26.81	19.44	28.2	23.56	26.41
5	12.05	29.6	16.67	26.79	21.17	26.41
6	25.05	25.08	10.53	28.9	24.97	25.39
7	22.29	25.48	17.75	27.51	15.27	28.61
8	16.51	27.61	21.63	25.79	22.45	26.81
9	21.38	26.51	20.97	26.6	24.31	25.39
10	10.69	28.79	15.07	28.1	14.28	29.11
11			14.2	29	21.63	25.9
12			20.8	26.89	14.94	28.29
13			22.82	26.41	15.52	28
14			25.96	25.8	23.98	26.09
15			22.33	25.6	13.29	28.5
16			19.27	26.78	27.03	25.2
17			11.23	29.61	19.52	26.4
18			14.61	28.8	23.56	25.99
19			22.29	25.48	20.18	27.5
20			23.4	26.2	19.11	26.5
21					24.43	25.29
22					19.03	26.91
23					20.76	26.69
24					17.91	27.3
25					10.12	29.3
26					25.05	25.08
27					24.64	25.6
28					19.56	26.4
29					17	28.21
30					20.72	27.1

Appendix B

B.1 Measurement data of vane shear test

For in-situ applications, the shear vane is the most reliable and readily-available device for measuring the undrained shear strength of cohesive soils (Knappett, 2012). It has been used extensively for the analysis of shear strength in soils. Vane shear equipment consists of two twin vanes perpendicular to each connected to a solid pushing rod. The test comprises inserting the vane to the required depth and rotating about the vertical axis which allows the soil to shear. Hand-held shear vanes are compact, portable devices that can be easily carried into the field and used *in-situ* as shown in Figure B.1.

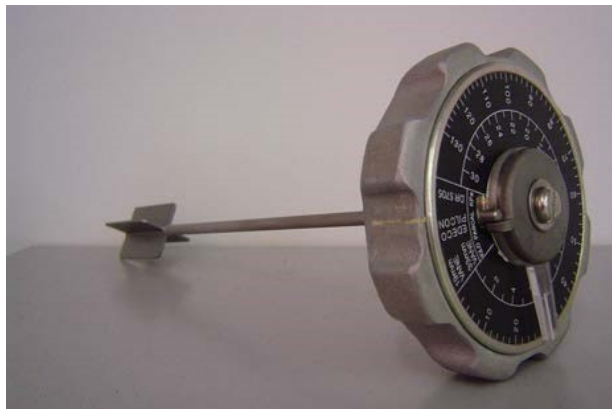


FIGURE B.1: A shear vane tester equipped with a 33mm diameter vane.

The test is conducted at the base of borehole. Since the test is carried out relatively fast, undrained conditions can be assumed and hence the shear stress at failure is the same as the undrained shear strength, c_u . The maximum torque T required to rotate the vane shear blades

and cause failure could be expressed as (Ameratunga et al., 2016):

$$T = M_{\text{top}} + M_{\text{base}} + M_{\text{side}}, \quad (\text{B.1})$$

where M_{top} is resisting moment at the top of the blades/cylinder; M_{base} is resisting moment at the base of the blades/cylinder and M_{side} is resisting moment at the sides of the cylinder.

The empirical equation for calculating the undrained shear strength c_u can be calculated by taking moments about the shaft axis:

$$T = \left[\frac{D^2 H}{2} + \frac{D^3}{6} \right] \times \pi c_u, \quad (\text{B.2})$$

when H/D ratio is 2 and then the Equation B.2 can become as

$$c_u = T / ((7/6)\pi D^3), \quad (\text{B.3})$$

where height (H) and depth (D) of the vane is different for different types of soil. Commercial shear vane testers are typically come calibrated for their standard size vanes so that c_u can be recorded directly in the field. The 33mm and 19mm diameter vanes are used in this research.

The shear vane is only recommended for use in cohesive soils, but that does not mean that the sample must be composed entirely of clay and silt, though, as fine sediments with relatively low clay contents can classified as cohesive. The presence of other types of material in the sediment, such as fine plant roots, is not as problematic as their tensile strength contributes to the shear strength of the sample.

The site investigations always include some errors. Bjerrum (1972) suggested some correction factors (λ) for shear strength $c_{u_{FV}}$ derived from field vane shear test from the relationship of plasticity index and mobilized shear strength as:

$$\lambda = 1.18 - 0.0107(\text{PI}) + 0.0000513(\text{PI})^2 \leq 1. \quad (\text{B.4})$$

The correction factor is taken as 0.985 to reduce the errors in test results.

Appendix C

C.1 Response surface method and sensitivity analysis

TABLE C.1: Different random variables design points adopted for developing the response surface

Design Point Scenario	c	ϕ	γ
1	μ	μ	μ
2	$\mu + 1.65\sigma$	μ	μ
3	$\mu - 1.65\sigma$	μ	μ
4	μ	$\mu + 1.65\sigma$	μ
5	μ	$\mu - 1.65\sigma$	μ
6	μ	μ	$\mu + 1.65\sigma$
7	μ	μ	$\mu - \sigma$

Note: μ = mean of a noise factor

$\mu + \sigma$ = mean plus one standard deviation of a noise factor

$\mu - \sigma$ = mean minus one standard deviation of a noise factor

A method for quantifying sensitivity is the sensitivity ratio (η SR). The ratio is defined as the percentage change in input for a specific input variable, as shown in equation (Peschl & Schweiger, 2003):

$$\eta SR = \frac{[(FS_{change} - FS)/FS]}{[(x_{change} - x)/x]} \quad (C.1)$$

where FS is the reference value of the output variable using reference values of the input variables and FS_{change} is the value of the output variable after changing the value of one input variable. The denominator x and x_{change} are respective input variables. For the sensitivity ratio, an input variable x_{change} is varied individually across the entire range requiring $2n + 1$

calculations, n being the number of varied parameters considered. An extension to the sensitivity ratio is the more robust method of evaluating important sources of uncertainty is the sensitivity score ηSS . This is the sensitivity ratio ηSR weighted by a normalized measure of the variability in an input variable, as given by equation:

$$\eta SS = \eta SR \cdot \frac{(\max x_R - \min x_R)}{x} \quad (C.2)$$

By normalizing the measure of variability, this method effectively weights the ratios in a manner that is independent of the units of the input variable. Performing a sensitivity analysis as described above allows to quantify the sensitivity score of each variable, ηSS , on respective results A, B, \dots, Z , (e.g. displacements, forces, P factor of safety, etc.). The total sensitivity score of each variable, ηSS , is the result of the summation of all sensitivity scores for each respective result at each construction step. It is mentioned that the results of the sensitivity analysis appear to be strongly dependent on the respective results used. Therefore, results of this analysis have to be chosen on sound engineering judgment. Finally, the total relative sensitivity $\alpha(x_i)$ for each input variable is then given by Peschl as:

$$\alpha(x_i) = \frac{\sum \eta SS, i}{\sum_{i=1}^N \eta SS, i} \quad (C.3)$$

References

- Abramson, L. W., Lee, T. S., Sharma, S., & Boyce, G. M. (2002). *Slope stability and stabilization methods*. John Wiley & Sons.
- Ali, N., Farshchi, I., Mu'azu, M. A., & Rees, S. W. (2012). Soil-root interaction and effects on slope stability analysis. *Electronic Journal of Geotechnical Engineering*, 17, 319–328.
- Ameratunga, J., Sivakugan, N., & Das, B. M. (2016). Vane shear test. In *Correlations of soil and rock properties in geotechnical engineering* (pp. 193–205). Springer.
- Ang, A. H.-S., Tang, W. H., et al. (2007). *Probability concepts in engineering: emphasis on applications in civil & environmental engineering* (Vol. 1). Wiley New York.
- Baker, R. (1980). Determination of the critical slip surface in slope stability computations. *International Journal for Numerical and Analytical Methods in Geomechanics*, 4(4), 333–359.
- Baum, R. L., Savage, W. Z., & Godt, J. W. (2002). Trigrs—a fortran program for transient rainfall infiltration and grid-based regional slope-stability analysis. *US geological survey open-file report*, 424, 38.
- Bin, L., & Songhong, Y. (2004). Simplified algorithm of structural reliability index under condition of correlated variables. *Journal of Southwest Jiaotong University*, 39(2), 147–151.
- Bishop, A. W. (1952). *The stability of earth dams* (PhD Thesis). University of London UK.
- Bjerrum, L. (1972). Embankments on soft ground, performance of earth and earth-supported structures. In *Proceedings ASCE Specialty Conference*. (Vol. 2, pp. 1–54).
- Breitung, K. (1984). Asymptotic approximations for multinormal integrals. *Journal of Engineering Mechanics*, 110(3), 357–366.
- Chen, C.-f., Zhu, J.-f., & Gong, X.-n. (2008). Calculation method of earth slope reliability based on response surface method and Morgenstern-Price procedure. *Engineering Mechanics*, 10, 031.

- Cho, S. E. (2009). Probabilistic stability analyses of slopes using the ann-based response surface. *Computers and Geotechnics*, 36(5), 787–797.
- Chowdhury, R., Flentje, P., & Bhattacharya, G. (2009). *Geotechnical slope analysis*. Crc Press.
- Christian, J. T., & Baecher, G. B. (2002). The point-estimate method with large numbers of variables. *International Journal for Numerical and analytical methods in geomechanics*, 26(15), 1515–1529.
- Christian, J. T., Ladd, C. C., & Baecher, G. B. (1994). Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering*, 120(12), 2180–2207.
- Cornell, C. A. (1969). A probability-based structural code. In *Journal Proceedings* (Vol. 66, pp. 974–985).
- Dawson, E. M., Roth, W. H., & Drescher, A. (1999). Slope stability analysis by strength reduction. *Geotechnique*, 49(6), 835–840.
- Dempster, A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *The annals of mathematical statistics*, 325–339.
- Deng, J., Li, X. B., & Gu, G. S. (2004). A distribution-free method using maximum entropy and moments for estimating probability curves of rock variables. *International Journal of Rock Mechanics and Mining Sciences*, 41, 127 – 132.
- Deng, J., & Pandey, M. (2008). Estimation of the maximum entropy quantile function using fractional probability weighted moments. *Structural Safety*, 30(4), 307–319.
- Deng, J., & Pandey, M. (2009). Using partial probability weighted moments and partial maximum entropy to estimate quantiles from censored samples. *Probabilistic Engineering Mechanics*, 24(3), 407–417.
- Deng, J., Pandey, M., & Xie, W. (2012). Maximum entropy principle and partial probability weighted moments. In *Aip conference proceedings 31st* (Vol. 1443, pp. 190–197).
- Dietzel, M., Kieffer, D. S., Schubert, W., & Schweiger, H. (2011). *Nasekbian, Ali: Application of Non-probabilistic and Probabilistic Concepts in Finite Element Analysis of Tunnelling*. Technische Universität Graz, Gruppe Geotechnik Graz, Institut für Bodenmechnaik und Grundbau.
- Di Matteo, L., Valigi, D., & Ricco, R. (2013, February). Laboratory shear strength parameters of cohesive soils: variability and potential effects on slope stability. *Bulletin of Engineering Geology and the Environment*, 72(1), 101–106.

- Dodds, R., Burak, J., & Eigenbrod, K. (1993). Nipigon river landslide. *International Conference on Case Histories in Geotechnical Engineering*.
- Duncan. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of geotechnical and geoenvironmental engineering*, 126(4), 307–316.
- Duncan, Wright, S. G., & Brandon, T. L. (2014). *Soil strength and slope stability*. Hoboken, New Jersey : Wiley, [2014].
- Er-Xiang, S. (1997). Finite element analysis of safety factor for soil structures. *Chinese Journal of Geotechnical Engineering*, 19(2), 1–7.
- Fellenius, W. (1936). Calculation of stability of earth dam. In *Transactions. 2nd Congress Large Dams, Washington, DC, 1936* (Vol. 4, pp. 445–462).
- Giam, S. K., & Donald, I. B. (1988). Determination of critical slip surfaces for slopes via stress-strain calculations. In *Fifth Australia–New Zealand Conference on Geomechanics: Prediction Versus Performance; Preprints of Papers* (p. 461). Institution of Engineers, Australia.
- Greco, V. R. (1996). Efficient Monte Carlo technique for locating critical slip surface. *Journal of Geotechnical Engineering*, 122(7), 517–525.
- Greenwood, J. A., Landwehr, J. M., Matalas, N. C., & Wallis, J. R. (1979, October). Probability weighted moments: Definition and relation to parameters of several distributions expressible in inverse form. *Water Resources Research*, 15(5), 1049–1054.
- Griffiths, D. V., & Fenton, G. A. (2004). Probabilistic slope stability analysis by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(5), 507–518.
- Haldar, A., & Mahadevan, S. (2000). *Probability, reliability, and statistical methods in engineering design* (Vol. 1). Wiley New York.
- Hamdy, F., Cai, F., & Ugai, K. (2003). Three-dimensional base stability of rectangular excavations in soft soils using FEM. In *Proceedings of the Japan National Conference on Geotechnical Engineering The 38th Japan National Conference on Geotechnical Engineering* (pp. 1007–1008). The Japanese Geotechnical Society.
- Harr, M. (1977). *Mechanics of particulate media a probabilistic approach*. New-York: McGraw-Hill.
- Harr, M. (1989). Probabilistic estimates for multivariate analyses. *Applied Mathematical Modelling*, 13(5), 313–318.

- Hasofer, A. M., & Lind, N. C. (1974). Exact and invariant second-moment code format. *Journal of the Engineering Mechanics division*, 100(1), 111–121.
- Holick, M. (2009). *Reliability analysis for structural design*. AFRICAN SUN MeDIA.
- Hong, H. P. (1996). Point-estimate moment-based reliability analysis. *Civil Engineering Systems*, 13(4), 281–294.
- Hosking, J. R. M., & Wallis, J. R. (2005). *Regional frequency analysis: an approach based on L-moments*. Cambridge University Press.
- Indraratna, B., Chu, J., & Rujikiatkamjorn, C. (2015). *Ground improvement case histories: Chemical, electrokinetic, thermal and bioengineering*. Butterworth-Heinemann.
- Jaksa, M., Kaggwa, W., & Brooker, P. (1999). Experimental evaluation of the scale of fluctuation of a stiff clay. In *Proceedings of the 8th international conference on application of statistics and probability, aa balkema, rotterdam* (pp. 415–422).
- Janbu, N. (1975). Slope stability computations: In Embankment-dam Engineering. Textbook. Eds. RC Hirschfeld and SJ Poulos. JOHN WILEY AND SONS INC., PUB., NY, 1973, 40p. In *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts* (Vol. 12, p. 67). Pergamon.
- Jiang, Y. (2013). *Slope analysis using boundary elements* (Vol. 52). Springer Science & Business Media.
- Julong, D. (1989). Introduction to grey system theory. *The Journal of grey system*, 1(1), 1–24.
- Kavzoglu, T., Sahin, E. K., & Colkesen, I. (2014). Landslide susceptibility mapping using gis-based multi-criteria decision analysis, support vector machines, and logistic regression. *Landslides*, 11(3), 425–439.
- Kendall, M., & Stuart, A. (1977). *The advanced theory of statistics. vol. 1: Distribution theory*. London: Griffin, 1977, 4th ed..
- Klir, G. J., & Folger, T. A. (1988). *Fuzzy sets, uncertainty, and information* (Vol. 448). Prentice Hall Englewood Cliffs.
- Knappett, J. (2012). *Craig's soil mechanics* (Vol. 8). Spon Press London, UK.
- Li, Jiang, S.-H., Cao, Z.-J., Zhou, W., Zhou, C.-B., & Zhang, L.-M. (2015). A multiple response-surface method for slope reliability analysis considering spatial variability of soil properties. *Engineering Geology*, 187, 60–72.

- Locat, A., Locat, P., Demers, D., Leroueil, S., Robitaille, D., & Lefebvre, G. (2017). The saint-jude landslide of 10 may 2010, quebec, canada: Investigation and characterization of the landslide and its failure mechanism. *Canadian Geotechnical Journal*, *54*(10), 1357–1374.
- Low, B., & Tang, W. H. (2007). Efficient spreadsheet algorithm for first-order reliability method. *Journal of engineering mechanics*, *133*(12), 1378–1387.
- Low, B., Zhang, J., & Tang, W. H. (2011). Efficient system reliability analysis illustrated for a retaining wall and a soil slope. *Computers and Geotechnics*, *38*(2), 196–204.
- Malkawi, A. I. H., Hassan, W. F., & Abdulla, F. A. (2000). Uncertainty and reliability analysis applied to slope stability. *Structural safety*, *22*(2), 161–187.
- Matsui, T., & San, K.-C. (1992). Finite element slope stability analysis by shear strength reduction technique. *Soils and foundations*, *32*(1), 59–70.
- Matsuo, M., & Kuroda, K. (1974). Probabilistic approach to design of embankments. *Soils and Foundations*, *14*(2), 1–17.
- Moore, R., & Lodwick, W. (2003). Interval analysis and fuzzy set theory. *Fuzzy sets and systems*, *135*(1), 5–9.
- Morgenstern, N. u., & Price, V. E. (1965). The analysis of the stability of general slip surfaces. *Geotechnique*, *15*(1), 79–93.
- Most, T. (2009). Estimating uncertainties in maximum entropy distribution parameters from small-sample observations. In *Proceedings of the European Safety and Reliability Conference (ESREL)* (pp. 1745–1752).
- Nemčok, A., Pašek, J., & Rybář, J. (1972). Classification of landslides and other mass movements. *Rock mechanics*, *4*(2), 71–78.
- Nowak, A. S., Yamani, A. S., & Tabsh, S. W. (1994). Probabilistic models for resistance of concrete bridge girders. *Structural Journal*, *91*(3), 269–276.
- Pandey, M. D. (2000). Direct estimation of quantile functions using the maximum entropy principle. *Structural safety*, *22*(1), 61–79.
- Peschl, G. M., & Schweiger, H. F. (2003). Reliability Analysis in Geotechnics with Finite Elements—Comparison of Probabilistic, Stochastic and Fuzzy Set Methods. In *ISIPTA* (Vol. 3, pp. 437–451).

- Phoon, K.-K., & Kulhawy, F. H. (1999a). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, 36(4), 612–624.
- Phoon, K.-K., & Kulhawy, F. H. (1999b). Evaluation of geotechnical property variability. *Canadian Geotechnical Journal*, 36(4), 625–639.
- Phoon, K.-K., Kulhawy, F. H., & Grigoriu, M. D. (2003). Multiple resistance factor design for shallow transmission line structure foundations. *Journal of Geotechnical and Geoenvironmental Engineering*, 129(9), 807–818.
- Rackwitz, R., & Flessler, B. (1978). Structural reliability under combined random load sequences. *Computers & Structures*, 9(5), 489–494.
- Rosenblueth, E. (1975). Point estimates for probability moments. *Proceedings of the National Academy of Sciences*, 72(10), 3812–3814.
- Salgado, R., & Kim, D. (2013). Reliability analysis of load and resistance factor design of slopes. *Journal of Geotechnical and Geoenvironmental Engineering*, 140(1), 57–73.
- Sarma, S. K. (1987). A note on the stability analysis of slopes. *Geotechnique*, 37(1), 107–111.
- Shrestha, B., & Duckstein, L. (1997). A fuzzy reliability measure for engineering applications. *Uncertainty modeling and analysis in civil engineering*, 121–135.
- Siddall, J., & Diab, Y. (1975). The use in probabilistic design of probability curves generated by maximizing the shannon entropy function constrained by moments. *Journal of Engineering for Industry*, 97(3), 843–852.
- Uzielli, M., Vannucchi, G., & Phoon, K. (2005). Random field characterisation of stress-normalised cone penetration testing parameters. *Geotechnique*, 55(1), 3–20.
- Van Gelder, P. (2000). *Statistical methods for the risk-based design of civil structures* (PhD Thesis). TUDelft.
- Vanmarcke, E. (1980). Probabilistic stability analysis of earth slopes. *Engineering Geology*, 16(1-2), 29–50.
- Wong, F. S. (1985). Slope reliability and response surface method. *Journal of Geotechnical Engineering*, 111(1), 32–53.
- Youssef, A. M., Pourghasemi, H. R., El-Haddad, B. A., & Dhahry, B. K. (2016). Landslide susceptibility maps using different probabilistic and bivariate statistical models and comparison of their performance at wadi itwad basin, asir region, saudi arabia. *Bulletin of Engineering Geology and the Environment*, 75(1), 63–87.

- Yu, L. (2008). A new calculation algorithm for quantile function with application to headway distribution. In *Service Operations and Logistics, and Informatics, 2008. IEEE/SOLI 2008. IEEE International Conference* (Vol. 1, pp. 452–456).
- Zadeh, L. A. (1996). Fuzzy sets. In *Fuzzy Sets, Fuzzy Logic, And Fuzzy Systems: Selected Papers by Lotfi A Zadeh* (pp. 394–432). World Scientific.
- Zhang, D.-b., Sun, Z.-b., & Zhu, C.-q. (2013). Reliability analysis of retaining walls with multiple failure modes. *Journal of Central South University*, 20(10), 2879–2886.