Malleable Multiplication: The Use of Multiplication Strategies and Gamification to Create Conceptual Understanding

Rachel Macintyre

Lakehead University

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Introduction

I have always considered math to be both my friend and foe. The parts of math I enjoy are algebra, trigonometry, and multiplication. I could find solace in math that was formulaic and had clear instructions. At times these concepts may be abstract, but there was always a formula to consult. However, for every unit I found comfort in, there were always more that distressed me. As much as math would stress me out and make me feel inadequate, I was always trying methods to make it more enjoyable. I tried the placebo effect (which was not successful because I knew it was a trick), positive affirmations, and bringing aesthetically pleasing math accessories to class (i.e., pens, notebooks, etc.). Unfortunately, during my elementary and high school years, I couldn't find a method that made learning math an experience I would enjoy.

The moment everything clicked, and I found a method I could utilize came to me during the second year of my Bachelor of Education degree. My teaching mathematics professor tasked my class to invent or bring in a math game that would help students understand a mathematical concept. I was interested in the project and had fun researching and presenting my game. I was amazed by all the games my peers brought to class. Every game was fun and helped instil concepts. I had never even considered gamification as a method that could solve my stress and anxiety concerning math. I honestly didn’t even know it was possible to teach such a wide array of mathematical subject matter using games as an aid.

That game project sparked my interest in math and how the subject matter can be made more accessible and fun for every student. After that, I started researching, reading books about math education, and listening to podcasts. I got excited by this world where math could be fun and inclusive instead of an anxiety-ridden subject. That one project has shaped the course of my Master of Education program and inspired me to create a math game. I would never have predicted that I would be focused on mathematics at any part of my academic journey. For me, math has transformed from a scary subject to something challenging but conquerable. I want every student to feel like they have the potential to understand math and have fun with the subject. My fun with math may have happened outside the confines of elementary and secondary school, but it still happened and has changed a lot of my misgivings concerning the subject.

Throughout school, math was a consistent struggle; even with supports in place, I still had an uphill battle. Even with the introduction of math games, there will be students who need support, but one of the most important things a math game can do is to show students that math can be fun. If a student can have fun while playing a math board game, it has the power to change their perspective on the subject. I know how I felt playing math games and how much it has changed my trajectory; I want to give students a chance to have that moment as they develop an understanding of multiplication.

Literature Review

Introduction

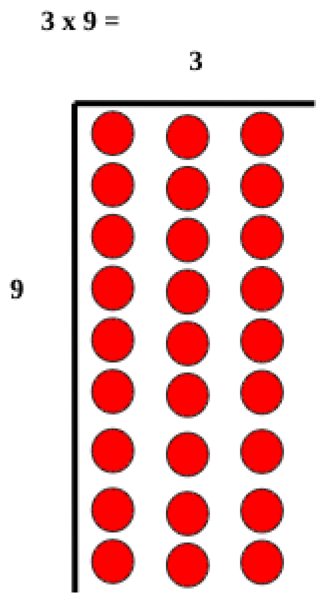
The following literature review presents articles that will help with the development of the multiplication board game discussed later in this portfolio. The main themes identified are multiplicative thinking/reasoning, the commutative and distributive properties of multiplication, conceptual and procedural understanding, various multiplication strategies (i.e., equal groups, additive strategies, number lines, open arrays, and algorithms), and the gamification of mathematics instruction. All of these themes have helped inform the creation of my board game. The main aim of my board game is to aid students in garnering a procedural and conceptual understanding of multiplication.

Multiplicative Reasoning

Multiplicative thinking/reasoning is more than just remembering and reciting multiplication facts; it is one of the foundational ideas concerning numbers (Hurst, 2015). Students who reason multiplicatively can use numbers flexibly and see the many relationships that numbers share (Siemon, Breed, & Virgona, 2005, as cited in Lu & Richardson, 2018, p. 240). Field (2021) states that multiplicative thinking forms when students can "unitize, understand equal and unequal groups, combine equal groups and understand the relationship between repeated addition and the times sign" (p. 21). For example, if a student were considering the number 10, they would be able to state that ten consists of two groups of five, five groups of two, ten groups of one, or one group of ten. It is essential that students learn how to reason multiplicatively in order to interact with mathematics outside of the confines of memorized facts. The other integral reason students should learn to think multiplicatively is that it will help them with mathematical concepts such as algebra, geometry, ratios, area, proportional reasoning, volume, data analysis, graphs, functions, and probability (Calabrese, Kopparla, & Capraro, 2020; Mulligan & Watson, 1998). All of these concepts tie into multiplicative thinking, which makes it imperative that students develop this reasoning to aid their future mathematical development. If students do not develop into multiplicative thinkers, their secondary school math experiences may be more difficult (Mulligan & Watson, 1998, p. 61).

The Commutative and Distributive Properties of Multiplication

Understanding the two essential properties of multiplication, the commutative property, and the distributive property, requires abstract thinking. Downton and Sullivan (2017) define abstract thinking as an individual's ability to visually imagine the sets of numbers they are using to solve problems. The commutative property is the understanding that switching the multiplier and multiplicand has no bearing on the outcome of the equation (Squire, Davies, & Bryant, 2004). The commutative property can make solving equations easier because some students might be intimidated by a certain multiplicand, and the multiplier might be the better lead number. To illustrate, if an individual had to solve 9 x 3, they might find it easier to flip the multiplicand and multiplier to solve 3 x 9 instead. Another way to represent the commutative property is with an array. Arrays are helpful when demonstrating the commutative property because students can see that no matter which number is on the horizontal or vertical axis, the answer remains the same (Day & Hurrell, 2015). Using the array example below, a student should see that if they switched the 9 to the vertical axis and the 3 to the horizontal axis that the number of dots does not change. Once students understand this concept through a visual representation, they should see that it is the same with multiplication equations – if you flip the multiplier and the multiplicand, the product remains the same.



Students can also decompose numbers to solve equations and do not have to rely on number facts. To illustrate, for the equation 3 x 9, students may know that 2 x 9 = 18 and then add one more group of 9 to get to 27. This ability to break the multiplier or multiplicand into friendlier numbers is called the distributive property of multiplication. For example, if a student approached the question 10 x 15 and was unsure of the answer, they could decompose the 15 to help them solve the problem. The 15 could decompose into a ten and a 5. Then the student could create two equations from the decomposed numbers: 10 x 10 and 10 x 5. Once the student solves both equations, they would add the answers to get the final result to the original equation. When understood, the commutative and distributive properties allow students to flexibly use numbers when multiplying.

Procedural Knowledge and Conceptual Understanding

Procedural knowledge and conceptual understanding are two essential components of mathematical understanding that students must develop. These types of knowledge help students understand multiplication outside of memorized facts (Squire et al., 2004). Procedural knowledge is evident when an individual has learned and can recall the rules of the operation and knows how to solve problems following the guidelines (Young-Loveridge, 2005). For example, if a student has procedural knowledge of multiplication, they will understand the rules and procedures that they can use to arrive at the correct answer. If a student has the equation 5 x 5, the student might use a times table chart or calculator to arrive at the solution (depending on the circumstances). The student may have memorized that 5 x 5 = 25, but with only procedural knowledge, that student will not understand other ways to look at the initial equation or why and how the procedures work. The same notion applies to arrays and other visual representations of multiplication. A student may know how to set up an array without knowing the 'why' and the 'how' concerning the array or how the array relates to the equation. They know where to place the numbers to start the calculation process but do not understand how it works to get the final answer. To fully understand the processes behind numbers and equations, individuals need to obtain a conceptual understanding.

Conceptual understanding goes deeper than procedural knowledge, although it is beneficial for students to grasp both knowledge types. Conceptual understanding "is about links and relationships and knowing how ideas are connected and why processes work as they do" (Hurst & Hurrell, 2016, p. 34). Essentially when students have a conceptual understanding, they become aware of the 'how' and 'why' and can see the flexibility and relationships between numbers/concepts. When students have a conceptual understanding, they will also have knowledge of the commutative and distributive properties (although they may not know these terms by name) (Bakker, van den Heuvel-Panhuizen, & Robitzsch, 2016). To illustrate, when students have a conceptual understanding, they will be able to grasp the commutative property. If a student struggles to solve 4 x 5 because they are not as confident multiplying the number four, they would know that they can switch the equation to 5 x 4, and the outcome will be the same. When a student only has procedural knowledge, they would resign themselves to the fact that they have to answer the equation in its original form (4 x 5). Since that student would not know the 'how' and 'why' that comes with the commutative property and conceptual knowledge, they would not know that they can switch the numbers around with multiplication questions and not alter the outcome of the equation. Also, when students conceptually understand multiplication, they will understand the distributive property. Once students understand the distributive property, it will allow them to see how numbers and equations are malleable (Kinzer & Stanford, 2014). For example, if a student is multiplying 8 x 3, if they have conceptual knowledge of multiplication, they will know that they can change the numbers to create an easier equation. The student can change the equation to 4 x 3 and complete that equation twice because 4 x 2 = 8. Having a conceptual knowledge of multiplication gives students more power with approaching questions (Squire et al., 2004). Students will know the rules/procedures because of their procedural knowledge, but when students conceptually understand multiplication, they can make the equations work with their mathematical strengths. This demonstrates that students need both knowledge types to fully understand multiplication outside of memorized facts.

Students must obtain procedural knowledge and conceptual understanding because they work together to help students gain a fuller understanding of mathematical concepts. Hurst (2018) discusses how students “who were taught conceptually outperformed those who were taught procedurally, then conceptually” (p. 31). He found studies claiming that students who knew procedures felt like they did not need to worry about anything more. Hurst implies that if students are taught conceptually and procedurally in tandem, they will obtain a deeper understanding and realize the importance of both procedural and conceptual understanding.

Multiplication Strategies

When students are learning about multiplication, there are a variety of strategies that they can use to conceptualize the concept and solve equations/problems. Procedures can include creating equal groups, additive strategies (i.e., repeated addition, skip counting, and doubling), number lines, open arrays, and algorithms. Researchers have found that many students will stick to procedures they are most comfortable using; however, some strategies may not be a good fit for particular problems. Thus, students must understand and learn multiple strategies (Downton & Sullivan, 2017, p. 303). A solution would be to introduce more strategies and pay attention to the difficulty level of the posed problems. If "problems and exercises posed are too easy then opportunities for students to develop more sophisticated approaches are delayed" (Downton & Sullivan, 2017, p. 303). Thus, students need to have exposure and practice using strategies that may initially be outside their comfort zone; and be introduced to problems that will appropriately challenge their use of new procedures. The following section will detail many strategies which will help children develop their multiplicative thinking skills.

Multiplication Strategies: Equal Groups

The first multiplication strategy discussed in this literature review is creating equal groups. An example of an equal group problem is if the student has to sort a number of items into groups of the same size. When solving, the student would ensure that each group contained the same number of items. Equal groups is commonly the initial model for multiplication in instruction (Greer, 1992) and involves "connecting multiplication to the calculation procedure of repeated addition" (Larsson, Pettersson, & Andrews, 2017, p. 2), which will be discussed in the next section.

When utilizing the equal groups strategy, students may develop more sophisticated multiplication strategies (Mulligan & Mitchelmore, 1997, p. 328). By using equal groups, students may recognize that equal groups are the essence of multiplication because multiplication is essentially "groups of" thinking. Students may also make connections to multiplication tables. To illustrate, if one is looking at a multiplication table, they will see that by looking at the columns, the values increase by the same amount (i.e., 3 x 2 includes two groups of three and 3 x 3 consists of three groups of three).

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

According to the literature on equal groups, this strategy usually involves manipulatives and materials to construct visual representations. Specifically, Roche, Ferguson, Cheeseman, and Downton (2020) discuss how they evaluated students' understanding of equal groups. The students in the study had not had formal lessons on multiplication and division before the start of the study. The researchers had the subjects listen to a story about twelve ducks. Then the students were tasked with creating equal groups of the twelve ducks without using manipulatives (after they were initially allowed to consult manipulatives) (p. 32). After the study, the researchers found that a number of the children could create equal groups and understood the task. The same number of children had a partial understanding of the expectations. Then, a small population of students did not understand what the question was asking of them (Roche et al., 2020, pp. 32-34). The authors' study demonstrated the need for explicit instruction concerning equal groups (i.e., how to create and understand them). Teachers must model equal groups with manipulatives so students can see and apply that strategy to problem solving. Thus, the equal groups strategy is excellent for students, but students need a foundation and modelling to access the procedure correctly.

Explicitly teaching students about equal groups will serve them in the future when they explore different areas of mathematics. Equal groups are foundational for other multiplication strategies, including repeated addition, arrays, and number lines. Whether students use repeated addition, arrays, and or number lines, equal groups are always present because they are rooted in multiplication as a concept. Equal groups can also be used to understand division, the inverse operation of multiplication. For example, 7 x 3 = 21 can be represented by three groups of seven. On the other hand, 21 ÷ 7 = 3 can be thought of as 21 divided into equal groups of 7 each, with a total of 3 groupings overall. The equal groups strategy is essential for students. It will help students construct their conceptual and procedural understanding of multiplication and to bridge their learning and understanding of other mathematical concepts, such as division.

Multiplication Strategies: Additive Strategies

Examples of additive strategies are repeated addition, skip counting, and doubling. Repeated addition involves the individual looking at the numbers in the multiplication equation and adding the multiplicand by the multiplier. To illustrate, "3 times 5 [would become] 5 + 5 + 5" when using the repeated addition model (Fischbein, Deri, Nello, & Marino, 1985, p. 4). Skip counting is similar to repeated addition. Skip "counting is done in multiples" (i.e., 5, 10, 15, 20, etc.) (Mulligan & Mitchelmore, 1997, p. 311). The last additive strategy mentioned above is doubling, which involves numbers and their multiples. For example, if an individual has to solve 3 x 4, they can use prior knowledge and remember that 3 x 2 = 6. Then they would double that number to get the correct answer (Downton & Sullivan, 2017, p. 316). Repeated addition, skip counting, and doubling are similar because they all take the multiplicand and add it as many times as the multiplier indicates. For this section, I will focus on repeated addition.

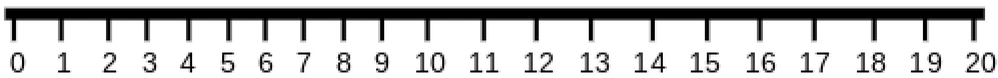
One drawback of introducing multiplication as “a faster way of doing repeated addition” is that it makes multiplication an extension of addition and can negate its impact and identity as a mathematical concept (Clark & Kamii, 1996, p. 41). The notion concerning the morphing of addition and multiplication has made researchers question how to introduce multiplication to students. Park and Nunes (2001) discuss their study concerning how children conceptualize multiplication. The goal was to see if students' understanding of multiplication was grounded in repeated addition (p. 763). They concluded that repeated addition should only be considered a strategy for developing multiplicative reasoning. Since repeated addition does not form the conceptual basis of multiplication (p. 771). Park and Nunes (2001) were not the only researchers who concluded that repeated addition should only be used as a strategy and not as the basis for multiplication. For instance, Calabrese et al., (2020) also discussed that teaching students multiplication only through repeated addition could be detrimental because it does not allow them to grasp the full scope of multiplication (p. 4).

Although repeated addition should not form the basis of students' understanding of multiplication, it can still be a helpful strategy (Park and Nunes, 2001). For example, it is essential that students know that when multiplying two numbers together, the multiplier indicates how many times to multiply the multiplicand to procure the answer. So, if a student sees the equation 6 x 3 after understanding repeated addition, they should also understand that equation as 6 + 6 + 6. When one recognizes that the above equations are the same, they can see how repeated addition connects to multiplication but is not the encompassing factor of multiplication. However repeated addition does allow students to consider the basis of multiplication and the inner workings of the equation. Then when students see 6 x 3, they know how to deconstruct and conceptualize the question.

Some children might gravitate towards additive strategies because they have had lots of practice with addition before learning about multiplication. Mulligan and Mitchelmore's (1997) study found that when their participants used the repeated addition strategy, their answers and use of the procedure were predominately correct (p. 323). Thus, by introducing multiplication as repeated addition, educators are trying to build on students' prior additive knowledge. However, as discussed above, it may hinder students' conceptual understanding of multiplication (Calabrese et al., 2020, p. 4). Secondly, although repeated addition can work as a strategy for a while, it can become cumbersome when multi-digit multiplication and decimals come into play (Larsson et al., 2017, p. 1). Since the numbers will be larger and may include decimal points, this strategy would still be effective but take the student too long to solve problems. Since repeated addition can be tedious, it is essential to expose students to various procedures so they can deduce the appropriate one to apply to specific equations. Some of those strategies may be other additive strategies (i.e., skip counting, doubling, etc.), and others may fall away from the additive models.

Multiplication Strategies: Number Lines

Another representation that students can use to solve multiplication equations is number lines. One of the benefits of number lines is that they are around us every day, allowing students to see their many functions within different contexts (Diezmann, Lowrie, & Sugars, 2010, p. 24). Number line examples can include thermometers, scales, and a marked ruler. When using a number line to solve a multiplication problem, the individual will make ‘jumps’ forward to determine the correct answer (Young-Loveridge, 2005, p. 36). There are two types of number lines: structured and blank. A structured number line has markings that indicate number placement (Diezmann et al., 2010). This type of number line is beneficial because the student does not have to place the numbers; the number line is ready to use. The only information the student would need to know is how to count to use this type of number line. The other kind of number line is a blank number line. Blank number lines allow students to use their number knowledge and estimate benchmarks and the numbers in between (Reinert, Huber, Nuerk, & Moeller, 2015, p. 96). This allows students to showcase their counting and knowledge of numbers while spacing them out evenly. Both the structured number line and the blank number line (physical or digital) can be helpful when solving problems and can test students' knowledge of numbers and proportions.



Structured Number Line

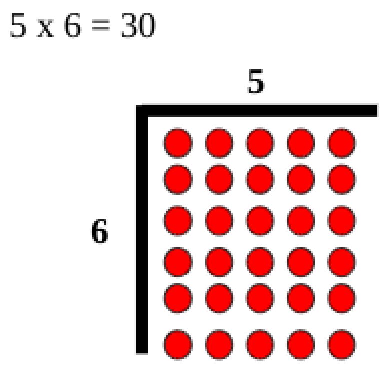
Blank Number Line

Number lines can be physical objects, or they can be digital. Some of the literature concerning number lines delved into the digital space. To illustrate, Tucker, Shumway, Moyer-Packenham, and Jordan’s (2016) article started by discussing virtual manipulatives, including virtual number lines, which have become more prevalent with the influx of technology in classrooms (p. 23). However, with any number line (i.e., structured, blank, and digital), the ways that students use the number lines to solve problems are the same. For example, if a student uses a number line (numbers 1-30) to solve 5 x 4, the student would start at zero and then skip count four groups of five to get to the answer of twenty.

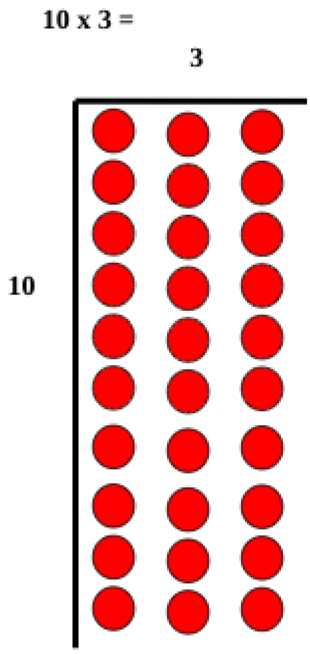
One of the main benefits of the number line is that it is a physical tool that allows the individual to see a visual representation (Diezmann et al., 2010, p. 25). Students can hold a number line and use their pencils to make marks when solving problems. Counting using a manipulative that the individual can see and grip hopefully will help an individual recognize number patterns and help with the accuracy of answers.

Multiplication Strategies: Arrays

An additional strategy that students can use to solve multiplication problems is an array. Arrays help students see the inner workings of multiplication problems in a visual format (Young-Loveridge, 2005, 37). Specifically, when it comes to the construction of arrays, "students need to combine both spatial (rows of squares) and numeric composites (number of squares in a row)" (Downton & Sullivan, 2017, p. 306). Essentially, arrays can look different depending on the numbers involved, but generally, they look like grids.

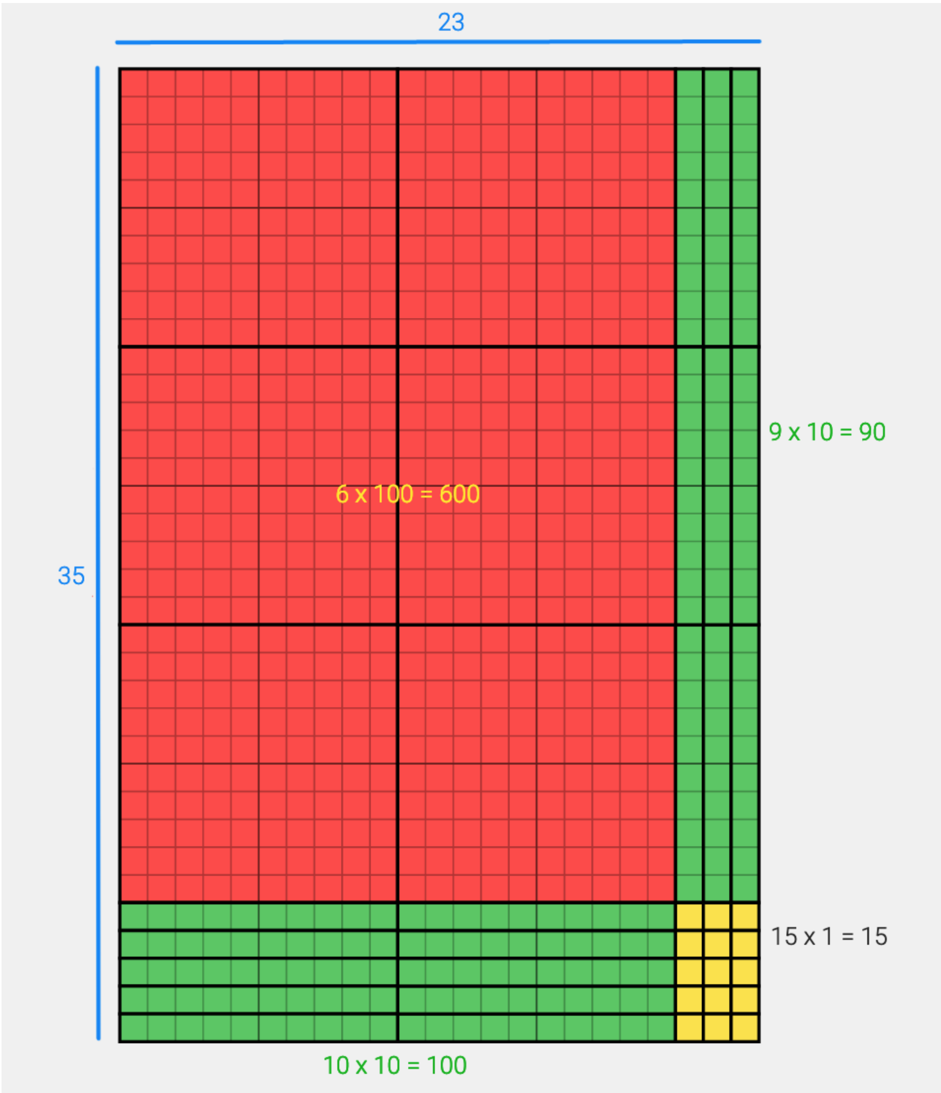


Barmby, Harries, Higgins, and Suggate's (2009) article states that arrays are "a key representation for multiplication" and are a vital representation because of their visual format (p. 223). When students can visually see the workings of an equation, it can aid understanding. Arrays are not just an excellent tool to demonstrate the parts of a multiplication problem/equation; they can also prepare individuals for algebraic reasoning and showcase the commutative and distributive properties (Day & Hurrell, 2015, p. 20). As stated, arrays can help students learn about the commutative property because arrays show individuals that it does not matter which order the numbers are in when multiplying. Arrays and the commutative property are linked together, which one can notice in "the capacity to rotate [arrays] to show that four lots of three [yields] the same total as three lots of four" (Day & Hurrell, 2015, p. 20). No matter how one looks at the rows and columns, the answer remains the same. Also, arrays can teach students about the distributive property. Arrays display the distributive property by decomposing numbers within equations (Day & Hurrell, 2015, p. 21). For example, when a student creates an array, it gives them a chance to break down numbers using their prior knowledge of place value. Suddenly thirty can be decomposed into three tens in an array (Young-Loveridge, 2005, p. 40).



All in all, arrays can help a student figure out the answer to an equation, can help build a conceptual understanding of the commutative and distributive properties, and prepare them for future mathematical concepts.

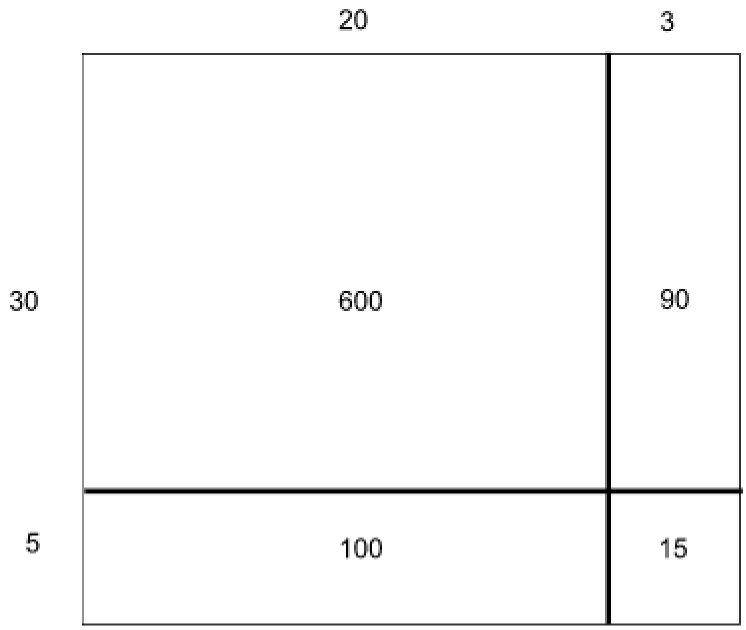
Another way to represent arrays is by using manipulatives. Base ten materials can help students when calculating equations that include big numbers. Base ten materials include flats (representing 100 units), rods (representing 10 units), and units (individual cubes). The base-ten materials can represent all the components of equations. If an individual uses base-ten materials to answer the following question, 23 x 35, they would create an array with 23 along the top and 35 along the side. They would then use base 10 materials to construct an array. Then they would count how many 100s, 10s, and 1s are included and add them to find the answer.



Essentially, the base ten materials allow individuals to see the 100s, 10s, and 1s that make up questions. Thus, manipulatives can be valuable tools to help students understand the components of arrays.

Another array that students can utilize is an open array. Open arrays also help students visually see the breakdown of a multiplication problem (Young-Loveridge, 2005, 37).

However, instead of including individual units (like the examples above), an open array represents quantities numerically. To illustrate, the open array below would be the next step from the base 10 model shown previously. The 23 is decomposed into 20 (two tens), and 3 (three ones,) and the 35 is decomposed into 30 (three tens) and 5 (five ones). This helps students recognize that numbers can be flexible, and they can connect their base ten knowledge to what they know about arrays in a more abstract representation.



Algorithms

Typically, when one thinks of a standard algorithm, one recalls neat columns and procedures that an individual follows to ensure the correct answer. Although standard algorithms are efficient, they also prohibit students from conceptually understanding multiplication (Dolk & Fosnot, 2001; Hurst & Hurrell, 2018). With algorithms, students are just following procedures, and they know that if they follow along correctly, they will get the answer. Standard algorithms do not allow students to explore what the numbers represent; it only has them memorizing a formula and plugging numbers into said formula. For example, with a standard algorithm, if a student had to tackle 55 x 64, they would automatically stack the 55 over the 64. Then they would follow the steps until they eventually got the answer. There is no room for exploration or decomposition with the numbers. The equation stays as is, and students must follow the procedures regardless of their understanding of what is happening with the numbers (Hurst & Hurrell, 2018). Also, standard algorithms ignore the distributive property, which is a concept that is integral for students to know to develop their conceptual understanding (Dolk & Fosnot, 2001). When students do not know that they can change the numbers in an equation, they are not getting the chance to use the distributive property. For example, with the above equation 55 x 64, students who have only ever followed the standard algorithm procedure would not know that they can change the equation to 55 x 32 and repeat that problem twice to receive the answer. They could also find 5 x 64 and then multiply that answer by 10 (to find 50 x 64) and then add the two products together. These are two examples of how students can change equations to suit their needs. Students need to be introduced to the distributive property to understand that they can alter equations; they do not need to stick to the standard algorithm. The more chances students receive to explore the distributive property, the more they will start to hone their conceptual understanding of multiplication.

Standard algorithms are not an inferior tool to utilize when students understand number sense, but typically educators introduce standard algorithms far too early (Dolk & Fosnot, 2001). Students get introduced to these algorithms before they have had a chance to explore and work with numbers. The Ontario Mathematics Curriculum Grades 1-8 (2020) explicitly mentions algorithms for the first time in the grade 3 section. Once students have learned about place value,the goal is that they will be able to apply that knowledge to algorithms and other mathematical concepts (p. 183). Students typically know about stacking and horizontal algorithms before developing number sense. They learn how to use a procedure (algorithms) that works without really understanding why or how it works. If these students know standard algorithms without truly understanding concepts of multiplication, they might be less willing to learn about the commutative and distributive properties.

Similarly to Dolk and Fosnot's (2001) perspective, Hurst and Hurrell (2018) believe that algorithms need to be developed through a thorough understanding of the distributive property, gradually increasing the size of the numbers and developing the grid or area representation for multiplication" (p. 18). This means that algorithms must be flexible, and students should see their inner workings. Partial product algorithms allow students to arrange, explore, and decompose numbers and equations. Hurst and Hurrell (2018) believe that when illustrating partial products, utilizing an array/grid model is best so students can see what the algorithms they are dealing with entail. Students can develop a conceptual understanding of multiplication using partial product algorithms, which provide a visual aid and allow students to manipulate numbers. For example, the partial products equation that would follow from the base 10 model and open array described earlier would be:

$$
\begin{array}{l}
\quad 23 \\
\underline{\times\ 35} \\
600\ (20 \times 30) \\
\ \ 90\ (30 \times 3) \\
100\ (20 \times 5) \\
\underline{\ \ 15\ (5 \times 3)} \\
805
\end{array}
$$

The partial products algorithm above demonstrates how the numbers in the initial equation can be decomposed to allow students to solve double-digit algorithms while acknowledging place value knowledge. When solving the partial products algorithm above, first, the student will start with the equation and then multiply the tens columns (20 x 30). Then the student will individually multiply the tens columns with the opposite ones column (30 x 3 and 20 x 5). Lastly, the two ones columns will be multiplied (5 x 3). Once all the multiplication is complete, the student will add the answers to get the final sum (600 + 90 + 100 + 15 = 805).

Partial products algorithms work in a similar way to arrays, especially when it comes to decomposing numbers.

Gamification

The last sub-topic to discuss is gamification. Gamification occurs when one applies various games and formats to settings in which game elements might not typically be found (Yildirim, 2017, p. 87). To illustrate, this portfolio centres on the creation of a math game to help students develop their procedural and conceptual understanding of multiplication.

As mentioned previously, students can use many strategies (i.e., additive strategies, number lines, etc.) to help them solve multiplication problems. It is paramount to find unique and accessible ways to teach students mathematical concepts because every student is different. For example, some students may automatically like math, and some may not be too passionate about the subject, so having games would give both groups an accessible and fun opening to various topics.

Math games can also take on different forms. Some math games can be digital, or the games can use traditional means (i.e., board games, etc.). Each format has its benefits and limitations. Digital games raise the question of access. One should determine if the school has the funds to allocate to technology and programs to outfit said technology. However, as Yildirim (2017) states, because of the influx of technology, at least at home, if not at school, students have more opportunities to engage with online math games (p. 86-87). Paywalls exist for some gaming software, but many free resources are accessible. Educators can get around a potential lack of technology by embracing physical math games that do not involve technology. Physical math games can include board games, cards, dice, and other materials that help express a concept enjoyably. The rest of this section will focus on physical math games since physical math games can be more accessible and are usually more cost-efficient than technology (Scalise, Daubert, & Ramania, 2020, p. 217).

An important aspect of board games/physical math games is that they employ cooperative play. Students can gain knowledge and work on social skills by playing these games with other students. As well as playing these math games with other students, students could play the same games or different math games at home with their parents/guardians and or siblings (Ramani & Siegler, 2008). At school, students get the instructions, and the steps will be modelled for them. Also, the teacher will know how to guide the gameplay to maximize learning potential. At home, some parents may not know how to properly support "their children's numeracy development during joint numeracy activities" (Cheung & McBride, 2017, p. 572). Since the parents probably have not received training on supporting this learning, they might just be playing the game thinking that that is enough numeracy support. One part of Cheung and McBride's (2017) study examines whether a parent receiving training to support numeracy would make a difference in their child's development (p. 572). The researchers concluded that when parents have training to play math games, it helps their children better develop the appropriate numeracy skills the game targets (Cheung & McBride, 2017, p. 585). Therefore, one element to keep in mind with math board games is that parents may need more specific instructions, so they can ensure that the playtime with these math games is equally productive to garner skills. Training parents to scaffold learning within the confines of a math game may be difficult, so having written instructions that target the parent's role in facilitating the game may be handy, especially for a parent or a teacher who is using the game to help numeracy development. The game that will be the focal point of this portfolio will include accessible instructions for educators and parents. Students should have the opportunity to practice integral skills both in and outside the classroom.

Another advantage of math games is that a child does not have to play the math game for long to reap the benefits. Scalise et al., (2020) discuss a study that used a linear board game, and the authors concluded that by playing for a short period, the pre-schoolers had improved their “symbolic numerical magnitude skills” (Ramani and Siegler, 2008). In a later study, Scalise et al., (2020) found that the knowledge garnered from playing math games lasted up to eight weeks for most participants (p. 215). One can deduce from the literature that playing a math game of any kind, as long as the instructions are clear and accessible, can help a child improve on a mathematical concept. This argument is reminiscent of the notion that if a child continues to practice reading during the summer, said child will maintain and potentially improve their literacy levels (Viox, 1963, p. 40). Another benefit of math games is that they may help struggling students with a concept they did not previously understand (Scalise et al., 2020, p. 217). Some students might not understand a mathematical concept because of how the educator teaches; it might not register for students, or the educator may not consider the students’ unique learning styles. A board game may help students unlock knowledge they previously could not receive and improve their work on particular mathematical concepts. Even though math games may help struggling students, they can also provide extra practice for students who are comfortable with math. Lastly, an additional benefit of math games is that they can connect the individual to a subject. Essentially, “the learner [becomes] the centre of learning, which facilitates a more fecund and interesting learning process” (Su, 2015, p. 10014). Instead of completing a worksheet and going through the motions, the outcome of a board game may have a greater impact on students. Since students would potentially have more motivation to win the game, they might focus on the subject more and see the usefulness of a concept sooner.

Some physical math games can be too complicated for the intended audience (Moomaw, 2015, p. 505). Games that are too complex can take away from the lesson. There are reasons why some board game developers might make a child-friendly version of their games (i.e., Monopoly and Monopoly Junior, etc.) One might be that it enables children to play these games with and without adults present. Thus, having child-friendly versions of games also allows children to be independent and have control. To ensure that children reap the benefits of board games, they must target their audience. The rules and instructions need to be simple so that the children playing the game do not get lost and can play. For example, Wang and Hung (2010) discuss the teacher's role in doling out rules for games students play. In their study, Wang and Hung (2010) discuss how the teacher, who was participating in their study, went over the rules of the game and gave students chances to amend the rules (with the teacher's help). The teacher went over the rules many times. This meant that once the students started playing, they were all familiar with the expectations (p. 25). The teacher could still engage with the students playing the games but did not need to go over the rules because they were easy to follow, and the students had had a lengthy introduction. Also, the same notion goes for over-complication. Essentially, the game should fit the intended audience and their capabilities.

Multiplicative Thinking and Gamification

As discussed previously, it takes time for students to become multiplicative thinkers (Hurst, 2015, p. 15). One way that teachers can support their students in multiplicative reasoning would be to introduce interactive and fun activities such as math board games. Creating the connections needed for students to elevate their logic does not just come from traditional activities (i.e., textbook lessons and worksheets). Students need rich discussion that allows them to dig into a concept from multiple angles (Hurst, 2015, p. 12). The board game presented in this portfolio will allow students to explore strategies they may not have tried before and will encourage discussion between players. Also, the strategies that students will use while playing the board game will help them become better with numbers which will aid their transition from additive reasoning to multiplicative thinking.

Mathematical board games are great for students because they allow students to see mathematical concepts visually and in many ways. Instead of continually completing problems on worksheets, students can see the same types of questions differently. To illustrate, when students are learning to reason multiplicatively, visuals can help scaffold students' mathematical thinking. Students may use visuals such as "groups‐of models, arrays, area models, and variations of length models" (Kosko, 2019). It is beneficial to expose students to multiple representations because it will help them garner a better understanding of mathematical concepts (Milton, Flores, Moore, Taylor, & Burton, 2019). Students will finally see that math is not stagnant; one can see an equation visually and see the malleability of numbers. The more representations students see, the more effortlessly they will notice the flexibility that numbers hold. The board game discussed in this portfolio allows students to learn about and use multiple representations in the hopes that practicing these skills will help them gain a conceptual understanding of multiplication.

Conclusion

To conclude, this literature review has discussed many elements that have helped to inform the creation of my multiplication board game. I had to consider multiplicative thinking, the difference between conceptual and procedural understanding, multiplication strategies, the commutative and distributive properties, and gamification. These elements will be present in the game and have influenced the creation and execution of my board game.

The Four Elements of my Game

My multiplication board game is focused on four main elements: procedural and conceptual understanding, strategies, and fun. First, I wanted students to have a procedural understanding of multiplication because they need that foundation. It is essential that students know the elements (i.e., where to put their answer, recognition of the times symbol, etc.) of a multiplication equation and understand how to solve it. Along with building this foundation, students must also develop a conceptual understanding of multiplication. Developing a conceptual understanding, enables students to recognize the “links and relationships and… [understand] how ideas are connected and why processes work as they do” (Hurst & Hurrell, 2016, p. 34). When designing my game, I wanted students to be able to see links and relationships outside the confines of memorizable facts. It is essential that students see how numbers and patterns are connected and that they can work with equations to obtain answers in a helpful way. For example, to begin to develop a deeper conceptual understanding of multiplication, students need to be introduced to the distributive property. To illustrate, if they encountered the equation 13 x 7, they would know that they could manipulate the question to make it more accessible. They could change the equation to 10 x 7 and 3 x 7 and add the products to find the answer. A conceptual understanding allows students to see and solve in different ways that extend beyond the act of memorization. If students have a predominately procedural understanding before playing my board game, after playing, I would hope that they have also developed a conceptual understanding. My game will challenge students and support them in recognizing that multiplication is more than memorizing facts.

An additional element of my board game is that it tasks players to use multiple strategies. Some students may stick to one strategy because they have only learned one approach to multiplication. By only utilizing one procedure, students miss out on other ways to calculate multiplication problems and do not have an opportunity to develop deep conceptual understanding. For example, Bobis' (2007) research found that “the worry with an early emphasis on standard algorithms is that students will shift their focus to executing convenient procedures rather than on understanding the mathematics” (Bobis, 2007, p. 23). I don’t want students to rely on only one method to solve multiplication problems. If students only use one strategy, that might stunt their understanding and limit their mathematical learning. My game will have students try procedures such as equal groups, repeated addition, number lines, open arrays, and partial products. Students won’t have a choice with the strategies they use; and will try some that they initially may feel uncomfortable using. It’s important to note that my game will have support, so students feel comfortable using all the required strategies (the specific supports will be discussed in a following section). Essentially, the goal of my board game is slowly to get students comfortable with taking risks and challenging themselves with math. Utilizing multiple strategies is one way that students can learn to challenge themselves and expand their math prowess.

The last element that I wanted to include in my board game was the inclusion of fun. I want all students, no matter their mathematical ability, to play my game with ease. That’s why I want to make sure the use of multiple strategies is accessible to all students so they are not concerned about remembering particular multiplication strategies. This math game will allow students to have fun because they will have assistance in multiple ways (i.e., strategy cards, classmates, teacher assistance). Students won’t need to concern themselves with anything but following the rules and enjoying themselves. Once the elements of my board game (i.e., procedural and conceptual understanding, strategies, and fun) were finalized, then the design and rules were formulated.

An Overview of the Malleable Multiplication Game

The importance of Strategies

When researching multiplication, I came across a poignant idea that made me reconsider how strategies would fit into my game. The idea was that students usually utilize only one procedure and don't explore others when they find one that works for them. When playing my board game, I want to encourage students to use multiple methods to solve problems (Anghileri, 1989; Bell et al., 1984; Calabrese et al., 2020; Clark & Kamii, 1996). I hope this stipulation will allow students of all abilities to play my board game with one another and garner a conceptual understanding of multiplication. Whether students are strong in math or struggle with math, most students tend to use only one strategy that they are comfortable with to solve multiplication problems. My board game will encourage both groups of students to use multiple procedures and challenge themselves. Even if a student is doing well academically, it's integral that they continue to be challenged and use different strategies. Every student who plays my board game can experience using other procedures and working together through challenges.

Since it's integral to use multiple strategies, I decided to focus on five main strategies: equal groups, repeated addition, number lines, open arrays, and partial products. I wanted students to use strategies that were promoted/discussed across various scholarly articles and ones they would be familiar with in some capacity. Another benefit of using these procedures is that students may see the similarities between particular strategies. For example, it'll be interesting to see if students see the similarities between repeated addition and the equal groups strategy. While using repeated addition to solve an equation, hopefully, students realize that they are adding equal groups of the multiplicand as many times as the multiplier indicates. For example, if students encounter the following problem 7 x 3, they know that they can add the multiplicand 7 three times (as the multiplier indicates) to get the correct answer. Students might realize that when they add 7 three times, they're adding three equal groups of seven. Thus, recognizing the similarities between both strategies. Therefore, it is vital that students get to experiment with multiple procedures, so they can get a better understanding of multiplication and expand their knowledge base.

Strategy Cards

The first thing I created for my game was the strategy cards. The strategy cards include a description of the procedure and a visual depicting whichever strategy the card focuses on (i.e., partial products algorithms). The following would provide students, at any level, with something to lean on in case they forget how to use a procedure they need for the game. For a student who struggles with math, the strategy cards might quell some anxiety they might have when playing the game with classmates that they don't usually interact with daily. It will also help students who generally do well in math because they don't have to have everything memorized. Students can lean on the cards until the strategies become second nature. Having students use the strategy cards, regardless of their mathematical acumen, will hopefully help instil those strategies in students allowing them more options with equations. My game isn't supposed to make students struggle or feel inadequate. My game will appropriately challenge learners with the aim that students will develop a better understanding of multiplication after playing. Even though the board game is supposed to help students understand multiplication, that doesn't negate the fact that games are supposed to be enjoyable; when students don't have to worry about remembering strategies, that will hopefully help them relax and have fun.

Supports

Teachers can implement two main supports with my game. One of the supports that teachers can put in place, besides the strategy cards, is to have audio recordings of the cards. Audio recordings will help students who are auditory learners or struggle with reading. The teacher could have the recordings labelled on a device (if the teacher has access to technology), and students could access those recordings easily. This will allow for more inclusion and make every student feel validated in their learning style. Another support would be having manipulatives available when students play. The role of manipulatives in my game is integral. The only equipment teachers must have include dice, which the students will use to take their turns. I wanted to ensure that my game didn't require specific manipulatives because all classrooms have different materials, and some may have more than others. I want all teachers to be able to provide this game for their students without feeling like they have to buy new materials or substitute for required materials. Some manipulatives that would work well with my board game are counting chips, mathlink cubes, square tiles, and number lines. Also, some manipulatives may double as game tokens that students will use to track their place on the board as they play the game. Teachers can use whatever manipulatives they think will help their students while playing my game.

Collaboration

One social element I wanted to emphasize with my game is collaboration. Students may feel comfortable collaborating with friends but may feel nervous asking other students for help. I want my game to encourage students to work together if they need help instead of sitting and struggling. I don't want students giving other students the answers to questions. Instead, I want students to work with others if they need help. My game is supposed to promote learning, not parroting answers. Students need to understand that they are helping when they assist students with problem-solving. Secondly, the other type of collaboration that would be ideal is encouraging students to be kind and praise others. Praise and kindness should be doled out regardless of a correct answer. Showing others kindness even when competing is a good skill that will stick with students throughout their lives and academic journey. To ensure that concept of collaboration, teachers can explain the acceptable types of collaboration.

Who should Play Malleable Multiplication?

My board game can be played by students from grade 3 (when multiplication is first introduced) to grade 8. The skills that students learn concerning problem-solving strategies and the basis of multiplication will be beneficial throughout their education. Students need to know and understand the commutative and distributive properties. When students practice multiple procedures, they will see that multiplication doesn't just consist of memorizable facts. Instead, they will see that multiplication is based on numeric relationships and that one can solve equations in many ways. When students have a well-rounded understanding of multiplication, it will help them with other mathematical concepts.

How to Play Malleable Multiplication

My game allows up to six players to play at one time. Students can play the game by themselves, or they can play with classmates. If a student plays alone, they don't need to keep score but will have a chance to practice their multiplication skills using the game. If two or more students are playing the game, the students will roll the dice to decide who goes first. The player who rolls the highest number will go first. Then the turns will go in a clockwise direction. Students can set their selected tokens (tokens can be buttons, manipulatives, or any small item) at the starting point on the board.

Students will notice that the board game has three distinct colours; orange, red, and purple. These colours are present on the board game squares and question cards and signify point values. Red question cards are worth 3 points, orange question cards are worth 2 points, and purple question cards are be worth 1 point. The cards will have different multiplication sets. The red question cards will have questions ranging from the 8 to 12 times tables; the orange question cards will have questions ranging from the 4 to 7 times tables; and the purple question cards will have questions ranging from the 0 to 3 times tables.

Once the students are familiar with the gameplay and board, they can start playing. Every student will begin their turn by rolling the dice and going to the indicated square. To illustrate, if a student rolls and lands on an orange square, said student will pick up an orange card and a strategy card and then attempt to answer the question; if they answer the question correctly, they will receive two points. Students will keep track of each player's scores on paper. Each square that students land on gives them a chance to earn points. The colour of the square will indicate the point value of the question the student is answering. Every time a student lands on a square and selects their question from the coloured pile, they have to pick a strategy card before they can answer the question on the question card. Also, students can't select a new strategy card unless they randomly pick up a strategy card with the words 'free choice' from the stack or land on a free choice square. The 'free choice' card/square allows players to pick any one of the strategies when answering their selected question card. The rest of the strategy cards will include a description of the strategy, a visual, and an example problem. Having this extra information on the strategy cards will hopefully ensure that all students can re-familiarize themselves with multiplication strategies and, or will help remind them of the procedure they need to solve. Students will continue to take turns and move around the squares on the board until they reach the finish square. When students land on the final square, they can pick a question card from the red, orange, and, or purple pile and choose a strategy. Once every player has landed on the last square (it has to be an exact roll to land on the final square) and answered their final question, the game will be over. Whichever student has the highest score wins the game. However, I would encourage teachers to emphasize that all students are winners in their own right by practicing multiple strategies and flexing their multiplication knowledge.

My board game has a few factors that might make it difficult to time how long one game will take. One factor that could make the game longer depends on the dice rolls. If students roll lower numbers, they will move around the board slowly, and they will have the potential to answer more questions. On the other hand, if students roll large numbers, they will go around the board more quickly and answer fewer questions. Another factor that affects the game length is the time it takes students to answer questions. If students are unfamiliar with any of the strategies they encounter, it might take them longer to work out how to use the procedure to answer the question; some students may take longer when answering a question with a larger multiplier. The last factor that would make it difficult to determine the game length depends on the students' prowess in multiplication. If a student is well-versed in multiplication, and comprehends the strategies quickly, that can move the game along more quickly. However, if a student struggles with multiplication, they might take longer using the procedures to answer the questions, which would impact the game length. Thus, it's difficult to determine the exact time a game can take. If the time to play the game runs out, then students can calculate the scores, and the highest score wins regardless of whether everyone completed the game.

Malleable Multiplication Manual

Number of Players: 1-6

Age Range: 7+

Contents: game board, strategy cards (teachers should print four sets of the strategy cards), and question cards.

Materials Needed (not provided with the game): 1 die (1-6), manipulatives (teachers choice), tokens (i.e., buttons, manipulatives, or any small item), and scrap paper (to work out problems; if needed).

Description: Malleable Multiplication will help students hone their procedural and conceptual understanding of multiplication. This game will help students garner a deeper understanding of multiplication by having them solve problems using multiple strategies. These various procedures will help students see the connection the strategies have with one another.

Game Objective: Students will solve multiplication problems and move around the board until they get to the "Finish" square. Whichever student has the most points at the end (earned when they correctly answer questions using the dictated strategy) will win the game.

Game Board Squares

- The game board includes a start square and finish square.

- There will be squares on the board that will say ‘Free’. These squares allow players to choose their question card (red, orange, or purple) and decide what strategy they want to use.
- The other squares on the board are either red, orange, or purple. The red, orange, and purple squares will indicate what question cards the players will answer.

Set-up:

i) Players will place the board in an accessible place so everyone can reach it when it’s their turn. The teacher can select one of four boards. One board will have the red, orange, and purple squares and the others will be split up by colour. For example, if the teacher wants students to focus on the times tables up to three they would use the purple board. Then one of the players will shuffle the strategy and question cards.

ii) Students will place the shuffled deck of strategy cards to the side of the board.

iii) Students will place the three shuffled question decks to the side of the board.

iv) Students will select the manipulatives they want to use to help them solve equations. (The manipulatives that students use will depend on what is in their classroom and what the teacher selects).

Getting Started

i) Before the game begins, the players will pick a scorekeeper. The scorekeeper will keep track of all players' points throughout the game. The scorekeeper will use tally marks to track scores on a piece of paper.

ii) All players will place their tokens on the “Start” square before the game begins.

iii) Next, the players will roll the dice to determine the playing order. The player with the highest roll will go first, and the subsequent order will be in a clockwise rotation.

How to Play

- At the start of each player's turn, they will roll the dice to determine how many squares their token will be moving on the game board. Depending on the dice roll, students will move their token 1 to 6 squares.
- When students land on the square, determined by their dice roll, they will select a question card that is the same colour as the square they landed on. For example, if a student rolled a 4 and landed on a red square, they would select a red question card from the deck.
- After students select a question card based on their role, they will pick a strategy card off the top of the deck.
- Students will answer their questions using the strategy they selected from the strategy card deck. Students can use scrap paper and manipulatives to solve their problems.
- If students get the correct answer, they will receive a point value based on the colour of the question card. For example, if students correctly answer a red question card, they will receive 3 points, an orange question card will be worth 2 points, and purple question cards will be worth 1 point. The scorekeeper will keep track of how many points players earn. Students will only receive points if they correctly answer the equation on the question card *using the strategy that the strategy card indicated* (no partial points).
- If students can't answer a question, they will remain on the spot they rolled to at the beginning of their turn. Although, their classmates can choose to help the student who is stuck answer the question as long as they don’t give away the answer.
- Students will continue rolling the dice, answering questions, and moving their tokens around the game board until they land on the final square.

- When players land on the final square, they will choose a question card and strategy of their choosing and answer their last question. If students get that question right, the points will be added to their total score.
- Once all players have landed on the final square and completed their last question, the scores will be added, and a winner will be determined.

Modifications

Teachers can introduce a few modifications depending on their circumstances. One modification teachers can implement is making the game shorter. For example, the teacher can shorten the game by telling students to skip the red spaces and only answer questions on the orange and purple squares. When students land on the red square, the teacher can either tell them to re-roll until they land on an allowed space or wait until their next turn for a chance to answer a question. The teacher could implement this rule for any of the other square colours. Also, the teacher may choose to have students only land on squares that reflect their current multiplication understanding. For example, if students have only been working on their times tables up until the seven times tables, then the teacher can tell the students that the only squares they will answer questions on are the orange and purple cards. Also, if teachers want students to focus on just one set of times tables they can use the other available boards that solely focus on one of each card colour (i.e., a purple board focusing on times tables up to 3). These modifications will allow students to practice their multiplication prowess and help students focus and not get overwhelmed.

Another modification that teachers can implement is with the strategy cards. If teachers are working on one strategy, they could remove the other strategy cards and have students only use one procedure to solve the multiplication problems. For example, if the teacher wants students to focus on repeated addition, they could have their students answer all the questions using that strategy. Although using the repeated addition strategy to answer every question might get tedious, it would meet the teacher’s aim to encourage students to become proficient at using multiple strategies. Another reason a teacher might want to control the usage of the strategy cards is if they have students who struggle with a particular procedure. To illuminate, if a group of students struggle with partial products, the teacher could give the students extra practice by having the students play my board game only using that strategy. Students may improve their skills and understand the procedure better because they are playing a game rather than completing a worksheet. Lastly, the teacher could let students play a round of the game where they get to pick their own strategies. The strategy cards could be available during the game for students who want to use them. Letting students choose their own procedures might give the teacher information about how their students are solving problems and whether the strategies students are using are right for the problems they're solving. Ultimately, when it comes to the strategy cards, the teacher can modify them as they see fit.

Intrinsic and Extrinsic Motivation and my Game

There are two main types of motivation: intrinsic and extrinsic. Intrinsic motivation is when the individual participates in a task and or activity only to participate and or gain requisite knowledge (Cerasoli, Nicklin, & Ford, 2014, p. 980). To illustrate, if an individual is completing a science project and is intrinsically motivated, the student would be happy just completing the task and knowing that they were learning something that would serve them later in their academic journey. Intrinsic motivation is the opposite of extrinsic motivation. Extrinsic motivation is when one completes a task to obtain a reward (i.e., a sticker or a prize) (Cerasoli et al., 2014, p. 980). Unlike intrinsic motivation, when someone is extrinsically motivated, they are not completing the task for the sake of knowledge. In that case, they need something else to complete the activity/task. Throughout the literature, there's still debate concerning which motivation is the best. Most articles lean towards intrinsic motivation as superior however, because without rewards, students see the knowledge they've obtained as the actual reward.

When it came to my board game, I wanted to give teachers the discretion to choose whether to keep the game as is, focusing on intrinsic motivation since the game comes with no prizes. Alternatively, teachers could opt to reward the student who wins the game, or all the game participants. I believe it's integral for teachers to assess the situation and their learners when deciding whether to intrinsically or extrinsically motivate their students. Each type of motivation has benefits and disadvantages. For example, if students are required to play the game five times before they receive a reward (i.e., a sticker or a prize out of a prize bin) this type of extrinsic motivation may also give students a chance to derive intrinsic motivation because they would have to commit to playing the game multiple times to derive a benefit. The students may find that at the end of playing all the required games, they want to keep playing because they had fun. Thus, teachers must consider a lot of elements with both types of motivation when deciding which motivation type they should use to introduce my board game. Ultimately, I believe the teacher is the best person to determine what motivates their students.

Reflection on the Process of Creating the Malleable Multiplication Game

My multiplication board game had quite a few iterations before the final product came to fruition. Throughout the process, I faced two main challenges. One challenge was creating a game that balanced fun game elements and ensured students would grasp a procedural and conceptual understanding of multiplication. The other challenge was creating a unique game. When I think about board games, it conjures thoughts of Monopoly, The Game of Life, and Sorry. These board games all have distinct elements that make each enjoyable/fun. This element made it a bit challenging to think ‘outside the box’ and design a game that didn’t resonate with games I’ve played previously. At first, to get away from classic game archetypes, I created games that would be too complicated for my target audience. The games I designed had too many steps for a young audience to balance without an adult leading. I was encouraged to remove a lot of the steps and rethink the structure. After taking steps away and simplifying the game, it started to resemble the game in this portfolio.

Once the two main design challenges were fleshed out and resolved I had to make creative decisions concerning the layout and rules of my game. One of the big decisions was determining if my board game squares would hold information or if I should create cards that students pick up that include the instructions. At first, I wanted each square on the board to have a point value and have the questions decided by the sums of dice rolls. After reflecting on the game, the better strategy was to have question and strategy cards. The question cards include a multiplication question. The card's colour determines the point value for that question. Depending on the question card, students can earn up to three points if they answer correctly. The strategy cards ensure that all players practice multiple multiplication strategies and don’t simply rely on known procedures.

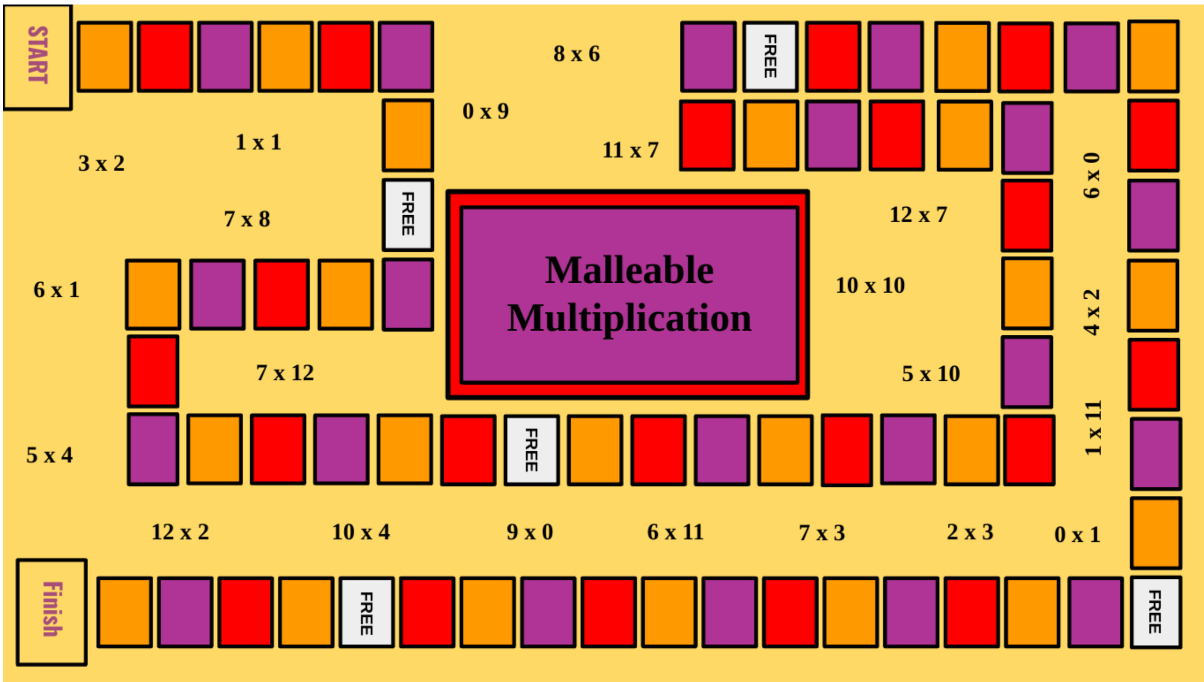
After I’d made decisions regarding the layout of the game board I focused on the aesthetics. When I was working on the physical design of my board game, I knew I wanted to use a bright colour palette. My eyes gravitate toward bright colours, and I hope that students will gravitate toward my game for the same reason. Also, I wanted the colours to be bright and distinguishable; so students would not get confused and mistake one colour for another. I wanted my game to look simple, because the knowledge students take away from the game is the most important aspect of the game. I want students to like the game's aesthetic, but I don't want it to serve as a distraction.

All in all, conceptualizing and designing my board game was challenging. I initially thought it would be easy to dream up and execute an original idea, but that wasn’t the case. It was hard to create design elements that meshed with the learning elements I wanted in my game. Initially, I was getting caught up in everything from the design to the learning factor. I later realized that the important part of my game is the learning aspect, and the other elements would align after that was fleshed out. My main goal was, and still is, that students take away fun memories from playing but also understand multiplication better.

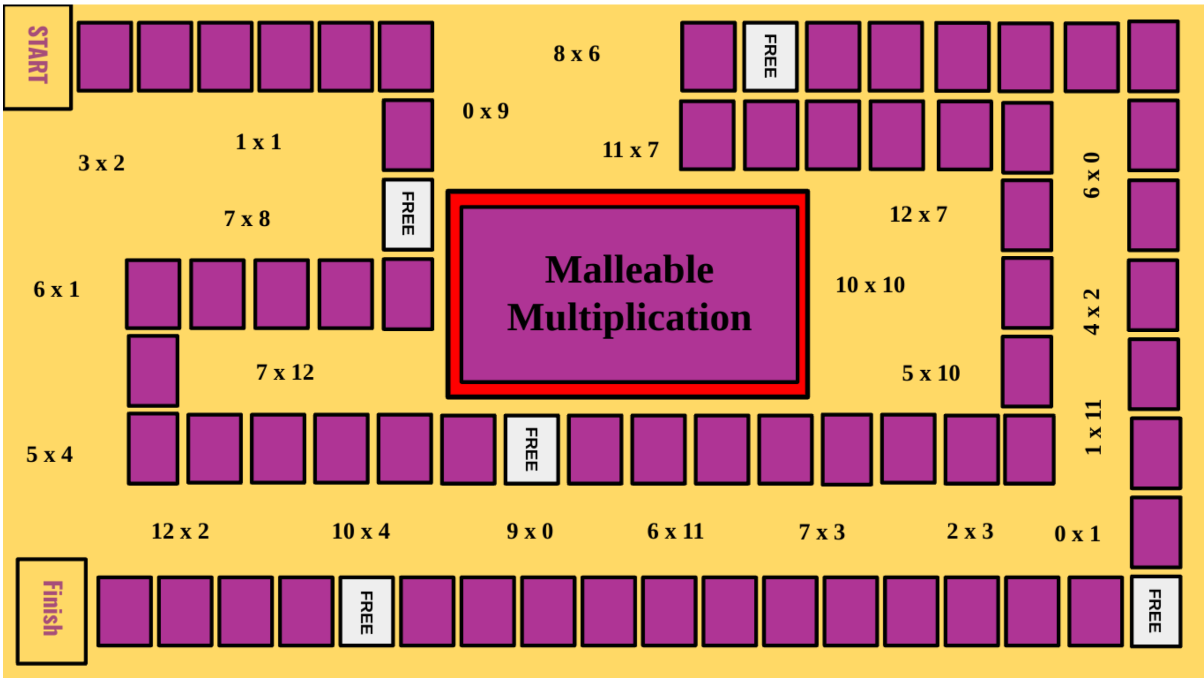
Final Thoughts

I wish that I had been exposed to multiplication games when I was younger. In elementary school, I was pulled out of math class and went to the resource room to practice multiplication one-on-one with my resource teacher. The methods my resource teacher used were flashcards and making me recite the multiplication table. I did not enjoy practicing multiplication this way, and it made me a bit nervous because I wanted to be able to recall the correct answers and make my resource teacher happy. When I couldn’t remember the answers, I didn’t feel good about myself. If I'd been exposed to alternative ways to learn multiplication and the associated strategies, I believe I would have been better served, and my confidence in math would have been greater. I assume that if students can learn and have fun simultaneously, it will foster their appreciation of the subject matter.

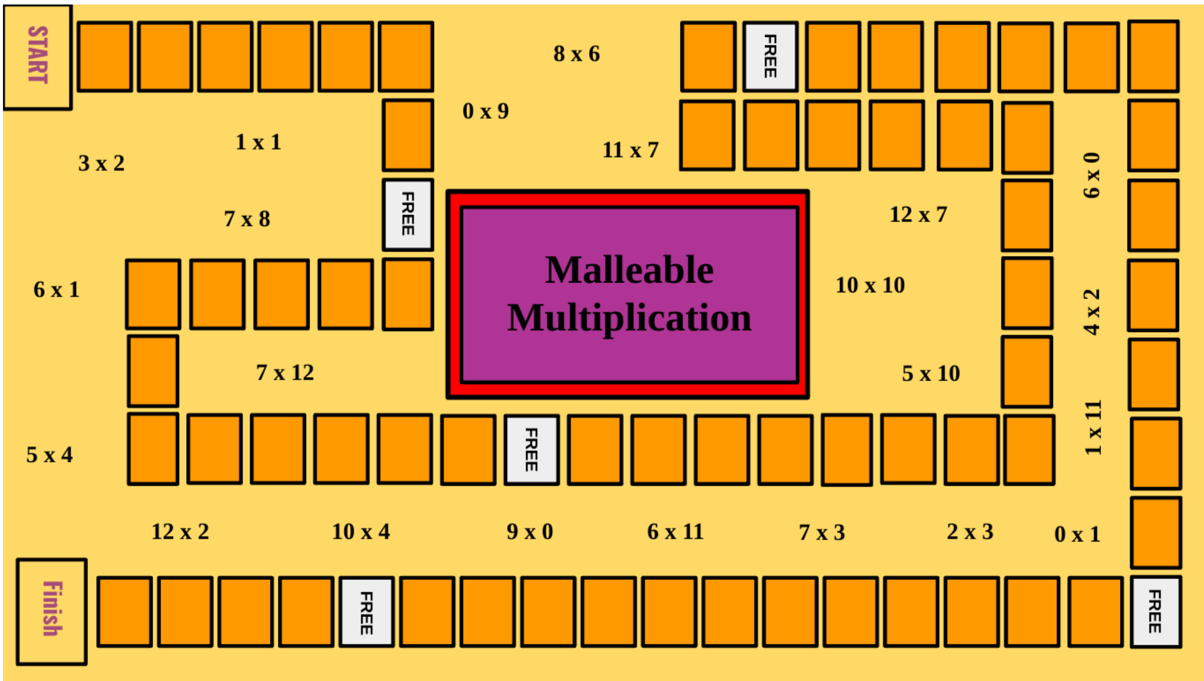
Appendix



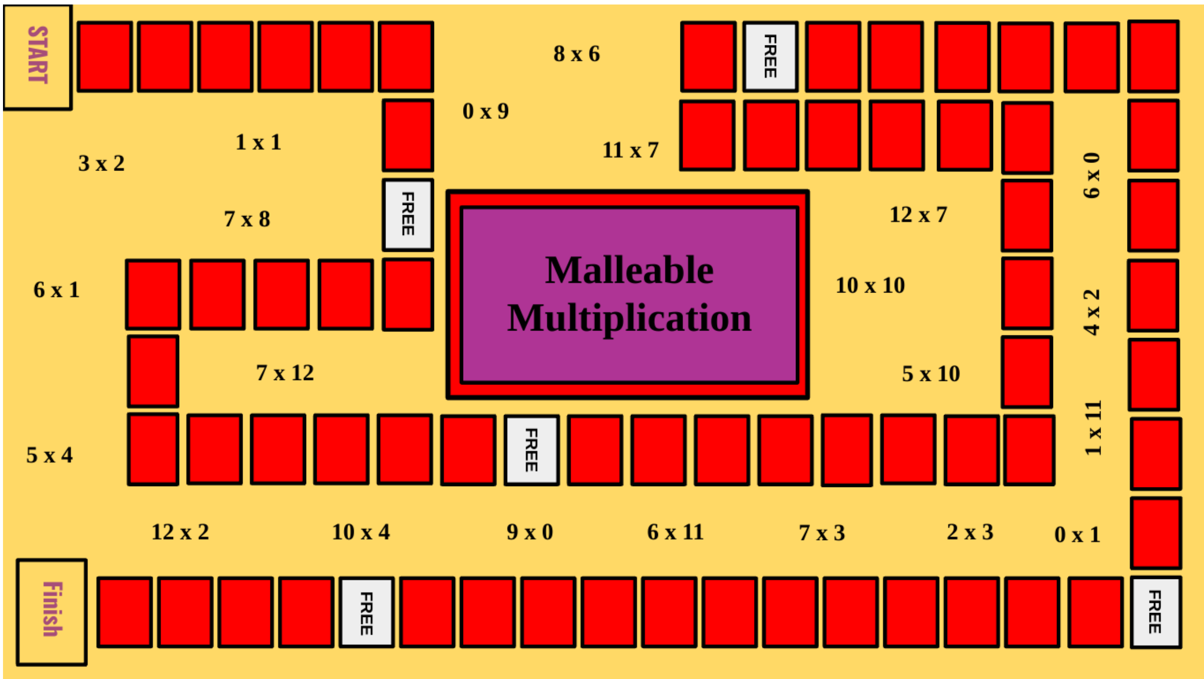
Malleable Multiplication Board (1.1)



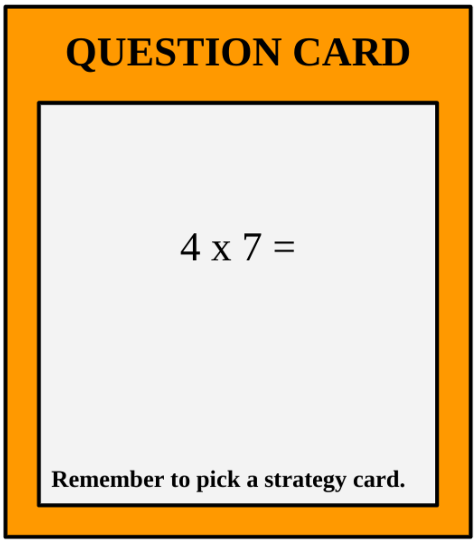
Malleable Multiplication Board (1.2)



Malleable Multiplication Board (1.3)



Malleable Multiplication Board (1.4)



Question Card Sample (1.2)

QUESTION CARD 0 x 0 = Remember to pick a strategy card.	QUESTION CARD 0 x 1 = Remember to pick a strategy card.	QUESTION CARD 0 x 2 = Remember to pick a strategy card.	QUESTION CARD 0 x 3 = Remember to pick a strategy card.	QUESTION CARD 0 x 4 = Remember to pick a strategy card.
QUESTION CARD 0 x 5 = Remember to pick a strategy card.	QUESTION CARD 0 x 6 = Remember to pick a strategy card.	QUESTION CARD 0 x 7 = Remember to pick a strategy card.	QUESTION CARD 0 x 8 = Remember to pick a strategy card.	QUESTION CARD 0 x 9 = Remember to pick a strategy card.

QUESTION CARD 0 x 10 = Remember to pick a strategy card.	QUESTION CARD 0 x 11 = Remember to pick a strategy card.	QUESTION CARD 0 x 12 = Remember to pick a strategy card.	QUESTION CARD 1 x 0 = Remember to pick a strategy card.	QUESTION CARD 1 x 1 = Remember to pick a strategy card.
QUESTION CARD 1 x 2 = Remember to pick a strategy card.	QUESTION CARD 1 x 3 = Remember to pick a strategy card.	QUESTION CARD 1 x 4 = Remember to pick a strategy card.	QUESTION CARD 1 x 5 = Remember to pick a strategy card.	QUESTION CARD 1 x 6 = Remember to pick a strategy card.

Question Cards (1.3)

QUESTION CARD
1 x 7 =
Remember to pick a strategy card.

QUESTION CARD
1 x 8 =
Remember to pick a strategy card.

QUESTION CARD
Press Esc to exit full screen
1 x 9 =
Remember to pick a strategy card.

QUESTION CARD
1 x 10 =
Remember to pick a strategy card.

QUESTION CARD
1 x 11 =
Remember to pick a strategy card.

QUESTION CARD
1 x 12 =
Remember to pick a strategy card.

QUESTION CARD
2 x 0 =
Remember to pick a strategy card.

QUESTION CARD
2 x 1 =
Remember to pick a strategy card.

QUESTION CARD
2 x 2 =
Remember to pick a strategy card.

QUESTION CARD
2 x 3 =
Remember to pick a strategy card.

QUESTION CARD
2 x 4 =
Remember to pick a strategy card.

QUESTION CARD
2 x 5 =
Remember to pick a strategy card.

QUESTION CARD
2 x 6 =
Remember to pick a strategy card.

QUESTION CARD
2 x 7 =
Remember to pick a strategy card.

QUESTION CARD
2 x 8 =
Remember to pick a strategy card.

QUESTION CARD
2 x 9 =
Remember to pick a strategy card.

QUESTION CARD
2 x 10 =
Remember to pick a strategy card.

QUESTION CARD
2 x 11 =
Remember to pick a strategy card.

QUESTION CARD
2 x 12 =
Remember to pick a strategy card.

QUESTION CARD
3 x 0 =
Remember to pick a strategy card.

Question Cards (1.3)

QUESTION CARD

3 x 1 =

Remember to pick a strategy card.

QUESTION CARD

3 x 2 =

Remember to pick a strategy card.

QUESTION CARD

3 x 3 =

Remember to pick a strategy card.

QUESTION CARD

3 x 4 =

Remember to pick a strategy card.

QUESTION CARD

3 x 5 =

Remember to pick a strategy card.

QUESTION CARD

3 x 6 =

Remember to pick a strategy card.

QUESTION CARD

3 x 7 =

Remember to pick a strategy card.

QUESTION CARD

3 x 8 =

Remember to pick a strategy card.

QUESTION CARD

3 x 9 =

Remember to pick a strategy card.

QUESTION CARD

3 x 10 =

Remember to pick a strategy card.

QUESTION CARD

3 x 11 =

Remember to pick a strategy card.

QUESTION CARD

3 x 12 =

Remember to pick a strategy card.

Question Cards (1.3)

QUESTION CARD 4 x 0 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 1 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 2 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 3 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 4 = **Remember to pick a strategy card.**
QUESTION CARD 4 x 5 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 6 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 7 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 8 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 9 = **Remember to pick a strategy card.**

QUESTION CARD 4 x 10 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 11 = **Remember to pick a strategy card.**	**QUESTION CARD** 4 x 12 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 0 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 1 = **Remember to pick a strategy card.**
QUESTION CARD 5 x 2 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 3 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 4 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 5 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 6 = **Remember to pick a strategy card.**

Question Cards (1.4)

QUESTION CARD 5 x 7 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 8 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 9 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 10 = **Remember to pick a strategy card.**	**QUESTION CARD** 5 x 11 = **Remember to pick a strategy card.**
QUESTION CARD 5 x 12 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 0 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 1 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 2 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 3 = **Remember to pick a strategy card.**

QUESTION CARD 6 x 4 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 5 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 6 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 7 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 8 = **Remember to pick a strategy card.**
QUESTION CARD 6 x 9 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 10 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 11 = **Remember to pick a strategy card.**	**QUESTION CARD** 6 x 12 = **Remember to pick a strategy card.**	**QUESTION CARD** 7 x 0 = **Remember to pick a strategy card.**

Question Cards (1.4)

QUESTION CARD

7 x 1 =

Remember to pick a strategy card.

QUESTION CARD

7 x 2 =

Remember to pick a strategy card.

QUESTION CARD

7 x 3 =

Remember to pick a strategy card.

QUESTION CARD

7 x 4 =

Remember to pick a strategy card.

QUESTION CARD

7 x 5 =

Remember to pick a strategy card.

QUESTION CARD

7 x 6 =

Remember to pick a strategy card.

QUESTION CARD

7 x 7 =

Remember to pick a strategy card.

QUESTION CARD

7 x 8 =

Remember to pick a strategy card.

QUESTION CARD

7 x 9 =

Remember to pick a strategy card.

QUESTION CARD

7 x 10 =

Remember to pick a strategy card.

QUESTION CARD

7 x 11 =

Remember to pick a strategy card.

QUESTION CARD

7 x 12 =

Remember to pick a strategy card.

Question Cards (1.4)

QUESTION CARD 8 x 0 = Remember to pick a strategy card.	QUESTION CARD 8 x 1 = Remember to pick a strategy card.	QUESTION CARD 8 x 2 = Remember to pick a strategy card.	QUESTION CARD 8 x 3 = Remember to pick a strategy card.	QUESTION CARD 8 x 4 = Remember to pick a strategy card.
QUESTION CARD 8 x 5 = Remember to pick a strategy card.	QUESTION CARD 8 x 6 = Remember to pick a strategy card.	QUESTION CARD 8 x 7 = Remember to pick a strategy card.	QUESTION CARD 8 x 8 = Remember to pick a strategy card.	QUESTION CARD 8 x 9 = Remember to pick a strategy card.
QUESTION CARD 8 x 10 = Remember to pick a strategy card.	QUESTION CARD 8 x 11 = Remember to pick a strategy card.	QUESTION CARD 8 x 12 = Remember to pick a strategy card.	QUESTION CARD 9 x 0 = Remember to pick a strategy card.	QUESTION CARD 9 x 1 = Remember to pick a strategy card.
QUESTION CARD 9 x 2 = Remember to pick a strategy card.	QUESTION CARD 9 x 3 = Remember to pick a strategy card.	QUESTION CARD 9 x 4 = Remember to pick a strategy card.	QUESTION CARD 9 x 5 = Remember to pick a strategy card.	QUESTION CARD 9 x 6 = Remember to pick a strategy card.

Question Cards (1.5)

QUESTION CARD 9 x 7 = Remember to pick a strategy card.	QUESTION CARD 9 x 8 = Remember to pick a strategy card.	QUESTION CARD 9 x 9 = Remember to pick a strategy card.	QUESTION CARD 9 x 10 = Remember to pick a strategy card.	QUESTION CARD 9 x 11 = Remember to pick a strategy card.
QUESTION CARD 9 x 12 = Remember to pick a strategy card.	QUESTION CARD 10 x 0 = Remember to pick a strategy card.	QUESTION CARD 10 x 1 = Remember to pick a strategy card.	QUESTION CARD 10 x 2 = Remember to pick a strategy card.	QUESTION CARD 10 x 3 = Remember to pick a strategy card.

QUESTION CARD 10 x 4 = Remember to pick a strategy card.	QUESTION CARD 10 x 5 = Remember to pick a strategy card.	QUESTION CARD 10 x 6 = Remember to pick a strategy card.	QUESTION CARD 10 x 7 = Remember to pick a strategy card.	QUESTION CARD 10 x 8 = Remember to pick a strategy card.
QUESTION CARD 10 x 9 = Remember to pick a strategy card.	QUESTION CARD 10 x 10 = Remember to pick a strategy card.	QUESTION CARD 10 x 11 = Remember to pick a strategy card.	QUESTION CARD 10 x 12 = Remember to pick a strategy card.	QUESTION CARD 11 x 0 = Remember to pick a strategy card.

Question Cards (1.5)

QUESTION CARD

11 x 1 =

Remember to pick a strategy card.

QUESTION CARD

11 x 2 =

Remember to pick a strategy card.

QUESTION CARD

11 x 3 =

Remember to pick a strategy card.

QUESTION CARD

11 x 4 =

Remember to pick a strategy card.

QUESTION CARD

11 x 5 =

Remember to pick a strategy card.

QUESTION CARD

11 x 6 =

Remember to pick a strategy card.

QUESTION CARD

11 x 7 =

Remember to pick a strategy card.

QUESTION CARD

11 x 8 =

Remember to pick a strategy card.

QUESTION CARD

11 x 9 =

Remember to pick a strategy card.

QUESTION CARD

11 x 10 =

Remember to pick a strategy card.

QUESTION CARD

11 x 11 =

Remember to pick a strategy card.

QUESTION CARD

11 x 12 =

Remember to pick a strategy card.

QUESTION CARD

12 x 0 =

Remember to pick a strategy card.

QUESTION CARD

12 x 1 =

Remember to pick a strategy card.

QUESTION CARD

12 x 2 =

Remember to pick a strategy card.

QUESTION CARD

12 x 3 =

Remember to pick a strategy card.

QUESTION CARD

12 x 4 =

Remember to pick a strategy card.

QUESTION CARD

12 x 5 =

Remember to pick a strategy card.

QUESTION CARD

12 x 6 =

Remember to pick a strategy card.

QUESTION CARD

12 x 7 =

Remember to pick a strategy card.

Question Cards (1.5)

QUESTION CARD	QUESTION CARD	QUESTION CARD	QUESTION CARD	QUESTION CARD
12 x 8 =	12 x 9 =	12 x 10 =	12 x 11 =	12 x 12 =
Remember to pick a strategy card.	**Remember to pick a strategy card.**	**Remember to pick a strategy card.**	**Remember to pick a strategy card.**	**Remember to pick a strategy card.**

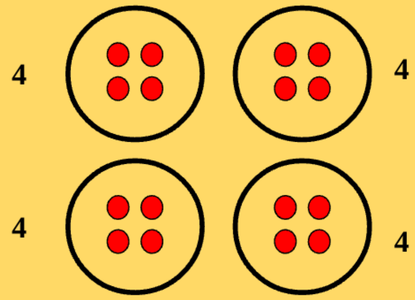
Question Cards (1.5)

Strategy Card - Equal Groups

Equal Groups: when using this strategy, you will **draw groups that show the same amount**.
Then you will **count all the numbers in the groups to get the final answer**.

In the example problem, you have to make 4 groups of 4. A good strategy is to put a circle around each group. Then you can see how many groups have been created. Once all 4 groups of 4 have been created, you will add up the items in the groups to get the answer.

For example: 4 x 4 = 16



Strategy Card (1.6)

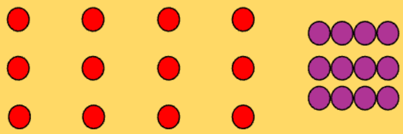
Strategy Card - Repeated Addition

Repeated Addition: A number is added multiple times (according to the numbers in the question) to get the answer.

For example: 3 x 4 = 12

Instead of multiplying, you would **add 3 four times** to get the answer.

3 + 3 + 3 + 3 = 12



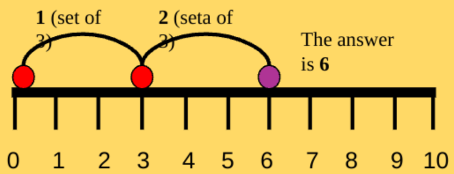
Strategy Card (1.7)

Strategy Card - Number Lines

Number Lines: are used to count up to the answer. When using a number line, the first step is to place the larger number in the question on the number line. The next step is to ‘hop’ using the large number by the other number in the question.

For example: 3 x 2 = 6

Place the largest number in the question on the number line. In this question, you would **place three first**. Then **count up by three one more time** because by starting at **three,** you already have one set of three, so you need to make **one more to have two sets.**



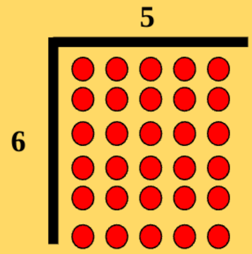
Strategy Card (1.8)

Strategy Card - Single-Digit Arrays

Single Digit-Arrays: help people see the numbers in problems. The single-digit array allows individuals to draw or use manipulatives to show what the numbers represent.

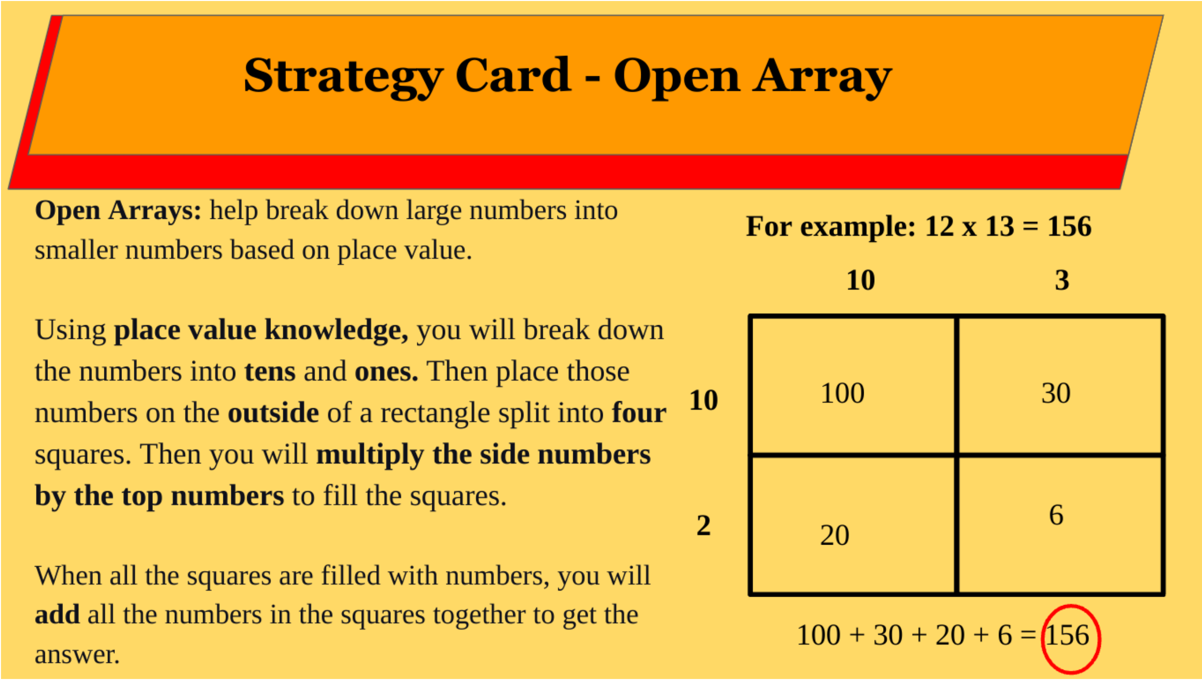
To start, you would **draw two lines,** one horizontal and the other vertical (make sure they’re attached). Then you would **place one number along the horizontal line and the other on the vertical line**. Next, you would **draw the amounts** in the lines. For this example, one would draw 6 rows of 5. Then **to get the answer, add up the amounts**.

For example: 5 x 6 = 30

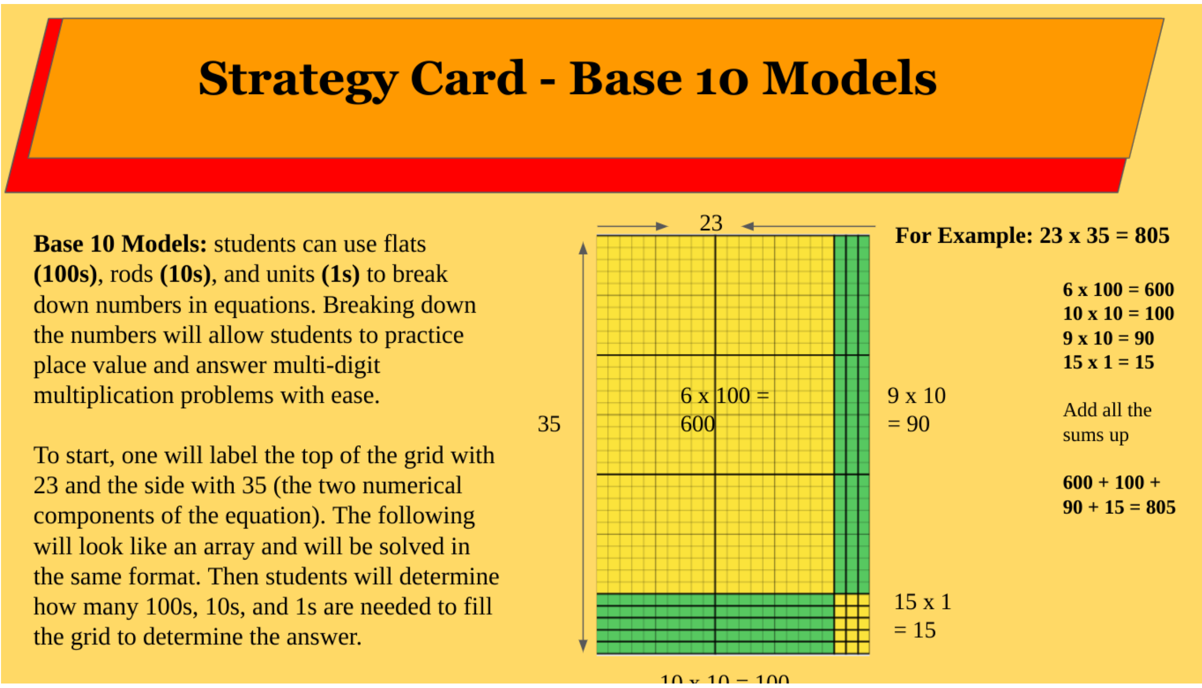


*If you’re using manipulatives, you can follow the drawing instructions but instead of drawing, lay out the materials.

Strategy Card (1.9)



Strategy Card (1.10)



Strategy Card (1.11)

Strategy Card - Partial Products

Partial Products Algorithms: are similar to arrays because one has to have **place value knowledge** to set-up an understand the algorithm.

First the student will start with the equation and then **multiply the tens columns** (20 x 30), then the next two equations will have students **individually multiply the tens columns with the opposite ones column** (30 x 3 and 20 x 5). Lastly, **the two ones columns will be multiplied** (5 x 3). Once all the multiplication is complete, the student will **add the answers** to get the answer to the equation (23 x 35 = 805).

For example: 23 x 35 = 805

23
x 35
600 (20 x 30)
90 (30 x 3)
100 (20 x 5)
15 (5 x 3)
805

Strategy Card (1.12)

9 x 9 = **1 x 0 =** **6 x 3 =** **2 x 8 =** **8 x 3 =**

3 x 5 = **7 x 12 =** **9 x 10 =** **6 x 4 =**

12 x 4 = **2 x 11 =** **0 x 0 =** **11 x 6 =** **10 x 4 =**

Free Choice

9 x 9 = **1 x 0 =** **6 x 3 =** **2 x 8 =** **8 x 3 =**

3 x 5 = **7 x 12 =** **9 x 10 =** **6 x 4 =**

12 x 4 = **2 x 11 =** **0 x 0 =** **11 x 6 =** **10 x 4 =**

Strategy Card (1.13)

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