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THE CONSEQUENCES OF ARRANGEMENT FOR VISUAL PERCEPTION: SUBJECTIVE NUMEROSITY AND DISCRIMINATION AMONG REGULAR, RANDOM AND CONTAGIOUS DISPLAYS

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Abstract

Judgements of the numerousness of dots vary depending on their arrangement. It has been demonstrated that regular patterns are perceived as more numerous than random ones. Labeled the regular-random numerosity illusion (RRNI), explanations of this phenomenon are based on the faulty premise that features are randomly distributed in nature. Natural features tend to be contagiously (systematically clumped) rather than randomly distributed. There is, in fact, a continuum from regularity through randomness to contagiousness, which can be mathematically represented. To more completely investigate the consequence which arrangement has for visual percreption, this study yields numerosity estimates for four levels of arrangement (Regular, Random, Contagious 1, Contagious 2) at three levels of number (N = 37, 74, 111), each spread over a hexagonal display field. A second experiment obtained numerosity estimates for random and contagious displays spread over a square display field. The third task required participants to sort random versus contagious stimulus cards into homogeneous sets. Speed of sorting determined whether one class of arrangement was more easily discriminated than was another. Results indicate that 1) numerosity estimates are highest for regular, lower for random, and lowest for contagious, and 2) superior facility in discriminating among random versus contagious displays.

Results are discussed in relation to memory, contrast with expectancy, and ease of subitizing distinct clusters. Clustering is proposed as an ecologically valid means of specifying stimulus structure.

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The arrangement of stimuli has consequences for the estimation of their number. It has been known for some time (Piaget, 1965) that young children's numerosity perception is highly dependent on item arrangement. While young children (aged four to five years) appreciate numerical equality when there is a one-to-one spatial correspondence, they fail to do so when stimuli of equal number differ in density. Piaget's young subjects believe that objects spread over a greater area are also greater in number. He described this phenomenon as centration. The young child centrates on length to the exclusion of other stimulus-characteristics and thus, fails to conserve number.

With the advent of formal operations (ages eleven to twelve) comes the ability to simultaneously process along the dimensions of space and density, and adult observers are not deceived by simple manipulation of stimulus-items. When presentation time is very brief, and numerosity is beyond the range of subitizing ($n = 5$; Oeffelen van & Vos, 1982a), however, there is evidence that adult observers fail to conserve (eg. Frith & Frith, 1972; Krueger, 1972).

Stimulus area and subjective numerosity

Krueger (1972) demonstrated that of two random dot displays of identical number, the one spread over a larger area would be perceived as more numerous. Krueger (1972) varied the spread of dots over a background of fixed size. When dots are spread over a background of varying size, the trend is reversed (Birnbaum & Veit, 1973; Birnbaum et al., 1974). Dots were arranged randomly and distributed uniformly over backgrounds of various sizes. For a given level of number, as background size increased, subjective numerousness decreased.

When trained to expect a positive relationship between background size and dot numerousness, observers compensate by judging dots spread over a larger background as less numerous (Birnbaum & Veit, 1973). Conversely, preexposure to a negative size-numerosity relationship decreases subjective numerosity for stimuli on smaller backgrounds. These findings support a contrast with expectancy model (Birnbaum & Veit, 1973), whereby observers expecting a certain display to be more numerous, compensate by judging it less numerous.

In the absence of experimentally induced preexpectancies (i.e. zero correlation between background size and numerosity) observers judge dots spread over a larger area as less numerous. Birnbaum et al. (1974, p. 539) conclude that, "everyday experience produces a positive correlation between size and number" with observers compensating by judging stimuli subtending greater area as less numerous.

The discrepency in findings between Krueger (1972) and Birnbaum and colleagues (1973, 1974) has not been adequately resolved. Birnbaum and Veit (1973) speculate that changing dot dispersion with background size constant, results in a surround of varying size which may affect the subjective size and density of the dot pattern. Given the present state of knowledge, any research designed to detect the pure effect of item arrangement on perceived numerosity, should control for both background size and overall area of pattern dispersion.

Figural goodness and subjective numerosity

Frith and Frith (1972) constructed six figures, each having a vertical row of twelve equally spaced dots of one colour. In addition, each stimulus contained twelve dots of a second colour, having the same vertical extent, but arranged to yield clusters of one, three, or six elements. Exposure time was one second, with the task being a verbal report of the colour which appeared more numerous. The results supported the hypothesis that a single large cluster appears more numerous than several small ones which do not form a Gestalt. This finding was significant (Chi-square: $.001 < p < .01$) for both eight year-old and adult observers. Frith and Frith (1972) labeled their finding the solitaire illusion.

In a more thorough investigation of the solitaire illusion, Ginsburg (1982) found that a single cluster of dots appeared more numerous than an equal number organized into two or three clusters. For these regular patterns (n = 30, 60, 90) estimates for single clusters exceeded those for multiple clusters by sixteen per cent [F(2,168) = 11.8, $p < .001$]. This extension of the solitaire illusion supports earlier findings (Frith & Frith, 1972; Ginsburg, 1976) that better Gestalten appear more numerous.

Pattern goodness is related to the informational concept of redundancy (Garner, 1974). Good figures are those with regularity, simplicity and predictability. These figures are redundant to the extent that the observer is able to accurately extrapolate the entire stimulus configuration on the basis of exposure to a subset of that stimulus:

> Redundancy is correlational structure and exists in a set of stimuli whenever we can define that set of stimuli as a subset from a larger total set of stimuli.... Good patterns exist in small subsets and are thus very redundant. Poor patterns exist in large subsets and are thus not very redundant (Garner, 1974, p. 11).

To demonstrate the relationship between pattern goodness and redundancy, Garner and Clement (Garner, 1974) constructed 90 five-dot stimuli, each on an imaginary three-by-three matrix. One group of participants was required to rate the goodness of each stimulus on a scale from one to seven, with one being the best pattern. The second group sorted the cards into homogeneous groups on the basis of perceived similarity. There was a large positive correlation (.84) between mean goodness rating and mean subset size, strongly supporting the hypothesis that pattern goodness and redundancy (measured as subset size) are strongly interrelated.

Each of the elements of a good figure is "strongly implied or suggested by the other elements of the figure" (Bear, 1973, p. 32). Bear's stimuli were a set of 149 cards with four dots spread over a three-by-three matrix. Participants were required to draw a fifth dot in the position "implied or suggested" by the dots already in the pattern. Placement of this dot was highly predictable for the subpatterns rated as "good" by a second group of subjects. When confronted with increasingly poor four-dot patterns, there was a systematic decrease in the ability to predict the placement of the fifth dot. The degree of predictability of the fifth dot accounted for 98.8 per cent of the variance in the goodness ratings:

> This finding accords with the Gestaltist concept of a good figure as one whose elements are well organized, and it is the state of affairs required by Garner's hypothesis that better figures are perceived to have fewer alternatives than poorer figures (Bear,

1973. p. 39).

The Regular-Random Numerosity Illusion

Observers tend to judge good (regular) dot patterns as more numerous than poor (random) ones (Ginsburg, 1980). Stimuli were ten regular dot patterns containing between twenty-eight and forty-six dots each, and ten random displays of corresponding number. The regular patterns were judged to be more numerous than the random ones, with a mean illusion of 5.5 per cent. This tendency to judge regular patterns as more numerous persisted for both circular and rectangular displays. Ginsburg (1980) labeled this finding the regular-random numerosity illusion (RRNI). That numerosity estimation should favour good Gestalten is consistent with Frith and Frith's (1972) earlier finding with the solitaire illusion.

When interviewed, observers believe that a random pattern would appear more numerous than a regular pattern of the same number (Ginsburg, 1978). Expecting random arrays to appear more numerous, observers compensate by judging them less numerous than regular patterns of equal number. This is the contrast with expectancy hypothesis (Birnbaum & Veit, 1973) proposed by Ginsburg and Deluco (1979) as a plausible interpretation of the RRNI. Empirical support for this hypothesis comes from a study (Ginsburg & Deluco, 1979) in which second graders failed to show the RRNI. Having less experience with the coincidence of high numerosity and randomness, their judgements are not mediated by preexpectancies.

Thus, the RRNI may be an illusion of negative context, that is, some contextual feature (randomness) has a negative effect on numerosity estimation. Another illusion of this type is the size-weight illusion (Anderson, 1970). If weight is held constant as size increases, then judged heaviness decreases. This is analogous to the RRNI, where increased randomness (with number constant) serves to decrease perceived numerosity.

The observer as organizer

Observers are able to accurately perceive up to five items regardless of the brevity of the presentation. This is considered to be the limit of direct perception or seeing-at-a-glance. Kaufman and colleagues (1949) proposed the term subitizing for the discrimination of stimuli containing fewer than seven elements. Beyond the span of subitizing, it is assumed that the observer must rely on either counting or estimation, or a combination of both (Klahr and Wallace, 1976).

It has recently been demonstrated, however, that observers may be able to directly abstract number far beyond the range of subitizing (Oeffelen van & Vos, 1982a, 1982b; Smitsman, 1982). Two random dot displays will be perceived as different in number so long as the ratio (max-min)/min exceeds the Weber fraction of .162 (Oeffelen van & Vos, 1982b). Subjects were able to discriminate, above chance, simultaneously presented six and seven-dot displays (Weber fraction = .167 > .162), but failed to do so for seven and eight-dot displays (Weber fraction = .143 .162). So long as the difference between the two numbers exceeds the Weber fraction, discrimination above chance levels was observed for numerosities far beyond the range of subitizing. The authors conclude: "The idea that the mind can grasp only a small number of objects at once remains quite unsupported by the evidence, if indeed it has any meaning at all" (Oeffelen van & Vos, 1982b, p. 109).

In discriminating numerous (n 5) stimuli, the observer may perceive and take advantage of "higher order structure" (Smitsman, 1982, p. 5). Smitsman's stimuli were composed of 120 elements of two types, small circles and small squares. For each stimulus, one geometric figure formed clusters of either one, two, or four, amidst single randomly arranged figures of the second type. Each stimulus was presented for one, four, or seven seconds. Subjects (aged six through adult) were required to verbally indicate which figure appeared more numerous, the circles or the squares. For subjects eight years and older, estimates favoured the grouped category (Newman-Keuls, $p <$ 01).

When such objective structure is absent, observers may impose their own structure on an ambiguous stimulus, the elements of which are perceived in small, subitizable clusters, each summed to a running total (Oeffelen van & Vos, 1982a). As group size increases beyond n = 5 (the limit of subitizing) number is progressively underestimated. At n = 8, observers are more likely to report seeing six or seven than eight elements. This is consistent with an earlier finding (Indow & Ida, 1977) that dots are underestimated for objective numerosities beyond n ¬ 10. For objective numerosities between 25 and 300 underestimations fit a power function with an exponent of .83 (Krueger, 1982).

With these numerous displays, the observer will likely generate subsets beyond the span of subitizing. As the proportion of the groupings increases, we expect a progressive underestimation of objective number. It seems that clustering increases subjective numerosity so long as subset size is small (eg. Smitsman, 1982). Beyond about n ¬ 5 group size is underestimated with a consequent decrease in the perception of total number.

While the preceding may explain the observer tendency to underestimate highly numerous displays, it does not account for random arrangements being judged less numerous than regular ones. The elements of a good pattern have uniform, predictable spatial relations which resist abstraction in unique subsets. The observer encountering a regular (good) pattern would experience considerable overlap among the perceived subsets (Smitsman, 1982). This non-exclusivity would allow a single element to belong to more than one subset, thus increasing the perception of number for good Gestalten. This model may be an alternative to contrast with expectancy, or may operate in conjunction with same.

An ecological approach

The present research investigates the consequences which item arrangement has on perceived numerosity. Both regular and random dot-displays are incorporated, along with a third type of display containing mathematically definable levels of clustering. All stimuli correspond to the spatial arrays in the ecosystem which are usually classified as either random, regular or contagious (Stiteler & Patil, 1971). The elements of a contagious display appear clumped or aggregated, terms which will be used interchangeably throughout the paper. From an ecological perspective, natural phenomena (such as trees in a forest) do not fulfill the criterion for randomness (Pielou, 1977). Rather, the spatial arrays in nature are described by the contagious distribution (Taylor, Woiwood, & Perry, 1978).

A formal definition of contagiousness is required for experimental purposes. A distribution is a set of objects or events divided among a set of samples (in space or time). We can obtain an empirical classification of dispersal structure by comparing the sample mean with the variance of organisms across sample units (Stiteler & Patil, 1971). When the mean number of events per sample of space is equal to the variance of these events across samples, the distribution is random. When the variance exceeds the mean, the distribution is contagious. Should the number of events be equal for each sample of space (variance = 0), the distribution is regular.

Taylor et al. (1978) examined the relation between the variance and the mean for 156 sets of field data. From a survey of 3,840 samples from 102 species (ranging from protozoa to plants to humans), only two data-sets were found to be random, and most of the data were significantly more clumped than random. This finding invalidates the presumption that high numerosity and randomness coincide in nature (eg. Cousins, 1979), discounting this relationship as mediating contrast with expectancy.

Ecological science has found a continuum of arrangement from regularity to randomness to contagiousness (see Fig 1) which has been only partially explored by psychological research. Experiments 1 and 2 undertake a more comprehensive analysis of the consequences which item arrangement has on the subjective experience of number. Experiment 3 investigates the ease of discriminating among random versus contagious dot patterns.

Experiment 1: Subjective numerosity for regular, random and contagious sets

Method

Subjects. A total of 63 subjects participated in Experiment 1. Fifty-four were taken from the Introductory psychology subject pool at Lakehead University. The remaining nine subjects were the author's colleagues in a fourth year Perception class. Two subjects were dropped due to ambiguous handwritten responses. The remaining subjects ranged in age from 18 to 49 years, with a mean of 23. There were 22 males and 39 females.

Stimuli. Dots 6mm. in diameter were punched out of black bristol board and spread over a field of 37 contiguous hexagons (see Appendix 1). Each hexagon constituted a single sample of space, within which were seven possible dot locations (six vertices and one central point). There were four different arrangements: Regular, Random, Contagious 1 (variance = twice the mean), Contagious 2 (variance = four times the mean), at each of four levels of number (37, 74, 111). In total, there were 12 different dot stimuli (see Appendix 2).

Regular patterns. For N = 37, there was one dot in position seven of each hexagon. At N = 74, these positions were one and four; and one four and seven for N = 111. Once dot position was determined, the hexagon field served as a template for gluing the dots to a 21.6 by 27.9cm. sheet of blank typing paper.

Random patterns. These were determined by the Poisson distribution: $P(n) = C(m^n/n!)$, where n = number of events (dots) per sample of space, m = mean number of events per sample, and $C = 1/e^m$ (e = 2.7183). Given 37 samples of space, $37P(n) = F(n)$, where F(n) = frequency of n to the nearest whole number. A probability table was constructed for each level of number and adjusted so that F(n) = N, where N = Grand Total. This was necessary for N to be a whole number (i.e. whole dots). For each random pattern, the mean number of dots per sample of space was equal to the variance of dots across samples, thus fulfilling the criterion for randomness. To ensure that all displays were approximately equal in perimeter, six peripheral hexagons (1, 4, 16, 22, 34, 37; see Appendix 1) were occupied. More frequent events had a proportionally greater chance of peripheral assignment. If F(2) = 10 and F(3) = 5, for example, hexagon one would be twice as likely to have two dots than three. Thus, events were assigned ranges corresponding to their frequencies of occurence and selected by a random number table. Once peripheral assignment was complete, the remaining hexagons, along with dot position within each, were determined by random numbers.

Contagious 1. Variance equalled two, four and six for N = 37, 74 and 111, respectively. Tables were constructed to fulfill these criteria, with the procedure for dot placement being identical to that for regular patterns.

Contagious 2. Variance equalled four, eight and twelve for N = 37, 74 and 111, respectively. The procedure for dot placement was identical to above.

Procedure. All stimuli were photographed to yield 35mm positives. Slides were presented using a Kodak Carousel projector and a projection screen. A timing device presented each slide for 2.24 seconds followed by a blank screen of five second duration. To ensure that results were not an artifact of a single ordering, two sequences were used: Order 1 (74C1, 37 Reg, 111C2, 74 Ran, 111C1, 37 Ran, 37C2, 74 Reg, 111 Ran, 37C1, 111 Reg and 74C2), and Order 2 (74 Reg, 37C1, 111 Ran, 74C2, 111 Reg, 37C2, 37 Ran, 74C1, 111C2, 37 Reg, 111C1, 74 Ran). Due to a shortage of subjects, only 13 of the 61 participants received the second ordering. Ideally, an equal number of subjects would have served under each condition.

All experimental stimuli were preceded by two practice slides (Random, n = 7 and n = 19). This was deemed necessary to accustom observers to their task prior to responding to the experimental slides. Data collection allowed group testing, with participants run on three separate sessions. Once seated, each subject was given a response sheet (see Appendix 3) and instructed as follows:

> This is an experiment on how people

> perceive number. I'm going to show you some slides with dots on them. Each slide will be on for about two seconds followed by a five second blank screen. During the blank screen interval, estimate the number of dots that were presented and record this on your answer sheet. Do not discuss your responses with your neighbour. Before we begin, please record your age and sex at the top of the answer sheet. Any questions?

Questions were fielded and/or instructions repeated until all subjects were believed to have a complete understanding of task requirements.

Results

The main effect of arrangement was significant [$F(3,177) = 22.18$, p .001], indicating differential responding contingent on stimulus arrangement (see MANOVA Summary Table, Appendix 4). Regular patterns were judged most numerous, followed by random (Newman-Keuls, $p < .01$) and contagious (Newman-Keuls, p .05) patterns, respectively (see Table 1; Fig 2). There was no difference in numerosity estimation for the two levels of contagiousness. Independent of sex and arrangement, an increase in objective numerosity increased subjective estimates [F(2,118)= 157.57, p <.001]. A Number by Arrangement interaction was observed [F(6,354) = 2.23, p = .04], but failed to reach significance after applying the Greenhouse-Geisser correction for repeated measure designs (see Keppel, 1973).

When collapsed across sex and number, regular stimuli were judged more numerous than random stimuli (Newman-Keuls, p <.01). Numerosity estimates did not differ between contagious arrangements, but both were judged lower in number than the random stimuli (Newman - Keuls, p < .05). There was no effect of sex on numerosity estimation.

Experiment 2: Subjective numerosity for random and contagious sets

Rationale

To increase the generalizability of the findings from Experiment 1, subjects were run on a second set of dot stimuli spread over a different field. Having already replicated the RRNI, regular patterns were excluded from the present design. This simplified statistical analysis while further exploring the difference between random versus contagious numerosity estimation.

Method

Subjects. A total of 115 subjects participated in Experiment 2. All participants were Introductory psychology students at Lakehead University. Fifteen subjects were dropped due to ambiguous handwritten responses, and one more for failing to record age and sex on the response sheet. Of the remaining 99 subjects, 36 were males and 63 were females. Subjects ranged in age from 17 to 46, with a mean of 23 years.

Stimuli. Dots 6mm. in diameter were punched out of black bristol board and spread over a ten-by-ten matrix of squares (see Appendix 5). Each of the 100 squares constituted a single sample of space, each divided to form a three-by-three matrix with nine possible dot locations. As in Experiment 1, subjects did not see this construction grid which served as a template for gluing the dots to a 21.6 by 27.9cm. sheet of blank paper.

There were two different arrangements, Random and Contagious (variance = four times the mean), at three levels of number (N = 40, 60, 90). This constituted the six initial stimuli, from which six more were generated, having an identical frequency distribution but different selection procedure (i.e. different random numbers).

Random patterns. As in Experiment 1, these stimuli were based on the Poisson distribution. The procedure for selecting dot location was identical, again with the constraint that all peripheral samples (squares 0, 9, 90, 99; see Appendix 5) be occupied. All remaining squares, along with dot position within each, were selected from a random number table.

Contagious patterns. For each level of number, these patterns were constructed so the variance of dots across samples was four times the mean number of dots per sample of space. The dot placement procedure was as described in Experiment 1.

Procedure. All 12 stimuli were photographed to yield 35mm. positives and projected with a Kodak Carousel projector. Exposure time for each slide was 2.24 seconds followed by a five second blank screen. The two practice stimuli preceded the experimental slides: 90C, 40C, 60R, 60C, 40R, 90R, 40R, 60C, 90R, 60R, 90C, 40C. There are two different stimuli (having the same frequency distribution) for each arrangement and level of number. Thus, the experimental slides may be considered two different sequencings of six frequency distributions, run consecutively. This helped control for an order effect, and allowed testing in a single session.

All subjects were run simultaneously and instructed as in Experiment 1.

Results

When collapsed across number, there was a strong tendency for observers to judge random stimuli as more numerous than contagious stumuli of corresponding number [Chi-square(1) = 32.67, $p < .005$; see Table 2]. The effect of sex was non-significant [Chi-square(1) <1].

Experiment 3: Discrimination among random versus contagious subsets

Rationale

This experiment explored another dimension in responding to item-arrangement, the critical measure being the speed with which subjects were able to sort random versus contagious stimulus cards into homogeneous groups. This task determined whether there was a difference in discriminating among random versus among contagious stimuli. More rapid card sorting would indicate greater ease of discrimination among members of that set.

Method

Subjects. Participants were 54 Lakehead University students. Ten subjects were the author's colleagues in the Graduate psychology programme, with the remainder taken from the Introductory subject pool. Seven subjects, having errors two standard deviations above the mean on either card-sort were excluded from the analysis. Of the remaining 47 subjects, there were 19 males and 28 females with a mean age of 22 years.

Stimuli. Stimuli were 44 contagious (variance = four times the mean) and 44 random dot displays of equal number (N = 74). Contagious stimuli were generated using the 37 hexagon field from Experiment 1. There were four contagious stimuli, having a single frequency distribution with corresponding hexagons occupied. Each was subject to a different sampling for dot location within each hexagon, generating four similar (same frequency distribution and occupied samples) but non-identical stimuli. Using a single frequency distribution, four random stimuli were constructed in an identical manner.

Dots were glued to a 21.6 by 27.9cm. sheet of blank paper and photographed to yield eight (four contagious, four random) 8.9 by 12.7cm. photographs. Each photograph was photocopied 11 times, and copies cut and pasted to 8.9 by 12.7cm. index cards. In total, there were four identical groups of 11 contagious cards and four identical groups of 11 random cards (see Appendices 6 and 7)

Procedure. Once seated across from the Experimenter, four model cards (either random or contagious) were placed face-up and spread left-to-right in front of the subject. Instructions were then read as follows:

> Here we have four cards with dots on them (pointing to the four models at the top of the desk). These are the model cards. Your task is to select from this pile (presenting stack of forty random or contagious cards, face up) the cards which correspond to each of the four models. Every card in the pile will correspond to one of the

> models. Place each of the cards in the pile below its model so that there is an identical match. Work as quickly as you can without making mistakes. Any questions?

Instructions were repeated as necessary to ensure a complete understanding of task requirements. Upon completing the sort for the first arrangement, subjects were instructed to perform the identical task for the second set of stimulus cards.

Elapsed time was recorded with a digital stopwatch, and the order of random versus contagious sortings was counterbalanced across subjects. An error was scored for each card placed under the incorrect model. Stimulus cards were thoroughly shuffled after each trial. To minimize the effect of extreme scores, times were transformed [speed = 1/time(seconds)] prior to data analysis.

Results

Subjects took a mean of 10.4 seconds longer to sort contagious than random stimuls cards (see Table 3). This was significant for both time (seconds) and speed (1/seconds) scores [$t(46) = 1.8$, $p < .05$; $t(46) = 2.9$, $p = .005$, respectively]. Mean errors were .98 and 2.17 for random and contagious card-sorts, respectively [$t(46) = .73$, n.s.]. Sorting time for random versus contagious stimulus cards was independent of sex [Chi-square(1) 2.5, n.s.].

Discussion

These series of experiments found, 1) a decrease in subjective numerosity along the continuum from regularity to randomness to contagiousness, and 2) less latency to sort random versus contagious displays into homogeneous subsets. That regular displays are judged more numerous than random ones replicates previous findings with the RRNI (eg. Ginsburg, 1976), and establishes this illusion for hexagonal fields. Experiment 1 found contagiousness to further decrement subjective numerosity, a finding supported and extended to include square display fields (Experiment 2).

Using 64-dot patterns spread over a square grid, Goldstein (1982) found a decrease in subjective numerosity along the continuum from regularity to contagiousness. The present findings are consistent with Goldstein's (1982) preliminary research and extend the phenomenon to different levels of number spread over square and hexagonal grid matrices. An attempt to account for these findings will propose three models which may be profitable avenues for future research.

The third experiment addressed another class of response to clustering, that of discrimination. Random displays were sorted into homogeneous subsets more rapidly than were contagious displays, indicating greater facility in discriminating among members of the former.

Contrast with expectancy

Previous attempts to account for the RRNI (eg. Ginsburg, 1980) have proposed contrast with expectancy as mediating higher estimates for regular arrays. Presumably, there is a natural coincidence of randomness and high numerosity which observers perceive and compensate for when estimating number. Though observers do expect random arrays to appear more numerous (Ginsburg, 1980), ecological research invalidates the premise that natural features are randomly distributed (Taylor et al., 1978). Natural phenomena tend toward contagiousness, allowing the observer to form an association between this arrangement and greatness of number. Thus, the observer may expect clustered (rather than random) displays to seem most numerous and compensates by judging them least numerous.

While this theoretical model is consistent with the present findings, it is unknown whether observers do, in fact, expect contagious arrays to be (or appear) more numerous than either random or regular patterns. Confirming such an expectation would suggest the operation of a contrast with expectancy which decrements numerosity estimates for contagious phenomena.

Cousins (1979) conducted a study attempting to modify the RRNI by experimentally inducing subjects to expect either a positive, negative, or no correlation between numerosity and regularity. Contrast with expectancy would predict estimates for regular patterns to be decreased, increased and unaffected, respectively. These hypotheses were not confirmed, the RRNI being unaffected by the experimental manipulation. It would be interesting to determine whether estimates for contagious displays are equally resistant to such pretraining.

Numerosity estimation and ease of subitizing

For all levels of number and clustering, the results did not concur with Smitsman's (1982) finding that clustering increases subjective numerousness relative to random stimuli. Smitsman required subjects to verbally indicate whether random or clustered displays appeared more numerous. The task was not numerosity estimation, as such, and this, combined with other aspects of his design, may account for the discrepancy between this findings and those of the present study.

The clustering of Smitsman's stimuli (see Fig 3) was systematic and unnatural (i.e. all clusters were identical in number and orientation). Clusters were also spatially removed from adjacent groupings. In a natural setting samples of space would be continuous, with several small clusters combining to yield larger clusters. Such large clusters would exceed the span of subitizing and tend to be underestimated. This would not occur with Smitsman's groupings which are spatially distinct and readily subitizable (n = two and four). As clustering increases, grouped elements become more numerous, further resisting breakdown into subitizable subsets. On this basis, we may expect a further decrease in perceived numerosity from Contagious 1 to Contagious 2. The present study failed to detect such an effect.

At the other extreme of the arrangement continuum, regularity may pose its own problems for the subitizing of distinct subsets. Good patterns are characterized by uniform spatial relationships which resist breakdown into unique groupings. An element of a regular display may belong to more than a single abstracted subset (Smitsman, 1982; see Fig 4) incrementing subjective numerosity for good Gestalten.

Developmental trends. There seem to be developmental differencess in the ability to impose structure on one's visual perceptions. Young children may be less able than adults to impose organization on a visual display and thus fail to reduce display elements to subitizable subsets. This may account for the RRNI not occuring among second-graders (mean = 7.5 years, Ginsburg & Deluco, 1979). When structure is explicit (as with Smitsman's discrete, subitizable clusterings), young children utilize such information to generate numerosity estimates similar to those for adults: Smitsman (1982) failed to detect response-differences between eight year-old and adult subjects. This was not true for Smitsman's six year-olds who did not report clustered stimuli as being more numerous.

There may be a change with age in the way of estimating (Smitsman, 1982) with children below seven years incapable of utilizing objective structure, those between seven and ten incapable of imposing structure, and those older than ten having the capacity to do both. Such a progression is consistent with Piaget's stage theory (Ginsburg & Opper, 1969) of cognitive development. Prior to attaining concrete operations (ages five to seven) the child is unable to perform the complex operations (eg. summing clusters to a running total) required to profit from explicit structure. With formal operations (ages eleven to twelve years) comes the ability to go beyond the observable (eg. imposing one's own structure on an ambiguous environment).

The role of memory in numerosity estimation

Stimuli having regular properties seem to be available for encoding longer than those having random properties. This corresponds to better memory for good Gestalten and may be associated with an increase in subjective numerousness for these arrangements. There is better reproduction memory for regular arrays (Attneave, 1955), with the visual trace (icon) being more stable and less vulnerable to interference than those resulting from chaotic ones (Koffka, 1935).

Horne and Turnbull (1977) found that a brief (.5 second) exposure yielded an underestimation, and a long (1.0 second) exposure an overestimation of objective number. A more persistent icon (Koffka, 1935) for regular displays increases the time available for encoding and remembering regularity. This would be equivalent to an increase in objective presentation time, with a corresponding increment (Horne & Turnbull, 1977) in numerosity estimation.

This memory model may account for Smitsman's (1982) symmetrical clusters being judged more numerous than were his random displays. Whereas the present study presented stimuli in succession, Smitsman superimposed clustered and random arrangements. A more persistent icon (Koffka, 1935) for Smitsman's redundant clusterings may have masked or interfered with the observer's perception of the random display. This would render the former more salient and confound a pure comparison of subjective numerousness for these arrangements.

An empirical test for a memory model would present equally numerous random and regular displays, followed by a powerful masking stimulus. A consequent decrease in the RRNI would implicate the icon trace as contributing to higher estimates for regular stimuli. Alternately, presentation time for the regular display could be progressively decreased until observers judge it equal in number to a random stimulus. The difference in presentation time required to reach this point of subjective equality would be equal to the difference in iconic persistence between regular and random patterns.

The ecological approach

J. Gibson (1960) criticized psychology for its misguided conception of the stimulus: "We... define our stimulus by certain operations of physical science, not by the judgement of our subject" (p. 694). In this article Gibson calls for greater appreciation of the relationship between natural stimuli and the observer, specifically, the laws of stimulus information in the organism's natural environment. The laboratory stimulus is too often divorced from the ecological laws "relating organisms to the affordances of [their] environment" (Turvey, et al., 1981, p. 237).

J. Gibson considered the best exemplars of orderly relations in the world to be the world's surfaces, objects and events (E. Gibson, 1982). This must be similar to what Tolman and Brunswik (1935) had in mind when they spoke of the environment as a "causal texture" in which events are systematically related to each other. Brunswik's (1951) doctrine of "ecological validity" called for experimental stimuli to be more representative of the ecological relationships in the natural environment.

The present study is, in part, an attempt to reconcile the stimulus with natural phenomena, giving the observer the opportunity to respond to the full complex of arrangements in his or her environment. Given that clustering decreases subjective numerosity, animals may cluster into contagious groupings so as to seem less numereous and presumably less attractive to predators (Goldstein, 1982). As a natural event, and as a continuum which may be perceived by the observer (Goldstein, 1982), clustering is proposed as an ecologically valid means of specifying stimulus structure.

Conclusions

These series of experiments are an initial exploration of the observer's response to the continuum of stimulus-arrangement from regularity through contagiousness. Experiments 1 and 2 investigated the consequences of arrangement for the perception of number. The third experiment explored the effect of arrangement on discrimination among members of a single frequency distribution. Results indicate that regular patterns are perceived as most numerous, folowed by random and contagious patterns, respectively. There was greater ease in discriminating among random versus contagious subsets. It seems that three factors may be mediating the outcome of the present study:

Contrast with expectancy. Expecting random patterns to appear more numerous, observers compensate by judging them less numerous. Confirming that contagious displays are expected to be (or appear) most numerous would further support contrast with expectancy.

Ease of imposing subjective structure. As clustering increases, observers may find it more difficult to subitize subsets, resulting in an underestimation of number for contagious displays. The elements of regular patterns may be subitized in overlapping subsets, with this non-exclusivity increasing the perception of number for these arrangements.

Memory and figural goodness. Visual memory is optimal under conditions of regularity and this may be related to good figures being judged most numerous.

Depending on the task, these three variables may operate exclusively or in combination. Suggestions are made for isolating the consequences which each has for visual perception.

Finally, clustering is proposed as a means of relating stimulus structure to the natural features in the organism's environment.

References

Anderson, N. Averaging model applied to the size-weight illusion. Perception and Psychophysics, 1970, 8(1), 1-4.

Attneave, F. Symmetry, information and memory for patterns. American Journal of Psychology, 1955, 68, 209-222.

Bear, G. Figural goodness and the predictability of figural elements. Perception and Psychophysics, 1973, 13, 32-40.

Birnbaum, M., and Veit, C. Judgemental illusion provided by contrast with expectancy. Perception and Psychophysics, 1973, 1, 149-152.

Birnbaum, M., Kobernick, M, and Veit, C. Subjective correlation and the size-numerosity illusion. Journal of Experimental Psychology, 1974, 102(3), 537-539.

Brunswik, E., and Herma, H. Probability learning of perceptual cues in the establishment of a weight illusion. Journal of Experimental Psychology, 1951, 41, 281-290.

Cousins, J. Subjective correlation and the regular-random numerosity illusion (RRNI). Unpublished Honours Thesis, Lakehead University, 1979.

Frith, C., and Frith, U. The solitaire illusion: An illusion of numerosity. *Perception and Psychophysics,* 1972, *2(6),* 409-410.

Garner, W. *The processing of information and structure.* New York: Halsted Press, 1974.

Gibson, E. Contrasting emphases in Gestalt theory, information processing, and the ecological approach to perception. In T. Beck [Ed.], *Organization and representation in perception.* New Jersey: Lawrence Erlbaum Associates, Inc., 1982.

Gibson, J. The concept of the stimulus in psychology. *American Psychologist,* 1960, *15,* 694-703.

Ginsburg, H. and Opper, S. *Piaget's theory of intellectual development.* Englewood Cliffs, New Jersey: Prentice-Hall, 1969.

Ginsburg, N. Effect of item arrangement on perceived numerosity: Randomness vs. regularity. *Perceptual and Motor Skills,* 1976, *43,* 663-668.

Ginsburg, N. Perceived numerosity, item arrangement, and expectancy. *American Journal of Psychology,* 1978, *91(2),* 267-273.

Ginsburg, N., and Deluco, T. A developmental study of the regular-random numerosity illusion. *Journal of Genetic Psychology,* 1979, *135,* 197-201.

Ginsburg, N. The regular-random numerosity illusion: Regular patterns. *Journal of General Psychology,* 1980, *103,* 211-216.

Ginsburg, N. Solitaire illusion: Replication and extension. *Perceptual and motor skills,* 1982, *54,* 209-210.

Goldstein, S.R. *Perception of dispersion.* Unpublished manuscript, Lakehead University, 1982.

Horne, E., and Turnbull, C. Variables of colour, duration, frequency, presentation order, and sex in the estimation of dot frequency. *Journal of General Psychology,* 1977, *96,* 135-142.

Indow, T., and Ida, M. Scaling of dot numerosity. *Perception and Psychophysics,* 1977, *22(3),* 265-276.

Kaufman, E., Lord, M., Reese, T., and Volkman, J. The discrimination of visual number. *American Journal of Psychology,* 1949, *62,* 498-525.

Keppel, G. *Design and analysis: A researcher's handbook.* New

Jersey: Prentice- Hall, Inc., 1973.

Klahr, D., and Wallace, J. Cognitive development: An information processing view. New York: Wiley, 1976.

Koffka, K. Principles of gestalt psychology. New York: Harcourt, Brace World, Inc., 1935.

Krueger, L. Perceived numerosity. Perception and Psychophysics, 1972, 11, 5-9.

Krueger, L. Single judgements of numerosity. Perception and Psychophysics, 1982, 31(2), 175-182.

Oeffelen van, M., and Vos, P. Configurational effects on the enumeration of dots: Counting by groups. Memory and Cognition, 1982a, 10(4), 396-404.

Oeffelen van, M., and Vos, P. A probabilistic model for the discrimination of visual number. Perception and Psychophysics, 1982b, 32(2), 163-170.

Piaget, J. The child's conception of number. New York: Norton, 1965.

Pielou, E. Mathematical ecology. New York: Wiley, 1977.

Smitsman, A. Perception of number. International Journal of Behavioural Development 5, 1982, 1-31.

Stiteler, W., and Patil, G. Variance-to-mean ratio and Morisita's index as measures of spatial patterns in ecological population. In G. Patil, E. Pielou, and W. Waters [Eds.], Statistical ecology (volume 1): Spatial patterns and statistical distributions. Pennsylvania: Pennsylvania State University Press, 1971.

Taylor, L., Woiwood, I., and Perry, J. The density-dependence of spatial behaviour and the rarity of randomness. Journal of Animal Ecology, 1978, 47, 383-406.

Tolman, E., and Brunswik, E. The organism and the causal texture of the environment. Psychological Review, 1935, 42, 43-77.

Turvey, M., Shaw, R., Reed, E., and Mace, W. Ecological laws of perceiving and acting: In reply to Fodor and Pylyshyn (1981). Cognition, 1981, 9(3), 237-304.

	Number			
	37	74	111	Totals
Regular	44.7	74.0	90.9	12,786
Random	34.5	64.6	83.9	11,170
Contagious 1	34.2	50.6	82.2	10,183
Contagious 2	33.3	59.0	76.1	10,275

Table 1. Means and totals for the 3 x 4 matrix of Number and Arrangement, based on an n-of 61.

		Arrangement		
		Random	Contagious	
Sex	Male	26	9	35
	Female	50	11	61
		76	20	96

Table 2. Two-by-two matrix of Sex and Arrangement from Experiment 2. Each cell contains the number of subjects judging that condition more numerous. Note grand total of 96, as three of the 99 subjects judged both arrays equally numerous.

Arrangement

	Random	Contagious
Mean time	86.57	97.00
Mean errors	.98	2.17

Table 3. Mean time and mean errors for sorting random versus contagious displays.

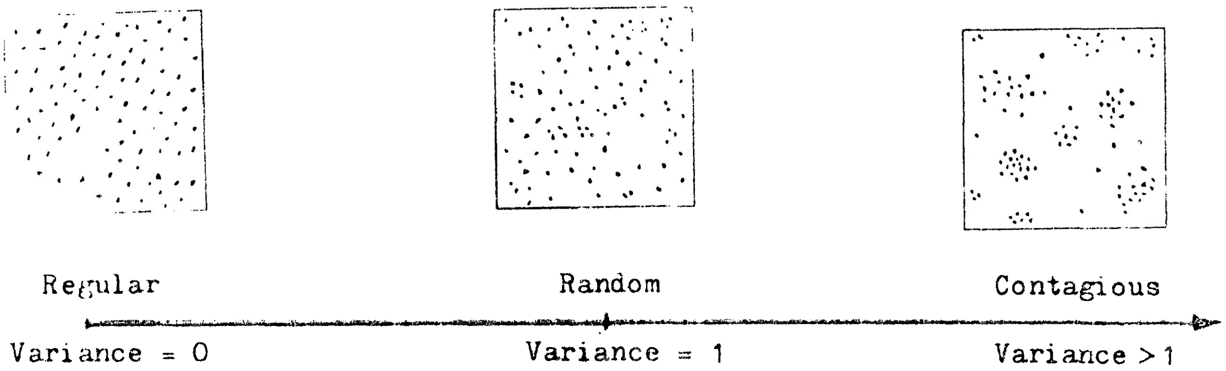


Figure 1. The change in variance along the continuum from regularity to contagiousness for a mean of one event per sample of space.

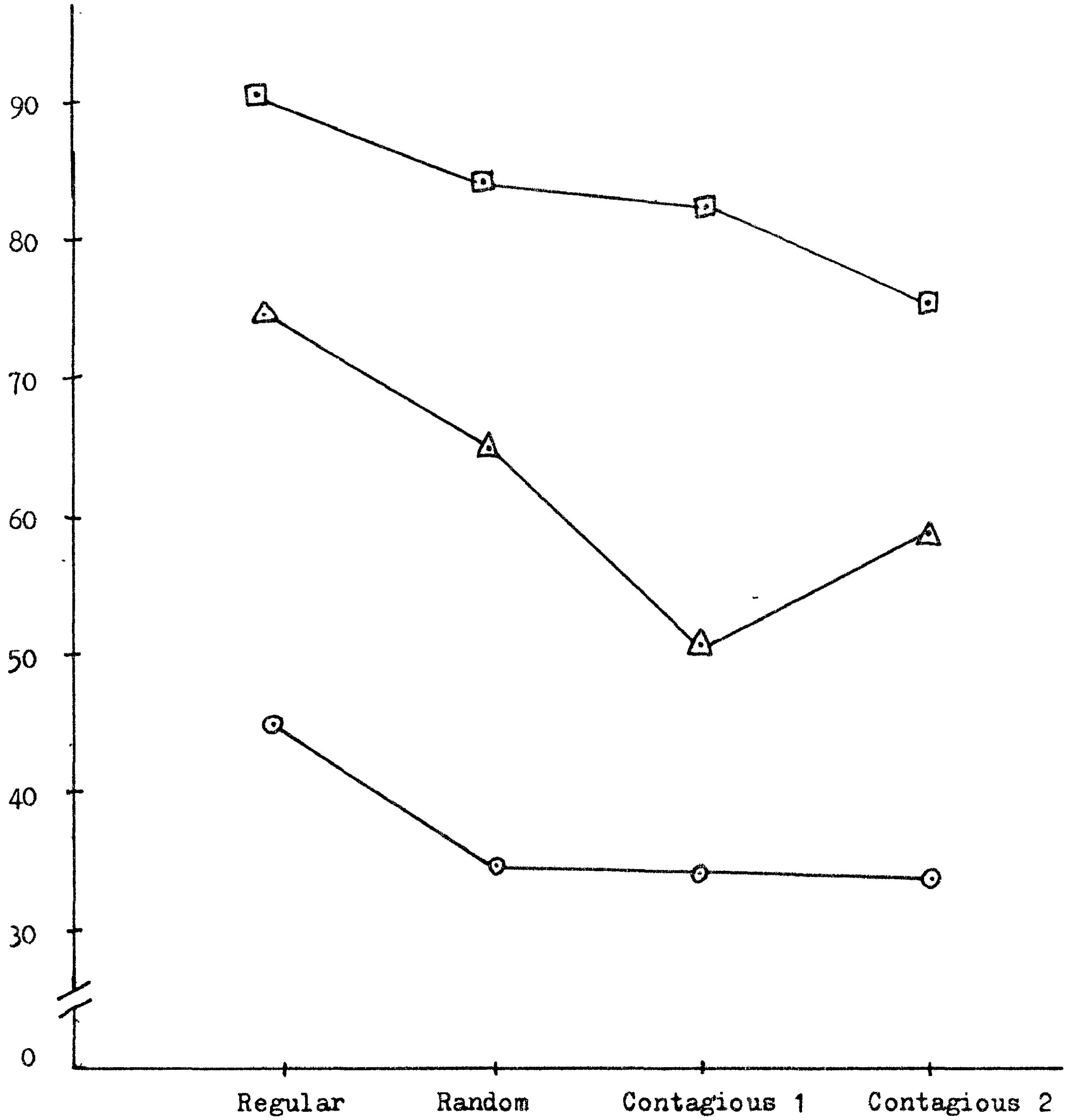


Figure 2. Mean estimates fot the three levels of number in Experiment 1.

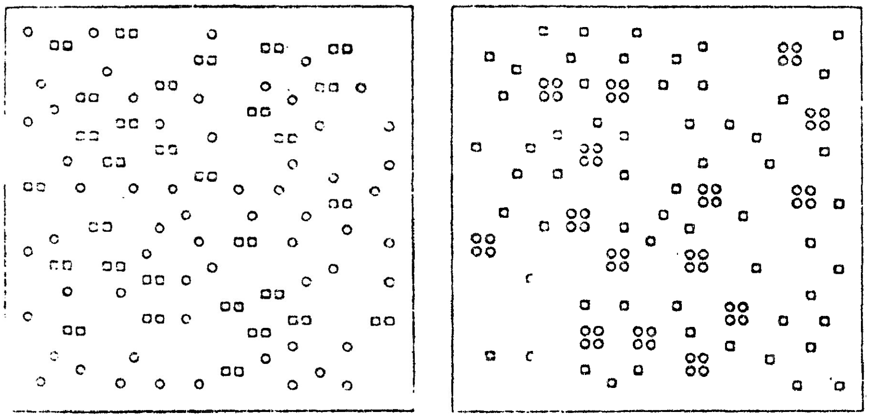


Figure 3. The two levels of clustering (n =2, 4) from Smitsman (1982).

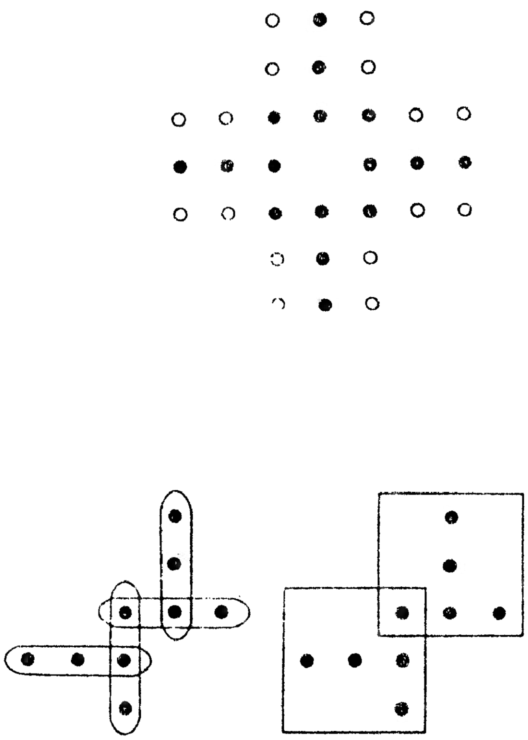
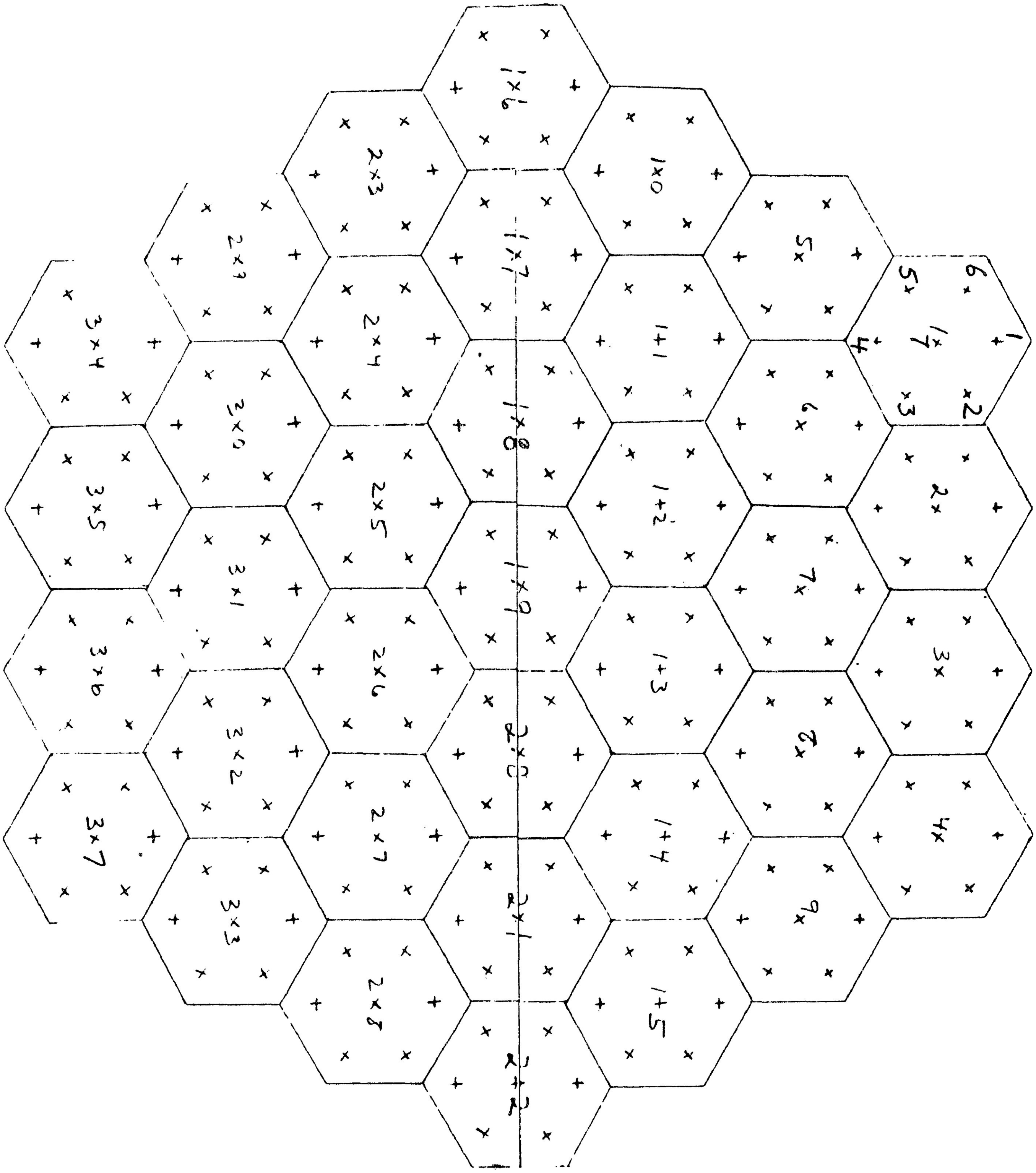


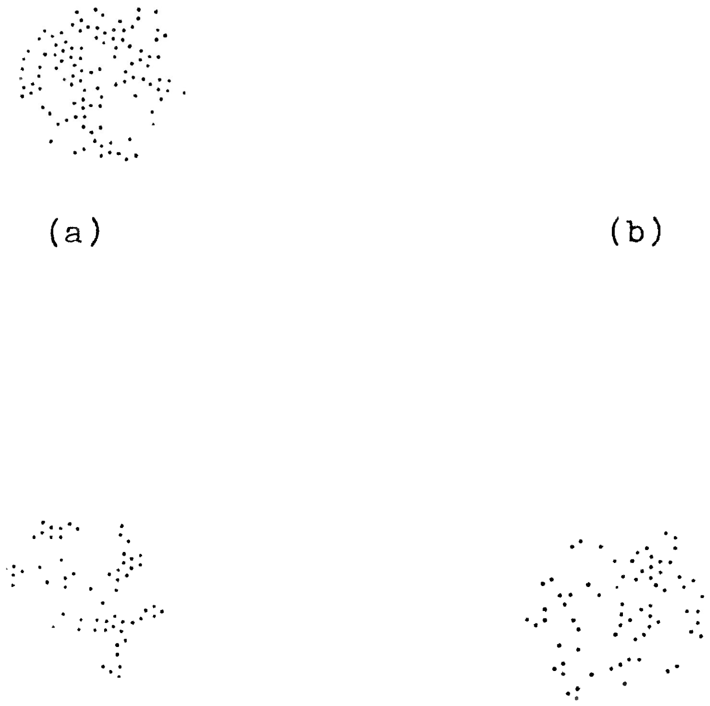
Figure 4. Frith and Frith's solitaire illusion (top) exemplifying how good Gestalten (black dots) resist abstraction in unique subsets. Two possible groupings are depicted (from Smitsman, 1982).

Appendix 1



Appendix 1: Hexagon field used to construct all experimental stimuli. To equate overall area, hexagons 1, 4, 16, 22, 34 and 37 always occupied.
All loci numbered as in hexagon 1.

Appendix 2



(a) (b)

(c) (d)

Appendix 2: A sample from the twelve stimulus slides used in Experiment 1: (a) N=111, Random, (b) N=37, Regular, (c) N=74, Contagious 1, (d) N=74, Random.

AGE: ________

SEX: ________

______

______

______

______

______

______

______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

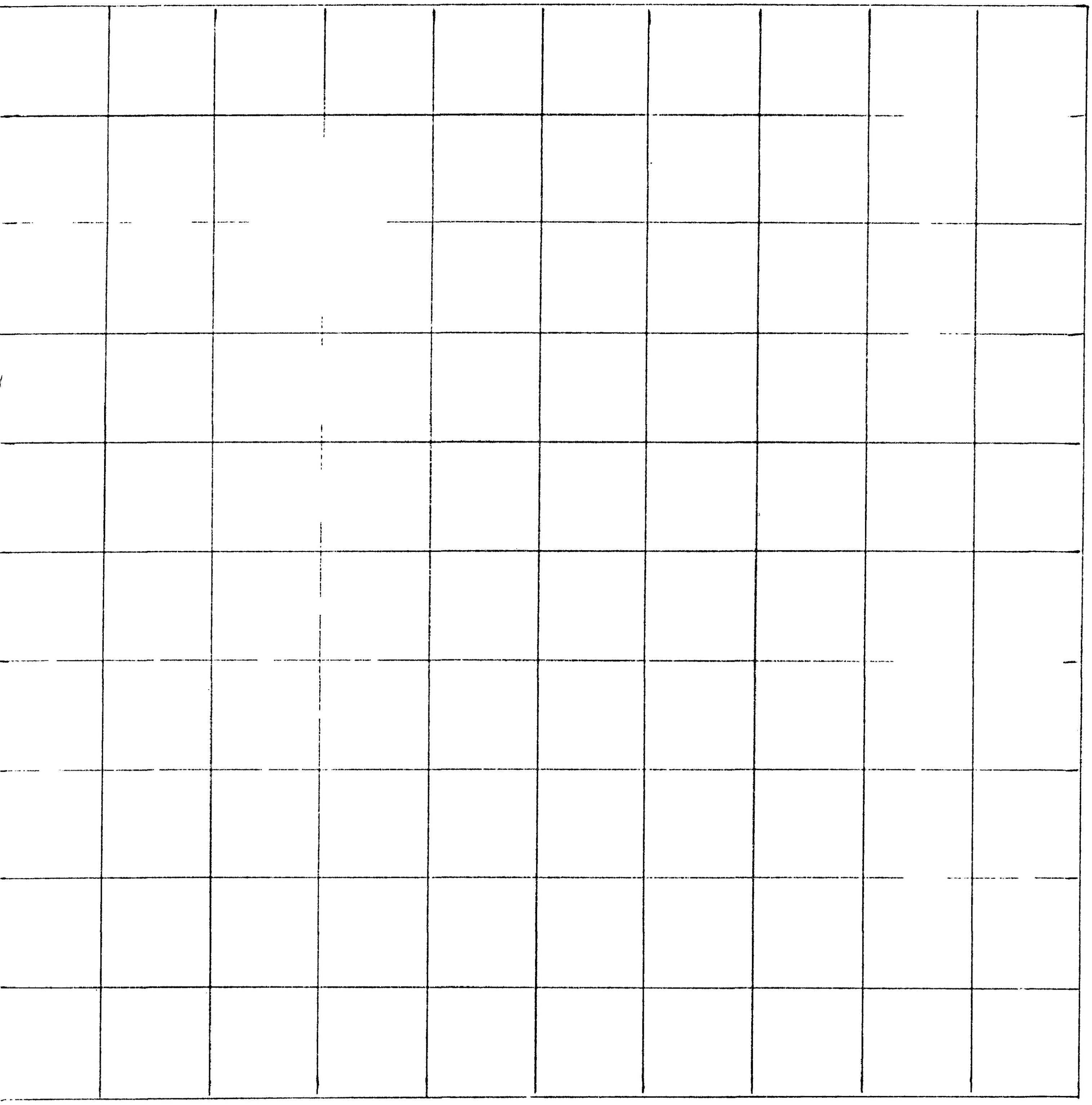
15. ______

<u>Appendix 3</u>. Answer sheets for Experiments 1 and 2.

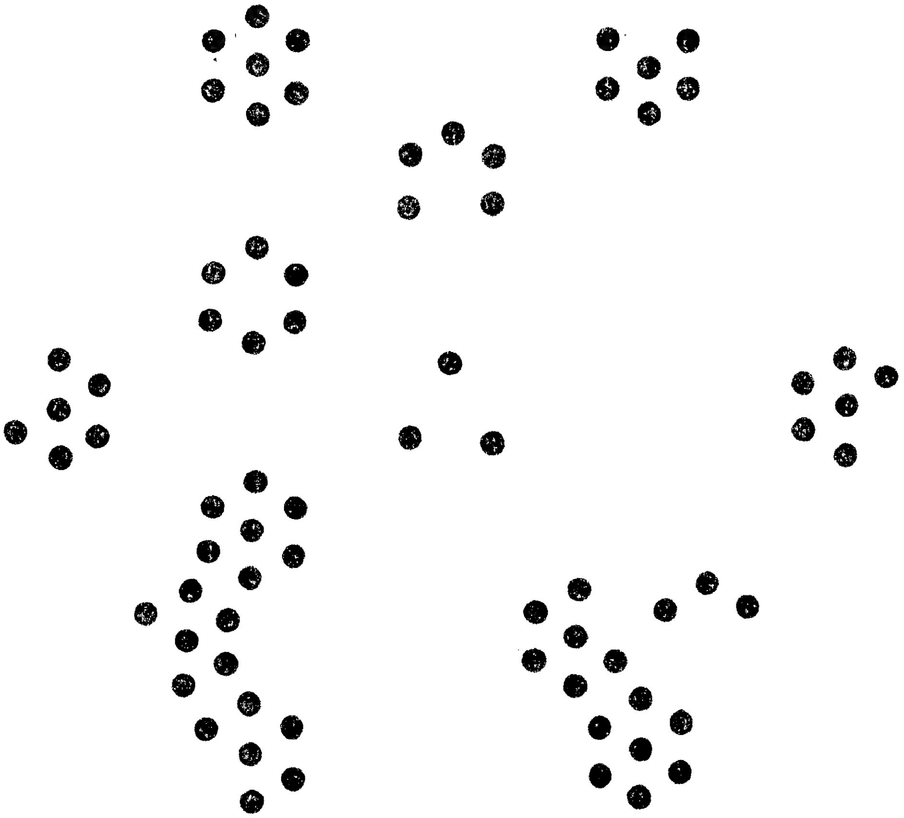
Source	df	MS	F
Between Subj			
A (Sex)	1	4608.56	.81
Ss within groups	59	5648.69	
Within Subj			
B (Number)	2	132969.45	157.59**
AB (Sex x Number)	2	1110.44	1.31
B x Ss within groups	118	843.74	
C (Arrangement)	3	7941.30	22.18**
AC (Sex x Arrangement)	3	195.48	.65
C x Ss within groups	177	358.12	
BC (Number x Arrange)	6	991.93	2.23*
ABC (Sex x Numb x Arrang)	6	835.70	1.88
BC x Ss within groups	354	444.98	

*p .05
**p .001

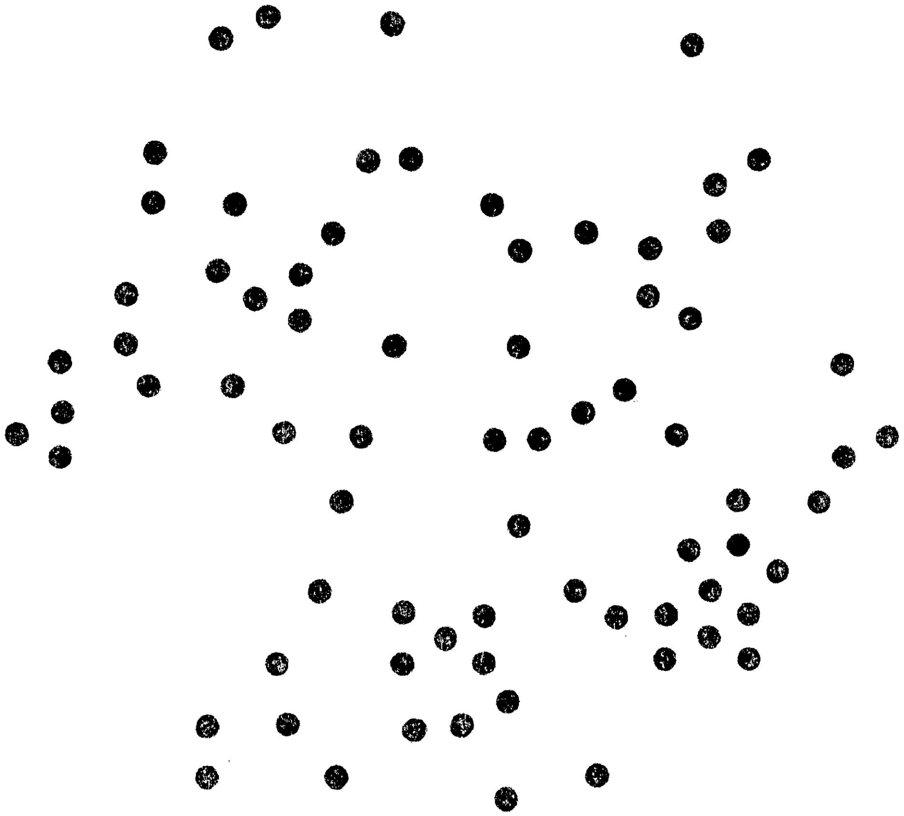
Appendix 4. MANOVA table for data in Experiment 1, summarized from SPSS output.



Appendix 5. Ten- by- ten matrix field for stimuli in Experiment 2.



Appendix 6. One of four model cards for the contagious (n = 74, variance = 4 x mean) card-sort in Experiment 3.



Appendix 7. One of four model cards for the random (n = 74, variance = mean = 2) card-sort in Experiment 3.

Subjects	Regular	Random	C1	C2	
1	160	183	145	160	Males
2	160	150	120	123	
3	190	245	215	190	
4	182	146	121	116	
5	224	169	191	186	
6	175	165	160	155	
7	190	153	115	115	
8	200	235	165	130	
9	140	114	115	170	
10	169	121	122	127	
11	180	125	130	120	
12	245	215	180	186	
13	170	150	135	150	
14	245	314	226	290	
15	163	123	135	135	
16	174	170	120	167	
17	200	140	130	128	
18	260	200	185	160	
19	170	169	180	165	
20	245	190	173	235	
21	212	158	215	200	
22	215	180	193	175	
23	230	165	125	150	Females
24	177	170	155	140	
25	374	215	145	195	
26	340	275	185	220	
27	110	100	88	105	
28	154	240	170	155	
29	300	185	223	195	
30	332	265	305	252	
31	148	101	93	131	
32	130	115	120	120	
33	155	155	215	140	
34	628	545	376	340	
35	120	100	80	100	
36	91	110	105	95	
37	350	300	250	240	
38	145	155	135	125	
39	310	260	258	242	
40	145	142	107	156	
41	365	365	355	280	
42	180	177	154	170	
43	214	165	177	199	
44	170	140	150	150	
45	304	119	149	190	
46	215	165	185	190	
47	95	105	90	100	

(con't)

Subjects	Regular	Random	C1	C2	
48	122	103	89	98	Females (Cont'd)
49	175	142	136	161	
50	260	200	200	170	
51	254	160	180	165	
52	160	145	170	110	
53	140	140	125	140	
54	200	200	235	174	
55	295	330	190	190	
56	180	180	185	195	
57	152	145	116	222	
58	193	208	177	185	
59	300	270	235	237	
60	74	73	74	63	
61	230	225	205	205	
	12786	11170	10183	10275	Totals
	69.9	61.0	55.6	56.1	Means

Appendix 8. Subject totals collapsed across number for Experiment 1.

	Subject	Random	Contagious
Females	1	310	229
	2	270	258
	3	470	240
	4	318	259
	5	340	320
	6	397	298
	7	350	350
	8	290	270
	9	315	370
	10	275	285
	11	190	174
	12	430	322
	13	380	262
	14	370	310
	15	267	277
	16	320	272
	17	240	190
	18	380	255
	19	320	305
	20	154	123
	21	285	260
	22	470	345
	23	558	586
	24	295	243
	25	735	683
	26	665	405
	27	269	210
	28	345	248
	29	267	240
	30	463	313
	31	251	220
	32	230	260
	33	472	371
	34	310	321
	35	385	277
	36	474	460
	37	284	246
	38	352	381
	39	315	270
	40	280	225
	41	280	306
	42	375	385
	43	255	260
	44	208	176
	45	265	255
	46	200	170
	47	365	318
	48	337	350
	49	461	336
	50	398	343
	51	365	209
	52	486	313
	53	203	194
	54	293	293
	55	450	332
	56	275	225
	57	290	175
	58	470	341
	59	327	299
	60	264	217
	61	481	370
	62	231	146
	63	321	244
Males	64	503	360
	65	230	249
	66	325	349
	67	406	324
	68	237	220
	69	250	270
	70	218	190
	71	440	460
	72	313	271
	73	342	287
	74	236	184
	75	330	330
	76	220	148
	77	360	325
	78	333	193
	79	261	273
	80	260	302
	81	295	181
	82	335	292
	83	284	264
	84	220	285
	85	269	251
	86	185	138
	87	387	348
	88	520	345
	89	255	220
	90	225	215
	91	405	365
	92	218	270
	93	360	305
	94	361	343
	95	170	165
	96	212	235
	97	259	235
	98	170	157
	99	330	290
	Total	32,190	27,829
	Mean	325.2	281.1

Appendix 9. Subject totals collapsed across number for Experiment 2.

Subject	Random	Contagious	
	Elapsed time (sec.)/ Errors		
1	76/1	63/1	Males
2	88/0	222/1	
3	138/1	102/3	
4	88/0	87/0	
5	73/0	98/4	
6	276/0	149/8	
7	90/4	77/0	
8	114/4	75/0	
9	58/0	69/0	
10	74/0	125/0	
11	158/6	122/0	
12	71/0	99/1	
13	79/0	95/3	
14	72/1	59/0	
15	83/0	121/5	
16	65/2	108/2	
17	94/1	74/0	
18	94/0	197/0	
19	143/0	75/0	
20	72/1	100/1	Females
21	102/0	84/7	
22	54/0	81/0	
23	75/1	110/1	
24	59/0	62/0	
25	43/0	54/0	
26	64/0	58/1	
27	80/0	96/5	
28	98/2	90/0	
29	90/0	143/4	
30	76/1	117/11	
31	71/0	68/8	
32	50/0	47/0	
33	67/2	67/3	
34	54/0	70/0	
35	86/4	72/0	
36	75/0	109/2	
37	74/5	110/4	
38	71/0	98/0	
39	125/2	126/0	
40	83/1	73/0	
41	81/4	111/3	
42	81/0	116/4	
43	59/0	76/1	
44	54/3	60/11	
45	88/0	102/2	
46	76/0	122/2	
47	127/1	120/4	

Appendix 10. Sorting time and errors for subjects in Experiment 3.