Height Growth and Site Index of Trembling Aspen in North Central Ontario

Kerry C. Deschamps ©

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fulfilment of the requirements for the Degree of
Master of Science in Forestry

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ABSTRACT

Deschamps, K.C. 1990. Height-growth and site-index of trembling aspen in north central Ontario. 101 pp. Major Advisor: Dr. W.H. Carmean.

Key Words: Polymorphism, site-index curves, non-linear regression, stem analysis.

Height-growth and site-index curves were developed for estimating site quality of trembling aspen (*Populus tremuloides* Michx.) in north central Ontario. These curves were developed from stem analysis data of dominant and codominant, uninjured aspen trees obtained from 89 plots covering a wide range of site quality in north central Ontario. The actual height-growth patterns were modelled using several non-linear biological growth models: Chapman - Richards function, modified Weibull function, Monserud logistic function and an expansion of the Chapman - Richard function. In addition, a new height-growth model was developed using a similar approach to that of Cieszewski and Bella.

Height-growth patterns of aspen varied with level of site-index. Height growth curves show an almost linear growth pattern for poor sites (SI < 16 m) to a highly curvilinear pattern on good sites (SI > 24 m). Medium sites (SI 16 -24 m) show a rapid linear surge of height growth before 40 years followed by a slowing curvilinear pattern.

Height-growth curves, site-index curves and a site-index prediction equation were constructed from trembling aspen stem analysis data. Goodness of fit tests were computed using a modified Chi-square test. In addition, the accuracy of the height-growth curves, site-index curves and site-index prediction equation were tested using independent stem analysis data from 19 plots supplied by the Ontario Ministry of Natural Resources. Comparisons with the independent data source shows close agreement; the 95% site index error prediction interval for the site-index curves and the site-index prediction equation are 0.19 ± 1.37 and 0.21 ± 1.35 respectively.

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INTRODUCTION

Trembling aspen (*Populus tremuloides* Michx.) is the most widely distributed tree in North America, spanning 110 degrees of longitude and 47 degrees of latitude. It occurs in all the Canadian provinces, in the Lake States and the western Mountains of the United States (Figure 1). In Canada, commercial

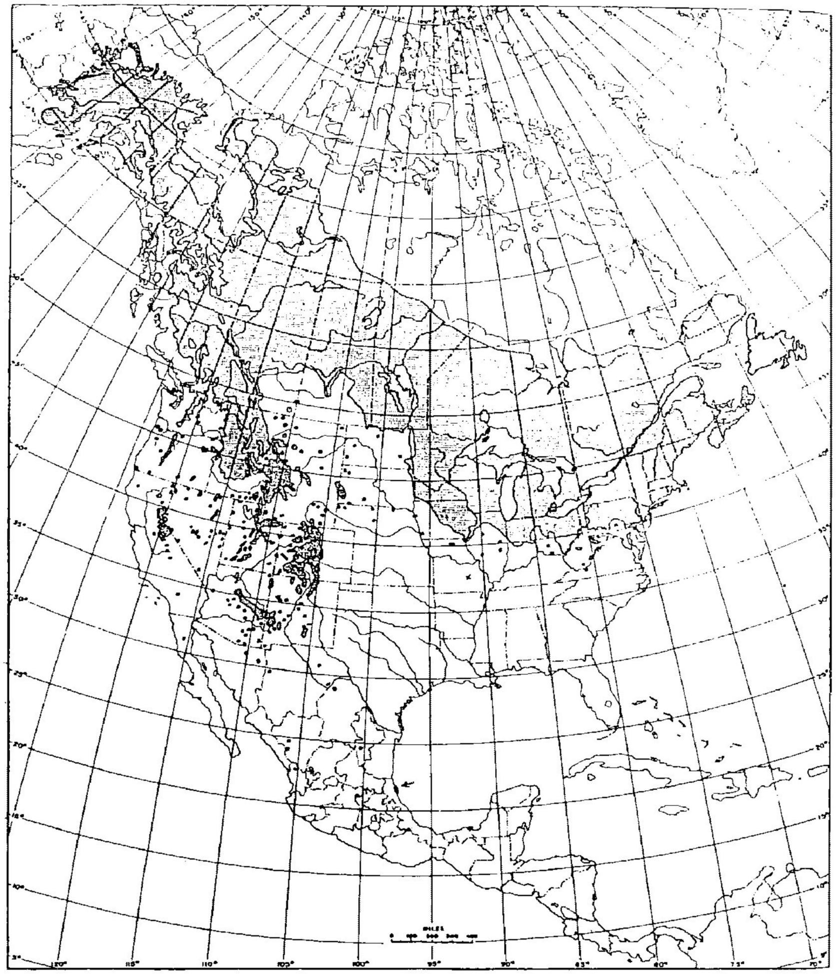


Figure 1. Trembling aspen range in North America (Fowells, 1965).

quantities of aspen are found in British Columbia, in the three Prairie Provinces, in Ontario, and in Quebec (Jarvis, 1968). Trembling aspen dominates much of the provincial landscape of Ontario, ranging from the Quebec border on the east to the Manitoba border in the west and from the southern border of Canada to a northern limit near Hudson Bay (Maini, 1968b).

In terms of gross total volume and land base coverage, the poplar working group represents the second largest working group in the Boreal Forest and Great Lakes - St. Lawrence Forest Regions in Ontario (OMNR, 1986; OMNR, 1988). Figure 2 shows the gross total volume for the poplar working groups in each Ontario Ministry of Natural Resources (OMNR) Region (OMNR, 1986).

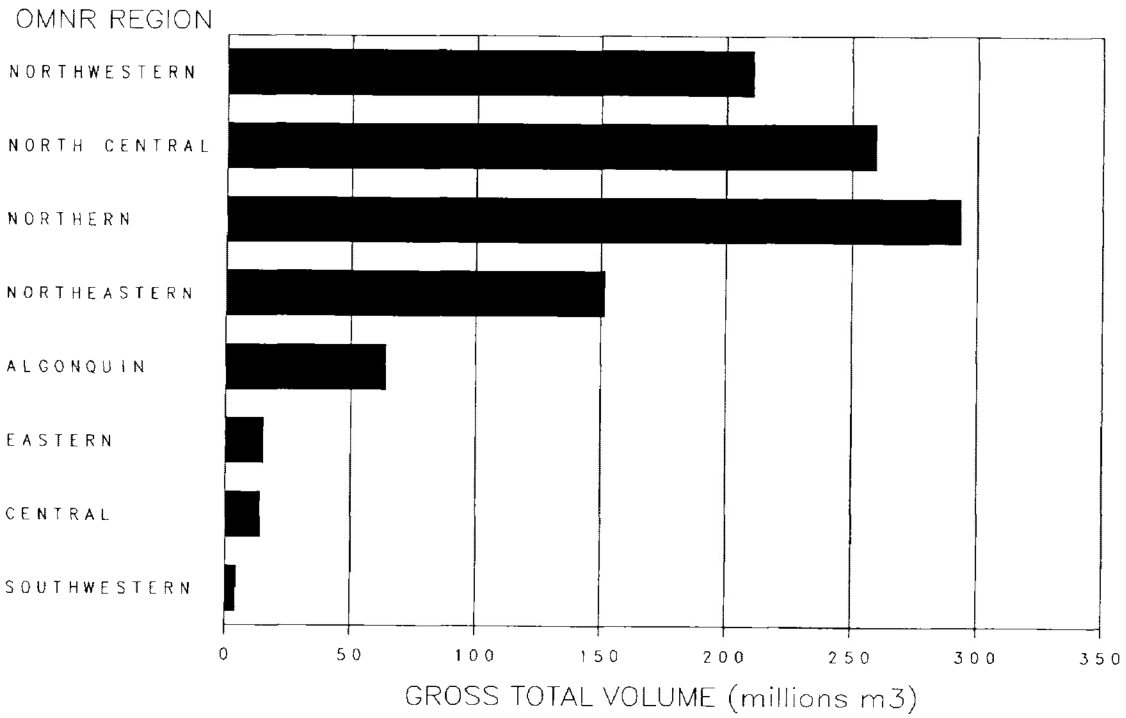


Figure 2. Gross total volume of the Poplar Working Groups in each Ontario Region (OMNR, 1986).

The bulk of the volume occurs in the Northern, North Central and Northwestern Regions (Figure 2). Despite the wide range and large volumes of aspen in Ontario, only a small portion of the annual allowable cut was harvested in 1972 (OMNR, 1988). However, the total volume harvested from Crown lands in Ontario has increased more than 300 per cent from 1976 to 1986 (Smyth and Campbell, 1987; Balantinecz and Morley, 1987) (Table 1).

Table 1. Poplar timber harvested on provincial crown lands from 1976 - 1986 (Smyth and Campbell, 1987).

Year	Harvested Gross Total Volume (m^3)	Percentage Increase Based on 1976 Cut
1976	687,164	0
1977	792,672	15
1978	1,059,586	54
1979	1,439,082	109
1980	1,509,922	120
1981	1,414,955	106
1982	1,800,431	162
1983	1,505,038	119
1984	2,243,088	226
1985	2,641,390	284
1986	2,768,308	303

This increased aspen utilization occurs because of expanding markets, economically accessible stands and because of major technological developments in the pulp and paper, particle board and fibreboard industries (Maini and Cayford, 1968; Neilson and McBride, 1974; USDA, 1976; DeByle and Winokur, 1985). Aspen is suitable for a variety of products including pulp and paper, fibreboard, plywood, particleboard, lumber and veneer (Maini and

Cayford, 1968; Neilson and McBride, 1974; USDA, 1976; DeByle and Winokur, 1985; Wong and Szabo, 1987; Ondro, 1989).

Many problems are associated with aspen management in Ontario:

1. Scattered and often inaccessible location of aspen stands.
2. High incidence of decay and stand break-up caused by diseases including *Hypoxlon* spp. and *Fomes igniaris* (Basham and Navratil, 1975; Kemperman *et al.,* 1978; Basham, 1979, 1981).
3. Small tree size for aspen and Canadian dependency on coniferous species (Balatinecz, 1979).
4. Lack of research in site quality and growth and yield of trembling aspen over its range in Ontario.

This aspen site quality study is based on the need for increased knowledge about aspen site quality in Ontario. The objective of this study is part of a larger site quality evaluation project designed by Dr. W. H. Carmean at Lakehead University, Thunder Bay, Ontario (Carmean, 1986, 1987, 1990). The specific objective of this project is to develop mathematical models for expressing height-growth curves and site-index curves for aspen in north central Ontario. Studies with many forest species usually show polymorphic height growth, that is, height-growth patterns are not proportional but may differ greatly for varying soil conditions and levels of site quality (Carmean, 1968, 1975). An additional objective of this study is to determine whether height polymorphism exists for aspen or not.

LITERATURE REVIEW

ASPEN SILVICS

Habitat

The wide geographic range shows that aspen can tolerate a great variety of climatic conditions. Key climatic factors that may affect the range and growth of aspen are temperature and moisture (OMNR, 1988). Within the range of trembling aspen in Canada, climatic factors vary as much as 23° to 5° C for mean July daily temperature, 13 to 445 cm for mean annual precipitation, and 40 to 260 for growing degree days above 5.6° C (Maini, 1968a).

Aspen is found on a variety of soils, from wet clays to dry sands and peat (Kittredge and Gevorkiantz, 1929; Kirby *et al.,* 1957; Steneker 1976; Fowells, 1965). However, growth is generally best on fresh to moist clay loams and moist sandy loams that have a water table within reach of the roots (Stoeckeler; 1948, 1960; Strothman, 1960; Steneker, 1976). Steneker (1976) illustrated good, intermediate and poor site on a matrix of soil texture and soil moisture (Figure 3). Several systems have been developed to relate aspen site index to soil features such as texture, moisture, depth to water table, and topographic features such as aspect, slope, and slope position (Kittredge, 1938; Stoeckeler, 1948; Meyer, 1956; Voigt *et al.,* 1957; Strothman, 1960; Graham *et al.,* 1960; Fralish and Loucks, 1967, 1975; Steneker, 1976).

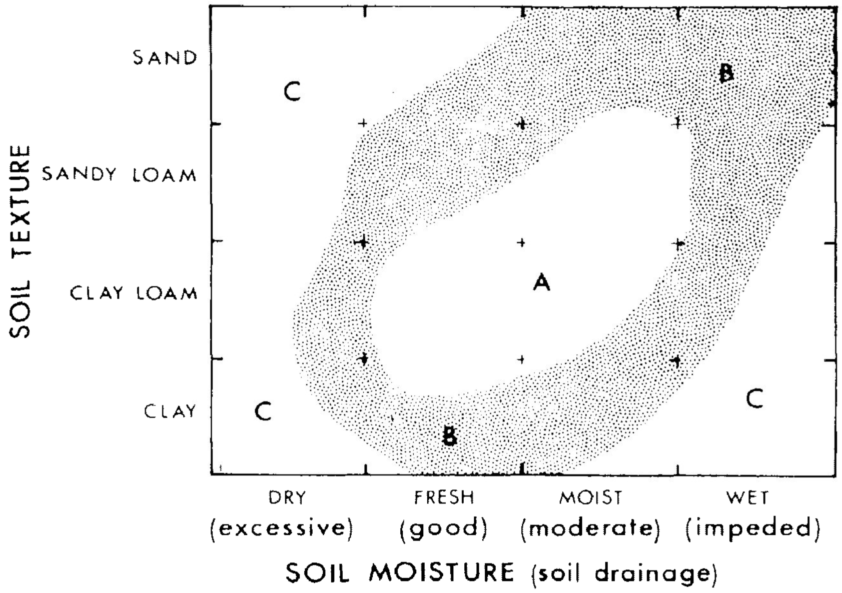


Figure 3. A soil texture, moisture and drainage matrix showing good (A), intermediate (B), and poor (C) aspen sites (Steneker, 1976).

Clonal Concept

Trembling aspen typically occurs throughout its native range as a clonal species (Barnes, 1966, 1969). A clone consists of genetically identical individuals (ramets) that have initiated from a root system of a single parent tree (ortet) (Barnes, 1966). Aspen clone size is generally small (less than 1 ha) in the northern part of its range (Kemperman and Barnes, 1976; Steneker, 1973; Lehn 1979). Larger clone size (10 to 43 ha) have been found in Utah (Kemperman and Barnes, 1976).

Identification of clones is possible by using a combination of the following features: flowering, leaf flush, leaf shape, leaf fall, bark colour and texture, stem form, branching and susceptibility to injury (Barnes 1969, 1975; Kemperman, 1977; Steneker and Wall, 1970). Aspen clonal differences have been reported for many features including tree biomass (Lehn, 1979), height-growth (Jones and Trujillo, 1975; Heidt, 1983), frost (Egeberg, 1963), defect (Wall, 1969, 1971; Copony and Barnes, 1974; French and Manion, 1975; Kemperman *et al.,* 1978; Weingartner and Basham, 1985) and suckering ability (Garrett and Zahner, 1964; Schier, 1974, 1976; Zasada and Schier, 1973).

Regeneration

Trembling aspen is a short-lived, fast growing pioneer species that regenerates by seed, and regenerates vegetatively by root suckers, root collar sprouts and stump sprouts. The trees are dioecious, meaning, the flowers on a single tree are the same sex (Maini, 1968b, 1972; Steneker, 1976; McDonough, 1985). Trembling aspen begins producing seed at about 20 years with good seed crops every 4 or 5 years thereafter (Fowells, 1965). It is generally assumed that aspen regeneration by seed in the field is uncommon (Maini, 1968b; Brinkman and Roe, 1975; and many others). However, preliminary results by Navratil and Bella (1988) show clear evidence of aspen seeding primarily on sites with mesic and subhygric moisture regimes. More commonly, aspen regenerates by root suckers following disturbances such as fire, logging, insects or disease, as long as the root system is not killed. Suckers originate from pre-existing primordia on lateral roots located below the soil surface (Schier, 1973). Suckering is triggered by breaking of apical dominance, that is, changing the hormonal balance (ratio of auxins and cytokinins) in the roots (Navratil and Bella, 1988). A high ratio of auxins/cytokinins suppresses suckering while a low ratio stimulates suckering (Navratil and Bella, 1988). Once the apical dominance is broken, soil temperature, carbohydrate reserve and clone genetics are the key factors in controlling the density of sucker development (Garrett and Zahner, 1964; Maini and Horton, 1966; Steneker and Walters, 1971; Zasada and Schier, 1973; Steneker, 1976; Navratil and Bella, 1988). High soil temperatures and high carbohydrate reserves will stimulate suckering while a low temperature and low carbohydrate reserves will suppress suckering. Suckering density is also affected by site, amount of aspen in the parent stand, and harvesting technique (Perala, 1972, 1977; DeByle and Winokur, 1985; Navratil and Bella, 1988; OMNR, 1988).

Development, Growth and Yield

Light is an essential factor for controlling the development of aspen stems (Perala, 1977; Schier and Smith, 1979; OMNR, 1988). Aspen is a very intolerant species and requires full sunlight to achieve maximum growth. Although it is common for first-year sucker density to be as high as 200,000 per ha canopy differentiation is rapid and natural thinning is usually quick and effective (Steneker, 1976). OMNR (1988) in a comprehensive review of aspen sucker density changes over time, show fairly rapid decreases to between 20,000 and 40,000 stems per ha by six years of age.

On good sites in Ontario, aspen grows to an average height of 30 m with an average diameter of 30 cm (Hosie, 1979; Plonski, 1981). Height and diameter growth on good sites is very rapid to maturity. However, Bella (1975) showed that diameter growth in young, high density aspen stands may be reduced by as much as 50 percent. Although young high density aspen stands thin naturally and produce maximum fibre yield, precommercial thinning may be a treatment option if large size trees or shorter rotations for specific tree sizes are required (Perala, 1978; Mowrer, 1987; Navratil and Bella, 1988). But thinning might involve risks from injuries, diseases and invasion by grass and shrubs (Walters *et al.,* 1982; Navratil and Bella, 1988).

Several yield tables have been prepared for aspen, but in Ontario aspen normal yield tables are limited to Plonski (1960, 1981) (Table 3). Many of these yield tables including Plonski (1960, 1981) use height-age curves (site-index curves) for relating height to volume.

Table 2. Inventory of aspen yield tables over its native range.

Area	Reference
Ontario	Plonski (1960, 1981)
Manitoba	Johnson (1957)
Saskatchewan	Kirby *et al.* (1957)
Alberta	MacLeod and Blyth (1955), Johnstone (1977), Grabowski (1981), Beck *et al.* (1982)
Prairies	Bella (1970, 1972)
Newfoundland	Page (1972)
Lake States	Schlaegel (1971), USDA (1979), Belcher (1981), Miner *et al.* (1988)
Rocky Mountains	Hinds and Wengert (1977), Edminster (1978), Edminster *et al.* (1982), Edminster and Mowrer (1985), Mowrer (1986).

Pathological Agents

Mortality and growth losses in stands of trembling aspen can be caused by several insects including forest tent caterpillars (*Malacosoma disstria* Hubner), aspen tortrix (*Choristoneura conflictana* Walker), leaf tiers, leaf rollers, leaf miners, and borers. Diseases affecting aspen include trunk and butt rots, stains, cankers, and leaf diseases (OMNR, 1988). The two most important pathological agents throughout the natural range of aspen are *Fomes igniarius* and Hypoxylon canker (*Hypoxylon* sp) (Fowells, 1965; Anderson and Anderson, 1968; Anderson, 1972; and many others). *Fomes igniarius*, a white heartwood rot with characteristic black zone lines infects stems, mainly through dead branch stubs and stem wounds and accounts for 35 to 50 percent decay in aspen (Anderson, 1972; Steneker, 1976). *Hypoxylon* canker, rapidly invades dead branch stubs and stem wounds causing girdling of the stem tissues (Anderson and Anderson, 1968). If boles become infected trees usually die within four to eight years (Baranyay, 1967).

FOREST SITE QUALITY EVALUATION

Carmean (1975) defines forest site quality as " the ability of forest land to grow trees " thus, evaluating site quality corresponds to land capability estimation for various tree species. Spurr and Barnes (1980) consider site quality as the sum total of all environmental factors: climatic, edaphic, and biological affecting the capacity to produce forests or other vegetation.

In terms of timber management, site quality can be defined as " the timber production potential of a site for a particular species or forest type under a given management regime" (Clutter *et al.,* 1983).

Historical Perspective

Traditionally, timber management in North America revolved around fire protection and harvesting of what appeared to be inexhaustible virgin forests (Carmean, 1975). Population growth and industrial expansion caused enormous demands for timber products, even though the area of forested land was reduced because of agriculture, urban and recreational needs (Carmean, 1975). Intensive forestry management soon replaced exploitative forestry practices and the need to forecast future timber yields became essential. Intensive forestry practices usually are concentrated on the most productive forest lands and on the most valuable forest species. Accordingly, one of the first steps for intensive forestry is the development of site quality evaluation methods capable of identifying productive forest lands where intensive silviculture can be applied.

From 1910 - 1925, the urgent need for a standard system of site quality evaluation for forest management was recognized (Mader, 1963). Intense controversy involved three methods of site quality evaluation: (1) site types, (2) volume growth and (3) height-growth.

Zon (1913) advocated a comprehensive system of forest site types based on Cajander's earlier work in Finland who later published his work (Cajander, 1926). This method is an ecological approach to site classification based on plant indicators. Differences of opinion existed as to whether site types should be based solely on vegetation or whether they should also encompass soils, topography and climate (Mader, 1963). Coile (1938) pointed out many shortcomings of the plant indicator method for estimating site quality. Even so, successful use of this method has been reported from the northern Rocky Mountains (Daubenmire, 1976), Quebec (Gagnon and MacArthur, 1959), Alberta (La Roi *et al.,* 1988), and British Columbia (Spilsbury and Smith, 1947; Green, *et al.,* 1989).

Bates (1918) strongly advocated setting up a comprehensive system of forest site classification based on current annual cubic foot volume increment of fully stocked stands of the species under management. Bates (1918) was the most vocal of the supporters for volume saying " any other criterion of site quality is a compromise or makeshift ... ". Mader (1963) has advocated the use of volume as a method of site quality evaluation.

Roth (1916, 1918) initiated the movement for site classification based on height growth, but the main supporters were Frothingham (1918, 1921a, 1921b), Sterrett (1921) and Watson (1917). They all recognized volume as a desirable standard of site classification but saw practical problems involved indirectly using volume. Supporters of height growth for site-quality evaluation state that (1) height is a sensitive measure of differences in site, (2) height is independent of stocking and species mixture within broad limits, and (3) height-age relationship of trees are simple and easily determined in the field.

The Society of American Foresters organized a committee in 1920 to evaluate the various methods of site quality evaluation and to recommend a standard method for use in the United States. In 1923, the committee recognized that volume production was the ultimate measure of site quality and recommended construction of yield tables but did not recommend one site quality evaluation method over another (Carmean, 1975). However, they did favour the use of height growth because of its simplicity and convenience (SAF, 1923).

Reviews of the history and philosophy of forest site quality evaluation are given by Coile (1952), Mader (1963), Ralston (1964), Jones (1969), Carmean (1975), Pritchett and Fisher (1987), and Daniel *et al.* (1979) for the United States, Burger (1972) for Canada, Spurr and Barnes (1980) for North America and Europe, and Hagglund (1981) for all areas.

Site Index as Site-Quality Estimation

Though the ultimate measurement of site quality is the total volume of wood under a given management regime at rotation, volume may be affected by many factors other than site quality. For instance, stand density, past cutting practices, and species mixture tend to limit the usefulness of volume as a measure of site (Clutter *et al.,* 1983; and many others). Height growth is sensitive to differences in site quality, little affected by stand density and species mixture, and is strongly correlated with volume (Clutter *et al.,* 1983). This generally accounts for the widespread use of site index as a measure of site quality in North America (Carmean, 1975) and in Europe (Hagglund, 1981).

Direct Estimation of Site Index From Forest Trees

Directly estimating site index from forest trees involves two fundamental conditions; (1) reliable site trees and (2) accurate site-index curves (height-age curves). Several free-growing, uninjured, dominant, or dominant and codominant trees are directly measured for height and age. Such trees commonly occur in even-aged, fully-stocked, older forest stands. Height and age measurements are used with site-index curves to estimate the total height of trees at a standard index age. The index age is most commonly 50 years, but younger index ages are common for short-lived species or young plantations, and older index ages are used for longer-lived species. A basic assumption when determining site index is that site trees now dominant have been dominant throughout their lives. Daniel *et al.* (1979) suggests this assumption is valid in even-aged stands especially for shade intolerant species. Thus, site index is simply a height-growth prediction of those site trees either forward or backwards in time to a standardized index age. See Carmean *et al.* (1989) for a discussion of the above points as well as an inventory of site-index curves for the eastern United States.

Many authors have developed site-index curves for trembling aspen (Table 3). Several older curves were developed only from total height-age measurements and from harmonizing methods; however, most newer curves are based on stem analysis methods (Table 3).

Two other methods for directly estimating site index from forest trees are the species site-index comparison method and the growth intercept method. The species site-index comparison involves estimating site index for a species not present in the stand using site index of a species present (Carmean and Vasilevsky, 1971; Carmean, 1975, 1979; Carmean and Hahn, 1983). Commonly, paired site indices of different species are gathered from sample plots then

Table 3. Inventory of site-index curves for trembling aspen.

Area	Data	Methods	Reference
Ontario	Stem analyses	Harmonized	Plonski (1960)
Ontario	Stem analyses	Polymorphic	Payandeh (1974a) based on Plonski (1960)
Ontario	Stem analyses	Polymorphic	Deschamps (1985)
Saskatchewan	Total height/ age points	Harmonized	Kirby *et al.* (1957)
Manitoba	Total height/ age points	Harmonized	Johnson (1957)
Prairies	Stem analyses	Polymorphic	Bella and DeFranceschi (1980)
Alberta	Stem analyses	Harmonized	Heidt (1983)
Alberta	Stem analyses	Polymorphic	Alberta Dept. Energy and Natural Resour. (1985), Cieszewski and Bella (1990)
Newfoundland	Total height/ age points	Harmonized	Page (1972)
New Brunswick	Total height/ age points	Harmonized	Longphee (1984)
Lake States	Total height/ age points	Harmonized	Gevorkiantz (1956)
Lake States	Total height/ age points	Polymorphic	Lundgren and Dolid (1970) Laidly (1979) based on Gevorkiantz (1956)
North Lower Michigan	Total height/ age points	Harmonized	Graham *et al.* (1960)
North Wisconsin and Upper Michigan	Stem analyses	Polymorphic	Carmean (1978)
North Central Minnesota	Growth plot remeasurement	Harmonized	Schlaegel (1971)
North Central Rocky Mountains	Total height/ age points	Polymorphic	Edminster *et al.* (1985)
Central and South Rocky Mountains	Stem analyses	Polymorphic	Jones (1966; 1967)
Alaska	Stem analyses	Harmonized	Gregory and Haack (1965)

correlated statistically using linear regression. Resulting information is usually presented in species site-index comparison equations or graphs to simplify ease of use in the field. This method permits a quick and easy way of extending direct site-index estimation to areas where decisions are required to select the most desirable tree species for management from among many possible species (Carmean, 1975). In addition, this method can be applied to areas where soil and site vary greatly for estimating the most valuable species for that piece of land.

" The growth intercept method uses a selected period of early height-growth as an index of site quality rather than the long-term height-growth portrayed in site-index curves " (Carmean, 1975). Growth intercept is commonly accepted as the total length of the first five internodes above breast height (Alban, 1972). However, Thrower (1986a, 1987) found that growth intercept for both white spruce and red pine were more precise when internode measurements were started somewhat higher than breast height. Thrower found that the best precision in estimating site index resulted by using the average length of three to five internodes above 2.0 m for white spruce, and above 1.5 m for red pine. This method can only be used in young plantations and natural stands of tree species having well defined whorls marking annual height growth (Alban, 1972; Carmean, 1975; Thrower, 1987).

Indirect Estimation of Site Index From Soil, Vegetation and Topography

Indirect methods for estimating site index are used for areas where trees and stands are not suitable for directly estimating site index. Indirect methods for estimating site index include soil surveys, ecosystem classifications and soil-site studies.

Soil surveys were originally developed to classify soils for agricultural use and as such have a continuing built-in agronomic bias on their development (Rowe, 1962). However, in recent years, soil surveys have expanded to forest lands for the purpose of classifying soils into similar units (Carmean, 1975). Soil surveys are very valuable for land use planning in that they provide a comprehensive survey of land capability for many purposes including agriculture, forestry, recreation and wildlife. The soil survey approach has the potential for classifying forested landscapes into units that are relatively similar in site quality. However, frequently difficulties occur because soil types are too broadly defined and often vary greatly in features such as depth, stone content, subsoil conditions, and topographic conditions that are closely related to site quality (Carmean, 1975).

Ecosystem land classification attempts to break complex forest landscapes into progressively more homogeneous units having similar vegetation, soil and topographic conditions. There are many different approaches to forest ecosystem land classification including the efforts of Krajina (1965) in British

Columbia, Corns and Annas (1986) in Alberta, Hills (1960), Rowe (1972), Jones *et al.* (1983), Nicks (1985), Greenwood (1987) and Sims *et al.* (1986) in Ontario. In the north central region of Ontario, a Forest Ecosystem Classification (FEC) study was begun in 1983 with the goal of providing foresters and other resource managers with a workable, field-based classification system for silviculture planning (Sims, *et al.* 1986). This work followed very closely the original FEC work in the clay belt of northeastern Ontario conducted in the late 1970's.

In general, the FEC can be characterized by a two-way grid with nutrient regime gradients and moisture regime gradients as the two axes. Other criteria that are used together with the moisture-nutrient grid to define operational groups (homogenous land units) are vegetation and soil properties of the specific site. The operational group level of classification is thought to have some value in defining site factors that are important for forest site quality. These important site factors include moisture, nutrients, and microclimatic conditions. Although intuitively, the ecological land classification approach has merit in defining important features of site quality, this method as with the soil classification method, often use classification units that are only generally related to site quality.

The soil-site method involves an estimation of site index for a species based on soil, topographic and climatic variables (Carmean, 1975; Schmidt, 1986; Schmidt and Carmean, 1988). Soil, topographic and climatic information gathered from sample plots are statistically correlated with site index measured on the plots using multiple linear regression, and more recently multivariate techniques. Resulting relationships are used to develop site prediction tables and graphs for use in the field. This method is particularly useful where areas are extremely variable in site quality, soil and stand conditions, or have no suitable site-index trees present (Carmean; 1975).

SITE-INDEX CURVES

Data Sources

Site-index curves can be developed using various kinds of data:

1. Measurement of total height and total age from dominant and codominant trees located in temporary sample plots.
2. Measurement of height growth over time from dominant and codominant trees found in permanent sample plots.
3. Stem analysis can be used to reconstruct individual tree height-growth patterns for trees representing a wide range of soil and site quality conditions.
4. Internode measurements also can be taken from tree species having recognizable internodes. These measurements can then be used to reconstruct individual tree height-growth patterns covering a wide range of soil and site quality conditions.

In the first case, Clutter *et al.* (1983) suggested that temporary plots provide the cheapest data for site-index curve development. However, they go on to state that " the use of such data involves the assumption that the full range of site index is well represented in all age classes within the sample ". However, if this assumption is false, the resulting height/age curves will be seriously biased. To illustrate this point, refer to Figure 4. In this example, trees in the younger ages generally are taller than trees in the older age classes. This may occur because younger ages are mostly represented by plots having good site quality, and older ages are mostly represented by poor site plots. This abnormal distribution of site quality within age classes will cause the average curve through the points to be biased upward at young ages and biased downwards at older ages.

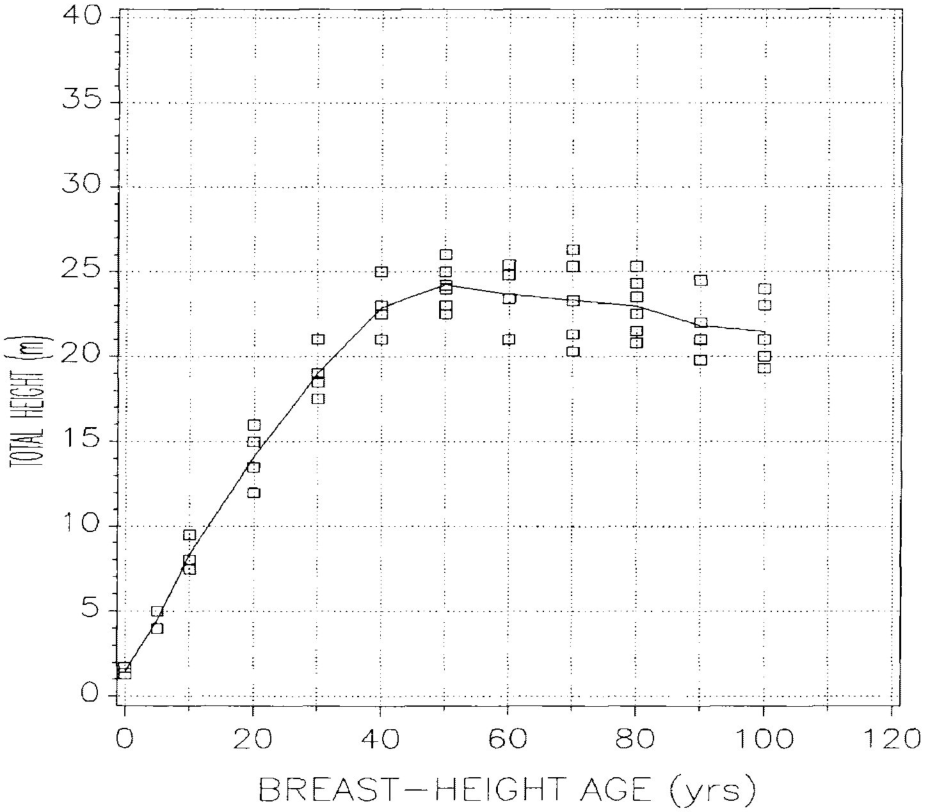


Figure 4. A biased average height-age guide curve.

The second source of information for construction of site-index curves is from remeasurement of permanent sample plots. This information usually consists of remeasured heights at known ages for individual dominant or dominant and codominant trees within the plots. This method is limited due to the expense of obtaining data, time element before data becomes available and lack of permanent sample plots (Curtis, 1964; Clutter *et al.,* 1983). Also existing permanent sample plots may not represent the full range of soil and site quality conditions within a forest region.

The most favoured source of data for constructing site-index curves is from stem analysis of dominant or dominant and codominant site trees. This technique expresses actual tree height-growth patterns and interpolation or approximate site-index can be made with reasonable accuracy (Beck, 1971; Newnham, 1987). The data consists of age counts at various points on the bole of each sampled tree. Costs are high to obtain data in this fashion, however, the information can be obtained very quickly. Detailed techniques used in stem analysis work are given by Curtis (1964), Strand (1964), Heger (1968), Carmean (1972), Lenhart (1972) and Dyer and Bailey (1987).

The fourth source of information for construction of site-index curves is from internode measurements. This method is limited in applicability to only tree species, and to tree ages showing distinct internodes. Data usually consists of internode measurements from individual dominant or dominant and codominant young trees within a stand. Often this method is done together with stem analysis techniques to verify the position and age at each internode (Thrower, 1986a). Similar to the stem analysis method, costs are high but information can be gathered very quickly.

Desirable Properties

Several desirable properities have been listed by Devan and Burkhart (1982) and Borders *et al.* (1984) for the construction of height-growth and site-index curves. These properities include:

(1) height is zero when age is zero;
(2) height at index age equals site index;
(3) the curves have no specific index age, that is, they are invariant with respect to choice of base age;
(4) the upper asymptote increases with site index so that trees on better sites reach a higher ultimate height than those on poorer sites;
(5) the curves should also be polymorphic.

Besides these properities, another may be added to allow for the height-growth model to be solvable for site-index; then a site-index model compatible with a height-growth model can be formulated (Cieszewski and Bella, 1989). Few site-index curves possess all these properties; however, the goal should be to include as many as possible.

Tree growth process are seldom linear, thus investigators have used polynomial regression and logarithm transformations to approximate tree height growth. However, these linear models of tree height growth are not very flexible and usually require a greater number of parameters to describe height growth.

Non-linear least squares regression techniques have been commonly used for formulating published harmonized site-index curves for many species (Payandeh, 1974a,b; Lundgren and Dolid, 1970; and many others) and for developing polymorphic site-index curves for many tree species (Ek, 1971; Monserud and Ek, 1976; Monserud 1984a, Cieszewski and Bella, 1989; and many others). Generally non-linear models are more flexible, often biologically based and often provide reasonable estimates beyond the data range (Pienaar and Turnbull, 1973; Payandeh, 1983).

Construction Techniques

A wealth of literature is available on site-index curve development techniques. Based on the literature, it is apparent that many approaches and analytical techniques can be used to model height over age. However, Clutter *et al.* (1983) suggest most of the techniques can be viewed as special cases of the three following development methods: (1) the guide curve method, (2) the difference equation method, and (3) the parameter prediction method.

The Guide Curve Method

Most older site-index curves included with normal yield tables studies were developed using the guide curve method described by Bruce (1923, 1926).

Generally this technique involves:

1. Collecting one-time total height and total age data from dominant and codominant trees located in yield study plots spanning a wide range of site quality.
2. Plotting all the height/age information on a scattergram.
3. Drawing an average guiding curve through the scattergram of height/age points.
4. Using the guide curve to draw specific proportional site-index curves for high and low levels of site quality.

Analytical models soon replaced the hand drawn average guide curve. One of the first commonly used models with the guide curve method was the " ln(height)/reciprocal of age " model originally suggested by Schumacher (1939) (Clutter *et al.,* 1983). This model has the following form:

$$\ln (H_t) = \beta_0 + \beta_1 (Age_t^{-1}) + \varepsilon \qquad [1]$$

where

ln = natural logarithm
H_t = total stand or tree height at Age_t
Age_t = total stand or tree age
β_0, β_1 = model parameters to be estimated.
ε = error of the model

This resulting equation represents the average height-age guide curve for the sample data. The equation is then translated to the following in order to produce a family of height/age curves:

$$\ln (H_t) = \ln (SI_t) + \beta_1 (Age_t^{-1} - Age_{t\,0}^{-1}) + \varepsilon \qquad [2]$$

where
SI_t = site index (total height at $Age_{t\,0}$)
$Age_{t\,0}$ = base index age (usually 50 years)

More recently, Newberry and Pienaar (1978) presented a complex model form for use with the guide curve method (Clutter *et al.,* 1983). They used the following form of the Chapman-Richards function (Richards, 1959; Chapman, 1961) to develop the mathematical relationship between tree height and tree age.

$$H_t = \beta_1(1-e^{\beta_2 \, Age_t})^{\beta_3} + \varepsilon \qquad [3]$$

Regardless of how the average guide curve is developed, the fact remains that the resulting family of site-index curves have exactly the same shape as the guide curve, differing only in amount by a fixed percent. Site-index curves developed using the guide curve method typically have been called " anamorphic " or " harmonized " site-index curves.

Many weaknesses of harmonized site-index curves have been reported by several authors (Spurr, 1952; Carmean, 1956, 1975; Curtis, 1964; Beck and Trousdell, 1973; and many others). These include the following:

1. The technique is only sound if the average site quality is the same for each age class (Spurr and Barnes, 1980). However, in most cases, young stands are found on better sites while older stands are found on poor sites. This is most likely because of early harvesting on better sites and easily accessible stands. The resulting average guide curve will be warped upwards at young ages and downward at older ages (see Figure 4).

2. The assumption that the shape of the average guide curve is the same for all sites. Bull (1931) was one of the first authors to demonstrate that anamorphic site-index curves introduced large errors in red pine curves because they misrepresented the actual trends of growth. He found the shape of the height-growth curve varied by site, with maximum height-growth occurring sooner on the good sites than on the poor sites. Several methods have been used to show the evidence of many-shaped " polymorphic " patterns of height-growth (Bull, 1931; Beck, 1971; Carmean, 1968, 1975).

The Difference Equation Method

This technique involves the development of a difference form of the height-age equation (Clutter *et al.,* 1983). The difference form expresses future height as a function of future age, initial age and height at initial age. An example of this approach presented by Clutter *et al.,* (1983) uses the Schumacher (1939) " ln (height)/reciprocal of age " model (see eq. 1). The difference equation form for the Schumacher model is:

$$\ln (H_{t2}) = \ln (H_{t1}) + \beta_1 (Age_{t2}^{-1} - Age_{t1}^{-1}) + \varepsilon \qquad [4]$$

where

H_{t2} = total tree height at remeasurement
H_{t1} = initial total tree height
Age_{t2} = total tree age at remeasurement
Age_{t1} = initial total tree age

This equation was then fitted using linear regression as follows:

$$Y = \beta_1 X + \varepsilon \qquad [5]$$

where

$Y = \ln(H_{t2}) - \ln(H_{t1})$
$X = Age_{t2}^{-1} - Age_{t1}^{-1}$

This equation was then translated to the following to produce a site-index equation.

$$\ln (SI_t) = \ln (H_{t1}) + \beta_1 (Age_{t0}^{-1} - Age_{t1}^{-1}) + \varepsilon \qquad [6]$$

Clutter *et al.* (1983) present the development of the following difference equation form of the Chapman - Richards model:

$$H_{t\,2} = H_{t\,1}\left(\frac{1-e^{B_2\,Age_{t\,2}}}{1-e^{B_2\,Age_{t\,1}}}\right)^{B_3} + \varepsilon \qquad [7]$$

This equation was then fitted using non-linear least-square regression to obtain the parameters. Once the parameters are estimated, replacing the Age_2 value by index age (Age_0) and H_2 by site-index (S) gives the following site-index equation:

$$SI_t = H_{t\,1}\left(\frac{1-e^{B_2\,Age_{t\,0}}}{1-e^{B_2\,Age_{t\,1}}}\right)^{B_3} + \varepsilon \qquad [8]$$

Examples of site-index curves developed using this method include Clutter and Lenhart (1968), Clutter and Jones (1980), Devan and Burkhart (1982) and Borders *et al.* (1982).

The Parameter Prediction Method

The parameter prediction method has recently been the most frequently used method for developing site-index curves. This method requires remeasurement, stem analysis, or internode data. This technique involves relating the parameters of the height/age points to site-index through linear or non-linear regression procedures (Clutter *et al.*, 1983). Several biological growth functions have been used with this method (Table 4).

Table 4. Inventory of growth functions used in the parameter prediction method of constructing site-index curves.

Function Name	Source	Model Form	Equation Number
Logistic	Robertson (1923)	$H_t = \beta_1(1+e^{\beta_2 Age_t})^{-1} + \varepsilon$	[9]
ln (height)/ reciprocal of age	Schumacher (1939)	$H_t = \beta_1 e^{\beta_2 Age_t^{-1}} + \varepsilon$	[10]
Gompertz	Medamar (1940)	$H_t = \beta_1 e^{\beta_2^{e^{\beta_3 Age_t}}} + \varepsilon$	[11]
Chapman-Richards	Richards (1959) Chapman (1961)	$H_t = \beta_1(1-e^{\beta_2 Age_t})^{\beta_3} + \varepsilon$	[3]
Modified Weibull	Yang *et al.* (1978)	$H_t = \beta_1(1-e^{\beta_2 Age_t^{\beta_3}}) + \varepsilon$	[12]
Modified Weibull	Bailey (1980)	$H_t = \beta_1(1-e^{\beta_2 Age_t^{\beta_3}})^{B_4} + \varepsilon$	[13]

One of the most widely used non-linear growth functions is the Chapman-Richards generalization of von Bertalanffy's function (von Bertalanfy, 1941). This function was developed to describe mathematically the relationship between animal body size and metabolic rate. Pienaar and Turnbull (1973) introduced this function into the forestry literature showing its adequacy for use in tree and stand growth odelling. Each parameter of this function [eq. 12] has a biological meaning: β_1, governs the upper asymptote; β_2 describes the growth rate after the inflection point; and β_3, defines the increase in the growth rate before the inflection point.

Due to the ease of biological interpretations of the parameters, many expansions have been developed to incorporate other variables that influence height-growth. Commonly, the β_1 and β_3 are power functions of site index (aSI_t^b) although linear functions that include site index can be found (Lundgren and Dolid, 1970; Beck, 1971; Burkhart and Tennett, 1977; Ek, 1971; Payandeh 1974a, 1974b and Monserud and Ek, 1976). The β_2 parameter is usually a single negative parameter, although linear functions that include site are found. Expansions of the Chapman - Richards function include:

Ek (1971) $$H_t = \beta_1 SI_t^{\beta_2} \left(1-e^{\beta_3\, Age_t}\right)^{\beta_4 SI_t^{\beta_5}} + \varepsilon \qquad [14]$$

Biging (1985) $$H_t = \beta_1 SI_t^{\beta_2} \left(1-e^{\beta_3\, Age_t}\right)^{\beta_4} + \varepsilon \qquad [15]$$

$$\text{Alemdag (1988)}\ SI_t = \frac{1}{\beta_1 H_t^{\beta_2}\left(1-\left(1-\left(\frac{1}{\beta_1 H_t^{1+\beta_2}}\right)\frac{1}{\beta_3 H_t^{\beta_4}}\right)\frac{Age_t}{Age_{t\,0}}\right)^{\beta_3^{H_t^{\beta_4}}}} + \varepsilon \qquad [16]$$

These expansions allow height growth to vary with the level of site-index. The Ek (1971) model has been successfully used by Payandeh (1974 a,b) to formulate height-growth curves for the major tree species in Ontario and in Canada. Hahn and Carmean (1982) and Carmean *et al.* (1989) also used the Ek model to formulate site-index curves for many tree species in the United States. However, these expansions were not flexible enough for other species and locations to remove all bias across site and age (Monserud and Ek, 1976; Monserud, 1984a). It has been pointed out by Goudie (1984) that the inflection point in the Chapman - Richards model and its expansion is constrained to be between 0 and 0.367 of the distance between the X-axis and the upper asymptote.

Monserud developed a more flexible height-growth model by investigating the logistic function, which is seldom used in forestry. He found that the b_1S^{b2} was good for the upper asymptote and selected the following β_2 after investigating several alternative functions; $b_3 + b_4 \ln(Age_t) + b_5 \ln(SI_t)$. This function has performed very well for several species in western Canada (Goudie, 1984; Dempster and Associates, 1983 and Alberta Dept. Energy and Natural Resour., 1985). This function has the following form:

$$H_t = \beta_1 SI_t^{\beta_2} \left(1 + e^{\beta_3 + \beta_4 \ln(Age_t) + \beta_5 \ln(SI_t)}\right) + \varepsilon \qquad [17]$$

Cieszewski and Bella (1989) provided an interesting approach to developing polymorphic height and site-index curves for lodgepole pine in Alberta. They defined a suitable base model for identifying the underlying height-growth pattern of a small subset of the fastest growing trees. They modified the base model to test several hypotheses about growth rates and polymorphism on the stem analysis of 970 lodgepole pine trees resulting in this variable-age-site-index height model form:

$$H_t = 1.3 + \frac{h_{t\,x} + A + \sqrt{(h_{t\,x} - A)^2 + B\ h_{t\,x}\ Age_{b\,0}^{-1-\gamma}}}{2 + B\ Age_b^{-1-\gamma} /\ (h_{t\,x} - A + \sqrt{(h_{t\,x} - A)^2 + B\ h_{t\,x}\ Age_{b\,0}^{-1-\gamma})}} + \varepsilon \qquad [18]$$

where

$h_{t\,x}$ = site index (total height at $Age_{b\,0}$)
$Age_{b\,0}$ = breast height base index age
Age_b = breast height age
$A = 20\ \beta_1 / Age_{b\,0}^{\ 1+\gamma}$
$B = 80\ \beta_1$
γ, β_1 = parameters to be estimated

This functional form constrains the curves to pass through the origin and site height at any index age. It also provides compatible site-index and height estimates and can predict height without prior knowledge of site index (Cieszewski and Bella, 1989).

Another common approach to height-growth and site-index curve development is that of Dahms (1975) and others. This approach is built upon

Linear relationships are developed relating height as a function of site index for each decade and then age is re-entered by relating the regression coefficients to the function of age (Dempster and Associates, 1983; Lenthall, 1985). However, the problem of multicollinearity may result in several variables not needed in this technique as shown by Monserud (1984a).

To overcome the limitation of site-index curves not passing through the site height at index age, investigators have constrained the functions to pass through the site height at index age. Newnham (1987) presents the following version of the Ek model that constrains the curves to pass through the appropriate site height at index age:

$$H_t = \beta_1 SI_t^{\beta_2} \left(1 - k^{Age_t \,/\, Age_{t\,0}}\right)^{\beta_4^{SI_t^{\beta_5}}} + \varepsilon \qquad [19]$$

where

$$k = 1 - \left(SI_t \,/\, \beta_1 SI_t^{\beta_2}\right)^{\frac{1}{\beta_4 SI_t^{\beta_5}}}$$

He also developed and tested another constrained version of the Ek model by applying a constraining technique displayed by Burkhart and Tennent (1977). The model had the following form:

$$H_t = SI_t \left[\left(1 - e^{\beta_3 \, Age_t}\right) / \left(1 - \beta_3^{Age_{t\,0}}\right)\right]^{\beta_4 SI_t^{\beta_5}} + \varepsilon \qquad [20]$$

However, he found that this model gave a poorer fit and showed noticeable

However, he found that this model gave a poorer fit and showed noticeable bias in places compared to the previous constrained Ek model [eq. 19]. Dempster and Associates (1983) constrained the Monserud logistic model [eq. 17] to pass through site height at index age by applying the same technique used by Burkhart and Tennent (1977). This resulted in the following model form:

$$H_t = 1.3+(SI_b-1.3)\left(\frac{1+e^{\beta_3+\beta_4\,\ln(Age_{b\,0})+\beta_5\,\ln(SI_b-1.3)}}{1+e^{\beta_3+\beta_4\,\ln(Age_b)+\beta_5\,\ln(SI_b-1.3)}}\right)+\varepsilon \qquad [21]$$

where

SI_b = site index (total height at breast height age)

In trying to develop more flexible functions that better describe the data, the resultant models have become so complicated that it is very difficult, if not impossible, to solve for site index as a function of height and age. To solve this problem with the Ek model, Payandeh (1974b) proposed the following model that closely approximates the height-growth model solved for site index:

$$SI_t = \beta_1 H_t^{\beta_2}\left(1-e^{\beta_3\,Age_t}\right)^{\beta_4 H_t^{\beta_5}} + \varepsilon \qquad [22]$$

However, for many height-growth functions there is no model form that would closely approximate the height-growth model solved for site index, so multiple linear regression techniques are used instead to relate site index to height and age.

Accuracy and Validation

As the level of forest management increases, so does the need for more accurate site-quality estimates. Information on site-index prediction errors can help the forest manager use site quality information wisely in forest management plans. However, many site-index curves do not come with estimates of precision or accuracy for use in application; in fact the user has no idea of the size of error involved in any given estimate of site index (Beck and Trousdell, 1973). Error associated with site-index prediction equations or site-index curves can be any of the following forms:

(1) inaccurate equations or curves;
(2) insufficient or poor quality data used during equation or curve construction;
(3) site-index prediction of individual trees;
(4) variations in sample tree height and age;
(5) measurement error (McQuilkin and Rogers, 1978);
(6) improper use of the procedure (Monserud, 1984b).

Heger (1971, 1973) and McQuilkin and Rogers (1978) presented confidence intervals for assessing effects on site-index estimates due to stand age, level of site index, sample of heights used with the curves, index age and sample size underlying the curves. Generally, the width of the confidence decreased with increasing (i) number of sample trees; (ii) proximity to index age and (iii) precision of tree measurement. Lloyd and Hafley (1977) estimated precision and the probability of misclassification in site-index estimation. They quantified the effects of stand age, base age, and sample size in the variance of site index. The probability of misclassifying site index is greatly affected by changing the width of class intervals or the value of stand age. Hagglund (1975) estimated the accuracy of Scots pine (*Pinus sylvestris* L.) site-index curves by means of simulation.

"Validation attempts to increase the confidence in a model's ability to provide useful and correct inferences about growth of trees rather than prove a model correct" (Goulding, 1979). Snee (1977), suggests four common approaches to validating models:

(1) comparison of model prediction and coefficients with theory;
(2) collection of new data to check model predictions;
(3) comparisons of results with theoretical model calculations;
(4) data splitting in which a portion is used to estimate the model coefficients, and the remainder of the data are used to measure prediction accuracy of the model.

The best way to validate the predictive ability of a model is to apply it to new data and compare predictions with actual observed values (Berk, 1984). Freese (1960) developed a statistical method for determining whether the accuracy of a model or estimation technique is adequate to meet the requirements of the user. This goodness of fit test uses the Chi squared probability function to predict the accuracy of the model against the observed data. In Freese's (1960) procedure, the model is assumed adequate unless there is evidence to suggest the contrary. The statistical procedure has the following form:

$$\chi^2_{(\alpha_1,n)} = \frac{\sum_{i=1}^{n} (x_i - u_1)^2}{\sigma^2} \qquad [23]$$

where x_i = the value of the i^{th} observational unit as estimated by the model.
u_i = the true value of the i^{th} observational unit as measured by the data.
n = the number of units observed (i = I...n)
σ^2 = the required accuracy as described below.

The required accuracy for considering whether a model is acceptable has the following form:

$$\sigma^2 = \frac{e^2}{Z^2_{(\alpha_1)}} \qquad [24]$$

where e = units within the true value
Z = standard normal deviate for probability of α_1

The null hypothesis of the model adequately describing the data can be rejected if the test statistic is greater than the tabulated value of χ^2 (α_1,n).

Reynolds (1984) reviewed the assumption and derivation of Freese's procedure and consequently developed an alternative and more conservative procedure to test the accuracy of models. In Reynolds approach, if the test statistic [eq. 23] is less than the tabulated value of χ^2 (α_2,n) where $\alpha_2 = 1 - \alpha_1$, then the null hypothesis of the model adequately describing the data is not rejected. Although the user wants to know how well a model meets certain standards, it might be more valuable to know the magnitude of errors that will result when the model is applied to predict future values (Reynolds, 1984).

Rennie and Wiant (1978) modified Freese's (1960) test to an approach of confidence limits to allow users to specify their own level of required accuracy. Their critical error limit formula is shown in the following equation.

$$e^{*} - \left[\frac{(Z^2)\left(\sum_{i-l}^{n} (x_i - u_i)^2\right)}{\chi^2_{(\alpha_1, n)}} \right]^{1/2} \qquad [25]$$

The critical error, e** for the more conservative Reynolds test is shown as:

$$e^{**} - \left[\frac{(Z^2)\left(\sum_{i-l}^{n} (\chi_i - \mu_i)^2\right)}{\chi^2_{(\alpha_2, n)}} \right]^{1/2} \qquad [26]$$

However, these critical errors cannot be directly interpreted into probability statements that future residuals will be below the critical levels. Instead, the critical errors are confidence bounds on the upper 95% quantile of the distribution of residuals under the assumption that the model is not biased.

In developing site-index curves for Jack pine, Lenthall (1985) applied a technique developed by Hahn and Nelson (1973) for constructing prediction intervals around future predicted residuals. This technique uses the following formula for a $(1-\alpha_1)$ 100% interval around a future predicted value:

$$\overline{D} \pm \left(1 + \frac{1}{n}\right)^{1/2} S \, t_{(1-\alpha_1 /2)} \qquad [27]$$

where

$\overline{D}$ = mean difference between observed and predicted value
S = the standard deviation of the residuals
t = the Student's t statistic with n-1 degrees of freedom

METHODS

STUDY AREA

The area for this study is entirely within the OMNR North Central Region, spanning most of the Thunder Bay District and small portions of the Nipigon and Ignace Districts (Figure 5). It is located within the rectangular area defined by 48° 30' to 50° 15' N latitude and 88° 15' to 91° W longitude. The study area was selected based on feasible travel to and from Thunder Bay.

The North Central Region has a microthermal and humid climate according to the Thornthwaite system (Sims *et al.*, 1986). Most of the region lies within the 2W, 3W and 4W physiographic site regions of Hills (1960), and the Boreal forest region of Canada according to Rowe (1972). Within the North Central Region, a variety of bedrock types are present with the precambrian rocks of the superior and southern provinces being the most prevalent (Sims *et al.*, 1986). Surficial features of the region reflect the effects of four major Pleistocene glaciations, the last ending about 10,000 years ago (Zoltai, 1967). Common glacial landforms include shallow to bedrock areas, ground moraines, undulating hills, end moraines, drumlins, eskers, kames, outwash deltas and plains, lacustrine deposits, and organic accumulations.

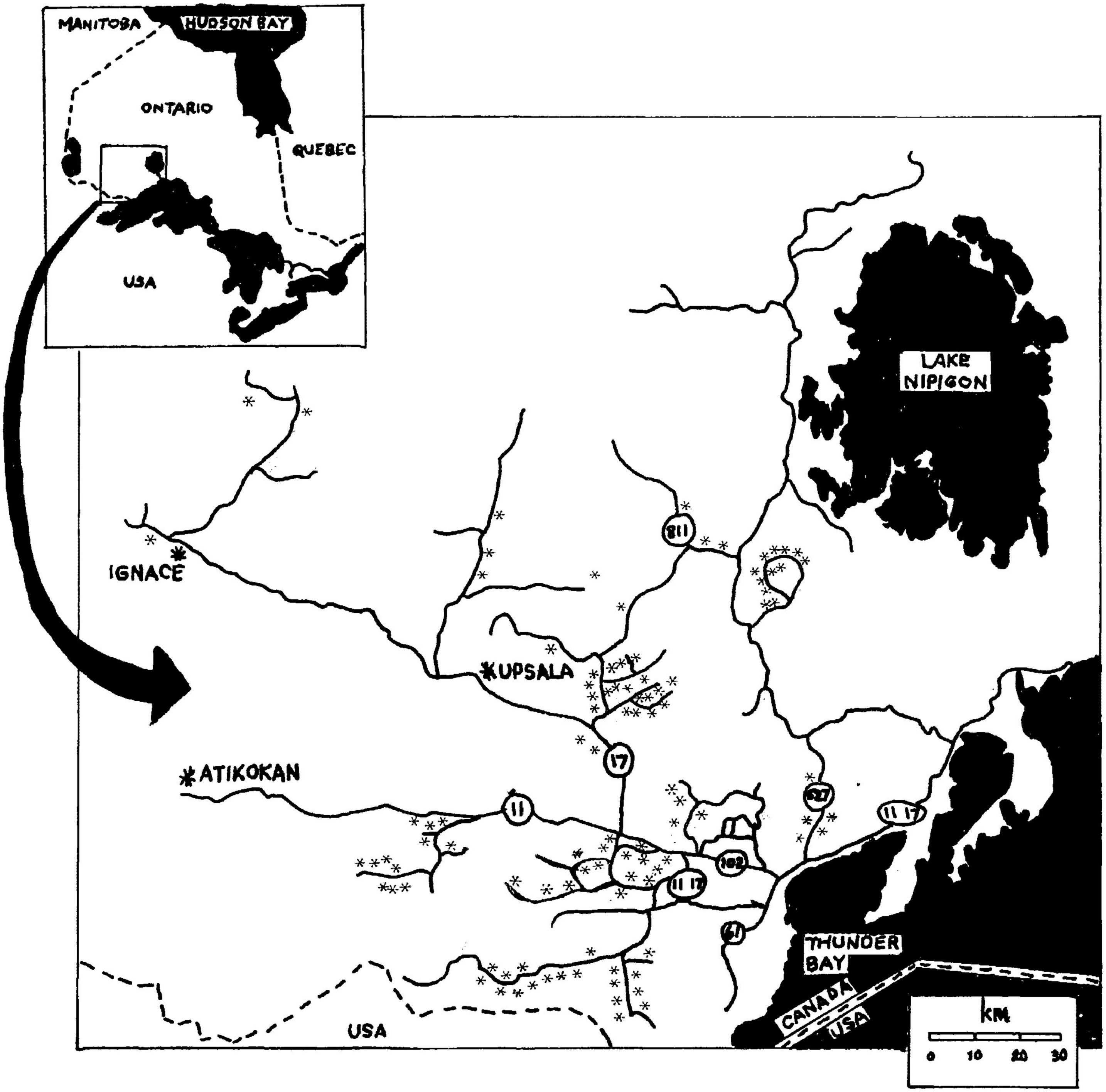


Figure 5. Study area and aspen sample plot locations.

SAMPLE PLOT LOCATIONS

One hundred and one sample plots were located throughout the study area (Figure 5). Their general characteristics are described in Appendix I. Most of the study plots were located in the summer of 1985 as part of this study; additional plots used in this study were located by Lenthall (1985) in the summer of 1983 and 1984, Deschamps (1985) in the summer of 1984, and Dr. W.H. Carmean in the summers of 1981, 1982 and 1983 (Table 5).

Table 5. Trembling aspen stem analysis data sources.

Source	Year/season of collection	Total number of sample plots	Total number of stem trees analysis
This study	1985/summer	81	256
Lenthall (1985)	1984/summer	4	16
Deschamps (1985)	1984/summer	2	8
Carmean (1986,1987)	1981, 1982, 1983/summer	14	44
Total		101	324

The Ontario Ministry of Natural Resources supplied stem analysis measurements from 87 aspen trees collected from 33 plots that were set aside for validation purposes (Appendix II).

The primary criterion for sample plot selection was the presence of dominant and codominant aspen in the stand. The Deschamps (1985) and OMNR data source were essentially in pure, even-aged aspen stands. The Lenthall (1985), and Carmean (1986, 1987) data sources were in jack pine stands that also included occasional dominant and codominant aspen trees. Other criteria observed in locating plots were:

(1) Plots were selected to represent a wide range of soil, topography and geologic landform conditions;

(2) Plots were in stands that were even-aged, fully-stocked (crown cover was nearly or fully complete) and undisturbed (no evidence of cutting or fire);

(3) Trees on each plot were a minimum of 50 years breast-height age, but were preferably older;

(4) Plots were in stands covering at least 4 ha in size; and

(5) Plots were in areas that were reasonably accessible, for instance, no more than 500 m from the nearest road.

Circular plots (0.08 ha in size) were located in portions of stands having uniform microtopography. These plots were at least 25 m from clearings and roads to avoid any influences on tree growth. A map was drawn for each plot showing location, so that plots could be relocated for future soil-site work. On each plot, 3 to 4 dominant and codominant aspen trees were selected for stem analysis. Each tree selected for stem analysis showed no significant evidence of defect, deformity, or injury. The stem analysis procedures were as follows:

1. The sample trees were felled, limbed and total height of each tree recorded;
2. Discs were cut at the stump, 0.75 m, 1.3 m and 2.0 m of height; above 2.0 m discs were cut at 1.0 m intervals to 13.0 m, then at 0.5 m intervals to the tip of the tree;
3. The discs were labelled for identification as to plot number, tree number, and the height at which the disc was cut; and
4. All the discs were bagged and transported to the laboratory where growth rings were counted.

LABORATORY PROCEDURES

Discs were cut near the pith transversely into two pieces as shown in Figure 6. The piece without the pith was discarded. The upper edge of the piece with the pith intact was shaved at a 45° angle with a sharp utility knife. Annual growth rings were carefully counted on each disc using magnification and illumination. The number of rings were recorded on the sample tally sheet found in Appendix III. If discs had large amounts of heart rot accurate ring counts were not possible and consequently values were left blank on the tally sheet. After the ring counts were determined and tallied for a tree, they were checked for obvious counting errors. If counting errors occurred, the questionable discs were reshaved and growth rings recounted. One year was added to the stump ring count to estimate total tree age.

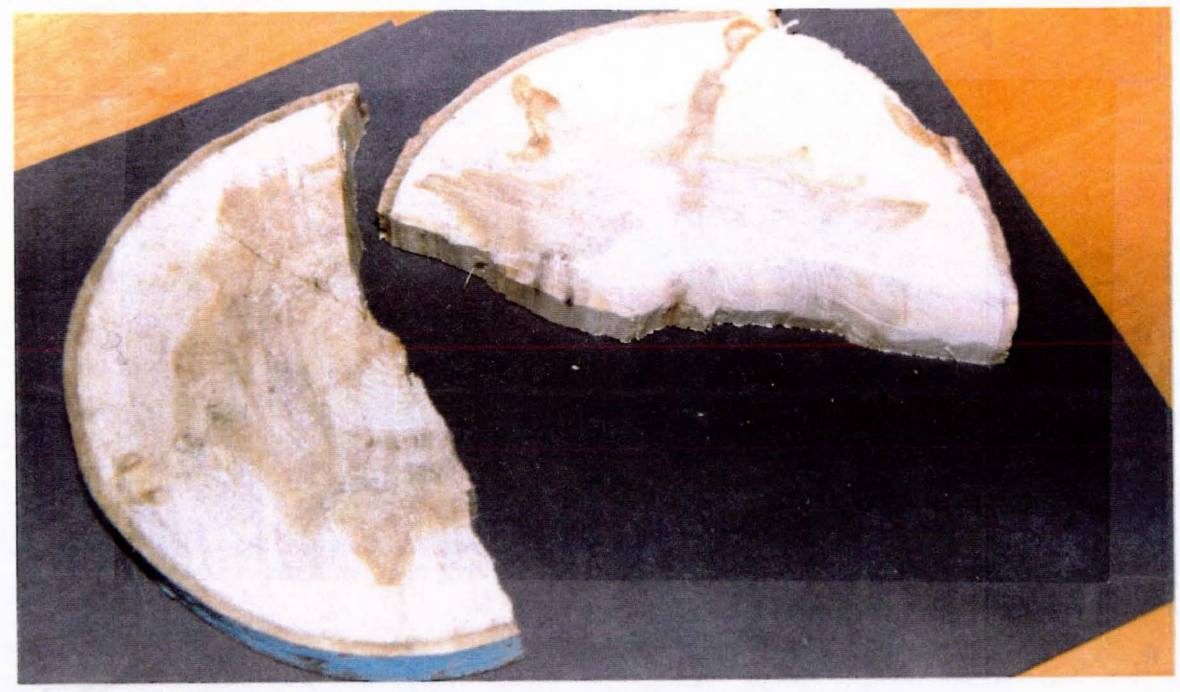


Figure 6. Aspen disc preparation for counting of growth rings.

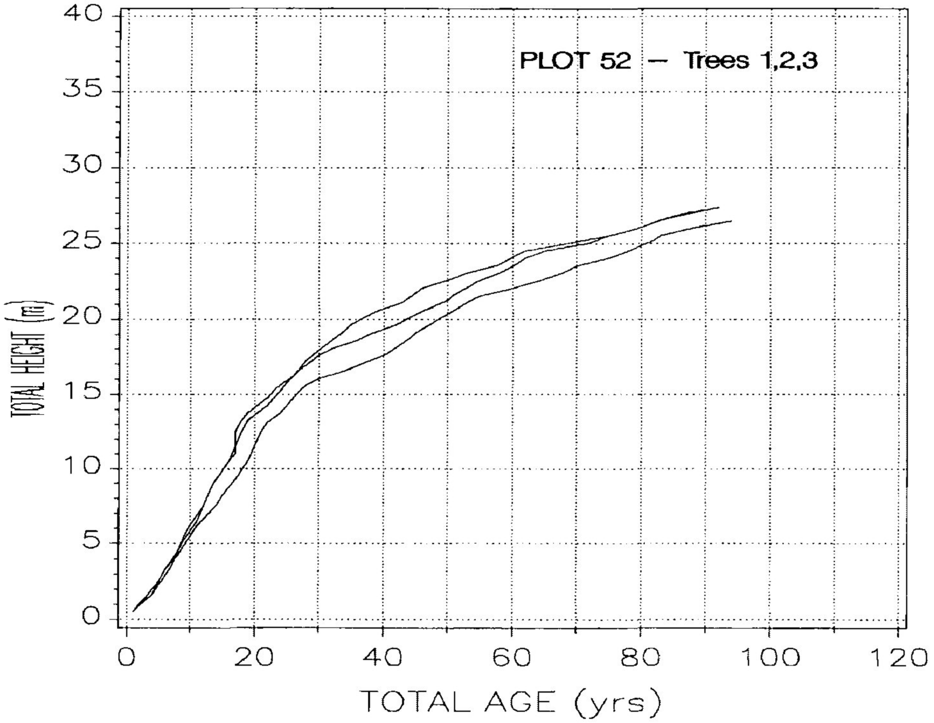
The ring counts on each disc section were then subtracted from the total tree age to obtain age at each section height. The total height-total age points for each sampled tree were adjusted upward by removing a bias caused by the section point not coinciding with the tip of the annual leader. This adjustment is based on the assumption that the section point (on the average) occurs in the middle of the leader (Carmean, 1972). In all likelihood, the adjusted height data point should be more representative of the tree height values. In an independent test of six methods for estimating tree heights from stem analysis data, the most accurate was the method proposed by Carmean (1972) (Dyer and Bailey, 1987). This adjustment can be represented as follows:

Adjusted tree height = section tree height + (bolt length/age difference)/2.

Height over age curves were then plotted for each sectioned tree on each plot using adjusted section height and age at each section (Figure 7). Missing height-age data for trees having rot in less than four discs were extrapolated to get a complete height-age curve. Trees having more than four consecutive rotted discs were dropped from further analysis.

Tree height-age curves for species such as spruce and red pine show erratic and slow early height-growth before reaching breast height (BH), but after BH, trees usually have a smooth regular pattern of height-growth (Husch, 1956; Thrower, 1986a). Trembling aspen does not usually have this pronounced slow and erratic early height-growth, but instead has rapid early height-growth (Figure 7). Nevertheless, aspen height-age curves were converted to total height-BH age curves because many stump discs had rot, thus accurate total ages could not be determined. Also BH age was used so that site-index curves would be comparable to curves for other species including jack pine, black spruce, and plantation grown red pine and white spruce (Carmean, 1986, 1987, 1990).

The total height over BH age curves were inspected for any early signs of height suppression (Figure 7). Badly suppressed trees were rejected from further analysis because their abnormal growth patterns would not provide a realistic indication of the height-growth potential for the site. The height of each sample tree at 50 years BH was linearly interpolated from the total height-BH



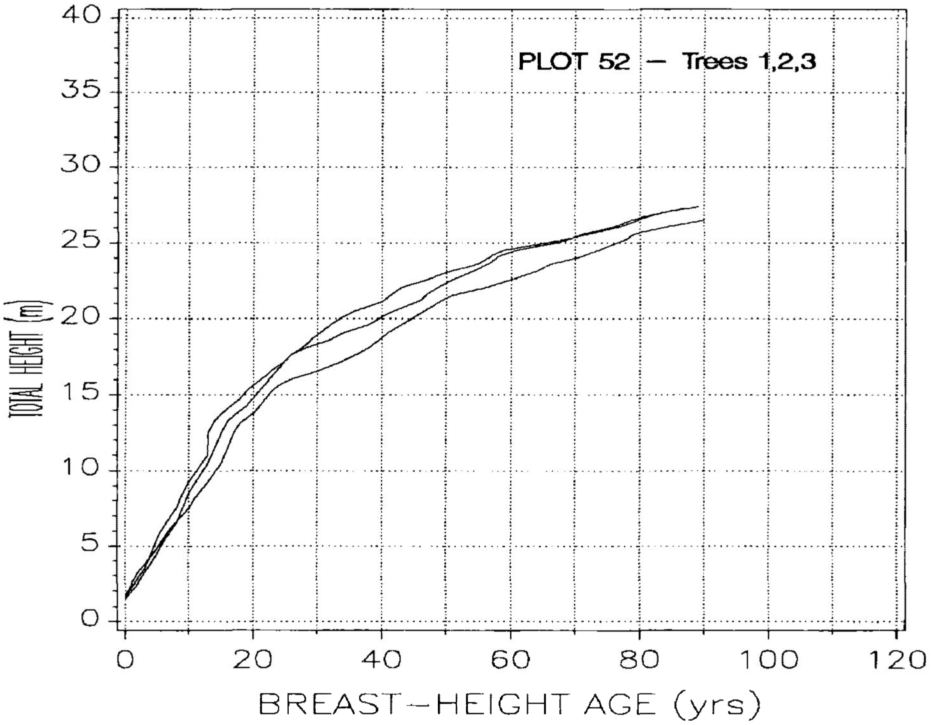


Figure 7. Height over total age (upper) and height over breast-height age (lower) for three aspen stem analysis trees on a single plot.

age curves and this height value was used as site index. Trees with a BH age less than 50 years were discarded from further analysis because a site-index value could not be reasonably estimated. Stem analyses were made on a total of 324 trees on the 101 plots, however, approximately one-quarter of all the aspen stem analysis trees were discarded from further analysis (Table 6). Most of the 77 rejected trees were suppressed or showed abnormal height-growth patterns. This left 247 usable aspen stem analysis trees on 89 site plots for developing height-growth and site-index curves.

Table 6. Trembling aspen stem analysis data source after tree screening.

Tree screening criteria	Number of stem analysis trees rejected	Percentage of stem analysis trees rejected
Tree suppressed or showing abnormal height-growth pattern	41	13
Trees less than 50 years breast-height age	18	6
Trees have more than four consecutively rotted discs	13	4
Only one tree on a single plot	1	< 1
Other (age difference greater than 10 years)	4	< 1
Total rejected trees (rejected plots)	77(12)	23
Total retained trees (retained plots)	247 (89)	77
Total trees (total plots)	324 (101)	100

Almost 43 per cent of all the aspen stem analysis trees in the OMNR validation data set were discarded (Appendix II). These trees were discarded because of 1) less then 50 years breast-height age (15%), 2) suppressed or abnormal height-growth patterns (20%), and only one tree on a single plot (8%). This left 40 aspen stem analysis trees on 19 plots for validating the site-index curves and prediction equations.

An average total height - BH age curve for each plot was calculated by averaging all the sample trees on the plot. The modelling data set and validation data set average total height - BH age curves for each plot are presented in Appendix IV and Appendix V respectively. Plots having less than two sample trees were rejected from further analysis (Table 6). Average site index (the average total height of the site trees at 50 years BH age for each plot) was linearly interpolated from the average total height - BH age curve for each plot. Paired total height - BH age points were also linearly interpolated at 2-year increments. These points were used in the analysis for constructing height-growth and site-index curves for aspen.

EXPLORATORY DATA ANALYSES

Summary statistics including the sample size, mean, standard deviation, minimum, maximum and coefficient of variation were computed for the plot site index and BH age variables. A scatterplot of site index and BH age were subsequently plotted to reveal general relationships between the two variables. A linear regression between site index (S) and BH age ($S = \beta_0 + \beta_1$ BH age) was computed.

Two metre site-index classes were computed by grouping the data from the 89 modelling plots. The Chapman - Richards model [eq. 3] and the modified Weibull model [eq. 12] were separately fitted to the data in each site-index class. The rationale for fitting the height-growth patterns within each site-index class was to examine if height-growth patterns varied in a definable way with level of site index. Scattergrams and correlations between each model parameter and site index were computed.

HEIGHT-GROWTH AND SITE-INDEX CURVE DEVELOPMENT

The development of height-growth and site-index curves was split into two separate parts. The first part was to investigate polymorphic height growth in model development. The second part was to investigate models for developing site-index curves and prediction equations for aspen.

If the exploratory phase suggested a pattern between model parameters and level of site index attempts were then made to expand the model to express this polymorphism. In addition, polymorphic height-growth patterns were investigated during the development of a new height-growth and site-index model for aspen following a similar approach used by Cieszewski and Bella

(1989) for lodgepole pine. This approach involved using five of the highest site-index trees from the OMNR data set to identify the underlying height-growth pattern for the fast growing trees and to select a base model for its description. Several height-growth functions were used to define the base model; the Chapman - Richards model [eq. 3], modified Weibull model [eq. 12], Schumacher, (1939) ln (height)/age model [eq. 10], and the logistic model [eq. 9]. These models were adjusted to use BH age instead of total age by replacing (Age_t) with (Age_b) and adding 1.3 to the right-hand portion of the equation. The two following models were also used to define the base model:

Tait et al. (1988) $$H_t = 1.3 + \frac{\beta_1}{1+\frac{\beta_2}{Age_b}} + \varepsilon \qquad [28]$$

Zakrzewski (1986) $$H_t = 1.3 + \left(\frac{Age_b}{\beta_1+\beta_2\ Age_b}\right)^2 + \varepsilon \qquad [29]$$

The half-saturation model [eq. 28] showed the best fit to the small subset of fast growing aspen trees. For this model, the β_1 value governs the upper asymptote and the β_2 value is the age at which height is half way to maximum height (Cieszewski and Bella, 1989). Hypotheses were then tested using this base model; to account for possible reduction in growth with age, the age variable was allowed to have an exponent ($1 + \beta_3$) and resulted in the following model:

$$H_t = 1.3 + \frac{\beta_1}{1+\left(\frac{\beta_2}{Age_b^{1+\beta_3}}\right)} + \varepsilon \qquad [30]$$

If the β_3 parameter is greater than zero, then this suggests that the dominant and codominant trees of the modelling data set had slower height growth with age than the five high site aspen trees.

For anamorphic curves, the β_2 parameter (the age at which half the maximum height is obtained) would be the same for all sites. To account for polymorphism, the β_2 parameter was allowed to be a function of site index. A polymorphic hypothesis might be that β_2 is inversely proportional to site index ($\beta_2 = \beta_4/SI_b$). This would imply that trees growing on good sites reach their half-way maximum height in a shorter time compared to trees growing on poorer sites. In developing this hypothesis the β_2 has been modified in such a manner that it has lost its original interpretation of being the mid-point of maximum height. To keep this parameter's interpretation it was multiplied by 27 (rounded off maximum height of the modelling aspen data set) resulting in $\beta_2 = 27 * \beta_4/SI_b$. To allow the hypothesis to be more flexible an additional parameter was added to SI_b ($SI_b^{\beta 5}$). Replacing $27 * \beta_4/SI_b^{\beta 5}$ in the model for β_2 results in the following model form:

$$H_t = 1.3 + \frac{\beta_1}{1+\left(\frac{\frac{27\ \beta_4}{SI_b^{\beta_5}}}{Age_b^{(1+\beta_3)}}\right)} + \varepsilon \qquad [31]$$

If $\beta_5 = 0$ the curves would be anamorphic, whereas if $\beta_5 = 1$ the curves would be polymorphic as hypothesized. However, if β_5 is neither 0 or 1 the curves would be polymorphic in a different way than hypothesized.

To account for the assumption that the upper limit of height growth increases with site index, equation [31] can be solved for β_1 and substituted back into equation for β_1. Replacing height for site index and age for index age at BH age gives the following model:

$$H_t = 1.3 + \frac{SI_b+\left(\frac{27\ \beta_4}{Age_{b0}^{1+\beta_3}}\right)}{1+\left(\frac{\frac{27\ \beta_4}{SI_b^{\beta_5}}}{Age_b^{1+\beta_3}}\right)} + \varepsilon \qquad [32]$$

Previous site-index curve studies at Lakehead University for aspen (Deschamps, 1985), jack pine (Lenthall, 1985) and black spruce (Thrower, 1986b) have shown the Ek model [eq. 14] to be very flexible and a good fit was provided to the data. However, height-growth and site-index curve studies in British Columbia (Goudie, 1984) and in Alberta (Dempster and Associates, 1983) suggests that the Monserud logistic model is very useful for developing site-index curves. These two models were investigated for use in constructing height-growth and site-index curves.

The parameters for all the models were estimated using data from the average total height - BH age curve for each plot. Parameters were estimated using a Statistical Analysis System (SAS) non-linear curve fitting procedure (PROC NLIN) on a Compaq 386/25 microcomputer. PROC NLIN fits non-linear regression models by least squares using one of these five iterative methods: (1) modified Gauss-Newton method, (2) Marquardt method, (3) gradient or steepest - descent method, (4) Newton method, or (5), multivariate secant or false position (DUD) (SAS Institute Inc., 1988).

The user must specify the following for each non-linear model to be used in the analysis:

(1) the names and starting values for the parameters being estimated;
(2) the model;
(3) partial derivatives of the model with respect to each parameter (except METHOD = DUD); and
(4) the second derivatives of the model with respect to each parameter (only for the Newton method).

Since it is very difficult, if not impossible to determine, the parameter partial derivatives for the Ek model and the Monserud logistic model, the METHOD = DUD was chosen. In this method, the partial derivatives are estimated from the history of iterations rather than being supplied analytically (SAS Institute Inc., 1988). Usually only one starting value is specified for each parameter; however, if several values are specified, then NLIN evaluates the model at each point on the grid. Initial estimates of the parameters for the Ek model were taken from previous site-index curve studies for aspen (Deschamps, 1985), and from previous site-index curve studies for aspen (Dempster and Associates, 1983) using the Monserud logistic model. The parameters were allowed to vary on a starting grid by as much as 50%. For instance, a set of possible starting parameter values for the EK model could be as follows:

PARAMETERS B1 = 2 to 3 by 0.25
B2 = 0.2 to 1.2 by 0.2
B3 = -0.01 to -0.03 by -0.005
B4 = 2.0 to 3.0 by 0.25
B5 = -0.1 to -0.5 by -0.1

Residual sum of squares were calculated for each 3750 (5x6x5x5x5) combinations of possible starting values. The single combination of starting parameter values that have the smallest residual sum of squares was used to start the iterations. The program iterates until a 10^{-8} change in the residual sum of squares or in the model parameters is reached.

ACCURACY AND VALIDATION

The final height-growth curves, site-index curves and site-index prediction equation were tested for goodness of fit using the modified Chi-squared test developed by Freese (1960) and Reynolds (1984). These goodness of fit tests determine whether the model accurately describes the data, given a user specified allowable error limit. The height-growth curves, site-index curves and site-index prediction equation were tested using the OMNR validation stem analysis data set. This data set had known heights, ages and site index, therefore, predicted site index was calculated using the height-growth curves, site-index curves and prediction equation for comparisons with site index observed for the validation data set. To predict a site index value from the height-growth curves and site-index curves, a search routine was applied to equation 17 and 21. In addition, prediction limits for the mean residual were computed.

RESULTS

EXPLORATORY DATA ANALYSES

The aspen plots covered a wide range of site quality throughout north central Ontario. Although the site quality of the plots was unknown at time of selection, attempts were made to sample a wide range of site index without introducing a bias between site index and age. Summary statistics of the average plot site index and average plot BH age variables are given in Table 7.

Table 7. Summary statistics of the average plot site index and average plot BH age variables.

Statistic	Data set			
	Modelling		Validation	
	SI_b (m)	Age_b (yrs)	SI_b (m)	Age_b (yrs)
Sample size	89	89	19	19
Mean	19.77	66.29	19.87	65.84
Standard deviation	1.83	15.88	2.47	9.98
Minimum	15.59	50	14.23	54
Maximum	27.03	112	23.81	93
Coefficient of variation	9.25	23.96	12.43	15.16

Figure 8 shows the linear regressions of the average plot site index and average plot BH age ($SI_b = B_0 + B_1 \, Age_b$) for the modelling data set and the validation data set. Less than one percent of the variation in the average plot site index is explained by the average plot BH age for the modelling data set as compared to twenty one percent for the validation data set (Table 8).

Table 8. Regression statistics relating the average plot site index to the average plot BH age.

Data set	B_0	B_1	$S_{y.x}$	R^2
Modelling	20.171	-0.006	1.837	0.0028
Validation	27.323	-0.113	2.260	0.2094

It is clear from Figure 8 and Table 8 that there is very little apparent bias between average plot site index and BH age.

Height-growth Patterns for Different Site-index Classes

Height-growth patterns were examined to determine if they changed with site-index class. As previously mentioned on page 49, the data were stratified into two metre site-index classes and then fitted with the Chapman - Richards [eq. 3] and the modified Weibull model [eq. 12]. The modified Weibull model described the average plot total height BH age growth pattern slightly better than the Chapman - Richards model for each site-index class except for the 24 and 26 m classes (Table 9).

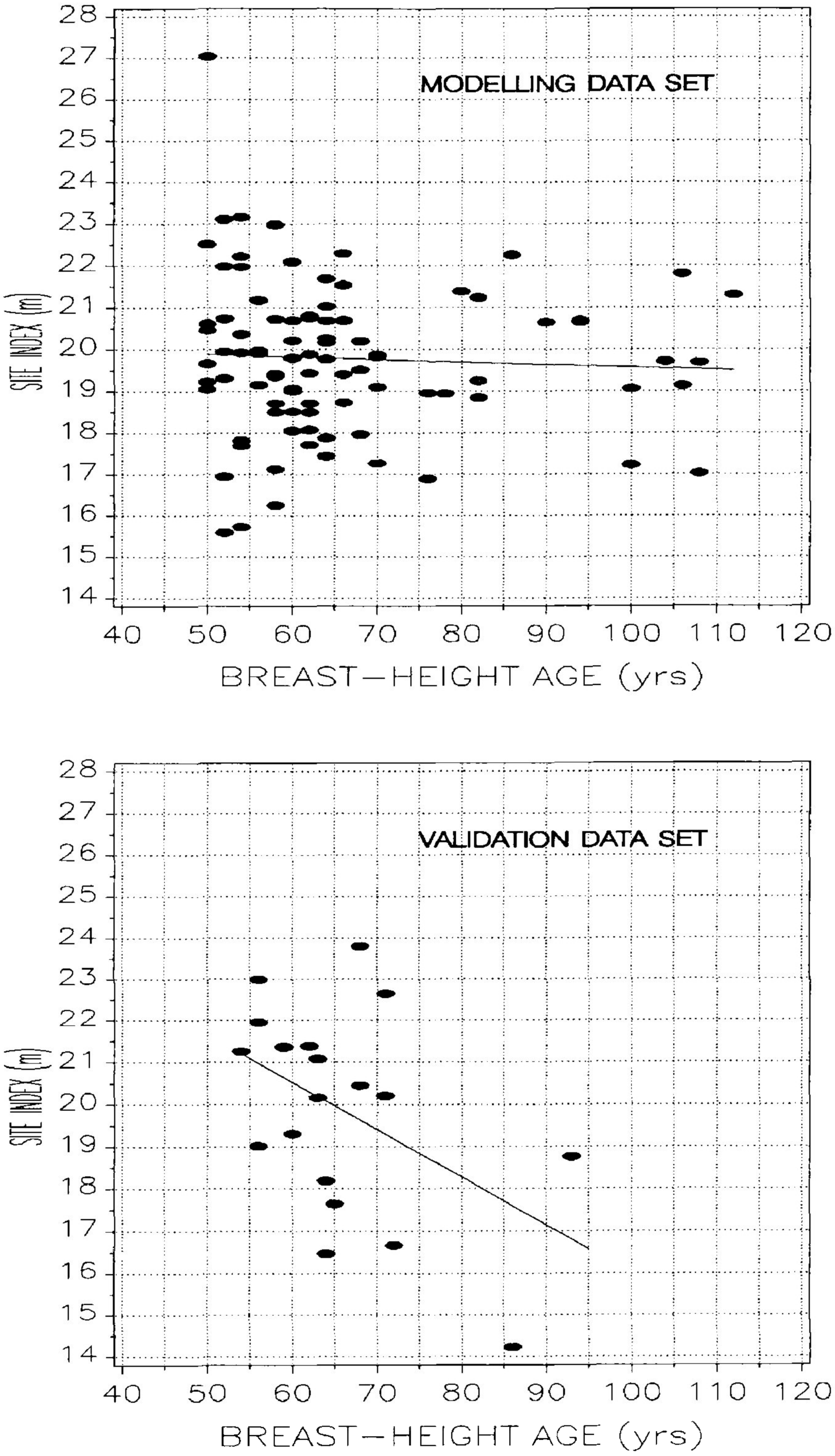


Figure 8. Regression of average plot site index and breast-height age for the modelling and validation data sets.

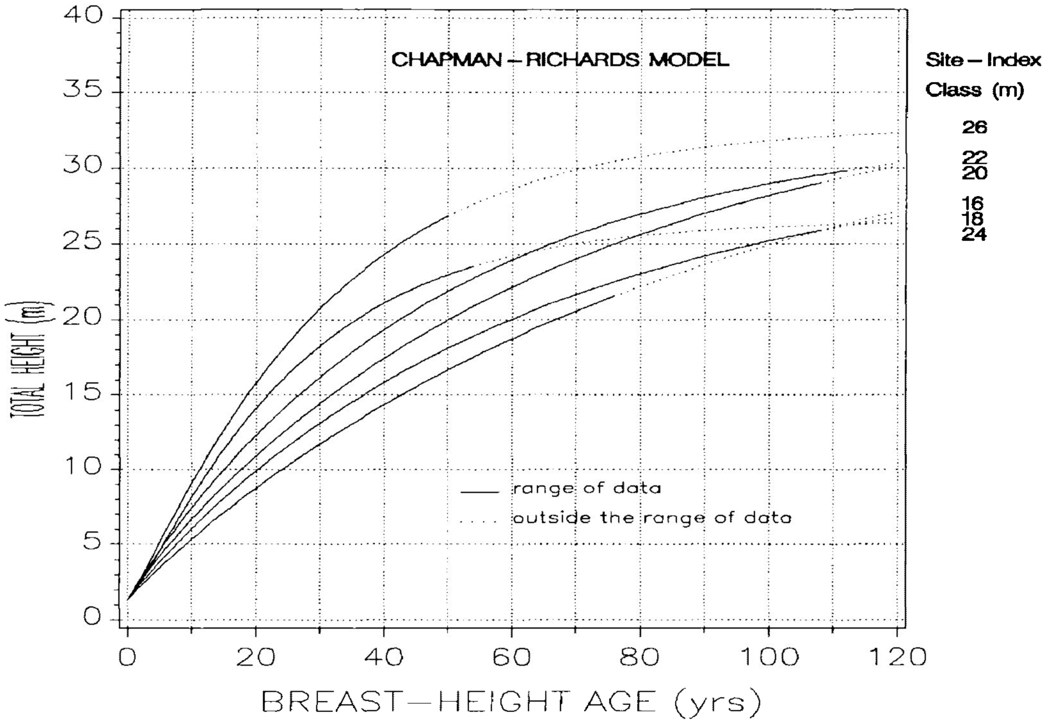
Figure 9 shows the height-growth patterns predicted by the Chapman - Richards model, [eq. 3] and by the modified Weibull model [eq. 12]. The coefficients of these non-linear regressions are given in Table 10. All coefficients are statistically different from zero at the 95% level of confidence.

Table 9. Residual statistics of the Chapman - Richards model and the modified Weibull model fit to the data by site-index class.

Site-Index Class(m)	Chapman-Richards model [eq. 3]			Modified Weibull model [eq. 12]		
	SSE	dfe	MSE	SSE	dfe	MSE
16	54.552	143	0.381	54.448	143	0.381
18	403.043	710	0.568	402.371	710	0.567
20	1051.088	1455	0.722	1049.108	1455	0.721
22	346.562	552	0.628	345.483	552	0.626
24	6.842	50	0.137	6.999	50	0.140
26	1.617	22	0.074	1.719	22	0.078

Table 10. Regression coefficients of the Chapman - Richards model and the modified Weibull model fit to the data by site-index class.

Site-Index Class(m)	Chapman-Richards model [eq. 3]			Modified Weibull model [eq. 12]		
	β_1	β_2	β_3	β_1	β_2	β_3
16	34.622	-0.01110	0.951	35.889	-0.01337	0.954
18	29.496	-0.01638	0.964	29.873	-0.01872	0.968
20	34.393	-0.01461	0.928	35.166	-0.01911	0.941
22	31.542	-0.02079	0.978	31.868	-0.02278	0.976
24	25.250	-0.04320	1.242	24.661	-0.02268	1.159
26	31.466	-0.03633	1.172	30.768	-0.02232	1.118



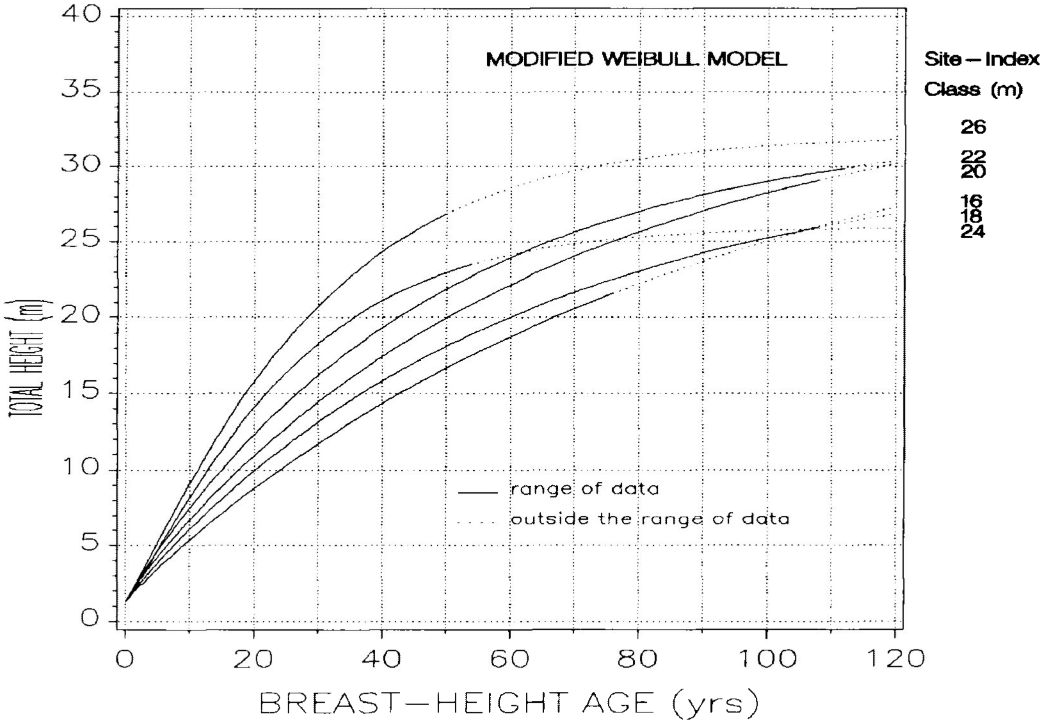


Figure 9. Average height-growth curves for aspen fit with the Chapman - Richards model [eq.3] and the modified Weibull model [eq. 12] to data stratified by two metre site-index classes.

HEIGHT-GROWTH AND SITE-INDEX CURVES

The Chapman - Richards and modified Weibull model coefficients fit to the site-index class data (Table 10) were examined for consistent trends related to site index. No strong trends were observed by plotting each coefficient (β_1 - asymptotic height, β_2 - growth rate and β_3 - initial height-growth) by site-index class except for the β_2 coefficient in the modified Weibull model. Expressing β_2 as a linear function of site index in the modified Weibull model did not predict height growth of the modelling data set very well. The two remaining parameters of the modified Weibull were then expressed as linear functions of site index. This expansion of the modified Weibull function has the following model form:

$$H = 1.3+\beta_0+\beta_1 S \left(1-e^{\beta_2\ Age}\right)^{\beta_3+\beta_4 S}+\varepsilon \qquad [33]$$

Residual statistics of these models fit to the modelling data set are shown in Table 11.

Table 11. Residual statistics of the modified half saturation model [eq. 32] and the expanded modified Weibull model [eq. 33] fit to the modelling data set.

Model	SSE	dfe	MSE	SEE
Modified half saturation [eq. 32]	1376.914	2947	0.467	0.684
Expanded modified Weibull [eq. 33]	1426.115	2945	0.484	0.696

The plot average total height - BH age height-growth curves fit with the modified half saturation model [32] are shown in Figure 10. The residual statistics are presented in Table 11. The modified half saturation model showed a 4% reduction in the residual mean square as compared to the expanded modified Weibull model. The coefficients for both models are given in Table 12. All the coefficients were statistically different from zero at the 95% level of confidence.

The standardized residuals for this model showed no apparent heteroscedastic trends or systematic lack of fit (Figure 11).

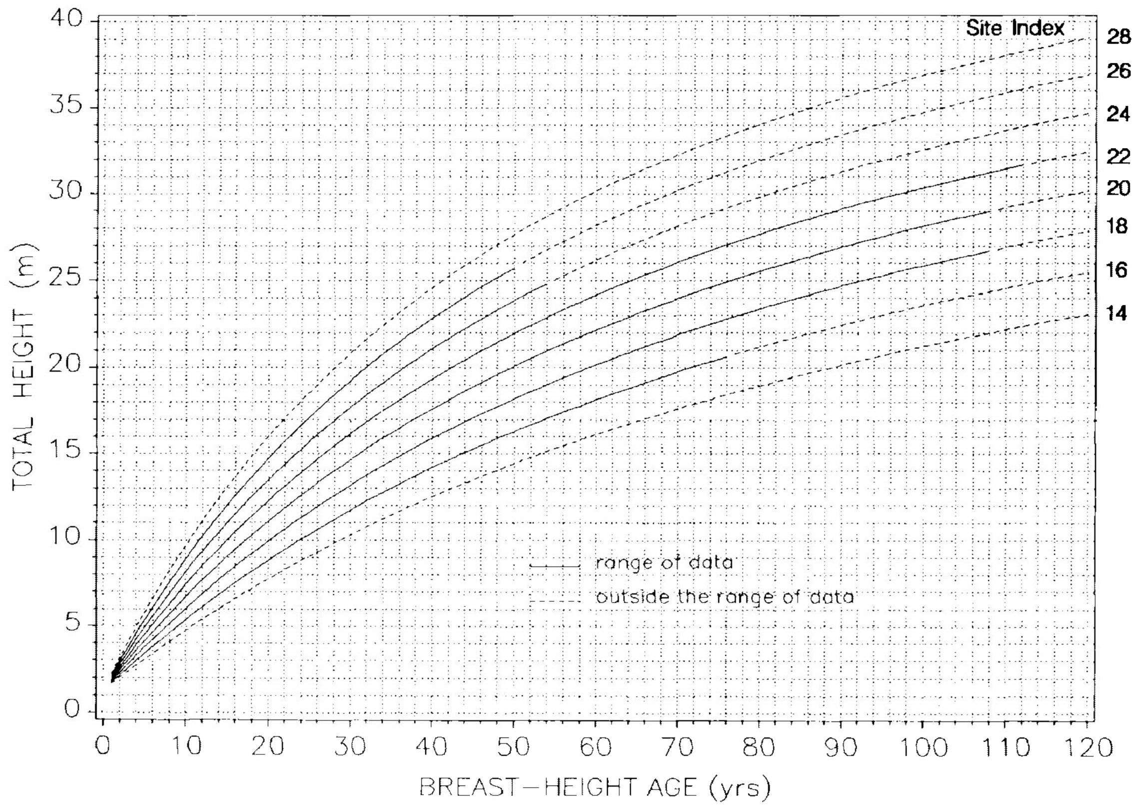


Figure 10. Height-growth curves developed for trembling aspen using a newly modified half-saturation model [eq. 32].

Table 12. Regression coefficients of the models [32] and [33] fit to the plot average total height - BH age growth data.

Model	Parameter	Estimate	Standard Error	t-Value	95% Confidence Limits	
					Lower	Upper
Modified half saturation [eq. 32]	β_3	0.0219	0.0076	2.9	0.0069	0.0369
	β_4	52.3881	0.5866	89.3	51.2378	53.5383
	β_5	0.9635	0.0005	1878.3	0.9625	0.9646
Expanded modified Weibull [eq. 33]	β_0	-0.8066	0.1034	- 7.8	-1.0092	-0.6039
	β_1	1.9082	0.0386	49.5	1.8326	1.9838
	β_2	-0.0013	0.0000	-37.0	-0.0013	-0.0012
	β_3	1.1134	0.0102	99.0	1.0933	1.133
	β_4	-0.0128	0.0002	-58.2	-0.0132	-0.0123

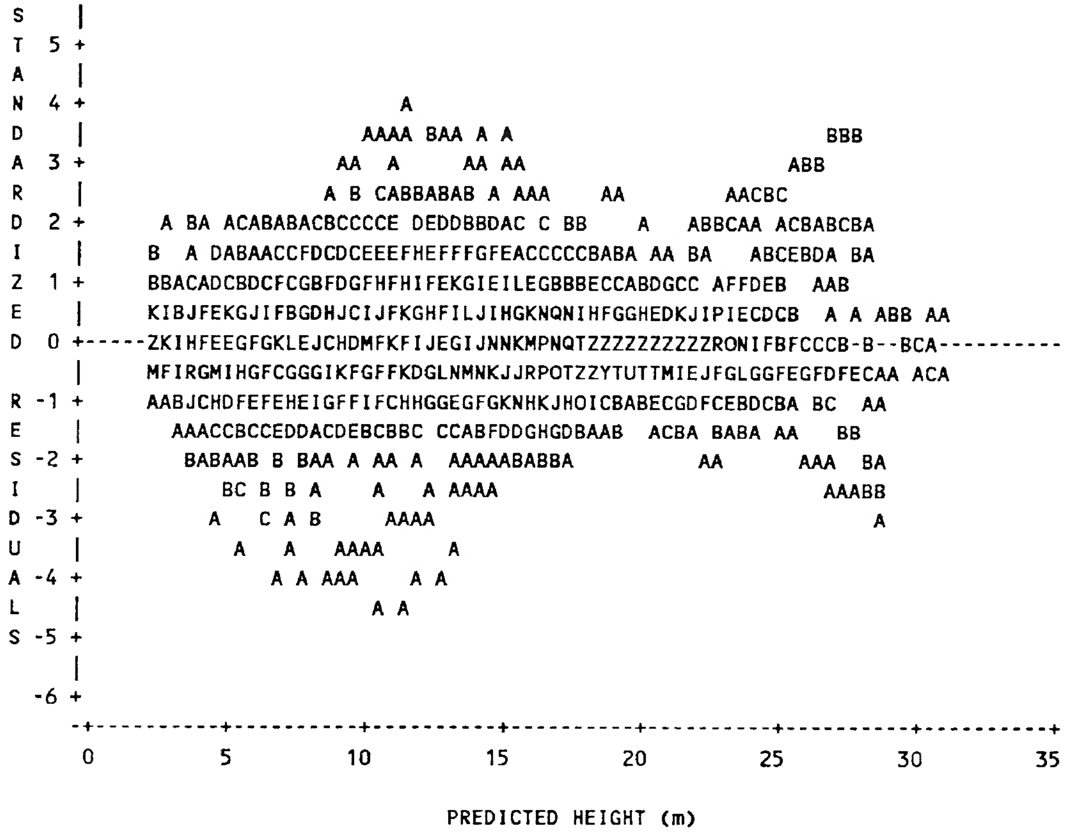


Figure 11. Standardized residuals of the newly modified half-saturation model.

Average total height - BH age growth patterns were fitted with the Monserud logistic model [eq. 17] and the Ek model [eq. 14]. The height-growth patterns are best described by the Monserud logistic model (Table 13).

Table 13. Residual statistics of the Monserud and Ek non-linear models fit to average total height - BH age growth data for each plot.

Model	SSE	dfe	MSE	SEE
Monserud logistic [eq. 17]	1350.036	2945	0.458	0.677
Ek [eq. 14]	1413.625	2945	0.480	0.693

The Monserud model showed a 5% reduction in residual mean square as compared to the Ek model. Figure 12 shows the total height - BH age height-growth curves fit with the Monserud logistic model [eq. 17]. The coefficients of both models are given in Table 14. All the coefficients were statistically different

Table 14. Regression coefficients of the models [17] and [14] fit to the plot average total height - BH age growth data.

Model	Parameter	Estimate	Standard Error	t-Value	95% Confidence Limits	
					Lower	Upper
Monserud Logistic [eq. 17]	β_1	41.9030	4.2179	9.9	33.6325	50.1735
	β_2	0.0357	0.0357	1.0	-0.0342	0.1057
	β_3	8.9580	0.2217	40.4	8.5234	9.3926
	β_4	-1.0168	0.0117	-87.0	-1.0398	-0.9939
	β_5	-1.5684	0.0667	-23.5	-1.6991	-1.4377
Ek [eq. 14]	β_1	3.7632	0.2678	14.1	3.2381	4.2882
	β_2	0.7288	0.0232	31.4	0.6832	0.7743
	β_3	-0.0155	0.0003	-48.5	-0.0162	-0.0149
	β_4	4.3301	0.4143	10.5	3.5176	5.1425
	β_5	-0.5127	0.0317	-16.2	-0.5749	-0.4505

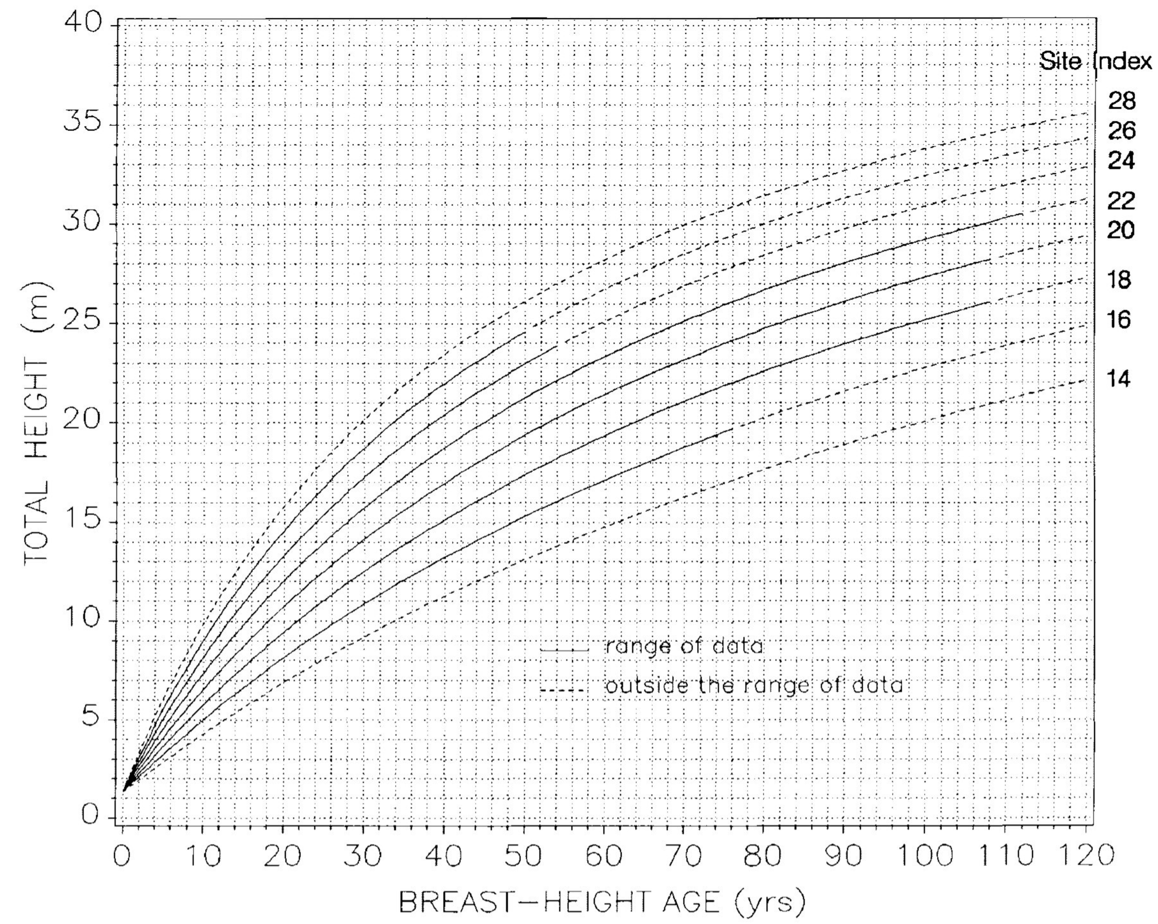


Figure 12. Height-growth curves developed for trembling aspen using the Monserud logistic model [eq. 17].

from zero at the 95% level of confidence except for the β_2 parameter of the Monserud logistic model. The standardized residuals of the Monserud logistic model [eq. 17] showed no major bias when fit to the modelling data set (Figure 13); this scattergram of standardized residuals showed no apparent heteroscedastic trends or systematic lack of fit. Several points lie outside the main group of standardized residuals and were a result of random error and did not indicate a lack of fit for the model.

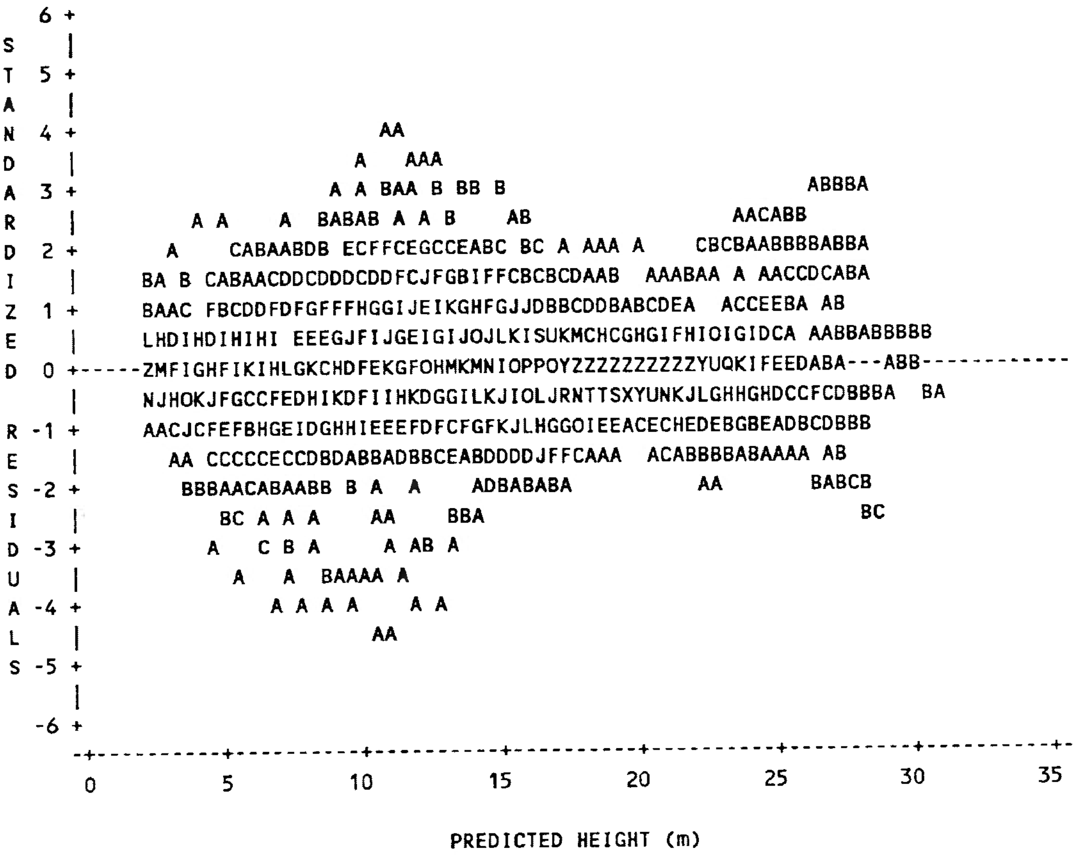


Figure 13. Standardized residuals of the Monserud logistic model [eq. 17].

The height-growth curves presented in Figure 12 showed the average expected pattern of height growth for each level of site index. The upper asymptote and early height growth before the inflection point of these height growth curves are dependent on the level of site index. These height-growth curves are not constrained to pass through the specified height at index age. However, within the range of the data, predicted height at age 50 was within measurement error of the height-growth curve, but differences became greater with the distance from the mean site-index.

By definition, site-index curves should pass through the specified height at index age (50 years BH age in our case). The two previous growth models [eqs. 17,14] were constrained to pass through the specified height at 50 years, thus creating site-index curves. The constrained Monserud logistic model best described the height-growth patterns for different levels of site index (Table 15).

Table 15. Residual statistics of the models [21], [20] and [19] fit to the plot average total height - BH age growth data constrained through site-index at 50 years.

Model	SSE	dfe	MSE	SEE
Constrained Monserud logistic [eq. 21]	1341.750	2947	0.450	0.675
Constrained Ek [eq. 20]	2402.886	2947	0.815	0.903
Newnham constrained Ek [eq. 19]	3420.728	2946	1.161	1.078

The constrained Monserud model showed a 79% reduction in the residual mean square as compared to the next best model [eq. 20]. Both constrained Ek model versions [eqs. 19,20] showed considerable increases in residual mean square compared to the unconstrained Ek model [eq. 14]. Figure 14 shows total height - BH age height-growth curves constrained to pass through the specified height at 50 years BH index age using the constrained Monserud logistic model. Constraining the Monserud logistic model caused very little change in the shape of the curves and the explained residual sum of squares decreased slightly compared to the unconstrained Monserud logistic model [eq. 17]. The coefficients for the three constrained models are presented in Table 16.

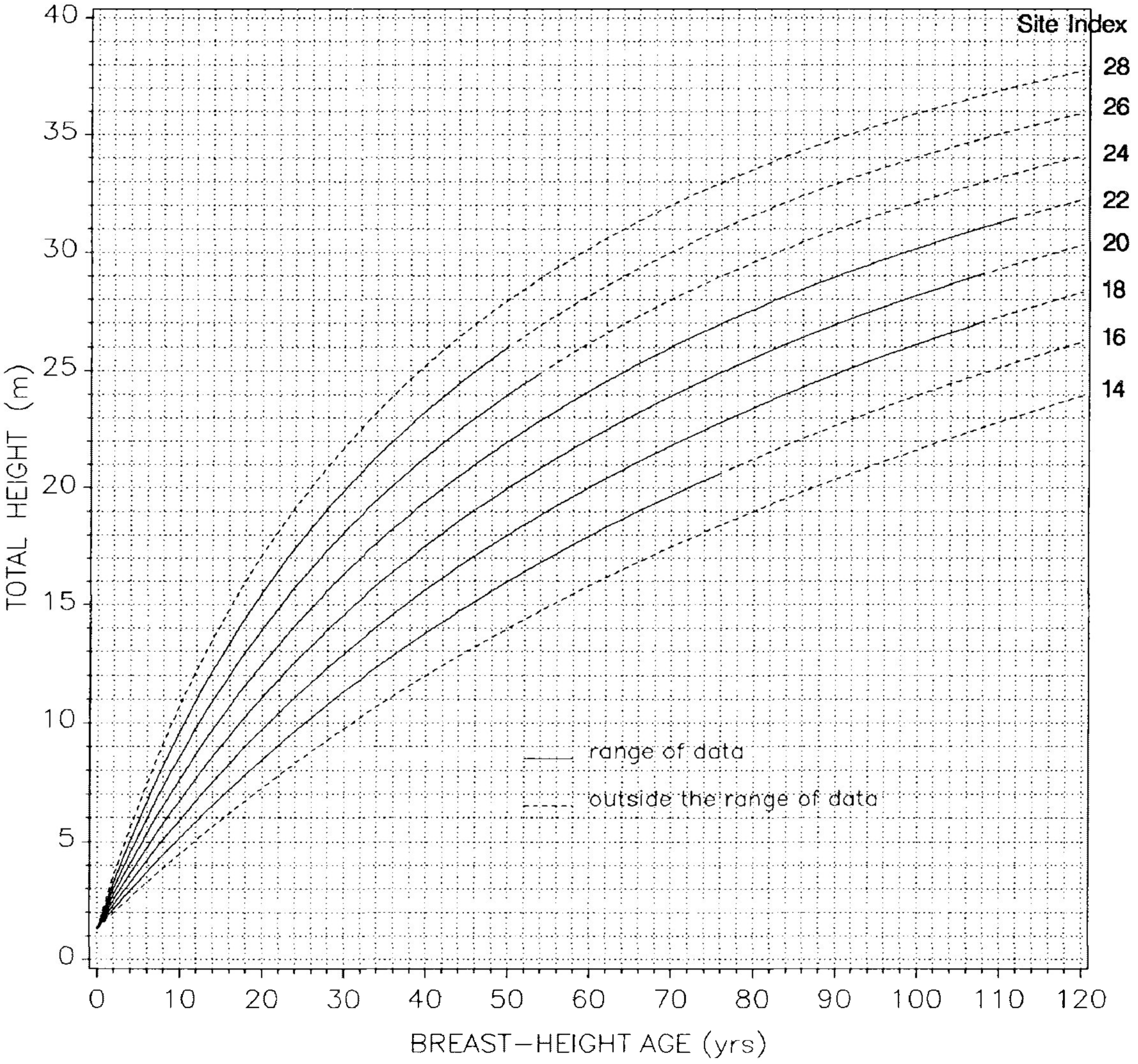


Figure 14. Site-index curves developed for trembling aspen using the constrained Monserud logistic model [eq. 21].

Table 16. Regression coefficients of the models [21], [20] and [19] fit to the plot average total height - BH age growth data constrained through site index heights at 50 years.

Model	Parameter	Estimate	Standard Error	T-Value	95% Confidence Limits	
					Lower	Upper
Constrained Monserud Logistic [eq.21]	β_3	9.3955	0.2577	36.5	8.8903	9.9007
	β_4	-0.9996	0.0065	153.0	-1.0124	-0.9868
	β_5	-1.7205	0.0878	-13.3	-1.8927	-1.5483
Constrained Ek [eq.20]	β_3	1.1086	0.0000	28245.7	1.1085	1.1086
	β_4	2.5607	0.2678	9.6	2.0356	3.0858
	β_5	-0.4531	0.0348	-13.0	-0.5213	-0.3849
Newnham constrained Ek [eq.19]	β_1	6.3803	1.4818	4.3	3.4748	9.2858
	β_2	0.5095	0.0776	6.6	0.3574	0.6617
	β_4	1.6101	0.5686	2.8	0.4953	2.7249
	β_5	-0.0985	0.1177	-0.8	-0.3292	0.1322

All the coefficients were statistically different from zero at the 95% level of confident except for β_5 in Newnham's constrained Ek model [eq.19].

The standardized residuals of the constrained Monserud logistic model showed no major bias when fit to the modelling data set (Figure 13). This scattergram of standardized residuals showed no apparent heteroscedastic trends or systematic lack of fit.

As mentioned previously, a site-index equation was needed to relate site index to height and age because it was impossible to solve the Monserud logistic model for site index. Using the stem analysis data, a series of logarithmic and quadratic transformations of height and age were investigated. A model using multiple linear regression techniques was formulated to predict site index as a function of height and age; this equation is given in Table 17. The standardized residual of this equation showed no apparent heteroscedastic trends or systematic lack of fit. For ease of use in the field, a site-index prediction table was constructed (Appendix VI).

```
   6 +
S    |
T  5 +
A    |
N  4 +                       A A
D    |                      A    AA
A  3 +                    A AAAABAA BABA                       ABABBA
R    |           A       BABBB A A C    AB               AABCBAA
D  2 +        AA  BABABBDBACDCFEDGDCECBCA C    A AA   ABBCAA BBBBABBA
I    |     BA B CABAACDDCCCFCDCFEFGGDEGFBBACDBCAA  A BB    BAACBDBABA
Z  1 +     A AB EBCDCEDDHGDGFFJGJHKJIHDIJNEDC DDBABBDBA  AA AAAB AB
E    |     GGBGGCGFIDK GDEHJIJFHFIEIKLMMKKPSPOHIGGGHIHHJLLIFDFC  ABABBBBBB
D  0 +-----ZNGIFEHJKJENFKCHDGFLGHNHNNIPMLMQSWZZZZZZZZZZZZSOOJGCEEDB----AB-------------
     |     PMHNMKDIEDFFDIIJEGGJFHGGIIJLJEQKKQLORQPTPPJGIHGLIEGDCFCEBBAA BBA
R -1 +     ABCLEFHFAHGEHEFGGICEFGBGBEEGKJJJGJIFDB DCDDFECCBCDCBDADBBBA
E    |       ABACCDDDCDDCBEBADBBBBAEBAEEFEHFDCAAA  AAAA BBBAAAA  AB
S -2 +        BBB ABABBBBC BAA AAA   BBBAAA            AA      AAA BB
I    |           BCAAAA AA  A A    BBA                           AABAC
D -3 +          A A B AAA     A AB A                                A
U    |            A A A  BA A  A
A -4 +               A A A B A  AA
L    |                       AA
S -5 +
     |
  -6 +
    -+---------+---------+---------+---------+---------+---------+---------+-
     0         5        10        15        20        25        30        35

                              PREDICTED HEIGHT (m)
```

Figure 15. Standardized residuals of the constrained Monserud logistic model [eq. 21].

Table 17. Equation and summary statistics relating site index to total height and breast-height age.

Equation	SSE	dfe	SEE	R^2
SI_b = 12.3922 + 0.9335 (H_t-1.3) -2.3632 ln (H_t-1.3) + 4.9393 ln (Age_b) - 1.6009 ln(Age_b)^2 + 6.5636 (H_t-1.3)/Age_b 0.0002 (H_t-1.3)* Age_b [eq. 34]	2511.629	2943	0.924	0.72

ACCURACY AND VALIDATION

The final height-growth curves, site-index curves, and site-index prediction equation were tested for goodness of fit using a modified Chi-square test (Freese, 1960; Reynolds, 1984). The calculated χ^2 [eq. 23] assuming a $\pm$ 1.5 m error and the tabulated χ^2 are presented in Table 18. According to the Freese procedure the calculated χ^2 for the height-growth curves and the site-index curves does not exceed the tabulated $\chi^2_{(\alpha 1)}$ value. Therefore the height-growth model and site-index model adequately describe the data within the $\pm$ 1.5 m allowable error limit at a 95 % level of confidence. Since the calculated χ^2 is greater than the tabulated $\chi^2_{(\alpha 1)}$ for the site-index prediction equation, the null hypothesis that the model adequately describes the data within an allowable error of 1.5 m is rejected. Using the Reynolds method the calculated χ^2 does not exceed the tabulated $\chi^2_{(\alpha 2)}$ value for the height-growth curves and site-index curves. Since the calculated χ^2 is lower than the tabulated $\chi^2_{(\alpha 2)}$, the null hypothesis

Table 18. Goodness of fit tests for the height-growth curves [eq.17], site-index curves [eq. 21], and site index-prediction equation [eq. 34].

Model	dfe	Calculated χ^2 [eq. 23]	Tabulated χ^2 $\alpha_1=0.05$	$\alpha_2=0.95$
Height-growth curves - Monserud logistic [eq.17]	2945	2305.0	3072.4	2819.9
Site-index curves - Constrained Monserud logistic [eq. 21]	2947	2290.9	3074.4	2821.9
Site-index prediction equation [eq. 34]	2943	4288.3	3070.3	2818.0

that the model is adequate is not rejected, indicating that the model adequately describes height growth and site index of aspen. The site-index prediction equation does not adequately describe the data within ± 1.5 m using the Reynolds method.

Critical errors (Freese, 1960; Reynolds, 1984) and future error prediction intervals for the height-growth curves, site-index curves and site-index prediction table are shown in Table 19. The critical error for the model is the minimum allowable error where the model is statistically valid.

The height-growth curves, site-index curves and site-index prediction equation were applied to the OMNR validation data set to obtain a prediction interval for future site index errors (Table 19). The prediction interval is the expected range of error for future site-index prediction. Comparing the prediction intervals for the different models shows that the constrained Monserud logistic model [eq. 21] and the site-index prediction equation [eq. 34] can predict site index better than the unconstrained Monserud logistic model [eq.17].

Table 19. Critical errors and prediction intervals for the height-growth curves [eq.17], site-index curves [eq. 21], and site-index prediction equation [eq. 34].

Model	Critical e* [eq. 25]	Critical e** [eq. 26]	95% prediction interval for D [eq.27]
Height-growth curves - Monserud logistic [eq.17]	1.30	1.36	-0.63 ± 1.58
Site-index curves - Constrained Monserud logistic [eq. 21]	1.29	1.35	0.19 ± 1.37
Site-index prediction equation [eq. 34]	1.77	1.85	0.21 ± 1.35

DISCUSSION

EXPLORATORY DATA ANALYSES

Exploratory analyses were conducted on the modelling and validation data sets before the development of aspen height-growth and site-index curves began. The bulk of the data lies between site index 17 to 22 m and between ages 50 - 70 years (Figure 8). Scattered data can be found with site-index values greater than 23 m and less than 17 m, and with ages greater than 70 years. Very few stands of fully stocked, uninjured aspen below 15 m site index were found throughout the study area. Aspen stands on poor sites tend to break up and decay before stands reach the breast-height index age of 50 years used in this study. In almost all cases, this early aspen break up and decay was found on poorer sites (site index < 15 m). Even on medium to good sites, aspen tends to be short-lived, and therefore stands usually start to break up before they reach 100 years old. Consequently, only a few, fully stocked, very old stands supporting uninjured aspen trees were found throughout the study area. The oldest stand was growing on a good site (site index=19.0) and was 112 years old BH age. Future studies that develop mathematical models for expressing height growth and site index may fill these data gaps for poor sites and for older ages. A regression of average plot site index to BH age showed very little sampling bias between site index and BH age (Table 8).

The average height-growth patterns vary in shape and size with differing levels of site index (Figure 9). This is evidence that polymorphic height-growth patterns occur for trembling aspen over the range of poor to good site quality. Each curve represents the average predicted height growth using the Chapman - Richard's model and the modified Weibull model for data stratified by two metre site-index classes. Computations showed that the modified Weibull model estimated height growth slightly better than the Chapman - Richard's model. No anomalies were evident within the range of observed data; however, extrapolating the 24 m height-growth curve beyond the range of data causes this site-class curve to intersect all other curves except the 26 m site-class curve. Therefore, extrapolation of height-growth curves beyond the range of observed data should be avoided. Height-growth curves have a relative linear pattern for poor sites (SI_b = 16) and a highly curvilinear pattern for good sites (SI_b = 24,26). The medium sites (SI_b = 18,20,22) have an early rapid linear surge of height growth before 40 years followed by a slowing curvilinear pattern after 40 years.

HEIGHT-GROWTH AND SITE-INDEX CURVES

Polymorphic height-growth patterns were tested using an expanded modified Weibull function, and by developing a new model using an approach similar to the Cieszewski and Bella (1989) model. A modified Weibull function was expanded to allow the parameters β_1 (upper asymptote), β_2 (growth rate after the inflection point) and β_3 (growth rate before the inflection point) to be linear functions of site index. This expanded model showed overall lack of fit and severe bias in older ages.

A new height-growth and site-index model, the half-saturation model, was the best fit to a small subset of fast growing aspen trees. The β_1 parameter represents the upper asymptote and the β_2 parameter represents the age at which height is halfway to maximum height. The β_1 and β_2 parameters can be interpretated biologically, so these parameters were expanded to express possible growth reduction with age and for testing the polymorphic hypotheses.

The β_4 in the new model represents the age at which an aspen tree on site $SI_b=27$ will reach half its maximum potential height growth. This age will proportionally vary from site to site as expressed by $27/SI_b^{0.964}$. For instance, on a poor site ($SI_b=16$), the trees would have to be 1.9 ($27/16^{0.964}$) times older to reach half their potential. To estimate the upper asymptote for different site-index values, equation $\beta_1 = SI_b + 27\ \beta_4\ /\ Age_{b\,0}^{1+\beta 3}$ would be used. Given base index age $Age_{b\,0} = 50$, $\beta_3=0.0219$, $\beta_4=52.388$, the maximum potential aspen height will be $SI_b + 26.00$. Therefore a good site ($SI_b = 24.0$) would have a maximum asymtotic height of 50 m whereas, a poor site ($SI_b = 14$) would have a maximum asymtotic height of 40 m. Since the $\beta3$ parameter was not equal to 0, polymorphic height growth exists in aspen in north central Ontario. Further, if the β_3 parameter was equal to 1, polymorphic height growth would be inversely proportional to level of site index. The 95% confidence level of β_3 does not cross 1 (Table 12), therefore height growth is not being inversely proportional to level of site index. Nonetheless, polymorphic height growth exists but in a different way than investigated.

This study shows that the Monserud logistic model is superior to the other models that were investigated, or that were expanded or developed for computing height-growth curves for aspen. Furthermore, constraining this model to pass through the appropriate site-index height at index age, showed its superiority over two other constrained functions for computing site-index curves. This constrained Monserud logistic function contains four of the six desired characteristics of a site-index model. It is constrained to go through zero at age zero and site-index height at the index age. It represents polymorphic growth and has increasing maximum heights with site quality. However, the model is not base age invariant nor solvable for site index.

None of the models investigated allow examination of variation in height-growth patterns for the same level of site index. For instance, trees may have differing patterns of height growth but may have identical site-index values. Therefore, predicted height growth from the model is only an estimate of average height growth for each level of site index. Deviations from the average height-growth curve might be a result of clonal differences, climatic change, and moisture and nutrient availability.

Genotypes of most species that exhibit slow height growth might be eliminated from dense stands where they are mixed with other trees that are rapidly growing genotypes. However, this competition may not occur for species that develop from clones. Barnes (1969, 1975) states that aspen regenerates mostly from clones on areas that may be as large as one hectare in size. Therefore, slow growing aspen clones will have a better chance of survival because severe competition only occurs on the boundaries where slow growing clones are bordered by faster growing clones (Carmean, 1975). Thus, variations in height growth and site index due to genetic differences would be more likely to occur with clonal species such as aspen and birch.

Aspen clonal differences in height growth have been shown to be important (Jones and Trujillo, 1975; Heidt, 1983) in estimating site index. Zahner and Crawford (1965) suggest that several clones of largetooth aspen (*Populus grandidentata* Michx.) should be measured to partly reduce these clonal errors in estimating site index. Future studies that compare variations in height-growth patterns for trees having the same site index may give insight to the causative factors of differing height growth for aspen.

Intensive forest management for aspen might include identifying superior aspen clones and superior aspen sites. Sites having superior aspen clones might be managed using treatments that eliminate poorer clones, thereby promoting the superior aspen clones. For superior aspen sites, superior clones might be established that have the potential for better form, and better quality, and that also are disease resistant. Such goals would require site quality, genetics and pathological research leading to the production of superior aspen planting stock for the very best aspen sites (Jones and Trujillo, 1975).

USING THE SITE-INDEX CURVES AND SITE-INDEX PREDICTION EQUATION

Before these curves are applied operationally, they should be validated with independent data for north central Ontario. If the site-index curves and site-index prediction equation are within allowable error limits defined by the user, then the curves and equation may be applied in north central Ontario. To apply these height-growth curves, follow these steps:

(1) Locate 0.08 ha plots within a fully-stocked, evenaged undisturbed aspen stand.

(2) Select 3 to 4 dominant or dominant and codominant aspen trees that are free from defect or other injuries.

(3) Measure total height and breast-height age from the selected sample trees.

(4) Calculate average total height and average breast-height age for the plot by averaging the total heights and breast-height ages of the sample trees.

(5) Site index can be graphically estimated for each plot by using the site-index curves (Figure 14). To do this, relate the average total height and breast- height age values on the site-index curves and then interpolate between the curves.

(6) Site index also can be estimated for each plot using the prediction equation [eq. 34]. Simply use average total height and breast-height age with the equation for a prediction of site index. This formula can be

easily added to a programmable calculator or computer for increased speed and efficiency, and also for reducing small errors and biases that might occur when interpolating between curves.

EVALUATION OF TECHNIQUES

A major strength of this study is the availability of a large amount of stem analysis data for use in the evaluation and hypothesis testing of height-growth patterns for trembling aspen. Further, height-growth patterns revealed by stem analysis eliminates the assumption of proportional height growth assumed in anamorphic site-index curves. Stem analysis of individual trees also permitted the examination of height-growth patterns for signs of suppression and abnormal growth. Trees showing signs of suppression and/or abnormal height-growth patterns were discarded because such abnormal growth would not provide a realistic indication of the height-growth potential for the site. Stem analysis provided paired height and age values for the remaining trees for use in constructing height-growth curves, site-index curves, and site-index prediction equations.

The paired height and age data were fitted with several biological growth functions using non-linear regression techniques. The Monserud (1984a) logistic model was the most precise model for expressing height-growth patterns of aspen. Site-index curves were developed by constraining this model to pass through the specified site-index height at index age. Model parameters can be interpreted for certain biological meaning that may lead to a better understanding of aspen height growth. A site-index prediction equation also was developed permitting rapid and efficient site-index prediction and reducing the errors and biases that may occur when interpolating between site-index curves.

Another strength in this study is the use of an independent data source supplied by OMNR for accuracy and validation purposes. A modification of the Chi-squared distribution proposed by Freese (1960) and Reynolds (1984) was used to test goodness of fit and to calculate prediction errors of the height-growth curves, site-index curves and site-index prediction equation.

Weaknesses in this study include:

(1) Sample plots were not randomly allocated within the study area, but instead were selected to cover a wide range of site quality and soil conditions. Therefore, the height-growth curves, site-index curves and site-index prediction equation only apply to the sample plots. The curves should be validated with independent data if they are to be applied operationally in north central Ontario.

(2) Lack of additional information to supplement the height/age data from stem analysis. The height/age data were collected to meet the specific goals of this study. But additional information could have been collected to allow linking site index to growth and yield. Such data could have included estimates of stand density, analysis of several additional trees on a plot, and ring widths on the sections. Clonal identification also might have been useful for evaluating height-growth patterns.

A difficulty encountered in this study is the identification of aspen annual growth rings. Although many techniques were tried, the annual growth ring identification technique described on page 43 worked the best. Nonetheless, it was still difficult to distinguish annual growth rings for aspen.

RECOMMENDATIONS FOR FURTHER RESEARCH

The aspen site-index curves (Figure 14) and site-index prediction table (Appendix VI) can provide a framework for intensive forest management. One of the first steps for aspen intensive forest management is the development of methods capable of identifying productive forest lands (high site indices) where intensive silviculture can be applied. The following additional research is recommended.

(1) Further aspen stem analysis data collection to supplement the existing data base. This would include older aspen stands occurring on very good and very poor quality sites.

(2) Evaluate the causative reasons for different aspen height-growth patterns (including effects of clones, soil type, and topography).

(3) Evaluation of how site index measured in one rotation compares to site index measured in the following rotation.

(4) Site quality evaluation studies are needed in mature aspen stands. Such information would be very useful in the present FEC program for the North Central Region. This would include soil-site studies that identify soil and topography features closely related to aspen site quality. A soil-site study for trembling aspen is now almost completed for north central Ontario (Li, 1991).

CONCLUSIONS

Height-growth and site-index curves were developed for estimating site quality of trembling aspen in north central Ontario. These curves were developed from stem analysis data from dominant and codominant, uninjured aspen trees growing in 89 plots covering a wide range of soil and site quality. Analysis of data from these 89 plots, and from an additional 19 independent validation plots led to the following conclusions:

(1) Trembling aspen site index in north central Ontario can be estimated from fully-stocked even-aged stands using either site-index curves (Figure 14) or by using a site-index prediction equation [34] (Table 17).

(2) Trembling aspen in north central Ontario varies greatly in site index and in pattern of height growth. The average height-growth curves were relatively linear for poor sites ($SI_b=16$), and highly curvilinear for good sites ($SI_b=24,26$). Medium sites ($SI_b=18,20,22$) show almost linear height growth until about 40 years of age, but after 40 years a slowing curvilinear pattern occurs.

(3) The Monserud (1984a) logistic model is superior to the other models investigated for computing height-growth and site-index curves for aspen.

(4) Critical errors (e*, e**) were calculated for the height-growth curves, site-index curves, and site-index prediction equation using a modification of the Chi-square distribution proposed by Freese (1960) and Reynolds (1984). Site index can be accurately estimated within an allowable error of ± 1.5 m using the site-index curves (Figure 14).

(5) An independent aspen stem analysis data set supplied by the OMNR were used for validating the height-growth curves, site-index curves and site-index prediction equation. Comparisons with the independent data set shows close agreement. The 95% prediction intervals for future site index error is --1.18 to 1.56 m.

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APPENDICES

APPENDIX I

ASPEN SITE PLOT SUMMARY - LAKEHEAD UNIVERSITY DATA SET

PLOT	TREE 1			TREE 2			TREE 3			TREE 4		
	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)
1	22.6	70	19.7	S/Ab			21.9	64	19.0	---		
2	15.8	53	15.6	S/Ab			15.8	52	15.6	---		
3	17.1	53	16.9	16.9	54	16.4	17.6	52	17.5	S/Ab		
4	20.7	65	18.8	R			20.6	65	18.9	20.6	62	18.4
5	24.4	56	23.4	23.2	55	22.6	23.4	56	23.0	24.1	55	23.6
6	LT50			LT50						S/Ab		
7	19.7	58	18.3	19.8	62	81.1	21.6	66	19.1	R		
8	22.7	71	17.8	20.4	70	16.8	---			---		
9	21.9	69	18.6	20.8	70	17.4	21.2	72	17.8	S/Ab		
10	23.4	66	19.8	24.0	67	20.8	23.3	65	20.3	---		
11	26.9	67	22.9	26.7	71	23.3	27.9	82	20.6	---		
12	24.2	68	20.4	24.4	65	20.9	24.4	64	21.3	23.1	63	20.4
13	R			22.2	77	16.8	23.4	80	17.0	S/Ab		
14	24.4	77	19.2	23.7	77	18.8	24.0	80	18.7	---		
15	25.1	72	20.6	23.7	70	21.1	23.9	78	17.9	---		
16	29.3	94	20.6	29.4	95	20.8	S/Ab			---		
17	25.6	87	18.3	25.9	84	19.3	R			25.3	83	20.1
18	23.0	78	18.8	23.1	78	19.1	S/Ab			S/Ab		
19	23.1	74	18.8	22.1	69	20.0	23.0	70	19.8	S/Ab	72	21.8
20	23.3	63	20.6	22.6	60	21.6	23.5	68	19.1	23.5	65	19.6
21	23.1	61	21.3	23.8	64	21.4	22.3	63	20.2	22.6	68	19.9
22	20.6	66	18.3	20.7	64	18.4	21.6	63	18.9	S/Ab		
23	S/Ab			S/Ab			S/Ab			S/Ab		
24	20.1	56	19.0	S/Ab			20.1	53	19.6	---		
25	20.4	55	19.6	20.8	55	19.9	19.4	50	19.4	---		
26	17.0	57	15.6	16.5	55	15.8	---			---		
27	R			S/Ab			S/Ab			---		
28	22.8	55	22.3	22.9	51	22.8	22.7	53	22.5	---		
29	27.1	83	21.4	26.6	84	21.6	27.4	86	20.4	---		
30	25.4	88	19.2	24.1	83	19.6	23.0	85	17.7	---		
31	27.3	95	20.8	27.4	93	20.5	28.0	91	20.8	---		
32	19.5	51	19.3	LT50			19.6	53	19.1	---		
33	23.3	59	21.3	23.7	59	22.1	23.3	55	22.5	---		
34	21.6	56	19.9	21.3	55	20.4	21.9	56	20.8	---		
35	17.7	54	17.1	19.1	59	18.2	19.4	60	17.9	---		
36	27.3	50	27.3	27.4	51	27.3	26.6	51	26.6	---		
37	25.6	52	25.3	LT50			LT50			---		
38	24.4	58	22.6	24.8	58	22.9	25.0	59	23.4	---		
39	26.0	68	24.8	25.7	67	22.1	25.5	61	22.4	---		
40	23.4	66	19.8	24.1	65	20.9	23.3	67	19.9	S/Ab		
41	S/Ab			24.5	66	21.3	23.5	66	21.4	23.9	73	19.4
42	21.2	64	17.9	21.4	64	18.6	20.6	67	17.1	---		
43	21.5	54	20.8	20.9	54	20.3	21.1	58	18.8	---		
44	24.0	60	21.9	24.5	55	22.9	23.4	59	21.7	---		
45	22.4	53	21.9	21.9	60	19.0	21.8	53	21.3	---		

PLOT	TREE 1			TREE 2			TREE 3			TREE 4		
	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)
46	20.2	56	19.3	19.2	50	19.2	20.1	58	18.6	---		
47	18.0	54	17.5	18.3	55	17.7	19.0	55	18.3	---		
48	21.3	58	20.1	22.2	61	20.4	20.7	57	19.3	---		
49	22.4	65	19.1	22.7	62	20.1	21.8	56	20.7	---		
50	29.0	114	19.1	28.9	100	19.4	28.5	105	18.8	---		
51	26.4	89	16.9	27.3	108	17.2	R			---		
52	27.4	102	23.1	27.3	87	22.3	26.5	90	21.3	---		
53	25.1	109	16.6	26.5	101	17.9	---			---		
54	27.1	95	18.1	27.4	112	21.4	S/Ab			---		
55	30.3	95	22.3	29.3	104	19.8	30.5	98	20.0	---		
56	17.6	59	16.0	16.9	59	16.1	18.1	60	16.7	---		
57	20.8	60	19.1	20.2	59	18.5	209	62	18.6	---		
58	22.5	65	19.6	22.9	68	19.4	21.6	63	19.4	---		
59	S/Ab			23.1	70	18.9	23.5	71	19.3	---		
60	21.6	59	19.9	22.6	66	19.3	21.6	65	18.9	---		
61	24.8	68	21.2	24.6	67	21.9	---			---		
62	22.8	66	19.8	23.2	64	20.9	23.7	64	21.4	---		
63	18.5	61	16.7	20.1	64	17.6	19.6	66	17.3	---		
64	LT50			LT50			LT50			---		
65	23.9	63	21.9	22.8	69	19.4	23.4	66	21.2	---		
66	20.6	53	20.1	S/Ab			20.6	55	21.8	---		
67	32.0	106	21.2	29.8	110	21.4	31.9	110	22.8	---		
68	30.8	113	20.8	R			30.2	114	21.8	---		
69	31.1	113	19.1	30.5	109	18.6	29.7	107	19.6	---		
70	27.9	106	19.8	27.5	105	19.6	28.8	107	19.8	---		
71	22.0	56	21.4	23.1	53	22.9	23.0	57	21.6	---		
72	LT50			LT50			LT50			---		
73	22.5	67	18.9	21.1	67	18.8	22.8	69	18.4	---		
74	21.9	68	19.8	21.1	67	19.1	21.7	69	19.4	---		
75	23.7	64	21.3	23.4	66	20.8	23.7	67	21.1	---		
76	20.8	51	20.7	LT50			20.2	50	20.2	---		
77	23.3	52	23.1	LT50			23.3	52	23.1	---		
78	LT50			LT50			LT50			---		
79	26.0	81	20.9	26.8	81	21.9	26.4	81	21.3	---		
80	24.7	65	22.7	24.5	68	21.3	23.7	66	21.1	---		
81	22.3	70	18.8	23.9	74	20.4	24.0	72	20.3	---		
82	24.7	69	20.4	24.8	68	20.4	24.0	68	20.6	23.8	71	19.4
83	22.9	62	20.4	21.4	61	18.8	R			---		
84	20.3	73	17.1	20.4	63	18.4	20.0	64	17.7	S/Ab		
85	22.9	58	20.9	23.0	60	20.9	22.7	57	21.2	---		
86	S/Ab			S/Ab			S/Ab			S/Ab		
87	24.8	70	21.3	24.1	69	19.8	R			S/Ab		
88	20.6	64	18.8	19.5	61	18.1	22.3	68	20.3	---		
89	18.0	59	16.6	R			18.2	58	17.7	---		
90	20.2	60	19.4	19.5	63	17.6	---			---		
91	21.3	69	16.8	S/Ab			21.5	63	19.4	---		
92	21.6	62	20.2	S/Ab			21.5	59	19.6	---		
93	20.0	61	17.9	S/Ab			20.9	65	18.3	---		
94	21.1	60	19.1	21.0	60	18.9	OTH			---		
95	S/Ab			S/Ab			OTH			---		
96	20.8	51	20.7	20.7	53	20.5	OTH			---		
97	R			R			19.9	59	19.2	19.9	56	19.1
98	S/Ab			R			S/Ab			---		
99	S/Ab			R			OTH			S/Ab		
100	22.3	68	18.9	22.8	70	20.6	22.6	63	20.1	---		
101	S/Ab			S/Ab			S/Ab			LT50		

where S/Ab = suppressed or abnormal growth — — — = no tree was sampled OTH = other
LT50 = less than 50 years breast height age R = rot

APPENDIX II

ASPEN SITE PLOT SUMMARY - OMNR DATA SET

PLOT	TREE 1			TREE 2			TREE 3		
	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)	TOTAL HEIGHT (m)	BH AGE (yrs)	SITE INDEX (m)
1	S/Ab			24.4	59	23.0	23.0	56	20.9
2	20.9	68	18.1	19.5	68	17.2	19.6	65	17.6
3	S/Ab			18.5	64	16.1	17.9	69	16.9
4	S/Ab			29.3	68	23.8	27.3	69	23.9
5	24.0	57	23.4	23.2	56	22.2	24.6	58	23.4
6	LT50			LT50			LT50		
7	23.5	66	18.1	20.0	64	17.1	23.5	64	19.4
8	20.8	59	18.1	22.6	56	19.6	22.3	60	19.3
9	LT50			LT50			LT50		
10	LT50			LT50			– – –		
11	LT50			LT50			LT50		
12	S/Ab			S/Ab			S/Ab		
13	23.5	68	19.8	24.4	70	19.4	24.9	69	22.1
14	20.8	87	19.7	24.7	91	19.9	25.0	90	20.1
15	23.7	72	19.2	23.4	71	19.8	– – –		
16	24.4	60	20.6	22.4	59	18.3	– – –		
17	24.8	82	22.1	S/Ab			– – –		
18	23.1	63	19.2	23.5	65	21.7	– – –		
19	26.9	71	24.3	27.3	73	21.9	25.4	72	21.8
20	23.0	64	20.1	24.3	64	22.0	23.3	54	21.7
21	LT50			20.8	52	20.5	LT50		
22	S/Ab			20.0	75	15.3	21.6	72	18.0
23	S/Ab			S/Ab			S/Ab		
24	22.1	63	20.6	22.1	65	20.0	24.3	66	18.0
25	22.0	64	19.4	20.0	60	19.2	– – –		
26	23.1	67	20.9	S/Ab			S/Ab		
27	25.0	90	18.1	S/Ab			– – –		
28	25.0	92	18.1	– – –			– – –		
29	26.3	83	21.7	– – –			– – –		
30	25.5	93	17.9	24.6	94	19.6	– – –		
31	18.4	56	17.2	S/Ab			S/Ab		
32	23.1	62	21.7	23.7	63	21.7	22.2	62	20.8
33	20.1	91	14.4	S/Ab			21.5	86	14.1

where S/Ab = suppressed or abnormal growth

– – – = no tree was sampled

LT50 = less than 50 years breast height age

APPENDIX III

ASPEN SITE PLOT TALLY SHEET

PLOT NUMBER:	STEM ANALYSIS	DATE:
LOCATION:	▨	SPECIES:
REMARKS:	▨	PARENT MATERIAL:
	▨	

▨	TREE 1			TREE 2			TREE 3			TREE 4		
HT (m)	RING COUNT	TOTAL AGE	DBH AGE	RING COUNT	TOTAL AGE	DBH AGE	RING COUNT	TOTAL AGE	DBH AGE	RING COUNT	TOTAL AGE	DBH AGE
STUMP			▨			▨			▨			▨
0.75			▨			▨			▨			▨
1.3												
2.0												
3.0												
4.0												
5.0												
6.0												
7.0												
8.0												
9.0												
10.0												
11.0												
12.0												
13.0												
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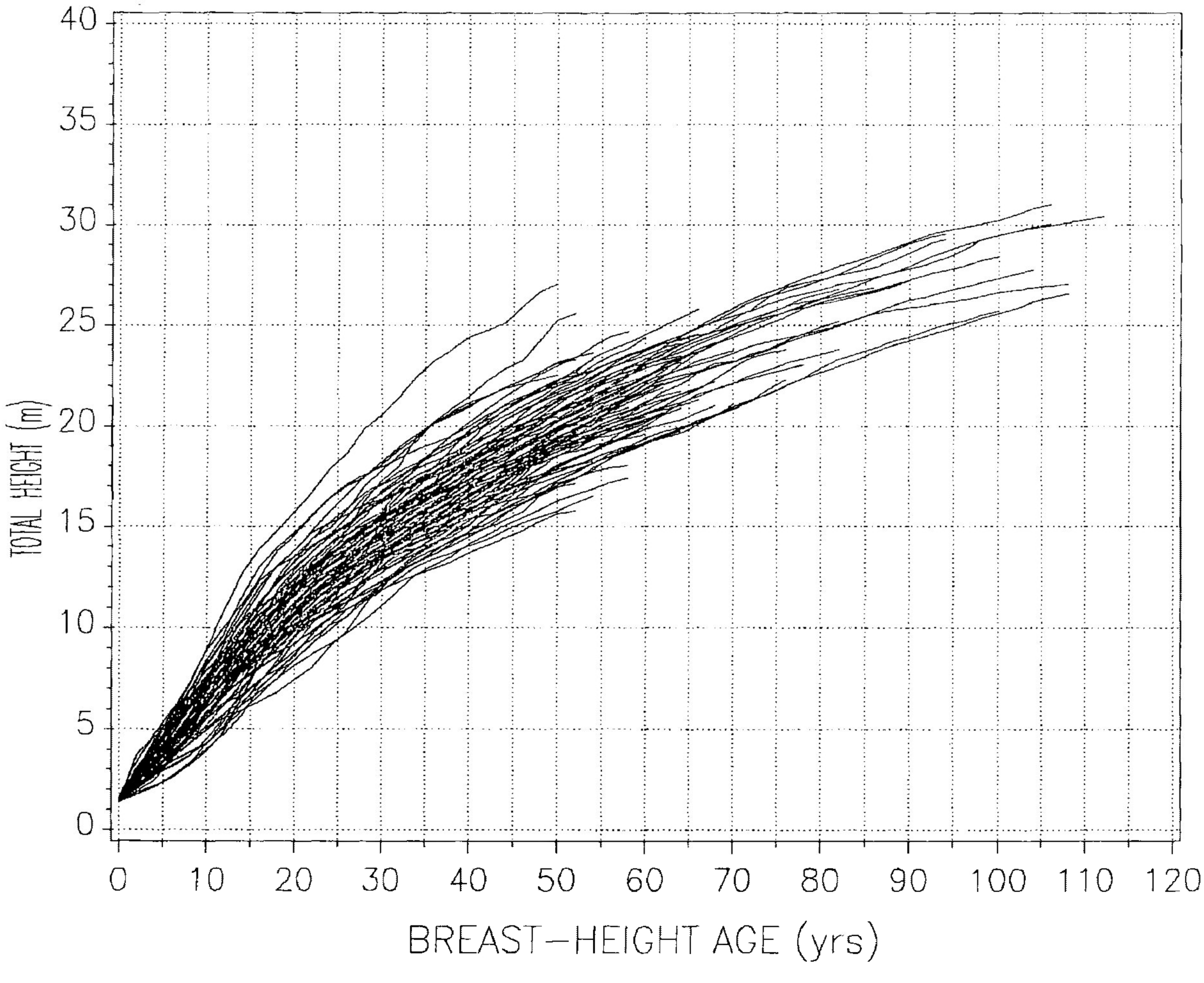
30.0												
30.5												
31.0												
31.5												
32.0												
TOT HT												

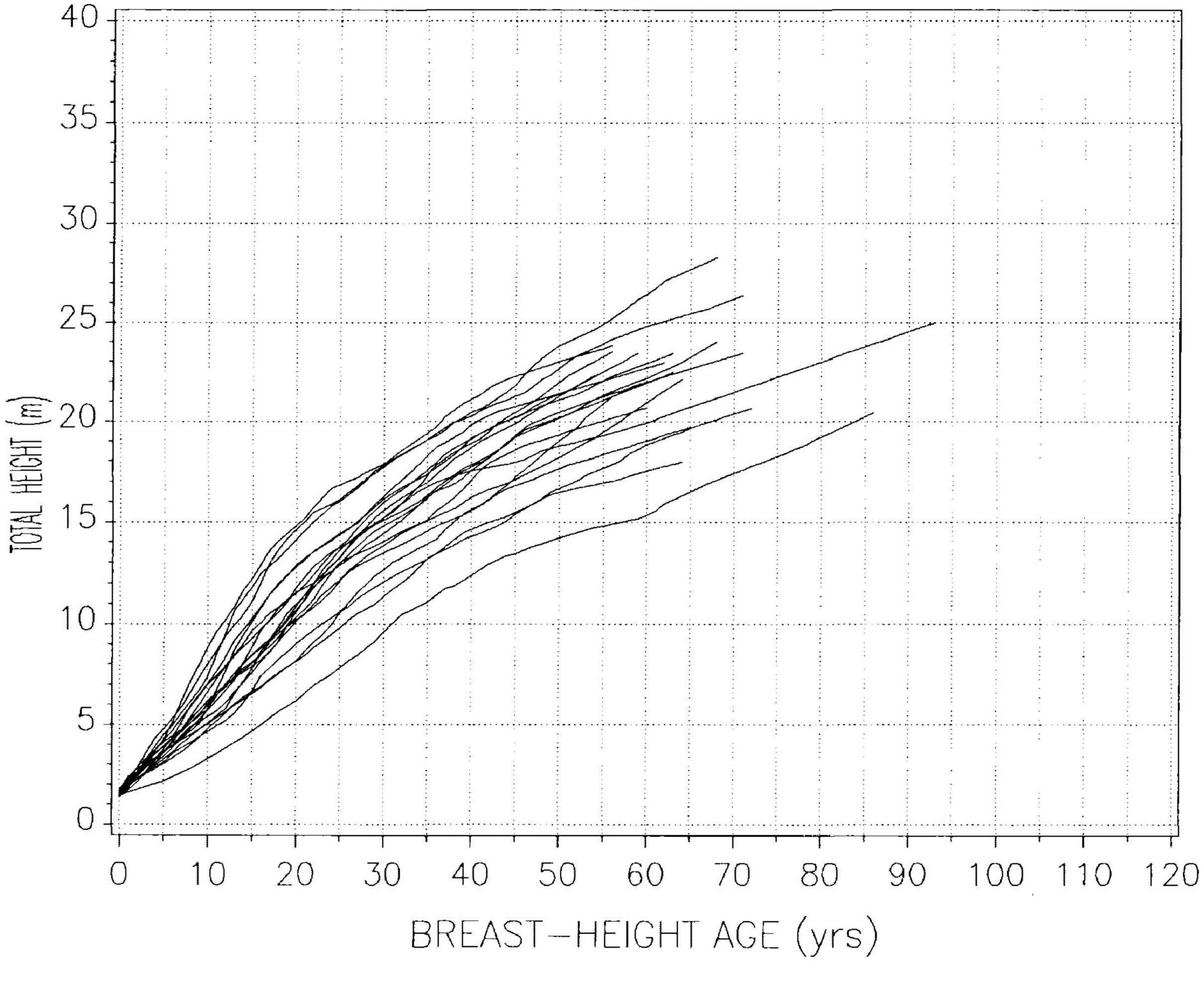
APPENDIX IV

INDIVIDUAL PLOT HEIGHT-GROWTH CURVES FOR TREMBLING ASPEN IN NORTH CENTRAL ONTARIO - LAKEHEAD UNIVERSITY DATA SET



APPENDIX V

INDIVIDUAL PLOT HEIGHT-GROWTH CURVES FOR TREMBLING ASPEN IN NORTH CENTRAL ONTARIO - OMNR DATA SET



APPENDIX VI

SITE-INDEX PREDICTION TABLE DEVELOPED FROM THE MUTIPLE LINEAR REGRESSION EQUATION [eq. 34]

BREAST HEIGHT AGE (yrs)		TOTAL HEIGHT (m)														
		4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
					Predicted site index				(BHSI 50 (m))							
10		17	19	21	23	27										
15		15	17	19	21	23	25									
20		14	15	18	19	21	23	25	27							
25			14	15	17	19	21	23	25	27						
30				14	16	17	19	21	23	25	27					
35					14	16	18	20	22	24	26					
40						15	17	18	20	22	24	26				
45						14	16	17	19	21	23	25	27			
50							14	16	18	20	22	24	26			
55							14	15	17	19	21	23	25	26		
60								15	16	18	20	22	24	25	27	
65								14	15	17	19	21	23	25	27	
70									15	16	18	20	22	24	26	27
75									14	16	17	19	21	23	25	27
80										15	17	19	20	22	24	26
85										14	16	18	20	21	23	25
90										14	15	17	19	21	23	24
95											15	17	18	20	22	24
100											14	16	18	19	21	23
105											14	15	17	19	21	22
110												15	17	18	20	22