STUDENTS' UNDERSTANDING OF FRACTIONS WITHIN A REFORM-BASED INSTRUCTIONAL PROGRAM: AN ACTION RESEARCH ANALYSIS.

by

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Abstract

Students have a difficult time understanding the concepts behind fraction procedures. The purpose of this study was to determine which teaching method, traditional or reform, helped students to develop a conceptual understanding of fractions as well as to examine the specific aspects of the reform teaching method that fostered this understanding. A pre-test and post-test were administered to two grade 6 classes. The tests were divided into two sections: Computations and Word Problems and were marked and coded to evaluate conceptual understanding. Similar results were found between both groups in computation skills. The results in the problem solving post tests were quite different between the two groups. The results revealed that the students in the reform teaching class developed more flexible ways to problem solve, were able to apply their own reasoning to solve computation questions, effectively used manipulatives to further their understanding and strengthened their ability to reason mathematically. The students in the traditional classroom, however, relied on faulty procedures to solve problems and the focus on algorithms hindered their problem solving ability. This study indicated that there were many benefits to the students that learned in the reform classroom and further research in this area would be beneficial.

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Chapter 1: Introduction

1.1 Context of the Study

Fractions are an area of mathematics that has generated much research, especially in the past fourteen years. Research has indicated that children have very little conceptual understanding of fractions (Mack, 1990). This is because fractions are traditionally taught as a set of procedures and rules to be followed with very little emphasis on the reasoning behind these algorithms (Aksu, 1997; Lamon, 2001; Moss & Case, 1999).

Despite extensive research evidence that students do not understand fractions, many teachers are still teaching fractions using this traditional approach. This is detrimental to a student's mathematical understanding; without an opportunity to learn about fractions conceptually, students have a hard time deciphering why an answer to a question may be unreasonable. All they have done is apply the algorithm with little attention to the reasonableness of their answer (Hasemann, 1981). Therefore, "when rules and procedures are not learned with meaning, students forget them or do not always realize when to use them" (Lamon, 2001, p.162). If the goal of teaching fractions was to have students simply regurgitate answers without sense-making, then teaching fractions in this manner would be sufficient, however, the emphasis in the Ontario Curriculum is on understanding (MET, 1997).

When students have been given problems to solve and have a chance to explain their own reasoning for their answers, before they are taught procedures and algorithms, they have a much stronger understanding of the concepts (Bulgar, 2003; Kamii & Warrington, 1999; Lappan & Bouck, 1998; Warrington & Kamii, 1998). When teaching is based on building on their informal knowledge, they have more of an opportunity to make sense of the information and learn to mathematically symbolize their thinking over time (Mack, 1990). When students learn through a reform method approach they have a broader conceptual base of fractions (Lamon, 2001, Moss & Case, 1990) and they have more flexibility and ease in adapting to various types of situations (Lamon, 2001; Warrington & Kamii, 1998).

Of the research that has been conducted in fractions, the message is very clear that students need time to develop their own reasoning for fraction methods and have the time to explore their solutions (Ball, 1993; Bulgar, 2003; Fosnot & Dolk, 2002; Niemi, 1996). It is the expectation of the Ontario Ministry of Education that these methods be used (MET, 1997).

1.2 Purpose of the Study

The purpose of this investigation was to find a teaching method that helps students develop a deeper understanding of fractional concepts as opposed to knowing only a series of procedures and methods to use when working with fractions. A comparison of two different teaching methods was employed. One class was taught fractions using a reform method approach (a problem solving approach that begins with students' own solution methods) while another class was taught fractions using a traditional direct-instruction of methods approach. The results from a pre-test and post-test were then compared to see whether there was a difference in students' understanding of fractions. Also investigated were which specific methods in the reform teaching classroom best helped students develop a conceptual understanding of fractions.

1.3 Research Questions

There were two specific questions examined in this project:

(1) What teaching method, traditional or reform, was most beneficial in helping students develop a conceptual understanding of fractions?

(2) What were the specific aspects of the reform teaching method that helped students develop a more accurate and meaningful understanding of fractions?

1.4 Significance of the Study

Investigating ways to improve student learning is a goal for educators. This research helped to indicate a method that may be more beneficial to helping children learn a deeper and more meaningful understanding of fractions. The comparisons that are made between the pre-test and post-test indicated the areas of conceptual understanding that students have difficulty with or have gained through a particular teaching method and some of the common errors students make or the misconceptions that students have regarding fractions. It also gave teachers specific insights into which methodologies proved effective in a reform teaching environment.

1.5 Limitations of the Study

There are some limitations of this study that need to be considered. First, the pre-test and post-test were designed slightly differently. Some of the questions may not warrant as valid a comparison had the questions been left exactly the same between both of the tests. The two sample populations that were being compared were also different in nature. The size of the classes was different along with the language that the mathematics was being taught in. A better comparison would have been made if the classes were made up of more similar characteristics.

1.6 Plan of the Thesis

The project is organized in the following manner:

- Chapter 2 consists of the background literature that helps foster an understanding of the goals for the thesis.
- Chapter 3 includes the methods that were used to execute the study and how each prospective class was taught.
- Chapter 4 thoroughly analyzes the results from the pre-test and post-test questions along with the effective methods that were used in the reform teaching class.
- Chapter 5 discusses the findings in relation to the research questions and summarizes the data that has been obtained.

Chapter 2: Literature Review

2.1 Introduction

In day to day life, fractions are used in very simple ways. They are used when cooking, baking, measuring and even when referring to amounts such as, "I'll just have half a piece of pie." Why then, in the mathematics classroom does the word "fraction" take on a whole different meaning? Students are taught how to find common denominators in order to add two fractions together, how to make equivalent fractions by multiplying the numerator and the denominator by the same number and are taught that shading in three out of four pieces will result in ¾. Suddenly, when dealing with fractions, students are taught a series of steps, rather than reasoning, in order to calculate answers. The context of the fraction number is lost and students are left with trying to remember simple formulas that are devoid of meaning. Should there be any question as to why so many students struggle with the understanding of fractions?

Traditionally, fractions have been taught as most mathematics concepts are taught: the teacher teaches a lesson, the students and the teacher do a few examples together and the students practice the concept that they have just learned (Baroody & Hume, 1991). Fractions are taught as a set of procedures and rules to be followed with very little emphasis on the mathematical reasoning behind the algorithms[1] (Aksu, 1997; Lamon, 2001; Moss & Case, 1999). An emphasis is placed on obtaining right or wrong answers and there is little chance for the students to create and use their own ideas to solve a problem.

[1] An algorithm is a set procedure that is followed to complete a calculation. This is the traditional way of teaching students. For example, when dividing fractions, invert the second number and multiply the fractions together.

2.2 Lack of Understanding

2.2.1 Procedural versus Conceptual Understanding

Students who learn fractional concepts through traditional direct instruction of rules often develop procedural errors based on superficial understandings. They memorize how to follow a procedure without understanding, which sometimes results in memorizing the procedure incorrectly and applying it inaccurately (Hasemann, 1981; O'Brien, 1999; Suydam, 1984). As a result, "many students' understanding of fractions is characterized by a knowledge of rote procedures, which are often incorrect, rather than by the concepts underlying the procedure" (Mack, 1990, p.17). Baroody and Hume (1991) give the example that "Joey remembers you have to invert and multiply something, but he is not sure what" (p. 54). Students as a result try to solve a problem based on what they think they remember from a procedure regardless of whether the answer makes sense or not.

When students are taught a series of procedures to solve fractions computations, the focus of their learning is on whether or not they have applied the algorithm correctly as opposed to understanding the reasonableness of their answer (Hasemann, 1981). A student may solve a problem using a faulty procedure but be unable to determine that his or her answer does not mathematically make sense. Students will often rely on an answer that they get by solving a problem with an algorithm first before trying to understand a problem conceptually (Khoury & Zazkis, 1994). Mack (1990) worked with eight Grade 8 students on adding and subtracting fractions. She found that these students "often trusted answers obtained by applying faulty procedures more than those obtained by drawing on informal knowledge" (p. 27). For example, one of the students was asked to solve the question, “If you had 3/8 of a pizza and I gave you 2/8 more of a pizza, how much would you have?” (p.27). The student first wrote down the numbers from the question and came up with an answer of 5/8. He thought that this did not look right so he changed his answer to 5/16. Even when he had an opportunity to use fraction circles, he was still confused by the answer. The student tried to follow a procedure to back up his answer as opposed to looking at why his original answer was reasonable.

Students learn procedures in isolation and therefore they have a very difficult time applying these procedures to word problems (Heller, Post, Behr & Lesh, 1990). Students are able to solve problems by using their own reasoning, but as soon as the same problem is represented in a symbolic matter, they resort to the traditional procedure as opposed to what makes the most sense (Mack, 1990). The focus on procedures therefore creates three problems: first, students learn not to use their own reasoning to work with fraction problems; second, they use the procedures incorrectly; third, they develop misconceptions about not only how to work with fractions but even the basic concept of fractions. This results in a number of students having misconceptions with fractions.

2.2.2 Representation of a Fraction as a Number

We have evidence that students have a difficult time with the representation of a fraction as number. Students often think of the numerator and denominator as two separate numbers instead of one entity (Behr & Post, 1992). For example, one student when asked which was smaller, 4/5 or 5/6, answered that 4/5 was smaller because it had a smaller denominator. Upon further reflection, the student said that both fractions were equal because 4/5 and 5/6 were both missing one piece (Mack, 1990).

Di Gennaro, Picciarelli and Rienzi (1990), in their study of 5th and 7th Grade students, found that 34 percent thought of fractions as being a representation of a part-to-part relationship. For example, a fraction of two thirds would be two stars shaded and three stars unshaded as opposed to two out of three stars being shaded. Students think of the symbolic representation of a fraction as being a ratio (Pa, 1991) instead of a part of a whole. When students do not understand what a fraction represents, they are unable to determine the size of a fraction. Leutzinger and Bertheau (1989) found that "seventeen out of twenty students in a fourth-grade class responded that ½ is the largest fraction less than 1" (p. 111).

Since the students do not have an accurate understanding of what a fraction is, they do not see the necessity of the fractional parts being of equivalent size (Suydam, 1984). A whole that is broken up into three different unequal pieces may still be considered by some students to be thirds.

They are also unable to flexibly think about fractional parts. Typically, fractions are shown in textbooks as having parts of a whole shaded side-by-side (Figure 1). When the parts of a whole are not shaded side by side (Figure 2), students can then become confused (D'Ambrosio & Campos, 1992). Students do not recognize that the parts of a fraction, while they need to be equivalent, do not need to look identical (Figure 3). If the students have a true understanding of fractions, they should be able to recognize a fraction in a number of different ways.

Figure 1. *Parts of a whole shaded side by side*

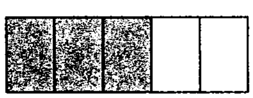
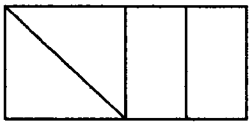


Figure 2. *Parts of a whole not shaded side by side*

Figure 3. *Equivalent quarters shaded which are not congruent*



2.2.3 Adults' Understanding of Fractions

These misunderstandings persist into adulthood. We have extensive evidence that prospective teachers have a hard time understanding fraction concepts because they themselves have been taught through traditional methods. Ball (1990) conducted a study that investigated future elementary and secondary teachers' understanding of mathematics in the area of dividing fractions. She found that "many children and adults perform mathematical calculations without understanding the underlying principles or meaning" (p. 458). Even the secondary teachers, who had recently or currently been taking math classes and were confident with their math knowledge, had a hard time explaining the rationale behind certain calculation procedures. For example, the prospective teachers were asked to solve the problem, 1 ¾ divided by ½. They then had to explain how they solved the answer and write a word problem for it but "very few secondary teacher candidates and no elementary candidates were able to generate a mathematically appropriate representation of the division" (Ball, 1990, p. 454). For teachers to be able to teach their students effectively, they need to have a conceptual understanding of fractions themselves.

In summary, we find that a sizable number of students have difficulty learning fraction concepts, including the meaning of fractions, how to solve mathematical problems with fractions and work with procedures related to fractions. We find these misunderstandings persist in some adults. Traditional direct instruction which focuses on procedures was implicated in creating this situation. Before exploring more promising instructional methods, we consider whether the instruction of fractions remains an important mathematical topic in today's classroom.

2.2.4 To Teach or not to Teach Fractions

Students and adults alike have had problems understanding fractions when taught traditionally. Groff (1996a; 1996b) feels that teaching fractions to students is not necessary. He feels that the only reason fractions are still being taught is because they have always been taught even though "common judgment among those who write about fractions [state] that they are extremely difficult for children to learn" (1994, p. 552). He argues that fractions are not used in many situations so setting up realistic problems for the students to solve is difficult (1996a). Also, teachers do not know how to teach fractions effectively and if fractions are not taught in the curriculum, there would be more time to spend on other math concepts (1994).

While Groff seems to be the only researcher that has clearly stated the teaching of fractions is unnecessary, Esty (1991) discusses one particular fraction area that he feels is irrelevant to students' understanding of fractions. Esty finds that teaching students to reduce fractions is complex and unnecessary. He states that "the real reason is historical: Reducing fractions makes the evaluation of the decimal equivalent by long division easier," and asks, "do we evaluate fractions by long division any more?" (p.6). Therefore, Esty (1991) questions the reasonableness and validity of teaching this concept.

Hecht (1998) feels that there needs to be a balance of both conceptual knowledge and procedural knowledge. If students only have a procedural understanding of fractions, they are more likely to apply an algorithm incorrectly, but if they have a conceptual knowledge they can still make mistakes when working with fraction computations. A balance of both would help to ensure the best of both worlds.

Even though these researchers have evaluated the necessity of teaching particular fraction concepts or if fractions should be taught or not taught at all, many researchers and mathematicians see a continued importance for the topic and therefore have investigated how students can be and should be taught fractions more effectively.

2.3 Reform Movement: Teaching Math for Understanding

The reform movement of mathematics, which started in the mid-1980s, grew out of the belief that the traditional ways of teaching math were not effective. Students were developing an often superficial understanding of mathematics rather than genuine knowledge. Students needed more of an opportunity to build on their own understanding of mathematics and methods of problem solving rather than simply following a set of rules outlined by the teacher. This movement is focused on having students learn the concepts behind mathematics as opposed to being taught a series of procedures and rules (Battista, 1999).

In 1989, the National Council of Teachers of Mathematics (NCTM) published a work entitled, "Curriculum and Evaluation Standards for School Mathematics," which encouraged the use of problem solving to help students learn and understand math (Van de Walle & Folk, 2005). There was a call to "reform" how math is being taught because many students who learn math through traditional means struggle with the meaning behind their calculations, and as a result, the "growth of students' mathematical reasoning and problem-solving skills" (Battista, 1999, p.426) was not strong. Adults who have been taught using traditional methods often have a difficult time understanding why they calculate the algorithms the way they do (Battista, 1999). The NCTM called for the following changes in classroom practice:

> They urged teachers to shift their mathematics instructional practice:
>
> - toward classrooms as mathematical communities – away from classrooms as simply a collection of individuals;
> - toward logic and mathematical evidence as verification – away from the teachers as the sole authority for right answers;
> - toward mathematical reasoning – away from merely memorizing procedures;
> - toward conjecturing, inventing and problem solving – away from an emphasis on mechanistic answer-finding;
> - toward connecting mathematics, its ideas and its applications – away from treating mathematics as a body of isolated concepts and procedures (NCTM 1991:3).

What do these basic tenets look like in the classroom?

2.4 Methodology for Teaching in Reform

To teach in a reform method is to teach students to develop their own understanding of concepts with guidance from their classmates and their teachers. It is hard to instruct teachers to teach in this method because there is no set textbook that describes exactly what you have to teach and in what order. The essential point of the reform movement is to begin instruction at the students' level (Prie & Kieren, 1992), rather than at the teachers'. Reform teaching is not easy and in order for teachers to be successful at implementing this teaching style, they need a lot of support from their colleagues and other professionals (Gearhart et al., 1999). This teaching style helps teachers to foster students' natural ability to think mathematically. It is important for teachers to remember that:

> almost all students enter school with an interest in mathematics and a belief that they are capable of understanding it, which means that they believe they can think mathematically. By teaching in ways that tend to *validate* rather than appear to *contradict* their intuition, we can maintain that belief and open the way to mathematics that is natural, understandable, and accessible to them. (Howard, 1991, p. 713)

Teachers need to promote this thinking and expanding of students' mathematical knowledge.

2.4.1 Classroom Atmosphere

In order for reform teaching to take place, there has to be a certain atmosphere present in the classroom and "establishing these environments in the classroom is a basic challenge for reform in teaching and learning fractions" (Steffe & Olive, 1991, p.24). Open communication is very important because the classroom has to be a place where students feel comfortable sharing and justifying their answers (Huinker & Freckmann, 2004). If the students are on the wrong path, it is the teacher's role to help redirect the students onto a different thinking path without dictating or telling them what they need to do (Ball, 1993). Rather than directly correct them, the teacher can ask students various questions to help them reexamine their work and come up with a different conclusion, one that they are ready and able to make.

In a reform classroom environment, "the teacher is constantly in the position of having to listen to what her students are thinking and understanding and, at the same time, keeping her eye on the mathematical horizon" (Ball, 1993, p. 185). A student needs the opportunity to learn at his or her own pace. The classroom environment should enable the students to develop in this way (Prie & Kieren, 1992). This type of class will encourage students to "justify their answers, to apply both spoken and written language, and to use both manipulatives and pictorial models to explain solutions" (Dorgan, 1994, p. 154). Overall, a reform classroom environment has to encourage student learning through problem solving, discussion and teacher guidance.

2.4.2 Types of Problems

To engage the students in worthwhile mathematical discussion of problems, the problems must be realistic and have rich mathematical content (Ball, 1993). It is important to note that "just one interesting problem and thoughtful teacher questioning can result in a rich learning experience" (Yang & Reys, 2001, p.166). In a reform classroom, the lessons and units normally begin with a problem before any formal learning takes place (Huinker, 1998). The problem should be challenging and reach students at all of their different levels; when the math is not challenging, students tend to misbehave (Houssart, 2002). The teacher does not provide a method for solving the problem. Posing problems allows students to come up with a variety of different solutions that can then be expanded upon and strategies for working with fractions can be solidified.

2.4.3 Student Interaction

Students need time to interact with one another in order to express their thoughts mathematically and to try to make sense of the new concepts that they are exploring. Students can come up with great answers to problems if they are given the opportunity to discuss and share ideas with their classmates (Yang, 2002). Steffe and Tzur (1994) do caution that "an important realization is that not all social interaction between two or more human beings leads to learning" (p.115). Even though students gain a lot from having discussions with one another, they also need time to come up with their own understanding and to make the concepts they are learning their own (Steffe, 2003; Tzur, 2004). Steffe and Tzur (1994) have "found that social interaction does not provide a full account of children's mathematical interaction" (p.99).

2.4.4 The Teacher's Role

The teacher plays an integral role in developing a child's mathematical understanding. As the teacher is walking around, he or she has to challenge the students, even if their answers are correct in order to help them make meaning of their solutions. Encouraging involvement from other class members also helps the students to be conscious of their solutions, as Alcaro, Alston and Katims (2002) write: "It is important for educators to help students connect concrete, verbal, and symbolic representations in ways that build meaning and help students develop precision in their use of mathematical language" (p.566). This helps the students to challenge themselves, which is an important aspect of the successful reform classroom (Kazemi & Stipek, 2001).

Once the students have had the opportunity to discuss their problems with a partner, it is important for the teacher to bring the entire class together to have students share their ideas with each other. A class discussion can help introduce the students to the idea that there are a variety of methods to solve one particular problem (Fosnot & Dolk, 2001). Students will explain how they came up with a particular answer so that other students can listen to various ways of solving the same problem (Carpenter, Carey & Kouba, 1991). It will help them to moderate and solidify their individual mathematical ideas (Fosnot & Dolk, 2002).

Fosnot and Dolk (2002) entitle this process a "math congress." This "congress continues the work of helping children become mathematicians in a mathematics community" (p.34). The teacher leads the class in a discussion, asking the students if they agree or disagree with what other students have stated to help each student build on his or her informal knowledge (Empson, 2003), leading towards more sophisticated thinking. It is very important that the teacher does not tell the student if they are correct or incorrect so that the student has the opportunity to decide for him or herself (Empson, 2003; Lehrer & Franke, 1992).

There are many different factors that are necessary in order to make reform classroom a successful and engaging learning environment for the students. It must be stressed again that teachers need a lot of guidance in order for this type of teaching to take place (Gearhart et al., 1999).

2.4.5 Benefits for Students Learning in a Reform Environment

If taught effectively, there are many benefits and advantages for the students involved and partaking in a reform classroom environment. The students learn to be more flexible in their thinking because "when children are accustomed to thinking and reasoning without rules, what numbers they are given makes little difference" (Lamon, 2001, p.162). They are able to apply the concepts that they have learned to a variety of situations (Saxe, Gearhart, & Seltzer, 1999) and move freely between different problems without being limited to a single algorithm (Clark, Berenson, & Cavey, 2003). Students become more confident in their mathematics and are more willing to take risks in their learning and development (Kamii & Warrington, 1999). Empson (2003) even found that lower achievers who were in a reform classroom environment could be part of the class discussions, stating whether they agree or disagree with their classmates. A reform environment empowers students to think mathematically instead of regurgitating memorized formulas (Battista, 1999). Burns (2004a) made a list of the "10 Big Math Ideas" that teachers should follow when teaching math:

1. Success comes from understanding.
2. Have students explain their reasoning.
3. Math class is a time for talk.
4. Make writing a part of math learning.
5. Present math activities in context.
6. Support learning with manipulatives.
7. Let your students push the curriculum.
8. The best activities meet the needs of all students.
9. Confusion is part of the process.
10. Encourage different ways of thinking. (pp. 17-19)

If teachers follow these guidelines, then they will be teaching in accordance with the reform movement and, most importantly, will be teaching mathematics for understanding.

2.5 Problem Solving and Student Invented Algorithms

When teachers encourage students to investigate math by allowing them to work out their own solutions to problems, they are allowing their students to gain a deeper understanding of math concepts (Fuson, 2003). "Children will go much further, with depth, pleasure, and confidence, if they are allowed to construct their own mathematics that makes sense to them every step of the way" (Warrington & Kamii, 1998, p.343). Students are often capable of tackling complex fractional problems at an earlier age when they have the opportunity to use their own methods for solution. For example, Sharp, Garofalo and Adams (2002) posed a division of fractions problem typically reserved for intermediate grades in a Grade 4 class.

The problem was,

> When I got home last night I found my puppy not feeling so well. So, I took her to the veterinarian. Our vet said to give our dog some medicine. She gave us 15 tablets. Because our dog is very large (100 pounds), the vet said to give the dog 1 2/3 tablet each day. For how many days will the medicine last? (p. 23)

A Grade 4 student solved the problem by drawing out 15 circles (pills) and drawing three sections to each one. She then circled how many parts of pills she would to give to the dog for one day. She was able to come up with a solution to the problem without having any prior instruction on how to divide fractions.

When students are posed with real life problems they can often solve them even if they have not been taught to make equivalent fractions (Empson, 2001) or how to add, subtract, multiply and divide, especially when the problems are open-ended allowing for a variety of responses from students (Streefland, 1982). In a study done by Huinker (1998), students in two Grade 5 classes were able to grasp an understanding of how to add, subtract, multiply and divide in four weeks by being given various problems to solve. For example, the teacher posed this problem to the students: "Right now you and your partner have one whole candy bar and one-fourth of a candy bar. I want you to share that amount between the two of you and see if you can tell me what part of a whole candy bar each person gets" (p. 176). Even when a concept seems complex, when given the opportunity, students can create their own solution to fraction problems (Sharp *et al.*, 2002).

If a problem is well-designed, it "stimulates thinking, encourages multiple approaches, and often results in different solutions" (Yang & Reys, 2001, p. 614). A group of students will likely come up with a number of different responses as they work through the problem. For example, Toni Cameron asked her class if they were to invite people to come and have "large flat pancakes" in their classroom and if "they want everyone to have three fourths of a pancake, how many people can be in the group in relation to the number pancakes?" (Fosnot & Dolk, 2002, p.66). The students went about solving the question and they all came up with different ways to solve the problem. Two students used "repeated addition" (Fosnot & Dolk, 2002, p.66) to solve the problem, some girls used doubling as a strategy, some students used multiples, and one student set up a ratio table. Finally, one student solved the question by setting up an algebraic equation. Even though the way the students solve this problem varied from simple to complex, the students were able to come up with an answer that made the most sense to them (Fosnot & Dolk, 2002).

As students are solving problems, they will come up with their own methods. They sometimes create their own algorithm that is a rule that can be generalized and used in all situations for solving a particular type of fractional problem. For example, traditionally students are taught to change a mixed number into an improper fraction by multiplying the whole number and the denominator together and then adding the numerator. A student in a study by Mack (1990) was able to come up with her own algorithm. The question was to change 3 5/8 into an improper fraction. The student responded, "twenty nine-eighths, eight goes into three, I mean 8/8 goes into one, so it's 8, then 16, then another one is 24, plus 5 is 29" (p. 26). The student was still able to get to the same answer but had her own path to get there. Students are able to learn more effectively if they have a chance to solve problems by creating their own algorithms (Kamii, Lewis & Livingston, 1993).

We have strong evidence that it is more beneficial for students to invent their own algorithms before they are taught traditional algorithms (Anghileri, 2001; Baek, 1998). Teaching in this way helps to "develop children's confidence in their own methods rather than replicating taught procedures" (Anghileri, 2001, p.80). When students develop their own algorithms they also have a chance to develop and learn at their own pace (Fosnot & Dolk, 2002). Even though "letting the students wrestle with making sense of situations takes more time than showing them an algorithm, but the payoff in the long run is that students learn to think and to reason about mathematical situations" (Lappan & Bouck, 1998, p.184).

Children can learn to create their own solutions by manipulating, drawing a picture, or figuring the problem out in their minds (Carroll & Porter, 1998). This way the student is thinking of *how* they are going to solve a problem as opposed to only thinking of *what* the correct answer is (Carroll et al., 1998).

Fosnot and Dolk (2002) argued that "asking children what they know and what they are trying to find out can be a powerful tool" (p.57). Encouraging students to think on their own without being taught a traditional method allows them to explore their understanding of math, develop a stronger number sense, and, increase their understanding of place value (Kamii et al., 1993). The more experience that the students have working with the numbers, the more opportunity they will have to develop more sophisticated algorithms, including working with fractions. Students are then more flexible in their thinking and learn to solve problems mathematically without relying on a particular procedure (Kamii & Warrington, 1999).

The research has proven that, if given the chance, students can create solutions that allow them to add, subtract, multiply, divide and create equivalent fractions (Sharp et al., 2002). They will be able to "meaningfully learn, or even create for themselves, appropriate fraction algorithms" (Sharp et al., 2002, p.18). They are learning to understand instead of learning to memorize.

2.6 Big Fraction Ideas

While allowing students to develop their own methods of solving problems is necessary, it is not sufficient; in order for students to have a good conceptual understanding of fractions, there are certain ideas that they need to comprehend to help them in their development of fraction knowledge. Researchers have tried to pinpoint these "big ideas" that are essential for students' fractional knowledge.[2]

2.6.1 Part/Whole Relation

The first major area that researchers see as an important means for constructing fractional knowledge is to understand the concept of equivalency. Students need to know that when a whole is divided into parts, all of those parts have to be equal (Van de Walle & Folk, 2005). The easiest way to help students to understand this concept is have them think of sharing. If they were to share a chocolate bar between the two of them, they would divide it into two equal pieces. If the chocolate bar was split into four pieces and that was to be shared between two people, each person would get two pieces, still half of the same chocolate bar (Van de Walle & Folk, 2005). When fractions are presented through the idea of sharing, students begin to think of fractions as division right from the beginning (Fosnot & Dolk, 2002).

Students also have to consider the importance of the whole in a fraction representation. The parts that a whole is divided into have to be equal in relation to the

[2] The titles for sections 2.6.1 – 2.6.5 are quoted from Fosnot and Dolk (2002, pp. 55-58).

size of the whole. Fosnot and Dolk (2002) give an example of a boy who is trying to divide a piece of paper in half. The boy was able to easily divide the paper into two equal pieces and then four equal pieces. When asked to divide the page into three equal parts he struggled. Finally, he was able to divide the paper into three equal pieces but he had a small amount of paper left over. He cut off the end of the paper, so that only three equal pieces were showing. He felt that he had successfully divided the page into three parts. He did not consider that this "extra piece" left over was part of the whole. This example clearly illustrates that the students have to understand the relationship that the part of something is based on the representation of the whole. The whole has to be conserved (Biddlecomb, 2002).

2.6.2 Equivalency versus congruency

Fractional pieces have to be equal in size and these pieces can have many different names (Huinker, 1998). The most important thing for students to understand is that the pieces have to be equal (Biddlecomb, 2002), but they do not have to be the same shape or divided in the same way (Armstrong & Larson, 1995; Van de Walle & Folk, 2005). Students need time to explore "how the quantity stays the same, even though the pieces look different" (Fosnot & Dolk, 2002, p.56).

Van de Walle and Folk (2005) use the example of sharing again. If a pizza is cut up into twelve pieces and it is being shared among four people, each person is getting three pieces which is equivalent to one fourth of the pizza. The students can still see that the pizza is still being split into four equal parts with each person receiving the same number of pieces, therefore $\frac{1}{4} = 3/12$.

2.6.3 The Whole Matters

When comparing two different fractions, students need to think of what the whole was originally. In order for two fractions to be compared for the size, the original whole plays a factor (Fosnot & Dolk, 2002). For example, if a student were to compare the ½ and 7/8 of an object, then how big the whole of each object was would affect the answer. If the whole was the same size then the obvious answer would be 7/8. However, if the whole for the 7/8 was smaller, than the ½ could potentially be bigger. Fosnot and Dolk (2002) emphasize that "the whole matters because fractions are relations" (p.57) and that this knowledge "is critical as children explore operations" (p.57). In order for fractions to be compared, the whole needs to be the same.

2.6.4 Connecting Multiplication and Division in Fractions

Researchers have found that recognizing how fractions are related to multiplication and division (Huinker, 1998; Saenz-Ludlow, 1995) will help students have a better understanding of "the relationship of the numerator and denominator" (Fosnot & Dolk, 2002, p. 57). An example that Fosnot and Dolk (2002) used to show how multiplication and division are related to fractions is through the sharing of submarine sandwiches. Students need to understand that "three subs shared among four kids (three divided, or *partitioned*, out of four) results in three out of four parts of one sub (*quotative division*)" (p.56). Quotative division is when students group the pieces of the sub together, for example, each person would receive ¾ of a sub. This can be taken further so that students understand that three fourths of a sub, is one fourth times three. This relationship will help them understand the fraction as represented symbolically.

2.6.5 Relations on Relations

When dealing with fractions, especially when working with the operations of multiplication and division, students have to understand how the numbers relate to each other. For example, when learning how to divide fractions, two wholes are being dealt with at the same time. The students have to develop an understanding of how to relate one whole to the other and to understand which whole they are talking about at a given time. For example, Fosnot and Dolk (2002) give the example of a child sharing five candy bars with six people. The first three chocolate bars she divides into half, the next chocolate bar and a half she divides into quarters and she has to divide the last half of the chocolate bar into six more pieces. "The half is now the whole, which is why we say 1/6 of it. But there are *two* wholes to consider: the candy bar is one whole, half the candy bar is another whole. The sliver is 1/6 of the half, but it is 1/12 of the whole candy bar" (p. 59). Determining what each whole represents is very challenging and requires a good understanding of fraction relationships.

2.6.6 Symbolic Relationship of Fractions

There is some controversy regarding when the symbols of fractions should be introduced to children. They have to be introduced at some point so that students can communicate their mathematical knowledge through fractions. When they are introduced, the numerator and the denominator should be explained very clearly. The two numbers need to be treated as one entity, not as two separate whole numbers (Behr & Post, 1992; Behr, Post & Wachsmuth,1986). Students also need to be aware that the numerator represents that number of parts within a whole that are being accounted for and the denominator represents the total number of parts the whole has been divided into

(Van de Walle & Folk, 2005; Huinker, 1998).

2.6.7 Relation of Fractions to Decimals and Percents

An understanding of how fractions are related to decimals and percents will help students to develop a strong sense of rational numbers. Decimals and percents have the same big ideas as fractions. In both instances the whole matters and this will have an effect on the outcome (Fosnot & Dolk, 2002). A teacher posed this problem to his students:

> Paul is telling his students about two advertisements he saw in a newspaper. Both are for department store sales. The first store, Van Merckesteijn's, advertises 25 percent off; the second, Doek's, advertises 40 percent off. "If you wanted to buy something, which store would you go to? Van Merckesteijn's or Doeks's?" Paul asks. (p. 63)

The students immediately assumed that Doek's, the store that gives 40 percent off would be the best because they offer the biggest discount. Upon reflection, the students found examples of when 25 percent may be cheaper, depending on what the original price was. Students are able to connect that the whole matters in the case of decimals and percents as well as in fractions. This would assist them in developing their rational number concept.

2.6.8 Place Value

One of the last big ideas that students must develop, according to Fosnot and Dolk (2002), is the relationship of fractions and decimals in place value. Certain relationships can be found when a fraction is converted to a decimal (dividing the numerator and the denominator). Some of the conclusions that students are able to come up with included that ½ will always be equal to 0.5 and that fractions that have a denominator of 10 have their numerator in the decimal answer (3/10 would be the same as 0.3).

There are many big ideas that students have to understand to have a good foundation of fraction knowledge. Researchers have tried many different approaches to help students grasp these big ideas.

2.7 Strategies to Help Improve Fraction Understanding

In addition to teaching through reform methods, researchers have been consistently trying different methods specifically related to fractions in order to help students develop a deeper and more conceptual understanding. The following is a list of different methods that researchers and teachers have found to be effective or ineffective. Some researchers disagree on strategies that will be beneficial for students' fractional understanding.

2.7.1 Whole Number Knowledge

Some researchers have found that students need to have a strong understanding of whole numbers before they move on with more complicated ideas such as fractions (Behr & Post, 1992; Saenz-Ludlow, 1995; Thompson, 1993) and further that their instruction of fractions should build on this knowledge. Saenz-Ludlow (1995) found that, "one of the main conjectures of the study was [that] the children would use and modify their ways of operating to generate natural number units to generate unit fractions as generalizations of part-whole relationships" (p.107). Ann was a third grader who was involved in this study and the first series of questions that she was asked dealt with determining her understanding of whole numbers. She was able to work flexibly with numbers, to iterate units, and, work with different quantities at once (money). As a result of this knowledge, she then was able to work with part-whole relationships to start to develop the construction of fraction ideas.

To build on students' whole number knowledge, the multiplication and division of fractions should be taught in relation to whole numbers. Thompson (1993) recommends that students need the time and practice to work with problems that allow them to draw out the answer such as "7 divided by 3" (quotient = whole plus fractions), and then, with problems like "2 divided by 3" (quotient = fractions)" (p. 32) because the first example is very similar to whole number division.

In contrast, Behr, Post and Wachsmuth (1986), in the early stages of their research, found that whole number knowledge interfered with fractional concepts. For example, students determined which fraction was bigger based on the size of the denominator. A student might state that 1/35 is bigger than 1/20 because 35 is larger than 20. The researchers referred to this as "whole-number dominance" (p.105). Children have to understand how fractions are written and what each number represents in order for them to have a clear understanding of what fractions represent and so their number sense does not hinder their fractional development (Behr, Post & Wachsmuth, 1986).

Mack (1995) also found that students made overgeneralizations about fractions based on their whole number knowledge. She found that "students drew on their prior knowledge of whole numbers and confounded whole-number and fraction concepts" (p.430) when working with fractions symbolically. Students had a different interpretation of what fractions meant when they were represented symbolically or represented in a word problem. When represented symbolically, students thought of the numerator as representing the number of wholes and the denominator representing the number of pieces that the whole were divided into. In the study "where students obtained different answers to problems presented verbally or symbolically, their explanations suggested that their responses were influenced by their prior knowledge to whole numbers" (Mack, 1995, p.432). This confusion may exist simply as a result of poor instructional methods. Whole number dominance, therefore, may contribute to students learning of fractions.

2.7.2 Informal Knowledge

Acknowledging informal knowledge means beginning with what students already know as opposed to poor instruction which begins at a typical stage of fractional development. This includes building on whole number concepts as mentioned above and on what students may have learned about fractions in their life up to the point of encountering them in the classroom (Mack, 1993). If fractional knowledge is built on students' informal understanding of fractions, the subject of fractions can be taught in the primary grades (Powell & Hunting, 2003). Teachers can start to teach primary students by having the students represent fractions using language only, beginning with having students sharing items (such as food) rather than symbols (Powell & Hunting, 2003). To start students on the development of the true meaning of fractions, situations should first be presented verbally, without the use of fraction symbols (Mack, 1993, 1995; Saenz-Ludlow, 1994). Students needed time to make the transition between whole number meanings for fraction meanings (Mack, 1995). Mack (1993) started to teach individual students about fractions by introducing real life problems to the students and then introducing the symbols later on. She found it very important to move "back and forth between problems represented symbolically and those in the context of real-world situations, removing symbolic representations when students were not yet ready to work with them" (p.364). She also found that students were able to relate their informal knowledge to fraction symbols as long as the fractions symbols were very clear (Mack, 1990). To teach multiplication with understanding, Mack went through the same process, building on questions and relating to the students informal knowledge when they were having a hard time moving ahead (Mack, 2001). This allowed the students to build from a point that they were comfortable with and learn more complex ideas when they were ready.

Building on students' informal knowledge is the basis for reform education. Steffe and Olive (1991) stated that teachers need to understand the informal knowledge that students have when they come into the classroom and be aware of the knowledge that the students construct when they are in the classroom. Sometimes students' knowledge may not be accurate and they need to be redirected on the right path to help them construct the correct knowledge. The teacher, therefore, is always looking at ways to improve the mathematical path that students are on, based on what they have learned both inside and outside the school environment in order to ensure that they are making realistic connections.

2.7.3 Manipulatives

Manipulative aids can help students develop a concrete understanding of fractions as they move from the concrete to the abstract. There are many different manipulative materials that students have used to help them develop a better understanding of fractions. Different types of manipulatives that can be used include fraction circles, Cuisenaire rods, counting chips, pattern blocks, paper folding and different coloured construction paper. Some researchers had a number of different manipulatives used in their study (Bezuk & Cramer, 1989; Behr, Wachsmuth & Lesh, 1984; Naiser, Wright &

Capraro, 2004). Teachers can help students to learn fractions by having them use manipulatives correctly (Naiser, Wright & Capraro, 2004). Post and Cramer (1987) found that a "variety of manipulative materials can help children overcome the initial influence that whole-number ideas have on their thinking and enable them to think quantitatively about fractions" (p.34). Other researchers, however, have focused their study on the use of one manipulative (Post & Cramer, 1987; Sinicrope & Mick, 1992). Burns (2004b), for example, has had students put together fraction kits (different colours of construction paper representing different sizes of fractions) and has had them use these strips to play different games. The fraction kit gives student a visual aid of what fractions can be compared based on the same whole. For example, two ½ strips would fit on the whole and two quarter strips would fit onto one half (Burns, 2001, 2004b).

Paper folding is an example of an activity can help students to learn how to multiply fractions (Sinicrope & Mick, 1992). Students first learn how to solve multiplication statements with whole numbers and then learn how to do multiplication with fractions. The paper folding activity involves folding and shading. "Discussion and framing the activities within a problem-solving format can help students connect the paper-folding activities with application of multiplication of fractions" (p. 121).

Harrison, Brindley and Bye (1989) tested two different methods of teaching of four hundred thirty-five 12 year old students. One half of the students were taught using a problem solving approach involving the use of concrete materials while the other group was taught in a more traditional fashion. The group that had the opportunity to learn with the use of concrete materials did much better not only in understanding fraction and ratio concepts but also in their general attitude toward mathematics.

Counting chips can be used to help students deal with discrete sets when dealing with fractions. Normally, textbooks represent continuous sets of fractions (a whole that is divided and then shaded into parts). A discrete set deals with individual items with a fraction of those shaded. Students should have the opportunity to be exposed to both types of sets to improve their fractional understanding and so that they can be familiar with different representations of fractions (Witherspoon, 1993). When students have an opportunity to work with discrete sets they seem to have a better understanding of fractions and they are easily able to transfer their knowledge to different types of situations including those that would be continuous in nature (Hunting & Korbosky, 1990). These researchers found "that discrete quantity materials have a valuable role to play in developing fraction knowledge in the elementary school" (Hunting & Korbosky, 1990, p.947). Any item that is represented as a single entity on its own can be used to represent a discrete set.

While manipulative aids can assist students in their understanding of fractions, the teaching method has to support the proper use of materials. Teachers can use manipulatives but may still not use them effectively or in a reform environment. All of the steps that are needed to teach effectively in reform are required as well. The ultimate goal is to have students moving from the concrete representation to the abstract symbolism, while using manipulatives that are appropriate to the given situation. Moreover, it may be that it is the act of creating the manipulative, as in the case of Burns (2004b) or Fosnot and Dolk (2002),that has the greatest impact. Students are given more of an ownership on their manipulatives when they take the time to make them and to work through how to create them. This in itself is an integral part of the learning process.

2.7.4 Benchmarks

Having students understand the use of benchmarks can help further their fraction understanding. Benchmarks are "important reference points" (Van de Walle & Folk, 2005, p. 232) that students use to help them make comparisons of fractions or use to estimate answers to questions. The important benchmarks for fractions are 0, ½ and 1. If students have a good understanding of benchmarks then they are able to develop a better conceptual understanding of fractions and are able to estimate the value of fractions (Reys, Kim & Bay, 1999). Research has shown that "this benchmark strategy had a higher success rate than the other two strategies of drawing a picture and finding a common denominator" (Reys, Kim & Bay, 1999, p.531). When students use ½ as a benchmark, they are able to more easily make conclusions about fraction relationships. For example, in a study conducted by Behr, Wachsmuth and Post (1985), students were given one minute to construct two fractions (when given the numerals) that would add up to be less than one but was closest to one. The students were told that they would not be able to work out their individual answer because of the time limit. This ensured that the students did not use an algorithm. The strategy that was most effective was when students were using the benchmark strategy.

Using benchmarks also helps students to estimate answers more accurately. When students were asked to estimate the sum of 7/8 + 12/13, when given the choices 1, 2, 19 and 21, they were not all able to accurately estimate the sum of two (Payne & Towsley, 1990). If the students understood the concepts of benchmarks, then they would easily be able to see that7/8 and 12/13 are both close to one which would result in a guess of two. Burns (2001) also uses the concepts of benchmarks in her fraction activities.

Benchmarks can assist students in their fraction understanding and it gives them another effective mathematical tool when problem solving. With the knowledge of benchmarks they can determine whether their answers are reasonable as opposed to relying on faulty procedures and algorithms.

2.7.5 Unit Fractions and Iteration

Unit fractions are fractions that have one as the numerator. It can also be a representation of a fraction that is to be iterated (repeated). For example, how many 2/6 to you need to make a complete whole? Historically, unit fractions were the first type of fractions that were used in the initial dividing of items. "In ancient Egypt, fair-sharing situations brought about the use of unit fractions – fractions in which the numerator is one" (Fosnot & Dolk, 2002, p.38). They used unit fractions to determine how much of a loaf of bread each person would receive. For example, 2 loaves for 15 people would equal 1/10 + 1/30 (Fosnot & Dolk, 2002, p.38).

Students naturally seem to have an understanding of unit fractions because these are fractions that they may more commonly see (Davis, Hunting & Pearn, 1993). Students may have a difficult time understanding a fraction when it is the iteration of a single unit. For example, 2/5 is the iteration of 1/5 twice (1/5 + 1/5 = 2/5). Even 2/3 was unnatural for students to see as two 1/3's (Davis, Hunting & Pearn, 1993).

Tzur (2004) investigated how students come up with unit fractions using a computer program, TIMA: Sticks. For example, students were asked to create what the original whole would look like for 5/8. They were given the piece 5/8. They then had to break that into five pieces, copy one of those pieces (1/8) and iterate it 8 times to produce the original whole. As the students were able to grasp this initial concept, they moved on to iterating more complicated fractions. Therefore, helping students have a solid understanding of a unit fraction has many benefits for developing some of the big ideas: they are able to keep the relationship of the part to the whole; they can produce the whole when only given a segment of the fraction; and understand that fractions are parts of whole.

2.7.6 Computers

Researchers have worked with various computer programs to help students develop a deeper understanding of fractions. The one that was mentioned above, TIMA: Sticks, can help students to segment fractions, copy fractions, cut fractions and paste different fractions together (Tzur, 2004). It should be noted that the success that the students in that particular study had was predicated on not only the program but also interaction with each other and the teacher. The computer programs were used to help the students figure out solutions to problems that were posed by the instructor as opposed to the computer programs themselves posing word problems that the students had to answer.

Tzur (1999) also used this computer program in an earlier activity to help students develop the concept of improper fractions. Students were once again required to come up with unit fractions and then iterate those fractions to produce a whole. Then Tzur (1999) had students iterate 3/5 twice to produce 6/5. The students then had to work through their thoughts to determine what the new fraction would be called (Tzur, 1999). In 2000, Tzur again worked with students and found that they changed what the fraction represented depending on how many times it was iterated. For example, if the students iterated 1/8 nine times then they thought the fraction was 1/9. The child had to learn to have a "shift of attention – from focusing on the whole as a single unit to focusing on the number of parts (produced in iteration) that constituted it as a partitioned unit" (Tzur, 2000, p.139).

Computers may be useful if you have the space and the right number of computers available. Some authors have tried to implement what they have used on the computer in Fractions Project to teach an entire class (D'Ambrosio & Mewborn, 1994). There were many limitations to this transition. The researchers ran into more obstacles than they anticipated, mostly regarding students' conceptual understanding of fractions and how it takes times in order to develop this type of understanding.

2.7.7 Diagrams and Other Visual Models

Diagrams and visual models can help a child have a better understanding of fractions or they can hinder students' development (Watanabe, 2002). It is important for teachers to choose the most appropriate fraction model to represent the concept that is being taught. Fractions need to be represented in a variety of different ways so that students can have a varied understanding of what a particular fraction represents. For example, Watanabe (1996) had a student look at a number of different visual interpretations and determine which ones depicted a ½. He needed further fractional development to recognize some diagrams of a half that were represented in different ways.

A common visual model that is used to teach students fractions is the bar model or the number line. Keijzer and Terwel (2003) did a long term "study of teaching and learning of fractions in two matched groups of ten 9-10 year old students" (p. 285). Fractions were introduced to the two groups using different types of models. Fractions were introduced to the experimental group, "using the bar and the number line as (mental) models" (p. 285) and introduced to the control group, "by using fair sharing and the circle model" (p. 285). A difference between the two groups was that the students in the experimental group were allowed to discuss their solutions and ideas with one another where as in the control group the students worked individually. The results were, "after one year, the experimental students showed more proficiency than those in the control group" (p. 285). However, one of the reasons for this difference may have simply been the fact that the experimental group students were allowed to converse as opposed to the number line or bar model being the most effective model.

Middleton, van den Heuvel-Panhuizen and Shew (1998) also recommend that the bar model be used to help students develop fractions. They suggest that "the bar is easily divided into key "benchmark" fractions such as ½, 1/3 and ¼" (p.304). The students in this study naturally divided things into halves and the bar model helped them to demonstrate this concept.

There is some caution that should be applied when using the number line model. Bright, Behr, Post and Wachsmuth (1988) found that the number line did not help students develop a deeper understanding of fractions. Even though there were significant improvements from the pre test to the post test, the students had a hard time representing the fraction information on the number line. They also found that being able to transfer their knowledge from one situation to another "was not particularly successful, especially when the representations of the fractions were in unreduced form" (p.227).

The number line and the bar model like any other visual model, in order to be effective, has to be appropriate to the fraction concept being taught and have relevance to the learning goal. If used properly, these models may be another tool for a student to further their understanding.

2.7.8 Having Fun with Fractions

Finally, some researchers have looked at ways for students to have fun when learning fractions and have come up with a number of different activities that students can be actively engaged in (May, 1999; Mosley, 1995; Naylomon, 2002a, 2002b, 2002c, 2003a, 2003b, 2003c; Moyer & Mailley, 2004; Rees, 1987; Sheero, Sullivan & Urbano, 2003; Stump, 2003). These activities offer students the opportunity to learn about fractions in a hands-on way as well as to investigate and problem solve to deepen their fraction understanding.

Moyer and Mailley (2004) discussed how a teacher used the story, "Inchworm and a half," to help teach students about fraction parts and equivalent fractions. Using the story, the students had to measure vegetables using different lengths of the worms (such as ½, 1/3, etc.). The next day the students had to come up with equivalent fractions using the worms as a guide. Then they moved onto representing these fractions symbolically. She used "fractional benchmarks such as ½, 1/3, and ¼ [because these] are integral to children's development of rational number sense" (p. 252). Further, when students have fun while they are learning, they are actively engaged.

It is important for teachers to remember, however, that while the students may be having fun, the fraction concept goal for that particular activity must always be kept in the forefront. Teachers need to continually engage the students in dialogue to ensure that they are taking from the activity what is needed for them to have a larger fraction sense.

2.8 Conclusion

As the literature suggests, there are ways to improve students' learning of fractions so that they develop a more conceptual understanding. The traditional methods of instruction have resulted in giving students a limited understanding of fractions. Students depend on faulty algorithms to solve problems as opposed to their own reasoning. In essence, the students are memorizing a number of different rules without any grasp of their relevance or significance.

In order for students to be able to develop a solid foundation of fractions, they need time to investigate problems. A major part of the reform teaching is posing problems to the students that they need to solve on their own without any prior guidance or instruction from the teacher. By talking through solution methods with a partner, students are able to develop their own concepts of fractions based on their developmental level. Enhanced with class discussions and mini-lessons, students can essentially move along the fraction continuum in a way that makes sense to them

Creating their own manipulatives to use when they are working through problems helps to extend their knowledge of fractions by giving them a greater sense of what a fraction represents. Before they can move into operations with fractions, they first need to have a grasp of what a fraction is and understand how the numerator and the denominator work together to represent one number as opposed to two separate whole numbers. If students learn to depend on their reasoning skills they will be more flexible in their application of fraction concepts and have a better sense of what a fraction means.

My action research was conducted to determine whether students could develop a better understanding of what it means to use fractions if they are given the opportunity to come up with their own solutions to problems (Bulgar, 2003; Kamii & Warrington, 1999; Lappan & Bouck, 1998, Warrington & Kamii, 1998) and when they are taught using a more effective instructional method.

Chapter 3: Methodology

3.1 Research Design

The purpose of the investigation was to answer the following two questions. First, what teaching method, traditional or reform, was most beneficial in helping students develop a conceptual understanding of fractions? Second, what were the specific aspects of the reform teaching method that helped students develop a more accurate and meaningful understanding of fractions? Two Grade 6 classes were used for this study; one of the classes was taught using the traditional method and the other class being taught in a reform teaching method. The study was qualitative in nature.

3.2 Design of the Study

3.2.1 Research Sample

The research was conducted with two Grade 6 classes in a Separate School Board in Northwestern Ontario. One of the classes was an English class that contained 30 students and the other was a French-Immersion class that contained 23 students. In the English class, all students participated in the research. Three of the thirty students were on modified programs (two identified students with a learning disability which means that the curriculum was modified for their learning needs; one remedial student who needed assistance to help work at the Grade 6 level in math). In the French-Immersion class, two students opted not to participate and four students were away for a length of time and therefore were unable to complete the post-test. Their data was not included in the study, so 17 students' results from the class were examined. There were no students that were on modified or remedial programs in this class.

3.2.2 Procedure

Before the fraction unit was started, ethical approval was granted by Lakehead University, by the school board and by the principal of the school where the study was conducted. Student data was being used, so letters (Appendix A and B) and parental consent forms (Appendix C) were sent home to both of the classes. To protect the anonymity of the school and of the teachers, pseudonyms have been used and some of the wording has been slightly altered or changed in the letters. As the researcher, however, I taught the fractions unit to Class A, using the reform method. This was my first time teaching in a unit in this fashion.

The unit on fractions was taught for approximately one month in each of the classes which started the second week of January and ended in the second week of February, 2005. The lessons were taught in the morning as much as possible.

The unit began with the pre-test which was completed in one session. For the pre-test, students were told to try their best and that this test would just give the teacher an indication of what they know about fractions at this point. The test was not timed; the students in Class A (reform class) finished the test fairly quickly compared to Class B (traditional class) where they took a lot longer to work through the test, especially the word problem section. All students were present in Class A for the pre-test and those students that were absent in Class B completed the test upon their return to school. See Figure 4 for a summary of the classes.

Figure 4. *Summary of Class Dynamics*

Teacher of Fraction Unit	Class Type	Class Identification	Number of Students	Special Education for Mathematics	Non-Participants
Mrs. Biesenthal	English	A	30	3	0
Mme Joans	French Immersion	B	23	0	6

Each teacher went on to teach her unit. Mme Joans (Class B; traditional method) was given an outline of which curriculum expectations and concepts were to be covered in the unit (Appendix D). She recorded what was taught during each lesson and as well completed a reflection on the students' understanding.

The teaching of Class A, the reform method of instruction, was recorded more in depth to help determine which strategies helped to improve students' learning. Each lesson was specifically outlined and points were made about students' responses to situations. Students' work was collected and/or photocopied to be examined further. Some students were tape recorded during the lessons to provide a record of the development of their thinking. A reflection was recorded at the end of each lesson that included some topics such as how the lesson was taught, improvements that could be made, the direction to be taken for the next lesson, and how the students responded to the concept being introduced and developed.

The unit ended with the completion of the post-test which was broken up into two sessions: one session for the computations portion and one session for the word problems. If students missed a portion of the test they wrote it as soon as possible once they returned. In Class B, where several of the students were away for a length of time, they did not complete the entire post-test because the completion of the test would cause inaccurate results in the final analysis.

3.2.3 Summary of Traditional Teaching Method

Mme Joans was instructed to teach the unit in a way that was most comfortable for her. The only difference from her regular teaching practice was that she was asked to record the lessons that were being taught and reflect on students' responses and understanding in those lessons. The textbook that she used most frequently throughout the unit was Mathématique 2000 (Addison-Wesley, 1997) and Houghton Mifflin Mathématique 6 (Houghton Mifflin Canada, 1984) as a supplemental text. The information from this section was taken from the journal (Joans, 2005) that Mme Joans wrote along with discussions to clarify information.

The unit was launched by having the students explore how fractions, decimals and percents are related. The students had to shade in the equivalent parts of fractions, decimals and percents from a sheet found in Mathématique 2000.

The following (Table 1) is an overview of how the unit was taught and how the lessons progressed through the different concepts.

Table 1. *Traditional Unit Plan Overview*

Lesson Number	Concept Taught
Pre-Test	Equivalent, comparing, adding and subtracting, word problems
1	Exploring how fractions, decimals and percents are related
2, 3, 4, 5	Solving problems using percent and fractions, comparing fractions, decimals and percents, exploring positive and negative integers
6	Equivalent fractions
7	Review vocabulary dealing with fractions
8	Simplifying fractions
9	Comparing fractions for half of the class
10	Computers: review of fractions
11	Mixed numerals
12	Playing with Cuisenaire rods (type of manipulative)
13	Test
14	Fun activity: restaurant menu and fractions
15	Review of fractions and decimals/part of a set
16	Adding fractions with same denominator
17	Adding fractions with mixed numbers
18	Test/review
19	First half of post-test
20	Second half of post-test (teacher was absent for this – supply teacher administered the test)

The overall teaching style was teacher-directed though she did have students work in groups a couple of times. She would teach a concept, have visual examples on the board whenever possible, and then have the students practice the concept that was just taught. For example, students were taught to simplify fractions by using a rule.

At this point in the unit, the class was divided in their understanding of simplifying fractions. Some students moved on to comparing fractions while the teacher continued to work with approximately half the class on simplifying fractions. To get the students to the same stage in their learning, the teacher worked with the students who were having difficulty earlier with simplifying fractions to teach them how to compare fractions at a table during a computer work period. She used a chart stand and paper to try to give the students more examples. She felt that working with them in a smaller group would help them develop understanding. The other students that already understood the concepts of simplifying and comparing fractions used the computer program "Math Trek" to independently work on fractions.

Mme Joans made use of diagrams and manipulatives to teach fractions. She used diagrams by showing them a picture of what a particular diagram meant and then had them work on their own examples. For example, when she taught Mixed Numerals, students had to draw figures and shade them in to show mixed numerals. Most students were able to grasp that concept and the teacher used peer mentors to assist those who were having difficulty.

Cuisenaire rods were used to further review the concepts and the students were paired up (stronger students with weaker students). Some students needed more time to get used to the manipulatives, but once they did it really helped those students who were having difficulty. Further, the students were reported to have enjoyed the activity. Some other manipulatives Mme Joans used in her unit were paper-folding and fraction bars.

Mme Joans made use of algorithms to teach the students how to solve fraction computations. For example, students were taught how to make equivalent fractions by multiplying the denominator and the numerator by the same number, and for simplifying fractions, to divide the numerator and the denominator by the same number.

Students were taught when they were adding fractions with the same denominator that they had to simplify their answer to lowest terms. The students grasped this quite readily although the teacher did comment that the students who were strong in simplifying did this easily while those students who were weaker in this area often left out this step. The teacher moved on to teaching the students how to add fractions with unlike denominators and adding fractions with mixed numerals. The teacher also explained that the same method would be used for subtracting fractions.

There were two points in the unit when a separate test was given to see how the students understood the concepts that were taught. The first test/quiz was given to determine how their understanding of comparing, simplifying and mixed fractions was up to this point. Most students did quite well. When reviewing decimals and fractions and working on part of a set, (for example, 1/3 of 15 is the same as 15/3 = 5), students seemed to have a good understanding.

Before the post-test, the teacher gave them one more test that reviewed comparing, simplifying, equivalent, mixed fractions, decimals and percents and a few word problems. To conclude the unit, the students completed the post-test. Some of the students reported feeling anxious because they didn't remember being testing on adding fractions in the pre-test.

At the conclusion of the unit, the teacher knew that it was time to move on to other math topics even though she felt that she did not spend enough time on adding fractions with her students. She felt more time was needed overall; however, the students needed a break from fractions.

3.2.4 Reform Teaching Method

The reform teaching method was taught very differently. The basis of the reform teaching method was for students to develop fraction concepts through problem solving, rather than direct instruction. The teacher acts as a guide to help the students' progress along the fraction continuum. Prior to the unit, the students' desks were arranged into pairs, based on a study done by Huinker (2004). Students were matched together based on similar abilities and personalities that would assist them in developing their fractions sense. The following is an overview of how that unit was taught (Table 2) taken from notes kept by myself (Biesenthal, 2005).

Table 2. *Reform Unit Plan Overview*

Lesson Number	Concept Taught
Pre-Test	Equivalent, comparing, adding and subtracting fractions, word problems
1	Introductory Problem
2	Working on Question 2 from the problem, took up Question 1
3	Finished taking up Question 1 and 2
4	Fraction Kits were made
5	Played games with the Kits (Cover-Up and Uncover)
6	Played the game Uncover again but with an altered rule
7	Drawing fractions with pieces that are equivalent but look different
8	Equivalent fractions – using fraction kits to show understanding
9	Equivalent Fractions
10	Equivalent Fractions, Building of Clock Fraction Kit
11	Finishing of Clock Fraction Kit
12	Comparing Fractions
13	Fraction Benchmarks, Addition Problems
14	Discussion of Results in groups of four, Results put onto chart paper
15	Groups results were presented to the class, Comparing Improper and Mixed Fractions
16	Adding Fractions using Equivalents
17	Subtracting Fractions; Subtraction Word Problems
18	Proving an answer to a problem; Ordering Fractions in a set; Put Fractions in Order Part 1
19	Taking up the answers to Subtraction Word Problems; Ordering Fractions in a set
20	Put Into Order Part 2
Post-Test	Equivalent, comparing, adding and subtracting fractions, word problems

The focus of the unit was on problem-solving, mathematical talk and the development of a range of student methods for problem-solving. Manipulatives and diagrams were used in a very different way from the traditional teaching method as students had to make their own fraction kits. Algorithms were not taught as solutions to problems; instead the students had to investigate and develop ways to solve problems, and later on, use computations efficiently.

Students were introduced to major fraction concepts usually by being given a problem to solve. To launch the unit, students were given a word problem to solve that brought up a number of different fraction ideas (see Figure 5). While the students were solving the problem, they were asked to come up with answers to the following questions (based on Fosnot & Dolk, 2002): How much did each child get, assuming all the subs were shared equally in each group? Which group got the most?

Figure 5. *Introductory Word Problem for Reform Class*

Lesson #1: Introductory Problem

Question adapted from Fosnot & Dolk, 2002, p.2

A few years ago I was taking my students on a field trip to different places in the city of Thunder Bay. There were several different places that I wanted my students to go so we had to split up. Some parents came to help out so we were able to go to four field trips in one day. Four students went to the Terry Fox Monument, five went to see the Sleeping Giant, eight went to the Thunder Bay Art Gallery and the last five students went to Marina Park. I had ordered submarine sandwiches for the students for lunch and I had seventeen of them in total. I gave three sandwiches to the four students going to the Terry Fox Monument, four sandwiches to the five students that went to the Sleeping Giant, seven sandwiches to the eight that went to the Thunder Bay Art Gallery and the last five students going to Marina Park received three sandwiches. We didn't eat together because we all went to different parts of the city. The next day after talking about our trips, several of the students complained that it hadn't been fair because some students got more to eat. What do you think about this? Were they right? If they were, what should I have done?

With your partner, take some time to work through the scenario and answer the questions. Be sure to use pictures, numbers and words to explain your answer. Be ready to present to the class.

I walked around while the students were working to ensure that they were engaged in the task at hand. I asked them questions about their ideas behind their solutions and asked them to explain certain details to me. Questioning the students allowed them to draw upon their own thoughts and make conclusions as opposed to telling them that they were right or wrong. Problems were also used to help students investigate adding and subtracting fractions.

For all of the problems the students worked with an assigned partner. They were encouraged to discuss their ideas with one another. If one student was making a conclusion or statement, I reinforced with them that their partner must also understand. To discuss problems even further before the class discussed various solutions together, students were paired up with another group so that there were at least four people discussing their strategies.

Class discussions were an integral part of the reform teaching method. Class discussions in the reform teaching class were based on student findings. This helped the students build on their current level of understanding as opposed to being told the answer. To take up a problem, groups or individuals were asked to present their findings to the class. For example, after the students spent some time on the first problem (approximately a day and a half), two specific groups that demonstrated two different ways of solving the problem were asked to go to the board and explain their solution method to the class. Students were encouraged to state their ideas out loud instead of raising their hands. During class discussion, I acted as a facilitator between the students, asking for clarification and more questions.

Throughout the unit, I directed the students to patterns or observations that were evident in their work to discuss with the class. This was done to help solidify some of the fraction knowledge that they were gaining as they worked through the word problems. For example, on the fourth day of the unit, this statement was written on the board: "When the whole is the same, the larger the denominator the smaller the piece." Students were then asked to give examples of fractions that were smaller than a ½.

Algorithms were not taught specifically, but I would discuss different strategies that the students came up with that were efficient in helping them work through problems. For example, after discussing a problem that involved the addition of fractions the strategy of using an equivalent fraction to help add the fractions together was discussed. When I asked them to solve, ½ + 1/8, the students were able to recognize that ½ was the same as 4/8. They changed the fraction and added the numbers together to end up with a fraction of 5/8. The students then continued to work on solving another problem as well as some straightforward computation questions.

Manipulatives were used extensively throughout the unit. Students made kits themselves so that they actively constructed fractions. They created two models of fractions. The first fraction kit they made was a Rectangular Fraction Kit (Burns, 2001). The students were given the coloured strips of paper needed to make the kit along with a plastic zip-lock bag to store it in. As a class, with my guidance, the fraction kit was built, using one strip at a time. The students were reminded to cut the strips vertically so that the pieces would be easier to interchange. The students then used the fraction kits to play various games, these were also taken from Burns (2001). The fraction kit games that they played helped them to be familiar with equivalent pieces so they were able to interchange them quite easily.

Students were then given templates to make a second fraction kit, a Clock

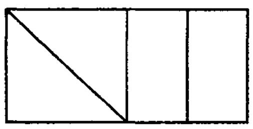
Fraction Kit. They were given blank clock outlines, different colours of construction paper and a zip-lock bag to store their kit in. This was more difficult then the making of the rectangular fraction kits. The students drew lines on each circle to divide it into the appropriate number of fractions parts. They then glued the circle onto the specified colour of construction paper, cut out the circle and the parts. They now had either of these kits to use throughout the unit to help them solve problems.

A few times a week, until the completion of the unit, the students had to complete a fraction assignment that was written on the board when they entered the class in the morning. This was entitled "Bell Work" and it helped to reinforce some of the concepts that were being learned. An example of a question on the board was: "Explain which fraction is the smallest (1/2, 1/3, 1/8 or 1/10)." Their books were then collected and photocopied to look for general understanding or lack of understanding of the concept. This work helped to guide the lessons and concepts that had to be covered as the unit progressed.

One time a concept was introduced in the Bell Work that students were not ready for. This was the comparing of Mixed Fractions and Improper fractions. When I realized that they were having such a difficult time with the concept, I left it, and returned to a point in the unit when they were able to grasp the idea more easily. To teach in reform, it was imperative that I move at a pace that the students were able to handle.

Even when there was not a specific word problem that students were trying to solve, they were engaged in activities that would challenge them to think about the true meaning behind fractions. For example, the students were asked to draw equivalent fractions with pieces that look very different (see Figure 6). They were to work through how they would draw their shapes in their "Bell Work" books. They had to draw one fraction that was a true equivalent fraction and one that was not. Once they were completed their two sets, they transferred shapes onto construction paper and had a partner figure out whether the two fractions were equivalent or not.

Figure 6. *A non-traditional way to show a half*



Another example was when the students learned to work with Benchmarks to give them another tool to use when working with fractions. They took a single index card, wrote the numbers 0, ½ and 1 and cut them into pieces. I gave them a fraction to make, for example 6/10, using their fraction kits and indicate whether it was closer to 0, ½ or 1 by using their index card. They shared their answer with their partner and then the answer was taken up with the class.

At the end of the unit there was the same sense that there were more fraction ideas that would have been beneficial to teach the students (such as discrete sets which are fractions using individual items such as chips as the whole), however, the students were ready to move on to a new topic.

3.2.5 Data Collection

To investigate the answer to the first question, "what teaching method, traditional or reform, was most beneficial in helping students develop a conceptual understanding of fractions?," a pre-test and post-test were designed to help analyze students' knowledge and to look at the students' level of achievement (see Figure 7 and 8). In each test, there were two sections: Computations and Word Problems. The computations section included questions for finding equivalent fractions, comparing fractions, adding and subtracting fractions. This section was set-up in a similar way for both tests except the numbers were changed. The major difference in the computation section was that in the post-test, students had to choose one question that they solved in each computation section to explain how they achieved their answer.

The word problem section included questions that evaluated the same computational areas. While the computations could be solved with procedures, the word problems required a deeper understanding of the concept. The intent of the design was to see how well students were able to solve problems when they were not always able to rely on algorithms. The word problems were drawn from a number of different sources; some of them from teaching guides and others from test questions that other researchers have posed (see Figure 7 and 8). The subtraction question used in the pre-test was changed to a division question in the post-test. Some questions were identical while in other cases the numbers or the wording was changed slightly. The pre-test was implemented in one session while the post-test was split into two (one session to complete the computations portion and another session to complete the word problems).

Figure 7. *Pre-Test on Fractions*

Number: _____
Class: _____

Pre-Test on Fractions

Section 1: Computations

Solve each of the computations listed below:

(a) Equivalent Fractions

1) $\frac{2}{5} = \frac{\_}{10}$ 2) $\frac{3}{4} = \frac{\_}{12}$ 3) $\frac{4}{5} =$

(b) Comparing Fractions

Indicate whether each fraction is <, >, or =.

1) $\frac{3}{8}$ $\frac{1}{5}$ 2) $\frac{1}{2}$ $\frac{9}{12}$ 3) $\frac{4}{5}$ $\frac{7}{15}$

(c) Adding Fractions

1) $\frac{3}{5} + \frac{1}{5} =$ 2) $\frac{7}{10} + \frac{1}{2} =$ 3) $\frac{1}{15} + \frac{1}{12} =$

(d) Subtracting Fractions

1) $\frac{9}{10} - \frac{3}{10} =$ 2) $\frac{1}{2} - \frac{1}{3} =$ 3) $\frac{3}{10} - \frac{1}{5} =$

Section 2: Word Problems

Answer the following word problems. Be sure to show all your work and use pictures, numbers and words to help explain your answer.

1. For each statement, decide if it can best be described as "exactly half," "about half," "less than half," or "more than half." (Burns, 2001, p. 150)

   (a) Maria received 13 birthday cards. Five of them arrived the day after her birthday.

   (b) Fifty-five students signed up to work on the school paper. Twenty-seven of them were girls.

2. A class is going to invite parents to come into their classroom for "Pancake Day." The pancakes are large and the students want each person to have ¾ of one pancake. How many people can be in the group in relation to the number of pancakes? (adapted from Fosnot & Dolk, 2002, p. 66)

3. Pat saved half a chocolate bar yesterday and a third of the same kind of chocolate bar that she got today. How much chocolate has she saved? (Kamii & Warrington, 1999, p. 87)

4. Paul has 7/8 of a piece of a chocolate bar. He eats one ½ of it. How much does he have left? (adapted from Van de Walle & Folk, 2005, p. 245)

5. There is 3/10 of a pie left at a bake sale. Someone buys 1/5 of the pie. How much is left? (adapted from Van de Walle & Folk, 2005, p. 246)

6. Suzy, her father, and her mother divided a chocolate bar equally among themselves. Suzy then gave half of her share to a friend who came over. Suzy's mother decided to give her share (the mother's 1/3) to Suzy. How much chocolate did each person get? (Kamii & Warrington, 1999, p. 87)

7. Raquel thought about this statement: When pitching, Joe struck out 7 of 18 batters. She said that is was better to say that Joe struck out about 1/3 of the batters than to say that Joe struck out about ½ of the batters. "I think that seven-eighteenths is closer to one-third than one-half," she said. Do you agree or disagree with Raquel? Explain your reasoning. (Burns, 2001, p. 152)

Figure 8. *Post-Test on Fractions*

Number: _____
Class: _____

Post-Test on Fractions

Section 1: Computations

Solve each of the computations listed below:

(a) Equivalent Fractions

1) $\frac{3}{5} = \frac{\_}{10}$ 2) $\frac{2}{3} = \frac{\_}{9}$ 3) $\frac{5}{12} =$

Choose your solution for either 1, 2 or 3 and explain how you know that one fraction is equivalent to another.

(b) Comparing Fractions

Indicate whether each fraction is <, >, or =.

1) $\frac{3}{7}$ $\frac{1}{5}$ 2) $\frac{1}{2}$ $\frac{3}{10}$ 3) $\frac{3}{4}$ $\frac{3}{5}$

Choose your solution for either 1, 2 or 3 and explain your answer.

(c) Adding Fractions

1) $\frac{1}{4} + \frac{3}{4} =$ 2) $\frac{5}{8} + \frac{1}{2} =$ 3) $\frac{1}{3} + \frac{1}{4} =$

Choose your solution for either 1, 2 or 3 and explain your answer in words and/or a diagram.

(d) Subtracting Fractions

1) $\frac{7}{10} - \frac{1}{10} =$ 2) $\frac{2}{3} - \frac{1}{2} =$ 3) $\frac{13}{15} - \frac{2}{5} =$

Choose your solution for either 1, 2 or 3 and explain your answer in words and/or a diagram.

Section 2: Word Problems

Answer the following word problems. Try to make use of your kit. Be sure to show all your work and use pictures, numbers and words to help explain your answer.

1. For each statement, decide if it can best be described as "exactly half," "about half," "less than half," or "more than half." (Burns, 2001, p. 150, 151)

   (a) Sally blocked 5 field goals out of 9 attempts.

   (b) Twenty- five students in the class have pets. Twelve of them have dogs. Nine have cats. Six have fish.

2. Jane saved two-thirds of a chocolate bar yesterday and one quarter of the same kind of chocolate bar that she got today. How much chocolate has she saved? (adapted from Kamii & Warrington, 1999, p. 87)

3. Paul has ¾ of a piece of a chocolate bar. He eats one ½ of the piece. How much does he have left? (adapted from Van de Walle & Folk, 2005, p. 245)

4. There is 7/8 of a pie left at a bake sale. Someone buys ½ of the leftover pie. How much is now left? (adapted from Van de Walle & Folk, 2005, p. 246)

5. Suzy, her father, and her mother divided a chocolate bar equally among themselves. Suzy then gave half of her share to a friend who came over. Suzy's mother decided to give her share (the mother's 1/3) to Suzy. How much chocolate did each person get? (Kamii & Warrington, 1999, p. 87)

6. Raquel thought about this statement: When pitching, Joe struck out 7 of 18 batters. She said that is was better to say that Joe struck out about 1/3 of the batters than to say that Joe struck out about ½ of the batters. "I think that seven-eighteenths is closer to one-third than one-half," she said. Do you agree or disagree with Raquel? Explain your reasoning. (Burns, 2001, p. 152)

7. A class is going to invite parents to come into their classroom for "Pancake Day." The pancakes are large and the students want each person to have exactly ¾ of one pancake. How many people can be in the group in relation to the number of pancakes? (adapted from Fosnot & Dolk, 2002, p. 66)

Each teacher recorded observations of students' understanding as the unit progressed. Mme Joans used a chart where she indicated the lesson that she taught and made observations about the students' responses to the concept. Teaching with the reform method, I used a journal and more specific documentations were made, not only to record information about how the students were grasping the concepts, but to help facilitate the development of the lessons in the unit.

In the reform teaching class, two other forms of data collections were utilized. Selections of the students' work were collected and/or photocopied. Once the students completed their "Bell Work", their books were collected and that section was photocopied. Some of the students' fraction duo-tangs were collected at the end of the unit to provide examples of various ways to solve the word problems or work through the activities. Throughout the unit, various student pairs or groups were tape recorded in order for me to take a closer look at how the students were developing an understanding of fractions. The tapes were listened to the same evening and summarized.

3.2.6 *Data Analysis Procedures*

In both classes, each student was assigned a number. The pre-test and the post-test were each coded with the class (Class A for the reform class and Class B for the traditional class) and the student number that was assigned to them. The students' tests were labeled once the tests were handed in so that the students themselves were not aware of what number they were assigned for this study. Each test was marked according to the student responses. The computation section was scored as 1 for a correct answer and 0 for an incorrect answer. In the post-test, the students had to explain their answer to one computation question in each section. The responses were evaluated and placed into categories of solution strategies. An example of a category would be the student using diagrams to try to solve the problem.

The word problem section was examined more in depth. Each problem that the student completed was evaluated as 0 (incorrect), ½ (partial understanding) or 1 (full understanding). Each section was then added up to determine what the level of understanding has been reached for that particular question for each class. For example, Class A may have scored 13.5 out of a possible 30, meaning that was the level of correct/partially correct answers for that question.

The coding for the word problems was done in the same fashion as the computation section. Depending on the problem, sometimes only the errors were analyzed. In some cases, all the answers were analyzed and were split not only into categories, but also into correct and incorrect sections. If a student received a ½ on a question, then their method of solving was counted as 1 in the correct category. For example, a student may have answered a question partially correct by (1/2) and used a diagram to solve their answer. This would then be charted as 1 in the diagram section under correct, indicating that the student used a diagram to help them solve the problem and they solved the problem correctly.

Once everything was coded and charted, the problems were examined to investigate the most common or least common strategies employed to work through a question. Sometimes the patterns were quite obvious while at other times there were very few or no patterns at all.

The other data sources were used minimally. The journals were used to examine the teacher's observations on students' understanding. The journal used in the reform teaching class was also used to help guide the lessons for the unit. The data sources are summarized in Figure 9.

Figure 9. *Summary of Data Sources*

Data Source	Analysis Used
Journal – Reform Teaching Class	Used to develop unit plan and record students' understanding of concepts
Journal – Traditional Teaching Class	Observations about students' response to concepts
Computations in Pre and Post Test	Evaluated 1 for correct and 0 for incorrect
Word Problems in Pre and Post Test	Evaluated 1 for correct, ½ for partially correct and 0 for incorrect
Written Responses	Coded for method used For example: Diagram
Photocopied Work – Reform Teaching Class	Examples of the way problems were solved and to further investigate students' understanding
Tape Recordings	Summarized to help develop unit and analyze students' thinking processes

Chapter 4: Results and Analysis

Each pre-test and post-test question was compared to examine students' achievement in each class. A comparison was then made between each of the two classes and each section, computations and word problems were analyzed separately. The classes are referred to as Class A and Class B. Class A is the class that was taught using the reform method and Class B is the class that was taught using the traditional method.

4.1 Addition of Fractions

4.1.1 Computation Results

Both classes showed very similar growth in the computation area for addition of fractions (Figure 10). In the pre-test, over 50 % of the students had errors in the addition questions. The majority of students with errors added the numerators together and then the denominators together to achieve an answer. An example can be seen in Figure 11.

Figure 10. *Addition Computation Questions from Pre-Test and Post-Test*

Pre-Test Computations

(c) Adding Fractions

1) $\frac{3}{5} + \frac{1}{5} =$ 2) $\frac{7}{10} + \frac{1}{2} =$ 3) $\frac{1}{15} + \frac{1}{12} =$

Post-Test Computations

(c) Adding Fractions

1) $\frac{1}{4} + \frac{3}{4} =$ 2) $\frac{5}{8} + \frac{1}{2} =$ 3) $\frac{1}{3} + \frac{1}{4} =$

Choose your solution for either 1, 2 or 3 and explain your answer in words and/or a diagram.

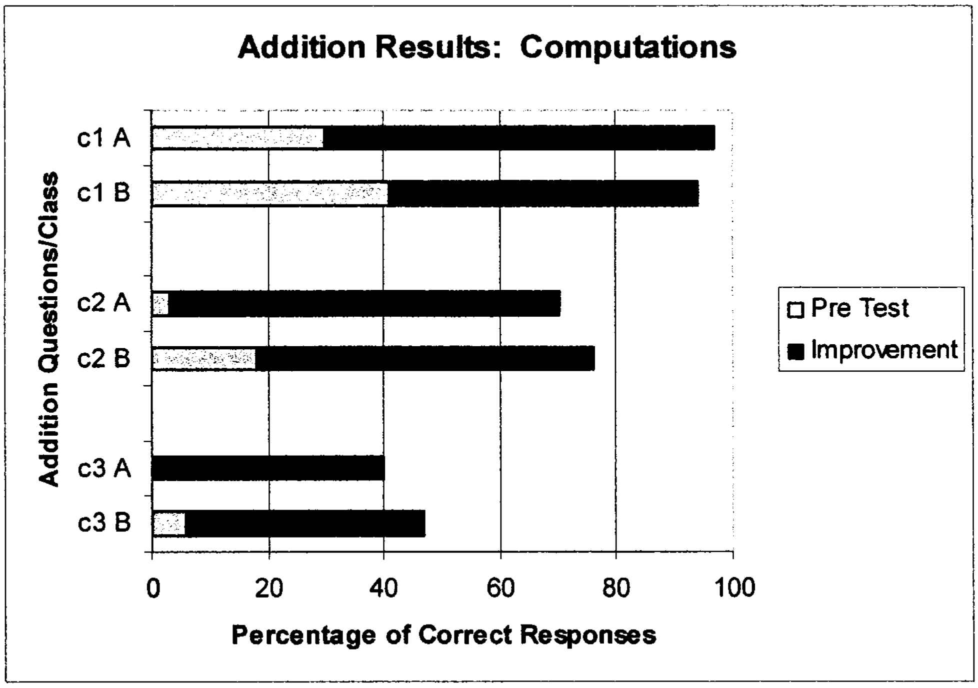
Figure 11. *Example of Student Error: Adding the numerators and denominators together*

$$\frac{3}{5} + \frac{1}{5} = \frac{4}{10}$$

Even when the denominators were the same, as in Figure 11, students followed this method, regardless of the fact that they had two years of learning how to add fractions with common denominators. A few students in Class A ended up with an answer of a whole number or left the question blank altogether (see Appendix E for further details.)

The improvement of both classes from the pre-test to the post-test was quite comparable (see Figure 12). In both classes, over fifty percent more students answered c1 and c2 correctly. Students showed the least amount of improvement (approximately forty percent more of the class) for question c3 which contained fractions with two uncommon denominators.

Figure 12. *Addition Results from Computation section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



Though the growth was similar, the methods that students used to solve the addition computations in the post-test were quite different. Of the students who answered the questions in Class A incorrectly, some of them left the questions blank, some added the numerator and kept the largest denominator as in Figure 13, and some incorrectly used equivalent fractions as in Figure 14. In Figure 14, the student used their fraction kit to make an equivalent fraction by lining up the pieces; however, the answer was not accurate. There were times when it was difficult to decipher what method the students were using.

Figure 13. *Added the numerator and kept the largest denominator (Class A, Number15)*

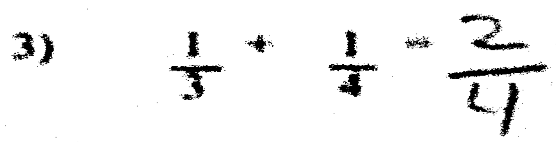
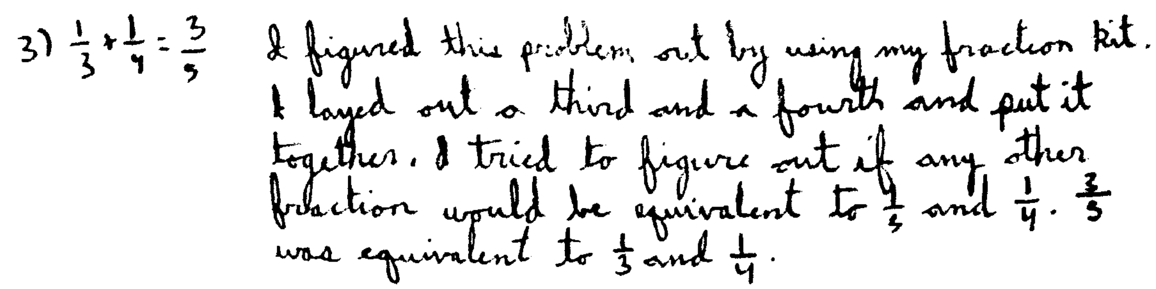
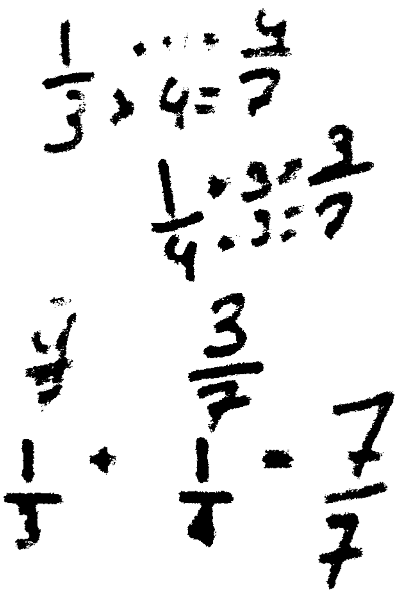


Figure 14. *Incorrect use of equivalent fractions (Class A, Number 20)*



The students who answered the post-test questions in Class B incorrectly used very different methods to try and solve the questions. Of the students that answered the questions incorrectly, the majority tried to solve the question by using a faulty procedure as in Figure 15 (refer to Appendix E for specific details).

Figure 15. *Faulty Procedure for adding fractions (Class B, Number 1)*



4.1.2 Word Problem Results

In the word problems section (Figure 16), many students solved the pre-test word problem in the same way that they added fractions in the computation section. The majority of students who answered the problem incorrectly added the numerator and the denominator together (see Figure 11).

Figure 16. *Word Problems from Pre-Test and Post-Test*

Pre-Test Word Problem

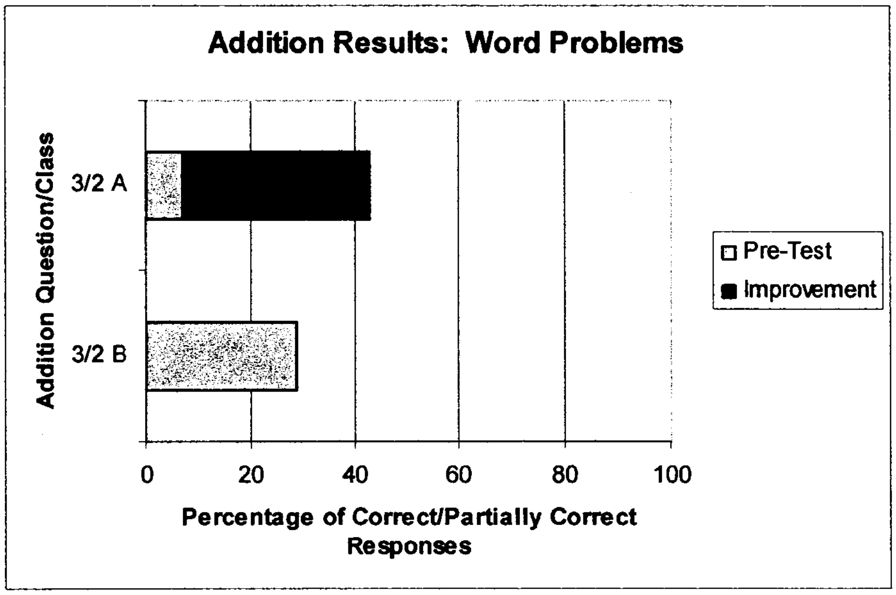
3. Pat saved half a chocolate bar yesterday and a third of the same kind of chocolate bar that she got today. How much chocolate has she saved? (Kamii & Warrington, 1999, p. 87).

Post-Test Word Problem

2. Jane saved two-thirds of a chocolate bar yesterday and one quarter of the same kind of chocolate bar that she got today. How much chocolate has she saved? (adapted from Kamii & Warrington, 1999, p. 87).

Even though the growth in the computation section for both classes was quite comparable, the word problem section depicted a gap (see Figure 17). In Class A, 43% of the answers generated were correct or partially correct, which was 36% more than in the pre-test, whereas in Class B, there was no improvement. In the post-test, 33% of the students in Class A used their fraction kits to solve the addition problem (of those 33%, more than two-thirds answered the question correctly) and approximately 17% of the students tried to solve the questions by making equivalent fractions. In Class B, 41% of the students used an algorithm to solve the problem and of those students, just over half of them used the algorithm correctly. Thirty-five percent of the students attempted to use a diagram to solve the problem, however, not one of these students solved the problem correctly.

Figure 17. *Addition Results from Word Problem section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



4.2 Subtraction of Fractions

4.2.1 Computation Results

The improvement of both classes varied slightly in the computation section (Figure 18). Both classes showed an increase from the pre-test to the post-test in all the questions. Class A showed a bigger increase (57% more of the class) than Class B (23% more of the class) for question d2 (see Figure 19). The most common error shown in the pre-test for the subtraction of fractions was subtracting both the numerator from the numerator and the denominator from the denominator as seen in Figure 20 (see Appendix F for further details).

Figure 18. *Subtraction Computation Questions from Pre-Test and Post-Test*

Pre-Test Computations

(d) Subtracting Fractions

1) $\frac{9}{10} - \frac{3}{10} =$ 2) $\frac{1}{2} - \frac{1}{3} =$ 3) $\frac{3}{10} - \frac{1}{5} =$

Post-Test Computations

(d) Subtracting Fractions

1) $\frac{7}{10} - \frac{1}{10} =$ 2) $\frac{2}{3} - \frac{1}{2} =$ 3) $\frac{13}{15} - \frac{2}{5} =$

Choose your solution for either 1, 2, or 3 and explain your answer in words and/or a diagram.

Figure 19. *Subtraction Results from Computation section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*

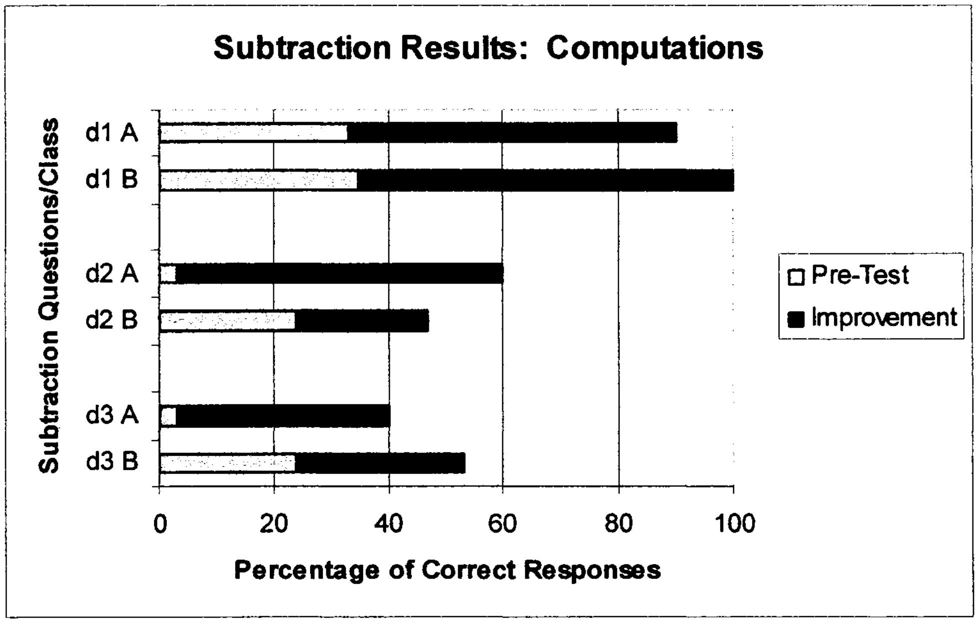
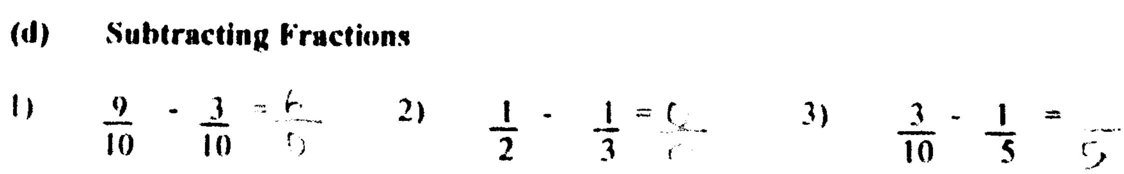


Figure 20. *Subtracting the numerator from the numerator and the denominator from the denominator (Class A, Number 3)*



The other errors that occurred in the pre-test did not fall into a particular pattern. Some students subtracted the numerators and kept the larger denominators (Figure 21) while other students subtracted the numerators and added the denominators (Figure 22).

Figure 21. *Subtracting the numerators and keeping the larger denominator (Class A, Number 11)*

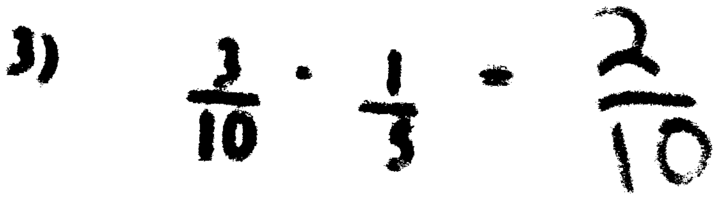
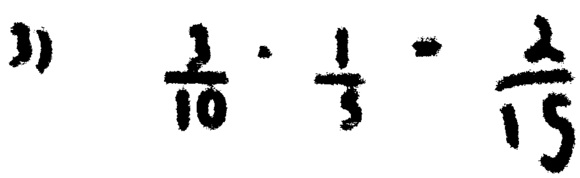


Figure 22. *Subtracting the numerators and adding the denominators (Class A, Number 4)*



In the post-test analysis it was very difficult to determine a specific pattern that students followed. In Class A there were no set patterns of errors and in Class B the biggest error was students used faulty procedures to solve their answers (d2 – 18%; d3 – 24%).

4.2.2 Word Problem Results for Subtraction/Division Questions

The questions from the pre-test to the post-test varied in difficulty (Figure 23). A comparison can be made but it is not as accurate as it would have been if the questions had been exactly the same. The results from the pre-test and the post-test showed a decrease in achievement for Class B in both questions (see Figure 24). It would seem that the wording for the questions had an impact on how the students answered them. For example, many of the students treated question 4 in the pre-test as a simple subtraction problem as opposed to a division problem. Thirteen percent of the students in Class A and 29% of the students in Class B stated that ½ was the correct answer (see Figure 25). The students took half away from the whole or 8/8 to end up with 4/8 or they simply wrote down ½. The subtraction that was often done was done inaccurately. The post-test showed a similar pattern for Question 3 (same question as the pre-test) in that the students treated the question as simply a subtraction computation from the whole bar. Fifty-seven percent of students in Class A and fifty-three percent of students in Class B stated incorrectly that the answer was 3/8 (see Figure 26). They changed ½ to 4/8 and subtracted this from 7/8 to end up with 3/8.

Figure 23. *Subtraction/Division Word Problems from Pre-Test and Post-Test*

Pre-Test Problems

4. Paul has 7/8 of a piece of a chocolate bar. He eats one ½ of it. How much does he have left? (adapted from Van de Walle & Folk, 2005, p. 245)

5. There is 3/10 of a pie left at a bake sale. Someone buys 1/5 of the pie. How much is left? (adapted from Van de Walle & Folk, 2005, p. 246)

Post-Test Problems

3. Paul has ¾ of a piece of a chocolate bar. He eats one ½ of the piece. How much does he have left? (adapted from Van de Walle & Folk, 2005, p. 245)

4. There is 7/8 of a pie left at a bake sale. Someone buys ½ of the leftover pie. How much is now left? (adapted from Van de Walle & Folk, 2005, p. 246)

Figure 24. *Subtraction/Division results from the Word Problem section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*

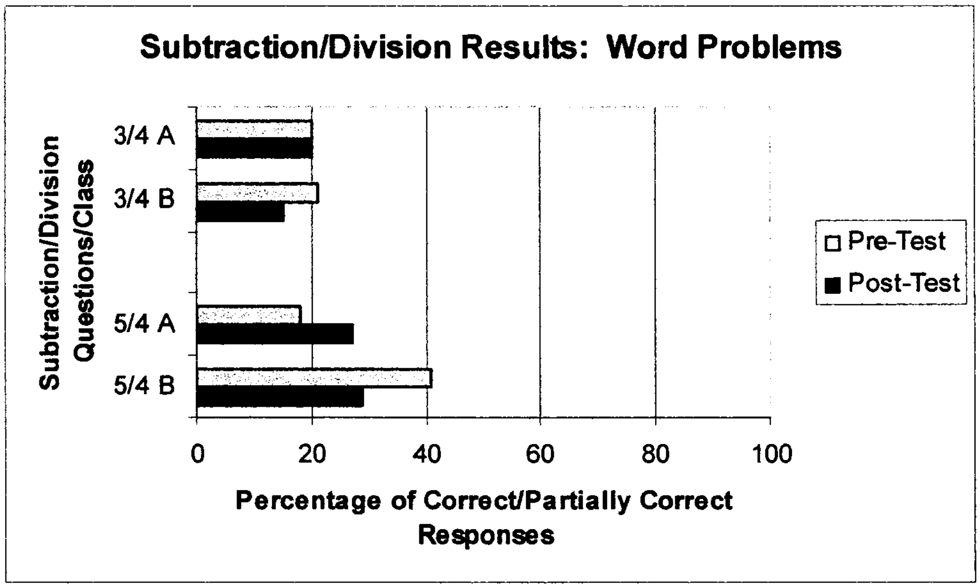


Figure 25. *Students answered question as being ½ or 4/8 in pre-test (Class A, Number 24)*

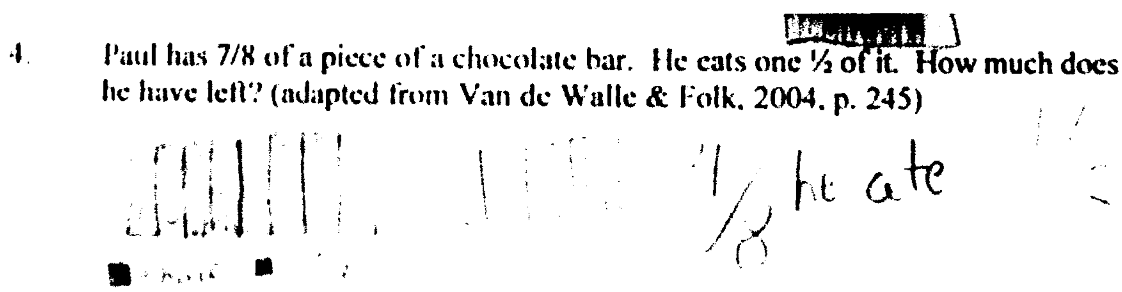

[redacted]
4. Paul has 7/8 of a piece of a chocolate bar. He eats one ½ of it. How much does he have left? (adapted from Van de Walle & Folk, 2004, p. 245)

Figure 26. *Students answered question as being 3/8 in post-test (Class B, Number 1)*

4. **There is 7/8 of a pie left at a bake sale. Someone buys ½ of the leftover pie. How much is now left? (adapted from Van de Walle & Folk, 2004, p. 246)**

The second comparison was made between a subtraction question (question 4 in the pre-test) and a division question (question 5 in the post-test). The post-test question was much more difficult than the pre-test question but Class A showed an improvement of 9% while Class B showed a decrease of 12%.

4.3 Addition/Subtraction of Fractions

4.3.1 Word Problem Results

Students in both classes had difficulty with this problem in the pre-test and the post-test (Figure 27). In the pre-test, only 13% of the answers generated in Class A and 21% of the answers generated in Class B were correct or partially correct. There was no specific pattern as to how the students attempted to solve the problem. The most often used strategy was picture drawing. In Class A, 27% of the students used this strategy and in Class B, 36% of the students used this strategy to try and solve the problem.

Figure 27. *Addition/Subtraction Word Problem from Pre-Test and Post-Test*

Pre-Test and Post-Test Question
Suzy, her father, and her mother divided a chocolate bar equally among themselves. Suzy then gave half of her share to a friend who came over. Suzy's mother decided to give her share (the mother's 1/3) to Suzy. How much chocolate did each person get? (Kamii & Warrington, 1999, p. 87).

There was a large difference in improvements between the two classes (see Figure 28). In Class A, there was a 39% of improvement while there was a only a 7% improvement in class B. In Class A, 10% of the class used their fraction kits to try to solve the problem. Another common strategy was for students to use a common denominator of 6, (Class A – 20%; Class B – 29%), as in Figure 29 and 30. (See Appendix G for more information).

Figure 28. *Addition/Subtraction results from Word Problem section in Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*

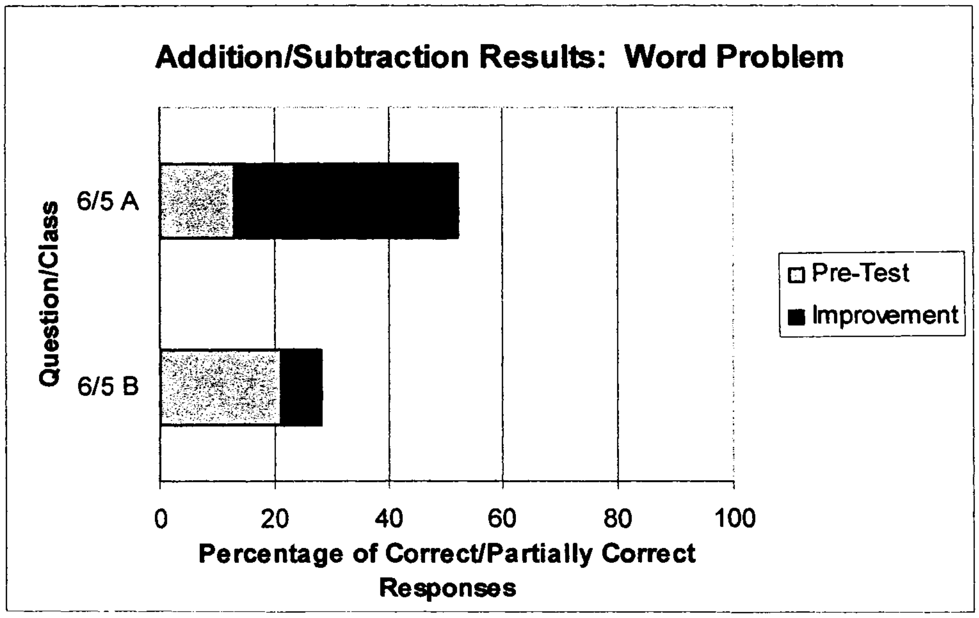


Figure 29. *Common Denominator of 6 (Class A, Number 23)*

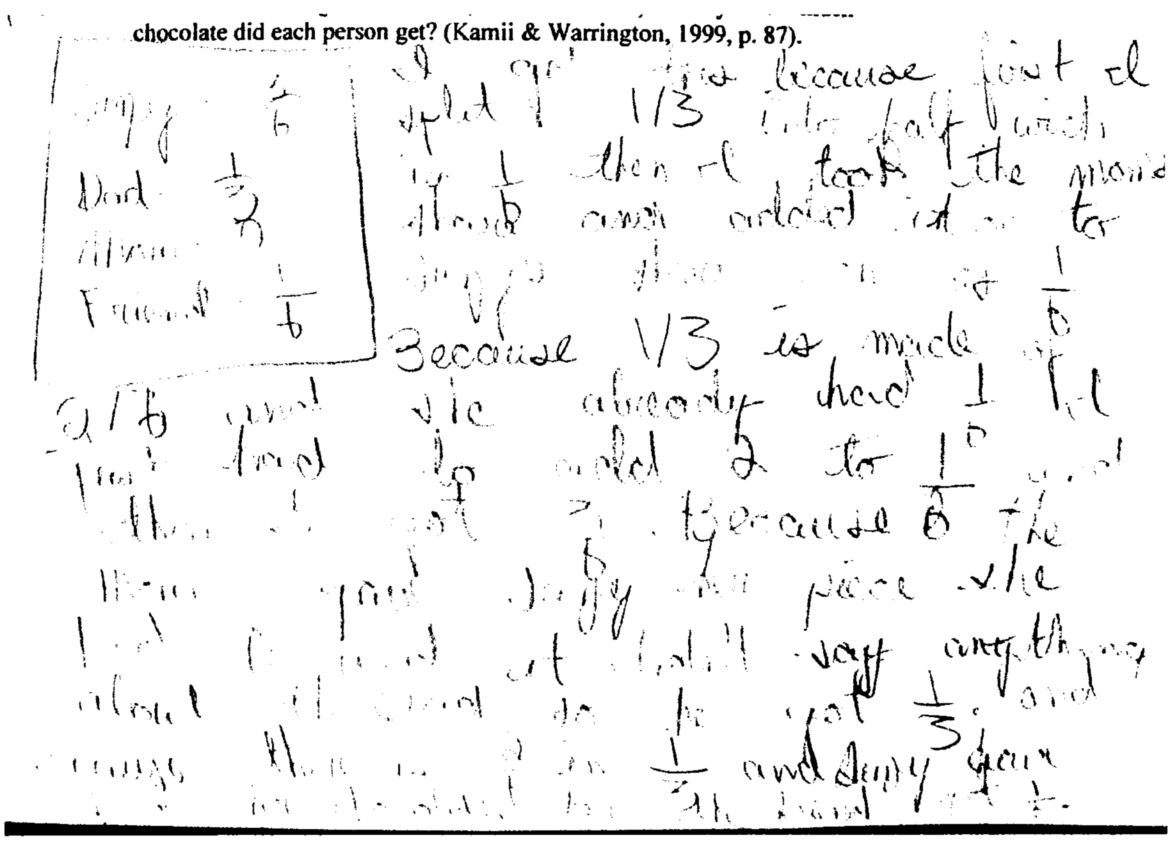
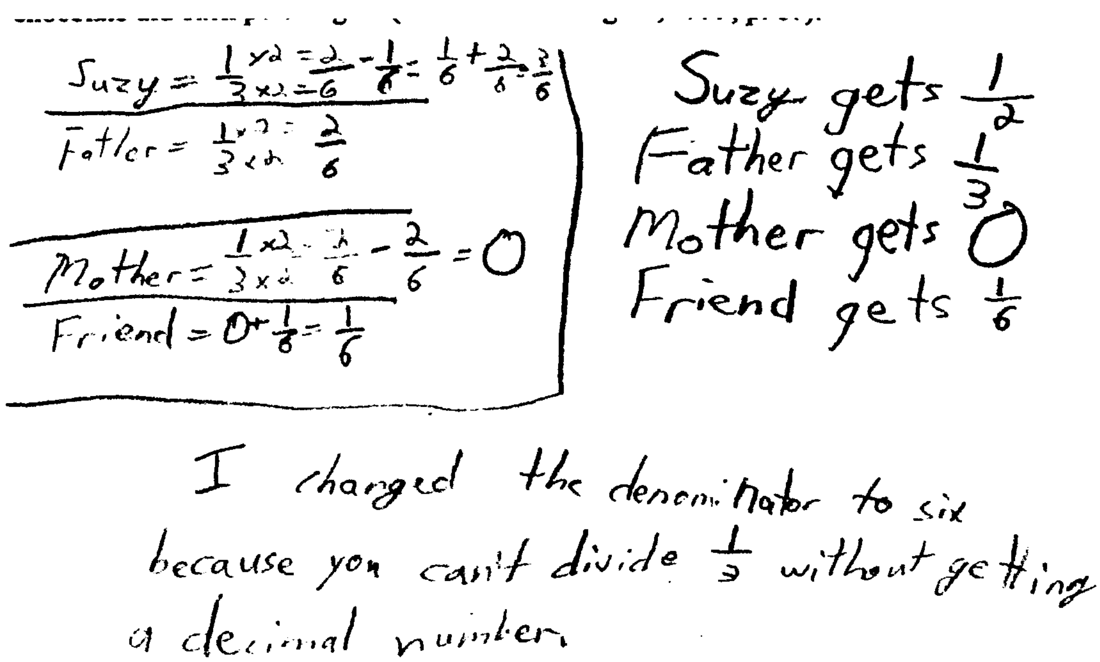

chocolate did each person get? (Kamii & Warrington, 1999, p. 87).

Figure 30. *Common Denominator of 6 (Class B, Number 8)*


Suzy = $\frac{1\times2}{3\times2} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

Father = $\frac{1\times2}{3\times2} = \frac{2}{6}$

Mother = $\frac{1\times2}{3\times2} = \frac{2}{6} - \frac{2}{6} = 0$

Friend = $0 + \frac{1}{6} = \frac{1}{6}$

Suzy gets $\frac{1}{2}$

Father gets $\frac{1}{3}$

Mother gets 0

Friend gets $\frac{1}{6}$

I changed the denominator to six because you can't divide $\frac{1}{3}$ without getting a decimal number.

4.4 Equivalent Fractions

4.4.1 Computation Results

Both classes showed a similar improvement between the pre-test and the post-test (Figure 31) for questions a2 and a3. The biggest improvement from Class A was with question a1 where 60% more of the class answered the question correctly (see Figure 32). Class B answered all three equivalent questions with a high degree of accuracy in the pre-test and the post-test. Over 90% of the class in the post-test was able to answer each of these three questions correctly. It was difficult to pinpoint particular strategies in the pre-test because the students were not required to give any explanation as to how they solved the questions.

Figure 31. *Equivalent Computation Questions from Pre-Test and Post-Test*

Pre-Test Computations

(a) Equivalent Fractions

1) $\frac{2}{5} = \frac{\_}{10}$ 2) $\frac{3}{4} = \frac{\_}{12}$ 3) $\frac{4}{5} =$

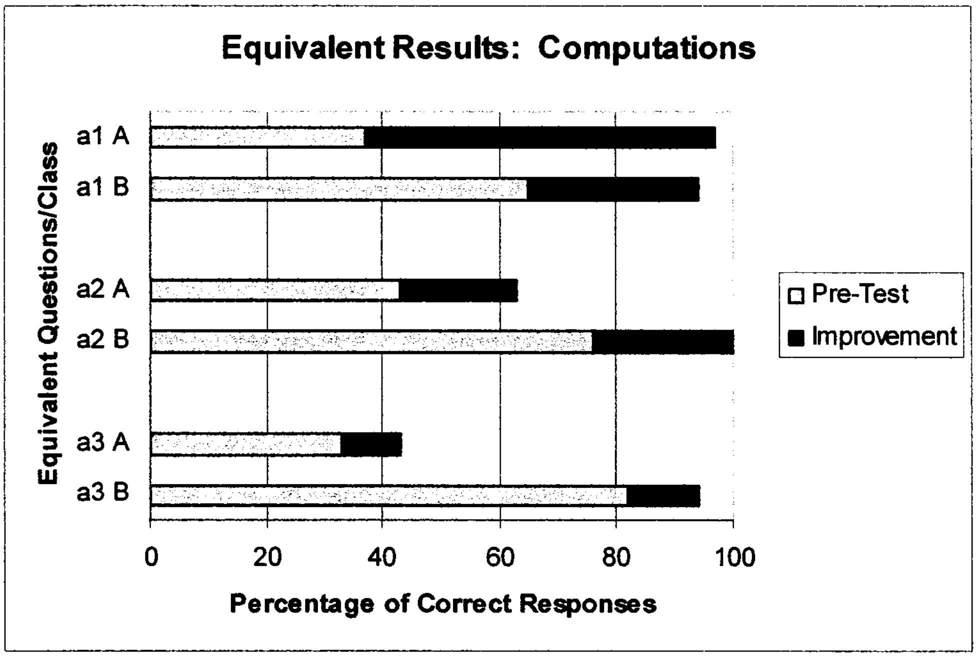
Post-Test Computations

(a) Equivalent Fractions

1) $\frac{3}{5} = \frac{\_}{10}$ 2) $\frac{2}{3} = \frac{\_}{9}$ 3) $\frac{5}{12} =$

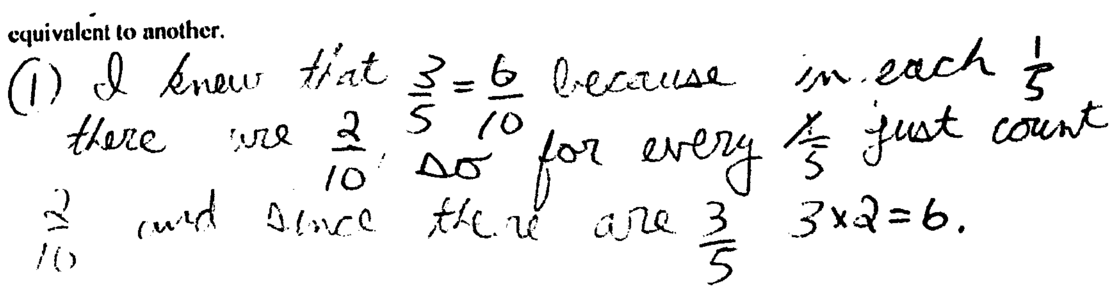
Choose your solution for either 1, 2 or 3 and explain how you know that one fraction is equivalent to another.

Figure 32. *Equivalent Results from Computation section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



In the post-test, however, there was a big difference between both classes as to how the students chose to explain their solutions to the problems. Class A used a variety of different strategies (see Appendix H for more details). The strategy that was used most often (37%) was the comparison of the fraction to a unit fraction in order to find what the equivalent fraction would be (Figure 33).

Figure 33. *Comparison to Unit Fraction (Class A, Number 27)*


equivalent to another.

(1) I know that $\frac{3}{5} = \frac{6}{10}$ because in each $\frac{1}{5}$ there are $\frac{2}{10}$, so for every $\frac{1}{5}$ just count $\frac{2}{10}$ and since there are $\frac{3}{5}$ $3 \times 2 = 6$.

The most common strategy for Class B was to use multiplication to find the equivalent fraction (the algorithm for finding equivalent fractions) and 82% of the class explained their solution in this way (see Figure 34).

Figure 34. *Algorithm for Equivalent Fractions (Class B, Number 4)*

Choose your solution for either 1, 2 or 3 and explain how you know that one fraction is equivalent to another.

For number 2, the fractions are equivalent because 3 × 3 = 9 and what you do to the denominator, you must do to the numarator, so if I did 3×3, I would also have to do 2×3, which = 6.

4.4.2. Word Problem Results

The results from both classes were similar, resulting in limited improvement between the pre-test and the post-test (Figure 35). The biggest growth was found in question 7 where 25% more of the answers generated in Class A were correct or partially correct (see Figure 36). Class A and Class B showed a decline in question 1b (Class A decline 1% and Class B declined 10%), however, these questions were not identical. The students were given a mark of 1 for a correct response or a ½ mark (partially correct) if they came up with a statement that qualified for one of the fraction parts (for example, 12 out of 25 is almost half). Originally, for question 1b, the question stated that six students had fish. The students were told to change this number to four so that there would be an equal number of animals to students in the class. This resulted in a confusing problem. There was no clear pattern or method that the students used to solve these particular problems.

Figure 35. *Equivalent Word Problem Questions from Pre-Test and Post-Test*

Pre-Test Question

1. For each statement, decide if it can best be described as "exactly half," "about half," "less than half," or "more than half." (Burns, 2001, p. 150)

(a) Maria received 13 birthday cards. Five of them arrived the day after her birthday.

(b) Fifty-five students signed up to work on the school paper. Twenty-seven of them were girls.

Post-Test Question

1. For each statement, decide if it can best be described as "exactly half," "about half," "less than half," or "more than half." (Burns, 2001, p. 150, 151)

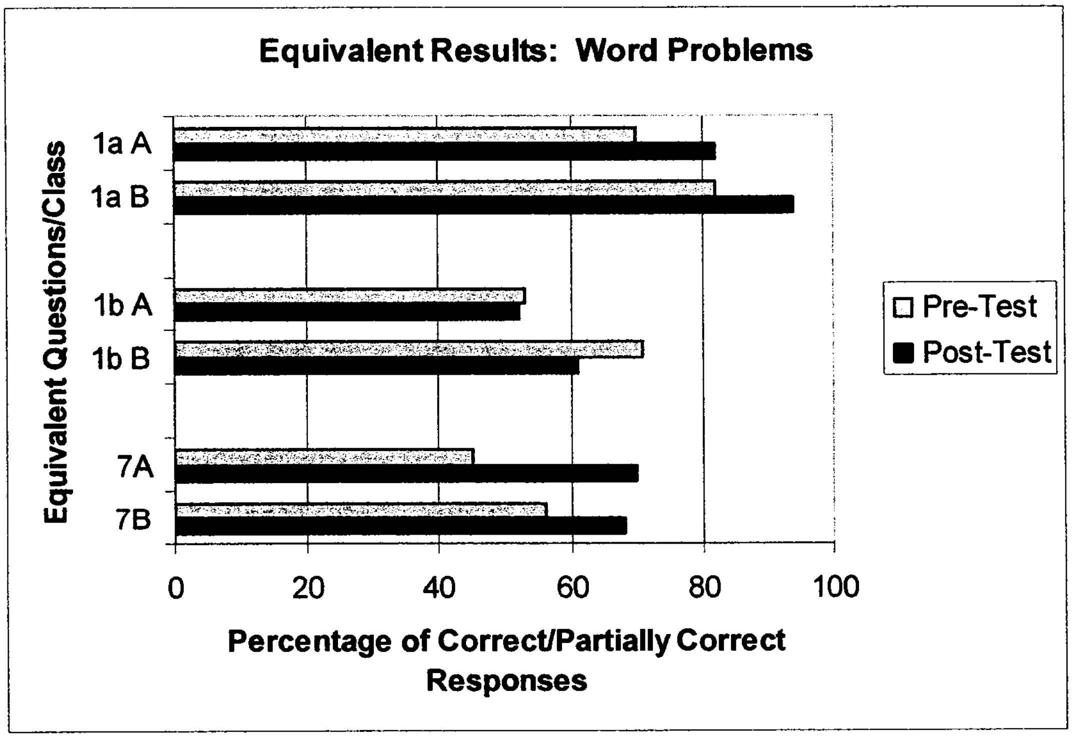
   (a) Sally blocked 5 field goals out of 9 attempts.

   (b) Twenty- five students in the class have pets. Twelve of them have dogs. Nine have cats. Four have fish. (question adapted)

Pre-Test and Post-Test Question

7. Raquel thought about this statement: When pitching, Joe struck out 7 of 18 batters. She said that is was better to say that Joe struck out about 1/3 of the batters than to say that Joe struck out about ½ of the batters. "I think that seven-eighteenths is closer to one-third than one-half," she said. Do you agree or disagree with Raquel? Explain your reasoning. (Burns, 2001, p. 152)

Figure 36. *Equivalent Results from Word Problem section in the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



4.5 Comparison of Fractions

4.5.1 Computation Results

The student improvement from both classes in the pre-test to the post-test (Figure 37) was very similar in both classes. Each class had another twenty or more percent of the class (except for Class B on question b2) answer each question correctly (see Figure 38).

Figure 37. *Comparison Computation Questions from Pre-Test and Post-Test*

Pre-Test Computations

(b) Comparing Fractions

Indicate whether each fraction is <, >, or =.

1) $\frac{3}{8}$ $\frac{1}{5}$ 2) $\frac{1}{2}$ $\frac{9}{12}$ 3) $\frac{4}{5}$ $\frac{7}{15}$

Post-Test Computations

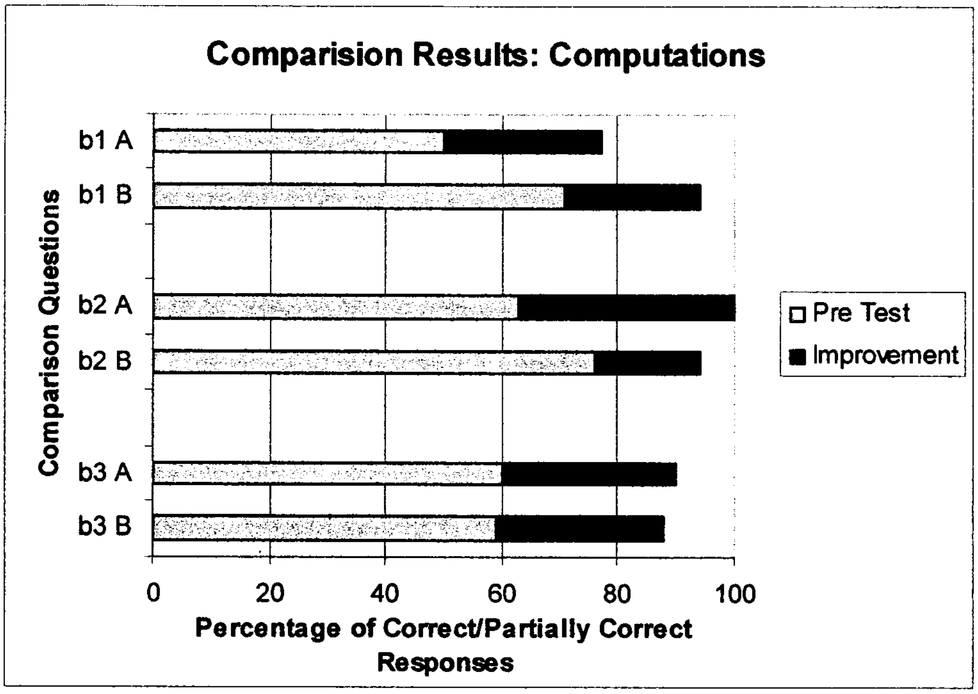
(b) Comparing Fractions

Indicate whether each fraction is <, >, or =.

1) $\frac{3}{7}$ $\frac{1}{5}$ 2) $\frac{1}{2}$ $\frac{3}{10}$ 3) $\frac{3}{4}$ $\frac{3}{5}$

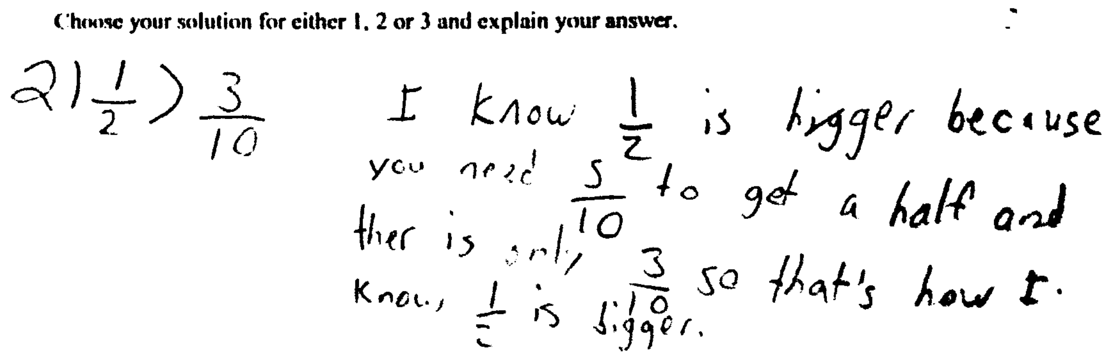
Choose your solution for either 1, 2 or 3 and explain your answer.

Figure 38. *Comparison Results from Computation section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



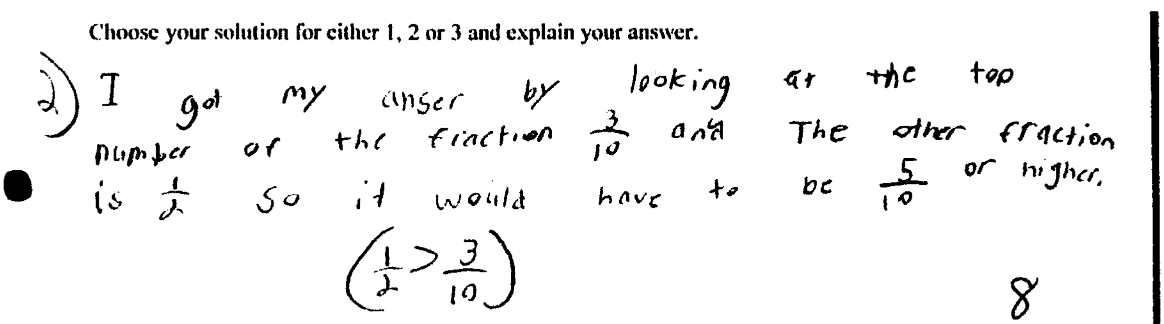
In the pre-test portion of the assessment, it was very difficult to determine common errors. There was no definitive pattern in the thinking processes behind each question. In the post-test, however, students had to pick one of the questions and explain how they came up with their answer. This is where there were notable differences as to how the students answered this question (see Appendix I for complete analysis). In Class A, 47% of the students used Benchmarks to explain their answer. Benchmarks are when students use 0, ½ or 1 to help compare two fractions. For example, the students compared ½ to being the same as 5/10. They then knew that 5/10 was bigger than 3/10 (see Figure 39 and 40).

Figure 39. *Use of Benchmarks to answer the question (Class A, Number 15)*


Choose your solution for either 1, 2 or 3 and explain your answer.

2) $\frac{1}{2} > \frac{3}{10}$ I know $\frac{1}{2}$ is bigger because you need $\frac{5}{10}$ to get a half and ther is only $\frac{3}{10}$ so that's how I know $\frac{1}{2}$ is bigger.

Figure 40. *Use of Benchmarks to answer the question (Class A, Number 4)*


Choose your solution for either 1, 2 or 3 and explain your answer.

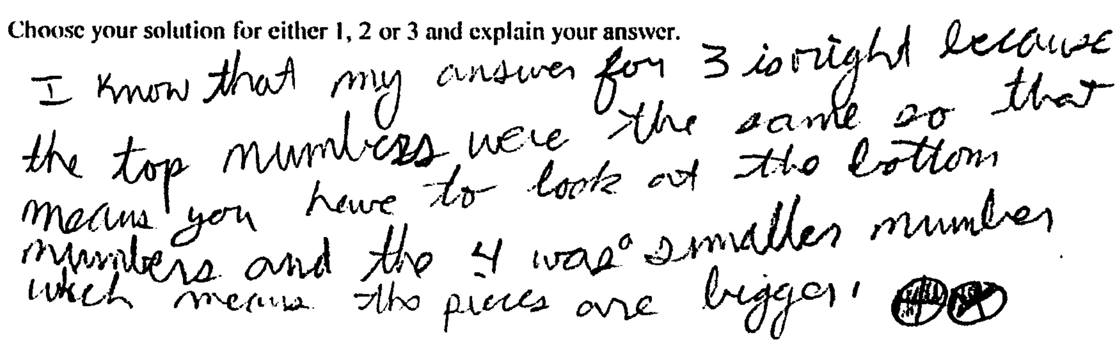
2) I got my anser by looking at the top number of the fraction $\frac{3}{10}$ and The other fraction is $\frac{1}{2}$ so it would have to be $\frac{5}{10}$ or higher.

$\left(\frac{1}{2} > \frac{3}{10}\right)$

8

Another explanation that Class A used was to refer to the numerator. In question b3, both of the numerators are 3. The students then looked at the denominators to determine which fraction would be larger (see Figure 41).

Figure 41. *Comparison of the denominators (Class A, Number 6)*


Choose your solution for either 1, 2 or 3 and explain your answer.

I know that my answer for 3 is right because the top numbers were the same so that means you have to look at the bottom numbers and the 4 was a smaller number wich means the pieces are bigger.

Almost fifty percent of Class B's explanations discussed making the denominators the same and then making a comparison. Many of the students showed how they would change ½ to 5/10 by multiplying the numerator and the denominator by 5 (see Figure 42 and 43).

Figure 42. *Algorithm for comparing fractions (Class B, Number 4)*

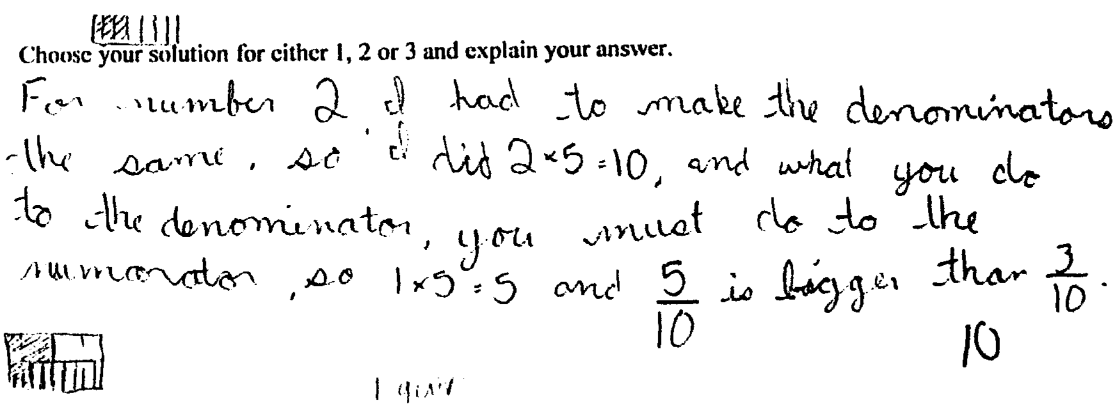
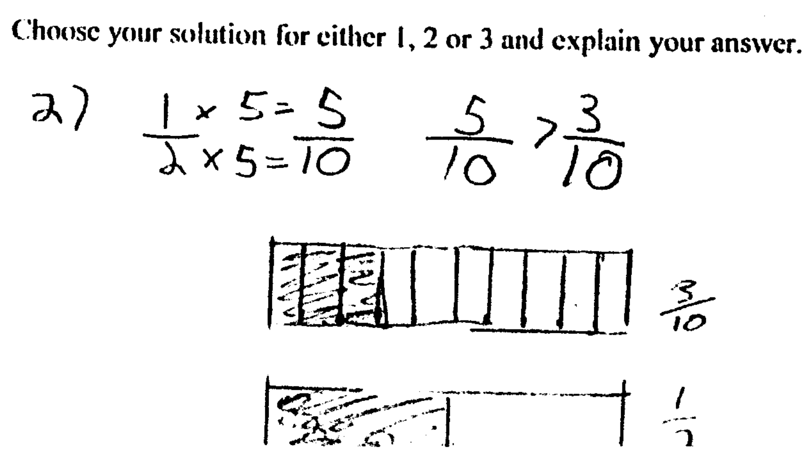


Figure 43. *Algorithm for comparing fractions (Class B, Number 8)*



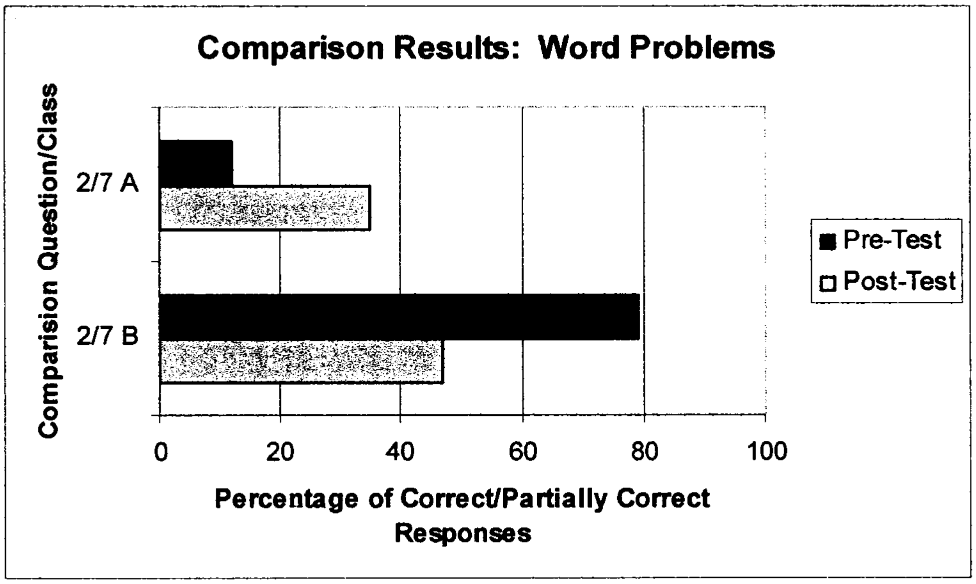
4.5.2 Word Problem Results

The results were very different for both classes between the two tests (Figure 44). Class B was much farther ahead in their ability to solve this question than Class A before any instruction was given. In the pre-test, 79% of the answers in Class B were correct or partially correct while only 12% of the answers generated in Class A answered were correct or partially correct (see Figure 45). That is almost a 60% difference between the two classes. In Class B, 53% of the class used a diagram to help them try to answer the question in the pre-test while only 20% of the students in Class A attempted to use a diagram to try to solve the problem (see Appendix I for further breakdown of the common strategies that were used.)

Figure 44. *Comparison Word Problem from Pre-Test and Post-Test*

Pre-Test and Post-Test Question This question was the same for the pre-test and the post-test. The only difference was the number of the question. 2/7. A class is going to invite parents to come into their classroom for "Pancake Day." The pancakes are large and the students want each person to have ¾ of one pancake. How many people can be in the group in relation to the number of pancakes? (adapted from Fosnot & Dolk, 2002, p. 66).

Figure 45. *Comparison Results from Word Problem section of the Pre-Test and Post-Test. Sample Size: Class A = 30 students, Class B = 17 students*



The post-test showed quite a dramatic result between the two classes. Class A improved, with 23% more of the answers generated were correct or partially correct, while Class B decreased, with 32% less of the answers generated were correct or partially correct. The use of a diagram was the most common strategy used by both classes (Class A – 50%; Class B – 65%) as seen in Figure 46 and 47.

Figure 46. *Diagram used to solve comparison word problem (Class A, Number 3)*

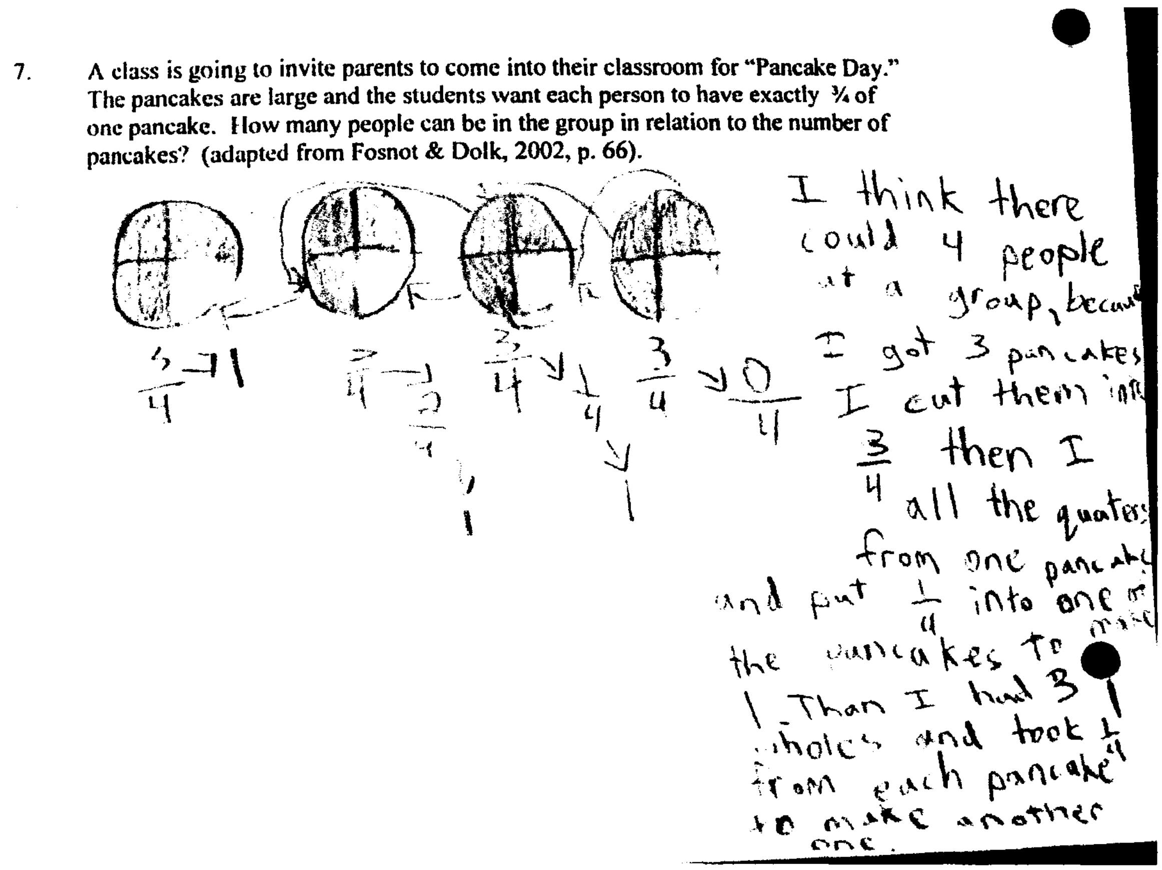
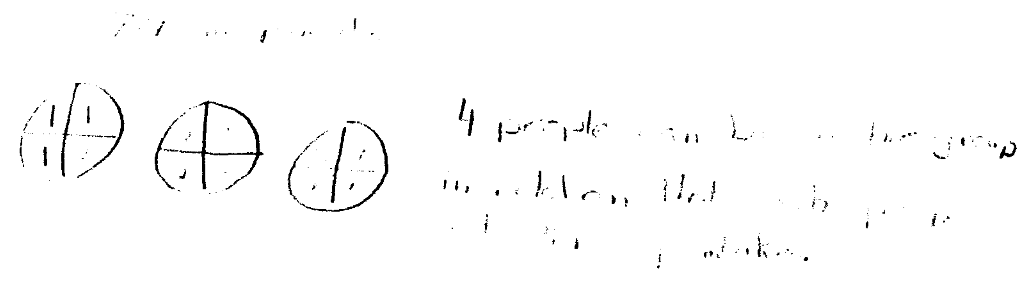


Figure 47. *Diagram used to solve comparison word problem (Class B, Number 19)*

7. A class is going to invite parents to come into their classroom for "Pancake Day." The pancakes are large and the students want each person to have exactly ¾ of one pancake. How many people can be in the group in relation to the number of pancakes? (adapted from Fosnot & Dolk, 2002, p. 66).

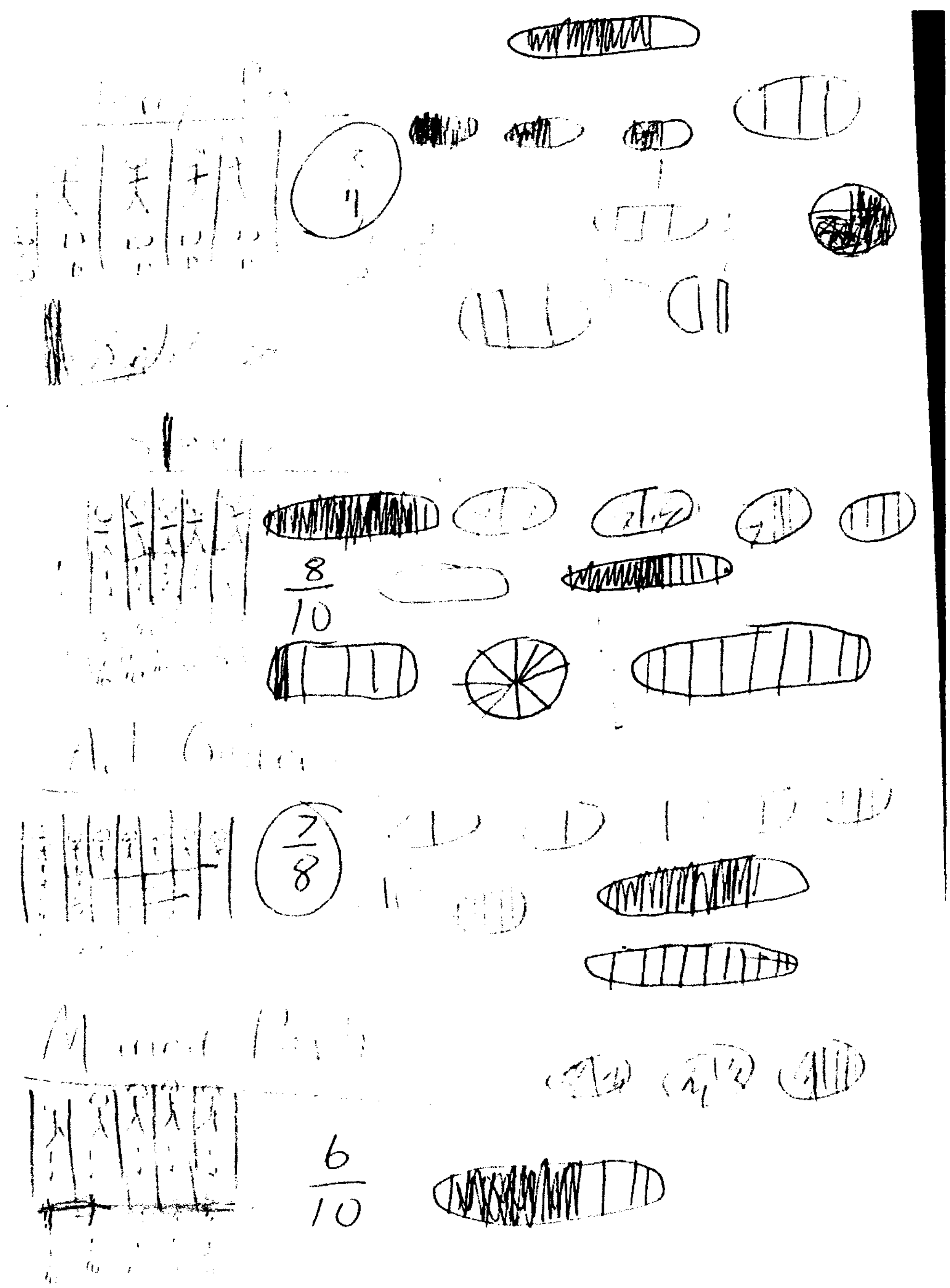


4.6 Results from Reform Teaching Method

The results from the reform teaching method varied depending on the student. Some students at the end of the unit had a solid conceptual base of fractions that could continue to be built upon over the coming years. Others needed more time to really grasp the foundation of fractions.

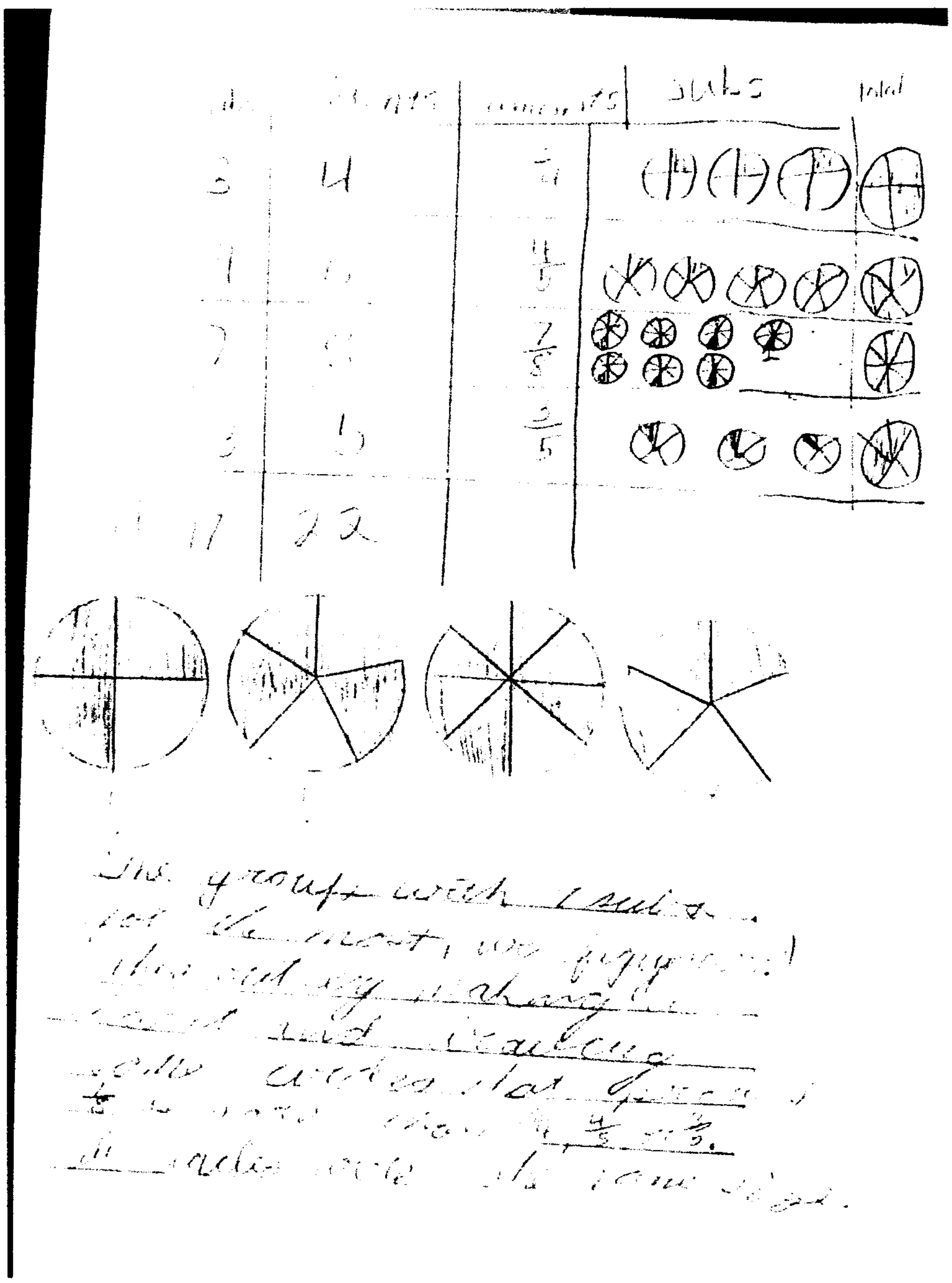
The first problem (see Figure 5) had the students immediately engaged as they came up with a variety of ways to answer how the sub sandwiches were divided up equally in each group. One strategy the students used was to divide the subs in half first and hand out an equal number of halves. The next step was to divide any leftover subs into smaller pieces so that everyone had a fair share. For example, at the Terry Fox monument, there were three subs for four students. Each group would receive ½ and ¼ of a sub (see Figure 48 for student work).

Figure 48. *Sample solution to the word problem*



Another strategy was to divide each sub by the number of people that were in the group and hand out one part of every sub to each person until all the subs were used up. This is a perfect example of partitive division. Quotative division was used as well where an entire piece of the sub was taken out and the leftover pieces put together made up one person's share. Figure 49 shows an example of how one group was able to see the pattern of how the number of people in the group and the number of subs made a fraction.

Figure 49. *Sample solution to the word problem*



This one problem alone brought up many different fraction ideas that the students had to work through such as equivalent fractions, addition of fractions and comparing of fractions. For addition of fractions, the class was asked if 7/8 + 1/8 = a whole. Everyone readily agreed that this made sense. The students were then shown what many of them did on the pre-test to solve an addition question: 7/8 + 1/8 = 8/16. The students were quite surprised to see what they did on the pre-test.

The next major part of the unit was the building and working with the fraction kit. Once the students had time to work with the pieces in the kit, they were able to interchange fractions to make equivalent fractions with ease. The students became so comfortable working with this kit that they were able to list off examples of equivalent fractions (that could be produced using the rectangular kit) with the same ease as if they were being asked to add 1+1. This enabled the students to build the foundation for finding equivalent fractions, comparing fractions and using benchmarks.

The use of the clock circle kit was not as effective for some of the students. Some of the pieces were very close in size and this gave them some difficulty when adding fractions or finding equivalent fractions. For example, fifths and sixths were sometimes confused which had some of the students using information inaccurately to solve the problem. The students needed not only to rely on the size of the pieces but to also utilize the minutes that were on the back of each of the pieces to confirm their conclusions. More time spent using the fraction pieces would have made the clock fraction kit a more useful and comfortable tool for the students to use.

In their work with equivalent fractions, the students had to find equivalent fractions to five of the fractions that were written on the board. Students really had to work through some of these questions. When the class was taking up the answers on the board one student was explaining that ¼ = 3/12 because 4 x 3 is 12 and 1 x 3 is 12. When the student was prompted to explain why this worked or how it made sense, the student was not able to explain. This led the class into a different direction and students were trying to explain and figure out why this procedure worked.

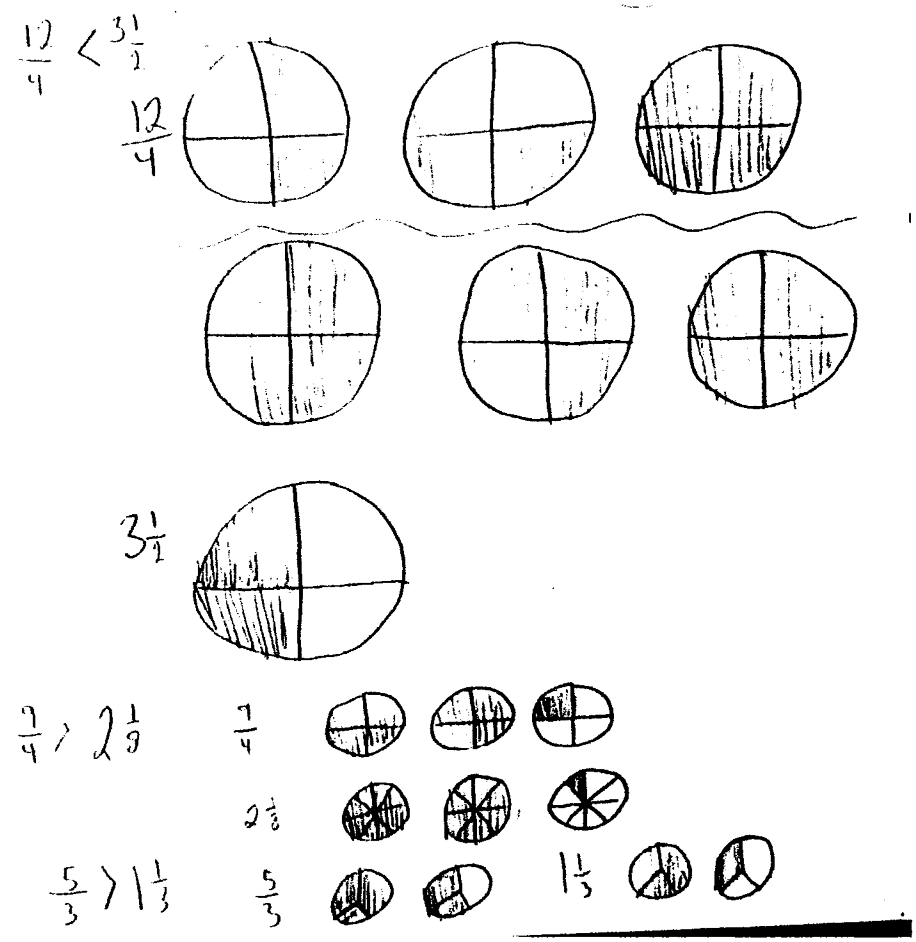
Many of the equivalent fractions that were written on the board for this activity were difficult to solve because they had not yet and as a consequence, manipulatives that could have been used for some of the questions were unavailable to them. This discussion would have been more beneficial to the students if they had had the time to build and use their circle kits. This does emphasize, however, that the learning of formulas does not constitute understanding. More details on this will follow in the next chapter.

When the students started to work with adding and subtracting fractions, they had both of their fraction kits built. The first addition question, Part A and B (Appendix K), that the students had to solve was very challenging. This problem was difficult for their first addition question and students struggled with how to add the fractions together even when they had their fraction kits. When they were guided to think of the minutes, this helped them to grasp the concept. As the students worked through the adding of fractions, some of them needed to be reminded and encouraged to use their kit to ensure that they were obtaining accurate answers. While they were becoming more comfortable solving the addition questions they were still having difficulty with the explanation of their answers.

Subtracting fractions along with putting fractions into the correct order were concepts that the students were able to grasp fairly easily because they were familiar with how to add fractions.

The use of improper fractions was introduced earlier than the students were ready to grasp. Even after some time when the concept was reintroduced, students struggled with having a larger numerator than denominator. For example, when asked to draw a picture to show 9/4, one interpretation of the fraction was to draw nine wholes and divide each whole into quarters. After some time and more work with fractions, the students were able to draw diagrams to determine which fraction would be larger when given an irregular fraction and a mixed number (see Figure 50). The students did not have to rely on a formula to find out their answer.

Figure 50. *Comparison of an irregular fraction and a mixed number*



Overall, the students were able to figure out answers to word problems and computation questions without having to rely on a formula for an answer. While some students still did this, the majority of the class used manipulatives or their own fraction reasoning to solve questions. This is evident in the post-test, as the students used a number of different strategies to solve the problems.

Chapter 5: Discussion and Conclusions

5.1 Summary of the Major Findings

This study was conducted to determine whether a traditional teaching method or a reform teaching method would help students develop a more conceptual understanding of fractions. The results from the pre-test and the post-test give strong indications of the benefits of teaching in the reform teaching method.

5.1.1 Faulty Algorithms

All students, before the start of this unit, were taught in a traditionally instructed class for the past two years and they demonstrated a far greater reliance on faulty algorithms than on conceptual understanding. The results of the pre-test, on its own, exemplified the reliance students developed for using poorly developed rules and procedures; despite prior fraction experience, the majority of them were unable to complete simple addition and subtraction questions accurately. These were similar findings to what Mack (1990) discovered in her research where students relied on using an inaccurate algorithm instead of determining whether the answer was reasonable. It was surprising to find that the faulty algorithm of adding the numerators together and then the denominators together was even followed for questions that had the same denominator. This alone demonstrates what little conceptual understanding students had of fractions even after they have been taught fractions in a traditional fashion for two years.

5.1.2 Algorithms and Computations

The accurate use of algorithms did not advantage the traditionally instructed class. Learning algorithms to solve fraction problems does not necessarily improve the development of computation skills. Students improved at a similar rate in both classes in the computation area from the pre-test to the post-test. Students in the reform teaching class were able to solve questions in the computation section without necessarily having to depend on an algorithm to figure out the answer.

This is not an argument that algorithms should not be used. There were a few students that used a formula from the reform class. After all, these algorithms were developed as an efficient way to solve problems and algorithms are still a valid strategy that can be used to obtain accurate answers. The difference is that the algorithms should be used once a conceptual understanding of fractions has been reached rather than preceding it. If not, students will become dependent on solving a problem by using an algorithm first without attempting to understand the problem conceptually (Khoury & Zazkis, 1994).

5.1.3 Reform Instruction and Flexible Thinking

Learning fractions through problem solving (reform method) fostered greater flexibility in students' thinking. As the literature supports, researchers have found in their studies that students are capable of finding their own ways to solve problems if given the opportunity (Sharp et al., 2002) and are able to learn more effectively (Kamii, Lewis & Livingston, 1993). Students in the reform teaching class used more diverse methods to solve and work through the problem solving section of the post-test than the traditional class. For example, when solving the addition problem in the post-test, almost three quarters of the students in the reform teaching class used some form of a manipulative to assist them in solving the problem whereas in the traditional teaching class, almost half of the students used an algorithm.

5.1.4 Algorithm Reliance Integrated with Problem Solving

Relying on algorithms can also hinder students ability to problem solve effectively. Based on the pre-test results alone, Class B (traditional) was much further ahead than Class A (reform). Even when working through the pre-test, the students in Class B were much more focused when working through the problems and they really took their time to complete the test. The students were motivated to work through the problems.

After instruction, one would think that a student would improve their problem solving methods and be more capable of answering the problem section on the test. This is what makes the results from the pre-test to the post-test very surprising. Students in Class B actually demonstrated a regression in certain fraction concepts in the post-test from the pre-test. In some cases, fewer students were able to solve the problems in the post-test (some of the questions were harder) but in Class A, students were able to show an improvement or at least maintain their results. The students in Class B were hindered because the focus of their learning was based on procedures instead of concepts. When fraction procedures were taught in isolation, the students had a very difficult time applying these procedures to word problems as found in Heller, Post, Behr & Lesh (1990).

5.2 Possible Sources of Differences in Findings

There are a number of variables that do have to be taken into account when looking at these large differences. The first one is simply how the students were taught. Even though Class B started off at a higher baseline than Class A, they did not show as much gain when it came to problem solving. The likely explanation is that the reform teaching methods fostered the greater improvement.

Some other factors that may have contributed to this difference were that the sample sizes were quite different. Class B's sample was almost half the size of Class A's so there was less data to be drawn from in Class B. Also, as a French-Immersion class their math program was taught in French while the pre-test and post-test was conducted in English. This, however, is the same way that Grade 6 testing is conducted. The students learn their mathematics in French but write the provincial assessment in English.

One final factor that needs to be considered is that Class B did not spent less time on the addition and subtraction of fractions than did the students in Class A. It needs to be pointed out that while Class B did not spend as long working on these particular concepts, nonetheless, there were still problems that they performed better on the pre-test than the post-test.

5.3 Sources of Differences due to Instructional Methods

5.3.1 Manipulatives

There were certain strategies that made the reform teaching method more effective in developing students' conceptual understanding of fractions. The effective use of manipulatives was important for the students' development of fraction concepts. In order for manipulatives to be useful they have to be relevant to the situation and be used in a manner that assists the students in developing their fractional understanding (Bezuk & Cramer, 1989). The fact that the students had to build their own fraction kits had them thinking about what the fraction pieces represented. These kits were available to them at all times and they became very familiar with the pieces and the fractions that could be made, especially in the rectangular kit.

It is important for students to be familiar with a number of different types of manipulatives (Post & Cramer, 1987) which is why the circle kit was also used. As well, the circle kit gave the students more of a variety of denominators to work with (for example sixths).

The rectangular kit was the most effective because the students worked with this kit the most and the fractions were simple (half, quarters, eighths, sixteenths) rather than factors up to sixty. It would have been beneficial to have had the students make thirds, sixths and ninths for this kit. As well, more time to become familiar with the circle kit would have benefited the students greatly.

When finding equivalent fractions in the post-test, the biggest improvement was in the two questions that the students were able to use their fraction kits for and the least amount of improvement was in the question that the kits were not able to be used.

5.3.2 Paired Work

It was not only important for the students to work with the kits but it was important for them to have someone to work with. This allowed them the opportunity to talk about their ideas, to have their ideas challenged by their partner and to help each other come up with solutions to the problems as Yang (2002) found in his research. When listening to the tapes of the students' discussions, it was interesting to see how they would come up with ideas and try to explain those ideas to each other. Sometimes the conversations would lead students off the right track or bring them back on track. Fosnot and Dolk (2002) use many examples in their work that refer to students discussing various solutions to problems and how they work through their ideas. The use of "partner talk" helps students to become involved in their learning because they need to explain their thoughts and discuss their ideas with their partner (Chapin, O'Connor & Anderson, 2003).

If students are having a hard time getting along with one another or one person is dominating the solving of the problem, both partners may not be receiving what each of them needs. The choosing of partners by the teacher had to be carefully selected and closely monitored. There were some instances where the partners may have benefited more if they had worked with someone else. Changing some partners during the course of the unit would have helped some of them learn and work more effectively.

5.3.3 Math Congress

Class discussions, or a "math congress" as Fosnot and Dolk (2002) refer to it, was an integral part of the students fraction development. A math congress is when the class discusses their solution to problems. During the discussion, different questions are raised by the students and the teacher. In order for the discussion to be effective, the teacher really has to listen to what the students are saying and to encourage other students to agree or disagree with what is being said.

These math conversations helped not only the students to question and begin to solidify their thoughts, but it helped the teacher know which direction to continue the learning. The subject of the discussion revolved around issues that the students brought up during the taking up of questions or ideas that the teacher wanted to reiterate or discuss even further. It is very important to note that these discussions were based on issues that the students developed and difficulties that the teacher observed the students having. This is very different from a teacher leading the discussion with a certain answer in mind to the question that is being posed.

During the unit, students were encouraged to call out their answers instead of always having to raise their hands and to go up to the board to draw or explain what they were thinking. The teacher would ask the class if they agreed or disagreed with what was being said and would sometimes reiterate what a student was communicating by putting this information on the chalkboard. These talks also allowed for students to see other solutions that were used to solve the problems instead of just their own and to try and develop their own understanding of the issue. Many researchers (Fosnot & Dolk, 2002; Van de Walle & Folk, 2005; Chapin, O'Connor & Anderson, 2003) theorize that mathematical defense of ideas is at the heart of effective reform teaching.

What may have made the class discussions even more effective was for the students to have a name tag or a piece of paper that they could use to communicate their thoughts on a question the teacher was asking (Lawson, 2005). For example, if the students placed the name tag correctly it would mean they agree with the statement, if the name tag was placed upside down it would mean they disagree with the statement and the name tag turned vertically would mean they were still undecided. This way at a quick glance the teacher would see what students were thinking individually.

5.3.4 Minilessons

The class discussion helped lead into "minilessons" (Fosnot & Dolk, 2002, p. 34) that the teacher used to help consolidate some of the fraction ideas that were being discussed. The teacher based these lessons on students' level of understanding or to help students along the fraction developmental continuum. These minilessons were effective because it helped lead the students to ideas that would be beneficial and important for them as they continued to learn about fractions.

Researchers contend that minilessons are an essential component in reform teaching because the teacher is building on the concept that the students are establishing (Fosnot & Dolk, 2002, Prie & Kieren, 1992; Mack, 1990) as opposed to directly teaching them a method such as an algorithm. The unit moved at a pace that allowed the students to absorb the information when they were ready. This prevented having the students "stuck" in one spot of their learning for a period of time.

In contrast, in the traditional teaching class, a large number of the students were stuck on learning how to simplify fractions. If the students had had an opportunity to discover why simplifying fractions made sense then maybe there would not have been as much of a struggle for them when learning this concept. When students have an opportunity to try and come up with their own solutions to problems they are better able to develop an understanding of the math concepts (Fuson, 2003).

5.3.5 Problem Solving with Limited Guidance

Students in Class A were able to solve problems in a variety of different ways because of the manner that the problems were introduced to them during instructional time. The students would be given a problem, without any prior instruction or directed teacher guidance, then they had to try and come up with a solution with their partner. The teacher did not give examples of how they should start the problem or how they should communicate their solution. If the students were having problems starting a question, the teacher would ask them a variety of different questions. This gave the students the freedom to explore the problem in their own way and at their own level of understanding. As a result, they became more flexible problem solvers by the end of the unit because they were not limited by a single algorithm as Clark, Berenson and Cavey

(2003) found.

Class B showed a strong ability to solve problems before the unit even began. In many cases, this class far exceeded Class A in problem solving in the pre-test. This demonstrates that without the formalized instruction, they were able to investigate their own methods to solve the problems and did so accurately. It was after they had formal instruction that their problem solving abilities diminished. Other researchers have also found that traditional instruction may produce poorer achievement than no instruction at all (Wearne & Hiebert, 1988).

5.4 Conclusion

This research study has indicated that teaching students in the reform teaching method has many benefits for the students when learning fractions. The students developed more flexible ways to problem solve, were able to apply their own reasoning to solve computation questions, effectively used manipulatives to further their understanding and strengthened their ability to reason mathematically. The students were able to apply the concepts that they learned to a variety of situations including word problems and procedural questions. Even though there were still areas that needed more time to develop, the students now have more of conceptual base of fractions to build on in future years.

The dangers of teaching math in a traditional fashion were evident from the onset. Even from the pre-test, the students in both classes relied on faulty procedures to add and subtract fractions without paying any attention to the reasonableness of their answers. This follows Lamon's point (2001) that "when rules and procedures are not learned with meaning, students forget them or do not always realize when to use them" (p. 162). As for the students that learned fractions in a traditional way, even though they were stronger problem solvers before the unit began, their ability to problem solve lessened by the completion of the unit. Learning traditional fractional algorithms hindered their ability to think flexibly and to answer questions based on their own level of understanding.

Students are able to make more sense of their mathematics when they are building on what they already know (Mack, 1990). This research alone has shown only a small glimpse of what students can achieve in mathematics when they are given the opportunity.

5.5 Considerations for Future Research

Many researchers have investigated how students can learn mathematics on a conceptual level as opposed to a series of procedures. It would be interesting to do a study similar to the one that I conducted to see how much knowledge the students have retained after a six week period. Another test could be given, similar to the pre-test and post-test that they would have already written, to see their achievement levels. This study would also be better served if the class dynamics were more similar in nature.

Future research could be done in teaching fractions to students who have not had any formal exposure to this topic in their schooling. A comparison could be made as to how the students initially develop these concepts in both a reform and traditional teaching classroom. This way the students have not been taught any prior algorithms or methods in which to solve various fraction computations.

Of course, investigating how the reform teaching methods could be used to teach other areas of the curriculum would benefit educators and students. It would continue to help educators develop a deeper understanding of how students learn to make sense of mathematics for the long term.

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Appendix A: Parent Letter for Students in Traditional Teaching Method Class

For students in Traditional Teaching Method Class

January 5, 2005

Dear Parent/Guardian,

My name is Mrs. Biesenthal and I have recently started working on my Master of Education Degree. One of my goals for working on my Master's Degree is to investigate an area in mathematics that children have a difficult time understanding and to find ways to improve the teaching of this area.

In discussion with my colleagues and in my reading of the research, fractions are an area in mathematics that children often have difficulty understanding. My research focus, therefore, is to investigate teaching methods that would be most appropriate to help students develop a deeper understanding of factions. To do this, I will be comparing the results of two different teaching methods to determine if one may be more effective than the other.

In order to make this comparison, I will be teaching fractions through a problem solving approach in a classroom and Madame Joans will teach fractions as they have been traditionally taught in her classroom. The fractions unit will be taught during the first three weeks of January, 2005. The students will take a pre-test and a post-test to determine how much they have gained from each teaching method. The results will then be compared for similarities and differences. The instructional methods I am using have been found to be very effective in supporting students' understanding.

At no point in my project will your child be identified. Their pre-test and post-test will be collected, names removed and numbered so that there are no identifiable marks on their work. The data that is collected will be stored in a locked cabinet at Lakehead University for 7 years and your child's results will be kept confidential. Upon the completion of my research, you will be welcome to view any results that I have obtained. Participation in this study is voluntary and you may withdraw the use of your child's information at any time.

Please note that this research does not affect classroom instruction time. It is being conducted in the same manner and time length of the regular math lessons which meet curriculum expectations. This research will not take away from the normal learning environment in the classroom and there is no risk to your child. The research is simply being conducted to help improve your child's education. If you choose not to have your child participate, they will still be engaged in the math lessons. The only difference is that their data will not be used.

If you have any questions, please do not hesitate to contact me at (phone number inserted).

Thank you for your cooperation in the above matter,

Mrs. Biesenthal

c.c. Principal of the School

Appendix B: Parent Letter for Students in Reform Teaching Method Class

January 5, 2005 For students in Reform Teaching Method Class

Dear Parent/Guardian,

My name is Mrs. Biesenthal and I have recently started working on my Master of Education Degree. One of my goals for working on my Master's Degree is to investigate an area in mathematics that children have a difficult time understanding and to find ways to improve the teaching of this area.

In discussion with my colleagues and in my reading of the research, fractions are an area in mathematics that children often have difficulty understanding. My research focus, therefore, is to investigate teaching methods that would be most appropriate to help students develop a deeper understanding of factions. To do this, I will be comparing the results of two different teaching methods to determine if one may be more effective than the other.

In order to make this comparison, I will be teaching fractions through a problem solving approach in your child's classroom while my colleague Madame Joans will teach fractions as they have been traditionally taught in her classroom. The fractions unit will be taught during the first three weeks of January, 2005. The students will take a pre-test and a post-test to determine how much they have gained from each teaching method. The results will then be compared for similarities and differences. The instructional methods I am using have been found to be very effective in supporting students' understanding.

During the fraction lessons, some groups will be taped so that I will be able to listen for how the students are working through their answers during the lessons. Some children may be interviewed to determine the reasoning behind their answers. The answers will be transcribed and possibly quoted to exemplify understanding or lack of understanding.

At no point in my project will your child be identified. Their work will be collected, names removed and numbered so that there are no identifiable marks on their work. The data that is collected will be stored in a locked cabinet at Lakehead University for 7 years and your child's results will be kept confidential. Upon the completion of my research, you will be welcome to view any results that I have obtained. Participation in this study is voluntary and you may withdraw the use of your child's information at any time.

Please note that this research does not affect classroom instruction time. It is being conducted in the same manner and time length of the regular math lessons which meet curriculum expectations. This research will not take away from the normal learning environment in the classroom and there is no risk to your child. The research is simply being conducted to help improve your child's education. If you choose not to have your child participate, they would still be engaged in the math lessons. The only difference is that their data would not be used.

If you have any questions, please do not hesitate to contact me at (phone number inserted).

Thank you for your cooperation in the above matter,

Mrs. Biesenthal

c.c. Principal of the School

Parent Consent Form

My signature on this form indicates that my son or daughter, ______________________ will participate in a study by Mrs. Biesenthal on how students understand fractions.I have received an explanation about the nature of the study and its purpose.

I understand the following:

1. My child is a volunteer and can withdraw from the study at any time.
2. There is no apparent danger of physical or psychological harm.
3. The data provided by my child will remain confidential.
4. I will receive a summary of the project, upon request, following the completion of the project.
5. In accordance, with Lakehead University policy, all information collected during the project will be securely stored at Lakehead University for seven years.

____________________________   ________

Signature of Parent   Date

Appendix D: Curriculum Expectations and Fraction "Big Ideas"

Number Sense and Numeration: Fractions

Overall Expectations:

- compare and order, and represent the relationship between, fractions with unlike denominators using concrete materials and drawings
- select and perform computation techniques appropriate to specific problems involving unlike denominators in fractions

Specific Expectations:

- order fractions on any number line
- explain processes and solutions with fractions using mathematical language
- compare and order mixed numbers and improper fractions with unlike denominators using concrete materials, drawings, and symbols (e.g. use concrete materials to show 3 ½ > 8/4)
- explain their thinking when solving problems involving fractions

Taken from:

MET. (1997). *The Ontario Curriculum Grades 1-8: Mathematics* (http://www.edu.gov.on.ca/eng/document/curricul/curr97ma/curr97m.html). Ontario: Ministry of Education and Training.

Big Ideas to be covered:

- comparing fractions
- equivalent fractions
- adding and subtracting fractions with like and unlike denominators
- mixed and improper fractions

Addition Results: Summary

Summary of Pre-Test Common Errors

Problem/Question	Class A (%)	Class B (%)
Adding Numerator and Denominator		
C1	57	71
C2	73	65
C2	63	71
Ending With a Whole Number		
C1	10	0
C2	7	0
C3	7	0
Blank		
C1	0	0
C2	7	0
C3	10	18
Other		
C1	3	0
C2	10	18
C3	20	6

Summary of Post-Test Common Errors: Computations

Problem/Question	Class A (%)	Class B (%)
Adding Numerator and keeping largest Denominator		
C1	0	0
C2	10	0
C3	10	12
Adding Numerator and Denominator		
C1	0	0
C2	0	0
C3	0	12
Equivalent Fractions		
C1	0	0
C2	3	0
C3	13	0
Faulty Procedures		
C1	0	6
C2	0	24
C3	0	35
Blank		
C1	3	0
C2	3	0
C3	10	6
Other		
C1	0	0
C2	13	0
C3	27	6

Analysis of Pre-Test Addition Problem: Common Errors

Question/Error	Class A (%)	Class B (%)
Adding Numerator and Denominator	43	12
Diagram	17	29
Blank	17	6
Other	17	24

Analysis of Post-Test Addition Problem

Question/Problem	Class A (%)		Class B (%)	
	Correct	Incorrect	Correct	Incorrect
Fraction Kit	23	10	0	0
Equation	0	7	0	0
Clock	10	0	0	0
Diagram	3	20	0	35
Equivalent Fractions	7	10	6	0
Algorithm	0	0	24	18
Blank	0	0	0	6
Other	0	0	0	12
No Explanation	0	10	0	0

Subtraction Results: Summary

Summary of Pre-Test Common Errors: Computations

Problem/Question	Class A (%)	Class B (%)
Subtracting Numerator and the Denominator		
D1	40	59
D2	47	59
D3	57	59
Subtracting Numerator Adding Denominator		
D1	0	0
D2	10	0
D3	3	0
Subtracting Numerator Blank Denominator		
D1	3	0
D2	3	0
D3	0	0
Subtracting Numerator Bigger Denominator		
D1	0	0
D2	3	6
D3	3	0
Diagram		
D1	0	0
D2	3	0
D3	3	0
Whole Number		
D1	10	0
D2	10	0
D3	10	0
Other		
D1	7	6
D2	10	6
D3	7	6
Blank		
D1	3	6
D2	7	12
D3	10	6
Adding Numerator and Denominator		
D1	3	0

D2	0	0
D3	3	0
Adding Numerator Subtracting Denominator		
D1	0	0
D2	0	6
D3	0	0

Summary of Post-Test Common Errors: Computations

Problem/Question	Class A (%)	Class B (%)
Subtracting Numerator Adding Denominators		
D1	0	0
D2	7	0
D3	0	0
Subtracting Numerator Bigger Denominator		
D1	0	0
D2	7	18
D3	0	6
Adding Instead of Subtracting		
D1	0	0
D2	3	0
D3	7	0
Faulty Procedure		
D1	0	0
D2	0	18
D3	3	24
Diagram		
D1	0	0
D2	3	0
D3	3	0
Blank		
D1	0	0
D2	0	0
D3	13	0
Other		
D1	7	0
D2	20	12
D3	30	6
Subtracting Numerator Smaller Denominator		
D1	0	0
D2	0	6
D3	0	6

Summary of Pre-Test Results: Word Problems Common Errors

Problem/Questions	Class A (%)	Class B (%)
Question 4		
Subtracting Numerator and Denominator	13	12
Adding Numerator and Denominator	3	0
Diagram	23	18
Blank	3	0
Other	17	6
Result of ½	13	29
Question 5		
Subtracting Numerator and Denominator	20	18
Adding Numerator and Denominator	7	0
Diagram	20	24
Blank	27	0
Other	7	12
Subtracting Numerator Bigger Denominator	0	6

Summary of Post-Test Analysis: Problems

Problems/Questions	Class A (%)		Class B (%)	
	Correct	Incorrect	Correct	Incorrect
Question 3				
Subtraction Resulting in ¼	0	63	0	53
Equivalent Fractions	10	0	0	0
Diagram	7	0	18	18
Fraction Kit	3	0	0	0
Multiplication	0	0	0	6
Other	0	13	0	6
Blank	0	3	0	0
Question 4				
Subtraction Resulting in 3/8	0	57	0	53
Fraction Kit	3	0	0	0
Diagram	13	7	13	6
Equivalent	30	0	0	0
Division	0	0	6	0
Multiplication	0	0	0	6
Other	0	10	0	6

Appendix G: Summary of Addition/Subtraction Word Problem

Addition/Subtraction Word Problem Results

Problem	Class A (%)		Class B (%)	
	Incorrect	Correct	Incorrect	Correct
Fraction Kit	3	7	0	0
Diagram	7	20	18	18
Chart	0	0	12	6
Common Denominator of 6	0	17	18	12
Common Denominator of 12	0	7	0	0
Subtract/Add	0	3	0	0
Blank	7	0	0	0
Multiplication	0	0	6	0
Other	27	3	12	0

**Correct responses include students that have a partially correct answer and a completely correct answer. Even if students scored a ½ on a question it was counted as 1 student under these categories.

Equivalent Fraction Strategies: Post Test Computations

Problem/Question	Class A (%)	Class B (%)
Doubling	10	6
Fraction Kit	13	0
Comparing Smaller Fractions	37	0
Multiplying	13	82
Diagram	7	12
Division	3	0
Blank	3	0
Unclear of Response	7	0
Other	3	0

All students' strategies are listed whether the answer was correct or incorrect

Appendix I: Summary of Comparison Results

Comparison Results: Post Test Computations

All Student strategies – correct or incorrect

Problem/Question	Class A (%)	Class B (%)
Numerator Same; Comparing Denominators	23	0
Comparing Pieces	7	0
Benchmarks	47	12
Diagram	17	41
Same Denominator	0	47
Other	17	0

Comparison Results: Pre-Test Problem

All strategies that were attempted – correct or incorrect

Problem/Question	Class A (%)	Class B (%)
Diagram	20	53
Formula	0	24
Unsure of Question	3	0
Other	38	24
Blank	37	0

Comparison Results: Post-Test Problem

Problem/Question	Class A (%)	Class B (%)
Diagram	50	65
Formula	3	18
Unsure of Question	10	6
Other	23	12
Blank	13	0

4.4 Equivalent Fractions

4.4.1 Computation Results

Both classes showed a similar improvement between the pre-test and the post-test (Figure 31) for questions a2 and a3. The biggest improvement from Class A was with question a1 where 60% more of the class answered the question correctly (see Figure 32). Class B answered all three equivalent questions with a high degree of accuracy in the pre-test and the post-test. Over 90% of the class in the post-test was able to answer each of these three questions correctly. It was difficult to pinpoint particular strategies in the pre-test because the students were not required to give any explanation as to how they solved the questions.

Figure 31. *Equivalent Computation Questions from Pre-Test and Post-Test*

Pre-Test Computations

(a) Equivalent Fractions

1) $\frac{2}{5} = \frac{\_}{10}$ 2) $\frac{3}{4} = \frac{\_}{12}$ 3) $\frac{4}{5} =$

Post-Test Computations

(a) Equivalent Fractions

1) $\frac{3}{5} = \frac{\_}{10}$ 2) $\frac{2}{3} = \frac{\_}{9}$ 3) $\frac{5}{12} =$

Choose your solution for either 1, 2 or 3 and explain how you know that one fraction is equivalent to another.