ON-LINE ESTIMATION OF FIRST ORDER PLUS DEAD TIME PROCESS PARAMETERS BASED ON UNDER OR OVER-PARAMETERIZED MODELS

by

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This thesis has been prepared
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with the Master's regulations.

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I dedicate this work to my family for their love and support, and to my great mentors Dr. K. Natarajan and Dr. A.F. Gilbert for their help and guidance.

Contents

List of Figures **ix**

Abstract **x**

1 PID Controllers and Model Estimation **1**
- 1.1 Introduction . . . 1
- 1.2 Relay Feedback . . . 1
- 1.3 Process Model Estimation . . . 3

2 Process Frequency Response Estimation **7**
- 2.1 Introduction . . . 7
- 2.2 Under or Over-Parameterized Models-Based Frequency Response Estimation . . . 8
- 2.3 Simulation . . . 9
  - 2.3.1 Process 1 . . . 9
  - 2.3.2 Process 2 . . . 12
  - 2.3.3 Process 3 . . . 14

3 Frequency Response-Based Model Identification **17**
- 3.1 Introduction . . . 17
- 3.2 Nelder-Mead Optimization Approach . . . 19
- 3.3 Simulation . . . 19
  - 3.3.1 Process 1 . . . 20
  - 3.3.2 Process 2 . . . 25
- 3.4 Conclusions and Observations . . . 33

4 Line Fitting Approach **35**
- 4.1 Introduction . . . 35
- 4.2 Line Fitting Approach . . . 35
- 4.3 Simulation . . . 38
  - 4.3.1 Process 1 . . . 38
  - 4.3.2 Process 2 . . . 42
- 4.4 Conclusions and Observations . . . 46

5 **Slowly Time-Varying FOPDT Process Estimation** **48**
5.1 Introduction 48
5.2 Frequency Response Estimation of Slowly Time-Varying Processes . . 49
5.3 Simulation 54
5.4 Conclusions and Observations 58

6 **Distillation Column Experiment** **59**
6.1 Introduction 59
6.2 DC Bias Removal 61
6.3 Distillation Column Bottoms Temperature Control Loop Process Estimation 62
6.3.1 Line Fitting Approach 62
6.3.2 Step Response Test 65
6.4 Conclusions and Observations 68

7 **Conclusions and Future Work** **70**

References **71**

List of Figures

1.1 The Relay Feedback System 2

2.1 A block diagram of a closed loop system with under or over-parameterized estimation mechanism 10
2.2 The estimated parameters of (2.4) 11
2.3 The frequency responses of continuous time process of (2.2) and discrete-time over-parameterized model of (2.4) at different sampling instants. 11
2.4 The estimated parameters of (2.7) 13
2.5 The frequency responses of continuous time process of (2.5) and discrete-time over-parameterized model of (2.7) at different sampling instants. 13
2.6 The estimated parameters of (2.10) 15
2.7 The frequency responses of continuous time process of (2.8) and discrete-time under-parameterized model of (2.10) at different sampling instants. 15

3.1 A block diagram of a closed loop system with under or over-parameterized estimation mechanism with FIR filters 20
3.2 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 14dB 22
3.3 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 10dB 22
3.4 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 6dB 23
3.5 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 4dB 23
3.6 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 3dB 24
3.7 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 1.5dB 24
3.8 The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 0dB 25
3.9 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 14dB 26

3.10 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 10dB 26
3.11 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 6dB 27
3.12 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 4dB 27
3.13 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 3dB 28
3.14 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 1.5dB 28
3.15 The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 0dB 29
3.16 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 20. 30
3.17 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 60. 31
3.18 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 100. 31
3.19 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 120. 32
3.20 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 140. 32
3.21 The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 200. 33

4.1 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 14dB 39
4.2 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 10dB 39
4.3 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 6dB 40
4.4 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 4dB 40
4.5 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 3dB 41
4.6 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 1.5dB 41
4.7 The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and SNR = 0dB 42
4.8 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 14dB 43
4.9 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 10dB 43

4.10 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 6dB 44
4.11 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 4dB 44
4.12 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 3dB 45
4.13 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 1.5dB 45
4.14 The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 0dB 46

5.1 Process 3 parameters versus sampling instant 50
5.2 Frequency responses of the processes of (5.1), (5.2) and (5.3) 51
5.3 The discrete-time over-parameterized model parameters. 52
5.4 The frequency response of the over-parameterized model and the process frequency response at sampling instant 1500. 52
5.5 The frequency response of the over-parameterized model and the process frequency response at sampling instant 3000. 53
5.6 The frequency response of the over-parameterized model and the process frequency response at sampling instant 5000. 53
5.7 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=14dB. . . . 55
5.8 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=10dB. 55
5.9 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=6dB. 56
5.10 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting Nelder-Mead approaches with SNR=4dB. 56
5.11 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=3dB. 57
5.12 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=1.5dB. 57
5.13 The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=0dB. 58

6.1 A schematic diagram of a distillation column [22]. 60
6.2 Distillation column bottoms process output and input with their estimated DC bias. 63
6.3 Distillation column bottoms process output and input after DC bias removal. 63
6.4 The estimated distillation column temperature control loop process parameters (k, τ, τ_d). 64
6.5 The estimated distillation column temperature control loop process parameters (k, τ, τ_d). 65

6.6 A step response of first order plus dead time process [25]. 66
6.7 The distillation column bottoms temperature output response to a step input change. 67
6.8 The distillation column bottoms temperature input step change. . . . 68

Abstract

Simple on-line process identification methods that can be used in tuning PID controllers or any other controllers have been studied in this work. The underlying continuous-time first order plus dead time FOPDT process model parameters (k, τ, τ_d) have been estimated effectively at every sampling period from the frequency response of an under or over-parameterized model assigned for the process and enhanced with recursive least squares. It is shown that for any under or over-parameterized model structure and with any SNR as low as 0dB, the parameters of the FOPDT model are estimated reliably. This has been accomplished by using either the Nelder-Mead optimization approach or the line fitting approach. Line fitting approach provides better estimation results and faster parameter convergence than the Nelder-Mead optimization approach especially when small sampling periods are used. Line fitting approach is applied experimentally to a distillation column and evaluated successfully in this thesis.

Chapter 1

PID Controllers and Model Estimation

1.1 Introduction

Proportional-Integrative-Derivative or PID controllers are without doubt the major control schemes that are being widely used in industry [1,2]. More than 90 % of the controllers in industry are of PID type. Although big steps have been taken in advancing control theory and implementation technology, PID controllers and their variations (P, PI or PD) remain the dominant controllers in industry. The reason for PID controllers being so popular is that PID controllers have considerable robustness to process parameter changes and incorrect process model order assumption as well as provide strong robust tracking of set-point. In addition, it provides good disturbance rejection [1,10]. Besides the reasonable performance of PID controllers for usual processes, Smith predictors and cascade controllers using PID controllers have improved the control performance for large time-delay processes [11]. The PID controller parameters depend on the characteristics of the process under control. Therefore, for good tuning, the process characteristics must be known. Ziegler and Nichols [9] developed a very simple heuristical method that requires the critical gain and critical period to regulate the PID controller parameters. A closed-loop with low damping is often achieved by these PID controller parameters. Better damping can be attained by slight modifications of the Ziegler-Nichols rules used in calculating the controller parameters [3]. A modified method of this type was first proposed by Astrom and Hagglund in 1984 where the process is activated by a relay [1,8] discussed next.

1.2 Relay Feedback

For the relay feedback arrangement shown in Fig. 1.1, it is possible to quickly

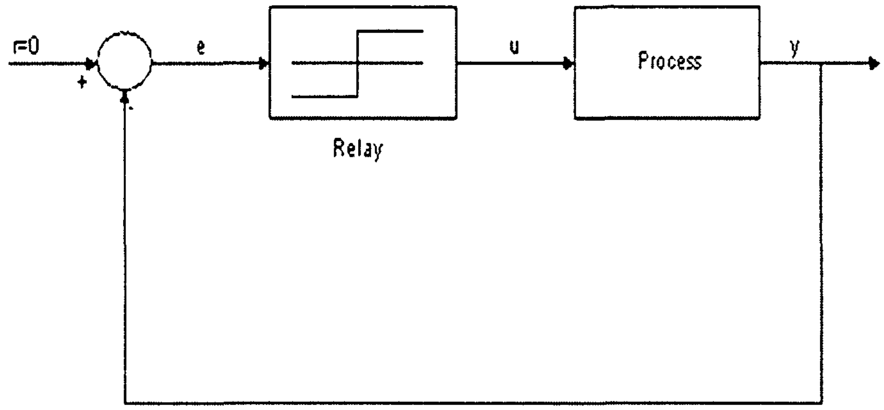

Figure 1.1: The Relay Feedback System

obtain the critical point viz. the frequency response of the process at the phase lag of $-180°$. The basic idea is the observation that the describing function of a relay is the negative real axis, where the system oscillates at the ultimate frequency or at a value very close to it [7]. The process ultimate gain is calculated by Astrom and Hagglund [7] as

$$k_u = \frac{4u_m}{\pi y_m} \tag{1.1}$$

Where u_m and y_m are the amplitudes of the relay and the process output, respectively.

Because of the simplicity of the relay feedback identification method as well as requiring no prior information on the process under estimation except the sign of the static gain, it has been applied widely in industry [11], and numerous developments and applications have been published based on it or methods derived from it. However, the basic relay auto-tuning method is considered insufficient to represent more general kind of processes such as processes with different time constants, in spite of its apparent use and success in industrial applications [12]. This is because of the approximations that are being used in the development of the estimation procedure. In particular, the describing function method is the basis of critical point estimation procedure of most existing relay feedback systems [49–51]. These relay based procedures could under some circumstances produce significantly different results as the describing function is approximate in nature. This could occur particularly in processes with significant dead-time and those with under-damped dynamics. There have been some theoretical attempts to investigate the accuracy and the validity of the limit cycles determined by the describing function DF method. However, unfortunately, these attempts haven't provided any practical results when applied on real systems [51, 52].

Recently, much effort to extract more information from the relay feedback experiment has been geared towards the extension of the basic technique in order to obtain accurate ultimate data that can be used to attain good control performance. However, the design of high performance industrial control systems significantly depend on the accurate modeling and identification of the relevant process dynamics over its entire frequency range rather than at its critical point alone. The need for industrial processes to function in a predictable manner, in order to meet demands and flexibility constraints, demands robust controller design. Such design requires a full knowledge of the relevant dynamics [45].

1.3 Process Model Estimation

Estimating the mathematical model of a real system will be always useful for predicting the performance of that system under different operating conditions, as well as for designing a controller that will make the system perform in a desired manner [46]. This can also be helpful to predict the closed loop (or open loop) process behavior due to set-point change, disturbance and noise. Most of the methods that are used for system identification involve changing input and measuring the output response of the system. In practice, disturbance and noise are always present; therefore, the process of changing the input and measuring the output has to be repetitive in order to separate the process response from the disturbance and noise effects. From the point of view of process regulation at an operating point, the process is identified assuming linear time invariance. However in such cases, small and symmetric repetitive changes of the input are needed to separate the effect of disturbances and noise and to average the process response on the up and down bumps of the manipulated variable. For instance, in a temperature control system, active heating performance is different from passive cooling behavior even in a small signal models [22].

There are two classes of system identification methods; parametric or non-parametric. The non-parametric models means that the plant frequency response function or impulse response is estimated directly; while parametric identification uses a model with parameters which are estimated by either minimizing the difference between the model frequency response and the actual frequency response or directly from time-domain data [4,5]. For non-parametric forms, identification without significant prior knowledge of the plant can be performed [30]. Obtaining Bode plots from applying sinusoids of different known frequencies at the input, and then measuring the output sinusoid amplitude and phase shift is a popular technique for this category. This approach can be sped up by using Fourier analysis (fast Fourier transforms FFT) [18,19]. However, in a practical set-up, the data used for frequency response estimation, notably the output of the system under test, are invariably corrupted by noise. Hence, for better frequency response estimation, these data have to be filtered first in order to reduce the noise. Another method that can be used as an alternative to the FFT-based approach is the quadrature correlation-based approach. It can be applied to obtain the amplitude and phase shift of the frequency response [44]. On the other hand, the parametric form often requires knowledge of at least the order of the process and is usually faster than non-parametric system identification.

Process identification, in the parametric form, can be carried out in the true open loop form, open loop form or closed loop form. In case of closed loop system identification, the changes from the set-point to the closed loop process output are used to identify the closed loop system. From this closed loop system identification, the process information can be obtained by factoring out the known controller dynamics. When using the true open loop identification configuration, the control system is configured in the manual mode. Changes in the manipulated variable and process response are used to identify the dynamics of the process. In the case of open loop identification, the system is still configured in a closed loop form, however, the controller output and process output are used to identify the system [22].

For a true open loop identification, the input has to be changed to perturb the process while operating the process in manual mode. The corresponding process output is used to calculate the process model parameters. Generally, a true open loop test is undesirable because this identification method is not robust against disturbances, since no control action can be relied on to reduce the effect of disturbances. In addition, true open loop identifications as well as non-parametric approaches such as frequency response schemes are often batch type identification schemes rather than on-line recursive schemes. Batch schemes often have the disadvantage of not revealing the information sought until the experiment is finished. If, while the experiment is in progress, the lack of information is detected, corrective efforts cannot be applied immediately [22]. In many practical situations, particularly when some type of adaptive control is needed for the system, it is necessary to obtain the model on-line. Hence, with efficient and cheap computers, on-line identification has become attractive [46]. Closed and open loop system identification tests can be either accomplished by relay tests or with the system under control of a PI(D) controller with suitable excitation. The second method is particularly appropriate for existing industrial plants, where tests may only be permitted during normal process operation in closed loop [20].

For closed and open loop tests with the system under PI(D) control, normally, a set-point change is utilized. One of the first approaches using set-point change in identification and controller tuning was developed by Yuwana and Seborg [36]. Since then, there has been much effort proposed to deal with shortcomings, such as the use of Pade approximation for dead-time modeling [20]. It is known that the Yuwana and Seborg (YS) method doesn't perform properly when processes have large time delays. This is because the time delay in the closed loop transfer function is replaced by a first order Pade approximation [37]. This has been modified in [38] by increasing the order of the approximation of the delay in the closed loop transfer function. The YS method was also improved in [39] by matching the poles of an apparent second-order process response with the dominant poles of the closed-loop system in order to determine the model parameters. The YS method was extended in [40] in another different way by proposing to obtain the process critical frequency response data directly from the closed loop step response. A comparative study in [37] on the previous four methods showed that the refinements on the original work originated by Yuwana and Seborg have led to a very practical identification procedure. It has also been shown that good estimates of the damped oscillatory closed-loop system parameters can be obtained if a suitably high feedback gain is applied during test such that the experimental data are higher than the normal noise level in the process [37].

An alternative closed loop approach is where the identification method is conducted with a relay replacing the controller [20]. Lately, much effort has been made to modify the basic relay feedback technique to extract the process model beyond just the ultimate gain and frequency. The parameters of a FOPDT or a higher order were found in [42] by using two points on the Nyquist curve obtained by using two relay feedback tests. Because it was assumed that the time delay could be calculated from the relay feedback test initial response, this method might not work properly in practice [41, 43]. Another modified relay feedback method to identify a FOPDT model was proposed in [34]. The A-Locus method, an exact method for limit cycle identification, to estimate the FOPDT and SOPDT (second order plus dead time) parameters using only one relay test was developed in [41,43]. The authors of the A-Locus method stated that the obtained parameters are exact if there are no measurement errors or disturbances. An identification technique that approximates long time-delay process via FOPDT or SOPDT models, whose parameters are obtained through a modified relay feedback identification method, was suggested in [32]. Based on the estimated model, a PI/PID controller is then designed according to the specified amplitude and phase margins. The authors of [32] indicated that this technique is very suitable for controlling long dead-time processes that require fast response but at the same time permit the presence of overshoot. A relay in parallel with P controller to induce limit cycle output that can be used to identify FOPDT processes was introduced in [33]. In this method, two parameters of the FOPDT process are estimated based on the information obtained from the limit cycle output, with the steady state gain assumed known (apriori). A proportional-integral-proportional-derivative (PI-PD) controller is then designed based on these estimated parameters according to gain and phase margin specifications. The authors of [33] indicated that quite satisfactory performances can be obtained if the gain and phase margins used are suitably chosen.

In general, in these methods a special configuration such as the PI(D) controller or the relay is used to generate the process input. Also, the identified frequency region by these methods is usually narrow compared with the wide operating frequency region of the controller. As a result of this, a satisfactory robust control performance might not be guaranteed. In addition, measurement noise might affect the identified models in the previous methods. In this thesis, a frequency response based process identification method is proposed. This proposed method is capable of utilizing any type of process input signal generator provided they contain sufficient frequency information and have enough signal amplitude because only the process input and the measured process output are required to identify the process.

In this work, a discrete-time under or over-parameterized model is assigned for the process. This discrete-time model is used to estimate the frequency response of the process. Nelder-Mead optimization approach has been used to fit this frequency response of the process to a FOPDT model to estimate its parameters. It has been shown that there are some parameters that need to be set properly when Nelder-Mead approach is used in such system identification task. Another technique, the line fitting approach, that does not require any parameter setting and can perform the process estimation efficiently is then proposed. This technique is capable of being used in industry as it shows good performance in online estimation.

Chapter 2

Process Frequency Response Estimation

2.1 Introduction

Frequency response of systems is of vital importance and can profitably be used in many engineering fields [17, 26, 27, 44]. In practice, the frequency response of any system can be estimated using different techniques. Each of these techniques has an excitation signal which might be different from method to method. In addition, advantages and disadvantages associated with each technique are different. The procedure of the frequency response estimation contains two stages: a testing stage and a processing stage [17]. The testing stage is the stage when the system under test (SUT) is excited by a suitable test signal. This SUT's output is then recorded in its steady state to be processed with the SUT's input in the second stage. On the other hand, the processing stage of the frequency response estimation method is the signal processing one where the gain $G(\omega)$ and phase $\phi(\omega)$ responses of the SUT at all test frequencies of interest are extracted by processing the SUT's output and input. The test signals used in the testing stage could be narrowband (sine) or broadband (noise-like). When using the narrowband signal (sine) test, the excitation signal of the SUT has to be repetitive and separate until the frequency range of interest is covered. On the other hand, one test of long enough duration could be sufficient when using the broadband signal (noise-like) test. The excitation signals often used for the system frequency response estimation, for instance, are the single frequency (SF) signal, the multi-frequency (MF) signal, the swept frequency signal, the impulse signal and a variety of random noise-based signals such as periodic noise, maximum length binary sequences (MLBS), discrete binary sequence (DIBS), random burst and random noise of some specific distribution [17, 35].

The amplitude spectra at the individual frequency can be completely and directly controlled in case of using the SF and MF excitation signals; while they cannot be individually controlled in case of using any other excitation signal. Beside this benefit, periodicity and simple generation are the other two advantages of these two excitation signals (SF and MF). The periodicity either allows complete rejection of the effect of all disturbances (including harmonics that are uncorrelated with these signals) on the frequency response accuracy in case of using correlation analysis based processing method, or gives no leakage in case of using fast Fourier transform (FFT) based processing method [17].

In case of using SF in the frequency response estimation method, the test signal, with user-defined amplitude and phase, is applied to the SUT, and then the SUT's output is recorded in its steady state mode. This procedure has to be repeated for the whole frequency range of interest in order to obtain the SUT's steady state output at each frequency of interest. However, this repetitive procedure might require large time to perform which might be considered as a disadvantage for the testing method if the dynamics of the SUT are slow. On the other hand, in case of using the MF testing method, a single multi-frequency test signal is generated. This single multi-frequency test signal consists of many single-frequency test signals, such as those created in the SF test method. Similar to SF test method, the individual frequencies, amplitudes and phases are defined by the user for all of the single-frequency components in the MF test method. The output of the SUT is also a MF signal, and it's recorded in the steady state mode. The gain and phase peculiar to every component frequency in the MF input are extracted from this steady state output and input as they contain all the frequency response information. The time used by the MF test method is considered low compared with the time needed by the SF test method which is a benefit in case of slow SUT. However, if the phases of the component signals in the MF test signal are not suitably selected, this feature could become worthless because of the detrimental loss in estimation accuracy [17]. The MF test method sometimes suffers from this issue which is known as the problem of "crest (or peak) factor" minimization. The MF test signal is called "peaky" if the energy of the signal is only concentrated at specific test frequencies, i.e. the crest factor is high. This could cause a shock to the SUT, and most likely damage it or drive its output into the nonlinear region of operation or both. In addition, some MF test signal component frequencies could have lower signal-to-noise (SNR) ratios than others if the crest factor is high. This would in turn lower the gain and phase estimation accuracy at these frequencies [17].

2.2 Under or Over-Parameterized Models-Based Frequency Response Estimation

Another approach which is completely different from the previous approaches can also be used to estimate the frequency response of a system. The first stage of this approach depends on assigning an under or over-parameterized model for the process to be subsequently identified by RLS algorithm. Over-parameterized models are the models which contain redundant parameters more than those needed by the discrete-time model of the real system; whereas under-parameterized models are the models whose parameters are less than those required by the discrete-time model of the real system. Under or over-parameterization is fundamentally an interesting approach because model order does not have to be precisely selected. The under or over-parameterized z-domain transfer function can be described as follows:

$$G(z^{-1}) = \frac{Y(z^{-1})}{U(z^{-1})} = \frac{z^{-N_{du}}(b_0 + b_1 z^{-1} + ... + b_{N_u} z^{-N_u})}{1 + a_1 z^{-1} + ... + a_{N_y} z^{-N_y}} \tag{2.1}$$

Where N_{du} is the delay, and $U(z^{-1})$ and $Y(z^{-1})$ are the process input and output respectively as a function of the back-shift operator z^{-1}. The numerator can be under or over-parameterized based on the selection of N_u. On the other hand, the denominator will be structured based on the selection of N_y. By using the recursive least squares technique (RLS), the numerator and denominator parameters can be easily estimated. Estimating the frequency response from the under or over-parameterized models then can be achieved by substituting $e^{j\omega_i T_s}$ for every z for a set of frequencies ω_i $(i = 1, 2, ..., n)$. It has been observed that the open loop frequency response of the identified plant exhibits convergence for a given process condition even though the coefficients of under or over-parameterized model identified by RLS vary and are not converging [6, 21]. This under or over-parameterized model based frequency response is practical and capable of being used in industrial control since any test signal can be used to excite the system. This is because only the process input and output are required to obtain the under or over-parameterized model that will be used to estimate the system frequency response. To show that the frequency response of the under or over-parameterized model for a process at different sampling instants matches properly the frequency response of the process, examples of different processes will be given in the next section. These processes will be controlled with PI-controllers in the closed loop, and an under or over-parameterized estimation mechanism will be associated with each process as in Fig. 2.1. The set-point is excited with a square-wave signal with amplitude equal to ± 0.5 and with a period T of 1200s for Process 1 and Process 2 while for Process 3 it is 900s.

2.3 Simulation

2.3.1 Process 1

Suppose a process described as follows

$$G(s) = \frac{2e^{-40s}}{200s + 1} \tag{2.2}$$

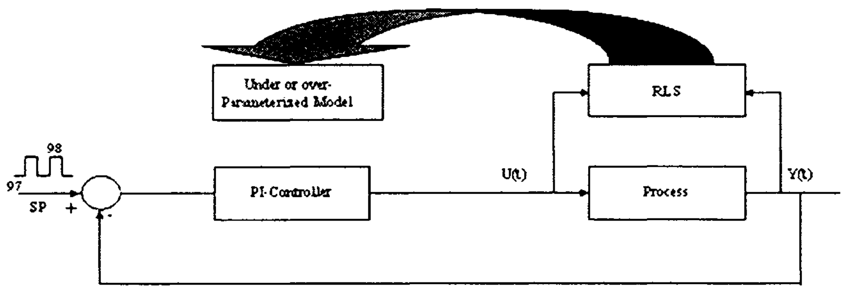

Figure 2.1: A block diagram of a closed loop system with under or over-parameterized estimation mechanism

is operating with a PI-controller in a closed loop form as in Fig. 2.1. The real discrete-time model of this process when a sampling period of 15s is used can be described like this:

$$G(s) = \frac{0.04938z^{-3} + 0.09513z^{-4}}{1 - 0.9277z^{-1}} \tag{2.3}$$

Since the delay is unknown quantity, the numerator of the discrete-time transfer function cannot be given in exact form. Because of this reason, the numerator is usually written as a series and consequently under or over-parameterization occurs in the result. Therefore, the discrete-time transfer function that is going to be used to represent this process in the discrete-time domain is assumed over-parameterized like this:

$$G(z^{-1}) = \frac{b_1z^{-1} + b_2z^{-2} + .. + b_5z^{-5}}{1 + a_1z^{-1}} \tag{2.4}$$

It is evident by comparing the numerators of (2.3) and (2.4) that the numerator of (2.4) has more extra roots than those the numerator of (2.3) has. The estimated parameters of this discrete-time over-parameterized model are given in Fig. 2.2. The frequency response of the process and the frequency responses of the over-parameterized model at different sampling instants are given in Fig. 2.3.

It can be seen from Fig. 2.2 that the discrete-time over-parameterized transfer function parameters are converging with time. The frequency responses of the discrete-time over-parameterized model at different sampling instants are almost identical with the frequency response of the continuous-time process, particularly at the low frequencies up to the bandwidth of the closed loop system. This can be seen clearly in Fig. 2.3, where the over-parameterized model frequency responses are computed at the sampling instants 100, 300, 600, 1500, 2000, and 3500. The differences in the frequency responses at much higher frequencies are due to aliasing

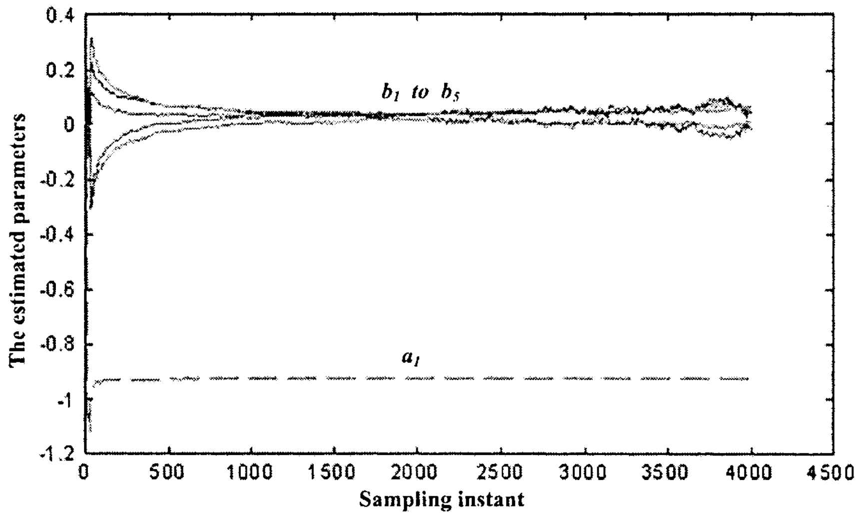

Figure 2.2: The estimated parameters of (2.4)

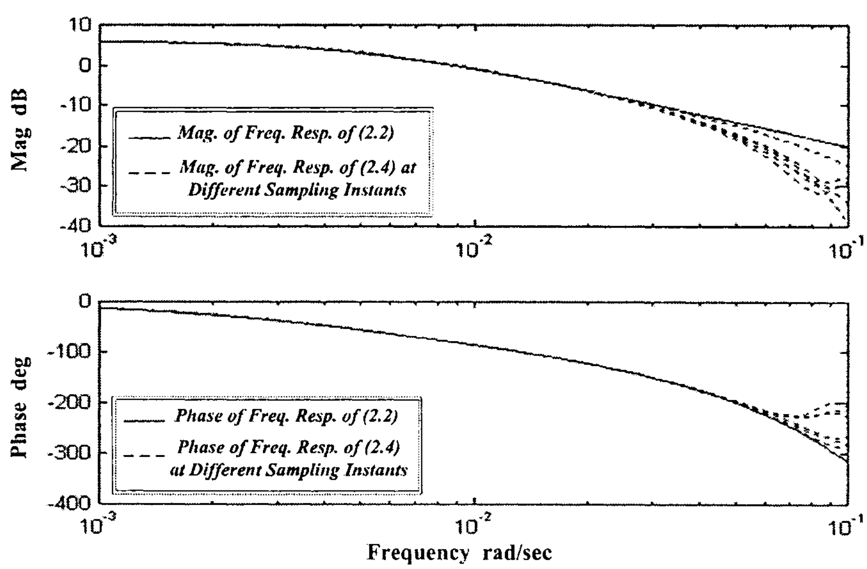

Figure 2.3: The frequency responses of continuous time process of (2.2) and discrete-time over-parameterized model of (2.4) at different sampling instants.

occurring due to sampling as well as noise corrupting the identification as the simulation is carried out with measurement noise of 10dB.

2.3.2 Process 2

Suppose a process is described as follows

$$G(s) = \frac{2e^{-25s}}{300s + 1} \tag{2.5}$$

and is operated in closed loop as in Fig. 2.1. The real discrete-time model of this process when a sampling period of 15s is used can be described like this:

$$G(s) = \frac{0.03306z^{-2} + 0.06448z^{-3}}{1 - 0.9512z^{-1}} \tag{2.6}$$

However, the discrete-time transfer function that is going to present Process 2 in the discrete-time domain is structured as an over-parameterized model as follows:

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + ... + b_7 z^{-7}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{2.7}$$

The estimated discrete-time over-parameterized model parameters are given in Fig. 2.4; while the frequency response of the continuous time process and the frequency responses of the over-parameterized model at different sampling instants are given in Fig. 2.5.

It is obvious from Fig. 2.4 that the discrete-time over-parameterized transfer function parameters are not converging with time. Different parameterized structures may give different results. The frequency responses of the discrete-time over-parameterized model at different instants, however, are almost the same with the frequency response of the real process, particularly at the low frequencies up to the bandwidth of the closed loop system. Fig. 2.5 shows the frequency response of the process along with the frequency responses of the discrete-time over-parameterized models calculated at the sampling instants 20, 290, 310, 400, 500, and 650.

Like Process 1, the differences in the frequency responses at much higher frequencies are because of noise corrupting the identification as well as aliasing effects. It is important to note that even though the discrete-time over-parameterized model coefficients (a_1 and a_2) change considerably between the sampling instant 290 and the sampling instant 310, the frequency responses at those sampling instants remain unchanged in Fig 2.5.

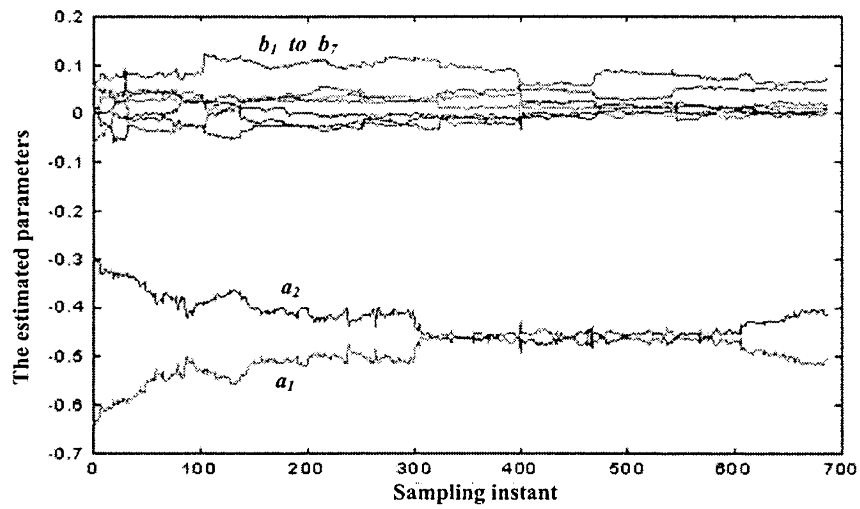

Figure 2.4: The estimated parameters of (2.7)

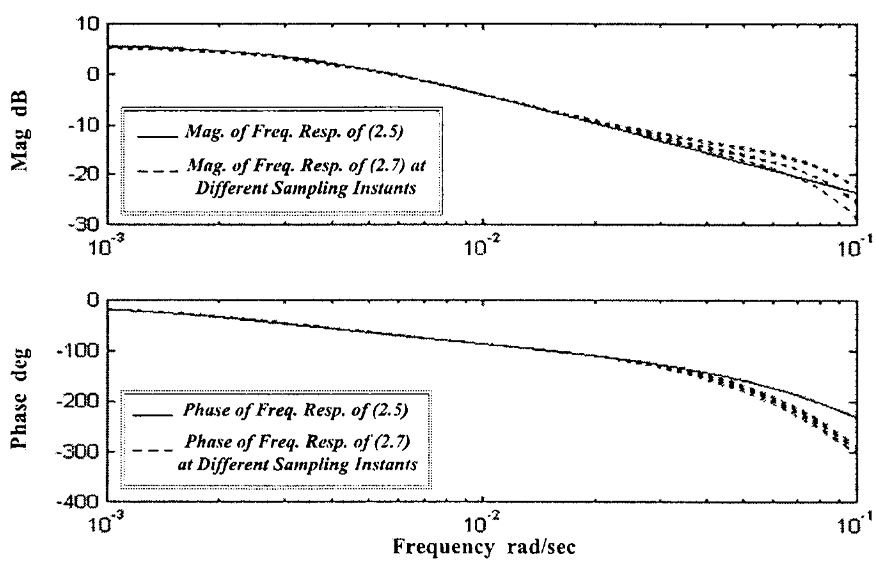

Figure 2.5: The frequency responses of continuous time process of (2.5) and discrete-time over-parameterized model of (2.7) at different sampling instants.

2.3.3 Process 3

Suppose a process described as follows

$$G(s) = \frac{2e^{-25s}}{60s+1} \tag{2.8}$$

is used to replace Process 2 in Fig. 2.1. The real discrete-time model of this process when a sampling period of 5s is used can be described like this:

$$G(s) = \frac{0.1599z^{-6}}{1-0.92z^{-1}} \tag{2.9}$$

Unlike the last two processes, the discrete-time transfer function that is going to represent Process 3 in the discrete-time domain is structured in an under-parameterized form as follows:

$$G(z^{-1}) = \frac{b_1z^{-1} + b_2z^{-2} + b_3z^{-3}}{1+a_1z^{-1}} \tag{2.10}$$

It is apparent from (2.10) that its numerator is under-parameterized when it is compared with the numerator of (2.9). Similar to the last two processes, the discrete-time model parameters are estimated using the RLS algorithm and plotted in Fig. 2.6; whereas the frequency response of Process 3 and the frequency responses of the under-parameterized model at different sampling instants are given in Fig. 2.7. Similar to Process 1 and 2, the frequency responses for the continuous-time process and discrete-time under-parameterized model at different sampling instants are almost the same even though the discrete-time under-parameterized model parameters are not converging over time.

In general, the change in coefficients is caused due to the nature of RLS scheme. Such variations in coefficients might cause variations in controller coefficients, if designed directly based on estimated process coefficients. These variations, however, are irrelevant as the process frequency response is invariant to these coefficient values. Advantages of using frequency response in control system design can be summarized as:

- The frequency response of a system is unique for the process condition, even if the coefficients of the identified process model through RLS change. The controller designed based on the frequency response will also be unique, irrespective of changes in coefficients of the identified process model.
- RLS is said to have converged when coefficients attain steady values for given process condition [31]. Figs. 2.2, 2.4 and 2.6 show that RLS convergence is

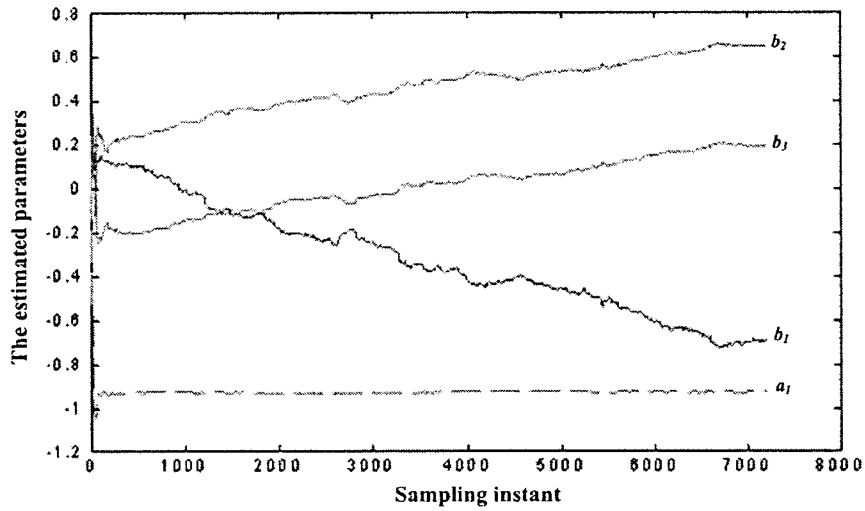

Figure 2.6: The estimated parameters of (2.10)

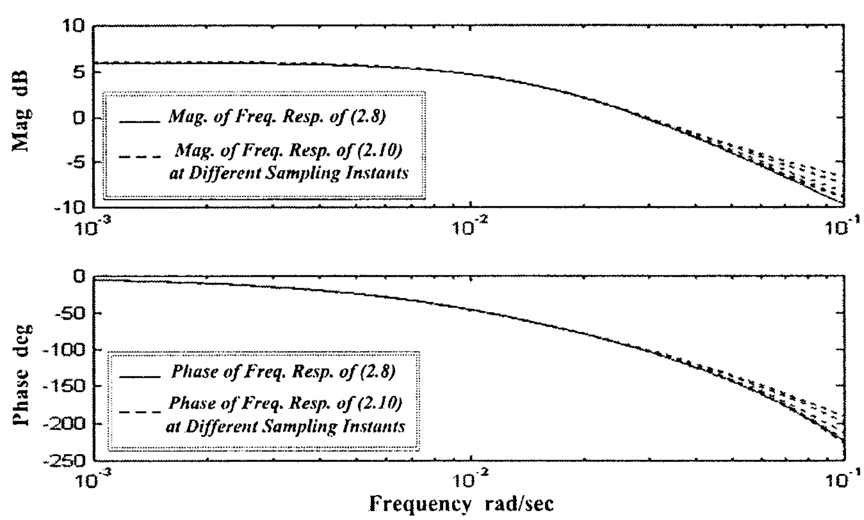

Figure 2.7: The frequency responses of continuous time process of (2.8) and discrete-time under-parameterized model of (2.10) at different sampling instants.

not guaranteed and might be worse, depending on the chosen structure of the under or over-parameterized model. However, RLS convergence is not a condition to identify reasonably the frequency response of the process. The faster convergence of RLS in terms of frequency response will be of a great benefit when the process dynamics change which will guarantee a prompt update of the controller parameters to meet required performance of the control system.

- The closed loop desired performance model can easily be formulated in accordance as a first order model with time delay, or as a second order model with time delay. For both first order and second order systems, the correlation is well established between the transient response and frequency response. This knowledge can be incorporated easily in specifying the closed loop performance model in frequency domain.
- The system can be designed such that the effects of the noise are minimal by controlling the bandwidth of the system.

While the frequency response of the process can be computed in terms of under or over-parameterized models and desired model frequency response can be computed and used for controller design in the frequency domain, engineers are more used to seeing a parsimonious description of such models using the gain k, time constant τ and time delay τ_d and design controllers with these parameters. The extraction of these parameters from such under or over-parameterized model frequency response is the main objective of subsequent chapters.

Chapter 3

Frequency Response-Based Process Model Identification

3.1 Introduction

Identification of a transfer function of a process from the available experimental or calculated process frequency response data is one of the fundamental issues in the design of practical control systems [5, 29, 45]. In case of noisy frequency response data, the objective of this identification is always to recover the model of the process with an acceptable error bound. There are several techniques developed in practice to deal with such situations to estimate frequency response-based nonparametric and parametric models [4, 5]. In case of nonparametric models, not only a *prior* knowledge may not be required to perform the identification, but also the identification might be performed even in presence of process noise and/or measurement noise. In such situations, the identification can be executed by optimizing the SNR at each measurement frequency by regulating the excitation amplitude to the limit of the process linearity. Also, frequency responses of a process from different experiments can be combined and used for the identification [29, 30]. On the other hand, developing an accurate parametric model from frequency response data has become an attractive research area for many researchers because of its vital importance in control design. Numerous methods have been developed to estimate the frequency response-based parametric models [4, 5, 28, 29, 45]. Some of these techniques depend on minimizing the difference between the process frequency response and the model frequency response using different optimization techniques such as in [28, 29, 45]. Further, the models to be estimated in some of these techniques are structured as ratios of polynomials in continuous or discrete-time domain, where the frequency response of the assumed models can be easily calculated by substituting $s = j\omega$ and $z = e^{j\omega T}$ [4, 5].

Seidel [28] has developed a technique to fit the model frequency response to the process frequency response. His approach minimizes the sum of the squared error between the model frequency response and the process frequency response using conjugate gradient search. This method has been modified by Sidman *et al* [29] by using a logarithmic conjugate gradient search to minimize the squared error, where the cost function used is described as follows:

$$J(P) = \frac{1}{2}\sum_{i=1}^{n_d} |\ln[H(j\omega_i, P)] - \ln[M(\omega_i)e^{j\theta(\omega_i)}]|^2 \times [\ln(\omega_{i+1}) - \ln(\omega_{i-1})] \tag{3.1}$$

Where,
$H(j\omega, P)$ is the complex frequency response of the desired model with parameters P. $M(\omega_i)$ and $\theta(\omega_i)$ are the magnitude and phase, respectively, of frequency response data. n_d is the number of data, and ω_i are the frequency values corresponding to the data. $[\ln(\omega_{i+1}) - \ln(\omega_{i-1})]$ is the logarithmic frequency separation weighting term which may be eliminated if frequency response data points are evenly distributed on a logarithmic basis with a constant number of data points per decade.

It is important to state that the logarithm of a complex frequency response is itself a complex function and can be separated into real and imaginary components [29].

$$\ln[H(j\omega, P)] = \ln|H(j\omega, P)| + j\angle H(j\omega, P) \tag{3.2}$$

Both techniques developed by Seidel and Sidman *et al*, however, might not work appropriately in practice because they have some weaknesses. One of their drawbacks is that the frequency range required for identification by these techniques is quite large. This is because they are assuming an ideal frequency response is available and they are fitting the desired model to it. However, this ideal frequency response, in reality, doesn't exist. Measurement noise falls into the high frequency range of the signal spectrum, while the underlying process signal usually lies towards the low frequency end. Therefore, the identification could be affected by this noise if it is performed based on the high frequency components. In addition to being corrupted by noise, identification could be influenced by aliasing occurred at high frequencies as well. This means that these approaches will always perform poorly in case of high measurement noise, which is the case in most real systems. Another frequency response-based identification approach, therefore, that can deal with these issues and ensures capturing of suitable process parameters is needed. This sought technique is preferred to work in the low frequency region to avoid identification corruption caused by both measurement noise and aliasing at high frequency region, which is no doubt one of the biggest issues system identifiers always face. In this thesis, two different techniques have been used to estimate the process parameters. Process model identification based on frequency response of under or over-parameterized models using Nelder-Mead optimization approach will be discussed in the next section; whereas process model identification based on frequency response of under or over-parameterized models using line fitting approach will be considered in the next chapter.

3.2 Nelder-Mead Optimization Approach

The Nelder-Mead optimization method or the simplex method was first proposed by J. A. Nelder and R. Mead [15]. It is a straightforward iterative algorithm for finding a local minimum of a function of several but not too many variables. Nelder-Mead algorithm doesn't need the derivatives of the function being minimized, but only requires the function evaluations at the simplex vertices. The method works with a number of rules and uses the concept of a simplex which is the geometrical figure in m dimensions consisting of $(m + 1)$ vertices. The starting point is to construct a simplex of $(m + 1)$ points, where m is the number of parameters of the function being minimized. Starting from the initial simplex, the algorithm evaluates at vertices the objective function being minimized and constructs a new simplex using operations of reflection, expansion or contraction, so that the final simplex is small enough to contain the optimal solution with the desired accuracy [15, 16].

In this work, the algorithm is used to minimize the error between the complex frequency response of the model being estimated and the complex frequency response of the process which is computed from the under or over-parameterized model as described in the last chapter. The cost function to be minimized is given as follows:

$$J(\theta) = \sum_{i=1}^{n} |G_p(e^{-j\omega_i T}) - G_e(j\omega_i, \theta)|^2 \tag{3.3}$$

Where $G_p(e^{-j\omega_i T})$ is the complex frequency response of the discrete-time under or over-parameterized model at the frequencies ω_i, and $G_e(j\omega_i, \theta)$ is the complex frequency response of the model, to be estimated in s-domain, at the frequencies ω_i. $G_e(j\omega_i, \theta)$ is assumed to be a first order plus dead time process, since the dynamics of the processes are often simplified to this [6]. However, in the future work, this can be developed to work with any order of $G_e(j\omega_i, \theta)$.

3.3 Simulation

The Nelder-Mead optimization approach has been applied on two different processes in order to see how good the estimated process parameters are compared to the real ones. In practice, filters are always required due to the presence of measurement noise. In this work, the noise filtration is performed before the RLS identification in order to ensure that the Nelder-Mead algorithm is fed by filtered frequency response data. There are different types of filters that might be used in this case. However, the filters used in this work to reduce the measurement noise and eliminate any bias in the process input and the process output are low pass FIR (finite impulse response) filters. FIR filters are one of the two primary types of digital filters that are often used because they are simple and easy to understand. The function describing the z-transform of an N-tap FIR filter with filter coefficients b_n is given as:

$$H(z) = \sum_{n=0}^{N-1} b_n z^{-n} \tag{3.4}$$

The filter coefficients b_n can be calculated by using standard MATLAB function. The number of taps N can be specified as 32 or 64, whereas the normalized cut-off frequency ω_n must lie between 0 and 1.0, with 1.0 corresponding to half the sample rate. Specifying the cut-off frequency depends on the number of the persistent excitation signal harmonics that are significant in the waveform spectrum. Since one harmonic can be used to estimate two unknown parameters, if the number of the unknown parameters of the under or over-parameterized model for example are five, then at least three harmonics would be required to estimate these five parameters. These three harmonics would be the (1^{st}, 3^{rd} and 5^{th}) if a square wave is used as an input. Fig. 3.1 shows a block diagram of a closed loop system with under or over-parameterized estimation mechanism with FIR filters.

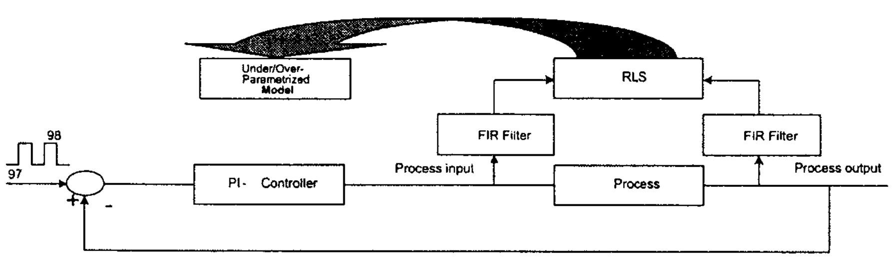

Figure 3.1: A block diagram of a closed loop system with under or over-parameterized estimation mechanism with FIR filters

3.3.1 Process 1

Let a process be described as follows:

$$G(s) = \frac{2e^{-25s}}{60s + 1} \tag{3.5}$$

Assume that the discrete-time transfer function for the process is under or over-parameterized with sampling period $T_s = 5s$ as follows:

$$G(z^{-1}) = \frac{z^{-N_{du}}(b_1 z^{-1} + ... + b_{N_u} z^{-N_u})}{1 + a_1 z^{-1} + ... + a_{N_y} z^{-N_y}} \tag{3.6}$$

An under-parameterized model is used in this case to represent the discrete-time model with the structure (1-3-0), where $N_y = 1$, $N_u = 3$ and $N_{du} = 0$. This structure is selected in order to observe how close the estimated process parameters (k, τ, τ_d) would be to the real ones. The process input and output are filtered by the FIR filters before being fed to the RLS algorithm. Since there are four unknown parameters to be estimated, three harmonics are preserved to identify them. The normalized cut off frequency of each FIR filter is therefore computed based on the fifth harmonic of the persistent excitation spectrum. With a sampling period of 5s and excitation period of 900s, the normalized cut off frequency of each FIR filter is therefore equal to ($5 \times \frac{1/900}{1/5/2} = 0.0556$). The frequency response of the discrete-time under-parameterized model is calculated at each sampling period for a certain range of frequency points. It is important to state that the maximum frequency point of this frequency range must not exceed the un-normalized cut off frequency specified for each of the two FIR filters used in this work. This un-normalized cut off frequency of each FIR filter is equal to the filter's normalized cut off frequency divided by the sampling period ($\frac{0.0556}{5} = 0.0111$ rad/sec). Therefore, the maximum frequency point of the frequency range is selected to be less than 0.0111 rad/sec. Figs. 3.2 to 3.8 show the estimated process parameters (k, τ, τ_d) using the Nelder-Mead approach versus sampling instants with different measurement noise. The SNRs used in these simulations are 14dB, 10dB, 6dB, 4dB, 3dB, 1.5dB and 0dB. The SNR used here is calculated as follows:

$$SNR|_{dB} = 20\log_{10}\frac{Signal_{RMS}}{Noise_{RMS}} = 10\log_{10}\frac{P^2}{\sigma^2} \tag{3.7}$$

Where:
$Signal_{RMS}$ and $Noise_{RMS}$ are the root mean square of the signal amplitude and noise variance, respectively; P and σ are the signal amplitude and the noise standard deviation, respectively.

It is apparent from all figures that the estimated process parameters with the different SNRs, up to 0dB, are satisfactory and reliable. This means that the discrete-time under-parameterized models can be a good source for the frequency response of the process irrespective of the measurement noise. This under-parameterized discrete-time model frequency response will be, therefore, the driving force to estimate profitably the process parameters (k, τ, τ_d) using Nelder-Mead optimization algorithm.

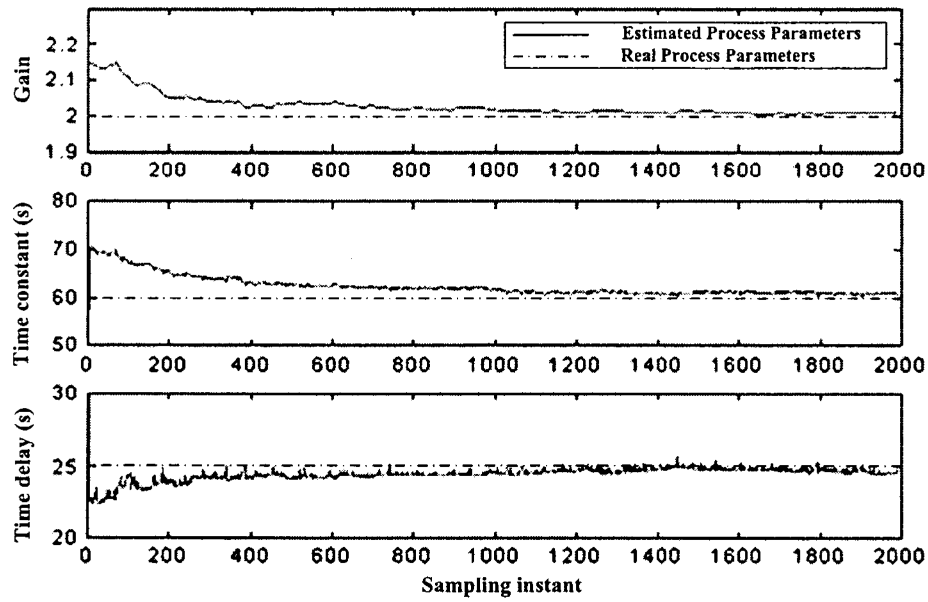

Figure 3.2: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 14dB

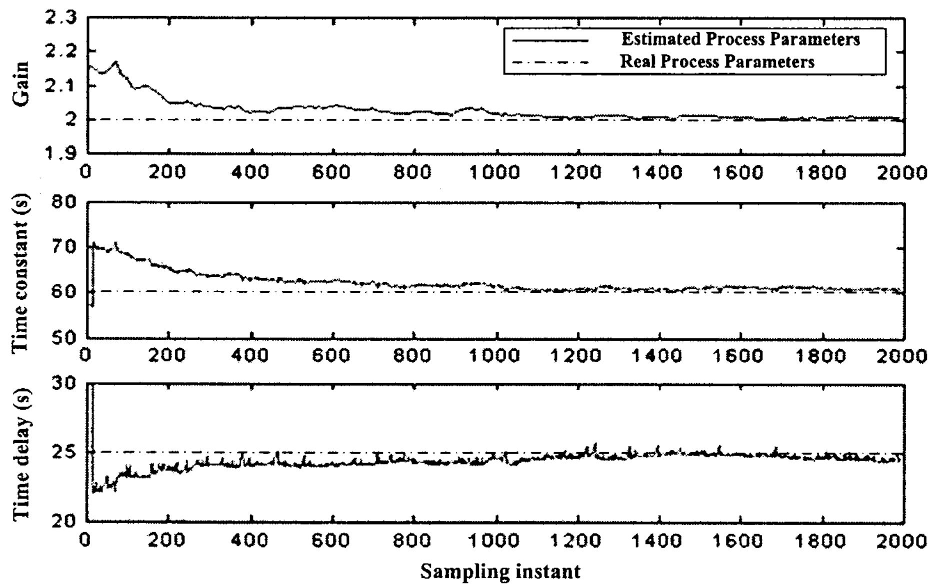

Figure 3.3: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 10dB

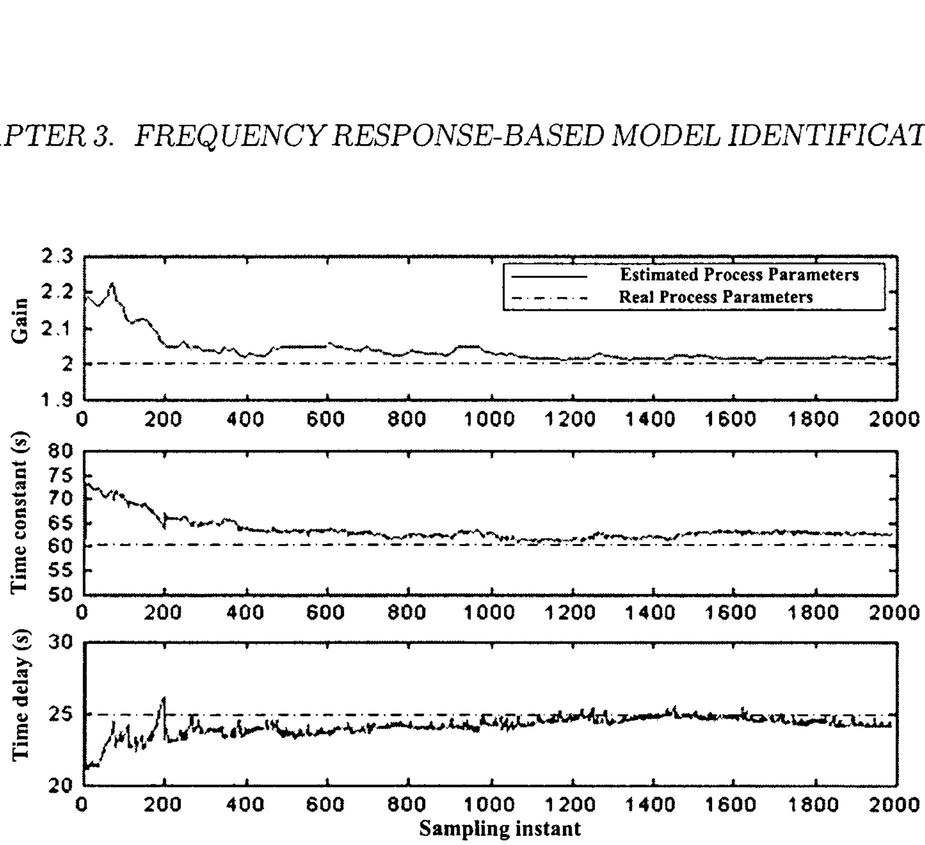

Figure 3.4: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and $SNR = 6\text{dB}$

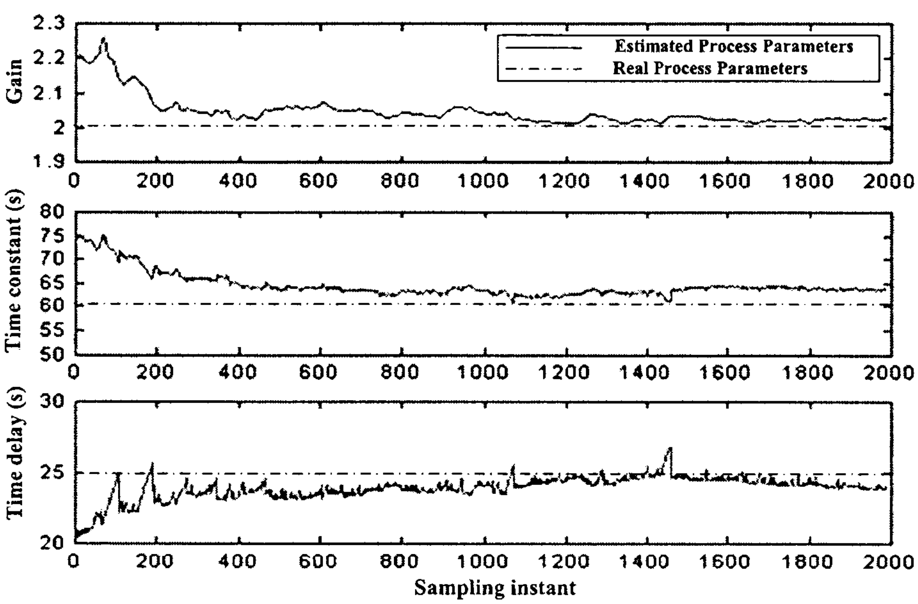

Figure 3.5: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and $SNR = 4\text{dB}$

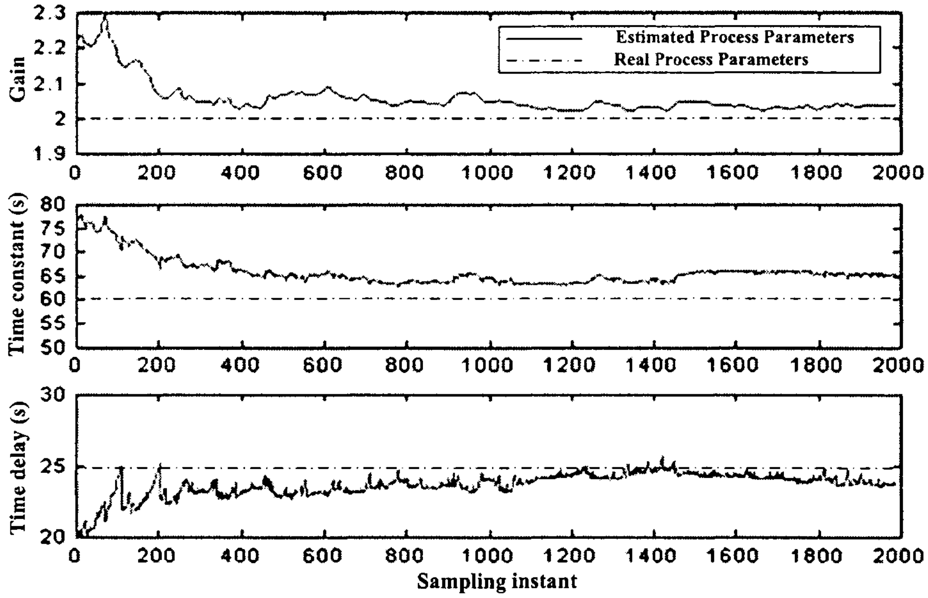

Figure 3.6: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 3dB

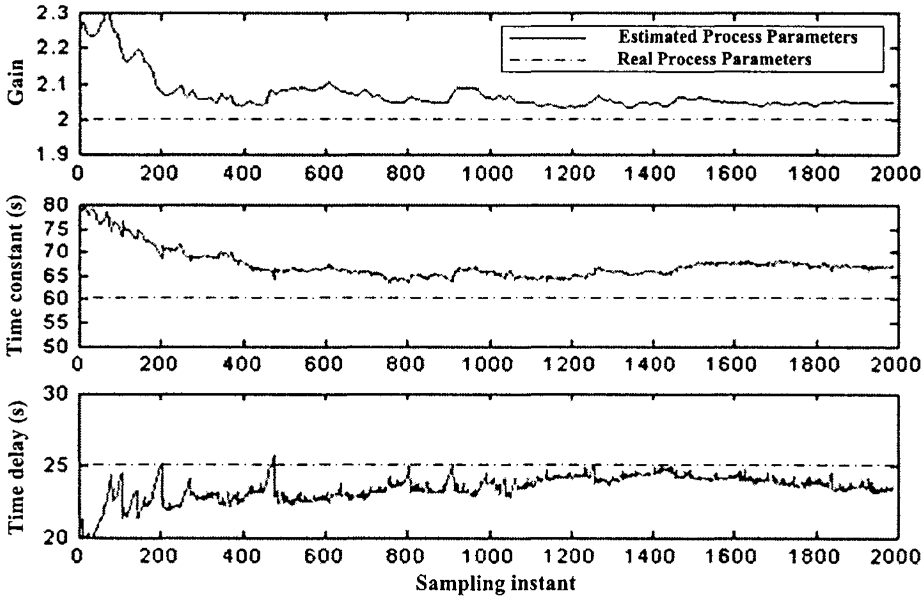

Figure 3.7: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 1.5dB

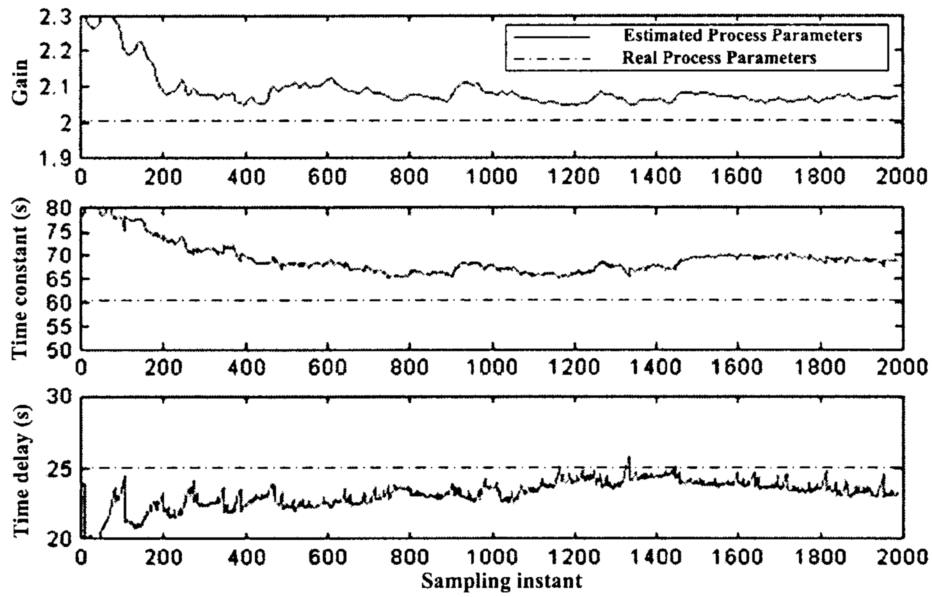

Figure 3.8: The estimated model parameters (k, τ, τ_d) with under-parameterized model structure of (1-3-0) and SNR = 0dB

3.3.2 Process 2

Let a process be described as follows:

$$G(s) = \frac{2e^{-25s}}{300s + 1} \tag{3.8}$$

Assume that the discrete-time transfer function is parameterized with sampling period of $T_s = 15s$ as follows:

$$G(z^{-1}) = \frac{z^{-N_{du}}(b_1 z^{-1} + ... + b_{N_u} z^{-N_u})}{1 + a_1 z^{-1} + ... + a_{N_y} z^{-N_y}} \tag{3.9}$$

Like Process 1, the estimated process parameters (k, τ, τ_d) using different SNRs are plotted versus sampling instants in Figs. 3.9 to 3.15. Although under-parameterized structures can be used as illustrated in case of Process 1, over-parameterized structures have been applied here in order to study the corresponding effect of using different SNRs on the estimated process parameters (k, τ, τ_d). The over-parameterized structure selected here is (1-5-0), with $N_y = 1$, $N_u = 5$ and $N_{du} = 0$. Similar to Process 1, the process input and output are filtered by the FIR filters before being

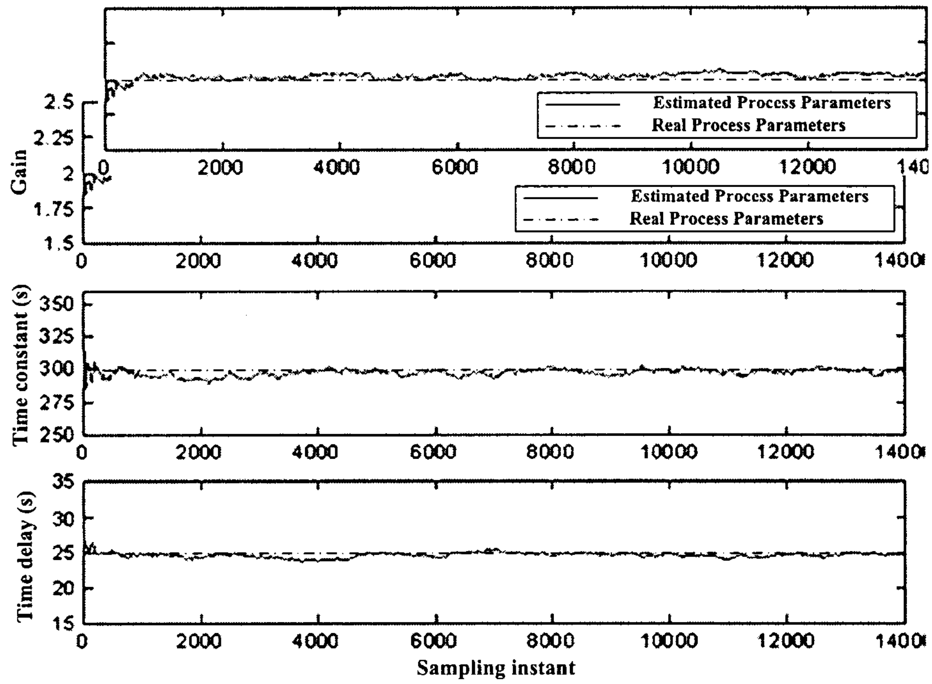

Figure 3.9: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and $SNR = 14$dB

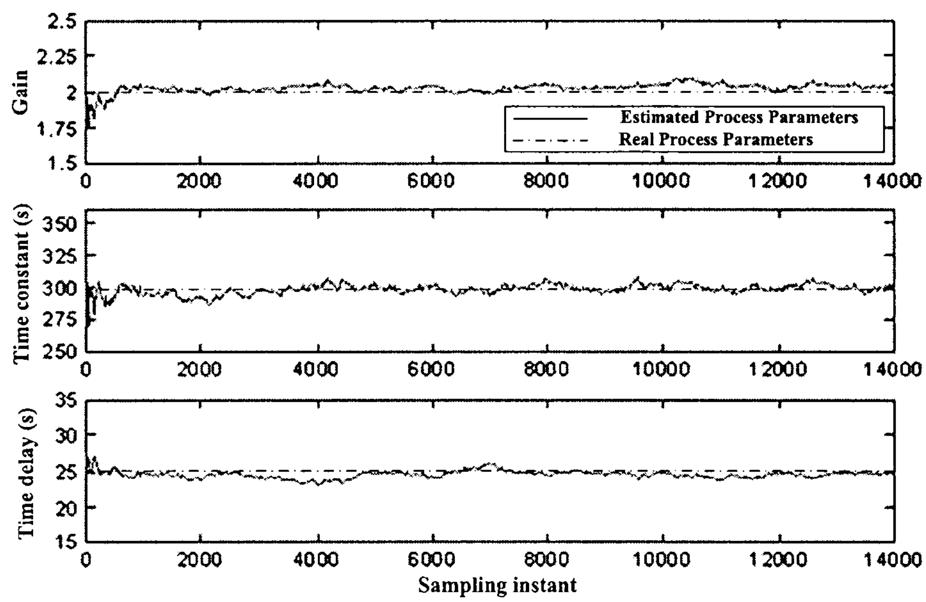

Figure 3.10: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and $SNR = 10$dB

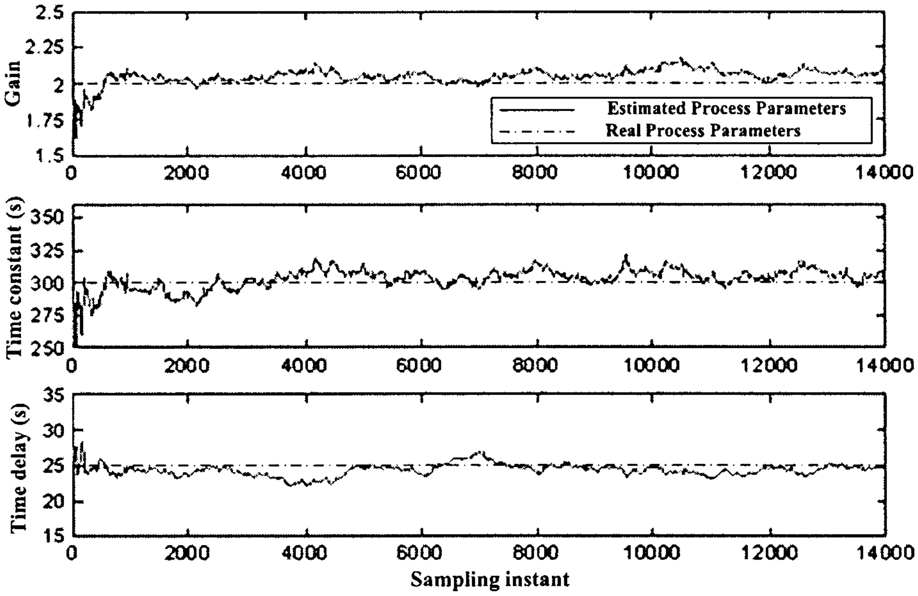

Figure 3.11: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 6dB

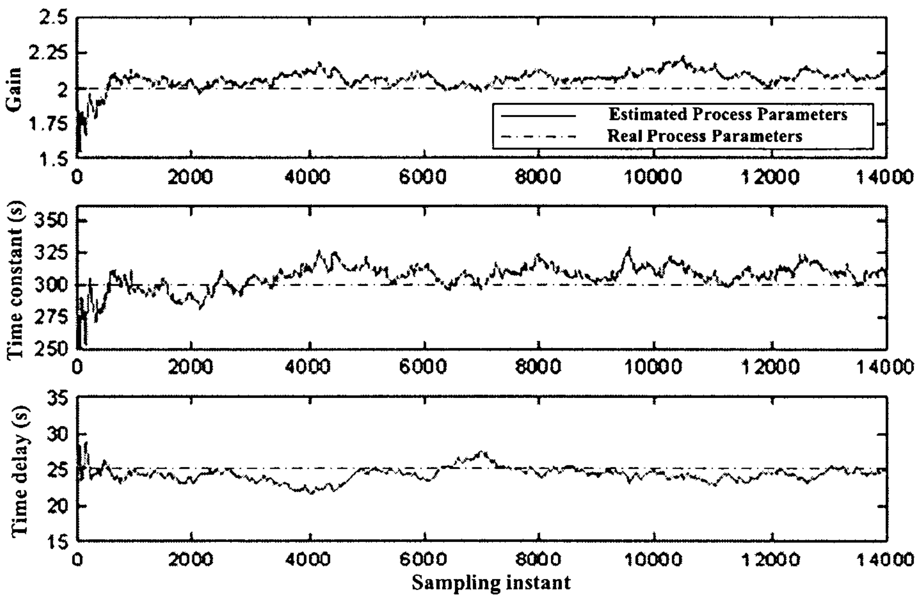

Figure 3.12: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 4dB

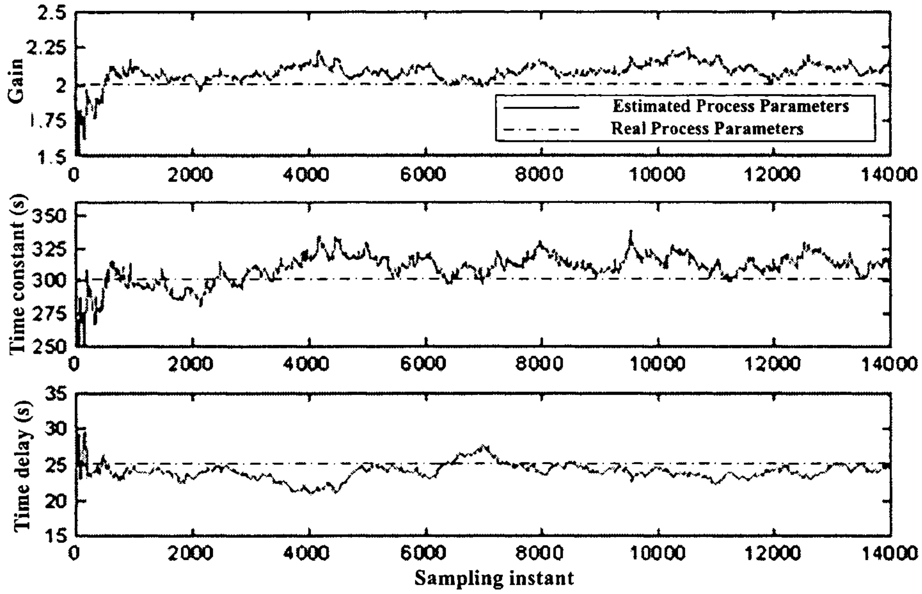

Figure 3.13: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and $SNR = 3$dB

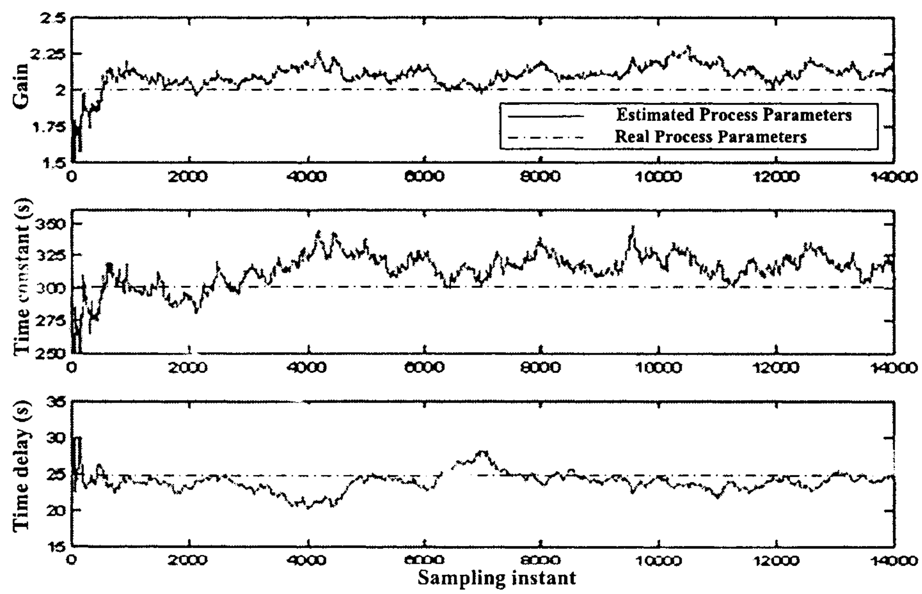

Figure 3.14: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and $SNR = 1.5$dB

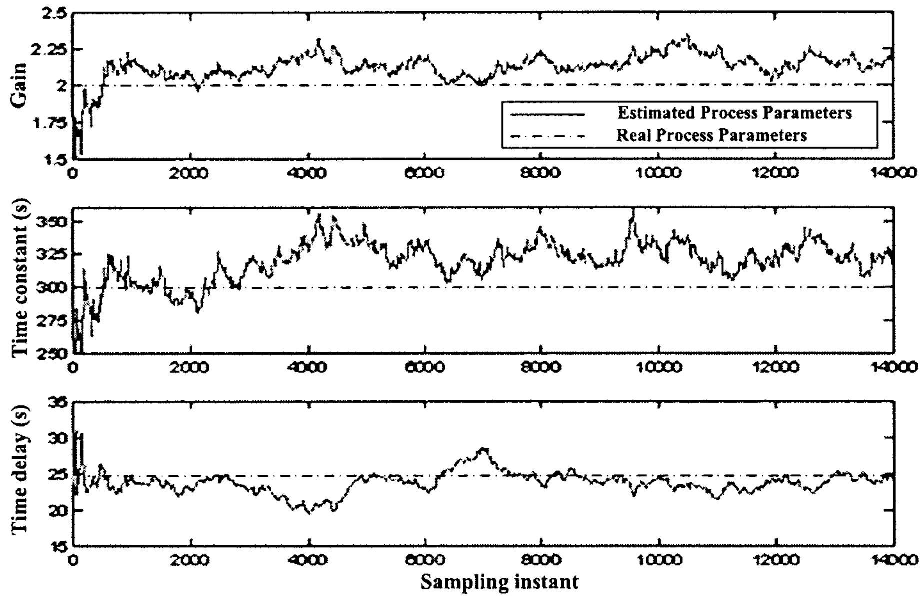

Figure 3.15: The estimated model parameters (k, τ, τ_d) with over-parameterized model structure of (1-5-0) and SNR = 0dB

fed to the RLS algorithm. Because there are six unknown parameters to be estimated by the RLS, the normalized cut off frequency of each FIR filter is calculated based on the seventh harmonic of the persistent excitation spectrum. With a sampling period of 15s and excitation period of 1200s, the normalized cut off frequency of each FIR filter is therefore equal to ($7 \times \frac{1/1200}{1/15/2} = 0.175$). As in Process 1, the maximum frequency point of the frequency range used to calculate the frequency response of the discrete-time model is chosen to be less than the un-normalized cut off frequency of the FIR filters. The SNRs used here in these simulations are 14dB, 10dB, 6dB, 4dB, 3dB, 1.5dB and 0dB.

The estimated parameters (k, τ, τ_d) of Process 2 as seen in all figures are very close to those real parameters of Process 2. These estimated parameters remain acceptable and reasonable even with low SNR (high noise) up to 0dB. In addition, Nelder-Mead optimization algorithm shows a good ability to fit the discrete-time over-parameterized model based frequency response to the frequency response of the desired FOPDT model of the process. However, there are some drawbacks associated with Nelder-Mead optimization algorithm that might lead to system identification failure in some cases. These drawbacks might be sometimes difficult to handle if the identification is carried out on-line. They may occasionally require a *prior* knowledge of the system being under estimation to be able to overcome them. One of these drawbacks is the iterations needed by the Nelder-Mead algorithm to effectively perform the minimization. These iterations should not exceed a maximum iteration number specified by user. This maximum iteration number has to be selected carefully to achieve a balance between accomplishing the minimal error and the time consumed to reach it. Nelder-Mead optimization algorithm either will not execute the minimization efficiently if this maximum iteration number is too small or will consume a lot of computing time if the maximum iteration number is too large. The later is not good in case of using small sampling period. How small or large the maximum iteration number should be depends on the number of variables of the function being minimized as well as the sampling period. In the trials conducted in this chapter, it has been noticed that the estimated process parameters (k, τ, τ_d) could be poor if the maximum iteration number is set to be a small number. Figs. 3.16 to 3.21 show the estimated process parameters of Process 1 with measurement noise SNR=4dB when applying different maximum iteration numbers (20, 60, 100, 120, 140 and 200) with different simplex step sizes (Delta=0.005, 0.05 and 0.1).

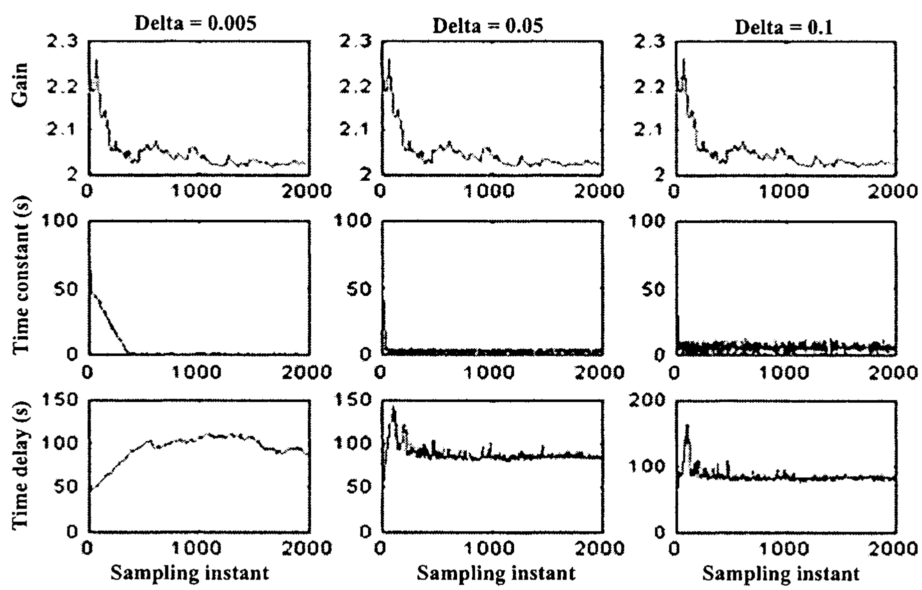

Figure 3.16: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 20.

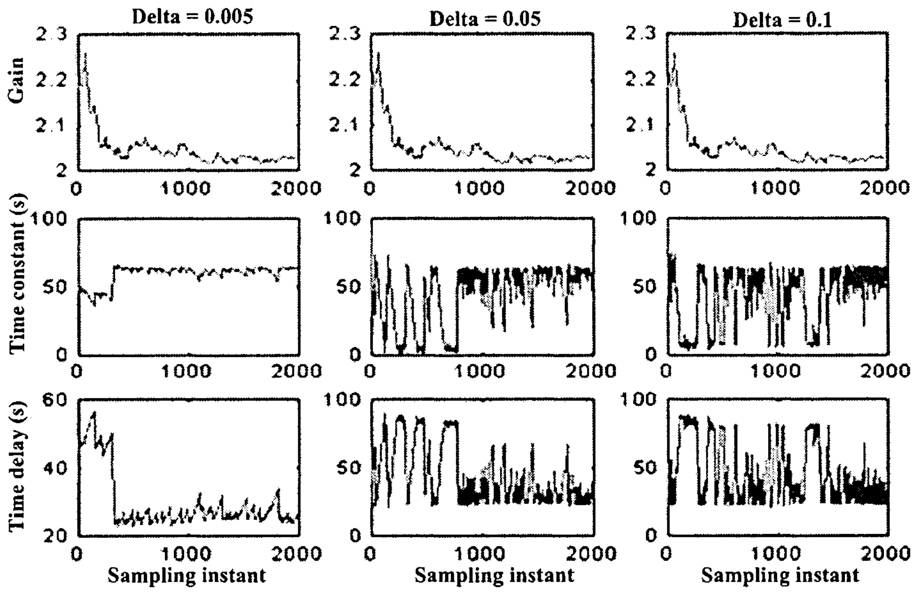

Figure 3.17: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 60.

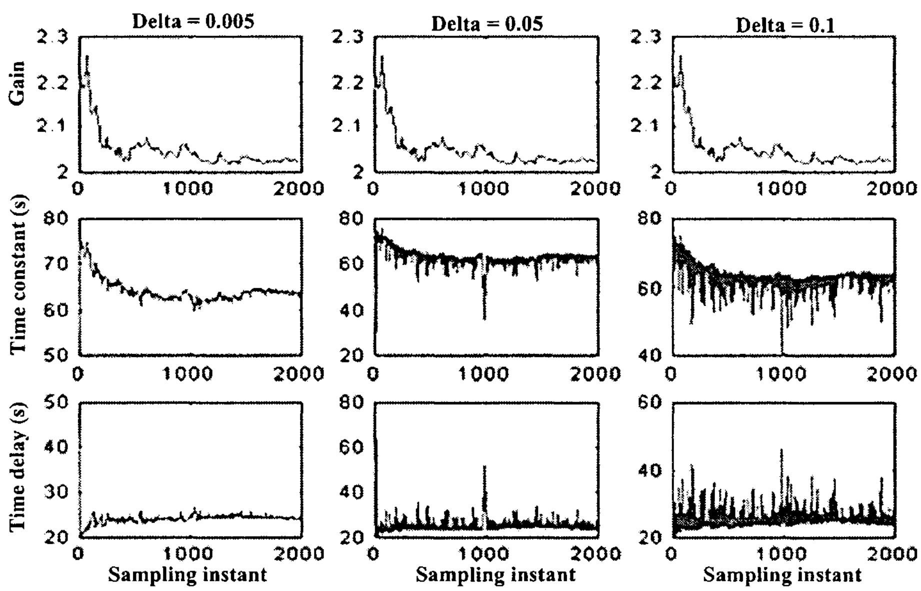

Figure 3.18: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 100.

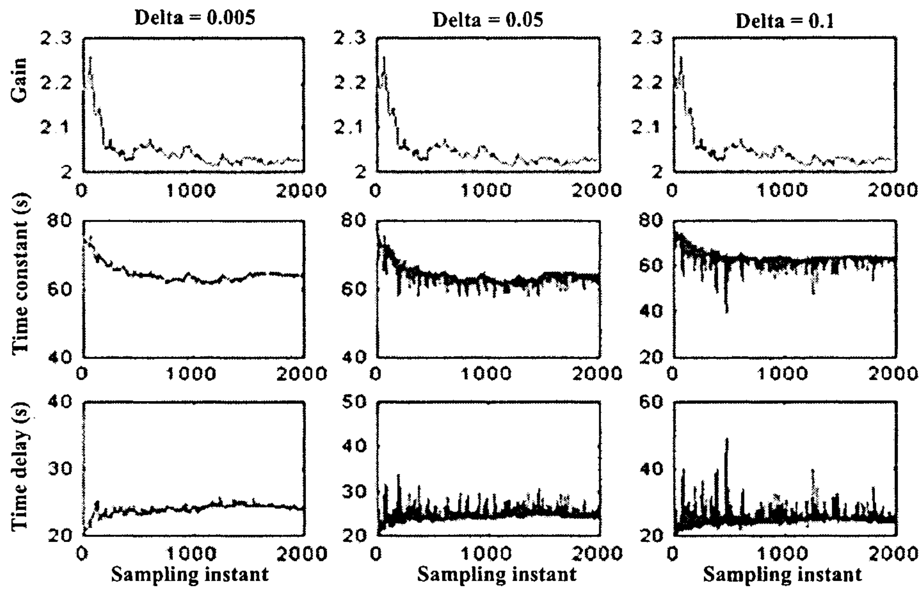

Figure 3.19: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 120.

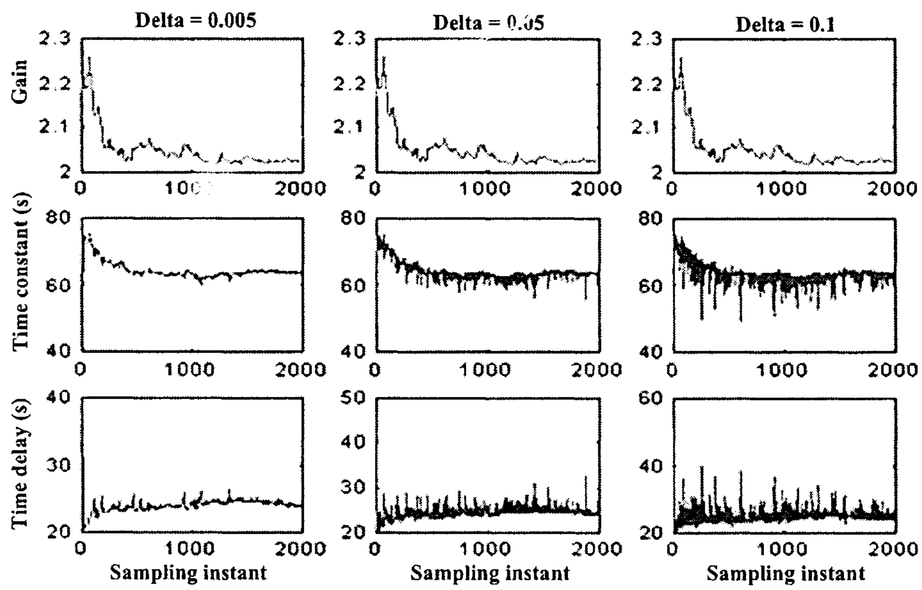

Figure 3.20: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 140.

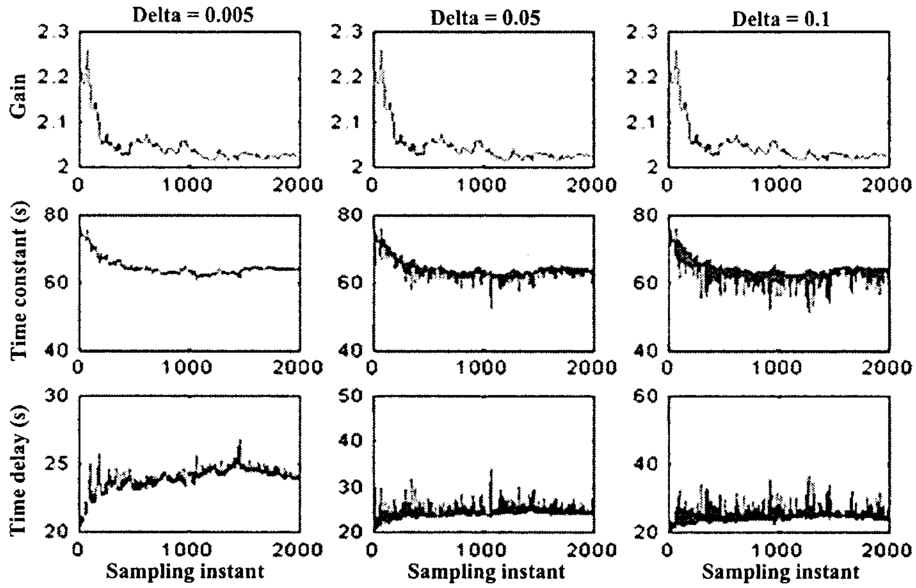

Figure 3.21: The estimated model parameters (k, τ, τ_d) with different simplex step size and maximum iteration number equal to 200.

As can be seen from the figures, in case of using small maximum iteration number, the estimated process parameters (k, τ, τ_d) can be completely different from Process 1 parameters. These estimated parameters could potentially cause undesirable performance if the controller is designed based on them. On the other hand, the estimated process parameters (k, τ, τ_d) are very close to Process 1 parameters when the maximum iteration number is assigned properly. It is clear from the previous figures that selecting the maximum iteration number and the simplex step size (Delta=0.005, 0.05 or 0.1) has a big effect on the minimization task. Selecting the starting point of the simplex vertices is also another issue that might be faced when using Nelder-Mead optimization algorithm. This is because this starting point might be assigned far away from the optimal solution which in turn will increase the time consumed by the algorithm to reach the optimal solution. In this case, if the maximum iteration number is not set correctly, the optimal solution may not be reached.

3.4 Conclusions and Observations

It is obvious now that no matter what the under or over-parameterized model structure is, its frequency response data can be used to estimate the process model parameters (k, τ, τ_d) of the assumed first order process plus dead time model. This can be done by fitting the frequency response of the under or over-parameterized model to the frequency response of the assumed first order process plus dead time model (FOPDT) by using Nelder-Mead optimization technique. However, the estimated process parameters (k, τ, τ_d) could be poor if Nelder-Mead optimization algorithm parameters of maximum iteration number, simplex step size and initial conditions are not set correctly. In practice and in case of on-line estimation, even a *prior* knowledge of the process under estimation may not be enough sometimes to guide the setting of these parameters when model parameters being estimated are large and the sampling period being used is small. Therefore, this approach can be applied in some cases but not in others. Another approach is, therefore, developed in the next chapter.

Chapter 4

Line Fitting Approach

4.1 Introduction

As illustrated in the last chapter, using Nelder-Mead optimization approach to fit the discrete-time under or over-parameterized model based frequency response to the frequency response of the assumed first order plus dead time model might provide poor estimation outcomes if Nelder-Mead algorithm shortcomings are not dealt with suitably. Line fitting approach, which is a form of least squares technique, can be applied to identifying properly the process model parameters (k, τ, τ_d) from the under or over-parameterized model frequency response without drawbacks that the Nelder-Mead approach faces. Moreover, the use of line fitting approach is more practical and beneficial than the use of Nelder-Mead optimization algorithm because of two reasons. The first reason is as pointed out earlier that the line fitting approach does not have any parameters that require to be set appropriately. Secondly, unlike the Nelder-Mead optimization approach, the line fitting approach can provide acceptable and reliable estimation results within a small time. The key factor that makes the line fitting approach provide good estimation results and be a powerful technique is the under or over-parameterized model based frequency response. Using under or over-parameterized model based frequency response has made the line fitting approach a superior method that can be used in "*on-line*" system identification. This is because at each sampling period, n frequency response data corresponding to n frequency points can be generated when using an under or over-parameterized model. This property, particularly, is the driving force for the line fitting approach to be one of the powerful "*on-line*" identification techniques.

4.2 Line Fitting Approach

Line fitting approach can perform effectively in presence of measurement noise up to 0dB. Further, it doesn't require large frequency region to estimate the process parameters (k, τ, τ_d). Three frequency points can suffice to perform the identification task. Moreover, this approach not only can be applied in case of large sampling periods, but also can be practical in case of small sampling periods.

Since the process that is going to be estimated, as assumed in the last chapter, is a first order process plus dead time model, let us describe it by the following equation:

$$G(s) = \frac{ke^{-\tau_d s}}{\tau s + 1} \tag{4.1}$$

The procedure is to estimate the gain k, time constant τ and time delay τ_d of (4.1) from the under or over-parameterized model based frequency response. The magnitude and the lagging phase shift of the frequency response of (4.1) can be expressed as follows:

$$g = \frac{k}{\sqrt{1 + \tau^2\omega^2}} \tag{4.2}$$

$$p = \tan^{-1}(\omega\tau) + \omega\tau_d \tag{4.3}$$

The parameters g and p are the under or over-parameterized model based magnitude and phase frequency response respectively, since the frequency responses of the under or over-parameterized model and the process are almost identical at low frequencies. Equation (4.2) can be rewritten as in (4.4) in order to find k by using the measured gains g at each frequency ω.

$$\frac{k^2}{g^2} = 1 + \omega^2\tau^2 \tag{4.4}$$

With one further manipulation, (4.4) can be recognized as a straight line relation of $y = mx + c$ with:

$$y = \frac{1}{g^2}, \qquad x = \omega^2 \tag{4.5}$$

$$c = \frac{1}{k^2}, \qquad m = \frac{\tau^2}{k^2} \tag{4.6}$$

Where m and c are the slope and y-intercept of the line, respectively. Therefore, for n points of the angular frequency ω_i (for $i = 1, \ldots.., n$) and their corresponding under or over-parameterized model frequency response magnitude data , the best least squares line fit [23] is given by:

$$m = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2} \tag{4.7}$$

$$c = \frac{\sum_{i=1}^{n} y_i - m\sum_{i=1}^{n} x_i}{n} \tag{4.8}$$

The coefficient of linear correlation r is given by:

$$r = \frac{n\sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{\{[n\sum_{i=1}^{n} x_i^2 - (\sum_{i=1}^{n} x_i)^2][n\sum_{i=1}^{n} y_i^2 - (\sum_{i=1}^{n} yi)^2]\}^{0.5}} \tag{4.9}$$

The closer the magnitude of r is to 1, the better the least squares line fits the data. On the other hand, the closer the value of magnitude of r is to 0, the poorer the fit by a straight line to the data.

The gain k and the time constant τ can be now calculated from (4.6), (4.7) and (4.8), while (4.9) can be used to estimate the goodness of the fit. Equation (4.3) can be processed now to calculate the time delay τ_d, using the obtained time constant τ , and can be rewritten as:

$$p - \tan^{-1}(\omega\tau) = \omega\tau_d \tag{4.10}$$

Equation (4.10) can be recognized as a straight line relation of $y = mx$ with:

$$y = p - \tan^{-1}(\omega\tau), \qquad x = \omega \tag{4.11}$$

$$m = \tau_d \tag{4.12}$$

Thus, least squares approach can be applied now to fit the line and obtain τ_d for n points of the angular frequency ω_i (for $i = 1, ..., n$) with their corresponding under or over-parameterized model phase p_i. To get a good estimation of τ_d, the parameter c calculated by (4.8) must be equal to zero or close to it as the line equation of (4.10) is of the form $y = mx$, where $c = 0$. In addition, the correlation coefficient r calculated by (4.9) must be close to one because when the least squares fits the measured data perfectly, this correlation coefficient r is equal to one. Therefore, during the on-line estimation of the process parameters (k, τ, τ_d), the correlation coefficient can be used as a rule to assess whether the estimated parameters (k, τ, τ_d) are good enough or not. Line fitting approach is applied on the same two processes used in the previous chapter in order to see how good the estimated process parameters are in comparison with the real ones.

4.3 Simulation

4.3.1 Process 1

Let a process be described as follows:

$$G(s) = \frac{2e^{-25s}}{60s+1} \tag{4.13}$$

Assume that the discrete-time transfer function of this process is parameterized with a sampling period of $T_s = 5s$ as follows:

$$G(z^{-1}) = \frac{z^{-N_{du}}(b_1 z^{-1} + ... + b_{N_u} z^{-N_u})}{1 + a_1 z^{-1} + ... + a_{N_y} z^{-N_y}} \tag{4.14}$$

As in the previous chapter, the process input and output are filtered by using FIR filters before estimating the discrete-time under or over-parameterized transfer function parameters. This discrete-time transfer function is prearranged in this sub-section with only under-parameterized structure. The under-parameterized structure that is being used here is the structure (1-3-0), with $N_y = 1$, $N_u = 3$ and $N_{du} = 0$. This structure is selected in order to observe how reliable and acceptable the estimated process parameters (k, τ, τ_d) would be with different SNRs, up to 0dB, when using the line fitting approach. The normalized cut off frequency of each FIR filter and the maximum frequency point of the frequency range used in this case are similar to those used for Process 1 in Chapter 3. Figs. 4.1 to 4.7 show the estimated process parameters (k, τ, τ_d) versus sampling instants with different SNRs. The SNRs used in these simulations are 14dB, 10dB, 6dB, 4dB, 3dB, 1.5dB and 0dB. Unlike the simulations performed using the Nelder-Mead optimization approach of the previous chapter, the simulations in case of the line fitting approach is performed concurrently with RLS since none of Nelder-Mead algorithm's drawbacks are present in this algorithm.

As can be seen from the figures, the line fitting approach together with discrete-time under-parameterized models can be used profitably to estimate the process parameters (k, τ, τ_d). The estimated parameters are always very close to the real ones even in case of low SNRs (high measurement noise) up to 0dB. This also confirms what was mentioned in Chapter 2 that discrete-time under or over-parameterized model could be used profitably to obtain the process frequency response.

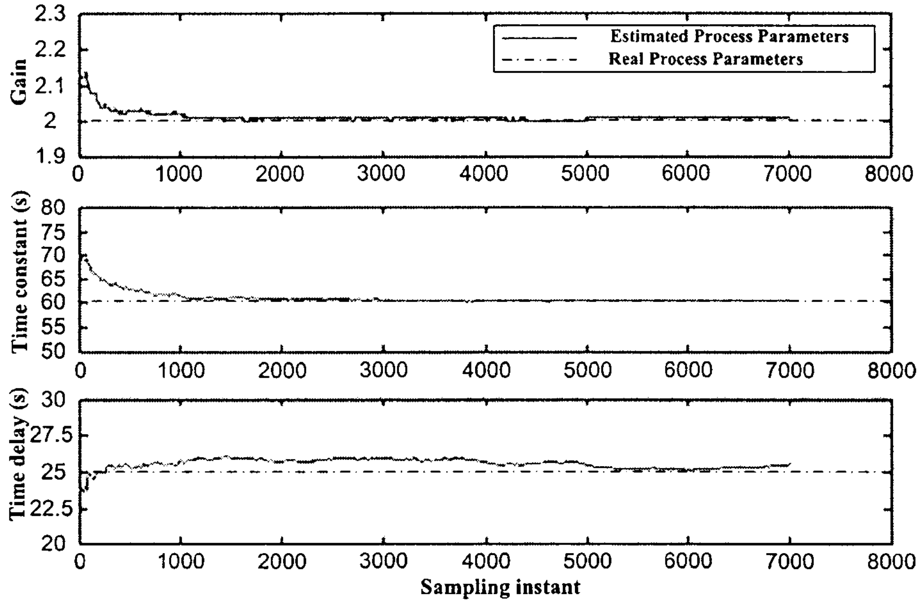

Figure 4.1: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 14$dB

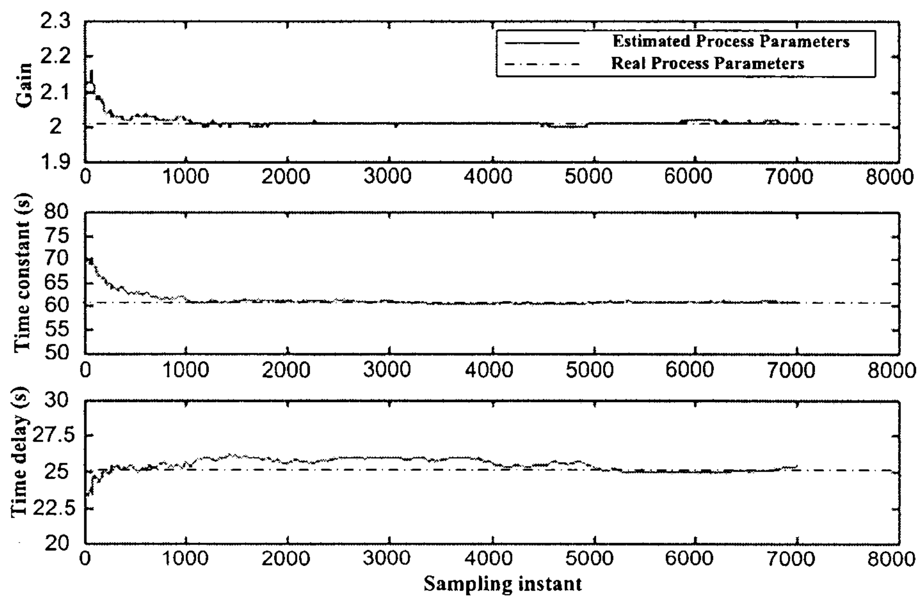

Figure 4.2: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 10$dB

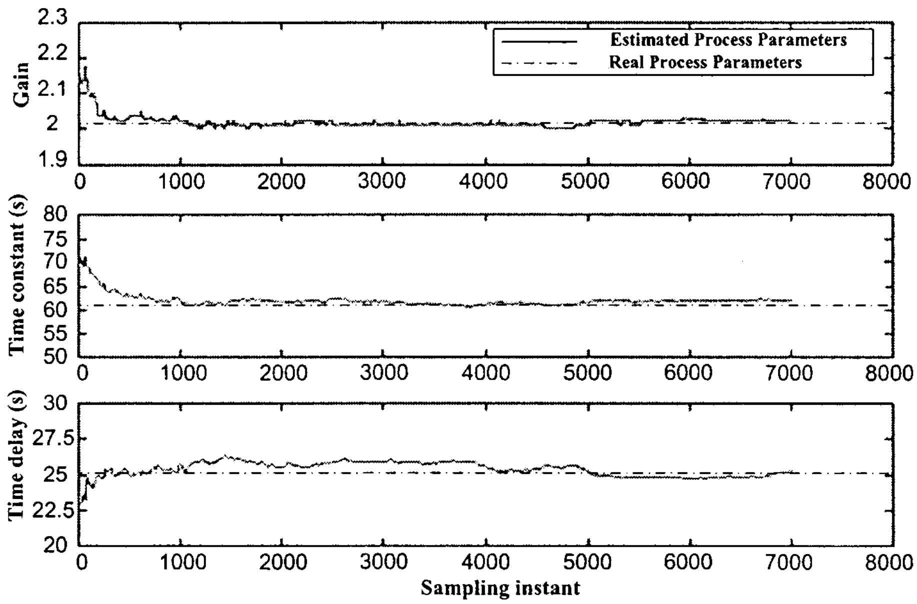

Figure 4.3: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 6$dB

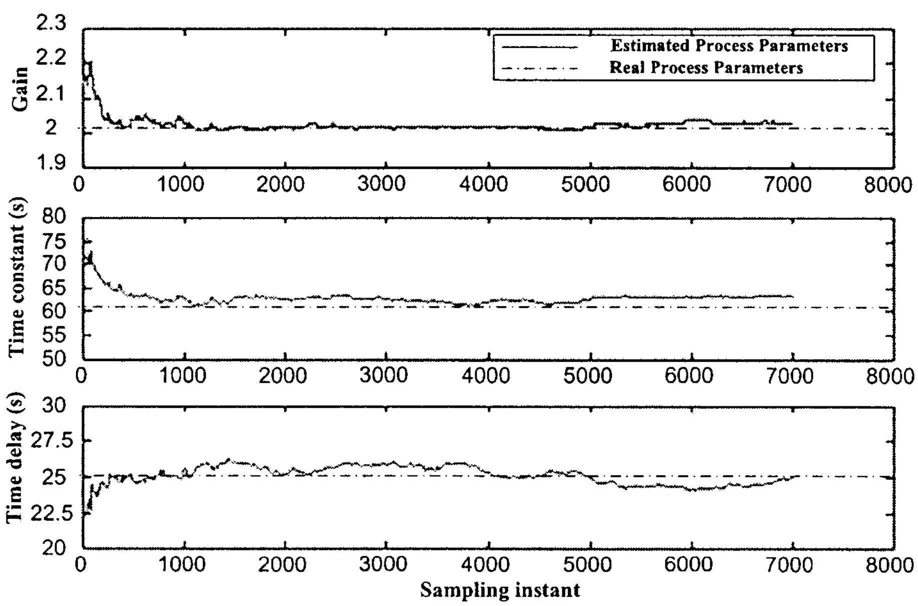

Figure 4.4: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 4$dB

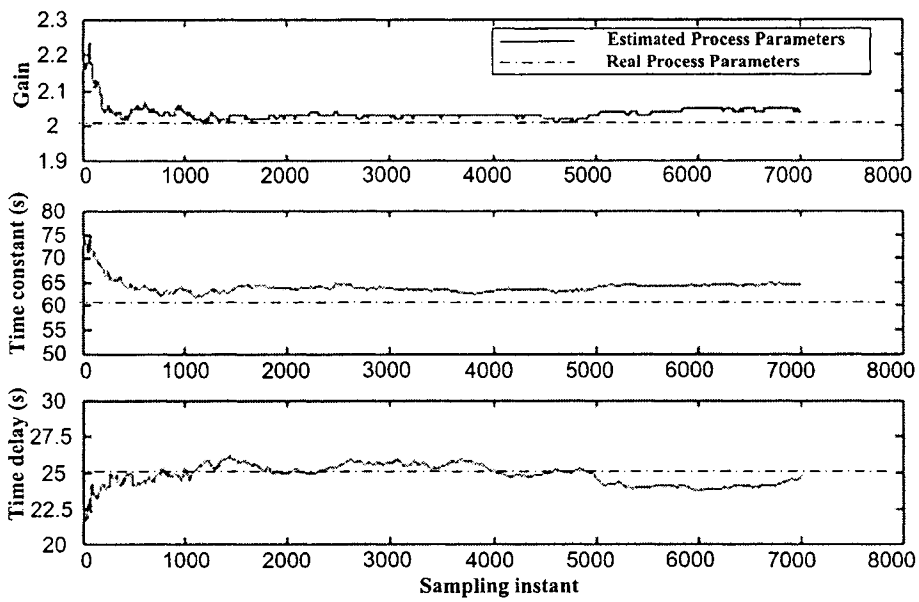

Figure 4.5: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 3$dB

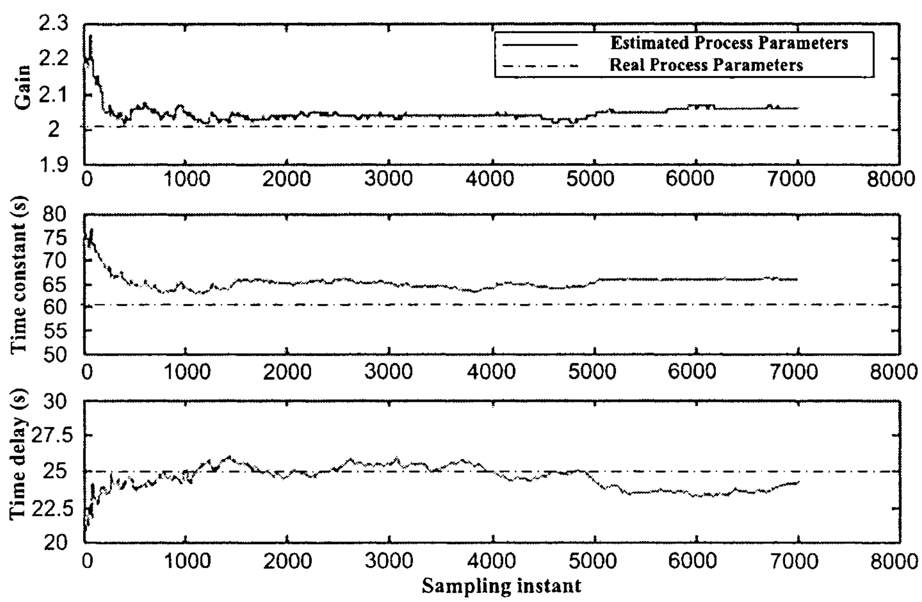

Figure 4.6: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 1.5$dB

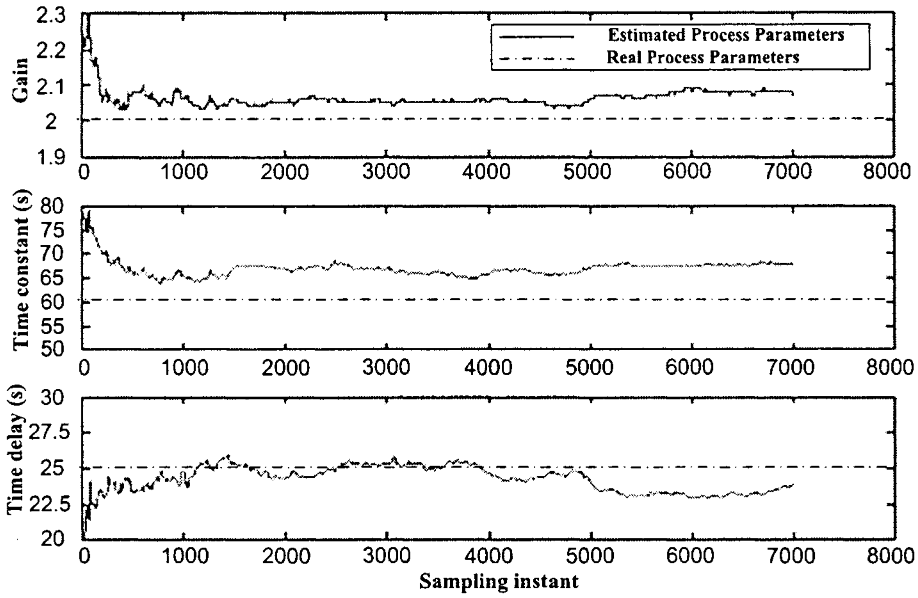

Figure 4.7: The estimated model parameters (k, τ, τ_d) with the under-parameterized model structure of (1-3-0) and $SNR = 0$dB

4.3.2 Process 2

Like in Chapter 3, consider

$$G(s) = \frac{2e^{-25s}}{300s+1} \tag{4.15}$$

with the discrete-time transfer function at a sampling period of $T_s = 15s$ described as follows:

$$G(z^{-1}) = \frac{z^{-N_{du}}(b_1 z^{-1} + ... + b_{N_u} z^{-N_u})}{1 + a_1 z^{-1} + ... + a_{N_y} z^{-N_y}} \tag{4.16}$$

Since under-parameterized structures are tested in Process 1, over-parameterized structures are selected here to evaluate the effect of different SNRs on the estimated process parameters (k, τ, τ_d) when using line fitting approach. The over-parameterized structure used here is (1-5-0), with $N_y = 1$, $N_u = 5$ and $N_{du} = 0$. The process input and output are first filtered by using low pass FIR filters before being fed to the RLS algorithm. Both the cut off frequency of each FIR filter and the frequency range used to calculate the frequency response of the over-parameterized model are set as those used in Process 2 in Chapter 3. Figs. 4.8 to 4.14 show the

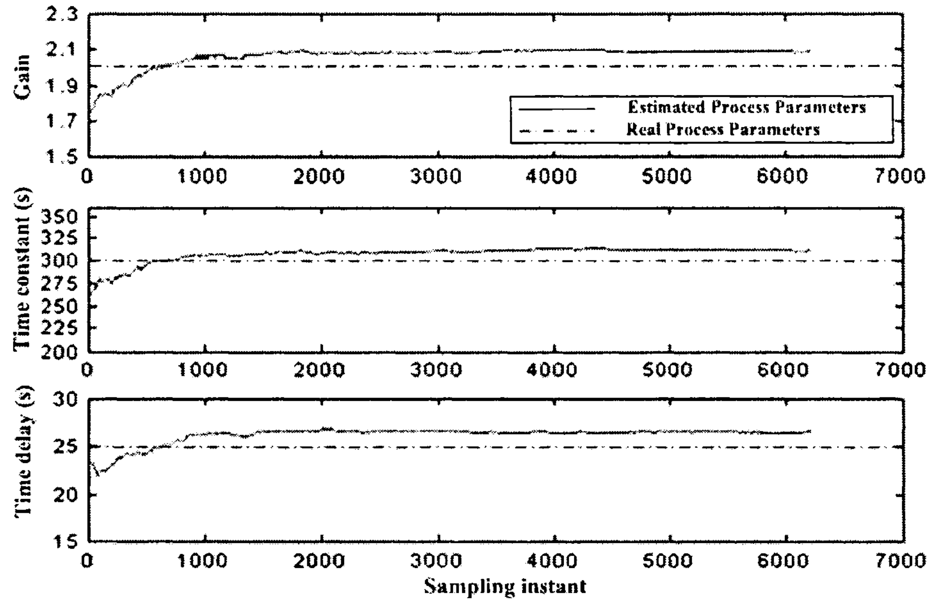

Figure 4.8: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 14dB

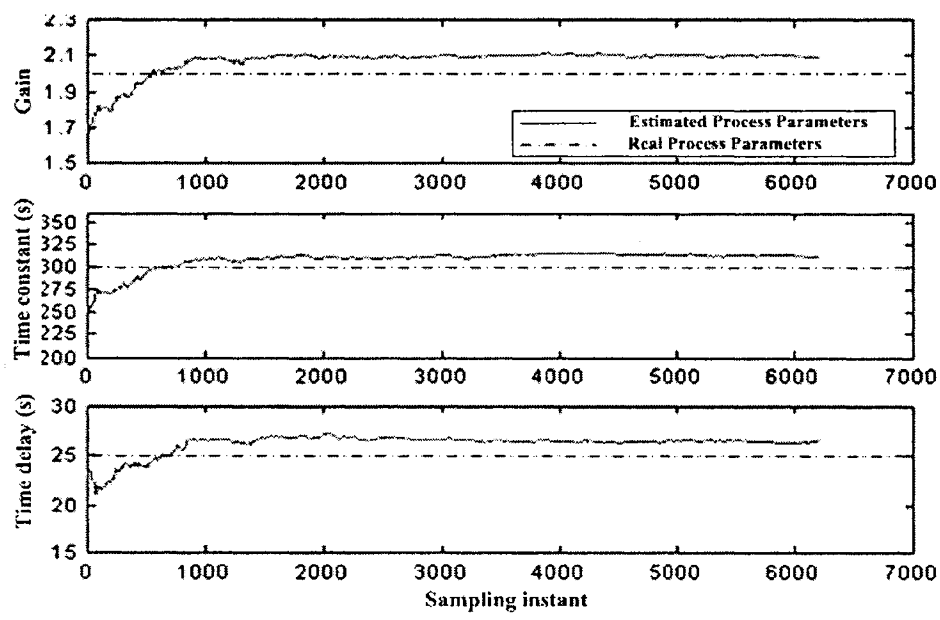

Figure 4.9: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 10dB

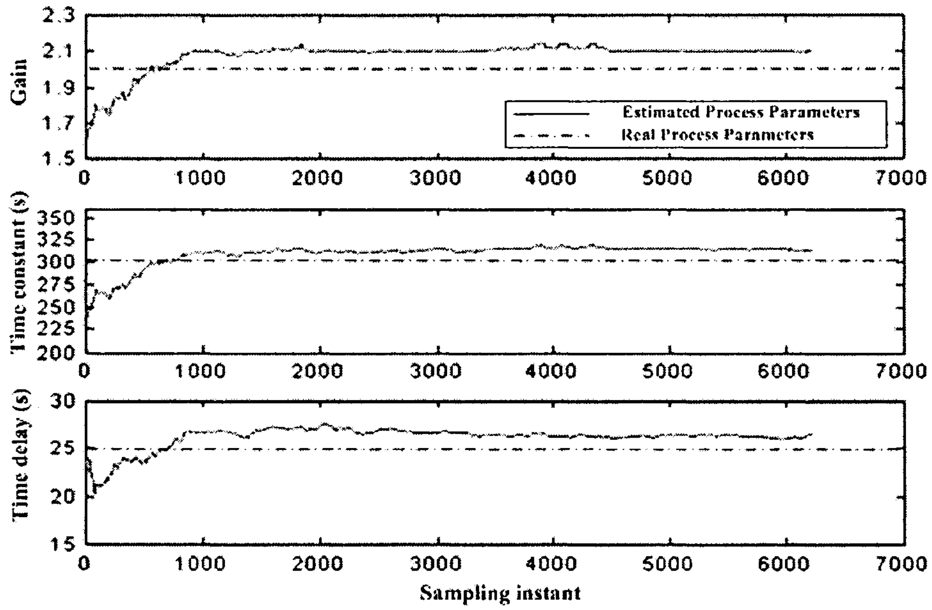

Figure 4.10: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 6dB

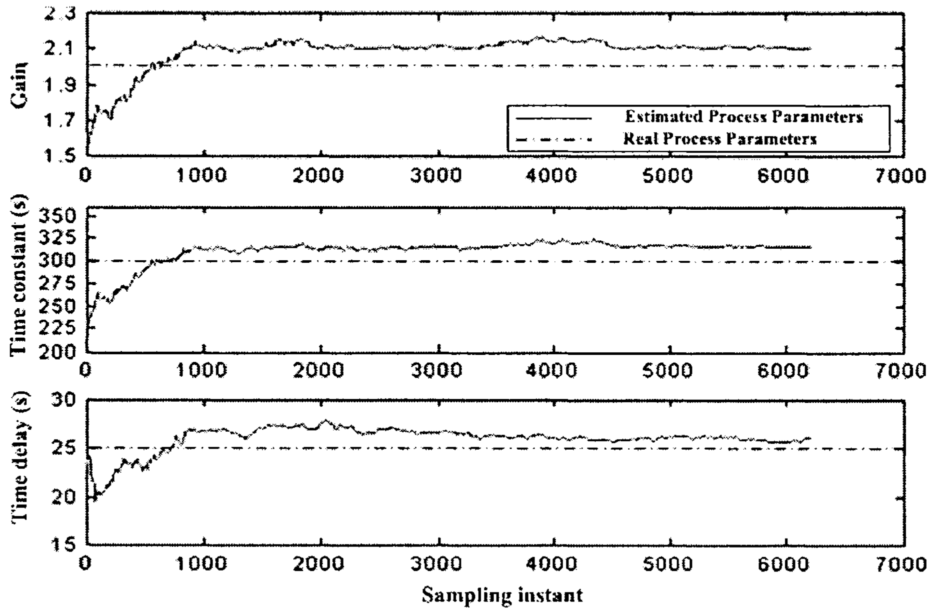

Figure 4.11: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 4dB

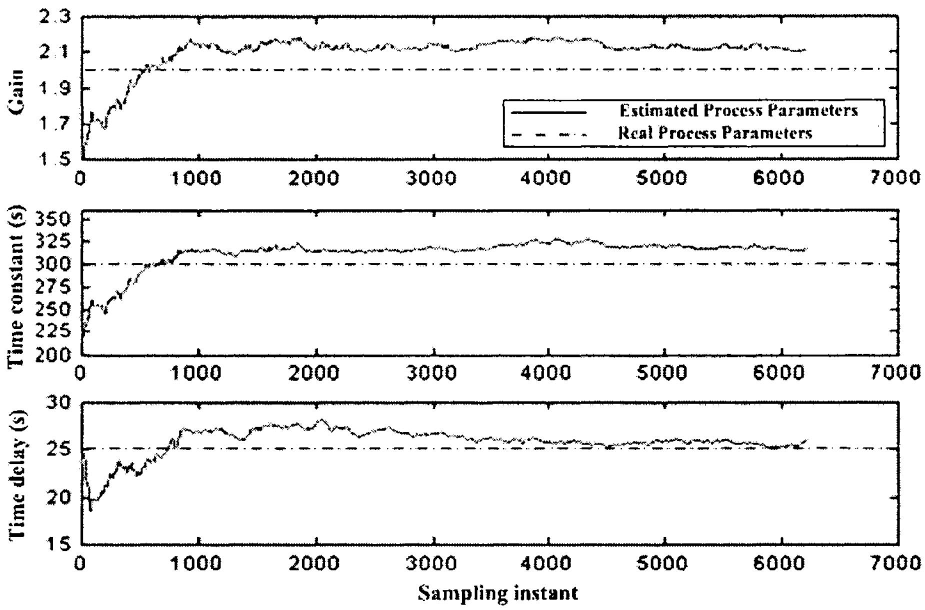

Figure 4.12: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 3dB

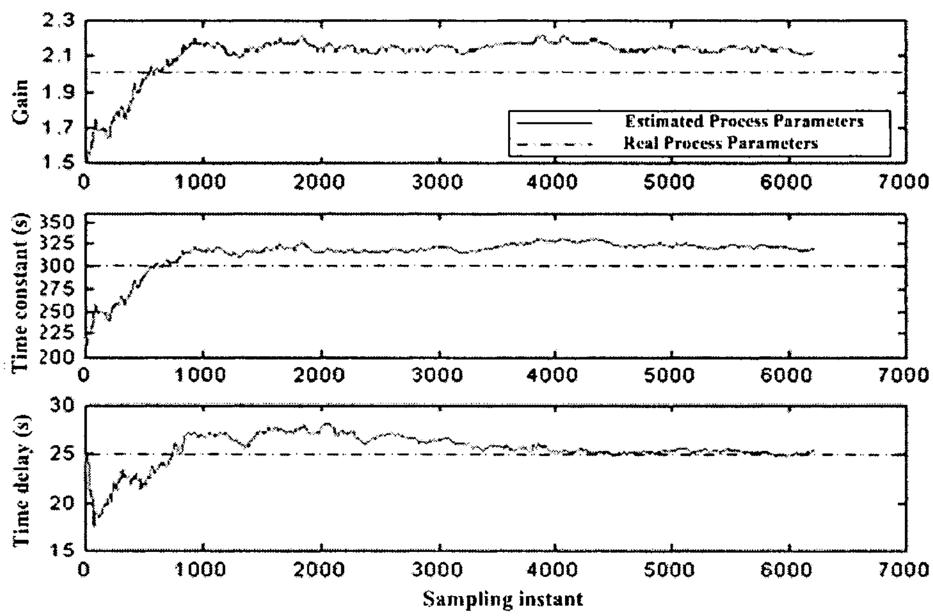

Figure 4.13: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 1.5dB

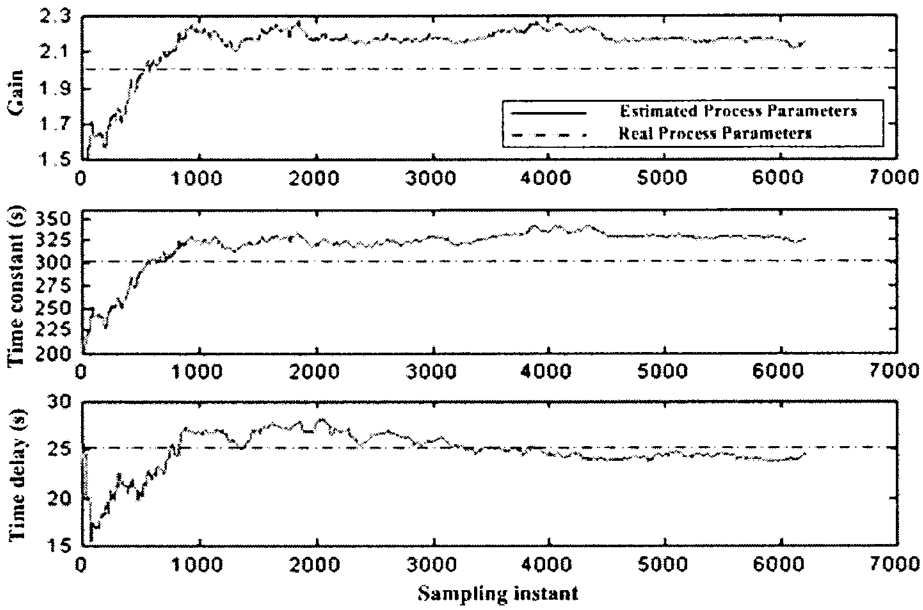

Figure 4.14: The estimated model parameters (k, τ, τ_d) with the over-parameterized model structure of (1-5-0) and SNR = 0dB

estimated process parameters (k, τ, τ_d) versus sampling instants with different SNRs. The SNRs used in this simulation are 14dB, 10dB, 6dB, 4dB, 3dB, 1.5dB and 0dB. It is apparent from all the figures that the estimated process parameters are converging to values that are very close to the real parameters. This means that the line fitting approach and discrete-time over-parameterized models can be employed together profitably to estimate these parameters (k, τ, τ_d). These estimated parameters, as can be seen from the figures, are always very close to the real ones even in low SNRs up to 0dB.

4.4 Conclusions and Observations

In this chapter, it is observed that no matter what the structure of the discrete-time model identified by RLS algorithm is (under or over-parameterized), its frequency response can be used with the line fitting approach to estimate the process parameters (k, τ, τ_d) of a first order plus dead time process. This line fitting approach has been tested on two different FOPDT processes with different discrete-time model structures and with different SNRs. The estimated process parameters (k, τ, τ_d) are always very close to the process parameters even with high measurement noise (low SNR) up to SNR=0dB. Also, line fitting approach can provide acceptable and reliable estimation results within a small time in comparison with the Nelder-Mead optimization approach. Moreover, using under or over-parameterized models to obtain the frequency response of process has made the line fitting approach a powerful *on-line* identification technique. Thus, these estimated process parameters identified by the line fitting approach can be employed efficiently to design a controller that can meet the desired specifications in a self-tuning approach. Last but not least, all simulations performed previously on Process 1 and Process 2 using either the Nelder-Mead optimization approach in Chapter 3 or the line fitting approach in this chapter assume the processes are linear time invariant processes. The application of the Nelder-Mead optimization approach and the line fitting approach to slowly time-varying processes will be discussed in the next chapter.

Chapter 5

Slowly Time-Varying FOPDT Process Estimation

5.1 Introduction

In several industrial problems, it is of interest to consider the situation in which the parameters are slowly time-varying [3]. In practice, there are many different sources of variations, and there is usually a mixture of internal and external influences [3]. The underlying reasons for the variations are in most cases not fully understood [3]. This is because most industrial processes are very complex and not well understood; it is sometimes neither possible nor economical to make a thorough investigation of the causes of the process variations [3]. This is an important reason for using recursive identification methods in practice so that the identification algorithm might track these variations [4]. This is handled in a natural way in the weighted recursive least squares criterion by assigning less weight to the older measurements that are no longer representative for the system. This means that the forgetting factor should be less than one in order to have the old measurements in the criterion exponentially discounted [4,13].

Process parameters, in practice, might change either continuously but slowly or abruptly but infrequently [4]. Parameters identification by using the line fitting and Nelder-Mead approaches of a FOPDT process whose parameters are changing continuously but slowly over time will be studied in this chapter. In case of linear time invariant parameters, the frequency responses calculated from the discrete-time under or over-parameterized model, which is assigned for the process by the user, at different sampling instants are almost identical with the frequency response of the process particularly at the low frequencies. This is true because the process parameters are not changing over time. On the other hand, in case of slowly time-varying parameters, the frequency responses of the discrete-time under or over-parameterized model at different sampling instants will not match each other. This is normal since the process parameters are not the same as time goes on.

However, the process parameters estimated by the line fitting or the Nelder-Mead approaches must follow these changing parameters. The line fitting and Nelder-Mead approaches perform the identification at every sampling period, and only depend on the corresponding frequency response calculated at the same sampling period from the under or over-parameterized model.

5.2 Frequency Response Estimation of Slowly Time-Varying Processes

Frequency response of the process used by the line fitting and Nelder-Mead approaches is estimated from a discrete-time under or over-parameterized model assigned for the process, as illustrated in Chapter 2. The parameters of this discrete-time under or over-parameterized model, as explained in Chapter 2, can be estimated from the process input and output by using recursive least squares algorithm [6, 21]. The frequency response calculated from this under or over-parameterized model at each sampling period, as long as the process parameters are not changing abruptly, can track the frequency response of the process at this particular sampling period. It can also be used by the line fitting approach or the Nelder-Mead approach to estimate the process parameters. To show how the frequency response calculated from the discrete-time under or over-parameterized model of a slowly time-varying process matches the process frequency response, an example is given.

Example:

Consider a process (Process 3) described by:

$$G(s) = \frac{2e^{-40s}}{200s+1} \tag{5.1}$$

Assume this process starts slow varying with time at a particular point and finally ends up as:

$$G(s) = \frac{2.5e^{-45s}}{250s+1} \tag{5.2}$$

Fig. 5.1 shows this variation of the parameters of this Process 3 versus sampling instants with sampling period of $T_s = 15s$. As can be seen from the figure, Process 3 remains linear time invariant (LTI) and can be described by (5.1) up to the sampling instant 2000. After that, Process 3 parameters start slow varying with time until the sampling instant 4000. Afterwards, Process 3 becomes again LTI and can be described by (5.2). In presence of these slowly time-varying process

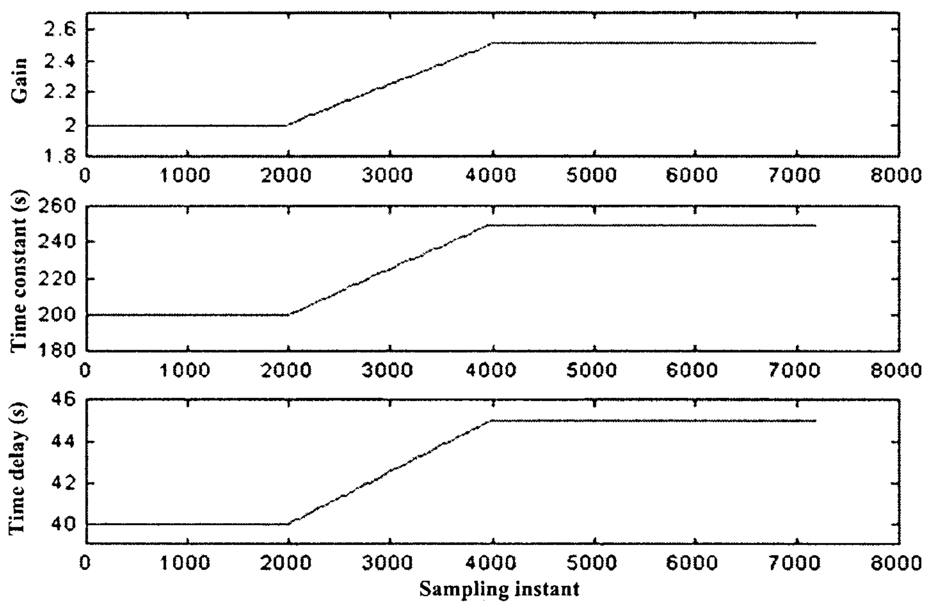

Figure 5.1: Process 3 parameters versus sampling instant

parameters, three different sampling instants have been chosen to see whether or not the discrete-time under or over-parameterized model based frequency responses are tracking or almost matching the real frequency responses at these three sampling instants. Two of these sampling instants are chosen at 1500 and 5000 where Process 3 is described by (5.1) and (5.2), respectively; whereas the third sampling instant is selected at 3000 where Process 3 is assumed to be described by LTI transfer function of (5.3) which is obtained by reading the values of parameters from Fig. 5.1 at this sampling instant. Fig. 5.2 shows the frequency responses of the processes described by (5.1), (5.2) and (5.3).

$$G(s) = \frac{2.25e^{-42.5s}}{225s + 1} \tag{5.3}$$

To estimate the frequency responses at the sampling instants 1500, 3000 and 5000, an over-parameterized model with the structure (1-5-0) as described in (5.4) has been assigned for Process 3 with a sampling period of $T_s = 15s$.

$$G(z^{-1}) = \frac{b_1 z^{-1} + ... + b_5 z^{-5}}{1 + a_1 z^{-1}} \tag{5.4}$$

Fig. 5.3 shows the discrete-time over-parameterized model parameters versus sampling instant. Figs. 5.4 to 5.6 show the frequency responses from the over-

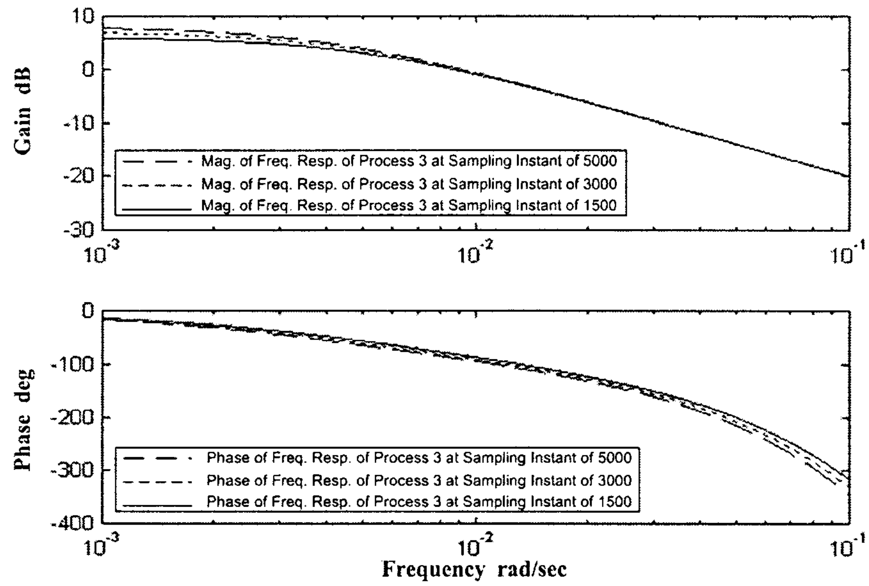

Figure 5.2: Frequency responses of the processes of (5.1), (5.2) and (5.3)

parameterized models at the sampling instants 1500, 3000 and 5000 respectively.

Although the discrete-time over-parameterized model parameters are not converging and varying over time, the frequency responses calculated at each sampling period are not affected by theses variations as explained in Chapter 2. As can be seen from Fig. 5.4, the frequency response of the over-parameterized model calculated at the sampling instant 1500 is almost identical with the frequency response of the process described by (5.1). Furthermore, the frequency response of the over-parameterized model computed at the sampling instant 5000, as can be noticed in Fig. 5.6, is almost identical with the frequency response of the process described by (5.2). In addition, the frequency response of the over-parameterized model calculated at the sampling instant 3000 and the frequency response of the process described by (5.3) are also very close to each other as can be seen from Fig. 5.5. However, this matching is not as good as the first two cases. But, generally, this is normal and acceptable as the RLS algorithm used to identify the discrete-time over-parameterized model parameters can often track the process parameters variations reasonably well if these deviations are slowly time variant [4,13,14]. This can be done by either selecting a suitable forgetting factor or resetting the covariance matrix. However, in this work, it is preferred to use a forgetting factor instead of resetting the covariance matrix as in processes it is neither possible to know when the process parameters variations occur nor whether such variations are abrupt.

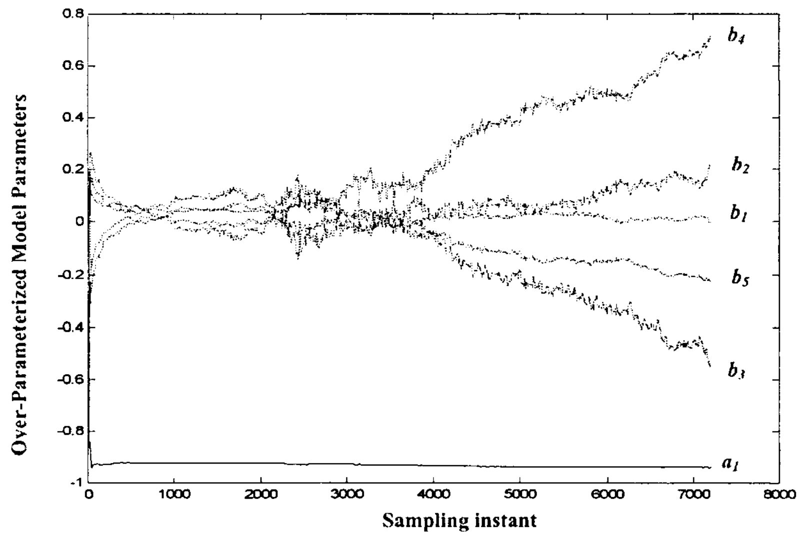

Figure 5.3: The discrete-time over-parameterized model parameters.

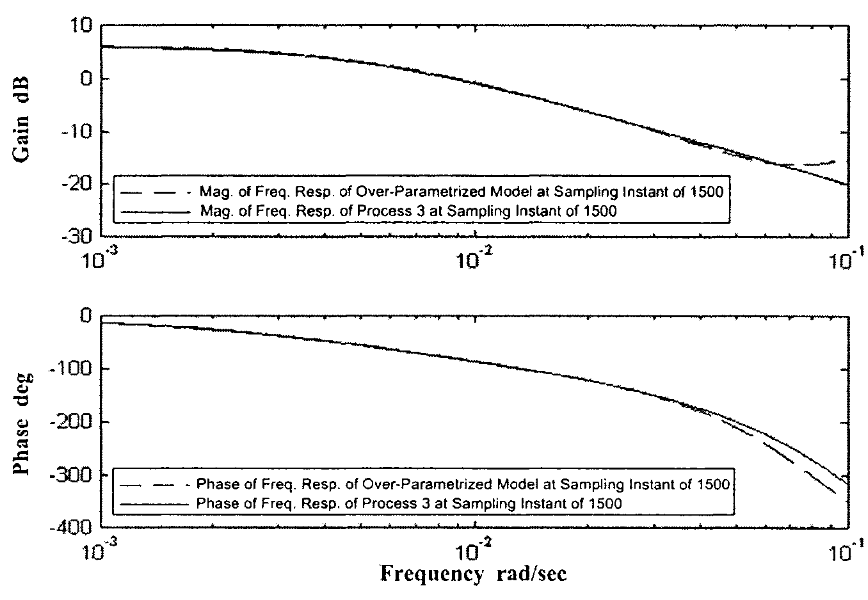

Figure 5.4: The frequency response of the over-parameterized model and the process frequency response at sampling instant 1500.

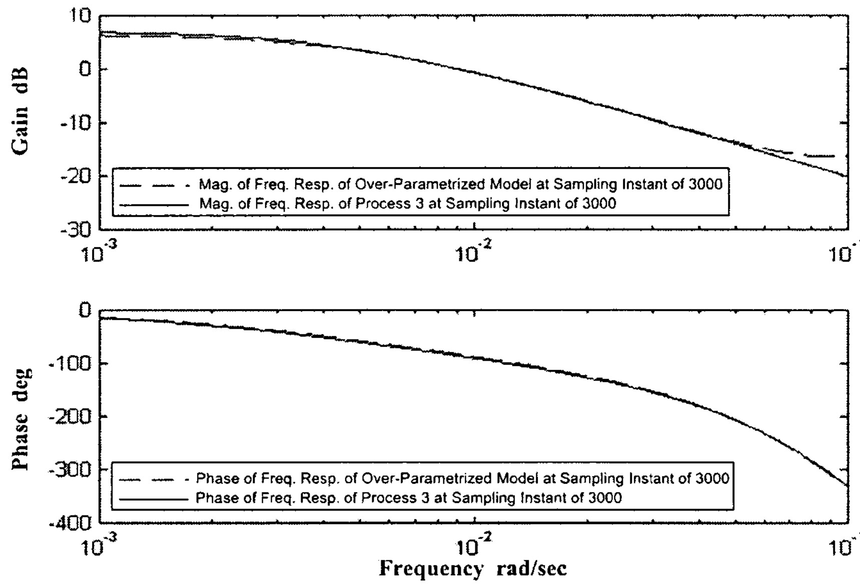

Figure 5.5: The frequency response of the over-parameterized model and the process frequency response at sampling instant 3000.

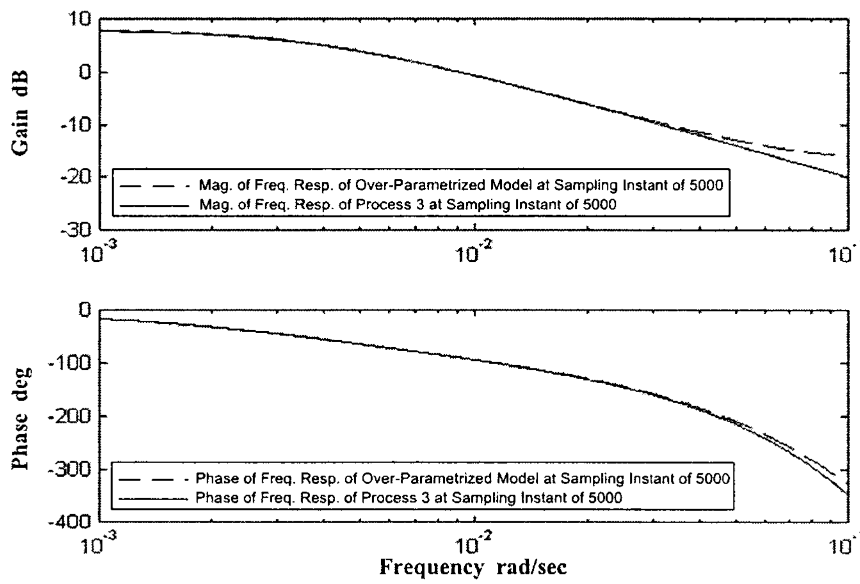

Figure 5.6: The frequency response of the over-parameterized model and the process frequency response at sampling instant 5000.

5.3 Simulation

The Nelder-Mead optimization approach and the line fitting approach have been employed to estimate the parameters of a slowly time-varying process (Process 3) with sampling period of $T_s = 15s$. The process input and output are first filtered using FIR filters before being fed to the RLS algorithm to identify the parameters of the discrete-time under or over-parameterized model assigned for Process 3 as in the previous chapters. This discrete-time model of this process is structured as in (5.4), with $N_y = 1$, $N_u = 5$ and $N_{du} = 0$. Since there are six unknown parameters to be estimated by the RLS, the normalized cut off frequency of each FIR filter is calculated based on the seventh harmonic of the persistent excitation spectrum. With a sampling period of 15s and an excitation period of 1200s, the normalized cut off frequency of each FIR filter is equal to ($7 \times \frac{1/1200}{1/15/2} = 0.175$). Similar to the last chapters, the frequency response of the discrete-time model is calculated at each sampling period for a certain range of frequency points. The upper limit of this range is chosen to be less than the un-normalized cut off frequency of each FIR filter. Both the Nelder-Mead optimization algorithm and the line fitting approach are applied to the same process with the same operating circumstances. Figs. 5.7 to 5.13 show the estimated process parameters (k, τ, τ_d) versus sampling instants with different SNRs. The SNRs used in this simulation are 14dB, 10dB, 6dB, 4dB, 3dB, 1.5dB and 0dB. The dotted lines in the figures represent the process parameters (k, τ, τ_d) of Process 3. However, the solid lines represent the estimated process parameters (k, τ, τ_d) when the Nelder-Mead and the line fitting approaches are applied.

It can be seen from the figures that the estimated process parameters (k, τ, τ_d) especially in case of SNRs greater than 1.5dB are converging to values that are very close to the real ones. On the other hand, in case of SNRs of 1.5dB and 0dB, the estimated process parameters (k, τ, τ_d) are converging to values close to the real ones, but are not as good as those for SNRs greater than 1.5dB. In general, these figures have shown that both Nelder-Mead and line fitting approaches can reasonably estimate the parameters (k, τ, τ_d) of slowly time-varying first order plus dead time processes even in presence of high measurement noise.

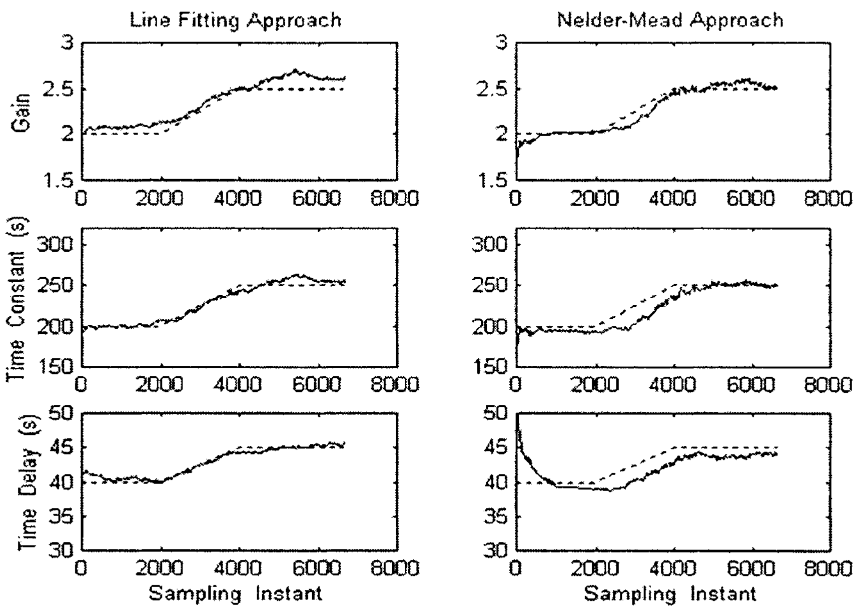

Figure 5.7: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=14dB.

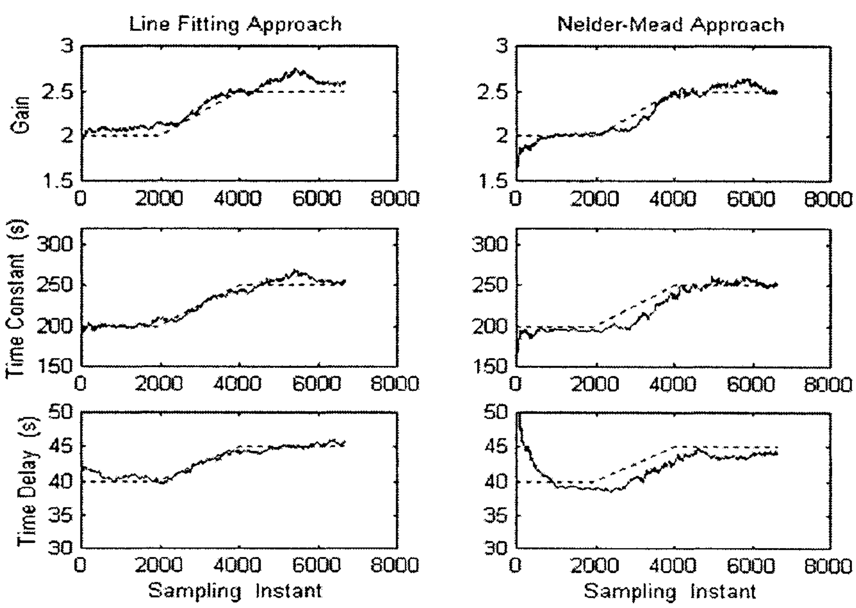

Figure 5.8: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=10dB.

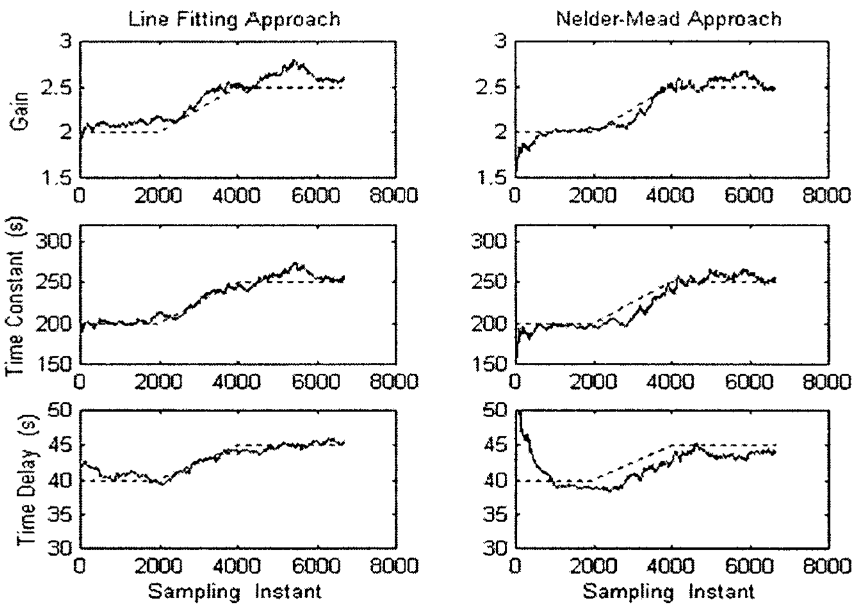

Figure 5.9: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=6dB.

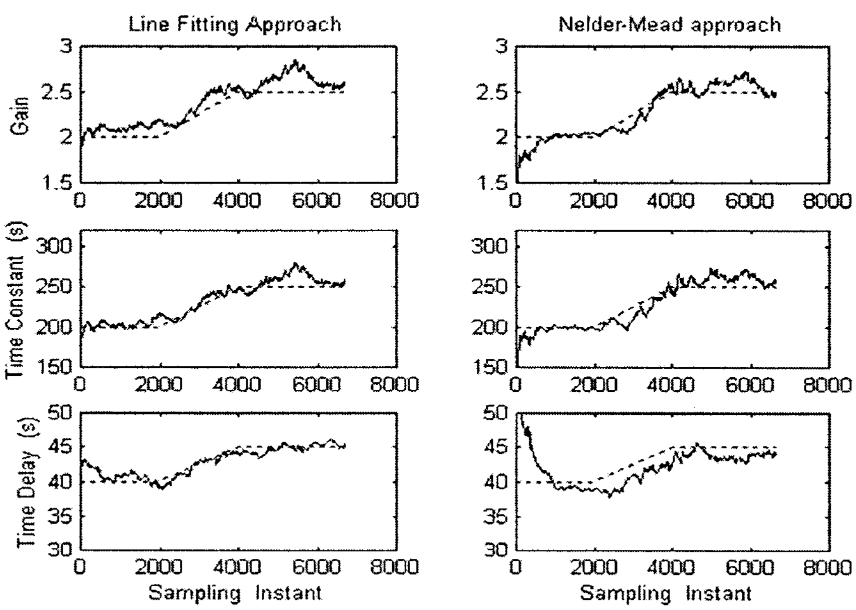

Figure 5.10: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting Nelder-Mead approaches with SNR=4dB.

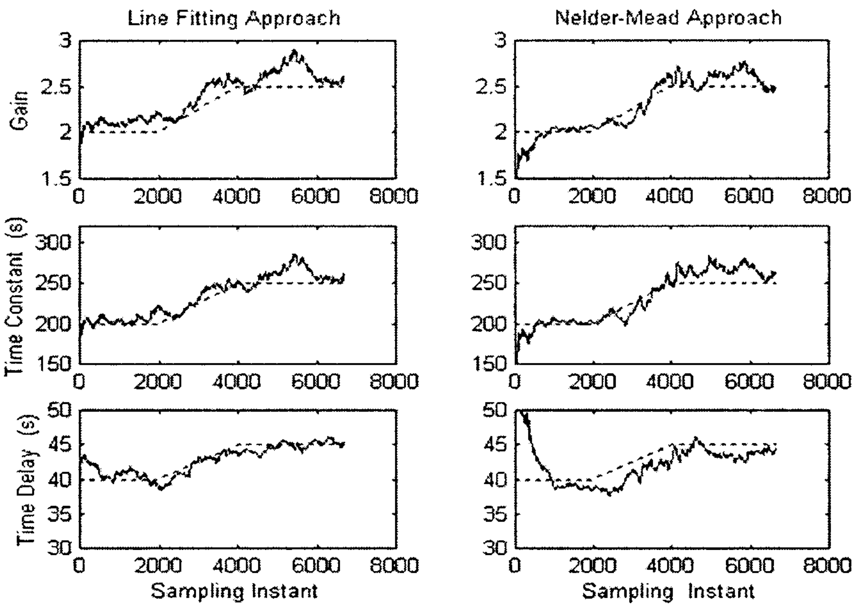

Figure 5.11: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=3dB.

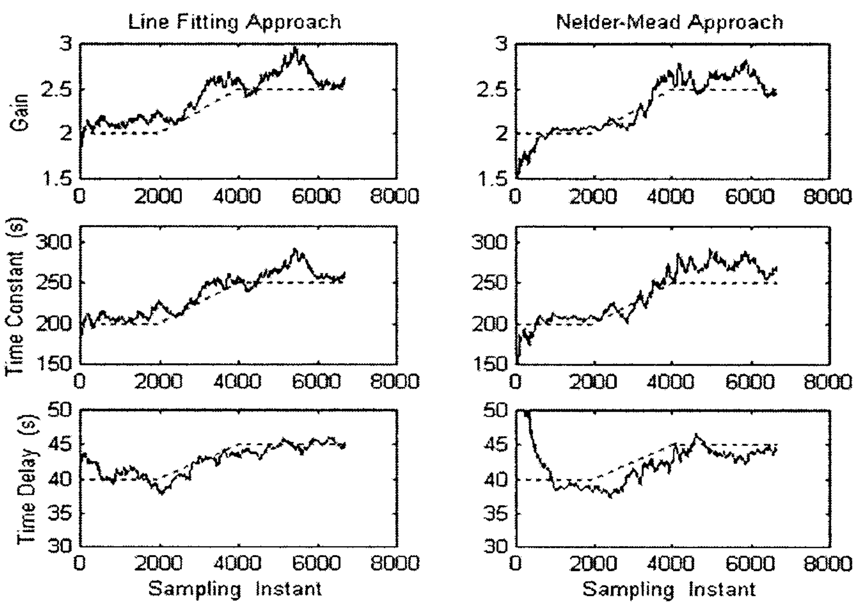

Figure 5.12: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=1.5dB.

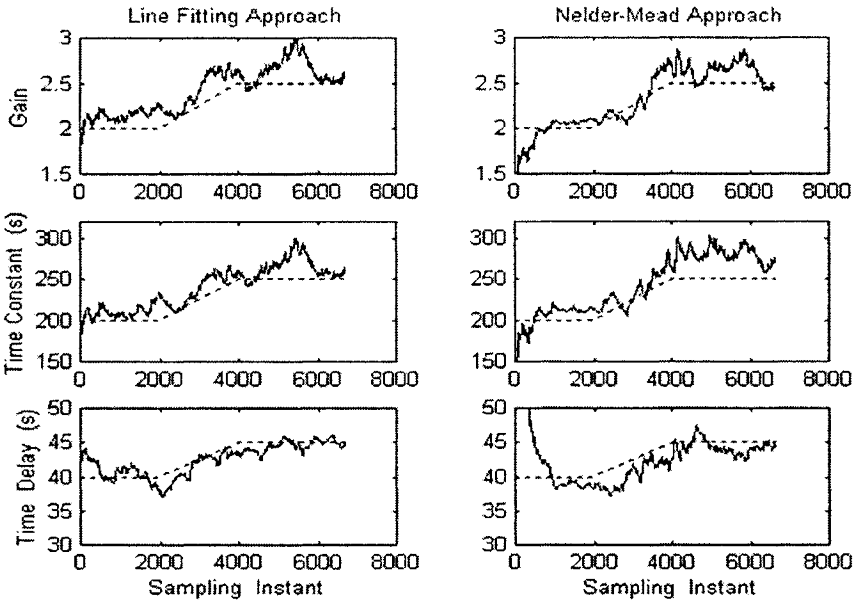

Figure 5.13: The real and estimated process parameters (k, τ, τ_d) identified by both line fitting and Nelder-Mead approaches with SNR=0dB.

5.4 Conclusions and Observations

It has been noticed that no matter what the structure of the discrete-time model whose parameters are estimated by the RLS is, its frequency response will fit or at least track the frequency response of the slowly time-varying process, provided a suitable forgetting factor for the RLS algorithm is used. Also, it has been observed that both line fitting and Nelder-Mead optimization approaches can be used effectively to estimate the parameters of a slowly time-varying first order plus dead time process. Both techniques are tested while taking into consideration different SNRs up to 0dB. It has been seen that the process parameters estimated by the two approaches are very close to the real ones even in presence of noise up to SNR=1.5dB. On the other hand, the estimated process parameters in case of measurement noise of 1.5dB and 0dB are still acceptable and reasonable, but not as good as those in case of SNRs greater than 1.5dB. Last but not least, it has been also noticed that the process parameters (k, τ, τ_d) estimated by the line fitting approach are much better than those estimated within the same time by the Nelder-Mead approach.

Chapter 6

Distillation Column Experiment

6.1 Introduction

In this section, the line fitting approach studied in the last few chapters is implemented on an experimental system, a distillation column, in the Chemical Engineering Unit Operations Laboratory at Lakehead University. The objective of this experimental study is to apply this technique on a real-world problem as opposed to computer simulation of transfer functions considered in the examples of the previous chapters. A schematic diagram of the distillation column is given in Fig. 6.1. The column usually runs on methanol-water mixture. The top and bottom temperature of the distillation column are controlled with individual SISO PID controllers. As can be seen from Fig. 6.1, a temperature controller is used to regulate the bottoms composition at tray 2. This temperature controller output is, then, cascaded to a reboiler steam flow control. Both the feed flow and the temperature of the top tray (tray 11) are controlled by a feed flow controller and a temperature controller based on the reflux ratio. The level of the mixture in the bottom, on the other hand, is regulated by a level controller.

Line fitting approach is only applied to the distillation column bottoms temperature to estimate the process parameters using both the controller output and measured temperature output obtained from the bottoms temperature control loop. A first order plus dead time (FOPDT) model was assumed to represent the process describing the distillation column bottoms temperature. It is normal that the data obtained from a real system are corrupted by noise, which will in turn affect the identification results if the noise is not reduced. Also, the recursive least squares (RLS) system identification algorithm used to identify the under or over-parameterized model employed to estimate the process frequency response requires persistent excitation. The set-point used should have sufficient magnitude to cause the excitation of system dynamics and should be persistent [3]. In this experiment, a square wave has been used for exciting the set-point, as in the previous chapters. The excitation, thus, is selected as a periodic temperature set-point bumps between

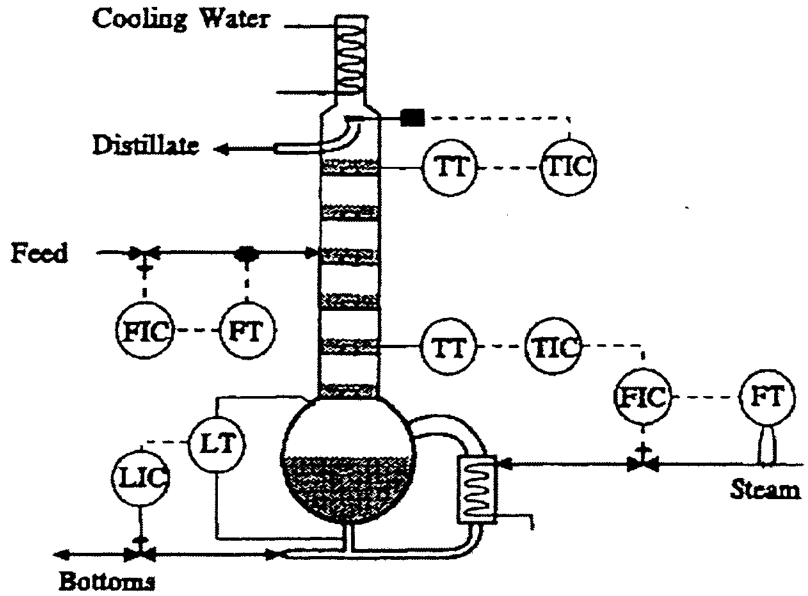

Figure 6.1: A schematic diagram of a distillation column [22].

97-98C for the bottoms temperature loop, with a period of 1500s for three cycles. The set-point period selected ensures that the closed loop system output tracks the set-point before the next bump is applied (i.e. steady state is achieved). It is required that the excitation frequency should be such that it provides rich excitation in the frequency range of interest. Also, the signal to noise ratio is affected by the magnitude of the excitation. This amplitude also controls the frequency range used for the identification, since the magnitude in the frequency spectrum will depend on it. At higher frequencies, the effective magnitude frequency spectrum drops and noise perturbs the process parameters estimation [18, 22, 48]. As the identification of the process parameters is to be carried out in terms of the frequency response, an adequate sampling rate is required. The sampling rate should be at least 20 times faster than the frequency range of interest [22, 47, 48]. The sampling period selected for this experiment is $T_s = 15s$.

RLS also requires initial values for both of the covariance matrix and the parameters that are to be estimated. The selection of these initial conditions will also affect the RLS estimate's convergence. A forgetting factor of 0.997 is used. The performance of the RLS algorithm will also depend on the noise corrupting the system. For RLS to converge quickly, the noise corrupting the system has to be reduced. This is because in case of no noise or at low noise condition, the RLS is solving a set of (mostly) linear deterministic equations for unknown parameters [3, 31].

RLS also requires that the data being used in the system identification (i.e. process input and output in open loop system identification, and set-point and process output in closed loop system identification) have to be free of DC bias. This is because the linearized models are valid around steady state operating conditions which are set by the DC bias applied to the process input. Perturbing the set-point applied to the closed loop system usually induces small changes on the process input and output without changing the DC bias, and the linearized model is only seeking to interpolate the behavior due to these changes. However, the change in DC bias in process variables, when process changes or in presence of disturbance due to controller action, affects the identification severely unless the bias is appropriately factored out [21]. In case of off-line process identification, for a slowly time-varying process, the DC bias can be eliminated on a cycle to cycle basis. Since the line fitting approach is to be implemented on-line in this experiment, an alternative approach for DC bias removal has been applied.

6.2 DC Bias Removal

The DC bias estimation can be implemented by taking the mean of set of data as stated previously, or by using a filter. The exponential filter used in digital control systems to attenuate noisy signals can be used to estimate the DC bias. This exponential filter is a standard form of low-pass filters. It is a first-order lag with unity gain [24], and can be described as follows:

$$Y(s) = \frac{1}{T_f s + 1} X(s) \tag{6.1}$$

Where $Y(s)$ and $X(s)$ are the filter output and input, respectively. T_f is the filter time constant, an adjustable parameter.

To obtain a discrete-time version of this filter, the idea is to start with the Laplace transfer function and replace the s variable with the z-transform variable. The key relationship [24] between the operators (s and z) is given as:

$$s = \frac{1 - z^{-1}}{T} \tag{6.2}$$

Substitute (6.2) into (6.1) and replace the Laplace transforms with the z-transforms of the sampled signals to obtain

$$Y(z) = \frac{1}{T_f(\frac{1-z^{-1}}{T}) + 1} X(z) = \frac{T}{(T_f + T) - T_f z^{-1}} X(z) \tag{6.3}$$

From (6.3), it is possible to obtain the recursive formula by replacing the z-transforms with the sampled values as per:

$$(T_f + T)Y_n - T_f z^{-1} Y_n = TX_n \tag{6.4}$$

Equation (6.4) can be rewritten as follows:

$$Y_n = \alpha Y_{n-1} + (1 - \alpha)X_n \tag{6.5}$$

Where $\alpha = \frac{T_f}{T_f + T}$ is always less than 1.

Equation (6.5) is used with $\alpha = 0.99$ to estimate the DC bias from the process input and output. Fig. 6.2 shows the process input and output, obtained from the experiment of the distillation column bottoms temperature control, with their estimated DC bias. Fig. 6.3 shows the process input and output after bias removal.

The figures indicate that the DC bias is estimated properly as time goes on. By using this type of DC bias removal, it is possible to estimate the DC bias at every sampling period. Also the DC bias estimated using this technique is not constant over a number of samples and it starts responding immediately to any process changes. The faster convergence of DC bias estimation has the potential benefit of achieving faster convergence in system identification.

6.3 Distillation Column Bottoms Temperature Control Loop Process Estimation

6.3.1 Line Fitting Approach

The process input and process output, obtained from the distillation column bottoms temperature control loop, are fed to FIR filters after removal of DC bias. The filtered data is fed to the RLS algorithm to identify the discrete-time under or over-parameterized model which is structured as follows (with a sampling period of 15s):

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1}} \tag{6.6}$$

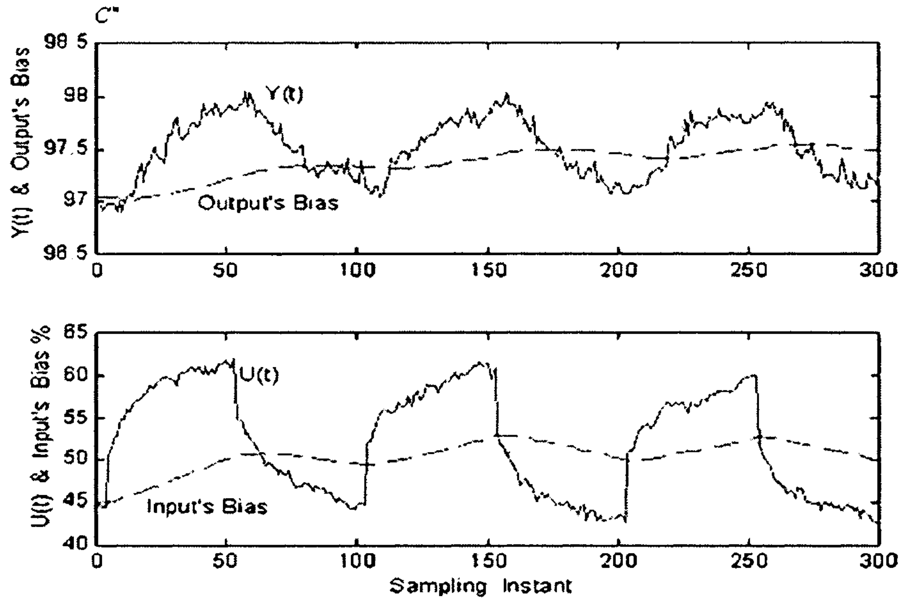

Figure 6.2: Distillation column bottoms process output and input with their estimated DC bias.

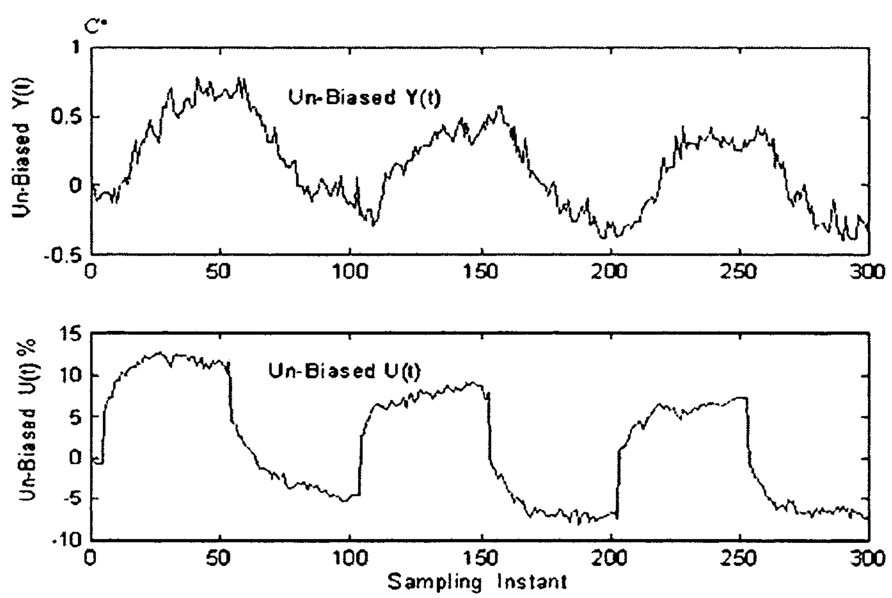

Figure 6.3: Distillation column bottoms process output and input after DC bias removal.

At each sampling period, the discrete-time under or over-parameterized model parameters are estimated by the RLS algorithm, and then used to calculate the frequency response of this discrete-time model at the same sampling period for a selected range of frequency points. Fig 6.4 shows these discrete-time model parameters versus sampling instants. The frequency range specified for this experiment consists of only three frequency points. As mentioned in the last chapters, this frequency range has to be chosen suitably, where the large frequency point of this range should be less than the un-normalized cut off frequency of each FIR filter. Since the discrete-time model has four unknown parameters to be estimated, the cut off frequency of the two FIR filters is selected at the fifth harmonic of the persistent excitation spectrum. With a sampling period of 15s and an excitation period of 1500s, the normalized cut off frequency of each FIR filter specified based on the fifth harmonic of the persistent excitation spectrum is equal to ($5 \times \frac{1/1500}{1/15/2} = 0.1$). Therefore, the frequency response of the discrete-time model is calculated up to a frequency less than the un-normalized cut off frequency of each FIR filter which is equal to ($\frac{0.1}{15} = 0.0067$ rad/sec). Fig. 6.5 shows the estimated process parameters (k, τ, τ_d) of the distillation column bottoms temperature control loop process when the line fitting approach is applied. To make sure these process parameters are estimated properly, an open loop step test is performed immediately after finishing the line fitting approach experiment.

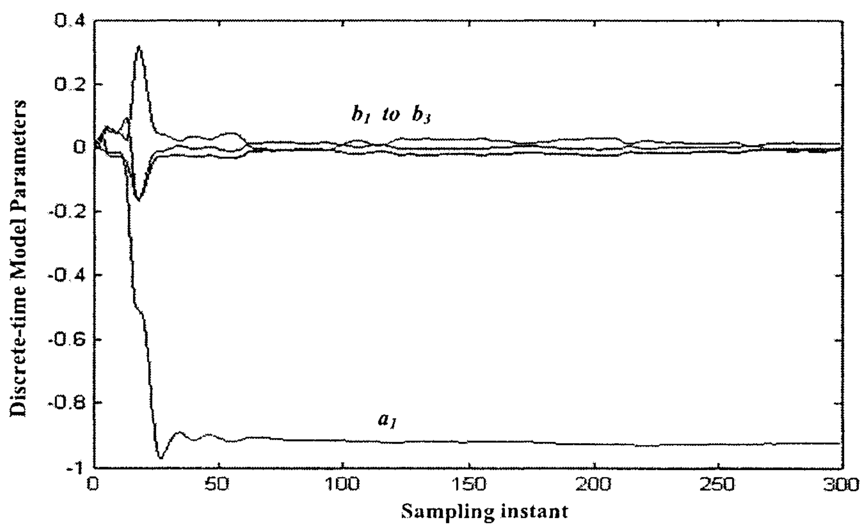

Figure 6.4: The estimated distillation column temperature control loop process parameters (k, τ, τ_d).

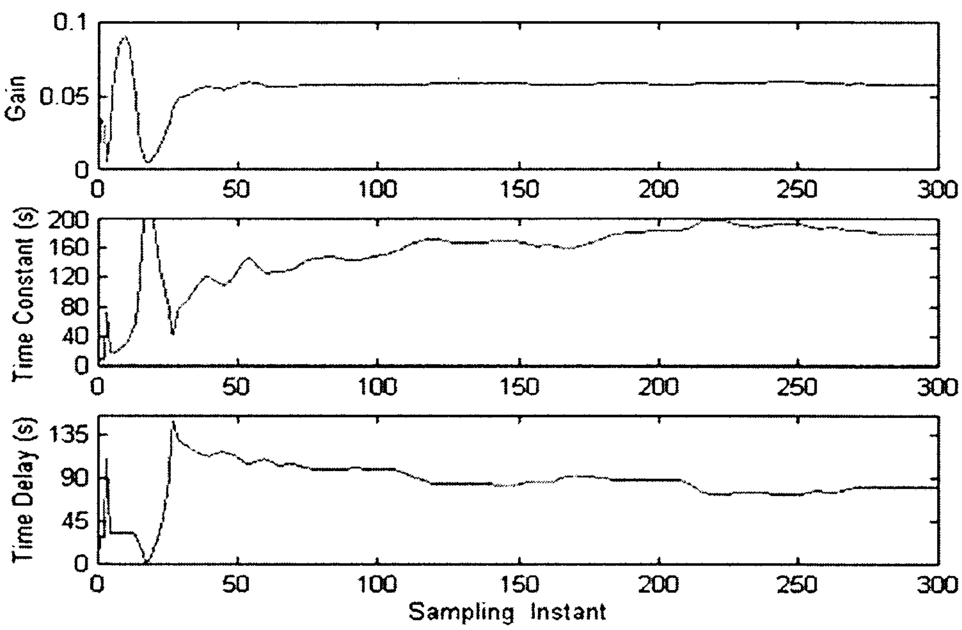

Figure 6.5: The estimated distillation column temperature control loop process parameters (k, τ, τ_d).

6.3.2 Step Response Test

The open loop output response of a process to a step change in input can be used to estimate the model of this process if its structure is known. This type of process identification is classified as one of the off-line techniques. Usually, performing and analyzing step response tests for first order plus dead time processes [25] have potential difficulties:

- In real systems, a perfect step change might not be formed properly. This is because the process equipment such as pumps and control valves take some time to perform the step change rather than instantaneously move. However, if this time is small compared to the process time constant, a reasonable step response may be obtained.
- The response curve to the step input will not represent the behavior of first order processes if the process is not first order .
- In case of high noise corrupting the data, it is difficult to estimate reasonably the FOPDT process parameters from this output.
- Since the step test is performed in an open loop sense, disturbance might affect the process response without the operator's knowledge.

To fit the first order plus dead time model (FOPDT) described by the following equation

$$G(s) = \frac{ke^{-\tau_d s}}{\tau s + 1} \tag{6.7}$$

to the distillation column bottoms temperature control loop measurements obtained from a step change in the input of magnitude $M\%$, the following steps as illustrated in [25] have to be taken to calculate the process parameters from a response shown ideally in Fig. 6.6:

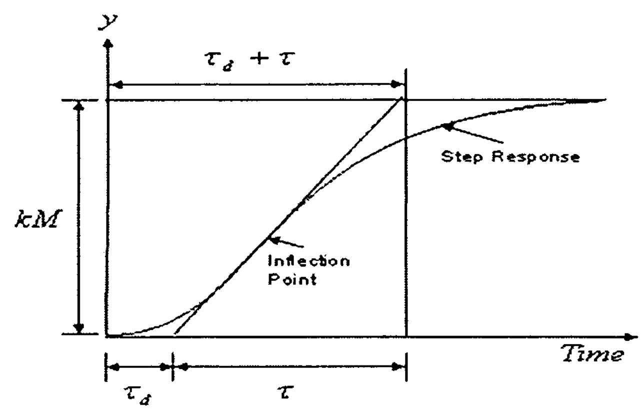

Figure 6.6: A step response of first order plus dead time process [25].

- The steady state value calculated from the step response is equal to the process gain k multiplied by the step change M. Thus, the process gain k can be calculated by obtaining the ratio between the step response steady state value kM and the step input change M.

- Process time delay τ_d can be obtained by calculating the time interval between the step change starting time and the time specified by the intersection between the time axis and the tangent line drawn at the step response inflection point.

- The time interval between the step change starting time and the time calculated at the intersection between the step response steady state line (where $y = kM$) and the tangent line drawn at the step response inflection point is corresponding to the time $(\tau + \tau_d)$. Thus, the process time constant can be calculated by subtracting the time delay τ_d from this time.

A step test is performed on the distillation column bottoms temperature control loop immediately after performing the line fitting approach experiment. This is done by making the controller work on the manual mode for sometime until the steady state response is reached. Afterwards, a bump test is done by the changing the controller output from 41.0 to 45.0%. Figs. 6.7 and 6.8 show the distillation column bottoms temperature control loop step response to the step change in process input. By applying the previous steps on the obtained step response curve, the distillation column process parameters are approximately estimated.

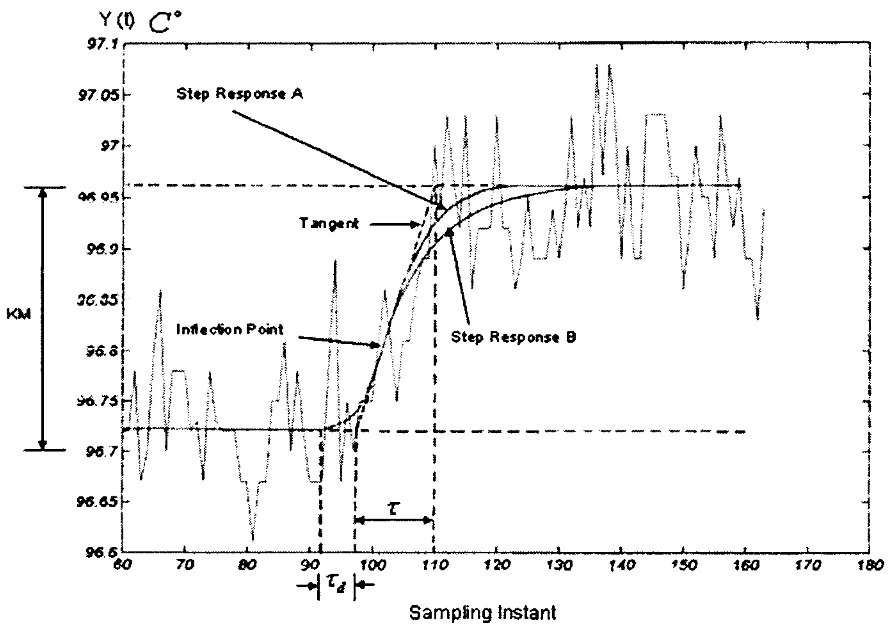

Figure 6.7: The distillation column bottoms temperature output response to a step input change.

As can be seen from the Fig 6.7, the step response curve is corrupted with noise and most likely disturbances as well. In such situations, typical of most industrial processes, it is impossible to estimate the process parameters accurately. This is because it is difficult to fit a first order plus dead time model to the measured step response. Thus, the accuracy of the estimated process parameters (k, τ, τ_d) will always depend on the precision of the drawn FOPDT model curve on the process response curve. As can be seen in Fig. 6.7, two different step response curves (step response A and B) have been drawn, and it is hard to decide which one fits the measurement data better. In addition, the process parameters could be estimated inappropriately if the inflection point is not selected reasonably. However, in this step test experiment, the objective is to know at least approximately the estimated process parameters. This is to confirm that the estimated process parameters cal-

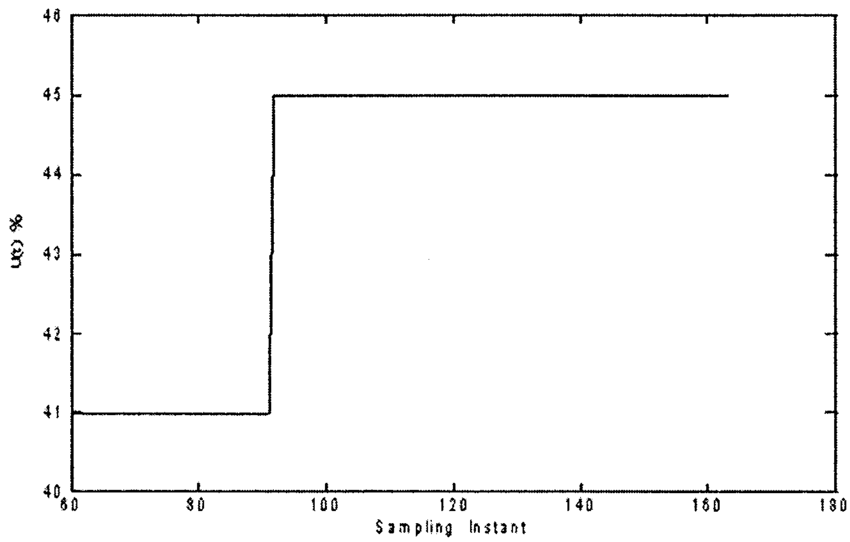

Figure 6.8: The distillation column bottoms temperature input step change.

culated by the line fitting approach are very close to those approximately obtained by the open loop step response test. Therefore, the three steps mentioned above are applied on Fig. 6.7 to approximately identify the FOPDT process parameters (k, τ, τ_d) , which are obtained as:

$$\text{The process gain } k \approx \frac{kM}{M} \approx \frac{0.24}{4} \approx 0.06$$

$$\text{the process time delay } \tau_d \approx (97 - 92) \times T_s \approx 75s$$

$$\text{and the time constant } \tau \approx (110 - 97) \times T_s \approx 195s.$$

These approximately estimated process parameters as can be seen are close to those parameters obtained by the line fitting approach which are plotted in Fig. 6.5.

6.4 Conclusions and Observations

Line fitting approach has been successfully applied on a distillation column bottoms temperature control loop to estimate on-line the process parameters (k, τ, τ_d). The process input and output are subjected to a discrete-time exponential filter to remove the DC bias before being fed to FIR filters to reduce the noise. An under-parameterized model has been assigned for the discrete-time model to estimate the frequency response of the process. An open loop step response test has been performed on the distillation column bottoms temperature control loop in order to compare the estimated process parameters with those identified by the line fitting approach. It has been noticed that the process parameters (k, τ, τ_d) estimated by the line fitting approach are very close to those parameters estimated approximately by a step response test. This indicates that the line fitting approach is capable of being used in industrial situations to estimate the process parameters and in turn these estimated parameters can be employed in tuning the controller.

Chapter 7

Conclusions and Future Work

In this work, the frequency response of a discrete-time under or over-parameterized model at each sampling instant has been used to estimate the process parameters. This estimation has been performed using two different techniques, Nelder-Mead approach and line fitting approach. In the Nelder-Mead optimization approach, the linear time-invariant or slowly time-varying process parameters are estimated reasonably even with SNR of 0dB. However, when using the Nelder-Mead approach, there are some parameters such as the maximum iteration number, the simplex size step and the starting point of the simplex vertices have to be selected properly in order to get good estimation results especially when the sampling period being used is small. On the other hand, when using the line fitting approach, the linear time-invariant or slowly time-varying process parameters are estimated properly and reasonably even with SNR of 0dB. These process parameters identified by the line fitting approach are much better than those obtained within the same time by the Nelder-Mead optimization approach. Also, the line fitting approach has been successfully applied on a distillation column bottoms temperature control loop to estimate on-line the process parameters (k, τ, τ_d). In addition, it has been noticed that the process parameters (k, τ, τ_d) estimated by the line fitting approach are close to those parameters estimated by an open loop step response test. To sum up, the line fitting approach and the frequency response of an under or over-parameterized model have been shown to estimate on-line FOPDT parameters, and are capable of being applied profitably in industrial situations.

The future work has two parts. The first part is to try to take advantage of the FOPDT parameters estimated by the line fitting approach to design and/or tune a PID controller or any other controller to meet the desired specifications. The second part is to extend the line fitting approach to work with second and high order plus dead time processes.

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