THE IMPACT OF THE TEACHER'S QUESTIONS ON THE LEARNING OF PART-WHOLE RELATIONS AND A BENCHMARK MODEL IN FRACTIONS

by

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Abstract

The focus of this case study was to explore how my questions as a teacher impacted my students' construction of part-whole relations and their use of a benchmark model in learning fractions. The research conducted in my classroom comprised of 12 Gr. 4 students and 12 Gr. 5 students. There were 13 boys and 11 girls. A pre-test, instruction and post-test sequence was used. The teaching unit was developed to assist the students in building a benchmark model for comparing and ordering fractions and to develop an understanding that a fraction is a relationship between its parts and the whole. A math class consisted of a small mini-lesson, which focused the students' thinking, a contextual problem that they solved in pairs, and a congress in which the strategies and solutions were debriefed and discussed. At the beginning of the unit most, but not all, of my students struggled with these concepts, but by the end of the unit most were comfortably using the benchmark model and had a very good understanding that a fraction was a relationship between its parts and the whole.

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The Impact of Teachers' Questions

Chapter 1: Introduction

1.1 Context of the Study

School mathematics programs have undergone significant changes over the last twenty years, and more importantly, the role of the teacher has begun to shift from a dispenser of knowledge to a facilitator of knowledge development, one who orchestrates learning through effective questions, contexts and discussion (Stein, Engle, Smith, & Hughes, 2008) prodding the learner to construct his or her own understanding. This change in emphasis, is central to the instructional practice known colloquially as, "reform." Though reform instruction is a relatively new practice for teachers, it has nonetheless undergone many transformations already. The one to which I am referring originated in the 1980s in response to a perceived failure of traditional teaching methods (Battista, 1999). Battista suggested that the instructional focus of most school mathematics classes had stressed an endless sequence of memorization of facts and procedures, which were often forgotten or misunderstood by students. It was for this reason that in 1989 the United States' National Council of Mathematicians (NCTM) conceived of five standards for teaching mathematics: 1) worthwhile mathematical tasks, 2) discourse between teacher and students, 3) discourse between student and student, 4) teacher acceptance of the use of different methods and manipulatives to solve problems, 5) teacher engagement in ongoing assessment and analysis of teaching and learning (NCTM, 1989, p. 25-63). Reformers hoped that these standards would create a shift towards developing, and deepening, students' conceptual understanding of mathematics instead of what Battista (1999) referred to as “mindless mimicry mathematics” (p. 427) which is to say, mathematics instruction that had students mimicking the teacher's lessons without an emphasis on understanding. In addition, this shift focused on presenting students with complex problems, whereby they could formulate and test the validity of their personally constructed mathematical ideas and draw their own conclusions.

1.1.1 Changes in reform - two "generations" of reform teachers.

Stein, Engle, Smith, and Hughes (2008) contend that over the last twenty years of mathematics reform there have been two waves of implementation: the first generation and the second generation. Their idea of first and second generation does not refer to the chronological age of the teachers, but rather to a philosophy or stage in the progression of understanding effective reform instruction. They observed that in the first generation of reform, the roles of the teacher and of the students were not well defined. The emphasis was placed on encouraging the students to think through problems, and then praising students for their unique strategies. The congress, or whole group discussion time, was used as an opportunity to practise listening skills; teachers' questions tended to focus on having students explain why they used a particular strategy, or asking students to explain their strategies further. Many teachers felt that in order for discussion to be focused on student thinking, teacher thoughts or interjections were to be avoided; both teaching and learning needed to come from the students (Stein et al., 2008). Stein et al. added that in the first reform generation, students' strategies often became inefficient; students and teachers would remain stuck on how to move towards more efficient strategies or how to move towards connecting the strategy to a bigger mathematical idea. In addition, Sherin, Mendez, and Louis (2000), contrary to popular belief at the time, suggested that student talk by itself did not necessarily improve students' learning. In much of the research, the role of the teacher has come under sharper scrutiny. Stein et al. (2008) proposed that teachers needed to move towards a second generation of instructional reform, in which the emphasis would be on directed and purposeful student talk, which is choreographed by the teacher but indirectly led. Some researchers have suggested that the key to creating this type of environment may be through effective class discussion and purposeful critical questions, asked by the teacher or by the students.

They contended that if teachers were able to implement the instructional reform aligned with second wave methods (in particular strong questioning techniques), then their students would construct a deeper understanding and be more proficient in mathematics (Stein et. al., 2008). This contention may extend to areas of mathematics such as fractions, which have proven particularly difficult for children (and adults alike) to learn well. At the elementary level children often have difficulty constructing a full understanding of the part-whole relationship in fractions. This knowledge is foundational to students' later ability to calculate using fractions (Fosnot & Dolk, 2000, Van De Walle, 2007). It may be that reform-oriented instruction, in particular well-constructed questions, will lead to greater student learning in this challenging area.

1.2 Purpose of the Study

The purpose of this study is to explore how teacher questions impact students' construction of part-whole relations and their use of a benchmark model in learning fractions. Although there are many problems that can arise, understanding part-whole relations is a key concept at the junior level and in future mathematical development.

1.3 Research Question

How do a teacher's questions impact student construction of part-whole relations and the students' use of a benchmark model in learning fractions? A benchmark model in the context of fractions is a model that showcases where pivotal fractions (0,1/2, 1/4, 3/4, 1) are located. In addition, students who use this model understand where other fractions are in relationship to these benchmarks.

Sub questions:

A) On what mathematical knowledge does a teacher draw in order to generate the questions used during the fraction math class? Mathematical knowledge in the context of this study is an understanding of mathematical big ideas, concepts and possible student models.

B) What planning is involved for the teacher when constructing these questions?

1.4 Key Terms

Within the context of the study the key terms are as follows:

Benchmark Model: A benchmark model in the context of fractions is a model that showcases where pivotal fractions (0,1/2, 1/4, 3/4, 1) are located.

Landscape of Learning: A developmental map of student progression through fractional number sense.

Teacher's Mathematical knowledge: An understanding of mathematical big ideas, concepts and possible student models.

Part-Whole Relationship in Fractions: A relationship between the part of a whole and its whole.

Reform Mathematics: A change in teaching practices. One to which I am referring originated in the 1980s in response to a perceived failure of traditional teaching methods.

Second Generation reform: Teaching emphasis is on directed and purposeful student talk, which is choreographed by the teacher but indirectly led.

1.5 Significance of the Research

As a beginner in reform practices, I started out as a first generation teacher and as I gained experience, read professional material, and observed reform practitioners, I have slowly moved towards becoming a second generation reform-based teacher. Over these years, I have observed teachers just beginning the process, teachers questioning the process, and teachers totally opposed to the idea of reform methods. Frequently, I have mentored teachers in the use of reform instruction and observed many of the challenges that come with implementing a new pedagogy in the classroom. Many of these struggles revolve around the use of questioning and "bringing out" the mathematics in the congress. Questioning is not a new topic of discussion. Teachers have been implementing this technique for years; however, questions have tended to be focused on initiation-response-evaluation (IRE) (Fuson, 2007) model of teaching. In this model teachers ask the questions and then wait for an anticipated response from their students. This pattern in turn prompts the teacher to ask further questions for clarification. Student communication often takes the form of answering the teacher's direct questions about gaps in learning, or what type of solutions need more practice; there is often no mathematical thinking involved except on the part of the teacher (Franke, Kazemi, & Battey, 2007). Ackles, Fuson, and Sherin (2004) suggest that for success to happen in reform education, teachers need to change their traditional teaching practices significantly, and develop a discourse community in their classrooms. In addition, Franke et al. (2007) note that although there is research about the role that the teacher plays in supporting discourse in the classroom, little is known about what teachers need to do to best support classroom discourse that uses students' discussion as instruction. In order for teachers to develop mathematical discussions that focus on the contributions of students, more research needs to be undertaken to understand the process, knowledge, and impact these questions have on students' development, understanding, and overall enjoyment of mathematics.

1.5 Contributions to the Community

This research will contribute to teachers, researchers and parents' understanding of the amount, and type, of work that is needed in order to use student talk as a focus for instruction, as well as the type of work necessary to focus that discussion on specific mathematical concepts. Fractions continue to be an area of great anxiety for adults and students alike. With effective use of questions in the classroom, teachers may be able to increase understanding of fractions in their classroom and reduce anxiety. In addition, questions may deepen understanding by creating dialogue among the students. By understanding how questions are created, the nature of the questions, and the impact they have on children's understanding, teachers may become more effective in their questioning techniques. This study may act as a model for future studies, the results of which may be of interest to classroom teachers and curriculum developers, possibly leading to further teacher professional development around instructional practices, and may promote reflective practices.

1.6 Bias and Limitations

There are some limitations to consider with this research. Firstly, I am the researcher and the teacher and therefore may be positively biased in my analysis of the results. Secondly, this is one case study of a Grade four/five classroom. Accordingly, the observations in this research are not representative of other classrooms. In addition, before this study was conducted I had established a collection of norms for the community; these norms gave students guidelines for discussion, because without them student discussion does not happen (Sherin, 2002). It is therefore a study that captures a period in time rather than the full development of questioning and its impact on the students.

Chapter 2: Literature Review

Introduction

The purpose of this section is to examine what the research has suggested concerning the challenges students may face when working with fractions, the role of the teacher in a reform context, the types of questions that teachers ask, and the impact teacher questions have on students' thinking.

2.1 Instructional Practices Leading to Poor Understanding of Fractions

The research suggests four underlying reasons that may result in students struggling with fraction concepts. The first is a natural progression of understanding which teachers tend to forget, moving too quickly through the curriculum for their students (Fosnot & Dolk, 2000; Lamon, 1996; Pothier & Sawada, 1983). Second is the instructional practices that result in poor understanding of part-whole relations (Cramer, 2002; Fosnot & Dolk, 2000; Kamii, 1999). Third is the teaching of algorithms and procedures without developing conceptual understanding (Battista, 1999; Cramer, 2002; Fosnot & Dolk, 2000; Kamii, 1999; Mack, 1999). Finally, emphasizing the teaching of and memorization of computational skills, resulting in poor problem solving skills with fractions (Asku, 1995).

2.1.1 Natural progression for learning fractions.

Much of the research has suggested that students progress through discernible stages while developing an understanding of fractions. The problem occurs when teachers either ignore these stages, or do not understand students' developmental path in learning fractions. Pothier and

Sawada's (1983) study of 43 students (Kindergarten to Grade 3) suggests that students progress through five stages when developing an understanding of fractions. The first stage is a *sharing stage*. At this stage students learn the basic language of fraction sharing along with a natural procedure for halving (Pothier & Sawada, 1983). Students understand that when there are two people, each person gets a piece of the whole but that piece does not necessarily equal a half. It is interesting to note that this is developed at a social level, which suggests that students do come to school with some fraction concepts and schemas from which to build their knowledge; this is in contrast to the traditional argument that the teacher imparts all knowledge to their students.

The second stage is a mastery of the *halving process* that students created in the earlier stage. The researchers contend that this is a critical stage in developing fraction concepts because it is here that students learn equivalency (1/2 = 2/4) at a basic level, when they start to share pieces equally and realize what happens to the whole piece as they share with more people, or that children can double the number of parts to obtain fractional parts whose denominators are half the size (Pothier & Sawada, 1983).

Pothier and Sawada's third stage is a development of *fair sharing*. At this stage students realize that partitions are classified as "fair" or "not fair." In addition, students also learn addition and subtraction of fractions (1/4+1/4= 2/4), when they give the pieces they create to each other. This critical stage was confirmed by Reyes (1999) who reported that students who struggled with creating benchmarks and equivalent fractions needed more practice representing equal fair shares. According to Reyes (1999) and the Ontario Curriculum benchmarks are critical fraction placements where students see other fractions are in comparison to those placements. Those critical fractions are 0, 1/2, and 1; sometimes, it also includes, 1/4, and 3/4.[1]

[1] When I refer to Benchmark or Benchmark model it is this definition to which I refer.

In the fourth stage students recognize the inefficiency in the doubling strategy when dealing with odd fraction denominators. At this stage students use a counting strategy to calculate thirds, fifths, ninths, and so on. Pothier and Sawada's developmental stages were replicated in later research by Lamon (1996) who followed students from grades 4 to 8 suggesting that students' progress from using inefficient calculation strategies to more efficient strategies. Lamon noted that although Pothier and Sawada's fifth stage, *using multiplication,* was described as theoretical in their research, her findings confirm that students can reach this fifth stage. In addition, Lamon also pointed out that students used their social sharing strategies to solve their particular problems. Ignoring the natural progression of student development is one aspect of teaching that often leads to further struggles in fractions (Pothier and Sawada, 1983).

2.1.2 Instructional practices that result in poor understanding of part-whole relations.

One of the most critical *big ideas*[2] that research explores is the issue with part-whole relationships. Fosnot and Dolk (2002) explain that this is one of the foundational pieces in their *Landscape of Learning,* which is a developmental map of student progression through fractional number sense (p.70). Researchers such as Fosnot and Dolk suggest that a problem occurs when educators begin fraction instruction with shading in diagrams and labelling the parts. This enables students to think only of the number of parts instead of thinking about the relationship between the whole and its parts. Cramer (2002) confirmed Fosnot and Dolk's theory when she looked at two students in her study, Jeremy and Annie. In the study she observed that Jeremy, a student in the *Rational Number Project* (RNP), compared 4/35 and 4/29 in relation to the

[2] A b*ig idea*: is the mathematical understanding, or mathematical principle, which is inherent within the student's strategies. Often, a big idea is the stepping stone or platform for the next stage in the student's learning process and is what underpins the learning process of young mathematicians (Fosnot and Dolk, 2000).

numerator and the denominator, whereas Annie compared the two with her whole number knowledge system, suggesting that 4/35 was bigger because 35 is greater than 29. Moss (1999) also observed this difficulty in her study. She suggested that educators did not spend enough time distinguishing between rational number systems and whole number systems. Another researcher who discussed this was Mack (1999) who found that overall, students were unable to see how fractional parts were different from whole numbers, often partitioning and referring to them as a number of pieces rather than the size of fractions. When students see only the individual parts they often compare fractions using whole numbers not realizing that as the denominator increases the size of the piece decreases. This becomes a problem when students are later expected to multiply and divide fractions or when trying to solve for equivalent fractions.

2.1.3 Teaching of algorithms and procedures without developing conceptual understanding.

A third instructional practice that causes difficulty is the practice of teaching algorithms and procedures before students understand the part-whole relationship of fractions. Asku (1995) asserts that a common type of error in teaching fractions is to have students begin computations before they have sufficient background to profit from such operations. Asku suggested that the reason procedures are taught first is that they are easier to teach. Students may be able to memorize rules and procedures, but no understanding is associated with them. In addition to Asku's findings, Mack (1990, 1995) found that students' initial knowledge frequently interfered with their attempts to give meaning to fractions in two ways: students were unable to use their informal or prior knowledge even for contextual problems; and, when using previously taught algorithms, students often ignored their prior knowledge in favour of an incorrect answer, trusting in the algorithm instead. In addition to this, Fosnot and Dolk (2002), in theory, echo this finding, contending that when rules and procedures are taught first, or in isolation, students often stop thinking, and give up on their own thinking, in order to perform the procedures.

The emphasis on the teaching of algorithms is a central instructional issue in the reform versus the traditional debate. As Fosnot and Dolk note, parents think that if students don't learn the traditional algorithms then they aren't learning anything. However, reformers (Asku, 1987; Fosnot &Dolk, 2002; Small, 2008; Van deWalle, 2007) do not suggest that computation procedures are unimportant; rather they disagree with the method and timing of teaching of procedures.

2.1.4 Problem solving difficulties.

A fourth issue arises in the observation that traditional instructional methods result in students who do far better on computation tasks than word problems, even when the problems contain the same numbers. Fosnot and Dolk (2000) contend that much of the issue lies in what educators think problem solving is meant to accomplish. Fosnot and Dolk believe that first, problems should allow students to explore and investigate a range of calculation strategies. They observe that many educators use word problems that allow one solution only and the problems are usually designed to teach and practise algorithms. Learning to solve problems by calculation in a variety of ways more closely resembles mathematicians' methods. Dowker (as cited in Fosnot and Dolk, 2002), asked 44 mathematicians to solve several typical math calculations. Dowker found that the mathematicians first looked at the numbers and then chose an efficient strategy for calculation. They varied their solutions from calculation to calculation, always looking for the most efficient strategy to solve the problem.

Secondly, Fosnot and Dolk contend that problems should begin with a context familiar to the student. This is supported by Sharp (2002), who studied a girl named Leah as she played fraction games with her father. The context of the fraction games was a discussion of fair sharing food with her friends. Once Leah constructed the conceptual knowledge by solving problems in a familiar context, her father taught her the fractional notation for those fractions. Furthermore, Fosnot and Dolk suggest that once children construct conceptual knowledge of fractions, they can meaningfully learn, or even create for themselves, appropriate alternative algorithms. This is supported in Cramer and Henry's (2002) research in the Rational Number Project (RNP). The Rational Number Project espoused four main tenets: 1) children's learning about fractions could be improved through involvement with concrete models; 2) children needed time to use these concrete examples to build mental images needed to think conceptually about fractions; 3) children would benefit from discussion with one another and with their teacher; and 4) teaching should focus on the development of conceptual knowledge prior to formal work of algorithms. Cramer and Henry looked at two fourth-grade classrooms, one in which fractions were taught using traditional methods and one in which the RNP project was being taught, using instructional practices based on the foundational principles previously outlined. Overall, RNP students' thinking depended on mental images for fractions and was directly related to their use of fraction circles. Cramer and Henry also observed that RNP students were better able to discuss and communicate their thoughts, whereas, traditionally taught children struggled with understanding fraction size or in estimating simple fraction problems (1/2 + 1/4 = ?). The RNP children who solved problems in a familiar context explained and defended their ideas and constructed a more flexible, deeper understanding of fractions than their traditionally instructed peers.

2.2 Constructivism

The research findings discussed in the previous section depict a very different picture about how children learn than what was traditionally thought. Traditionally, many researchers subscribed to a behaviourist approach to learning, one in which curriculum is broken into specific skills which are then sequenced into hierarchical parts. Assumptions are made that simply by listening to explanations from teachers followed by practice, the necessary skills to learn the concepts could be built (Fosnot & Perry, 2005).

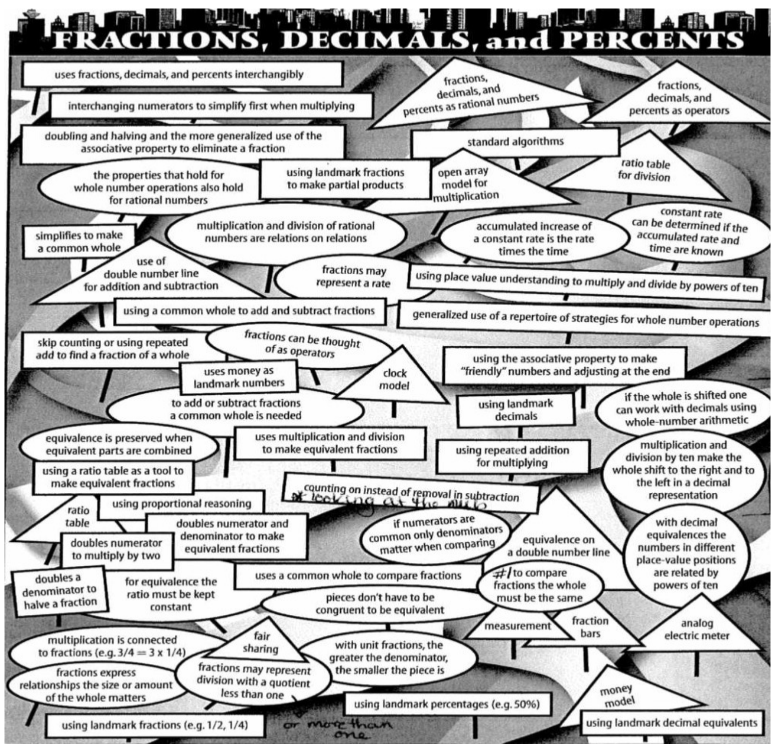
The researchers cited previously subscribed to more of a constructivist and sociocultural theory of learning. Constructivism is a theory about knowledge and learning. Knowledge is viewed as being personally constructed by learners as they try to make sense of situations (Fosnot, & Perry, 2005). Students construct new knowledge based on prior knowledge, they then reflect on, or actively think about, an idea, rather than passively absorbing it unaltered from the teacher. Because constructivism is about learning, it is a theory with important implications for the classroom. The sociocultural brand of constructivism is a theory about knowledge and learning where knowledge is thought to be gained through a series of human interactions, which emerge over extended periods of time (Cobb, Stephan, McClain, & Gravemeijer, 2001). Students construct new knowledge and learning with the teacher and with the whole class. Together they discover concepts through participation in dialogue with each other.

Constructivists, such as Piaget, proposed that the mind is not a blank slate. Even at birth, infants have organized patterns of behaviour, or schemas (Fosnot, & Dolk, 2002). Students learn at different rates; they use their personal schemas in various situations and re-structue them as different situations arise. Fosnot and Dolk suggest that at the very beginning these schemas are very specific and can be simply seen as representations of situations or problems by the learner. As students become more familiar with the topics and explore the connections between and across them, these schemas become more generalized to other situations.

2.3 The Role of the Landscape in Children's Learning: a Conceptual Framework.

Based on this constructivist theory of how students learn, Fosnot and others perceive learning not as an accumulation of concepts absorbed from the teacher but instead as series of understandings that are constructed over time. They view this progression as a landscape rather than a continuum thereby creating a developmental map of students' learning in mathematics. Fosnot and Dolk (2002) have developed a more complex understanding of fractions in *The Landscape of Learning: Fractions, Decimals and Percentages (See Figure 1)*. Although they built on the earlier work of Pothier and Sawada (1983), and later Lamon (1996), Fosnot and Dolk view development not as stages, but rather as a journey of learning. Students may move in a linear fashion, but they also can move back and forth as they develop and re-valuate their knowledge of fractions. In addition their landscape identifies possible *big ideas, strategies*, and *models* that may be created by students. They explain that the big ideas outline the major mathematical thinking that is happening within the students' minds (p. 36), the strategies are what the students are actually thinking and doing mathematically (p. 34), whereas the models are what the children use to represent their thinking (p. 73). Fosnot and Dolk point out that historically, curriculum designers did not use a developmental framework like the one they have devised, nor did the designers recognize mathematics as students *mathematizing:* that is both using and talking math; instead, in traditional curriculum, skills were viewed to be accumulated, and the accumulations or clusters would eventually turn into concepts. Fosnot and Dolk note that by perceiving student learning as developmental, teachers will no longer see strategies, big ideas, and models, as static points in a landscape but rather dynamic movements on the part of a learner in a mathematical development. In addition, they suggest that teachers must have this landscape in mind when they plan activities, when they interact, question, and facilitate discussions. In so doing, teachers can further facilitate the development of students' understanding of fractions. The role of the teacher in this type of reform classroom is very different from the traditional one based on behaviourist theory of learning.

Figure 1 Fosnot and Dolk's Landscape of Learning



from: Fosnot,C. 2002. <u>Context for Mathematics: Fractions, Decimals and Percentages.</u>

2.4 The Role of the Teacher in a Second Generation Reform-Oriented Classroom

Battista (1999) explains that historically, the role of the teacher was to show several examples to students of how they could solve certain procedural problems. The expectation was to have all students learn the same algorithms and practise applying them throughout the school year. Furthermore, Ackles, et al. (2004) state that: "Questions asked of students by the teacher were primarily answer-focused and at times the teacher did not even wait for an answer from the students, often giving the answer to the students instead of waiting for the response" (p.99). With a shift to reform practices, the emphasis was initially placed on encouraging the students to think through problems using their own methods, and then praising students for their unique strategies. The congress, was used as an opportunity to practise listening skills; teachers' questioning tended to focus on having students explain why they used a particular strategy, or asking students to explain their strategies further. Many teachers felt that in order for discussion to be focused on student thinking, teacher thoughts or interjections were to be avoided; all of the discussion needed to come from the students (Stein et al., 2008). They add that students' strategies often became inefficient and students and teachers were stuck on how to move towards more efficient strategies, or connecting the strategies, to a bigger mathematical idea. Stein et al. proposed that teachers needed to shift to a second generation in which the emphasis would be on directed and purposeful student talk, choreographed by the teacher but indirectly led. However, they noted that this was a challenge for teachers. Teachers lacked the knowledge to ask good questions which would achieve these goals. What should these questions look like at each stage of the lesson?

2.5 Effective Questioning

The three-part lesson as described by Van de Walle (2007), and endorsed by the Ontario Ministry of Education (2010) provides a useful structure for examining the theory and research on effective questioning in second generation reform-oriented mathematical classes. The first part of the lesson plan is structured to pose meaningful and purposeful problems to the students. The second part is designed to allow students to problem solve and work through the problems using their existing schema. The final part of the lesson is created to use student dialogue and careful teacher questions to consolidate, highlight and discuss the mathematics from the problem. For each section of the lesson various questioning techniques can be used. How does a second generation teacher effectively use questioning in each part of the lesson?

2.5.1 Part 1 of the lesson: Posing the problem

In a reform context, choosing a problem is as important as the teaching itself. Lampert (2001) suggests that the task should relate to the particular students in the classroom and with the particular mathematics the teacher wants the students to study. Furthermore, it needs to be both intellectually and socially possible for all of the students to work on the tasks in a way that supports that intended content. According to Fosnot and Dolk (2002) Part 1 of the three part lesson plan involves three unique components: using mathematical big ideas, models and strategies of the landscape as potential goals for learning; finding real life contextual problems for students to solve, and finally, anticipating students' thoughts and strategies in order to help with the next part of the lesson. The first component is to chose the goal for the lesson.

The second component is to chose a contextual problem. According to Fosnot and Dolk a rich problem is one that makes the students think about the big ideas in mathematics and is connected to a rich context or real life situation. The students shouldn't be thinking about just the numbers but rather about the contexts. Once a teacher has decided on a potential mathematical big idea, model, or strategy as a goal for the lesson, it is suggested that they think about an authentic, contextual problem for their students to solve. Fosnot and Dolk (2002) contend that the realistic nature of the context allows the students to understand the mathematics because it is grounded in their life. It also supports the students' ability to reflect and check whether their strategy makes sense. In addition, working within a familiar context allows students' to make sense of mathematics using their own lives and experiences. Finally, they note that in order for students to construct these relationships, a context needs to be open-ended enough for students to observe the patterns in the data.

The last component of Part 1 is anticipating students' responses. Stein et al. (2008) suggest that this involves actively thinking about what students might be doing mathematically when they approach the given task. This includes thinking about the different approaches, level of development, or current understanding of each student in the class. Both Lampert (2001) and Stein et al. (2008) note that it is critical at this stage for the teacher to solve the problem themselves and also to think of a number of different ways to solve the problem from a student's perspective. By anticipating students' strategies teachers are able to foresee potential difficulties the problem may pose for students and potential questions that could be asked in order to move students beyond their current developmental stage.

2.5.2 Part 2 of the lesson: students solving the contextual problem

Once the problem has been posed students enter into Part 2 of the lesson. It has been known by many different names. The Ontario Ministry of Education and Van de Walle (2007) name it the *during* section of the lesson; Fosnot and Dolk (2002) name it *working on the problem* and Stein et al. (2008) name this stage *monitoring student responses*. It is where, as Lampert (2001) states, students work simultaneously with their relationships amongst their partners, their teachers and with the content. She explains that as a teacher she needed to attend to each of these aspects to enable the relationships among the students, and between the students and the subject matter. She needed to learn more about how the students interacted, and what their communication skills were like, which then gave her information for potential use for later lessons or the congress. Lampert also observed that there were specific aspects of students' interactions during Part 2 that needed attention: 1) building a community of norms and accountability, 2) asking questions that facilitated discussion, 3) helping struggling students and, 4) monitoring student responses for the congress (Part 3 of the lesson).

Both Stein et al. (2008) and Lampert suggest that if teachers take the time to anticipate students' possible strategies, difficulties, and vocabulary, they will then be able to ask appropriate questions that guide students to learning mathematical concepts. According to Franke et al. (2007), and Sherin, Mendez, and Louis (2000) during Part 2 of the lesson these questions are often framed as an *interrogation* asking the students to give a reason for a particular idea, or to state how they arrived at the specific result. In addition, teachers can ask the other partner to clarify what is being said, or to rephrase the solution; in so doing they are balancing the accountability of the group with the mathematical learning. During this section of the lesson Lampert (2001) also notes that some teacher interaction takes the form of direct intervention. Lampert suggests that this often occurs when she notices her students heading down an unproductive path. These questions can be more direct, or open ended, depending on the challenges facing the student. In both cases, the teacher can re-voice students' responses for further clarification, interject with another idea, whether the teachers or other students, or scaffold the question depending on the level of student development (Small, 2010).

Finally, Stein et al. (2008) note that teachers can use response-monitoring to actively participate with the students, observing what is being said, the validity of students' ideas, as well as planning who will be participating in the whole class discussions. Furthermore, by closely observing, reflecting, and planning during this time, Stein et al. (2008) suggest that it will give teachers minutes instead of seconds to ask critical questions of students during the congress section.

2.5.3 Part 3 of the lesson: the congress

Whole class discussions play an integral role in the development of mathematical ideas and concepts. This is the place where teachers' questions play the most critical role. By carefully planning, watching and listening to their students, teachers can ask the appropriate questions and guide their students towards understanding and generalizations. In addition, during this time students also play an integral part in the construction of knowledge. As students participate in discussion, by answering the teachers questions, they reorganize their own and the class's beliefs about the mathematical notions being presented (Cobb, Stephan, McClair, & Gravemeijer, 2001).

During this time, teachers need to purposely select students' responses to present in whole class discussions, as well as select the sequence of students' strategies, and to connect the mathematical concepts between various students' responses (Lampert, 2001; Stein et al., 2008; Van De Walle, 2007). Stein et al. (2008) remind us that the teacher remains in control of which students present and therefore the mathematical content that might be discussed. During this whole class discussion time, teachers can air common misconceptions, introduce an important strategy, or increase the variety of strategies that are available to share. Each of the above suggestions brings about a variety of desired outcomes. This sequencing is a result of the constant monitoring that the teacher has done in Part 2 of the lesson. Stein et al. emphasize that it is the role of the teacher to connect the mathematical ideas for the students. Furthermore, they suggest that rather than having mathematical discussions consisting of separate presentations of different ways to solve particular problems, the goal is to build on each strategy presented by the students (p. 331). Franke, et al. (2007) contend that a teacher must find ways to make explicit the underlying mathematical similarities and differences in the solutions in a way that makes sense to the students and not by telling them the answer.

Sherin, et al. (2000) suggest two different questioning strategies to bring out the mathematics in the congress (Part 3): *build* and *go beyond.* Teachers would use the *build* strategy to enhance the nature of discussion further by comparing student strategies and building upon student ideas and mathematical comments. The authors note that for this strategy to be useful, it is critical that students learn how to listen and talk to one another. At this stage the guiding questions might include: What do we think? Can we make any comparisons between the strategies we just heard? Both these types of questions focus the discussion on the thoughts of the student presenters, as well as the mathematics in the students' strategies. This can also be accomplished with a *gallery walk*, which is a strategy used by Fosnot and Dolk (2002): students walk around the classroom looking at the different strategies presented on student posters. During this time the teacher and the students are often asking *explaining* type questions about the strategies given; however, at times the teacher can ask *building* type of questions though this is normally reserved for a whole class discussion. Their second technique, *go beyond*, occurs when teachers have students offer a response but also try to have the students generalize to bigger mathematical concepts or models. Sherin, et al. (2000) believe that the congress is also a time to connect the mathematical big ideas to what the students have done in their own work.

Franke, et al.(2007) and Colburn (2000) discuss three techniques to support the practice of *to build or go beyond* questioning. Firstly, provide students with *wait time* after asking leading questions. They propose that this is an important skill to remember because students need to struggle a little in order to build and understand concepts. This is also echoed in other researchers and theorists (Fosnot & Dolk, 2000; Lampert, 2001; Stein et al., 2007), when responding to students, paraphrase their learning or repeat what they have said. This strategy has two benefits: it gives validity to what students have created, and it also allows students to hear what they have just said, providing them with opportunities to re-evaluate their thinking. Franke, et al. (2007) believe that this rephrasing should be supplied by student voices more often than by teachers; however, at times, teachers must re-voice in order to redirect the discussion back to the big idea or plan of the congress (p. 228).

Franke, et al. (2007) conclude that teaching mathematics is about the teacher making decisions in the moment, decisions that serve both the individual student and the collective classroom's mathematical understanding. This is similarly found in the Ministry of Ontario's, *Growing Success (2009).* It suggests that primary purpose of assessment is to improve student learning. This can be done through a combination of assessment *of, as* and *for* learning (Ministry of Ontario, 2010)[3]. Franke et al. (2007) suggest that teaching is deliberate work, but it is deliberate work that takes into account the interaction among people and ideas and content (p.228). By orchestrating the three Part lesson as Franke et al. (2007) suggest, the teacher is moving his instruction towards a second generation of reform mathematics.

In addition to questioning during the three part lesson there is a second, short lesson, or mini-lesson structure which is also central to the second generation of reform instruction.

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[3] Assessment for learning is the process of looking for and understanding evidence in order to see where learners are in their learning, where they need to go and how best to get there. Assessment as learning focuses on monitoring and fostering student learning in the moment and, assessment of learning is a public display of learning (Ministry of Ontario, 2010, pg. 31).

2.5 Mini-Lessons

Fosnot and Dolk (2002) explain that *mini-lessons* or *strings* or *clusters* (Van De Walle, 2007) can be done at the beginning of the class for ten to fifteen minutes. They provide students with practice in applying specific mental strategies for the upcoming problems. They are also a place where students can discuss and debate strategies, learning concepts at the same time as learning effective and efficient procedures. An example of a string to focus on the strategy of using a clock model to add fractions is found in Figure 2.

Figure 2 String Lesson

1/3 + 1/4
1/2 + 2/3
10/60 + 1/2
1/3 + 25/60 + 1/4
1/6 + 1/2

from: Fosnot, C. (2002). Mini-Lessons for Operations with Fractions, Decimals, and Percents. Portsmouth, Ma: Heinemann.

Mini-lessons are yet another place for teachers to use effective questioning to highlight certain strategies or have students consolidate learning. During this time, Fosnot and Dolk (2002) view the teacher as using more direct instruction, often rephrasing students' comments and using them for instructional purposes. However, they also suggest allowing students enough wait time to solve the problems.

2.6 Summary and Statement

Franke, et al. (2007) and Stein et al. (2008), argue that teaching is not just about starting with mathematically rich problems, or just about listening to students' conversations, and asking them to describe their thinking. It is about shifting to a second generation of reform instruction in which teachers move away from a show and tell method and towards connecting students' thinking and conceptual understanding to the broader mathematics. There is limited research documenting the implementation and the impact of second generation instruction on student learning. Franke, et al. (2007) argue that the limited citations in their research clearly demonstrate that the research on building an effective classroom discussion has just begun (pg. 237). In addition, Tzur (1999) believes that studying teaching and learning together in a classroom setting would prove useful. Furthermore this limited discussion on second generation techniques has focused on mathematics instruction generally rather than specific concepts with a few notable exceptions (e.g. Franke, et al., 2000; Lampert, 2001; Stein et al., 2008). None of these studies has focused on the impact of these techniques on learning fractions and in particular, on childrens' constructions of part-whole relationships. Therefore, studying the impact of second generation instruction on children's development of the part-whole relationship in fractions could offer new and useful research to the field.

Chapter 3: Methodology

3.1 Research Questions

How do a teacher's questions in a second-generation reform-oriented classroom impact student construction of part-whole relations, and their use of a *benchmark model,* in the learning of fractions? (The benchmark model, in the context of fractions, is a model that showcases where pivotal fractions (1/4, 1/2 3/4, 1/1) are located in relation to each other, as well as other fractions.)

Sub questions:

A) What mathematical knowledge does a teacher draw on to generate the questions used during the fraction math class?

B) What planning is involved for the teacher when constructing these questions?

3.2 Research Design

This research project was designed as a qualitative case study (Bogdan & Biklen, 1998) investigating the impact of my questions as a teacher on students' construction of the part-whole relations and their use of a benchmark model as they learned to work with fractions. It was a case study because the focus of the study was bounded by one case: my students in a Grade 4/5 classroom and me (as defined by Creswell, 1998). This study also tried to answer a question of how my questions as a teacher impacted student learning of fractions. In so doing, a case study was an appropriate research design for this thesis because it was asking *how* questions would impact learning. It also had three other characteristics delineated by Baxter and Jack (2008) as part of a case study: 1) I did not manipulate the behaviour of those being studied, 2) I wanted to cover the contextual conditions because they were important to the study and, 3) the boundaries were not clear between the phenomenon and context. Finally, the study also includes various data sources over a period of five weeks. These sources included: pre- and post- assessments, video recordings of conversations and lessons, student work samples and, written self-reflections of the students and me.

These data sources allowed me to view the results of the students from different perspectives. The pre-and post-assessments allowed me to have a quick picture of the students' prior knowledge and areas of problems, as well as allowing me to see at a quick glance whether they made improvements in their learning. Collecting student work over time allowed me to capture student thinking at different times in the study. However, not all learning could be captured on a piece of paper, which is why videotaping, student work samples, and student and my reflections were needed in order to understand the full impact of my teaching and the student learning that happened during the case study. Table 1 is an overview of the data collected during the study as well as the alignment with the thesis questions the data was gathered to address.

Table 1 Overview of Data Sources

Type of Data	Type of Data Collected	Thesis Question it Answered
Pre-Assessment	Observations, Student Work Samples, Teacher Reflections	Main
Red Cross Problem	Observations, Video, Journal, Student Work Samples, Teacher Reflections	Main, a, and b
Mississauga Marathon #1	Observations, Video, Journal, Student Work Samples, Teacher Reflections	Main, a, and b
Mississauga Marathon #2	Observations, Video, Journal, Student Work Samples, Teacher Reflections	Main, a, and b

How much is Blue?	Observations, Video, Journal, Student Work Samples, Teacher Reflections	Main, a, and b
What fraction of the whole does each shape represent	Observations, Video, Journal, Student Work Samples, Teacher Reflections	Main, a, and b
Post-Assessment	Observations, Student Work Samples, Teacher Reflections	Main

3.3 Research Sample

Participants

The main focus of my research was on my questions of students and their impact on student learning. The research was conducted in my classroom. Data was collected from the whole class but, because of time limitation and large amount of data, the study was narrowed to four pairs of Grade 4 students. The students worked in homogeneous pairings. Pairings were established at the beginning of the year based on my observations, grades from the previous year and initial math assessments from the beginning of the year. For the case study I chose to keep the pairs together instead of creating new groups for this study based on the pre-test data. I thought the pre-test data would confirm my groupings and I felt that the trust and collaboration the pairs had already created far outweighed changing the groups for the case study.

I selected four pairs: two low, one middle, and one high achievement. These achievement groups were chosen based on report card marks, pre-test results and my observations. The first two pairs had had low achievement in mathematics. This group consisted of Holly, Rick, David and Erick. Because Erick was away for the beginning of the unit, David worked on the first two problems while partnered with Rick and Holly. In my mid-achievement pair were Nancy and Anita and my last pair James and Nick, had high achievement in mathematics.

3.4 Ethics

Ethics approval was required from Lakehead University. Since the research involved students, permission from the parents, students, and school principal was required by the school board. As I am an employee of the school board and involving my classroom only, I did not need permission from the board to conduct the research; however, guidance on this subject was given by the board. Letters and permission forms were sent to parents and guardians (Appendices A and B), to students (Appendices C and D), and to the school principal (Appendices E and F) during the month of March, 2012.

3.5 Instruction and Data Collection

The study was conducted over a period of five weeks in April 2012. Generally, a math class was 90 minutes and was scheduled in the morning before the first nutrition break; however, occasionally, scheduling conflicts occurred and lessons were not always in the mornings. A math class consisted of a small mini-lesson, which focused the students' thinking, a contextual problem that students solved in partners, and a congress in which the strategies and solutions were debriefed and discussed. My role as the teacher was to be a facilitator of the students' learning, questioning and offering suggestions when needed to extend the students' explorations. After each congress students answered a small reflective question on part-whole relations to document their understanding and growth.

3.5.1 Pre-test

Prior to the start of the unit introduction I administered a pre-test (Appendix I). The purpose of the pre-test was to obtain a diagnostic assessment and a general idea of the students' abilities before beginning the fraction unit. It set a baseline for comparison with the results of the final post-test. It also allowed me to see if group pairings needed to be changed, which was not the case. The test was divided into three main curriculum-based areas: representing, ordering and comparing fractions as well as three mathematical big ideas: fair sharing, part-whole relations and as the denominator gets larger the piece gets smaller. The test was composed of three questions using the following rationale. The first question was created because students often organize fractions by looking at the denominator, only or the numerator only, forgetting that fractions are a part-whole relation. It also tested the curriculum connection of representing, ordering and comparing fractions. The second question was used because it highlighted students' understanding of fraction sizes. For this question, students sometimes assume that because three is larger than two, it will always be bigger. Although this question did not deal mainly with the part-whole relation, students nonetheless needed to consider not only the individual numbers but also what the fractions actually represent. The third question was very similar to the second, as students will have to first create, and then organize, the fractions. Here students might struggle with the Pothier and Sawada's (1983) stages, as well as using only the denominator or the numerator to compare them.

Students were advised to do their best and that the results would help future planning of the units. The pre-test was not timed; students who required extra time to complete the test were able to do so during nutrition-breaks or the periods that followed.

3.5.2 Word Problems Lessons

During the instructional unit the five bolded word problem lessons (included in Table 2) were used to gather data from the unit. I focused on the five word problem lessons that dealt specifically with the part-whole relation. The rationale for each of the lessons and questions asked can be found in Appendix H.

The lessons were taught according to reform methods of instruction, (as generally delineated in Van de Walle (2008), and also followed the five practices set by Stein et al. (2008), (see Appendix L). First I used a field journal to document the planning that I did in order to conduct the lesson. I included my thinking around the types of strategies I looked for, my choice in partners, the types of questions I thought about asking.

Next I taught the lesson and recorded observations using anecdotal notes that I took while teaching. Additionally the lesson was videotaped (to capture a record of the instruction and students' reaction to my questions). During the period of instruction, the eight students were videotaped in order to obtain detailed data on what the students knew and could do. In order to minimize disruption to the class, the video camera was placed on a tripod at the side or back of the room or placed over the students as they worked. At the end of each day I reflected on the learning in the classroom. I would ask myself what went well, what didn't work, problems that my students had and possible next steps for the next day. I also reflected upon the eight students to assess where they were on the landscape and see what could be done in the next problem to move them along the continuum.

Table 2 Outline of Unit Plan

Day 1: Building your fraction kit and playing fraction games uncover and cover up (Adapted from Burns, 1999)

Day 2: Exploring Fractions with pattern blocks (Burns, 1999)

Day 3*: Red Cross Problem (Fosnot, 2002)

Day 4: Red Cross Problem Day two

Day 5: Using their fraction kit: play games

Day 6: Day three Red Cross Problem

Day 7: Congress of the Red Cross Problem

Day 8: Mississauga Marathon Version #1 (adapted from Burns, 1999)

Day 9: Mississauga Marathon Version #2 (adapted from Burns, 1999)

Day 10: Congress

Day 11: How much is blue? (Burns, 1999)

Day 12: What Fraction of the Whole Does Each Shape Represent? (Burns, 1999)

Day 13: Day seven of field trip: developing equivalence (Fosnot, 2002)

Day 14: Day eight of race for autism: (adapted from Fosnot, 2002)

Day 15: Bar capture game (Fosnot, 2002)

Day 16: If the world were a village

Day 17: Final assessment

* Days in bold indicated data sources.

3.5.3 Post Assessment

At the end of the unit a post-test (Appendix J) was administered to evaluate the impact of my questions on students' development. The purpose of the post-test was to see if any improvement could be in with the students' thinking. Although I obtained pre and post assessments no significant statistical analysis could be undertaken with a set of only eight students; instead, the assessments were used to look at individual growth. The post test followed the same parameters as the pre-test.

3.6 Data Sources and Analysis

The data was described, classified, interpreted and represented (Creswell, 2008) in the process described below. Student work, and pre- and post-tests were entered into Atlas.ti qualitative software. Table 3 outlines the data entered into Atlas.ti, the resulting number of primary documents as well as the link to the curriculum and the big ideas in mathematics.

Table 3 *Sources of Data and Relationship to Mathematical Big Ideas and Curriculum Connections*

Source	No. Students	No. Problems	No. Primary Documents	Big Ideas and Curriculum Connections
Pre-Test	8	3	24	Part-Whole Relationships, As the denominator gets larger the piece is smaller, fair sharing, representing, ordering and comparing fractions
Red Cross Problem	8	1	20	Part-Whole Relationships, As the denominator gets larger the piece is smaller, fair sharing, representing, ordering and comparing fractions
Mississauga Marathon	8	1	14	Part-Whole Relationships, As the denominator gets larger the piece is smaller, fair sharing, representing, ordering and comparing fractions
Mississauga Marathon Part 2	8	1	17	Part-Whole Relationships, As the denominator gets larger the piece is smaller, fair sharing, representing, ordering and comparing fractions
How much is blue?	8	1	13	Fair Sharing, Representing Fractions
What fraction of the whole does each shape represent?	8	1	14	Fair Sharing, Representing Fractions
Post-Test	8	4	32	Part-Whole Relationships, As the denominator gets larger the piece is smaller, fair sharing, representing, ordering and comparing fractions

Two general areas of coding were done. The first area was of student work, their thinking and their mathematical development. The assessments and student word problem samples were coded either: as correct, correct with support, or incorrect; and secondly, each was coded by type of solution strategy, model used and/or big idea addressed. The latter codes were based on the landscape for fractions, decimals and percents developed by Fosnot and Dolk (2002), as well as from Pothier and Sawada’s (1983) stages for fraction development. The pre- and post-tests was compared together in order to check for student growth in fractions.

The second general area of coding was on my own practice. The videotaped lessons were also entered into Atlas.ti. I also consulted my field journal on my planning process and thoughts about the lessons. I coded on my talk moves, questions and student responses to both. The codes on questioning were based on the literature from Ackles, Fuson, and Sherin, (2004); Franke, Kazemi, and Battey, (2007); Sherin, (2002); and Sherin, Mendez, and Louis, (2000). See the preliminary coding list included in Appendix K. In addition to the literature, codes were also developed during the course of the study using grounded theory as described by Bogden and Biklen (1998). These codes were derived from discussion with my supervisor. This was needed because some things happened in the classroom were not found in the research. For example, at times, it was unclear if my question was a direct interrogation (initiate-evaluate-respond) or a question trying to lead to a big idea. Through careful discussion it was decided to look at the wording of the question and end result to see if the question was an IRE or one designed to foster the development of a big idea. The final coding list is included in Appendix M. An explanation of the talk moves and question codes is found in Appendix N. Once everything had been coded, the students' solutions were re-examined to investigate evidence of the impact that my questions had on my students' development of part-whole relations and the development of a benchmark model.

After I coded each response I tallied the amount of times that I used each talk move and question. In addition, I would make notes in Atlas.ti on the reaction of my students had when I ask a question or performed a talk move. Finally, I tabulated whether the talk moves and questions were tied to big ideas (from the landscape) or talk. In doing so, I was able to get a better understanding of the types of moves and questions that I performed during a normal math class. It also allowed me to see if these moves and questions had a purpose and if they had any impact of the students.

The trustworthiness of this coding and analysis was checked through a variety of processes. First I gathered a variety of data (student work and video data). During the analysis, data was analyzed through discussion with my thesis advisor. This allowed me to check that my observations were less biased. In addition, the analysis was based largely on the codes, which were created from other research. This allowed me to verify my thoughts against other research. Using qualitative software allowed me to easily revisit video of the classroom lessons and the student work to verify my analysis.

Chapter 4: Findings and Analysis

I assessed the impact of my questions on students' construction of part-whole relations, and their use of a benchmark model in the learning of fractions, by evaluating students' responses to fraction problems, as well as analyzing video data of each lesson. Findings will be discussed in ability groups with lower students together, and then mid, and high groups. Furthermore, it will discussed within these groups by first their pre-test results, then the unit problems and finally their post-test results. In the unit problems the findings will include the planning, teaching results and the types questions I used in teaching the problem. The analysis will examine the learning that the students took part in and the impact these particular questions had on students learning

4.1 Results of the Pre-test and Analysis

The purpose of the pre-test was to obtain a diagnostic assessment and a general idea of the students' abilities before beginning the fraction unit. The test was divided into three main curriculum-based areas: representing, ordering and comparing fractions. Upon administering the test I realized that I had worded one question incorrectly and that it was asking the students to solve a problem that did not depend upon a big idea in fractions. In addition, it was not focused on comparing fractions or determining if they understood the part-whole relationship in fractions. I decided to ask another question, the next day and coded that instead. I have provided both questions in Table 4 below but will be discussing only the revised question. The pre-test was designed to capture what the students understood about fractions. It tested four mathematical big ideas: the greater the denominator the smaller the piece, fair sharing, the size of the whole matters, and part-whole relations. These related to the Ontario Math Curriculum: representing, ordering and comparing fractions. In addition, it also tested the students understanding of a benchmark model of fractions. It tested these big ideas because they are the foundations of fractions and the curriculum expectations are what I needed to cover and assess for the students in my classroom; however, they too are foundational expectations for students learning fractions. These big ideas and expectations also align with the research of Fosnot and Dolk (2002), Pothier and Swanda (1989), and others. It confirms that these are the developmental questions to be asking this set of Gr.4 students.

Most of the students, that is 6 out of 8, were unable to solve any of the pre-test questions, which showed that my students struggled with many of the concepts of fractions. In fact, only one student could answer the first question and then struggled with the other questions to follow. Table 4 contains a summary of the percentages of correct responses, responses correct with support, incorrect responses, and no response. This is followed by a description of each problem broken down by low, mid and high achievement group solutions.

Table 4 *Pre-Test Results: Question Wording and Percentages of Correct, Correct with Support and Incorrect Responses*

Question Number and Short Form	Question Wording	Correct	Correct, Needs Further Explanation	Incorrect	No Reply
#1. Ordering Fractions/ Benchmarks/ Part-Whole Relations	Order these fractions from greatest to least: 3/4, 5/12, 2/3, 3/2, 2/5, 5/8	1	1	6	
#2. Comparing Fractions/ Size of the Whole Matters	Jeremy and Fiona are eating pizza. Fiona has 1/2 of a pizza and Jeremy has 1/3 of a pizza. Is it possible that Jeremy has more? Explain your thinking				
#2 Revised. Comparing Fractions/ Benchmarks/ Part-Whole Relationships	Where does 3/6 fit in the list below? 1/16, 1/4, 3/8, 11/16, 1/1		1	6	1

#3. Ordering/ Part-Whole Relations	A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows: a) The first group had 4 people and shared 3 subs equally. b) The second group had 5 people and shared 4 subs equally. c) The third group had 8 people and shared 7 subs equally. d) The last group had 5 people and shared 3 subs equally. 1. Was the distribution fair – did each group get the same amount? 2. How much of a sub did each person get, assuming the pieces were cut equally?	7	1

4.1.1 Questions 1 and 2: ordering fractions

There are three main components to the Ontario Mathematics Curriculum (2005) for Grade Four: representing, ordering and comparing fractions. For all three components the most challenging area for students to understand tends to be: understanding fractions as a part-whole relationship. Often students will look at the numerator only or at the denominator only, rarely seeing the ratio between the two sets of numbers (Mack, 1999). This is exactly what occurred in the first two questions of the pre-test, thus echoing Mack's findings: overall, students were unable to see how fractional parts were different from whole numbers, often partitioning and referring to them as a number of pieces rather than the size of fractions.

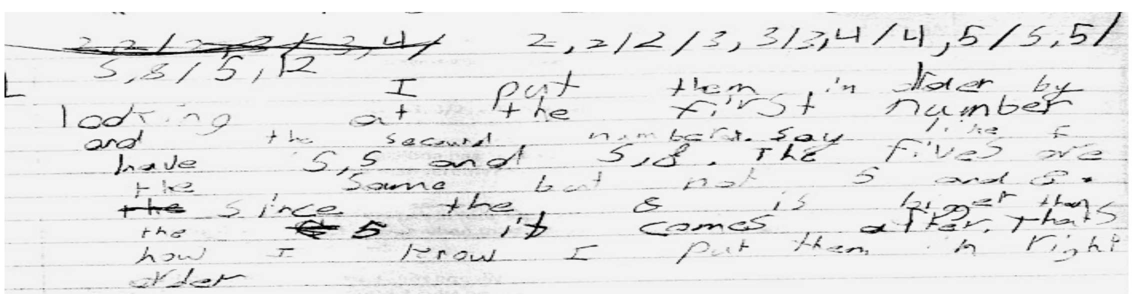
4.1.1.1 Lower students' results (Holly, Rick, David, Erick)

These four students struggled with the pre-test. None of them saw the part-whole relationship and looked only at the numerator or only the denominator. For most of these students their response was like Holly's who said, "I ordered them 5, 5, 3, 3, 2, 2" (PD37) see

Figure 3[4]. Only when the numerators were the same did they decide to look at the denominator (PD43[5]), see Figure 4. Here the students stated similar comments to Erick who stated, "I put them in order by looking at the first number and the second number. Say I have 5, 5 and 8. Since the 8 is the largest then it comes after the 5."

Figure 3 Holly's work for Question 1

Figure 4 Erick's Work for Question 1



For the second question the students followed the same strategy, whereby they looked at the numerator only and when the numerators were the same they looked only at the denominator. For example conferencing with James, he said, "I put 3/6 between 1/4 and 3/8 because it goes 1, 3, 3 and 4, 6, 8" (PD.45).[6]

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[4] Because of working in homogeneous groupings students for the majority have a similar level of thinking. They struggled with the same concepts and problems. Moreover because they worked together to solve the problems they often had similar reasoning. As a result I will often refer to a student's comment as a group's comment.

[5] In this example, Holly first looked at the numerator and ordered the numbers 5, 5, 3, 3, 2, 2. She has not looked at the denominator to see a relationship or even recognize that the numbers exist.

[6] Primary Documents (PD) are the name and number of a piece of data entered into Atlas.ti such as the video of a class or in this instance student's work sample.

4.1.1.2 Mid and high students (Nancy, Anita, James and Nick)

Of these four students only Nick was able to answer any of the questions correctly, but his responses contained insufficient detail to indicate how he arrived at the answer. Nancy appeared to answer the question correctly; but when I looked at her work more closely her answer was correct but her explanation didn't match her answer (see Figure 5). This discrepancy between her answer and explanation became even more apparent when looking at similar questions in the pre-test. When asked the revised Question 2, for example, she had some very interesting results. For both of the questions Nancy represented the fractions and looked at the shaded parts. However, she wasn't looking at the numerical relationships between the denominator and the numerator, only the visual representations of the fraction. In essence, Nancy was looking at the numerator and comparing it to the denominator and not the understanding the relationship between the two, see Figure 6. The other two, James and Anita, answered the question in a very similar way as the literature suggests: looking at one of the numbers only and not seeing the relationships, (Asku, 1999; Fosnot & Dolk, 2000; Mack, 1999). James and Anita's responses were very similar to those of the lower group, and are indicative of the types of misunderstandings that many students demonstrated.

In summary, all students, low, middle and high, except one, could not order the fractions from greatest to least.

Figure 5 Nancy's Response to Question 1

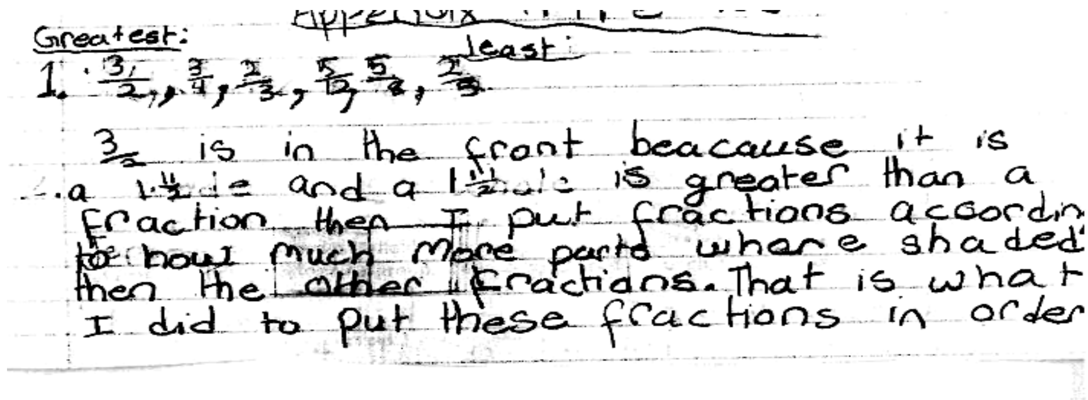
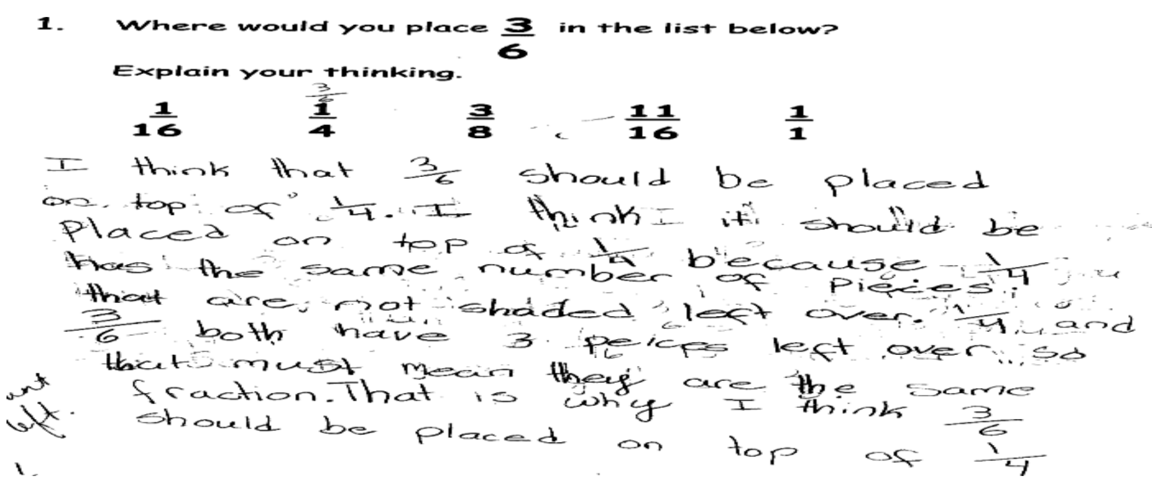


Figure 6 Nancy's Response to Question 2



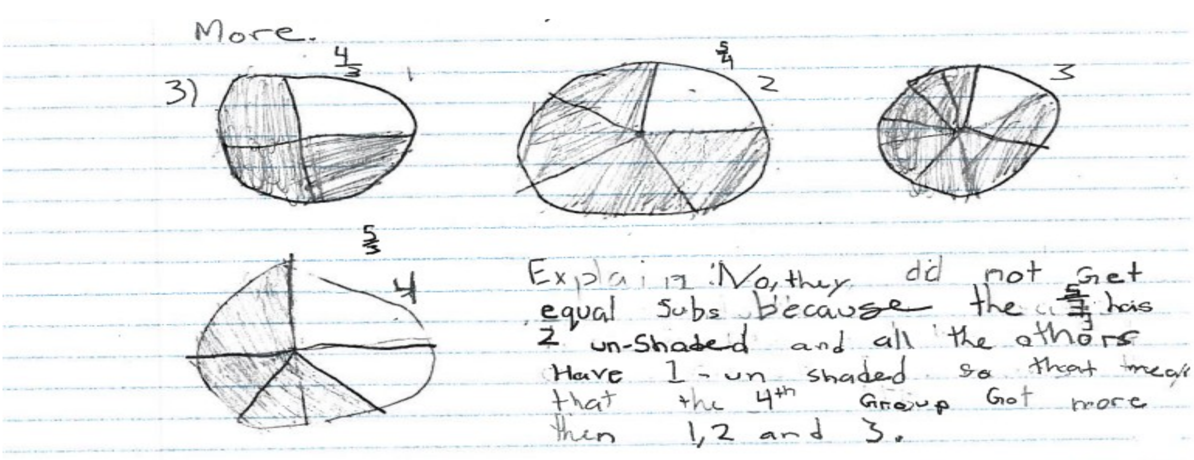
4.1.2 Question 3: comparing fractions

The last question in the pre-test was asked to determine how students represented fractions and if they could compare them. It also allowed me to obtain insight into what they understood about fractions. All of the students failed to answer this question correctly, and therefore I will analyze their results together.

Many of their areas of difficulty were the same as those referred to by Asku (1995), Empson (2002), Fosnot and Dolk (2002), and Mack (1995, 1999). As Pothier and Sawada

(1983) have suggested, there is a natural progression of representing fractions. Many of my students struggled to understand that the pieces had to be equal, which is the first stage in the natural progression of fractional development. Many of them struggled with odd numbered denominators (see Figure 7), often resorting to halving the piece and then splitting that half into odd denominators. Finally, all of my students represented fractions as a circle. In Cramer's RNP study (2002), she used only circles to represent fractions and found that the students were able to communicate better about the comparisons between fractions. In direct contrast, my results seem to suggest that using circles to represent fractions hindered the students when they were asked to explain who had the most or the least amount of subs. Students struggled to see how the whole had to be the same in order to compare fractions and when it came to odd numbers they were not able to see how they could represent the fraction (see Figure 7). Rick, for example, made all four circles differently and used a halving strategies to solve for odd denominators. In fact, this question was so difficult for my grade four students that one of them broke down in tears[7].

Figure 7 Rick's Work on Question 3



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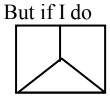
[7] Although this may appear as if the questions were too hard for Grade 4 students Fosnot and Dolk (2002) use this question in their grade four classes. It also is a very rich question that brings many of the big ideas and curriculum expectations in a Grade 4 classroom.

The pre-test focused on two aspects of fractional reasoning, part-whole relations and comparing fractions. These informed my practice alerting me as to where my students were and what I needed to do as a teacher to move them forward in their understanding. It allowed me to see that the majority of them did not understand the part-whole relation and that many of them were still in stage two of Pothier and Sawada's (1989) developmental stages: trying to master a halving process. Moreover, it affirmed that it is a challenge to learn fractions. I speculated that although students struggled with these concepts there was nonetheless a possibility that effective questions, asked at a pivotal moment in student's learning might increase their individual understanding. For this reason I had to look more closely at the problems and learning of the students as they worked in groups and congress. I turn to the next section of findings and analysis: the instructional unit and the five problems used to gather data.

4.2 Teaching Unit Problems: Planning, Questioning and Student Results.

Throughout the unit (begun after the pre-assessment) I used five word problems to assess how the students grasped the concepts being taught. Each problem will be discussed as ability groupings (lower group and mid/high group). For the full unit see Appendix G. Generally, as students progressed through the unit they understood more of the concepts (moving from one correct answer to the full groups solving the problem) and were able to solve the problem progressively faster (going from 80 minutes to 30 minute on average). The overall results of all students problems can be seen in Table 5; a discussion of each group will follow the table.

Table 5 Unit Assessment Problems Results

Question Number and Short Form	Question Wording	Correct	Correct, needs Further Explanation Or Support	Incorrect	No reply
Sub Problem: **One student's partner was away for this problem, and therefore the was put into another group. Hence why there are only three marks.**	At a recent rescue mission by the Red Cross, the Red Cross decided to hand out sub sandwiches to family groups: a) The first group had 4 people and shared 3 subs equally. B) The second group had 5 people and shared 4 subs equally. c) The third group had 8 people and shared 7 subs equally. d) The last group had 5 people and shared 3 subs equally. 1. Was the distribution fair? Did each person in each group get the same amount? 2. How much of a sub did each person get, assuming the pieces were cut equally?	2(full 80 minutes)	2(full 80 minutes)	2 (full 80 minutes)	
Mississauga Marathon Problem #1	There was a local marathon in Mississauga, and I found these stats about the people who ran the race. It was a very difficult race so some didn't finish, and I think the sun was getting to some so they ran more than they should have. Can you put them in order from who ran the least distance to who ran the most/farthest? Set 1 (put names to each fraction): 3/16, 5/8, 3/4, 1/4, 2/4, 1/2, 9/8, 1/1, 17/16, 15/16, 3/2	4 (60 minutes)	2 (full 80 minutes)		2 (full 80 minutes)
Mississauga Marathon Problem #2	There was a local marathon in Mississauga, and I found these stats about the people who ran the race. It was a very difficult race so some didn't finish, and I think the sun was getting to some so they ran more than they should have. Can you put them in order from who ran the least distance to who ran the farthest? Set 2 (put names to each fraction): 12/10, 9/8, 47/50, 6/9, 2/3, 9/16	4 (30 minutes)	2 (30 minutes)	2 (30 minutes)	
How much is Blue?	Page 97 in Marilyn Burn's <u>Introduction to fractions 4-5</u> book. The question is to determine from the shape what fraction is blue?	6 (20 minutes)		2 (75 minutes)	
Set of a Shape:	Shape: To create thirds in a circle you make a "Y." But if I do this in a square, what fraction is each piece? 	6 (20 minutes)	2 (30 minutes)		

4.2.1 Problem #1: The Red Cross (Sub Problem)

The first problem in the teaching unit was a reprise of one of the pre-test questions[8], Fosnot's Submarine Problem (2000). I changed it to include a social justice theme and the Red Cross (see Table 6: The Sub Problem). Before introducing this problem students had created fraction kits using a halving schema (1/2, 1/4, 1/8, 1/16) and played two fraction games, which were designed to build an understanding of a whole, a half, and equivalent fractions, at least with these denominators. I chose to do this problem because it lent itself to having the students explore many of the big concepts in fractions: fair sharing; when comparing fractions the size of the whole matters; as the denominator gets larger the pieces get smaller; benchmark fractions; unit fractions; and, part-whole relations. The goal of this problem was to build a linear model to compare fractions.

Before giving the students the problem I tried to anticipate many of the difficulties that could possibly occur. For example, I felt that many of the students would struggle with representing the fractions, especially when dealing with the fifths. In addition, when comparing fractions I anticipated that they would struggle with the concept that all of the fractions except 3/5 were one piece away from the whole, and therefore, most students would suggest that the fractions were the same until they compared them with a linear model. The majority of my questions and talk moves were derived from these anticipated difficulties and big ideas outlined in Fosnot and Dolks' Landscape of Learning. See Appendix G for a full list of learning difficulties that I anticipated students would have with this problem. As Fosnot and Dolk (2002) suggest, my questioning was not a random act but rather, a process that required me to have certain big ideas and models in mind when I planned the activities, interacted, questioned, and

[8] This is a typical teaching move as it allows the students to revisit the problems from the pre-test. Some improvement is going to happen but it is such a rich problem that it needed to be solved and discussed as a class.

facilitated the discussion (see Appendix G). When a student approached these big ideas and models, I would know how to direct and guide their learning, moving them toward the next big idea (p.23-24). For this particular problem the big idea that I was trying to have students construct was: understanding what the whole was and what parts had been used in each of the groups. In addition, they were required to compare these parts to each other. Along with these big ideas come a variety of potential areas of difficulty, most of which have been described in the literature. I anticipated that many of the students would be at different stages of development, and some would have issues with understanding the part-whole relationship. I also thought that there might be issues around comparing fractions as I know that most of their experiences with fractions had been shading in circles and determining which picture had more shaded parts.

4.2.1.1 Teaching, questioning and student results the Red Cross /sub problem: the lower group (Holly, Rick and David).

Unfortunately, Erick was not a part of this problem-solving activity because of his absence, and therefore my lower group consisted of Holly, Rick and David. The first challenge with learning fractions concerned appropriate representation of fractions as equal parts. Pothier and Sawada (1983) proposed that students learn fractions in five stages: fair sharing, representing in equal parts, halving, halving in repetition, and finally, multiplication for odd numbers. These stages were evident with these students as they solved the problem. As a result, it took numerous rounds of guiding questions to prod them into thinking about the fractions and what they represented. When I first came to check on Holly, Rick, and David, I noticed that the three of them had constructed the fractions incorrectly (see Figure 8), falling into stage two, which is typical for students at this age. I asked them, “How did you decide that it was 4 over 3?” (PD69). The response I received was, “There was three subs.” However, the students drew only one circle broken into 4 parts. To see if it would change their thinking by bringing them back to the context, I continued to ask them how many subs they had represented with the one circle. They were able to draw the correct amount of subs, but still represented them as circles instead of linear models, which meant that the fifths were a problem.

The group was able to represent the first set of fractions but then struggled to break the subs into fifths; this particular group was still in Pothier and Sawada's third stage of representation: using a halving strategy. As you examine Figure 9, take a close look at how they drew the lines for the fifths. In both cases they divided the image in half and then split the figure accordingly. Neither one is an equal nor a fair representation of the odd numbers.

Figure 8 Holly, David, and Rick's First Attempt

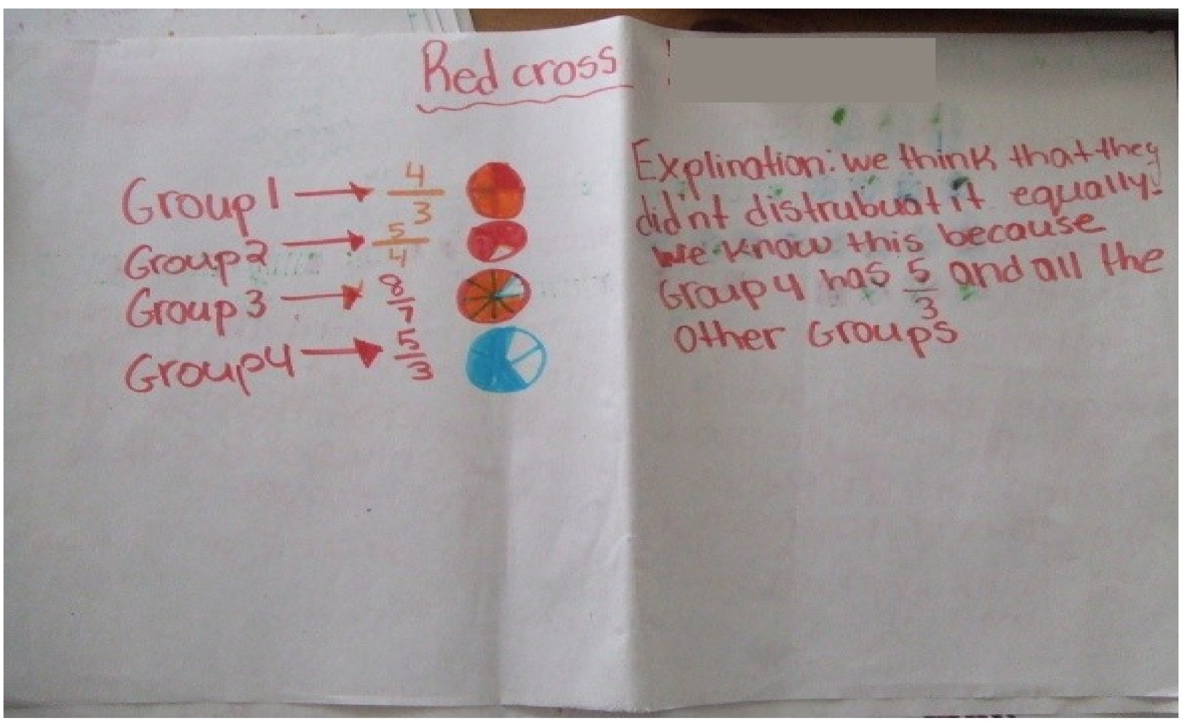


Figure 9 Holly, David and Rick's Second Attempt

After twenty minutes I came to check on them I had to spend another twenty minutes working with odd denominators, using a lot of scaffolding, re-voicing ideas, and interrogation of their thoughts around fair sharing and the part-whole relationship (PD. 69-71). I started off with the question, "How much did the first group get?" referring to the poster in Figure 9. I asked this question because I wanted to see if they understood what the fraction actually represented. Also, in their first attempt they seemed to understand this quite easily. Unfortunately, it took them quite some time to answer the question, so I followed up with, "How much does the blue represent?" (see Figure 9). Again this took them some time, so I decided to scaffold this with a small leading question, or word, "people?" pulling them back to the context as Fosnot & Dolk (2002) suggest: teachers need to notice how children are thinking about a problem, seeing if they stay grounded in the context. When the context is a good one, the children talk about the situation.

This sparked some discussion with a simple, "Yeah." This type of conversation went on for some time with my asking leading questions, about fractions, to help them understand what they had actually represented. For those leading questions, I used their fraction strips and asked them familiar fractions (1/2, 1/4, 2/4, 3/4) and had them represent them with their strips. I then asked them what they would call those fractions and I drew that representation out on their paper. I also modeled this representation as a linear model, hoping that they would see the connection to the subs and move away from circle notation. After some time they finally concluded that each blue section was a group of people and the green was another. This took some time because they struggled to understand that what they were doing was dividing each whole into four parts or the amount of people they had and that each person would get 1/4 of each sub for a total of 3/4. Unfortunately, when I asked them about how much sub the group of students received they continued to struggle with representation, saying that each unit piece was a quarter, or an eighth instead of 3/4. It was at this point I decided that these particular students needed more work with understanding how to represent fractions instead of comparing them. In fact they did not even get to compare the fractions; they did, however, achieve an understanding of what each group received.

In the end we worked through all of the fractions, relating everything back to our first group, and that the reason the answer was ¾ was because each sub had been broken into fourths based on the number of people and the fact that each person got a piece of each sub. This process helped them to identify that the answers for the remaining groups would be 4/5, 7/8 and finally 3/5. When I left them I did not know if at this point, whether they truly understood how to represent fractions. I did, however, want them to present what they had done in order to set up the problem and encourage their involvement in the subsequent class discussion.

Overall, Holly, David, and Rick struggled with representing fractions. All three were stuck in between stages 2 and 3 referred to by Pothier and Sawada (1989): struggling to understand how to divide by odd numbers using a halving strategy. For the equal fractions they were able to represent fair sharing and split all in half but when it came to odd denominators their strategy failed them, and they were unable to represent them. In addition, they were still unsure about the relationship between the parts and the wholes. They were struggling to understand how a fraction could be one number with two parts.

4.2.1.2 Analysing the types of questions I asked during the small group work

What role did my questions play in any new learning on my students' part? I found that of the 23 questions, the majority were interrogation questions (13) followed by *going beyond*[9] questions (7). The reason I tended to lean towards more interrogation questions was that these students were not able to work yet with many of the big ideas of fractions. They seemed to struggle with the basic representation of fractions and that fractions needed to be fair shares. This in turn led me to differentiate the lesson for them, scaffolding their work until they were able to work with the big ideas. As a result, I think Holly, David, and Rick were able to enter the problem and contribute to the congress[10]. In addition, they were able to build upon this lesson and use what we discussed in the next set of word problems. See Table 6 for a breakdown of question types across the three groups during the Red Cross/Sub problem.

[9] Going beyond is when a teacher pushes the students or class beyond the stage of development that they are at. (see appendix N for full definitions of questions and talk moves).

[10] A congress is a whole class discussion. As a class we came together to discuss the big ideas, problems and solutions that the students had created and thought about.

Table 6 *Questions I Initiated During the Red Cross/Sub Problem Set*

Types of Questions	Holly, Rick and David	Nancy and Anita	Rick and James	Congress
T- Building On	1 (5%)	1 (7%)	0	11 (25%)
T- Direct Teaching	2 (9%)	0	0	0
T- Go Beyond	7 (30%)	4 (29%)	0	10 (23%)
T- Compare	0	0	0	0
T- Initiation- Response- Evaluation	0	0	0	2 (4%)
T- Interrogation	13 (56%)	9(14%)	0	18 (41%)
T- Question Unclear	0	0	0	3 (7%)
T- Scafolding	0	0	0	0
T- Shares Strategy	0	0	0	0
Total Questions:	23	14	0	44

4.2.1.3 Teaching, questioning and student results: Red Cross /Sub Problem: the mid group Nancy and Anita.

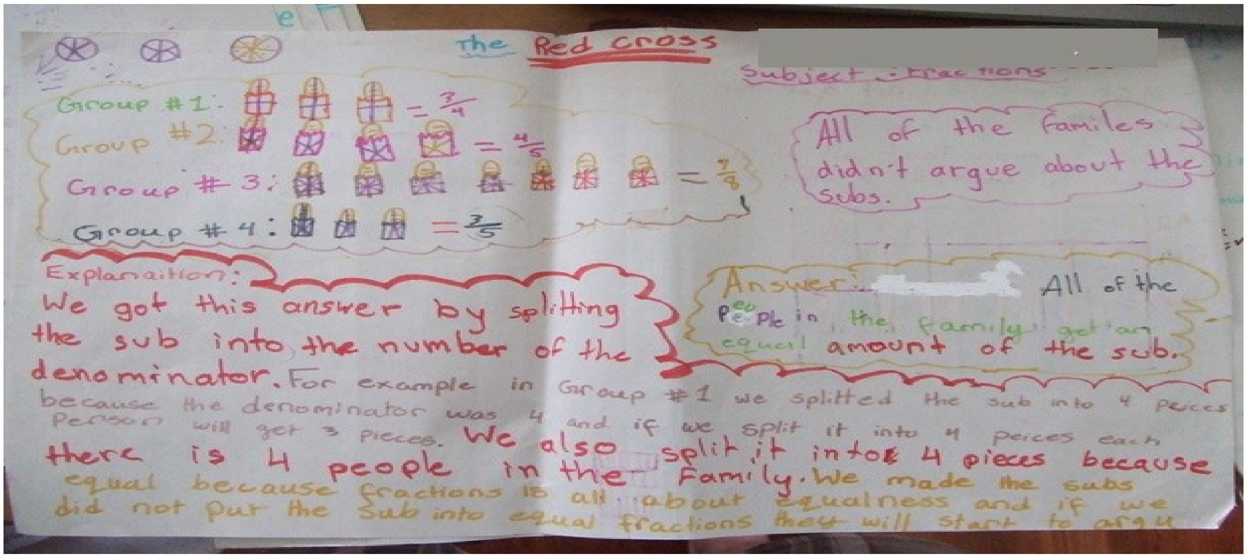
When I first came to these students, I saw that they had all of the correct fractions, with some error in their representations. They represented the three subs, broke one of them in half and said that each person would get 3/4. This piqued my interest so I asked them, "How did you figure out that it was 3/4?" Their response was, "Well... we had three subs and four people, which means one person wouldn't get a sub, so we divided a sub in half and each person would get a part: two people get one whole sub and the other two get a half" (PD.64). They would each get 1 of 4 pieces but the pieces were not the same size. This was an interesting statement because we had been talking about fair sharing with our fraction strips. For that problem they had to share a sub with friends equally, and as a result they had learned how to divide a sub equally; however, Nancy and Anita were struggling in this specific context to see how you could divide a sub equally without giving each person a whole. This may have occurred because I was moving away from the concrete strips. It was also the first time that they had tackled a fraction problem as a partner group instead of a whole class discussion.

It appeared to me that Nancy and Anita were struggling to understand how to divide the subs fairly. With this in mind, I asked them, “Does everyone need to get a whole sub?” This seemed to spark an idea in their head and brought about a discussion concerning the size of pieces and how the number of people seemed to determine the amount of the denominator. Once they realized that each person didn’t need to get a whole sub, I asked them, “Are your pieces the same?” I waited for them to do some thinking, watching them draw things in the air and talk to themselves. I asked them, “How many subs would each person get with your representation?” (PD. 64). They told me 2 1/2, which led me to ask, “Is this possible?” (PD. 64). They finally concluded that it was not possible but were struggling with why. It dawned on me that the two girls were struggling to understand how to divide something into parts because, with the kit, which they had been using, this was already done for them. Moreover, even though we had done some work with sharing using their fraction kits, their previous experiences with dividing had always been with quotients larger than one. Therefore I asked them, “Does everyone have to have a whole sub?” (PD. 64). This seemed to spark some thinking because Nancy concluded, “No,” and further, that each person would receive ¾ of the sub because there were four people and three subs, therefore each sub would be divided into fourths and each person would get one of those fourths for a total of 3/4s” (PD. 64).

When I visited them next I noticed that, like Rick, Holly and David, Nancy and Anita were able to represent the fractions of 3/4 and 7/8 but struggled to figure out the fifths. The difference was that Nancy and Anita were still able to understand what the fraction was and didn't seem to be bothered by their inaccurate representation. We had a brief talk about representing fractions and moved on to determining which group had the most subs and which one had the least (i.e. to order fractions). When I approached them this time I asked, “Are you saying that these fractions are all equal?” They suggested that they were. I then said, “But why would they argue?” relating everything back to the context of the problem (PD.66). Fosnot and Dolk (2002) suggest that using a contextual problem grounds students in real world thinking. It allows them to apply real world strategies, or applications, that they have used in the past. It no longer becomes a discussion about the numbers or procedures but more about the application of strategies (P.35). When Nancy and Anita realized that their thinking didn’t make sense within the context, they struggled to find a strategy for comparing fractions. They then realized that they could use their fraction strips, and made five different strips to represent the different subs and fractional amounts and then tried to compare them to how much they each had left to a whole (PD.67-68).

In the end, they created a benchmark model for comparing fractions. They understood that a fraction was a relationship between its parts and its whole. The interesting thing is that in their linear model they were able to make fifths properly, but when they had originally split it incorrectly as a square I believe they still were thinking of it as a circular model. If you look at Figure 10, you will notice that at the top of their paper are circles; these drawings look very similar to their square representations. Whereas on the back of the poster, they drew full rectangles in a true linear model. By the end of their group work these students understood how to represent fractions. They were able to see that fractions are a part-whole relation and that there are two different parts to a fraction. Unlike the first group, Nancy and Anita were able to move just past representing fractions and started to compare them to one another. At this time in the process, this level thinking was a work in progress. What role did my questions play in any new learning on their part?

Figure 10 Nancy and Anita's work



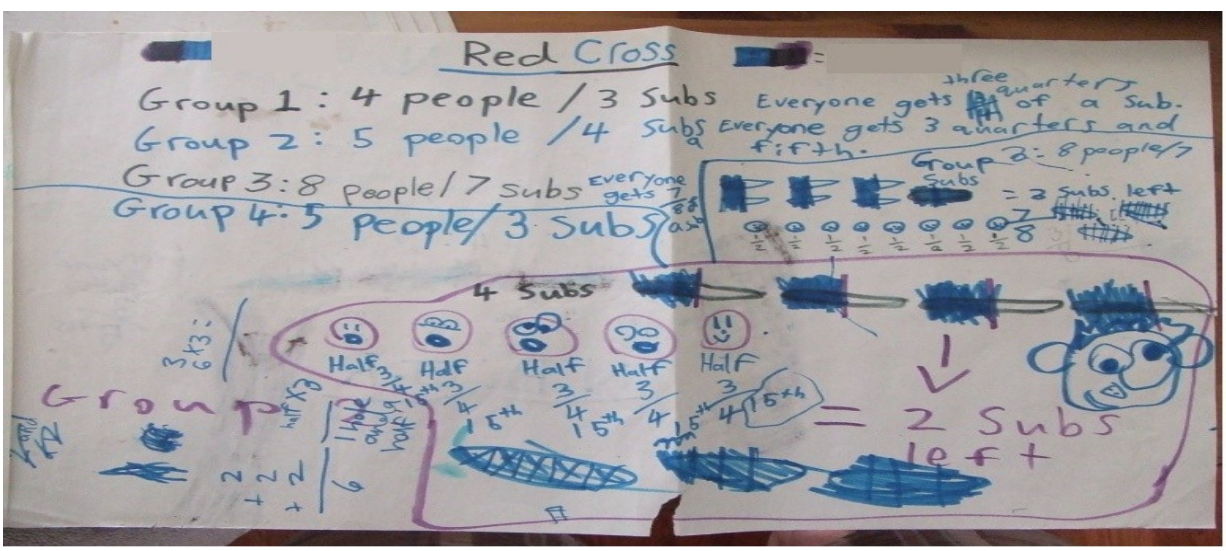
4.2.1.4 Analysing the types of questions I asked during the small group work

Examining my own role in questioning I found I used a similar proportion of interrogation (9) and going beyond (4) questions to those used with the lower group, but far fewer of them in total (14) thus reflecting their knowledge base of representing fractions. For Nancy and Anita I was able to ask more questions that stretched their thinking. While conversing with them I noticed that they grasped the concept, and my questions turned towards making them think, and communicate their strategies instead of trying to lead them to an answer through my questioning, as I had done with Rick, Holly, and David. In addition, Nancy and Anita were able to work with my questions and relate them to their work, whereas the first group really struggled with the basic big ideas of fractions and needed more scaffolding and a smaller task. This reduction in my questioning was even more pronounced with the high group.

4.2.1.5 Teaching, questioning and student results: Red Cross /sub problem: high group (Nick and James).

This was the only group that did not struggle with the problem. In fact, when conferencing with Nick and James they were able to articulate what the fraction was just by looking at the number of people and the number of subs. This was probably because they were able to represent the fractions as a line and therefore did not struggle with dividing their whole into an odd denominator. For them, it was a matter of breaking each sub into equal parts depending on the number of people; for example, for four people and 3 subs, they broke each sub into four parts and each person got 3 of them. Rick and James stated, “We knew what the answer was because we divided the amount of subs by the people and gave each person one piece of each sub”(P.D.80). See Figure 11.

Figure 11 Rick and James’ work on the Red Cross problem



Fosnot and Dolk (2002) note the strategy that they used is called a unit fraction, which is a strategy used by the Ancient Egyptians. However, unlike the Egyptians, Nick and James, broke the part into sections and then iterated those sections. The important variation is that Nick and James take it one step further by adding the units together to compose a final fraction (e.g. 1/4 +1/4 + 1/4 = 3/4). When I saw this particular strategy I knew that this would have to be the last strategy to show my students during the congress, in order to discuss this unit representation. Not only does this support the idea of a part-whole relation but it encourages students to realize that each section is worth a certain numerical amount and not just a section shaded in. Furthermore, it reflects the big idea that multiplication is related to fractions. This particular conversation happened as a whole class congress, or the part three in a three-part lesson plan.

Looking back on their discussion, these students understood the question right from the beginning and didn't need any help from me as the teacher. When I first approached them all I asked was, "What are you thinking?" and they were able to articulate their reasoning. In the end, Nick and James were able to represent each fraction and then compare them based on their constructed benchmarks. They knew how many pieces each group had left and, combining this with their understanding of the big idea that as the denominator got larger the pieces got smaller, they were able to figure out who had more and who had less, something that both the other groups and most of the class were not able to do.

4.2.1.6 Congress and concluding thoughts.

After monitoring all of the students and their thinking I realized that the majority of my students were struggling with understanding how much each group received in the problem. This discovery was surprising because I thought that after making our fraction strips and playing two days of games and mini-lessons on these games, the students would be able to compare these fractions correctly. However, this was not the case and therefore I decided that the congress would be focussed on a discussion around how they came up with the fractions and, more importantly, unit fractions. As previously mentioned, unit fractions prompt the students to think about the relationship between the whole and its parts in a fair sharing situation. In addition, it connects the students to multiplication and division, two familiar areas for my students. As the discussion unfolded, we were able to talk about what a numerator and denominator was and their corresponding relationship.

During the last thirteen minutes of our class congress we discussed what they noticed about each of the fractions. Reflecting back on their work from the previous day and some of the responses in the classroom, I noticed they had trouble with some of the fraction concepts. Due to this observation, I thought that even though they were not able to come to these conclusions as partners they possibly might do so as a community. As Cobb et al. (2001) suggest new knowledge is constructed with the teacher and the whole class. At times it takes the congress to consolidate the learning as students listen to others express their learning and opinions. Not only would this serve as a way for modeling fractions, and sharing our thinking, but it would also set-up the next day's lesson on comparing fractions using the Mississauga Marathon problem. As students worked together through many "think, pair and share" prompts[11], a lot of re-voicing on my part and wait time, they concluded that the subs were close but not equal. In fact one student in the class and one not being followed stated, "They all were one part away from a whole, that the one parts all had different values" (PD. 82). This prompted me to ask them why they could not look only at the numerators only. Their response was, "When you are looking at the numerators you are only seeing a part of the fraction and you need to see both parts because it is a relationship" (PD.82).

[11] Think, Pair and Share occurs when, in response to a teacher question or prompt, the students are asked to first think by themselves and then share with a partner and finally share their ideas with the whole class. When I have done this I have often found that students feel more willing to share ideas that have been confirmed by their peers. They also had a chance to try it out on their peers to see if their idea had any validity.

Reflecting on my own role as a questioner during the congress, I found I used 18 interrogation, 10 going beyond, and 11 building on type questions during this congress (see the earlier Table 6). For this particular congress, I felt that my students were having a hard time moving beyond the concrete representations; hence many of my interrogation type questions came from these pivotal moments. At the end of the congress I decided to do a whole group discussion on comparing fractions. This opportunity is where I tried to push the students beyond what they knew and build upon the concepts that they learned in the sharing section. At this time in the congress, I also implemented a lot of wait-time and think-pair share strategies. This situation allowed my students time to talk among themselves about the questions, which gave more opportunity for sharing.

To summarize: just as the literature suggested, this problem brought out many of the challenges that students face when learning about fractions. It developed an understanding of representing fractions, both even fractions and odd fractions. It allowed me to have a conversation about unit fractions and how they needed to think about the part-whole relationships. More importantly, it allowed us to have a guided discussion around how to compare fractions and that, as the denominator gets larger, the pieces actually get smaller. Through this process students were building a knowledge of fractional understanding. Although in the problem solving students struggled, (especially the lower group), it was not until the congress that students started to consolidate their learning and bring together many of the big ideas in fractions. Without this interaction between teacher and student or the questions that were asked the students would have continued to struggle. By having those pivotal questions during the congress it allowed the whole class to move along a continuum of learning (or trajectory) and prepared them for the next set of problems.

4.2.2 Problems #2 and problem #3: Mississauga marathon context.

The next two problems were chosen from a set of questions devised by Burns (2003). I used the context of the Mississauga Marathon because it provided me with a useful structure in which students could use a benchmark model or a number line. In addition, it also offered me a context that was practical for comparing fractions and one that students could relate to and have an interest in. These sets of problems also follow a natural progression from the Red Cross problem because students can work on the concept of comparing fractions, which we started to introduce in the congress.

Before teaching this lesson I reflected on the previous Red Cross problem that the students had done. During this problem I noticed that students were struggling with making fractions, and not seeing the relationship between the numerator and the denominator. In addition, many of the students struggled with actually comparing fractions because they were looking only at one of the fraction numbers, either the denominator or the numerator; or they forgot that as the denominator gets larger the piece actually gets smaller. The purpose of these two questions was to see if students would first learn how to compare fractions and also construct a linear model and benchmarks to perform those comparisons. Many of my questions for students were focused around how far away each fraction was from a whole, a half or zero. My aim in doing so was for students to construct an idea or benchmark model of how to compare fractions without using common denominators but instead a benchmark strategy. Furthermore, my questions were geared to leading my students to construct the big idea that as the denominator gets larger the piece gets smaller; therefore, if a smaller piece is closer to a whole it is the larger fraction. For example, 7/8 is smaller than 8/9 because 1/9 is closer to a whole because it is a smaller piece than 1/8.

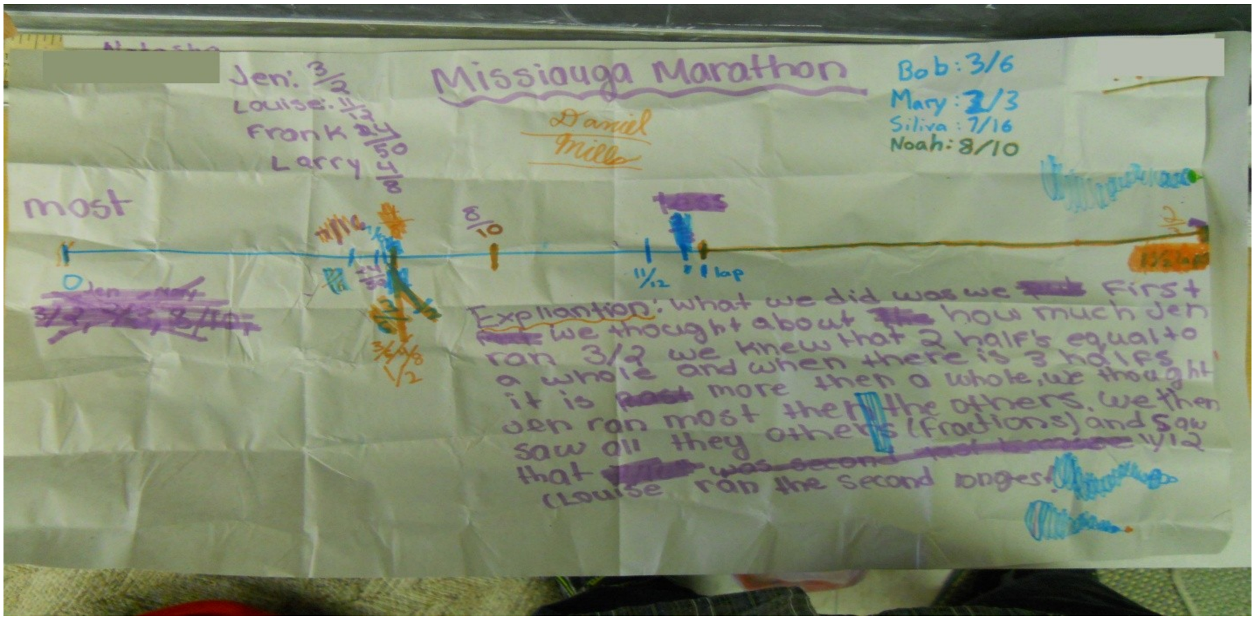
Before the actual problem was given out, I decided to do a small mini-lesson on how far away certain fractions were from the critical benchmarks of 0, 1/2, 1. Not only did this help set the tone and highlight certain strategies for the problem, but it allowed me to directly showcase why benchmarks are key when comparing fractions. I was really amazed at my students' responses and how much they had retained from the previous congress. Based on the last problem, I felt that some students were just starting to understand benchmarks, but upon reflection of this mini-lesson they truly understood the part-whole relation and how the fraction pieces could help determine how far away fractions were from these critical benchmarks. Holly stated, "Mr. So, I noticed that 7/16 is 1/16 away from ½ whereas 2/3 is over so it is larger" (PD.135).

4.2.2.1 Teaching, questioning and student results for the Mississauga marathon #1 and #2 children: lower group.

On their first attempt at this problem Holly, Rick and David seemed to struggle with the question; however it did take them significantly less time (27 minutes versus 49 minutes) to solve this question than the sub sandwich problem (see Figure 12). First, they attempted to use their fraction strips and find the fractions that they could. In addition, they were trying to figure out which fraction was greater than a half or less than a half; this part was challenging for them. When I approached them I asked them what this fraction meant, pointing to 3/2s (PD.86). They replied by stating that it was 3 twos, then 2/3, and finally coming to the conclusion that this was 3 halves. I asked them how much that was and they pulled out 3 half sections from their combined fraction kits. It was at this point I felt that they had learned from the last problem that a fraction is a relationship and that it is a unit of quantity, meaning that the 1/2 was not just one piece but that it was 1 x 1/2 and 3 halves is 3 x 1/2. I decided to let them continue the task. As they were working they came to the conclusion that they needed to establish benchmarks of a whole, a half and 0.

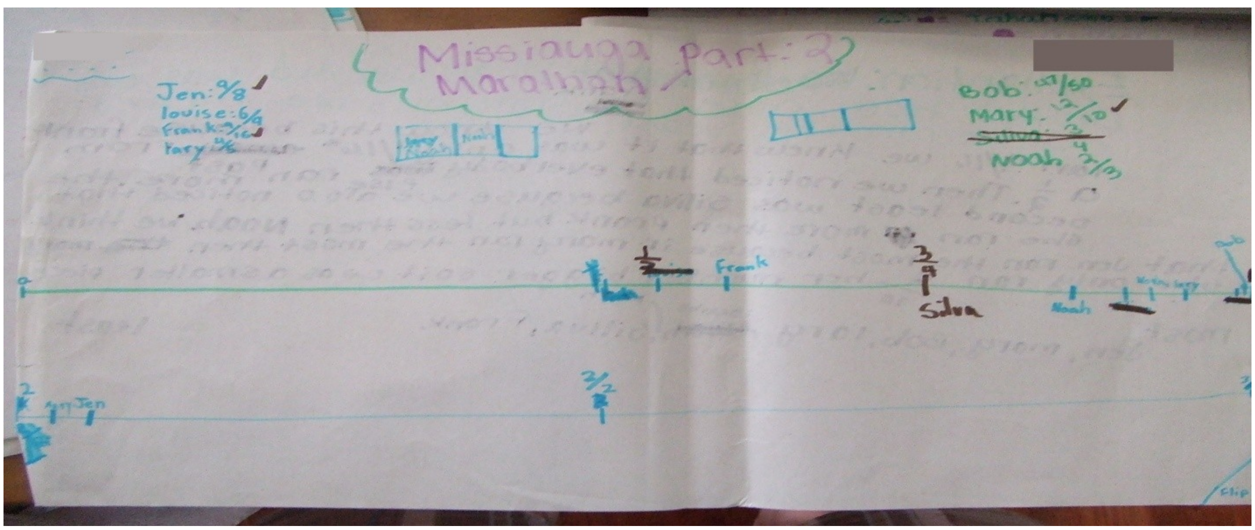
Throughout their discussion the group continued to use the idea of a benchmark and where each of those fractions lay according to this, often stating, "11/12 is only 1/12 away from the finish line," (PD.87) and then placing a line next to the finish line. Where they struggled was when the fractions were really close together (2/3 versus 11/12), not fully understanding that as the denominator gets larger the pieces get smaller. It was at this point that I came to them next. I asked, "Why did you place 11/12s here?" pointing to the position on their paper (see Figure 12). I continued with, "Why did you place 2/3 here?" (PD.87). The students responded by stating, "Cause 2/3 is smaller, and if the number is smaller, then the fraction is smaller." What they were trying to explain was that as the denominator increases the pieces get smaller, so because they were both one piece away from the whole, 2/3 was closer because the piece was smaller. Seeing that there was confusion in their understanding I asked, "So if the piece is bigger, does this mean it is closer or farther from whole?" (PD.88). Rick responded by saying, "farther, because the 11/12 has more pieces than 2/3, therefore it is bigger" (PD.88). Again I responded by asking, "Does the amount of pieces matter?" (PD.88). What I was trying to do was bring them back to the understanding of a part-whole relation and prompt them to see that because 1/12 was a tiny piece, it was in fact closer to a whole than 1/3. Holly finally saw this and stated, "a 12th was smaller than a 3rd so therefore it was closer to a whole" (PD.88).

Figure 12 Holly and Rick's work for Mississauga Marathon problem #1



The second Mississauga Marathon Problem was presented in order to give the students some more practise using a number line. For this second problem, Erick had returned from being sick, so I decided to break Holly, David and Rick apart by making Holly and Rick their own group. Furthermore, because of how far David was behind and how much Erick had missed I decided to do a small mini-lesson with just David and Erick. The mini-lesson focus was on using the fraction kit to compare small fractions within it. During this mini-lesson they did not do the problem, which is why I turned my attention to Holly and Rick. During their problem solving task, Holly and Rick did much better, starting right away with a number line and splitting the line into halves and wholes. They were also able to recognize that certain groups didn't improve, stating, "Jen didn't improve because 3/2s is more than 9/8. This is because 1/8 is smaller than 1/2 compared to 1" (PD. 96). The next time that I saw them they had completed the whole number line and accurately represented each of the fractions on the number line (see Figure 13). When I asked them to explain, they had some small minor errors in their representations, but they were able to sort these through while I revoiced their thinking to them. What really impressed me was that while in the first problem they had struggled to understand that as the denominator increased the size of the piece decreased, in this problem they were able to explain who ran the farthest distance and the least, based on this big idea. Not only were they able to do this, they were also able to articulate different equivalent fractions and identify with some scaffolding the location of all of the key benchmarks (PD.97/98).

Figure 13 Holly and Rick's Work in the Mississauga Marathon Problem #2

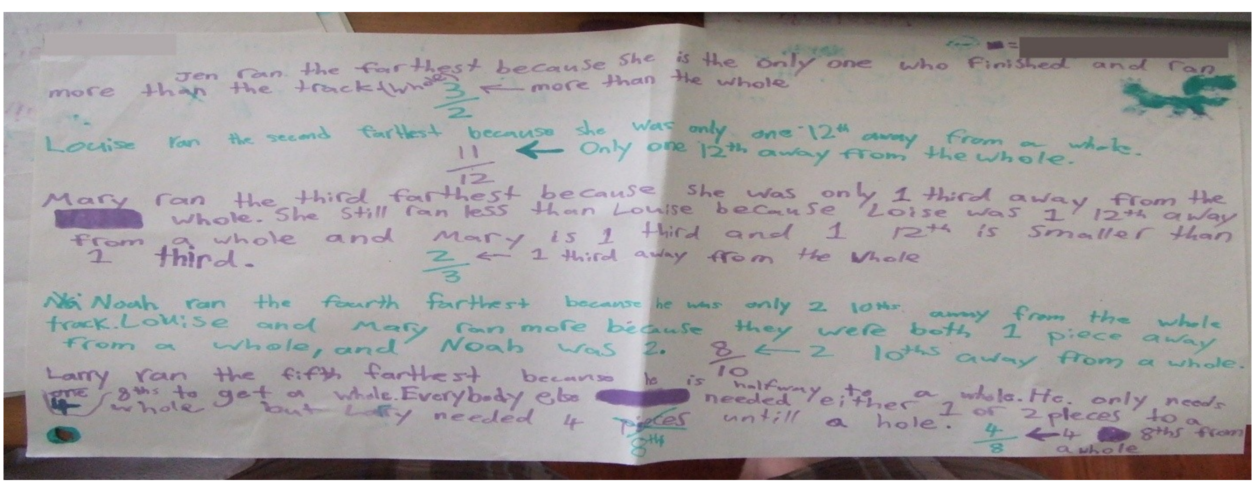


Examining my own role I found I used an equal proportion of interrogation (7) and going beyond (7) questions in the first problem and moved towards more building on questions (11) for the second problem. Though there wasn't as much of a decrease in the amount of questioning (23 vs 19), there was however, a decrease in the amount of interrogation questions that I needed to ask and more of an emphasis towards pushing their understanding beyond what they knew. This decrease might have been as a result of their growth in understanding fractions. Both Holly and Rick were starting to use the concepts they had learned in the congress and the mini-lessons. David and Erick were still struggling a little to work through the problems that were given, but that is understandable considering Erick's first day with fractions was this problem and David was still working towards making fair shares and representing fractions.

4.2.2.2 Teaching, questioning and student results Mississauga marathon #1 and #2 children: High and middle groups (Nick and James, Nancy and Anita).

The mid and high groups demonstrated two different paths towards understanding this question. James and Nick automatically saw the relationship between the fractions and were able to organize them based on how far they were over a whole or under a whole. For the smaller fractions they were able to compare those to one half (see Figure 14). In fact, they finished the question so quickly that I didn't even have time to come to them while they worked; at best I was able to ask them what they were thinking and use two building on questions to help them with their communication. Looking closely at their explanation they consistently used a benchmark model along with the big idea that as the denominator gets larger the pieces get smaller. Clearly these two students were able to carry their learning from the first problem into the second and had solidified their understanding of this big ideas in fractions.

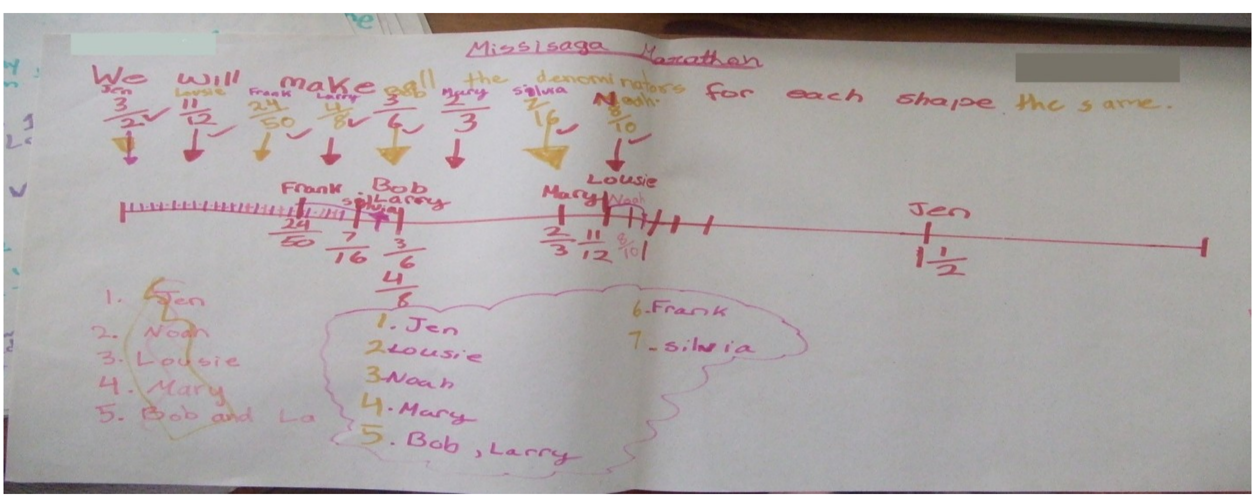
Figure 14 Nick and James' Work on Mississauga Marathon Problem #1



Nancy and Anita initially struggled with this question, even more so than Holly and Rick, though for different reasons. Their initial difficulty was a result of some procedural misconceptions, they were trying to make everything into a common denominator (see Figure 15). Both Mack (1995, 1999) and Asku (1999), suggested that this would happen, finding that students' procedural knowledge got in the way of their learning because students often would give up on making sense in order to rely on an algorithm. This is what happened with Nancy and Anita. When I first approached them, I found them discussing how they could make all of the fractions into a common denominator, mostly because Nancy insisted that this was the way it had to be done. Anita was very confused and just wanted to write out the fractions. In the end, and most likely because Nancy couldn't determine how to make a common denominator that worked, they settled on representing the fractions as circles. However, after the third representation they found it hard to compare to find which one was larger so they stopped this and decided to make a linear model (PD.91). But the girls ran into problems when they didn't make their wholes the same length, which is an essential big idea that we had stressed from the beginning of the unit; this is where I found them the second time. When I approached them I asked, "I noticed that your wholes look a lot different, are you able to compare them?" Their response was, "Yes, because if one is bigger, then they ran the most." I countered, "But isn't this whole different than this whole?" pointing to the two wholes. Nancy was still adamant that it was possible but Anita said, "No, because we don't know where one starts or ends" (PD.92). This small discussion prompted them to redo their strategy, but once again, Nancy wanted to make all of the denominators the same (PD.92). Again, the girls were at a standstill: Anita wanted to represent them as the same wholes and Nancy wanted to use like denominators. They decided to use the same denominators and spent the next ten minutes struggling to convert each fraction. When I came to them again I asked them what they were going to do, and when I found that they were going to make them into common denominators I stated, “Wow, that is a lot of multiplication.” Next, I asked them, “Where would you place the first fraction, 3/2, on a number line?” After some minor discussions about number lines, they then measured one out and accurately placed the fraction in the right spot. I queried, “How did you do that so quickly?” They replied, “Well, because it was over a whole by a half.” I continued asking them some of the easier fractions that I assumed they would know from the list, 2/4, 1/3, 4/16, and so on, and they continued to place the fractions on the number line in quick succession. Once on the number line they were able to finish the problem very quickly.

Looking back at Rick and Holly, it is interesting to see that even though they struggled from the beginning to conceptualize fractions, once they constructed an understanding of it, they could visualize the ordering. Nancy and Anita however, struggled to work through schemas and concepts that they had not properly constructed or were still working through. This was what Asku (1997) and Mack (1990) suggested would happen, when students relied on taught algorithms procedures versus learning through problem solving.

Figure 15 Nancy and Anita’s work for the Mississauga Marathon #1



For the second problem, both mid and high level groups were able to successfully apply their learning and do the problem rather quickly. I also decided that for both of these problems in the unit, we would only do a *gallery walk*. This allowed students to question each other's strategies and see how each of them solved the problem. After completing both problems the students were able to effectively use a number line and a benchmark model. Most of the students no longer used a circular model to compare fractions and many of them knew how much less or more a fraction was away from the critical benchmarks of 0, 1/2 or 1. Some of this success can be attributed to the work done with the fraction kits and the time spent on working with concrete models. This was similarly seen in the Cramer, Tate and del Mas (2002) RNP project, where they noted that students benefited from working with concrete models, and those students that did so had a better understanding of a part of a whole and its fractional amount.

In examining my own role and questioning across all the groups, I found, once again, I used an equal amount of interrogation (7) and building on (7) questions which were closely followed by going beyond (6) (see Table 7). This time around I observed that Nancy and Anita really struggled to find a solution. Many of their problems centered on using procedural ideas. As a result, I spent the majority of the time trying to refocus their thinking back to a working model to strengthen their understanding. Eventually, these students will be able to use common denominators effectively, but at that moment they did not have the fractional understanding or the multiplication skills to apply this strategy. Additionally, common denominators is not the most effective and efficient strategy, which is why I was trying to lead them towards a benchmark model. Left to their own devices, these students would have continued to struggle with this problem. They would also have tried the same procedure thinking as this is what was typically expected of them, getting discouraged in the end. This would confirm Asku's (1997) and Mack's (1990) findings on how reliance on procedural knowledge can lead to misconceptions in learning fractions. By using effective questioning skills (going beyond and interrogation) and knowing the landscape of fractional development, it allowed me to redirect the students so that they could focus on the fraction concepts and construct the big ideas. This could be seen in both the lower groups and the higher groups.

Table 7 *Questions I initiated during this problem set for Mississauga Marathon Problem #1 and #2*

Types of Questions	Holly, Rick, and David		Nancy and Anita (no #2 video available)		Nick and James	
Problems	#1	#2	#1	#2	#1	#2
T- Building On	4 (21%)	11 (41%)	7 (27%)	0	0	2 (100%)
T- Direct Teaching	1 (5%)	0	2 (8%)	0	0	0
T- Go Beyond	7 (37%)	5 (19%)	6(23%)	0	0	0
T-Compare	0	0	0	0	0	0
T- Initiation- Response- Evaluation	0	0	0	0	0	0
T- Interrogation	7 (37%)	4 (15%)	7 (27%)	0	0	0
T- Scaffolding	0	7 (26%)	5 (19%)	0	0	0
Total of Questions:	19	27	26	0	0	2

Overall, the majority of the groups handled these problems effectively. The only group that was still struggling was that of Erick and David because Erick was just starting the unit and David was my weakest student in the class for fraction understanding. Through the questioning the other groups were able to develop an understanding of a benchmark model, and were able to use this model to compare fractions effectively. In addition, the groups were using a lot of fractional big ideas to communicate their thinking, instead of relying on previously taught procedures or strategies they thought I wanted to see. They were able to articulate that because a fraction had a large denominator it, in fact, was a small piece, and that because it was a small piece, it was closer to the benchmark they were looking at. It is also interesting to note that the number of questions that I asked declined, and I was able to move away from interrogation methods and push the students' understanding beyond their initial schemas from the previous problems. Fosnot and Dolk (2002) would suggest that students are building upon their knowledge as they move through the various contexts and landscape. By this time in the unit plan, students had worked on three contextual problems, and played various fraction games. This has led to many opportunities for interacting with the students both individual, group and as a whole class. As Cobb et al. (2001) suggests the more interactions a student has the more learning is created. Moreover, Sherin et al. (2000) and Sherin (2002) suggest that these type of questioning (going beyond and building upon) helps students make deeper connections from one concept to the next. In so doing, students have assistance to bridge concepts faster than if on their own. This improvement in student learning can be seen here and in future problems as my questions move away from interrogation and scaffolding to go beyond and building upon.

4.2.3 Problem 4: How much is blue? Problem 5: What fraction is each piece?

I decided to group these two questions together because both explored the same big idea: that the size of the whole matters, and that fractions are multiplication, for example, 3/4 is also 3 x 1/4 and work towards understanding how fractions are a part-whole relationship. Both of these problems involved students figuring out the fractional amount of a given area in the whole. Problem #4 asked them how much of the shape was blue and Problem #5 asked them for the fractional amount of each shape. Both of these problems were an excellent way for me to see if my students understood the part-whole relation. If they did understand the relationship they would quickly see that the number of pieces was unequal and therefore not the denominator, forcing them to create equal pieces to find out how much of the shape was blue. The students that struggled to see this might still be having trouble seeing that fractions are a relationship between the whole and its parts. In addition, some students might have trouble recognizing how different students could have different answers but still be correct. A full description of my anticipated problems and questions associated with those problems can be found in Appendix G.

4.2.3.1 Mini-lesson with a small group of struggling students.

Between these two problems I inserted a small group mini-lesson for some of my struggling students, which included David, Erick, Holly, Rick and other students in the classroom. The other students in the class were playing fraction games taught at the beginning of the unit. I noticed that these students, were struggling with some of the basic big ideas we were talking about: 1) part-whole relation, 2) comparing fractions using the big idea that as the denominator gets larger the pieces get smaller and 3) benchmarks. Much of the time was spent having the students explore questions with their partners. I would often ask a building on question that was related to work we had already done in hopes of having the students make the connections between the class problems and the big ideas I was trying to have them work through.

Looking at the types of questions that I asked during the mini-lesson I noticed that they were predominantly going beyond and building on questions (see Table 8). In addition, there was a lot of opportunity for students to talk to each other with think, pair and share or just letting the students talk to each other for a long period of time, which enabled them to discuss without my interference. They already understood the basic concepts, but I was trying to build upon these skills and push their thinking so that they had a deeper understanding of the larger concepts. Moreover as students talked and interacted with me and the rest of the group it created more opportunities to consolidate their learning.

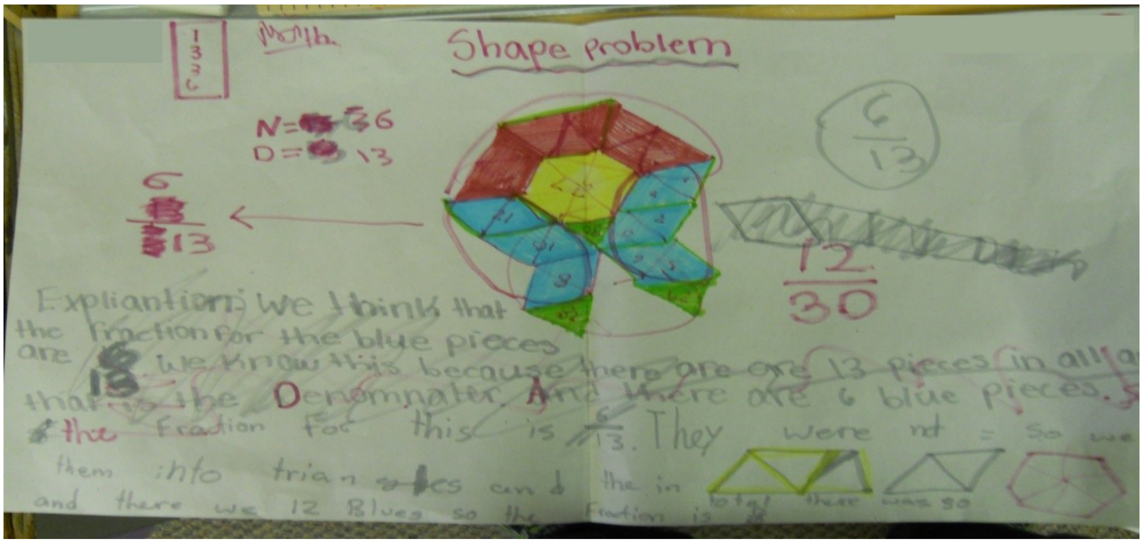
Table 8 *Questions I initiated during this problem set: Mini-Lesson, How much is blue and what fraction is each piece? (HMB/FEP)*

Types of Questions	Holly and Rick		David and Erick		Nancy and Anita		Mini-Lesson	Congress for HMB
	HMB	FEP	HMB	FEB	HMB	FEB		
T- Building On	0	2 (40%)	11 (31%)/	3 (37.5%)	0	2 (50%)	9 (39%)	6 (30%)
T- Direct Teaching	0	0	0	3 (8.5%)	0	0	3 (13%)	1 (5%)
T- Go Beyond	0	1 (20%)	0	8 (23%)	0	2 (50%)	5 (22%)	7 (35%)
T-Compare	0	0	0	0	0	0		2 (10%)
T- Initiation- Response-Evaluation	0	0	0	0	0	0	1 (4%)	0
T- Interrogation	0	0	3 (8.5%)/	2 (25%)	0	0	5 (22%)	4 (20%)
T- Scaffolding	0	2 (40%)	10 (29%)	3 (37.5%)	0	0	0	0
Total of Questions:	0	5	35	8	0	4	23	20

4.2.3.2 Teaching, questioning and student results problems 4 and 5: lower students (Holly, Rick, David and Erick).

This was a tale of two groups. Holly and Rick showed an enormous improvement and solidification of the big ideas and learning, David and Erick demonstrated smaller growth with more work needed in understanding the fraction big ideas. Unfortunately I was not able to video tape Holly and Rick's first session, but looking at their work and reflecting on the short discussion I had with them, they were able to identify what the whole and what fraction of the shape was blue (see Figure 16). Initially, Holly and Rick made the critical mistake that most students do when working with this problem, they counted the unequal pieces of the shape (13), which became their denominator and then counted the amount that were blue (6), which became their numerator. In the end, they arrived at a 6/13. I asked them, "Are your pieces the same amount?" The purpose of this question was to highlight the fact that they assumed that the triangular pieces and other shapes were all the same size, a common misconception with fractions (see Figure 16). When they looked at their answer they quickly realized, the pieces were not the same, and divided all of the pieces into triangles. In the end, they were quite comfortable with the concept, which translated into the next problem where they easily saw that the shape was a square and if you created four even triangles you could then derive the fractional amounts for each of the shapes. Improvement was not only seen in their understanding, but they also answered the question in 20 minutes, this was an improvement from the first question which had to be modified, and took them 85 minutes to solve. In addition, I was able to ask far fewer questions (5), and those questions were geared to help them explain their thinking more and to communicate their understanding.

Figure 16 Rick and Holly's work on how much is blue?



It was a very different story for Erick and David. Because of their struggles with fractions and the fact that Erick joined us part way through the unit, I decided to spend a lot of my time with them working through some of the concepts that we had already completed. I started by having them locate a variety of benchmarks on a number line. They were quickly able to identify where to place a half and a quarter on the number line but struggled to identify where to put 3/4. I asked them, "What relationship do you notice between 1/2 and 1/4?" They told me that 1/4 was a half of a half. We then continued to work through where to put 3/4, which they kept insisting was 1/4. I noticed that they knew that each space was 1/4 but were struggling to iterate them to make 3/4 (PD. 107). This led me to do some scaffolding and direct teaching to help them grasp the concept. Unfortunately, this discussion kept happening for each of the fractional benchmarks with which we were working: eighths, sixteenths. Every time Erick and David struggled to understand how unit fraction was not 1 part of the fraction. It wasn't until I compared it to a rational number line that they were able to see that a number line goes in a sequential order: 1/8, then 2/8, then 3/8, and so on (PD.108).

When they eventually understood where to put these fractional benchmarks I had them practise placing some quick fraction amounts, 5/8, 4/16. By the end of our little discussion I felt that they had worked with the concepts enough in order to start the problem. I realized that they might have been left with an impression that they were to use a number line for the problem because when I went to them again I found that they were lining up the shapes in a row and marking them on a number line (PD. 109). I asked them, "What are you thinking about?" They explained to me that they were trying to figure out how much each piece was worth. I asked them how they would figure that out. Erick said that the hexagon would be a quarter because it was a big piece. I asked him, "Is that the biggest fraction you know?" "No," he replied," but it can't be a half because it is not quite half of the whole shape" (PD.110). His answer prompted me to ask them, "Does the whole matter?" And then, "What would the whole be?" Erick told me that he was going to try and make them into one shape, and David suggested that they should count them because it would tell them how many pieces they would have.

I decided to leave them for a bit so they could work through things. When I came back to them I noticed that they were going back to the number line strategy, lining up the shapes in a line and then marking on the number line the space that each of these shapes occupied. Thus putting a line at the end of each triangle, then at the end of each hexagon, and so forth, forgetting that the space that these shapes occupied was not just a linear dimension. At this time, I thought it was best to scaffold the lesson and do some direct teaching as they did not understand and I wanted them ready for the congress. I started by asking them about what relationships they noticed, reminding them, "We have to make sure our pieces are equal. After some wait time and letting them work through it they were able to suggest that we could make all of them triangles (PD.110). Unfortunately, I did not realize the video camera's battery had died. But by the end of the problem they were able to understand that they needed to somehow divide the inside of the shape into equal parts. At this point I felt that they were ready for the congress and hoped that the whole class discussion would allow them to see other students' work and talk through some of their difficulties with their peers.

While working on Problem #5 both David and Erick were able to apply some of their thinking from the previous problems and, with help from me, actually solve the problem. When I first approached them they were working on trying to use a circle to draw the shape. I decided to ask them, “Why?” Their response was, “That is what the question asked.” I had them reread the question, and they realized it was asking them to use the same strategy for turning a circle into three equal parts, but apply this strategy to a square shape (see Figure 17). I asked them to think about the first problem, and what they had to do with the pieces. They told me that the pieces had to all be the same. I then asked them, “Can you apply that strategy to this problem (PD.126)?” This question got them thinking about how to break their shape into different equal parts. When I found them next, I asked them what they were thinking about. Erick told me that he noticed that with the "how much is blue" question, the trapezoid was twice the size of a triangle and, “if I split the square into triangles, I would get four triangles, and it would be 1/8 of the trapezoid (see Figure 18)”. I asked him how he knew that. He replied that he was still working on that (PD. 127). I decided to let them work through this problem because I saw that they were on the right track. When I came back for the final time, they had the representation completed but were struggling to figure out the fraction amount. They understood that the triangle was 1/4 because they split the square into four pieces. But they were struggling to see what the trapezoid would be. I asked them what they noticed about the middle line, pointing to how they had broken the square into four equal pieces. They told me that it broke the triangle in half. "Oh," I replied, "What is half of a quarter?" They responded that it was an eighth. We then discussed how they could combine these two fractions to create 3/8.

Figure 17 Erick and David's work, first attempt

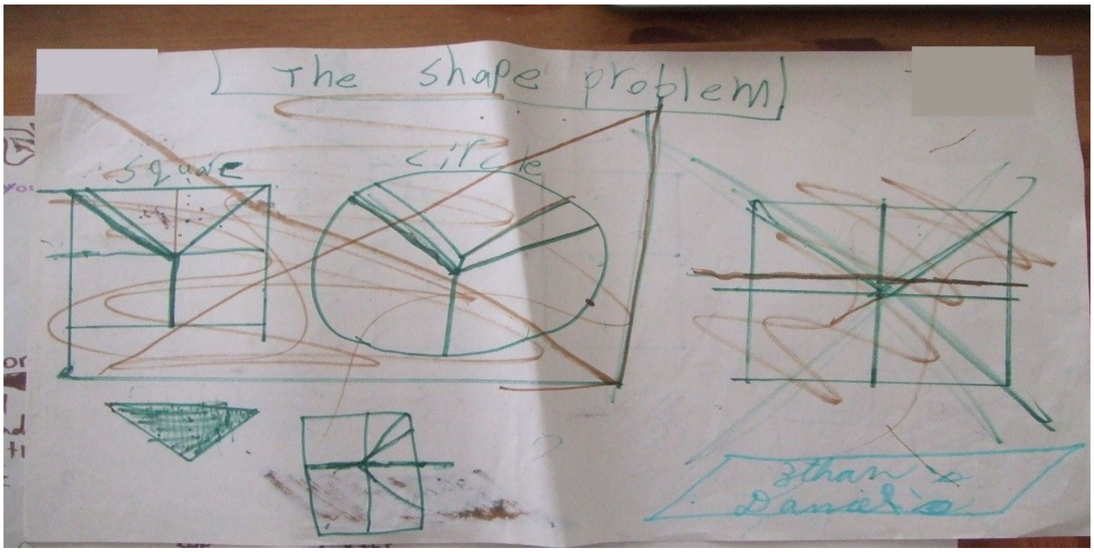
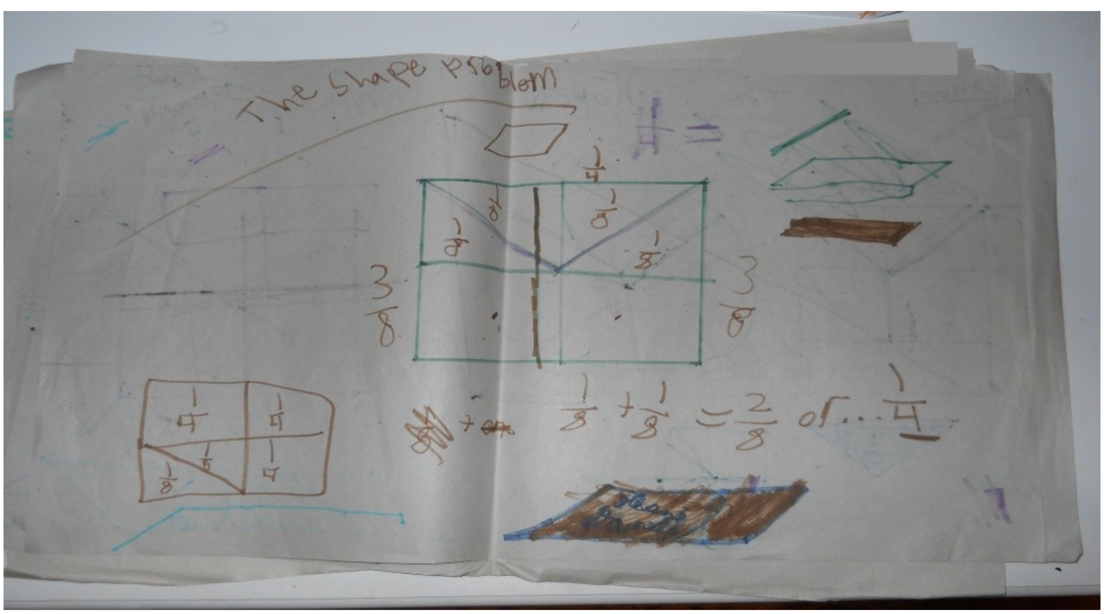


Figure 18 Erick and David's work, second attempt

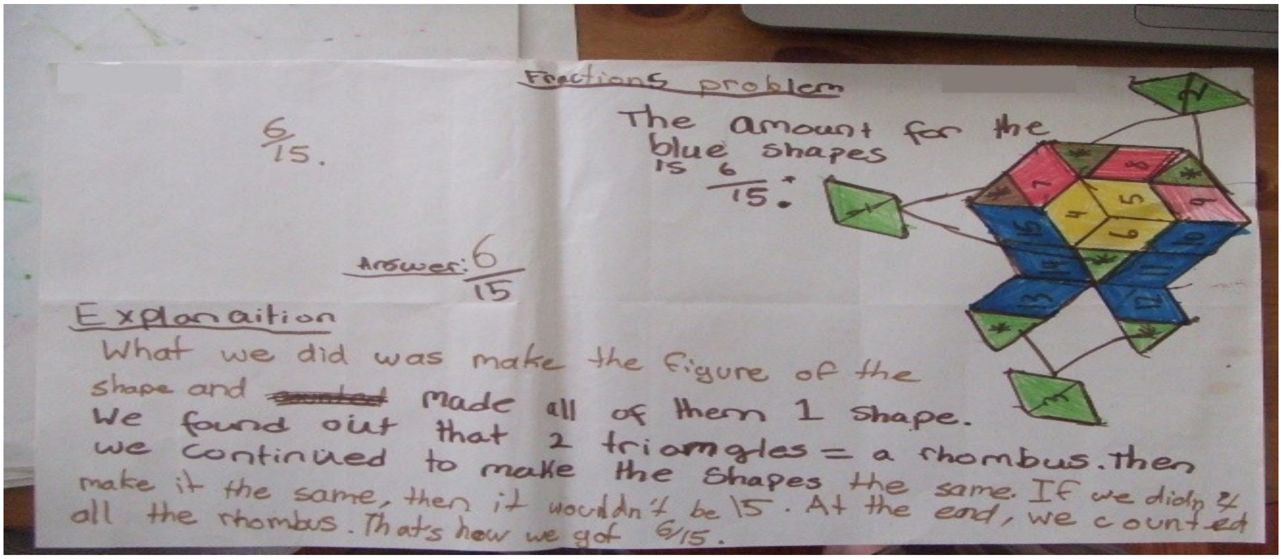


Overall, for Holly and Rick, and David and Erick, I asked a variety of questions depending on what was needed. At times, they needed a more direct approach, other times all they needed was a push to go beyond their pre-existing schemas. Looking back at the questions I used, I asked predominately building on (2 for Holly and Rick, 11 for David and Erick) and go beyond (2 for Holly and Rick, 8 for David and Erick). These questions enabled them to understand that the size of the whole matters, and that a fraction is a relationship between its whole and its parts (See Table 8). It is interesting to note that David and Erick were now moving away needing my scaffolding and interrogation types of questions, they were starting to articulate more of their learning without leading prompts. I was able to once again, transition to more questions that pushed them beyond or built upon the concepts they were working with.

4.2.3.3 Teaching, questioning and student results problems 4 and 5: middle and high students (Holly, Rick, Nick and James).

I analyzed the next two groups together because they both solved the problems in the same way as Holly and Rick, with very little difficulty. The only difference was that Anita and Nancy divided the shape into rhombuses and Nick and James used trapezoids. In the end both groups had little trouble answering the questions, and I only had to speak with Nancy and Anita's group in order to develop their communication. This situation was the same when it came to the last problem: both groups were able to look at the question and identify that the shape needed to be broken into four equal triangles, then the trapezoid would be 1/4 +1/8 which would be 3/8 altogether. The figures below are a sample of their work: Figure 19 is Nancy and Anita's and Figure 20 is Nick and James.

Figure 19 Nancy and Anita's work on How Much is Blue? (Top) and Fraction Amount Of Each Shape (Bottom)



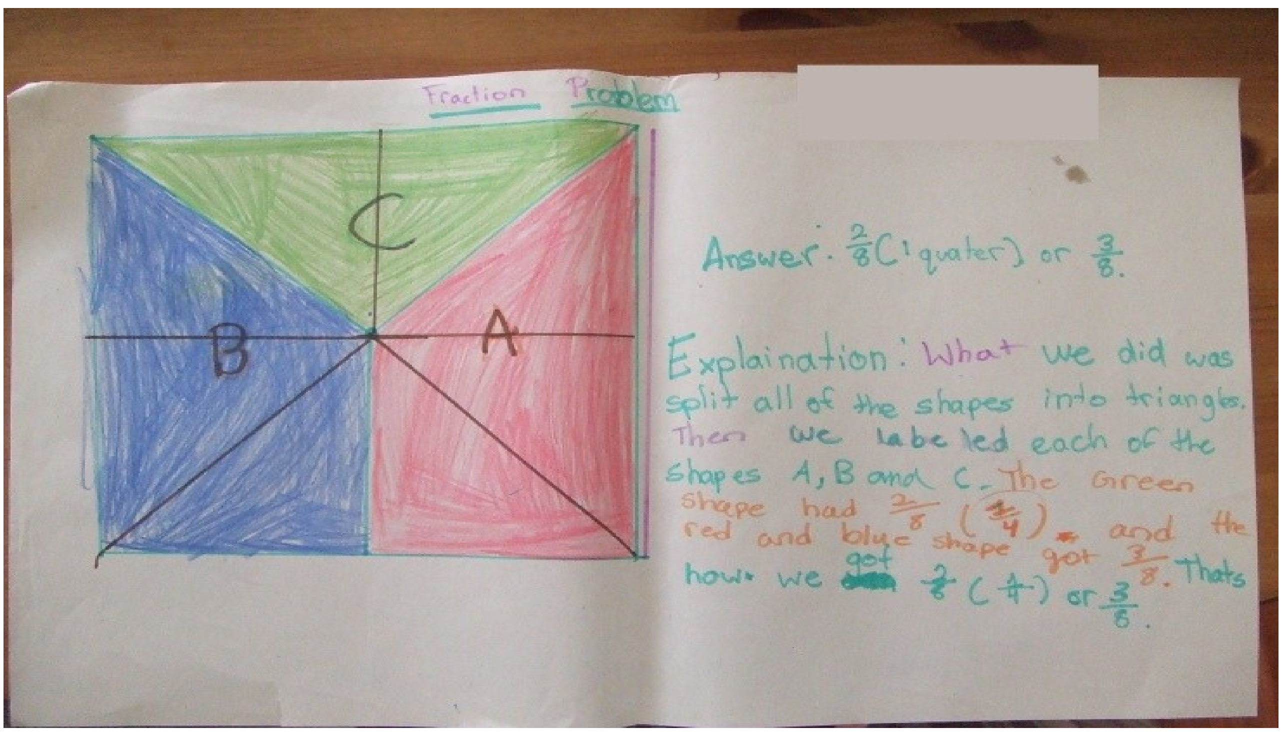
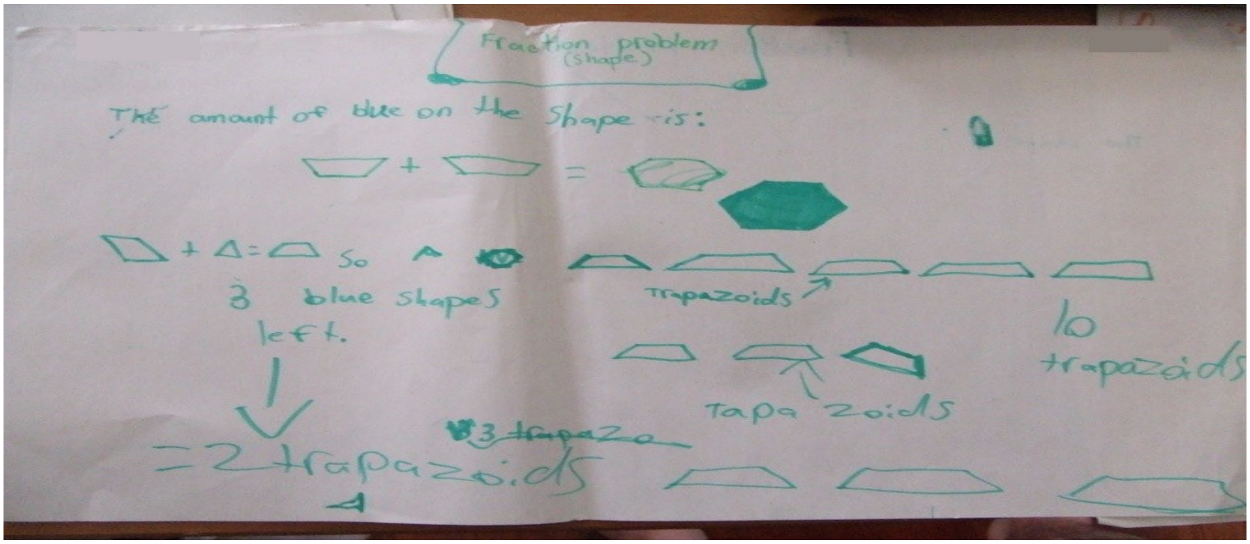
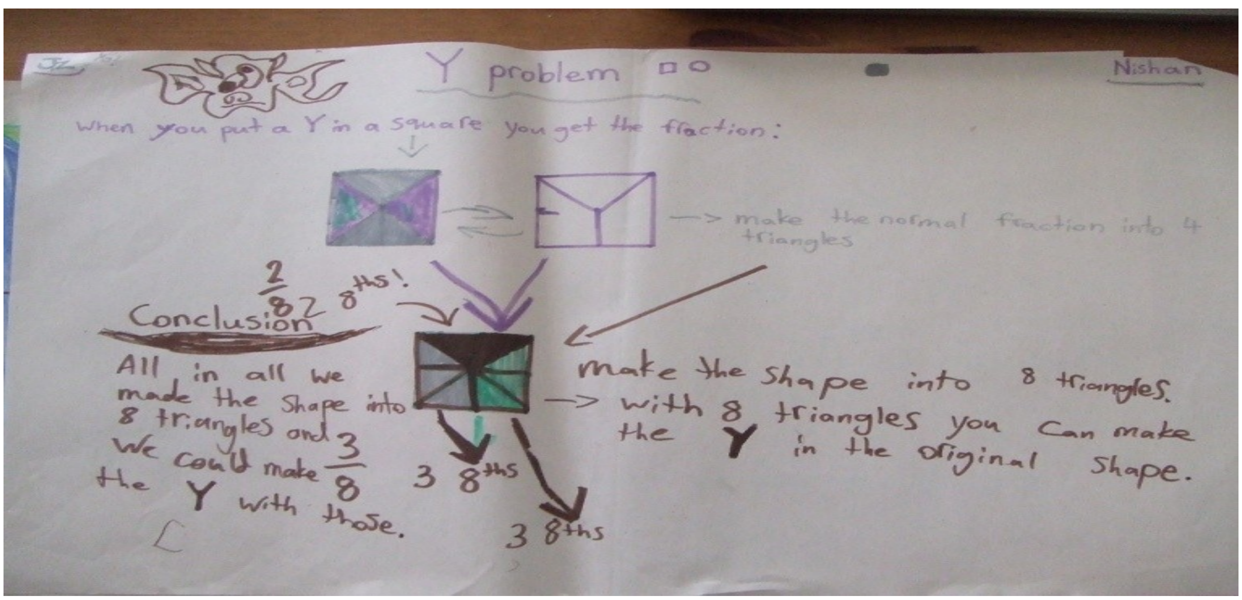


Figure 20 Nick and James' Work On How Much Is Blue (Top), And Fraction Amount Of Each Shape (Bottom)





4.2.3.4 Congress and concluding thoughts.

As I monitored my students, I noticed that either they were predominantly applying one of three strategies (using all triangles, using rhombuses, using trapezoids), or that the students initially struggled to see why they needed to do to make equal pieces. I decided to start with the latter issue and have the students identify why it was important that they break the pieces into equal parts. We then reviewed each of the strategies, and I finally asked them, “If we all got different answers, then who is correct?” This type of *going beyond* question allows me to push my students' thinking beyond a constructed schema. In this case, I wanted to see if they understood that fractions could be equivalent even if they had different shapes and took up different space (the big idea that pieces don't have to be congruent to be equivalent). I also wanted to see if they understood that the relationship was still the same regardless of the fractional representation. In the end, I was confident that my students had an understanding of a part-whole relation, and that a fraction needed to be in equal pieces.

4.3 Post-test Results and Analysis

The post-test was administered on May 12th, 2012, in order to see if the students had improved in their understanding of fractions and more importantly, the development of a benchmark model. The test comprised four questions asking generally the same types of questions as in the pre-test. All of the questions dealt with the part-whole relationship in fractions and the majority also dealt with having the students use their understanding relating to a benchmark to compare and order fractions. The results of the post-test (see Table 9) do not reflect the full growth of the students’ learning to the degree that can be seen in the unit problems and in the video discussions with the students. This is predominantly because they had some difficulty with the bare calculation problems, even though they fine with the word problems. These difficulties can be exacerbated by many factors, besides their understanding: test anxiety, English language learners, the wording of the problems, etc. This is examining only the pre and post would not give a sufficient or accurate representation of the students’ learning. For this one must look at the whole picture. The majority of the students had similar answers and problems, whether they were in the high, middle or low groups; David was the only exception.

Table 9 *Post-Test Results: Across all groups N=8*

Question Number and Short Form	Question Wording	Correct	Correct, needs further explanation	Incorrect	No reply
#1: Part-Whole Relations/ Fair Sharing	Share two pizzas among three people. Explain your thinking (Burns, 1998)	7		1	
#2: Benchmarks	Decide if each fraction is closest to 0, 1/2, or 1. Explain your thinking. 3/4, 3/9, 11/16, 1/4, 1/12	6	2		
#3: Comparing Fractions/ Benchmarks/ Part-Whole Relations	Joey and Robert each had the same pizza. Joey cut his pizza into 8 equal pieces and ate six of them. Robert cut his into five equal pieces and ate four of them. Who ate more pizza?		2	6	
#4: Comparing Fractions/ Benchmarks/ Part-Whole Relations	Raquel thought about this statement: When pitching, Joe struck out 7 of 18 batters. She said that it was better to say that Joe struck out about 1/3 of the batters than to say that Joe struck out about 1/2 of the batters. "I think that 7/18 is closer to 1/3 than 1/2,"she said. Do you agree or disagree with Raquel? Explain your reasoning (Burns, 1998)	3		5	

4.3.1 Question 1: fair sharing and representing fractions.

This question was used to assess the students' understanding of the part-whole relation. It is one of Burns' (2003) assessment questions on her book on fraction instruction. The students demonstrated the greatest success with this question (87.5%). Only one student could not answer this question correctly; however, he made other strides in his learning.

4.3.1.1 Low group Holly, David, Erick and Rick.

All, but David, were able to understand that each person would get 2/3 of the pizza because the denominator was what they were dividing the whole into, and it represented people; the numerator represented the slices in the whole. Holly and Rick used a unit fraction and then added the two fractions together (see Figure 21) proving to be a significant improvement on the pre-test in which they didn't know how to use unit fractions or make fractions with odd denominators. It is interesting to see that even though they could identify that each person would get 2/3 of the two pizzas they still struggled to represent the odd denominator as equal parts having two of the sections larger than the bottom third. Again, this issue seems to happen frequently, first when students are dealing with a circle model, and second when dealing with odd denominators. Nonetheless, as in the RNP project, the students still had a conceptual understanding of what the fraction represented and were able to identify the correct answer. As previously mentioned, David was the only student to struggle with this question. He represented a pizza cut into six equal parts, which then confused him when he tried to share those parts equally among three people (see Figure 22). Unfortunately, David is still trying to construct a concept of fair sharing and unit fractions, and is situated at the bottom of the landscape. This is still a big improvement from the pre-test in which David could not answer any questions because he had no conception of a fraction. In this situation, he may be thinking that there is a relationship, and that he has to divide the pizza amongst the three people, which is something he was not thinking about in the pre-test.

Figure 21 Holly and Rick's Correct response to Question 1

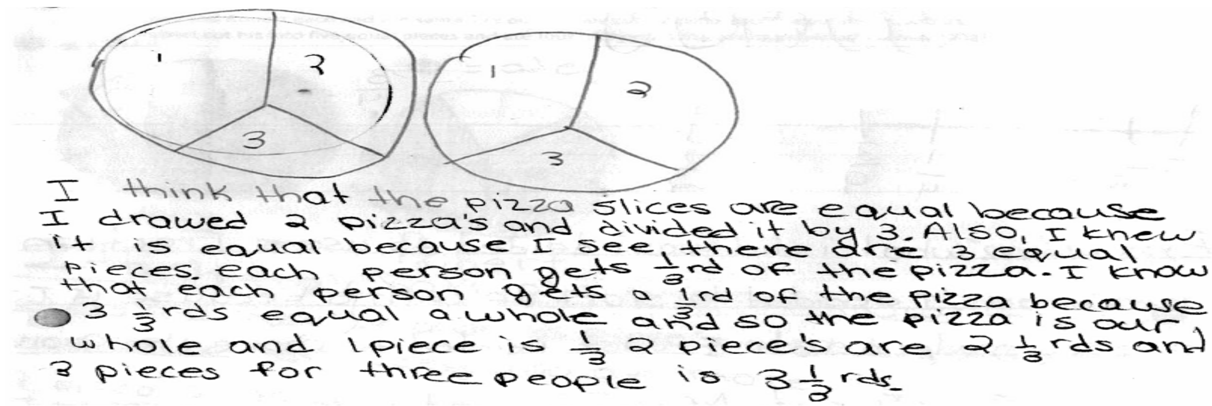
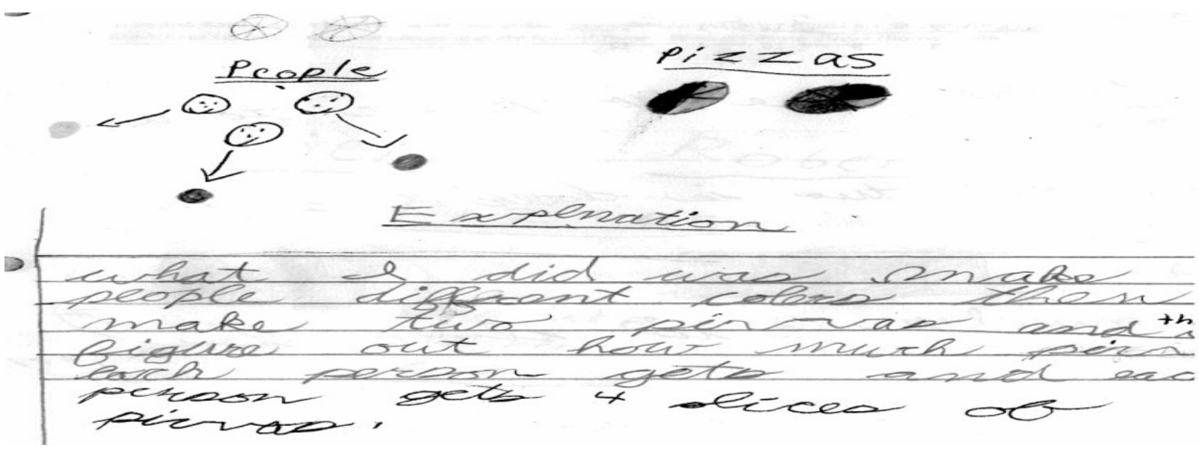


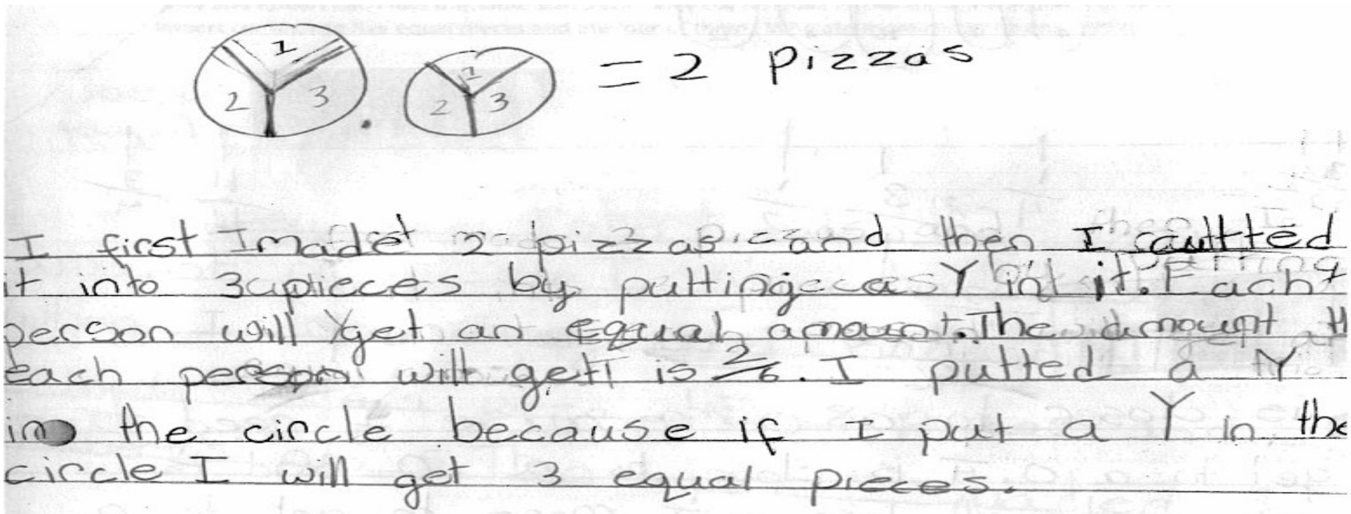
Figure 22 David's response to Question 1



4.3.1.2 Mid and high: Nancy, Anita, James, Nick.

For this group of students the answer was very similar to Holly and Rick's responses. The only difference was in the greater amount of communication (see Figure 23). In both cases, their answers to this question showed me their development in understanding fair sharing and how to represent fractions.

Figure 23 Nancy's response



In the pre-test, students struggled to represent a fraction whether it was even or odd. Here all were able to represent odd denominator fractions and divide them equally among three people.

4.3.2 Question 2: Benchmarks.

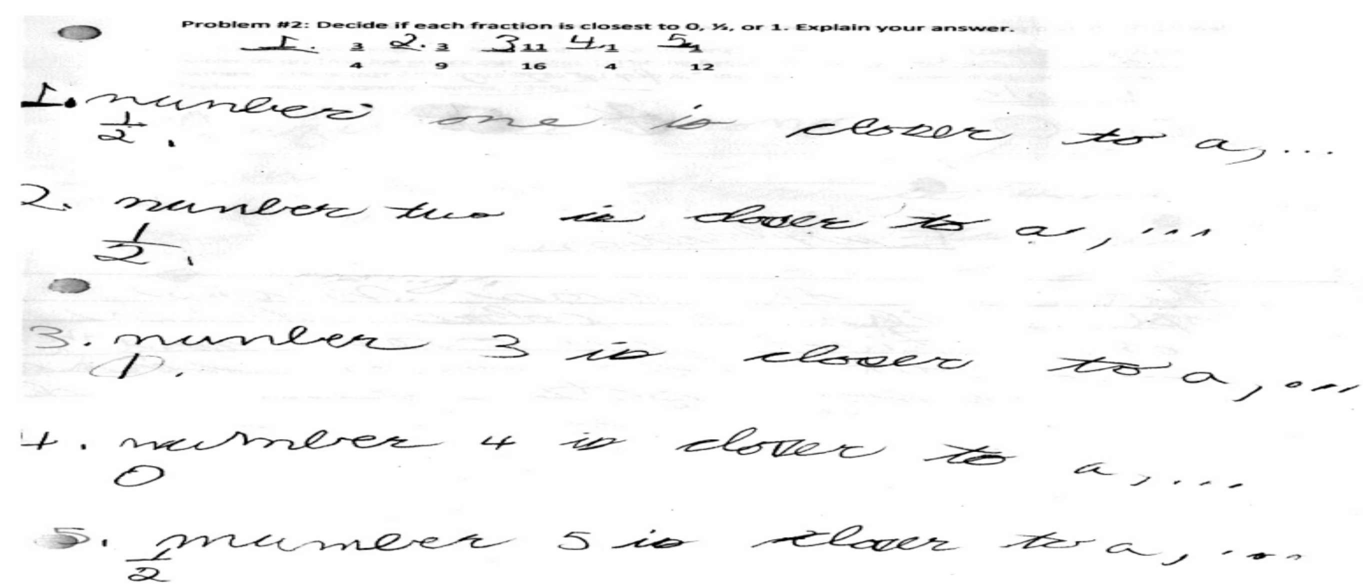
This question was chosen to determine if students were using benchmark fractions effectively, and if they understood how close fractions were to 0, 1/2, or 1. Again, for this question students showed improvement: six fully correct answers and two correct that required some further communication. All eight students, were able to answer the questions and for the majority of them, without my assistance.

4.4.2.1 Low students: David, Erick, Holly and Rick.

All of the students, David, Erick, Holly and Rick, clearly communicated their thinking and used their understanding of part-whole relations and benchmarks to place each fraction on a number line. The greatest improvement would be in David's response (see Figure 24). During the initial pre-test David was unable to draw a fraction or understand what fractions represented.

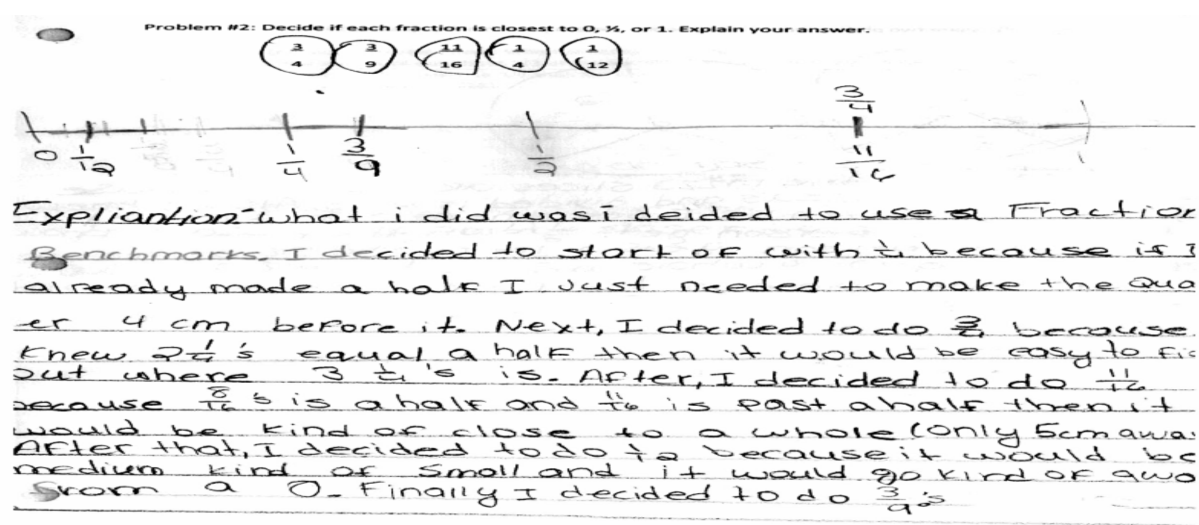
He saw the denominator and numerator as separate whole numbers instead of a part-whole relation. Even though he still struggled with the first question in the post-test he was able to identify where the fractions fit with common benchmarks. However, I am still unsure if they were lucky guesses or if he had some understanding of the concept. When I asked him to explain the fractions, 3/4, 1/4, 11/16 in his fraction kit, he was able to show me. But for the other fractions, 3/9, 1/12, he couldn't tell me how. This might have occurred because he was familiar with the fractions in his kit. We had been working extensively with them through math games and individual lessons, and he had been using some of the kit to assist with the numbers. In addition, when he had a concrete model, like the fraction kit, he could visually compare these to 1/2, 0 and the whole, whereas, the fractions that were not in the kit were harder to visualize for David because he was still in a concrete stage of learning fractions. Although this was the only area in which he grew, he nonetheless showed improvement.

Figure 24 David's response



The rest of the students were all able to show multiple ways as to why they thought the particular fraction was closer to 0, 1/2, and 1. In most cases they articulated how much closer to a 0, 1/2, or 1 a fraction was, and used that to define their answer. Again the largest improvement was in these students' ability to communicate their thinking. During the pre-test, many of the students gave up, one broke down in tears, and all of them lacked explanations of any kind. With this question in particular, their level of detail dramatically improved. In all but David's case, the students were able to use different models and explain their understanding about the big ideas. In figure 25, Holly is clearly articulating her use of a benchmark model; she has linear measurements, which show her understanding of a part-whole relationship, and she is breaking the question apart into sections that she can tackle easily. Not only has she demonstrated her understanding of fractions but she has also shown that she is becoming a mathematician and a patient problem solver. In addition, it is interesting to see how Holly is using a benchmark model to show her understanding of these critical benchmarks, a model that we examined closely in the congress, to compare fractions, and used by many of the other students as well.

Figure 25 Holly's (low group) response



4.3.2.2 Mid and high students: Nancy, Anita, Nick and James.

In this question, these students performed well. The main differences between these two groups and Holly and Rick's answers were in the vocabulary used, and the degree of communication. When the fraction was between two of the benchmarks they would often tell me that it was close to both, whereas Holly and Rick would pick only the higher of the benchmarks. In addition, these groups were able to clearly articulate their understanding that as the denominator gets larger the pieces get smaller. One student commented, " I think that 1/12 is closer to 0 because half of 12 is six, and 1 is 5/12 away from 6/12 and 1/12 is only 1/12 away from 0" (PD. 14). Some of the students even used a number line (PD. 13, 14, 15) onto which they placed each fraction and ordered them even though they were not asked to do this.

4.3.3 Question 3 and 4: Comparing fractions.

The final two questions investigated the students' understanding of how to compare two sets of fractions, which is one of the key curriculum expectations of Gr. 4. The questions also combine both an understanding of part-whole relations and benchmark fractions. Question 3 asked the students to compare two children who ate pizza and determine who one ate the most. Question 4 asked the students to determine whether the following statement was correct: 7/18 is closer to 1/3 instead of 1/2. The results for these two questions were the most surprising because after analyzing the students' work with the problems and listening to many of the conversations that we had had in the classroom, I thought that these two questions would be easy for them; however, this was not the case. For both questions the results were mixed. The low group struggled for various reasons, but my high and middle group had a lot of unexpected difficulties, as well.

4.3.3.1 Low group: Rick, Holly, Erick, and David.

In question 3, two main issues were evident, some of them were identical issues that occurred in the pre-test. David, Erick and Holly represented the fractions as circles, and though Empson (2002) and Cramer, Post & del Mas (2002) suggest that circles helped their students understand benchmarks and how to compare fractions, it negatively affected my students' understanding of which fraction was larger for two reasons. First, it was very difficult for my students to ensure the circles were the same size, and as one of the basic principles of comparing fractions is that the whole must be the same, this confusion affected the outcome of their results (see Figure 26).

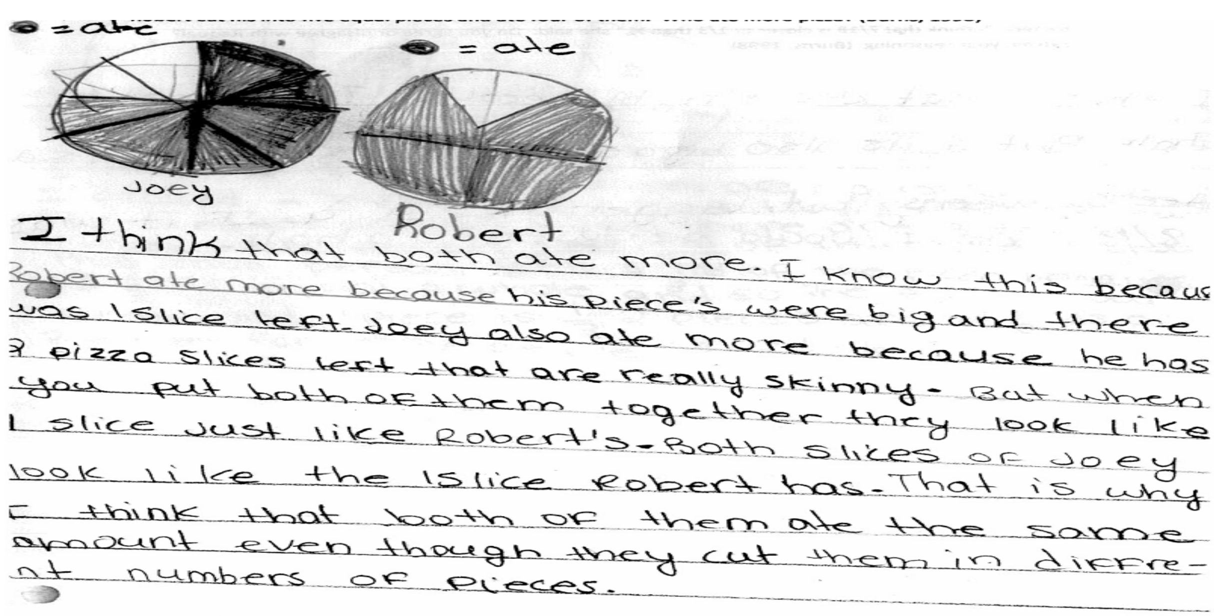
Secondly, it was difficult to represent odd fractions in circles as one must understand the degrees and angles that are in a circle in order to make the correct measurements, whereas in a linear model the students can measure the line and divide[12]. This discrepancy may be partially explained by Lamon's (1996) study in which she confirms Pothier and Sawada's (1989) research with the stages of fractions. However, she notes that these stages are very developmental, and the last stage in which students need to use multiplicative thinking takes a long time to develop. It might be that since my students are only in Gr. 4, they have not had enough time to work with partitioning activities to develop this final stage, which they needed for this particular question. However, I think the students had the most difficulty with the context of the problem and choosing the right model to represent their thinking. I did mention to the students during the unit that even though the context says pizzas their representations did not need to be circular; but it is hard to alter their strong personal experiences with pizza. Yet again, context is something that

[12] Out of the four lower students, only Rick (PD. 20) used a linear model, but that was only after he tried a circular model.

always must be considered when thinking about questions in general, let alone ones used for assessment. The reason I feel this assertion is true is that during the word problems, many of my students were comparing odd denominator fractions (see the Red Cross and Mississauga Marathon problems). For both of these problems the context directed them to a linear model. In many of the cases the students looked at what was left and compared those pieces to each other, which was what I was hoping for. In fact, two students tried to do just that in the next problem. In the end however, it was not what I was expecting with this answer. Having said that there were other improvements.

All, except for David, greatly improved in their level of communication. As evident in their work, they are trying to articulate the big ideas that they were learning in the class. In addition, not one of them felt aggravated or unsure about the problem. All four of them tried the problem and solved it to the best of their ability. This finding alone demonstrates a huge improvement from the pre-test to the post-test.

Figure 26 Holly's work for question 3



In question 4, only Holly and Erick were able to come close to a correct answer. Their only challenge was with unclear communication, Holly used a benchmark of one half, and stated that, “7/18 is not quite a half but 1/3 is greater than a 1/4 and that if it was 8/18 it would be closer to a half” (PD29). Although she didn’t fully complete the question, she has started to use a benchmark. She also demonstrated an understanding of where 7/18 was situated on that benchmark. Erick was the only one in the group to compare 7/18 to both 1/2 and 1/3, stating, “9/18 is half which is 2 away, and 6 (meaning 6/18) is 1/3 which is 3 away from nine” (PD. 28). Erick's mistake was that he compared the 1/3 to the 1/2 instead of relating it back to the 7/18. This mistake might be a small mental error, since he did know what the equivalent fractions were for the other groups. The other two in the low group struggled to even come up with an answer that made any sense. They used the numerators only, stating, “7+7=14 which is 4 away so it must be 1/3” (PD. 33) not yet understanding that a fraction is a part-whole relationship. This was a step back for Rick, who during the problems seemed to have a reasonable understanding of this relationship. Rick's possible struggles could have been the wording in the problem or that he still needed more work with part-whole relationships. At the beginning of the unit Rick struggle a lot with this problem and was starting to constructing the concept independently but most likely needed more concrete work with the concept.

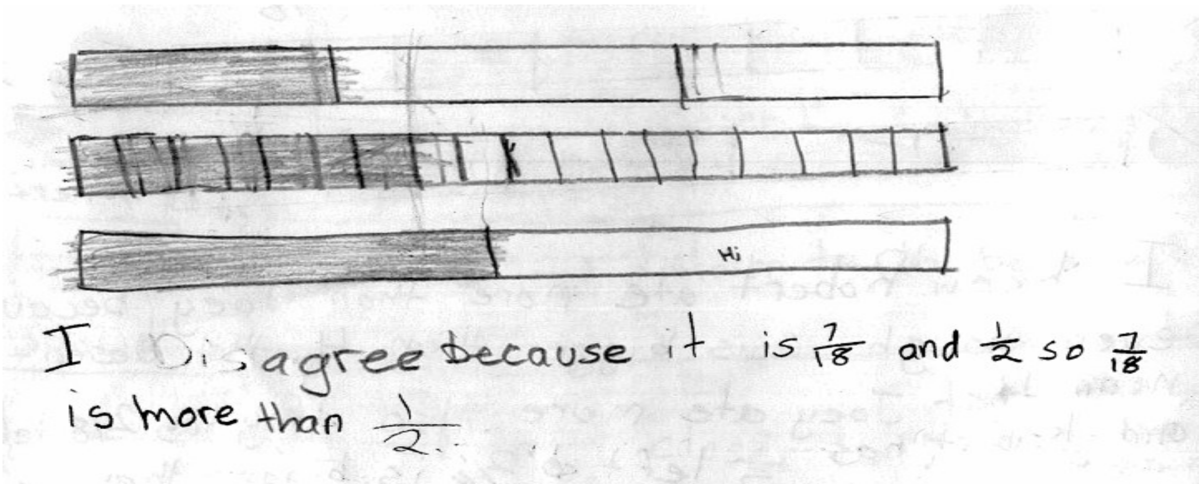
Finally, I realized that the fractions in Question 3 may have been too close together in comparison, being only 1/20th apart. However, more work with benchmark fractions might have allowed students to compare the pieces remaining as 1/5 vs. 1/4 and realize that 1/5 was smaller, meaning that Robert ate the most.

4.3.3.2 Middle and high-groups.

Only two students in the middle and high groups answered question 3 correctly (Anita and James). They were able to use the big idea that as the denominator gets larger the pieces get smaller, stating, "Robert ate more pizza because he had 1/5 left and Joey had 2/8, and 2/8 is 1/4 and because 1/4 is bigger than 1/5 so Robert is closer to the whole" (PD. 18). For the other students, however they all used a linear model and they struggled for various reasons. One of these errors might have been , as stated above, because the two fractions I chose were very close together, which is why I think Nancy made them equal in her explanation (PD. 23). Another reason the students had difficulty answering this question is that they may have constructed the big idea that larger denominators meant smaller pieces but were not able to use this to reason about who ate the most by looking at what remained. Nick, for instance, states that 1/4 is larger than 1/5 (thinking about what was left), but then says that Joey ate more because he ate the ¼; whereas the correct answer demonstrates that the opposite is true: he had more left, therefore he would have eaten less (PD. 24).

The students fared a little better on the fourth question as they could visualize a linear model more easily, and use a benchmark model as needed. In fact, all but Anita answered the question correctly by comparing 7/18 to both 1/2 and 1/3, stating that 6/18 = 1/3 and 9/18= 1/2 and 7/18 is only 1/18 away from a 1/3, so it's closer to 1/3 than 1/2 (PD. 32). Anita's response was the most interesting (see Figure 27) because she modeled the fractions correctly, but then stated that 7/18 is more than 1/2 even though in her picture it is clearly not.

Figure 27 Anita's answer



In summary, students made significant improvements from the pre-test to the post. Even though students may not have answered some of the question correctly, they persevered through the problem and communicated what they thought, which did not happen on the pre-test. It is possible that the interaction between the student and me, as well as the community, contributed to this improvement. In addition, all students attempted to use a benchmark model to some degree and some students had better success than others. These small growths show that students had improvements in their learning. Many of the students moved from no understanding to answering some to most of the problems and the post test questions. This could possibly be done on their own; however, looking at how students struggled at the beginning, this learning would have taken a long time to achieve. Some did have difficulties but perhaps some of their difficulty could be attributed to the way in which I worded the question or the numbers that I chose, ELL factors, and test anxiety. If learning happened because of these questions and interactions then the question then arises: what were the instructional methods that made the difference? What prompted the learning to take place in such a short three-week unit?

4.4 What were the instructional methods that made the difference? Teacher questions, talk moves and planning.

In such a short time span the majority of my students increased their knowledge and understanding of fractions. What enabled such insight or learning to occur? As I reflect on the process I can think of two possibilities: 1) the planning process, and 2) the types of questions and talk moves that I made during the unit. The following two sections briefly examine these two aspects of teaching.

4.4.1 Planning.

In their five practices, Stein, et al. (2008) (see Appendix L) suggest that teacher pre-planning is the key to moving from a first generation reformist to a second generation reformist. Reflecting back on the process and my field journal, planning was the key for all of the questions and talk moves that happened in the unit. Before the unit began I had identified most of the possible strategies that students might use to solve the problems. Within those strategies, I identified the misconceptions and problems that students might face, and then connected each of those challenges with particular guiding questions (see Appendix G). By thoroughly planning my unit, I was able to deal with my students' impromptu discussions, and accelerate or add more problems to the unit depending on what needed to happen and where my students were in their learning. In addition, it allowed me to make those split second decisions based on what I was noticing around the room. Because of that careful planning, I was able to quickly judge the students' zone of proximal development (Vygotsky as cited in Fosnot & Perry, 2005), and then ask those careful guiding questions to move them along in their understanding. After each lesson was presented and worked through, I reflected on how the lesson went and noted where the students' challenges were, what discussions happened, and if any new issues occurred that I hadn't thought about. This process then drove my next lesson and my next set of questions.

4.4.2 Questions and talk moves.

Chapin, O'Conner and Anderson (2009) suggest that teachers can spot misunderstandings much more easily when students are involved in discussion instead of sitting and listening to the teacher talk (p. 5). They propose that teachers can employ a variety of talk moves such as *revoicing*, *wait time*, *partner talk*, and so on, that can accelerate student discussions and communication. In addition, Sherin (2002) notes that the more students talk about mathematics, the more students learn about mathematics (p. 188). It is these types of talk moves, partnered with the types of questions suggested by Franke, et al. (2007) or Sherin, et al. (2000) that became an integral part of my teaching and the students' problem-solving process. Table 10 lists all of the talk moves that I used throughout the unit in interaction with the three groups. I made a total of 298 talk moves, 234 of them were used to promote talk and construct a big idea, 24 were just to promote talk and 40 to think about big ideas.

Table 10 *Types of Talk Moves and Resulting Student Focus*

Talk Moves	Amount of Times Asked	Talk Move	Big Idea	Both
T- Air Misconceptions	27			27
T- Answering With Another Question	32			32
T- Echos Students Words	15	15		
T- Letting Students Just Talk	9	9		
T- Monitoring Students	22		22	
T- No Confirmation/In Order to Push Beyond	14			14
T- Relate Back to Context	7		7	
T- Relate to Other Problems	11		11	
T- Revoicing	39			39
T- Student Revoicing	5			5
T- Think, Pair, Share	19			19
T- Wait Time	27			27
T- Checking for Understanding	71			71
Total Talk Moves	298	24	40	234

I also asked many questions throughout the problems to support student thinking. See Table 11 for a complete list of the questions that I used in the unit the type of question I asked, as well as whether the question resulted in student discussion. At the beginning of the unit I noticed that I was using more of an interrogation style of questioning, asking a lot of why questions or the talk move of revoicing the students' words back to them, hence, the high percentage of these types of questions (84). As the unit progressed I was able to ask broader questions that built upon the students' previous knowledge and the challenges that we had encountered, thus accounting for 64 building on questions and 85 go beyond questions. It is important to note that although I may have asked equal proportions of interrogation questions and go beyond questions, the majority of my questions dealt with a big idea in mathematics. This process may be very different than what is customary for teachers who typically ask questions to which they already know the response, or who ask a question expecting a certain response (Ackles, et al., 2004). It is possible that my talk moves and questions enabled many of my students to take over the discussion for themselves and do much of their own questioning within their group or with the groups beside them.

By the end of the unit students were asking questions of each other and interjecting their ideas into their discussions. This kind of interaction even happened at the beginning of the unit when making their fraction kits. For example during pair problem solving , James would often interject with questions like, "do you mean ... or so what you are saying...." (PD. 42)[13] These questions and conjectures would often come up during the congress where the students and I would be discussing the big ideas of the problems. In most cases, students would question what was being presented or disagree with a comment to my questions. As a teacher I was there to observe and help when needed but the students did the discussing with me as the teacher focusing their talk on the big ideas. The students' behaviour was close to what Ackles, et al. (2004) observed in their study when they noted that students who had a teacher asking more mathematically-focused questions began to defend and justify their mathematical ideas more confidently and thoroughly. I facilitated the atmosphere and context in which they learned. I gave them the problems designed to create the greatest learning. I also started the discussions at the beginning, but at the end the students led and developed their own models for thinking about fractions.

[13] James was not the only student to do this. Many of the other students in the classroom would offer similar conjectures and thoughts to the class.

Table 11 *Types of Questions and the Resulting Student Focus*[14]

Types of Questions	Amount of Times Asked	Talk Move	Big Idea	Both	Didn't initiate any discussion
T- Building On	64		64		
T- Introduce new strategy that has not been developed	14		14		
T- Direct Teaching	27				27
T- Go Beyond	85		85		
T- Compare	4		4		
T- Initiation- Response-Evaluation	8				8
T- Interrogation	87	23	64		
T- Question Unclear	3				3
T- Scaffolding	34		34		
T- Shares Strategy	8	8			
Total of Questions:	334	31	265		38

Some key observations can be made about the types of questions and talk moves that happened in the classroom. The first is that I had provided a lot of wait time (42 instances)[15], giving students time to think. I noticed that by doing so, I had more engaged students who were willing to participate. I also noticed at the beginning of the units, I would often re-voice students' communication or re-tell what they said (54 times)[16] over the course of the unit. These actions served two purposes: 1) it allowed students time to process what was being said, and 2) it allowed students to defend or clear up any miscommunication.

[14] A question can be linked to both a talk move and a big idea. It becomes a talk move when the question is asking surface information. Surface information is when you are trying to ask basic ideas (e.g. what did you do? Why? you did this? For this reason, I have included in this chart a column called talk move.
[15] This is combined with the think, pair, share talk move because I am not talking, but letting the students think first.
[16] I combined revoicing with echoing students' words to get the final result.

The final observation is that, although I used some questions that were unclear, 55/642, the majority of my questions had specific purposes and were linked to a big idea. Having a plan and a progression of learning for my students helped me to construct and use good questions. It is due to this progression of learning that I was able to ask the types of questions that pushed my students to build upon and move beyond their constructed schemas of fractions, and move towards developing an understanding and a conceptual knowledge of part-whole relations and building a benchmark model.

Chapter 5: Conclusion and Thoughts

5.1 Summary of Major Findings

The purpose of the case study was to examine, from my perspective as a teacher, how the instructional practice of questioning might impact my students' understanding of fractions, specifically their construction of part-whole relationships and their use of a benchmark model. Part-whole relations is one of the fundamental challenges that students face in understanding fractions, and it becomes the corner-stone to their development in comparing and ordering fractions, and later skills. In addition, the use of a benchmark model can also be helpful for students when comparing and ordering fractions, helping students to visualize a rational number and the magnitude of that number.

The results of this study seem to suggest that teaching through problem solving and using specific, key questions at critical times appropriate to students' development, helped my students understand these concepts in a very short time frame. In no way am I suggesting that my questions and talk moves were the only factors that had an impact on my students' learning. There were many variables at play within the dynamics of the unit, the students, and the case study. However, three main findings emerged: 1) students improved and were engaged and focused, 2) my questions and talk moves were linked to a developmental understanding of students' learning of fractions or a trajectory of learning, and 3) my practice was linked to the NCTM standards and the five practices of 2nd generation reform.

5.1.1 Student growth.

It is clearly evident in the unit problem and the pre-test (Table 2) and post-test (Table 9) that students developed an understanding of the part-whole relations in fractions and constructed a benchmark model understanding of fractions. Their growth in understanding ranged from one student, David, consolidating one concept, to the majority of the students understanding the concepts asked of them. Although David still continued to have difficulties, he did show some sign of growth when you consider that he could not even represent a fraction before the unit, and by the end, he could identify the fractions in his fraction kit. In a short three week period he moved from below Stage 1, (sharing, or understanding that a fraction is close to sharing), as described by Pothier and Sawada (1983), and Lamon (1996), to somewhere in between Stage 2, (students are equally fair sharing), and Stage 3, (using a halving strategy for all fractions). This finding was also true for the rest of the students in the study. Even though there was a consistent problem with question three of the post-test, the students showed growth in their development of fraction concepts, especially around understanding part-whole relations and making a benchmark model. In addition, student problem solving times decreased considerably. For most students, the first problem in the unit took 80-90 minutes to complete, and even then I had to modify the task for my lower groups. By the end of the unit the same groups took twenty minutes to solve the problems, although not always correctly. In addition, their confidence in problem solving rose. As evidence, two students who cried during the pre-test, were able, like all of the students, to attempt the post–test and write a strategy.

As previously mentioned, there are some other factors to consider in the development of my students' understanding. One factor is that the case study students belong to a community of other students, some in Gr. 5, and therefore, a year older. It is also important to note that I teach all subjects through a problem solving or inquiry based approach, which means that my students are accustomed to focussed discussions with one another. It is an expectation that we communicate our thinking. Many of my students are willing to take risks because they feel comfortable and understand that everyone's contribution is valued. I make it clear that we, me included, are all members of a learning community working together in disciplined inquiry and focussed collaboration.

Another factor that may have impacted this study is that some of my students have been with me for two years and, although I did not include any of them in this particular case study, they were still part of whole class discussions and may have influenced the case study students' learning. The last factor to consider in their learning journey is parent involvement. Some of my students have tutors or parents who are heavily involved in their education. They ask questions, come in for regular interviews and help their children at home. These are all factors that could have influenced and helped with the students' growth in understanding and development in the classroom.

5.1.2 Questions focused on a landscape of learning.

Fosnot and Dolk (2002) note that the framework encompassing teacher belief systems about teaching and learning, and teacher knowledge of mathematical understanding in children, can influence the ways in which teachers interact with their students. This framework may determine the types of questions they ask, the ideas that they try to present, and even what activities they design or select (p. 2). In this case study, and in all of my teaching, I tried to focus my attention on understanding and knowing the way in which students developed an understanding of fractions, and I used this as a lens to guide my instruction. Pothier and Sawada

(1983), and Lamon's (1996) work with stages of development, Fosnot and Dolk's Landscape of Learning and Empson's (2002) fraction progression were all key components in framing the problems that I selected as well as the big ideas that I presented in mini-lessons and focused on in congresses. It was from these trajectories that the majority of my critical questions were developed. My awareness of these trajectories enabled me to pose the questions or prompts at critical points in the development of my students. Without understanding students' development, or without thinking about the misconceptions and pitfalls that students may face, it would be improbable to think that I could ask critical questions at the right time in my students' learning. Pre-planning and reflection helped me construct a bridge between theory and practice. Also, as a teacher, I felt I could help my students draw mathematical conclusions about the strategies they and their fellow classmates made. Through the questions and talk moves I initiated, my students were able to construct their own understanding of some to many of the critical big ideas in fractions at the Junior level.

5.1.3 Connection to the NCTM standards and the five practices of 2nd generation reform teaching.

The National Council of Mathematics (NCTM) recommends five standards for teaching mathematics that would promote deep learning: 1) worthwhile mathematical tasks, 2) discourse between teacher and students, 3) discourse between student and student, 4) teacher acceptance of the use of different methods and manipulatives to solve problems, 5) teacher engagement in ongoing assessment and analysis of teaching and learning. In addition, Stein, Engle, Smith, & Hughes (2008) promote the idea that teachers need to move from a first generation of teaching reform towards a second generation using teaching practice that is more focused on highlighting and talking about the mathematics rather than sharing wonderful and inventive strategies.

I found all of the teaching that happened in this case study supports these suggestions. Through worthwhile tasks my students experienced effective discourse about mathematics. They participated in this discourse with me, the teacher, within the community congress, and with their learning partners during work time. By modeling and promoting questions and talk in the classroom, I helped to facilitate the learning so that they felt comfortable to talk and discover fractions. Could the students have learned fractions from a textbook and from a teacher using traditional rote and an IRE model of questions? The answer is: without a doubt, yes; there are studies that promote this style of teaching (Mighton, 2003). However, by following the practices suggested by the NCTM and from researchers such as Stein et al. (2008), all of my students developed not only accuracy but also some understanding of fractions as they worked, rather than some time in the future as Mighton (2003) contends they will. They were engaged in the learning and felt like contributors in the classroom. None of my students gave up on a problem; they felt confident that they had the skills necessary to tackle it and provide a solution. I felt that my students were empowered, or felt no fear when it came to fractions, and this discovery has motivated me to continue honing my observation and questioning skills so that I can become a better facilitator of student learning.

5.2 Conclusions

As this study showcases, developing students' understanding of fractions takes time and careful planning. Today, fractions remain one of the toughest subjects for students in elementary mathematics. This reality can be attributed to the various challenges and misconceptions that students may encounter in their development of the subject matter. However, by carefully planning problems and questions around a trajectory of learning, allowing students to talk and discover these concepts, and providing time to share and facilitate the learning, students will learn and, at their own pace, move forward in the journey towards an understanding of fractions, specifically the part-whole relationship and the use of a benchmark model.

However, questions such as the ones used in this case study do not happen on the spot. They take time to plan. First, I had to know the mathematics that my students needed in order to develop these skills. I had to know the learning trajectory of my students, and finally, I had to reflect on and anticipate the learning that might happen in the upcoming day and throughout the rest of the unit. Pre-planning the questions and congresses promoted frequent exposure to the terminology and concepts in the fraction unit. As a teacher, I was constantly reflecting on my practice: are my students understanding what is being asked of them, are they going where they need to go? As a result of this reflection, we participated in an everchanging curriculum plan that ebbed and flowed with the students' own development.

5.3 Considerations for Future Research

There is still much research to be done in the area of teachers' questions and the impact that they have on student development. This particular study was a small snapshot taken in a Grade 4/5 classroom. It was taught by a fairly experienced reform teacher. An extension of this study would be to see if it can be repeated using another group of students and a reform teacher, or to see if other non-experienced reform teachers can repeat the process and have the same success asking those types of questions. In addition, further research could be conducted to see how my own students perform with fractions the following year: did they retain the information that was taught, or were they only able to retain the information for this particular unit that was taught with me?

It would be interesting to examine teachers' question repertoires and tabulate the types of questions and their impact on student learning in other areas of mathematics. Does experience play a role in the types of questions asked: do first year teachers use different questions than second, third or ten year teachers? Does professional development make a difference in the types of questions asked? What training or types of professional development need to happen for questions to impact student learning?

Finally, it would be interesting to look at the other factors that existed in this study and if they contributed to the development more so than the questioning. Does being in a reform classroom all year impact students? How do student talk and discourse contribute to student learning? How does working with homogeneous partners for the whole year affect the development of student learning? Over time, what is this learning like? Does it improve or hinder development, especially when students transition to a classroom where reform practices are not used?

5.4 Final Thoughts

Although this was a very small study, limited in size and site, and designed by an inexperienced investigator, the results tend to support some of the findings and conclusions in the relevant literature that was reviewed in Chapter 2. That is that students do struggle with understanding part-whole relations and that they initially see the numerator and denominator as separate numbers. They progress through stages outlined by Pothier and Sawada's (1983) and finally, they follow a progression of learning very similar to that of Fosnot and Dolk (2002) if the teacher employs questions related to the landscape of learning. In addition, the findings suggest that specific types of questions do have a positive impact on student understanding of part-whole relations in fractions and the use of a benchmark model. In so doing, I have moved my students beyond their personal schemas of fractions, pushing them to understand that fractions are not just the shaded parts in pictures, nor that they should rely solely on taught algorithms, but to see that they are a relationship between two numbers. As a result, my students have started to conceptualize the big ideas in fractions and move forward in their learning development.

I hope that my students will utilize my provocative questions, such as: What are you thinking? Why is this happening? Describe the mathematics that you are doing. How does this compare to the person next to you? I hope that they will internalize this style of questioning for themselves, becoming more meta-cognitive about their own learning, eventually applying these questions to other areas of the curriculum.

As I reflect on this experience I realize how much reform teaching and, more importantly, asking critical questions have become a very significant tool in my teaching practice. At first this was an instructional practice that I started to employ at the beginning of my teaching career, seven years ago; but, the more that I see the outcomes and the young mathematicians that are produced, the more this practice has become a habit of the mind and a philosophy that I live by every day.

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Appendix A: Parent Letter Centre

[put on school letterhead]

March, 1st, 2012

Use one double space between date and salutation

Dear Parent/Guardian,

I am working on my Master of Education degree at Lakehead University. The goal for my thesis is to investigate an area in mathematics in which students have difficulty learning, and to find ways to improve the teaching of this topic. The focus of my research is on learning fractions and the impact that teacher's questions have on students' understanding of fractions concepts.

I will be observing mathematics lessons in the classroom during the unit on fractions. The unit will be taught for 4 weeks during April 2012. The students will take a pre-test before the unit begins and a post-test when the unit is completed. Some samples of students' work will be collected. I will be videotaping the lessons. Also, with permission, some groups of students will be videotaped so that after the lesson I will be able to listen carefully to how they have solved the problems. Their conversations may be transcribed and quoted in my final project in order to illustrate their understanding of fractions. I, or my supervisor Dr. Lawson, may also make use of some of the edited classroom footage and work samples for professional development for teachers and academics at conferences. I may also make use of this data for possible journal articles and further papers. Upon completion of the project, you will be welcome to obtain a summary of the research by contacting me at the school or by providing your mailing address on the consent form.

Your child will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If edited video data is used for professional development, your child will be identified by first name. The raw data that is collected will be securely stored at Lakehead University for five years and then destroyed.

Participation in this study is voluntary and you may withdraw the use of your child's data at any time, for any reason, without penalty. The research project has been approved by the Lakehead University Research Ethics Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 343-8283 or swright@lakeheadu.ca. The research has been approved by the Peel District School Board and the Principal of [name of school].

Please note that this research does not affect classroom instruction time. The lessons are being carried out in the same manner and length of time as they would be without the research project. This research will not take away from the normal learning environment in the classroom, and there is no apparent risk to your child. The research is simply being conducted to make note of the impact of my questions on students' development of fractions, which is a regular part of the fractions unit. If you choose not to have your child participate, he or she will still be engaged in the math lessons. The only difference is that his or her data will not be used. Even if you give permission for your child to participate, your child will also be asked whether he or she is willing to take part in this research.

You are welcome to contact me at 905-452-8296 ext: 505 or see me in person before or after school if you have any questions concerning this research project. I would be very pleased to speak with you.

Please complete the consent form below and return it to the classroom by March, 30th, 2012.

Sincerely,

Jonathan So
Grade 4/5 teacher
Master's Candidate

[insert Principal's name]
Name of school
(905)-

Dr. A. Lawson, Ph.D.
Master's Candidate Thesis Supervisor
Lakehead University
alawson@lakeheadu.ca

Sue Wright
School Research Ethics Board
Lakehead University
807-343-8283
swright@lakeheadu.ca

Appendix B: Parent Consent Form

(to be printed on letterhead)

I DO give permission for my son/daughter,

____,
(Student's Name/please print)

to participate in the study with Jonathan So as described in the attached letter.

I understand that:
1. My child will be videotaped in the classroom environment as part of the research.
2. My child's participation is entirely voluntary, and I can withdraw permission at any time, for any reason, with no penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then destroyed.
5. All participants will be identified by first name only in any publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by Jonathan So, [teacher], or Dr. Lawson. If my child appears in the video clips he/she will be identified by first name only.

☐ I initial this box to give permission for my child to appear in video clips which may be used for Professional Development purposes, as outlined in #6.

7. I can receive a summary of the project, upon request, following its completion, by calling or writing, or by providing my address below.

Please keep the introductory letter on file should you have any further questions. Bring sentence below up to here.If you agree to let your child take part in the study, please complete this page and have your child return it to [teacher].

Name of Parent/Guardian (please print): ___________________________________

Signature of Parent/Guardian: ____

Address (if you would like a summary of the findings):

Appendix C: Potential Participant Letter

(to be printed on letterhead)

March 2012

Dear Potential Participant,

In April, I will be videotaping in the classroom in order to do some research on how my questions impact your understanding of fractions. I will be paying attention and writing things down during your math classes because I am curious about what helps students to learn fractions best.

I will be teaching the lessons as usual and classes will be exactly the same as before. A difference you will notice is that during some lessons there will be a video camera in the classroom and a microphone on your work table. These tools will help me with my project by recording what you say and do while you are solving problems.

My supervisor Dr. Lawson may also want to use some video clips from the classroom and samples of my work for helping other teachers learn more about how to teach about fractions and in conferences. If you are in a video that will be seen by other teachers, I will use only your first name. I will use that name for any written part of my research and when showing the videos to other teachers.

The unit will start with a pre-test so that I can see what you know about fractions before any of the lessons. I will then teach the lessons and your work will be collected as usual. At the end of the unit I will have another test to see what you have learned. Please ask me any questions you have about my project, and I will be happy to answer them. You can decide whether or not to be part of my project. You will be doing the same work in math class whether you are in my project or not, the only difference is that I will not use your test results or your work or any video clips with you in them if you decide not to take part. Thank you for thinking about being part of my project.

Sincerely,

Mr. So

Appendix D: Potential Participant Consent Form

(to be printed on letterhead)

Potential Participant Consent Form
I, ____, want to take part in the project with
(Student's Name/please print)

Mr. So as described in the letter.

I understand that:

1. I will be videotaped in the classroom as part of the project.
2. I don't have to take part in the project, but I want to be part of it I know I can change my mind about that later, and it won't be a problem.
3. It is safe to be part of this project.
4. All of the information Mr. So collects for his project will be kept in a very safe place at Lakehead University for five years, and then it will be destroyed.
5. My real name will never be used in anything Mr. So writes about the project.
#6 ?
7. Mr. So or Dr. Lawson might want to use some of the videos or copies of my work to help other teachers learn about teaching fractions. My first name might be used in video clips of the classroom. My name will not be on any written copies of my work.

☐ I put my initials in this box to show that it is alright for me to appear in video clips which may be used for helping other teachers learn about teaching fractions. Double space between sentence above and this one. Align the If to the left. If you want to be part of my project, please fill in this page and give it to [teacher].

____

Name of Student (please print): ____________________

Signature of Student: _________________________

Appendix E: Principal Letter

(to be printed on letterhead)

March, 2012
Dear [Principal's Name],

I am working on my Master of Education degree at Lakehead University. The goal for my thesis is to investigate an area in mathematics in which students have difficulty learning and to find ways to improve how this topic is taught. The focus of my research is on learning fractions and the impact teachers' questions have on students' understanding of fractions concepts.

I will be observing mathematics lessons in the classroom during the unit on fractions. The unit will be taught for 4 weeks during April. The students will take a pre-test before the unit is taught and a post-test when the unit is completed. Some samples of students' work will be collected. During the lessons, I will be videotaping the teaching process. Also, with permission, some groups of students will be videotaped so that I will be able to listen carefully to how they have solved the problems. Their conversations may be transcribed and quoted anonymously in my final project in order to illustrate their understanding of fractions. I, or my supervisor Dr. Lawson, may also make use of some of the edited classroom footage and work samples for professional development for teachers and academics at conferences. Upon completion of the project, you will be welcome to obtain a summary of the research by contacting me at the school or by giving your mailing address on the consent form.

The students will not be identified in any written publication, including my master's thesis, possible journal articles or conference presentations. If edited video data is used for professional development, the child will be identified only by first name. The raw data that is collected will be securely stored at Lakehead University for five years and then destroyed.

The research project has been approved by the Lakehead University Research Ethics Board. If you have any questions related to the ethics of the research and would like to speak to someone outside of the research team, please contact Sue Wright at the Research Ethics Board at 343-8283 or swright@lakeheadu.ca. The research has been approved by the Peel District School Board.

Please note that this research does not affect classroom instruction time. The lessons are being carried out in the same manner and length of time as they would be without the research project. This research will not take away from the normal learning environment in the classroom, and there is no apparent risk to the students. The research is simply being conducted to make note of the impact of my questions on students' development of fractions, which is a regular part of the fractions unit. Students who are not participating will still be engaged in the math lessons. The only difference is that his or her data will not be used. Even if parents give permission for a child to participate, the child will also be asked whether he or she is willing to take part in this research.

The School Board, [name of] School, [teacher], and his students will not be identified in any written publication, including my master‘s thesis, possible journal articles or conference presentations. If video data is used for professional development, the students will be identified by pseudonyms; however, if students use the teacher‘s surname it may be revealed.

The raw data that is collected will be securely stored at Lakehead University for five years after completion of the project. A report of the research will be available upon request. I can be reached at 416-564-0231 or you can e-mail me at Jonathan.So@peelsb.com.

If you give permission for participation in the study, please sign the attached letter of consent and return it to me.

Sincerely,
Jonathan So
Master’s Candidate
Lakehead University

Dr. A. Lawson, Ph.D.
Master‘s Candidate Thesis Supervisor
Lakehead University
807-343-8720
alawson@lakeheadu.ca

Sue Wright
Research Ethics Board
Lakehead University
807-343-8283
swright@lakeheadu.ca 46

Appendix F: Principal Consent Form

(to be printed on letterhead)

Principal Consent Form double space between this line and next
I ____, do agree to participate in the study
(Principal's Name/please print)

with Jonathan So as described in the attached letter.

I understand that:
1. [teacher] and his students will be videotaped in the classroom as part of the research.
2. Their participation is entirely voluntary, and I can withdraw permission at any time, for any reason, without penalty.
3. There is no apparent danger of physical or psychological harm.
4. In accordance with Lakehead University policy, the raw data will remain confidential and securely stored at Lakehead University for five years and then will be destroyed.
5. The Peel District School Board, [name of] School, [teacher], and his students will remain anonymous in any written publication resulting from the research project.
6. The video clips of the classroom or student work may be included in Professional Development for teachers conducted by Jonathan So or Dr. Lawson. If students appear in the video clips, they will only be identified by first name. If [teacher] appears in the video clips, he may be identified by surname.

☐ I initial this box to give permission for [teacher] and his students to appear in video clips which may be used for Professional Development purposes and academics at conferences as outlined above.

If you approve of participating in my study, please complete this page and return it to me.
____

Name of Principal (please print): _________________________
Close extra spacing between to double space

Signature of Principal: _________________________

Date:

Appendix G: Detailed Unit Plan

Grade Five Fractions: Adapted from Cathy Fosnot *Field Trips and Fund-Raisers*, and Marilyn Burn's *Introducing Fractions* use italics for both books

Adapted by: Jonathan So

Day 1: Building your Fraction Kit and playing fraction games, uncover and cover up

Materials: Strips of paper 48cm long. One colour for each fraction. (Whole, 1/2, 1/4, 1/8, 1/16, 1/3, 1/6, 1/12, 1/9)

Problem: Helping Mom

So, I went over to my mom's house with 1 of my friends. She knew that we were coming, so she made us this huge submarine sandwich with everything on it. Well, I had a dilemma because I didn't know what to do. I mean, how do I split this submarine sandwich so that my friend and I get an equal share? What do you think I should do?

Part 2:

Well, you remember yesterday how I told you about going to my mom's house? Well I didn't tell you the whole truth. I had just figured out how much my friend and I get to eat when the doorbell rang. Well, without my knowledge, my mom invited some of my other friends to help out. At the door were seven more of my friends! Now I was really stuck. I was so worried; would I have enough sub to share equally? How much of the sub would each person get?

During:

This problem is easily completed as a whole class lesson, instead of working in partners. The congress and the during section can happen at the same time

Big Ideas	Anticipated Problems and Questions
Students have to build different fraction strips; though they will eventually be using them for equivalence, they must understand that fractions are relationships between the whole and its parts. Students will also be building how big a whole is in relation to all of its parts (1/2, 1/4, 1/8, 1/16, 1/3, 1/5, 1/10, 1/9, 1/6, 1/12)	1) Students often know that a half is two things, but they struggle to make a fair share of it a) How do you know this is a half? b) What strategies can I use to make sure it is exact? 2) They also struggle with identifying the fraction as a relationship. They see it has 2 and 1, not 1 part of 2. c) What is this fraction called? d) How many parts do you notice? e) How is this related to the fraction name? Notice anything? (**This question might take time; so start with some and then, as they build more fraction strips, build on everyone's answers)** 3) Students also struggle to use this strategy to make other fractional parts. a) What relationship do you notice between 1/2 and

	1/4? Or other fractions. b) Which fraction is larger? Why? 4) Students struggle to understand that the greater the denominator, the smaller the fractional part. This confusion is because they often only look at each part as a separate number forgetting that the fraction is a relationship between the two. a) Isn't 4 larger than 2? Why is it then that 1/2 is larger than 1/4? 5) Finally, students struggle to understand that these strips are only parts of this particular whole. Every whole has different sizes and different parts. A half of a fridge is different than half of the school. a) What if I applied these strategies to finding half of the school? Would they work? b) How does this compare to the half of our strip? c) Aren't they both halves? Why is one bigger?

Models expected: Measurement, fraction bars, fair sharing
Strategies: Using landmark fractions

Day 2: Exploring Fractions with Pattern Blocks

a) Which number is bigger 2/8 or 6/16?

b) Which number is the smallest 9/8 or 14/16?

c) Use your strips to find fractions that are equal to: i) 3/6 ii) 30/40

d) Write three ways to make 1 using different fractions from your fraction kit?

Marilyn Burn's lesson that follows up with the strips and a new game

Day 3: Field Trip Problem (Fosnot)

A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

a) The first group had 4 people and shared 3 subs equally.

B) The second group had 5 people and shared 4 subs equally.

c) The third group had 8 people and shared 7 subs equally.

d) The last group had 5 people and shared 3 subs equally.

1. Was the distribution fair? Did each person in each group get the same amount?

2. How much of a sub did each person get, assuming the pieces were cut equally?

Questions and Look-Fors:

Big Ideas	Anticipated Problems and Questions
This particular problem deals with having the students construct fractions and then compare them. Students have to understand what the whole is and what parts have been used. Then they have to compare these parts to each other. The problem lies in that they are not all using a reference of a half (some are 1/5s), and they are not all equal shares though they are really close. Some fractions are both one part away from the whole, but with different denominators.	1) Students a struggle with identifying the fraction as a relationship. They see it as 3 and 1, not 1 part of 2. a) How do you know this picture represents the fraction? b) What is the whole? c) What is the part? d) Can you use your strategy for all of the fractions? 2) Students struggle with odd fractions. a) What does the whole represent? b) What does each part mean? c) What does this mean altogether? 3) Students struggle to understand that the greater the denominator, the smaller the fractional part. This is because they often only look at each part as a separate number, forgetting that it is a relationship between the two. a) Why are you suggesting that these two are equal? b) What conclusion did we make from yesterday? How do these apply to what we are learning now? **Congress:** The focus will be on a debate between those students that think it is fair and those that don't. Students will be allowed to move back and forth depending on what is said. Students will have to think about why certain fractions are larger than others, and how a part-whole relationship helps us understand this concept. Questions will be very similar to the ones in the *during* section of the lesson.

Day 4: Day Two of Field Trip Congress (Fosnot)

Have students make posters of their solutions, and then, in a group, share three or four that have interesting points to discuss.

See above for congress set-up after the gallery walk and the questions to ask.

Day 5: Using Their Fraction Kit:

Compare 3/4 to 2/3 which is bigger? Explain.

Delete extra space between line above and below.

Day 6: Day 3 of Field Trips (Fosnot): Redistributing the Subs

Start the lesson with the mini string lesson: unbold this sentence

10 x 127	3 x 1/5
127 x 2	7 x 1/8
127 x 12	3 x 1/4
44 x 10	4 x 1/5
44 x 9	

Before thinking that students cannot do multiplication with fractions, think what the questions are asking, that three- 1/5s are really 3/5. Students should understand this concept. They should see the relationship between repeated addition and multiplication. They also can use their fraction kits to help. Record the possible strategies that students give for the answers.

Problem:

1. Three subs for 4 people is 1/2 +1/4, or 3/4, so 4 people each got 3/4 of a sub.

2. Four subs for 5 people is 1/2 +1/5+ 1/10 or 4/5, so five people got 4/5 of a sub.

3. Seven subs for 8 people is 1/2 + 1/4 + 1/8 or 7/8, so 8 people each got 7/8 of a sub.

4. Three subs for 5 people is 1/2 + 1/ 10 or 3/5, so five people each got 3/5 of a sub.

Ask the students to investigate if it would have been fairer if groups #1 and #3 combined and shared, and groups #2 and #4 combined and shared.

Day 7: Congress for the Previous Problem

Look at Fosnot's notes on how to setup the congress

Day 8: Comparing and Ordering Fractions

Using your fraction strips compare these numbers. Are the >, <, = asking how much more is needed?

3/8	9/16	3/16	1/2
1/2	3/8	1/2	2/4
3/4	5/8		

What do we need to compare fractions?

If students can do this activity, they will be able to discuss the relationship between the fractions. Often they will compare the wholes, saying "4/5 is bigger because it is one space away from a whole, and 1/3 is less because it is two spaces." They might also state as the denominator gets larger the piece gets smaller. Finally, they may resort to common denominators which happens when a) that is how they were taught and they go back to an algorithm without understanding, or the fractions are really close and pictures cannot help them.

Context: There was a local marathon in Mississauga, and I found these stats about the people who ran the race. It was a very difficult race; so some didn't finish, and I think the sun was getting to some so they ran more than they should have. Can you put them in order from who ran the least distance to who ran the farthest?

Set 1 (put names to each fraction): 3/16, 5/8, 3/4, 1/4, 2/4, 1/2, 9/8, 1/1, 17/16, 15/16, 3/2

Big Ideas	Anticipated Problems and Questions
Students have to understand what the whole is and what parts have been used. Then they have to compare these parts to each other. Students also have to start to think about landmark fractions that they have been building over the past couple of days. What relationships are all of the fractions in comparison to 1/2, whole and 0?	1) Students often only look at either the denominator or the numerator; they forget that fractions represent relationships of the whole. Students must first figure out what the whole is, and how each part forms a representation of that whole. a) What does the numerator represent? b) What does the denominator represent? c) What does this fraction mean? d) How close is this to 1/2 or a whole? e) Why is this fraction larger than this one? f) How does the fraction kit help you? g) What other strategies do you know of that are related to the strips? (number line) h) How does this help? i) What does it mean if the numerator is larger than the denominator? Congress: The focus of the congress will be on those students that have used a benchmark model. I want to highlight this and its use, but I also want to compare it to the students who have only used the fraction kits.

Models:

- use of benchmarks
- constant whole
- comparison of known fractions i.e.: 3/4 is greater the 1/2
- Order them on a number line using the benchmarks of 0, 1/2, 1

Big Ideas:

- The greater the denominator, the smaller the piece is
- To compare fractions the whole must be the same

Day 9: Day Five from Field Trips Working with Landmarks

Problem:

1. Three subs for 4 people is 1/2 +1/4 or 3/4, so 4 people each got 3/4 of a sub.

2. Four subs for 5 people is 1/2 +1/5+ 1/10 or 4/5, so 5 people got 4/5 of a sub.

3. Seven subs for 8 people is 1/2 + 1/4 + 1/8 or 7/8, so 8 people each got 7/8 of a sub.

4. Three subs for 5 people is 1/2 + 1/ 10 or 3/5, so five people each got 3/5 of a sub.

This distribution wasn't fair, and although it was a little fairer when two groups shared, it still wasn't fair. Now as, "If the 17 subs had been shared by 22 children fairly, about how much of the sub would each child have received?"

Day 10: Congress:

The purpose of this congress is to discuss strategies, concepts and big ideas that we have used over the last couple of days. Record all observations on chart paper. In their journals, have them record ten things that they have learned about fractions and create a representation to demonstrate their knowledge.

Day 11: How Much is Blue?

Page 97 in Marilyn Burn's *Introduction to fractions 4-5*, the question is to figure out from the shape what fraction is blue?

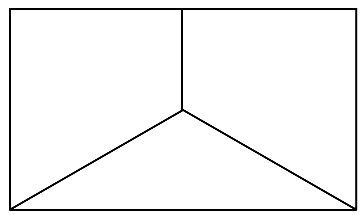
Big Ideas	Anticipated Problems and Questions
Students have to understand what the whole is. They often think that each piece is a part of the whole, but they soon realize that it is to do with the individual shapes. This is a great question to evaluate what students have really learned over the past three weeks. If they really understand that fractions are a relationship this question will not be an issue.	What is the whole? How do you know? What is the part? How did you figure this out? **Congress:** The focus of the congress will be on how students decided on what the whole was? How does having a different whole change the fraction? How do each of the fractions change the representation of what is blue?

Day 12: Set of a Shape

This question is very similar to the previous question. It is asking them what fraction of the whole does each piece represent. The shape is a rectangle with one triangle in the middle and two trapeziums on the sides. Students soon realize that they cannot use the shapes as the parts, but divide the whole into equal parts (triangles). The problems come when students think that each part is a third because there are three parts. Ask the same questions as above and have a debate around why students chose those particular wholes.

Shape:

What fraction is each piece?



Big Ideas	Anticipated Problems and Questions
Same as above just the shape has changed.	Students often think that because there are three parts showing then each part is a third, or the two trapeziums are slightly bigger than the triangle so it is a fourth, which is a problem all on its own. a) What do you notice about the shapes? b) Are they all equal? c) Can we create a fraction that isn't equal? d) What can we use as a fractional part? e) What is our whole? I am also going to use a lot of the questions from above.

Day 13: Day Seven of Field Trip: Developing Equivalence

Mini-lesson**:**

1/2 of 24 | 1/3 of 24

1/4 of 24 | 1/4 of 12

1/8 of 24 | 1/2 of 6

Problem: Tell students that you want to ensure that you won't ever make the same mistake as the teacher in the field trip story. So, you thought it might be a good idea to make a chart to keep track of the number of subs needed for future field trips. Tell them you think it is a good idea for everyone to get about 3/4 of sub. Ask them if they know how many subs that would be for four people. Pass out the recording sheet.

Congress for the last ten minutes of the period.

Day 14: Day Eight of Fundraiser:

Mini-lesson:

100/4	400/16
200/4	300/12
100/8	600/24

Wow, I think you came up with exactly what I was thinking because I decided to call up the organization and see if I could plan one. They told me that would be great, but what they need is a plan. So I thought that you could help me with the design of the course part of the plan.

This is what I have so far:

The total race is 60K and will happen over two days (hopefully a Saturday and Sunday). Since it is two days I will need to set up a rest station at the halfway point. I will also need:

- Resting points at every eighth of the course
- Food stations at every fourth of the course
- Water stations at every tenth of the course
- Media stations at every fifth of the course
- I need kilometre markers placed along the way so people can calculate how far they went. Remember people are pledging per kilometre. These markers need to be placed at every twelfth, sixth and third of the course, as well as at the above points. These markers should also tell how many kilometres they have gone and how many more they have to go.
- I also need a finish line

Fraction big ideas:

- With unit fractions the greater the denominator, the smaller the piece.
- Fractions express relationships; the size of the whole matters.
- Multiplication is connected to fractions (3/4= 3 x 1/4).
- To add or subtract fractions a common whole is needed.

Congress:

What locations have multiple landmarks? Why?

What do you notice about the relationships of the markers?

What strategies did you use to determine the locations?

Day 15: Bar Capture Game

Look in Fosnot day ten for rules and game boards.

Day 16: If the World Were a Village

Read the story *If the World were a Village*. Have students think about the whole (100). At each page have a discussion about the fractional amounts that they see. Discuss why they think they see those amounts. Stop at any page and have them work in partners to figure out the fractional amounts. Then work on the electricity problem

Day 17: Final Assessment

Appendix H: Rationale for the Five Focus Lessons

Lesson and data source	Focus question it will answer	My rationale of why this is a part-whole or bench mark model.	Possible problems and questions to overcome them
Lesson #1: Building the fraction kit Data source: Video of the partners working Video of the congress Student work Students' journals My field journal	 #1, b a	Students have to build different fraction strips; though they will eventually be using them for equivalence they must understand that fractions are relationships between the whole and its parts. Students will also be building how big a whole is in relation to all of its parts (1/2, 1/4, 1/8, 1/16, 1/3, 1/5, 1/10, 1/9, 1/6, 1/12)	1) Students often know that a half is two things but they struggle to make a fair share of it a) How do you know this is a half? f) What strategies can I use to make sure it is exact? 2) They also struggle with identifying the fraction as a relationship. They see it has 2 and 1, not 1 part of 2. 3) What is this fraction called? 4) How many parts do you notice? 5) How is this related to the fraction name? Notice anything? (**This question might take time, so start with some, and then as they build more fraction strips, build on everyone's answers)** 6) Students also struggle to use this strategy to make other fractional parts. c) What relationship do you notice between 1/2 and 1/4? Or other fractions. d) Which fraction is larger? Why? 7) Students struggle to understand that the greater the denominator, the smaller the fractional part. This is because they often only look at each part as a separate number forgetting that it is a relationship between the two b) Isn't 4 larger than 2? Why is it then that 1/2 is larger than 1/4?

			8) Finally, students struggle with understanding that these strips are only parts of this particular whole. Every whole has different sizes and different parts. A half of a fridge is different than half of the school. d) What if I applied these strategies to finding half of the school, would they work? e) How does this compare to the half of our strip? f) Aren't they both halves? Why is one bigger?
Lesson 3: Field trip problem Data source: Video of the partners working Video of the congress Student work Students' journals My field journal	#1, b a	This particular problem deals with having the students construct fractions and then comparing them. Students have to understand what the whole is and what parts have been used. Then they have to compare these parts to each other. The problem lies in that they are not all using a reference of a half (some are 1/5s), and they are not all equal shares though they are really close. Some fractions are both one part away from the whole, but with different denominators.	4) They also struggle with identifying the fraction as a relationship. They see it has 2 and 1, not 1 part of 2. e) How do you know this picture represents the fraction? f) What is the whole? g) What is the part? h) Can you use your strategy for all of the fractions? 5) Students struggle with odd fractions. d) What does the whole represent? e) What does each part mean? f) What does this mean altogether? 6) Students struggle to understand that the greater the denominator, the smaller the fractional part. This is because they often only look at each part as a separate number forgetting that it is a relationship between the two. c) Why are you suggesting that these two are equal? d) What conclusion did we make from yesterday? How do these apply to what we are learning now? **Congress:** The focus will be on a debate between those students that think it is fair and those that don't. Students will be allowed to move back and forth depending on what is said. Students will have to think about why certain fractions are larger than others, and how a part-whole relationship helps us understand

			this concept. Questions will be very similar to the ones in the *during* section of the lesson.
Day 8: Comparing and ordering fractions (Mississauga Marathon training) Data source: Video of the partners working Video of the congress Student work Students' journals My field journal	#1, b a	Students have to understand what the whole is and what parts have been used. Then they have to compare these parts to each other. Students also have to start to think about landmark fractions that they have been building over the past couple of days. What relationships are all of the fractions in comparison to 1/2, whole and 0?	9) Students often only look at either the denominator or the numerator; they forget that fractions represent relationships of the whole. Students must first figure out what the whole is, and how each of the parts change to fit the different representations of the whole. j) What does the numerator represent? k) What does the denominator represent? l) What does this fraction mean? m) How close is this to 1/2 or a whole? n) Why is this fraction larger than this one? o) How does the fraction kit help you? p) What other strategies do you know of that are related to the strips? (number line) q) How does this help? r) What does it mean if the numerator is larger than the denominator? Congress: The focus of the congress will be on those students that have used a benchmark model. I want to highlight this and its use, but I also want to compare it to the students who have only used the fraction kits.
Day 11: How much is blue? Data source: Video of the partners working Video of the congress Student work Students'	#1, b	Students have to understand what the whole is. They often think that each piece is a part of the whole, but they soon realize that it is to do with the individual shapes. This is a great question to evaluate what students have really learned over the past three weeks. If they really understand	What is the whole? How do you know? What is the part? How did you figure this out? **Congress:** The focus of the congress will be on how students decided on what the whole was? How does having a different whole change the fraction? How do each of the fractions change the representation of what is blue?

journals My field journal	 a	that fractions are a relationship, this question will not be an issue.	
Day 12: Set of a shape unbold Data source: Video of the partners working Video of the congress Student work Students' journals My field journal	#1, b a	Same as above; just the shape has changed	Students often think that because there are three parts showing, then each part is a third, or when the two trapeziums are slightly bigger than the triangle, it is a fourth, which is a problem all on its own. f) What do you notice about the shapes? g) Are they all equal? h) Can we create a fraction that isn't equal? i) What can we use as a fractional part? j) What is our whole? I am also going to use a lot of the questions from above.

Appendix I: Pre-test

Pre-test:

1) Order these fractions: 3/4, 5/12, 2/3, 3/2, 2/5, 5/8, from greatest to least.

2) Jeremy and Fiona are eating pizza. Fiona has 1/2 of a pizza and Jeremy has 1/3 of a pizza. Is it possible that Fiona has more pizza than Jeremy? Explain your reasoning (TIMSS Gr. 8 item).

3) A fifth-grade class traveled on a field trip in four separate cars. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows: (Fosnot and Dolk)

- The first group had 4 people and shared 3 subs equally.
- The second group had 5 people and shared 4 subs equally.
- The third group had 8 people and shared 7 subs equally.
- The last group had 5 people and shared 3 subs equally.

When they returned from the field trip, the children began to argue that the distribution of sandwiches had not been fair, that some children got more to eat than the others. Were they right? Or did everyone get the same amount?

Appendix J: Post-test

Problem #1: Share two pizzas among three people. Explain your thinking (Burns, 1998).

Problem #2: Decide if each fraction is closest to 0, 1/2, or 1. Explain your answer.

$\frac{3}{4}$ $\frac{3}{9}$ $\frac{11}{16}$ $\frac{1}{4}$ $\frac{1}{12}$

Problem #3: Joey and Robert each had the same size pizza. Joey cut his pizza into 8 equal pieces and ate 6 of them. Robert cut his into 5 equal pieces and ate 4 of them. Who ate more pizza? (Burns, 1998)

Problem #4: Raquel thought about this statement: When pitching, Joe struck out 7 of 18 batters. She said that it was better to say that Joe struck out about 1/3 of the batters than to say that Joe struck out about 1/2 of the batters. "I think that 7/18 is closer to 1/3 than 1/2," she said. Do you agree or disagree with Raquel? Explain your reasoning (Burns, 1998).

Appendix K: Preliminary Codes

	Fosnot Landscape
F/S	Fair Sharing Model
P/W	Part-Whole Relationship
CM	Concrete Model
FrS	Fraction Strips Model
Mea	Measuring Model
W/S	Whole the same
I/C	Incomplete
D	Dealing out scheme
W/N	Whole Number Scheme
#	Sick during post-est
LMF	Landmark Fractions
U	Only went up the number line
U/D	Up and down a fixed number line
	Wrong answer or wrong model/strategy
	Wrong answer, but parts are almost there
	Right, but needs further explanation
	Correct Answer
E	Equivalence Scheme
F/R	Fraction expresses relationships
Comm	Further communication needed
CW	Comparison with a common whole
GDSP	The greater the denominator the smaller the piece
Fset	Fractions as a set
(L)	Low
(M)	Middle
(H)	High
C/P	Context Problem
Chalf	Comparison with a common half

Pothier and Sawada's Stages	
Stage 1	
Stage 2	
Stage 3	
Stage 4	
Stage 5	
Tech	Instructional reliance on procedures
Teaching Codes	
ReV	Revoicing
Gb	Go beyond
Mon	Monitoring students
IRE	*initiation-response-evaluation*
inter	Interrogation
interject ide	interject with another idea
airmiscon	Air misconceptions
introimporstrat	Introduce important strategies that have been developed
intornewstrat	Introduce a new strategy that hasn't been discovered yet;
sharstrat	just share the strategies
compare	Comparing question
build	Building type questions
WT (sec)	Wait time (amount in sec)
echo	Echoing students' responses

Appendix L: Five Practises suggested by Stein, et al.

1: Anticipation (P.322)

The first thing is for the teacher to look and see how students might mathematically solve these types of problems. In addition, teachers should also solve them for themselves. Anticipating students' work involves not only what students may do, but what they may not do. Teachers must be prepared for incorrect responses as well.

2: Monitoring students' work (P. 326)

While the students are working, it is the responsibility of the teacher to pay close attention to the mathematical thinking that is happening in the classroom. The goal of monitoring is to identify the mathematical potential of particular strategies and figure out what big ideas are happening in the classroom. As the teacher is monitoring the students work, they are also selecting who is to present based on the observations that are unfolding in the classroom.

3: Selecting student work (P.327-328)

Having monitored the students, it is now the role of the teacher to pick strategies that will benefit the class as a whole. This process is not any different than what most teachers do; however, the emphasis is not on the sharing, but on what the mathematics is that is happening in the strategies that were chosen.

4: Purposefully sequencing them in discussion (P. 329)

With the students chosen, it is now up to the teacher to pick the sequence in which the students will present. What big ideas are unfolding, and how can you sequence them for all to understand? This sequencing can happen in a couple of ways: 1) most common strategy, 2) stage 1 of a big idea towards a more complex version or 3) contrasting ideas and strategies.

5: Helping students make mathematical sense (P.330-331)

As the students share their strategies, it is the role of the teacher to question and help them draw connections between the mathematical processes and ideas that are reflected in those strategies. Stein et. al. suggest that teachers can help students make judgments about the consequences of different approaches. They can also help students see how the strategies are the same even if they are represented differently. Overall, it is the role of the teacher to bridge the gap between presentations so that students do not see them as separate strategies, but rather as working towards a common understanding or goal of the teacher.

Appendix M: Final Code List

*Correct
*Correct with support
*Further communication needed
*In Correct
BI- Fair Sharing (equal) Stage 2
BI- Fair sharing (not always equal) Stage 1
BI- Fractions may represent division less than one
BI- Fractions represent a relationship (part whole relationship)
BI- Greater the denominator the smaller the piece
BI- Multiplication is connected to fractions Stage 3
BI- pieces don't have to be congruent to be equal
BI- Size of whole matters
BI_-BIG IDEAS
M- Algorithm Model
M- Benchmark model
M- Circle Fractions
M- dealing out
M- Fair Sharing Model
M- Linear Fraction Model (FS)
M- measurement
M- No Model
M- number line to compare fractions
M__ -MODELS
S- apply halving strategy to odd numbers (stage 3)
S- Dealing out
S- doubles a denominator to halve a fraction (Stage 2 halving)
S- doubles numerator to multiply by two
S- Landmark Fraction (when I do not know which one)
S- Landmark fractions (compare to a whole)
S- Landmark Fractions (compare to half)
S- Reliance on procedural learning and understanding
S- unit Fraction
S- Use multiplication to equally divide odd numbers has to re adjust (stage 4)
S- Uses a common whole to compare fractions
S- Uses Multiplication efficiently (stage 5)
S- uses proportional reasoning
S- Using a ratio table as a tool to make equivalent fractions
S- Whole number scheme instead of part/whole relation
S__- STRATEGIES
T- Air Misconceptions
T- answering with another question
T- Building a context
T- Building on
T- Checking for understanding

T- compares students work
T- direct teaching
T- Echo's students words
T- Go Beyond
T- Initiation- response- evaluation
T- Interjection with another idea
T- Interrogation
T- Introduce new strategy that has not been developed
T- Letting students just talk
T- linked to Big idea/ landscape
T- linked to talk move
T- Monitoring students
T- no confirmation/ in order to push beyond
T- question unclear
T- relate back to context
T- relate to other problems
T- Revoicing
T- Scaffolding
T- shares strategy
T- student revoicing
T- Think, Pair, Share
T- Wait Time
T-make students go beyond with their thinking
T_- TEACHING

Appendix N: Talk Moves and Questions

This is an explanation of the different talk moves and questions that I asked in the classroom.

Talk Moves:

Air Misconceptions: airing misconceptions is when the teacher will bring out a misconception in order to get more talk initiated. It will often be in the form of presenting a wrong strategy or making an incorrect statement.

Answering with another question: a strategy that is often employed by teachers. It is meant to get the students talk. By answering their statement with another question teachers are not stating that something is wrong but at the same time that the statement needs further clarification.

Letting students just talk: Often the best talk move is to say nothing and let the students talk it out.

Monitoring students: The talk move is to see if the students understand what is happening in their strategies or in the congress. This is often stated as a quick question, "What do you mean?" "Why did you do this?" It is a talk move because it normally is not related to a big idea but more of a diving board to create further and deeper discussion.

No confirmation/ in order to push beyond: Similar to letting students talk, with this talk move the teacher says nothing, which with time, will make the students want to explain more or keep going with the conversation.

Relate back to context: When students are stuck on the problem it is always good to bring them back to the context.

Relate to other problems: Like above sometimes there is not context, in this situation bring the student back to the problem.

Revoicing: A useful tool to make the students hear back what they have said. For this talk move all you need to do is state what the student said. "You are saying..." "Is this what you said...?" It is important to repeat as best as you can what the student said.

Student revoicing: Same as above but with the students.

Think, Pair, Share: This is good with reluctant talkers or participators in the classroom. For this move the teacher has the students first thing, then share with a partner and then share with the classroom.

Wait Time: Is exactly what the term says, wait. The more time the better.

Questions:

Building on: This type of questioning is when the teacher tries to build upon what a student has presented. This type of question looks like: "How is this related? Why did you do this? What big idea are you using? etc."

Compares students work: This type of question often is used to compare two strategies together. This type of question looks like: "How is this compared to this strategy? How is this similar...? How is this different?"

Direct teaching: This type of questioning is more teaching statements then questions. Direct teaching is when the teacher tells the students the answers or information.

Go Beyond: For this type of questioning the teacher is trying to bring the students beyond what they may understand. For this questioning the teacher may introduce a new strategy by asking students opinions. They may also ask if they understand a particular term. The teacher may also try to relate a problem to a term and see if the students understand.

Initiation- response- evaluation: This is traditionally found when the teacher asks a question they already know the answer to the question. The purpose of this is not to have students talk but to make sure that information is being disseminated. Once the teacher hears the appropriate response they often move on or ask another question.

Interrogation: This type of question is often used to gain information from the student. This is normally is in the form of "Why?" or "How come?"

Question unclear: This code was used more when I didn't know what type of question I asked or why I asked it.

Scaffolding: These type of questions are used when the students may not understand fully the big idea. Often the teachers will bring the questioning back to where the students are and then build on the knowledge and answers given. The first questions may be talk moves, relate to the context, or bring it back to the numbers the students are working with. To scaffold teachers need a good understanding of students progressions of learning.

Shares strategy: This is when a teacher, during a congress, just shares the students strategies. This would often happen in 1st generation reformists, according to Stein, M. K., Engle, R., Smith, M. & Hughes, E (2008).