Improving Mathematics Teaching Through Professional Learning Groups

by

Jennifer Holm

January 9, 2014

A dissertation submitted in partial fulfillment

for the requirements of the degree of

Philosophy of Education

Faculty of Education

Lakehead University

Thunder Bay, ON

Acknowledgements

It has been said “it takes a village” and I truly feel this doctoral journey embodied that idea. Thank you to my participants who let me poke and prod into their lives and who are at the heart of my research. As well, my friends and family deserve a huge thank you for their support and listening to me talk at lengths about this process. There are so many people who helped make all this possible, and I would like to recognize a few of them here.

I would like to thank my supervisor, Dr. Ann Kajander, for her tireless support during this entire experience. Without her reading and commenting on countless chapters and advice on dealing with so much data, my dissertation would not have been possible. Thank you for providing a critical ear throughout this journey.

Thank you as well to my committee members, Dr. Tony Bartley and Dr. Joyce Mgombelo, for contributing their advice and points of view. I would also like to share my appreciation for my examiners, Dr. Hope Fennell and Dr. Doug McDougall, for lending their critiques and comments to my final work. Without the comments, support, and guidance of so many respected individuals, my final dissertation would not be where it is today.

Finally, I must thank my Gramma Lyon and Grampa Holm for without their love and support I would not have finished with my sanity intact. My Gramma provided the support system and ear to listen to me throughout the years that was absolutely invaluable. I want to thank my Grampa for always believing in me, and although he did not make it to see this final product, I know he is immensely proud of what I have accomplished here.

So, a big thank you to the “village” who helped me accomplish my dream of finishing my doctorate.

Figures

Tables

Appendices

Abstract

In order to teach mathematics well, teachers must have a specialised knowledge of the content (Silverman & Thompson, 2008) and believe in effective teaching methods (Philipp, 2007). Research has indicated that teaching mathematics effectively may require teachers to use pedagogy that they have never experienced themselves (McNeal & Simon, 2000). To address the important issue of effective mathematics instruction, professional learning groups have been explored as a means to provide teachers the support they need to continue developing professionally. This research addresses a call by Johnson (2009) that professional learning groups need to be investigated further to ensure their effectiveness. A case study of one professional learning group was conducted to explore how the discussions provided the needed support for mathematics teachers in using research-based pedagogy in their classrooms. Professional learning group characteristics provided by the research literature were examined in relation to this case study in order to determine how such groups could be developed in mathematics. Conversations about beliefs and knowledge were also analysed, in order to provide an understanding of how the group focused on mathematics teaching and learning. Narrative inquiry was used to provide in-depth descriptions of five of the teachers, chosen in order to show a range of the members within the group. Through both the case study and narrative, a model was created, in order to provide a description of characteristics and dynamics needed in a professional learning group in order to support teachers in their mathematics teaching development. The potential of the model to analyse other research on mathematics professional learning groups was briefly examined. This research determined that to encourage teachers to make changes in their teaching, the professional learning group model should center on an action research type mentality with a "leader" pushing conversations toward more research-based pedagogy. Furthermore, professional learning groups in mathematics need to consider the beliefs and knowledge of the group members in order for the professional development to be effective.

CHAPTER ONE: BACKGROUND AND CONTEXT

In order to teach mathematics well, teachers require a depth of knowledge as well as a belief in effective teaching practices. Mathematics teachers benefit from participation in discussions about mathematics topics and may experience growth and support because, as research has shown, learning is a social process (Brown & Palincsar, 1989; Mullen, 2009; O'Donnell, D'Amico, Schmid, Reeve, & Smith, 2008). Best practices in mathematics teaching have been extensively discussed in the literature (e.g. National Council of Teachers of Mathematics [NCTM], 2000; Ontario Ministry of Education, 2005a; Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989), yet my personal experience with educating pre-service teachers indicates that many classroom teachers are not fully implementing these research based strategies. The lack of implementation of these strategies is linked to both a teacher's knowledge of the subject matter and beliefs of mathematics.

The literature indicates that the majority of pre-service teachers, no matter how removed from the school system, enter teacher education programs with a belief that mathematics education consists of rules and procedures that need to be memorised (Grootenboer, 2008; Holm & Kajander, 2012; McNeal & Simon, 2000; Szydlik, Szydlik, & Benson, 2003). Such beliefs suggest that past exposure to reform-based strategies in school experiences were minimal at best despite Ministry initiatives calling for changes in mathematics teaching practices (e.g. Ministry of Education and Training, 1997; Ontario Ministry of Education, 2005a). Teachers are the ones who ultimately enact the changes in the classroom (Blegen & Kennedy, 2000; Brahier & Schäffner, 2004), so it is important to provide professional development and support to in-service teachers.

Mathematics teachers require specialised knowledge of mathematics and how this knowledge applies to classrooms (Ball, Thames, & Phelps, 2008; Chamberlin, Farmer, & Novak, 2008; Davis & Simmt, 2006; Kajander, 2007; Ma, 1999; Shulman, 1986; Silverman & Thompson, 2008). Although this knowledge can be improved during teacher education programs, it is often not adequate when beginning teaching (Kajander, 2007, 2010). This lack of knowledge tends to lead to a vicious circle – many students have inadequate mathematics understanding and when they become teachers themselves without the knowledge base or confidence, as well as contradictory beliefs, they find it difficult to attempt new methods in teaching mathematics – and so the problems persist. Given challenges in pre-service programs, in-service professional development is important for many teachers who may be lacking the knowledge and corresponding beliefs needed to teach mathematics effectively. Ball, Hill, and Bass (2005) call for the need for teachers who understand mathematics and the curriculum to be teaching it, so ensuring effective professional development is especially important because high-quality professional development is needed in order to have high-quality, effective teachers in the field (Gojmerac & Cherubini, 2012). In order to support changes in the mathematics teaching practices of in-service teachers, professional learning groups[1] are one possible way of addressing teacher needs.

Professional learning groups have the potential to affect teachers to help fill the void in knowledge that teachers bring into their careers. As Schmoker (2006) notes, "Almost any team of teachers knows enough, collectively, to get started on adapting and refining [programs], with

[1] Professional learning communities, collaborative efforts, networks, study groups, and professional learning groups are among the many terms used to describe these groups of teachers working together. Professional learning group is the term used with the group studied because it was chosen by the members to differentiate it from other initiatives of the participating school board. This term is used for the current research study, although the terms may sometimes be used interchangeably to create a definition of the tenets necessary to support the professional learning group studied.

increasing success” (p. 116). By encouraging groups of novice and expert teachers to work together, there may be the skills and knowledge needed to teach mathematics using a more *Standards*-based, or reform-oriented, approach available within the group as a whole. A possible concern is raised in mathematics professional learning by the study of Heirdsfield, Lamb, and Spry (2010) which determined that having a support person was necessary for the mathematics professional learning group they observed, whereas other studies did not mention this finding (e.g., Allen, 2006; Brahier & Schäffner, 2004). As Kajander and Mason (2007) note in their study, defining success in a professional learning group is a highly contentious issue. Further, they note the difference between “success” as defined by the administration and “success” as defined by the group itself. Kajander and Mason determined that the personal nature of a professional learning group makes it essential to give importance to the groups’ definition of personal success. My research study gave credence to the definition of success provided by the group itself, but also established the professional learning group’s efforts in adopting a more reform-oriented approach to teaching. The dual definition of success is important because although professional learning groups are gaining rapid momentum, more research needs to be done to ensure the effectiveness of this initiative and appropriate implementation based on current research definitions (Johnson, 2009).

Purpose of Research

My interest in exploring professional learning groups stemmed from own experiences of being a participant, as well as being a researcher-observer in other groups. The groups I observed had varying impacts on the teachers involved in creating change, so it left with me questions about the effectiveness of professional learning groups that began with my own experience as a participant in this type of development. Not only through my own observation of professional learning groups in mathematics, but research indicates that there is a lack of consistency in what these groups look like and the impact they have on mathematics teaching practices, as well as on teacher knowledge and attitudes (Kajander & Mason, 2007). Although research has identified characteristics of effective professional learning groups (e.g., DuFour & Eaker, 1998; Eaker, DuFour, & Burnette, 2002b; Hord & Sommers, 2008), what conversations in mathematics these groups utilise to encourage growth of mathematics teachers is largely unexplored, so I chose this as the topic for my study.

My study examined the experiences of intermediate level teachers who participated in a professional learning group and the impact the dynamics and activities of the group had on their knowledge and beliefs. I posited that having teachers create their own community of learners supports improvements in teaching practices, therefore my research examined the professional learning group discussions, the structure of the group, and the impact on the teachers involved to discover the conditions needed for a professional learning group to support mathematics teachers. Bell, Wilson, Higgins, and McCoach (2010) note that “there is a critical need for research that investigates whether and how professional development programs can be scaled up to support desired teacher learning” (p. 482). My research sought to help fill this gap by examining the usefulness of a professional learning group in changing teacher practices since this form of professional development is being supported by the Ontario Ministry of Education (2007). By addressing the need to support teachers during their careers, professional learning groups have the potential to affect mathematics teaching in such a way as to allow teachers to grow to continue meeting the needs of their students.

Research Questions

In my research, I was mainly concerned with conversations that appeared to support mathematical growth in both teachers' mathematics knowledge and their pedagogy. My focus question was: What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching practices? In order to properly address this question, I targeted five sub-questions as being important to building an understanding of the effective professional learning group conditions.

Sub-questions

1. In what ways does the group adhere to or deviate from the characteristics of a professional learning group as defined in the literature?

In order to address this question, I first reviewed the literature on professional learning groups in order to determine the characteristics provided by previous research. The chosen professional learning group was analysed to determine whether or not all of the characteristics were present and to examine which appeared to be necessary for a successful group within the context of mathematics education. In order to accomplish this within my research, I inspected the field notes and transcripts from the professional learning group meetings to determine the adherence to or deviation from the defined characteristics.

2. In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?

An examination of both the foundational research and current studies of reform-based instruction provided the source for determining the impact of the professional learning group in supporting the teachers to make changes to their teaching practices. Interviews with the individual teachers as well as discussions held within the meetings were used to report on changes teachers were making as a result of the professional learning group meetings. Any artefacts the teachers brought to the meetings were also examined to show evidence of change by comparing them to artefacts from previous years or past discussions.

3. What are the beliefs about mathematics teaching and learning of the individual teachers in the group? How are these beliefs dealt with in the discussions of the group?

A review of the relevant literature on beliefs about mathematics was discussed in order to frame the importance of examining teacher beliefs. Grant, Hiebert, and Wearne (1994) set out "two dimensions of teachers' beliefs: (a) what kind of mathematics is important for students to learn, and (b) how this mathematics should be taught" (p. 9), and these were the areas that I focused on in my research. The continuum proposed by Grant et al. was also used to identify where the beliefs of the teachers fell in their support for reform teaching practices. Through interviewing and examining the transcripts of the meetings, I described the beliefs of the individual teachers and discussed how these beliefs impacted the discussions within the professional learning group meetings. Classroom observations of four of the professional learning group members also illuminated the discussion of the teachers' beliefs in teaching mathematics.

4. In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?

A discussion of the literature surrounding the specialised mathematics knowledge needed for teaching, as well as the intersections of knowledge with a teacher's beliefs, was created in order to ground discussions of knowledge that occur during the professional learning group meetings. Turner, Warzon, and Christensen (2011) discovered that "content knowledge remained an impediment" for implementing changes to classroom environments (p. 754), so my research considered evidence related to the effects of teacher knowledge on encouraging changes to teaching and learning. A framework for examining teacher knowledge was also discussed in order to aid in analysing discussions of knowledge that occurred during the discussions. Meeting transcripts and field notes served as the basis for determining how conversations about knowledge potentially impacted the success of the professional learning group.

5. What are the experiences of the individual teachers within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching?

Research in professional development indicates that it is imperative to consider the opinions of the teachers in making any development decisions (Anderson, 2005), so the voices of the teachers were essential to my research. In order to validate the importance of the personal successes of the teachers within the group, narrative inquiry was used to discuss the journeys of the individual teachers. Teachers were given opportunities to express how they personally believed the professional learning group impacted their teaching. Interviews, observations, and meeting transcripts were used to construct the narratives.

Situating the Researcher

In my research, I strove to keep the voices of my participants and their personal journeys at the forefront, but these journeys were viewed through the lens of my own personal experiences. In 2004, I was working as an elementary teacher in Tennessee. During that school year, I was directed to work with my fellow grade 2 teachers in a professional learning group in order to align our practices, as well as support each other. As the least experienced teacher, by far, in our grade level, I knew I could learn a lot from my colleagues. Our principal worked hard to organise time during the school day for the seven of us to sit down and talk. Armed with some ideas I was excited to share and get feedback on, we met for the first time. I will never forget what happened next: each of my colleagues handed each of us a file folder of all their October worksheets. My most senior colleague actually had copies of dittos included in the folder; that was how long she had been using some of the sheets. There was no discussion or comments, just handing out the folders. Following this “sharing”, one of my colleagues opened a newspaper and propped her feet on the table and that was it. Another proceeded to knit for the rest of the hour, and the remaining teachers gossiped about specific students. As I struggled to contain my surprise, I listened to the unproductive complaints about their classes, parents, or other teachers. I could not have imagined a more unproductive hour of being out of my classroom.

Prior to the next meeting, I was called in by my principal to discuss the upcoming professional learning group. She asked me to share some of my writing activities with the group because she felt that the other teachers could benefit from it. With a plan for the next meeting, I hoped to begin a pattern of sharing that would encourage changes in my practice, not just more worksheets for my file cabinets. The meeting started much like the first with November files being “shared”, and I explained my writing program. My senior colleague mentioned my ideas were “cute” before picking up her newspaper, and another noted it would never work in a “real” classroom, despite my having used it the previous school year with great success. One asked a few questions and then the discussions began about the students again, wasting the rest of time, no matter how many ways I tried steering the teachers. Sadly the meetings fell apart soon after this time due to lack of interest, and frankly I knew my hour would be more productive actually working with my students.

For me, this sparked my interest in professional learning groups and how they worked when I heard about the groups beginning in my area. I wondered what needed to change to keep them from moving down the same path as the one I had experienced. Is it a worthwhile initiative, or just a new buzz word in education? I wanted to know how to get teachers to buy in and keep them from being a waste of time, yet still maintain the integrity of the professional learning group characteristics.

Rationale

Mathematics education has been undergoing changes in order to align teaching practices with what has been termed “reform-based” pedagogy because the more traditional, or direct, methods of mathematics teaching have been shown to be less effective than other more constructivist-based learning paradigms (Askey, 1999; Van de Walle & Lovin, 2006). The NCTM (2000; Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989) has emphasised teaching students using a more social and exploratory method. Mathematics students should be given opportunities to experiment with and explore mathematical concepts in order to construct understanding of ideas. These teaching practices are very different from a traditional mathematics classroom in which a teacher demonstrates how to solve a question and then assigns multiple questions for students to “practice” the concept in the same manner previously shown. In a more reform-based classroom, groups of students would be working together to construct meanings while exploring a problem (Boaler & Humphreys, 2005; NCTM, 2000; Van de Walle & Lovin). Classroom discussion would follow in order to allow the students to share their ideas and thoughts in order to further construct meanings (Boaler & Humphreys; Wood, Cobb, & Yackel, 1991). Set in constructivist pedagogy, connections are made to previous mathematics concepts and students build knowledge about the relationships between different mathematics concepts (Bay-Williams & Meyer, 2005; NCTM, 2000). According to social constructivists, understandings are built through a combination of the actions of the individual and the social interactions the individual undertakes and that these two facets cannot be separated (Cole, 1985; Moll & Whitmore, 1993; Palincsar, 1998; Vygotsky, 1978). The social constructivist theory framed both my doctoral research as well as reform-based pedagogy because social constructivism discusses the importance of group dynamics in creating individual understandings (Palincsar, 1998), much like the purpose of the professional learning group. New directions suggested by research in mathematics education require teachers in a reform-based classroom to have different skills than they may have used or experienced in their past (McNeal & Simon, 2000). Research indicates that pre-service teacher education is not sufficient for fully developing the knowledge or skills needed to teach well (Baumert et al., 2010; Kajander, 2007, 2010). Effective professional development is an important component for work with current teachers to encourage changes, while also continuing to promote discussion and provide support for new teachers.

In order to address the needs of teachers, studies in the United States have shown that large amounts of money have been invested in offering professional development opportunities (Ball & Cohen, 1999; Desimone et al.). Due to the cost, boards often rely on "one size fits all" programs for professional development in order to reach large groups of teachers while minimising the cost, whereas long-term, high-quality professional development would increase the costs spent on programs (Desimone, Smith, & Ueno, 2006). As a result, "teachers have been considered as passive receivers of prescriptive programs, [and] given little time or incentive to integrate these new programs into their classroom practice" (Lieberman, 2000, p. 226). Teachers are often given information from administration or board-mandated workshops and told to go out and use the new strategies or programs. Buckner and McDowelle (2000) noted that this “top down” hierarchy is ineffective in today’s school systems. In an educational community where students are considered to be varied and require a breadth of different techniques for learning, why would professional development programs use the exact characteristics teachers are not to use on their students?

Another problematic characteristic of using the same professional development in all situations is that teaching is contextual, with each school or class being different from another. A “one size fits all” fix is simply recommending teachers employ the same strategies in all schools and is unlikely to be effective. Research supports the conclusion that these “one shot” or “top down” professional developments simply are not working (Arbaugh, 2003; Ball & Cohen, 1999; Brahier & Schäffner, 2004; Gojmerac & Cherubini, 2012; Hawley & Valli, 1999; Hofman & Dijkstra, 2010; Lieberman, 2000; Schmoker, 2006; West & Curcio, 2004; Wetzel, 2001); rather, professional development needs to encourage teachers to expand their pedagogical horizons and potentially make changes. Avoiding this traditional form of professional development in favour of what is being termed “sustained and significant” learning opportunities (Brahier & Schäffner) is the current best route for development. In mathematics, effective professional development is especially important as teacher content knowledge needs to be improved (Ball et al., 2008; Silverman & Thompson, 2008) at the same time as teachers need to examine their existing beliefs about teaching mathematics (Philipp, 2007; Wilkins, 2008). The spread of professional learning communities through educational systems attempts to inform and shape teaching practices in today’s changing field (Johnson, 2009; Laufgraben, Shapiro, & Associates, 2004). In mathematics in particular, more research needs to be done to explore why certain learning communities are more effective than others because as Kajander and Mason (2007) illustrated, there is a wide range.

Ontario Context

My research was set in an Ontario school board, so the Ontario context is very important for understanding the conversations of the teachers in this study. The teachers are required to use the curriculum documents produced by the Ontario Ministry of Education (2005a; 2005b). These curriculum documents were revised with a reform-based paradigm (NCTM, 2000) as the philosophy guiding framework. This curriculum revision was supported by many studies that examined the effects of the Reorganized Program in the prior curriculum on secondary school course choice and graduation (e.g. King, Warren, Boyer, & Chin, 2005). King, Warren, Boyer, and Chin found that although graduation rates were increasing, the rates were still below other provinces. The Ministry of Education strove to ensure that the newest curriculum revision was going to have a positive impact on student success in secondary school.

As part of the drive to increase graduation rates, the Ontario Ministry also began the “Student Success” or “Learning to 18” initiative (http://www.edu.gov.on.ca/eng/teachers/studentsuccess/strategy.html). This initiative was put into place to give resources to grade 7 to 10 teachers in order to provide support for their students. These grade levels were specifically chosen as critically related to graduation rates. The funding for the professional learning group in my research came from this initiative in order to facilitate a positive transition for students from elementary to secondary school. Factoring in the cost of occasional teachers, the cost of just allowing the teachers to meet in the professional learning group I observed was about $7,000 a year. Considering there were four other professional learning groups in the participants’ district, and this research continued for three years, the amount of money given by the province is staggering. Given the total bill of about $21,000 for this group alone, professional learning groups are not cheap to implement, so research ensuring the funds are both necessary and not being wasted is vital.

In Ontario, there are three choices that students can make for mathematics courses at the grade 9 and 10 levels: Locally Developed, Applied, or Academic. Not all school boards are required to have a Locally Developed (LDCC) course for their secondary students (LDCC Project, 2005), but the school board in this study has chosen to create one. “LDCC courses are intended to meet educational and career preparation needs of students that cannot be met by the courses authorised by the provincial curriculum policy documents” (LDCC Project, 2005, p. 2). By opting to take the LDCC course, students are able to satisfy their compulsory mathematics course credit for secondary school even if they are not able to meet the mathematics requirements of the Applied or Academic courses. According to the grade 9 and 10 curriculum document (2005b), “**Applied courses** *focus on the essential concepts of a subject, and develop students’ knowledge and skills through practical applications and concrete examples. Familiar situations are used to illustrate ideas and students are given more opportunities to experience hands-on applications of the concepts and theories they study*” (p. 6, emphasis in original). Academic courses, on the other hand, “*develop students’ knowledge and skills through the study of theory and abstract problems. These courses focus on the essential concepts of a subject and explore related concepts as well. They incorporate practical applications as appropriate*” (Ontario Ministry of Education, 2005b, p. 6, emphasis in original). In Ontario secondary schools, students are able to choose their “Pathway” of study through the mathematics courses (see Figure 1). Students who wish to enter grade 10 Academic courses must successfully complete the grade 9 Academic course. Similarly, students who wish to enter grade 10 Applied courses must complete either grade 9 Applied or grade 9 Academic successfully. Students who are successful in the grade 9 LDCC course may either take grade 10 LDCC or the grade 9 Applied mathematics course.

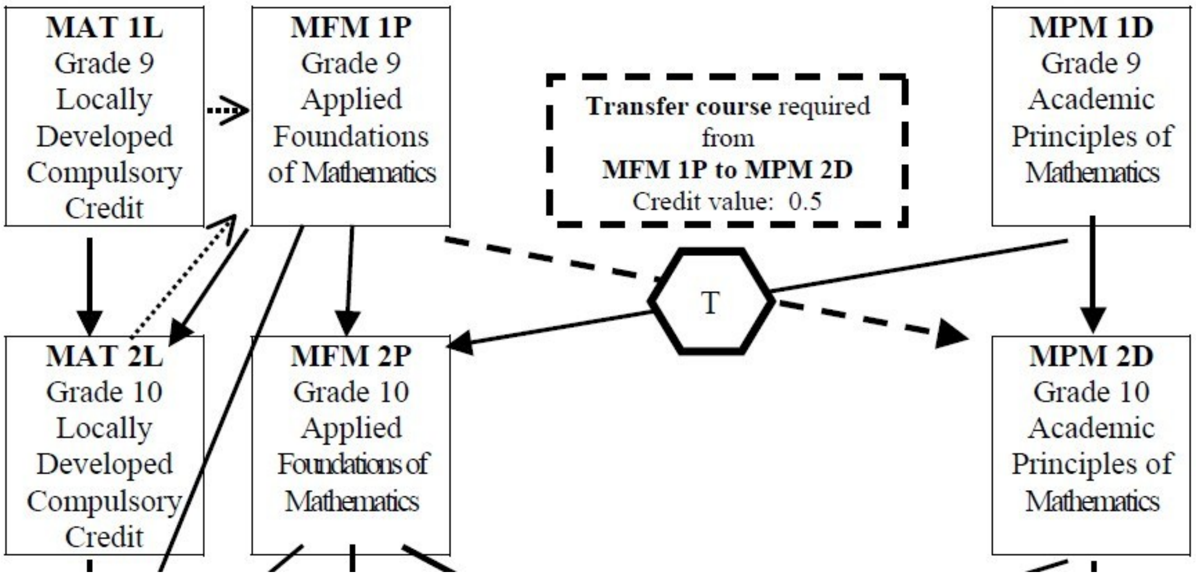


Figure 1. Mathematics "pathways" (2007) for grades 9 and 10 in Ontario.
Retrieved from http://nacimath.wikispaces.com/file/view/Math+Pathways.pdf

Another area of discussion in Ontario schools is that of the EQAO testing that occurs in all Ontario public schools. EQAO is the Education Quality and Accountability Office, and they are responsible for creating and grading the tests that are administered each year. The purpose of this arms' length organization was originally to provide data about curriculum effectiveness in mathematics in Ontario. Since the results are also available to individual schools, it is also being used as a way to track school improvement. The one area of the EQAO testing that was pertinent to my research is the grade 9 mathematics EQAO test. This exam is administered to all students enrolled in either Academic or Applied grade 9 courses, and it is given to students in either January or May, depending on which term they complete their mathematics course. There is a separate test for both Applied and Academic students that claims to test at the level the students are working within their course pathway. Depending on the school board or the individual school, teachers may use the grade from this exam as part of the mathematics mark in the course (Education Quality and Accountability Office, 2011). On the EQAO website (http://www.eqao.com), sample exam questions as well as statistical results from provincial, district, and school level exams for the past years are posted.

Kozlow (2012) studied the grade 9 EQAO and determined that students who liked mathematics and thought they did well in the subject, were more likely to meet the Ministry standard. He also found that students in the Academic courses (compare with those in Applied courses) were more likely to say that they enjoyed mathematics and have confidence in their abilities. Kozlow linked attendance to the scores in mathematics, and also found that students in the Applied courses were more likely to have a greater number of absences. Furthermore Kozlow determined that students who reached the Ministry standard in earlier grades were more likely to meet the standard in grade 9. Since grade 9 students were part of the focus of my research project, information about the EQAO results is important background for interpreting the conversations of the teachers.

Another resource particular to the Ontario context of my research is that of the eduGAINS website. In particular, the mathGAINS portion of the website was useful to discuss in my research (http://www.edugains.ca/newsite/math2/index.html). On this website teachers can find resources and lessons that are directly related to the Ontario curriculum which is guiding their mathematics teaching. Of particular interest on this website is the CLIPS, or Critical Learning Instructional Paths Supports, that teachers can use to provide support for their students in mathematics. Ross, Ford, and Bruce (2007) used research in order to develop the CLIPS program to target “lower-achieving mathematics students in grades 7-10” (p. 430). The purpose of the website is to create web-based activities for students and classrooms in grades 7 to 12 to explore and discuss mathematics concepts. For example, students can explore part/whole relationships in fractions through examining the activities and videos included in this website.

Limitations and Implications

A limitation of the study was the exploration of only a single professional learning group in the Northwestern Ontario area. By examining transcripts from past studies (e.g., Kajander & Mason, 2007), I felt the current group was relatively productive compared with others in the region studied, and a possible limitation could be that since it may have been a higher functioning group, I may have been allowed to participate because of the comfort. The only member in the group who seemed to strongly disagree with discussions in the meetings refused to be interviewed, so the lack of his perspective may have left something missing from the current discussion. My research allowed for the creation of a model for mathematics professional learning groups to make professional development successful but comparison with direct observations of another group would have made it more compelling.

Although my research focused on the specifics of a single professional learning group, there are some characteristics that were illustrated that have the potential to be generalisable to the field as a whole. Being able to describe the characteristics of this single, successful group does have the potential to support further study of professional learning groups in order to discover the significance of these characteristics. My study did point to concerns raised by members of the group that could have an impact on the field in ensuring proper implementation and use of professional learning groups.

Overview

The remaining sections of my dissertation are divided into seven chapters in order to address my research questions. My next chapter is a review of current literature in the fields pertinent to my research. The chapter begins with a discussion of the theoretical framework that underpins my entire research study. I begin with a study of the foundations of social constructivism, and then explore the evolution of the theory as it applies to my research work. Following this discussion of the theoretical framework, I explore research into professional learning communities. The chapter then examines the foundations of reform-based instruction practices, and describes the current practices being advocated in today's schools. I next review the literature that applies to mathematics knowledge for teaching and teacher beliefs that affect classroom decisions. The chapter concludes with a review of research studies that specifically address professional learning communities in mathematics and the questions still left to be answered based upon the previous research.

Chapter three explains the methodology used in my research. I then discuss how the methods of case study and narrative were blended to structure my study. The chapter concludes with a description of the data collected to answer each of my research questions.

The results section of my research has been divided into three chapters in order to properly address both the case study and narrative methods. Since the professional learning group, or case, is the most important aspect of my research, the first results chapter sets up the participants in the groups, the general meeting topics, and then the themes that arose from examining the case study data. This chapter also defines the success of the professional learning group based on the participants' perspectives of the experience. In the following chapter, I begin examining the data through my research lens with my specific questions in mind. This chapter explores the narratives of the chosen participants and then the beliefs of the group members. Both of these areas are explored using the words of the participants, and then linked to current research literature as possible. Chapter six concludes my results section focusing on the characteristics of the professional learning group and the knowledge of mathematics and teaching presented during the meetings. Each section of the characteristics and types of knowledge is concluded with a discussion of how the sections fit into or fill a gap in the current literature. The chapter ends with an examination of any changes seen in the participants through my study.

The next chapter begins with a summary of how each of my research questions was answered based on the examination of my data. As a culmination of my research, I developed a model to use in organising and evaluating professional learning groups in mathematics. I link my research with other research studies in order to set up the model. The chapter concludes with using my model to evaluate previous professional learning groups explored in another research study (Kajander & Mason, 2007).

The final chapter in this document presents a conclusion of my research study, including further implications of my study and suggestions for future research.

CHAPTER TWO: LITERATURE REVIEW

In this chapter, I review the literature relevant to my research focus. First, I describe how the theoretical framework of social constructivism provides a lens for my research. Next, I describe each of the characteristics of professional learning groups that have been determined by previous researchers. I examine literature related to mathematics education specifically in order to give a solid basis for my own study. I explore the foundational concepts for the reform-based pedagogy and then describe how this would be enacted in a classroom environment. Then, I describe how beliefs and knowledge are defined in mathematics and how they work together to influence pedagogical choices in classrooms. I end with a review the literature specific to mathematics professional learning groups to determine any gaps or questions remaining from previous studies.

Theoretical Framework

I have posited that my study is framed within social constructivist perspectives, hence I begin by examining tenets of constructivism in order to build a case for the relationship of social constructivism to my research. Constructivist theories stress how individuals construct their knowledge by engaging in new activities and fitting them into prior knowledge and experiences (Bartlett & Burton, 2007; Bredo, 2000; Copsey-Haydey, Zakaluk, & Straw, 2010; O'Donnell et al., 2008; Richardson, 2003). Bredo (2000) notes that in a constructivist paradigm the learner would take an "active role in learning" (p. 132). My research focused on how teachers work in a professional learning group and appropriate the new experiences and learning encountered during the meetings by connecting them with their existing knowledge structures. The broad umbrella of constructivism has since been divided into many different forms including educational constructivism, humanistic constructivism, sociocultural constructivism, and critical constructivism to name a few (Matthews, 2000). Burbules (2000) and Matthews (2000) note that there are a variety of definitions for constructivism making defining it accurately difficult.

Richardson (2003) points to the idea that although there are many forms of constructivism, there are two poles that define the outer limits: sociological and psychological constructivism. She further discusses how research is starting to focus on the two types as not being opposing views but as sharing traits between them. Sociological constructivists focus on public knowledge; whereas, psychological constructivists believe that individual knowledge is the focus (Bredo, 2000; Richardson, 2003). Radical constructivism falls under the category of psychological constructivism by allowing for multiple truths because individuals construct their own knowledge, arguing against a shared knowledge being created through group interactions (Howe & Berv, 2000). Since this knowledge is built in the experiences and mind of the individual (Matthews, 2000; McCarty & Schwandt, 2000), radical constructivists decrease or even deny the effects of social contexts in the creation of knowledge. This runs counter to the basis of my research where my concern was the social context of the professional learning group in supporting teachers developing their practices. On the other hand, according to social constructivists, all truth, language, morals, and knowledge come from the collective, not the individual (McCarty & Schwandt, 2000), which “gives primacy to the social sphere” (Gergen, 1996, p. 19). Gergen believes that “meaning is a product of assent and coordination among two or more people engaged in social relationships” (McCarty & Schwandt, 2000, p. 66). Social constructivists assert that knowledge is created only when it has been acknowledged and agreed to by more than one individual.

Social Constructivism

Cobb and Yackel (1996) questioned the idea that a classroom can be defined purely by examining the individual because the community is essential for learning so the social context of the classroom must be considered. This would also apply to a teacher professional learning group where the community is the central context for learning about teaching. Although some researchers set psychological and sociological constructivism as opposite ends of spectrum, Bredo (2000) argues that humans are not just individuals or completely social, but both, thus further hinting to a need for a middle ground between them. If I treat the sociological and psychological constructivist perspectives as the outer limits, it becomes apparent that my research into professional learning groups falls more in the social tenets of constructivism.

Social constructivism gives precedence to meaning being created as a group within a social context and has roots in the work of Vygotsky (Bartlett & Burton, 2007). Although Vygotsky focused solely on the learning of children, his ideas do provide a foundation for exploring an adult learning environment. According to Vygotsky, children learn within their own experiences based upon their own developmental level. The use of reform-based pedagogy is supported by this philosophy by having students solve problems from their own knowledge base and at their current developmental level. This developmental process was important to Vygotsky (1962) because he believed that when a new idea is encountered, students will adapt their previously learned schemas to accommodate the new knowledge. A skilled teacher would be able to help students reach new levels of understanding by taking students from their current knowledge base and pushing their thinking forward. A group of skilled teachers would also push each other’s thinking about teaching mathematics forward by challenging ideas and engaging in open discussion about teaching practices. Professional learning groups are further supported by the work of Vygotsky because teachers work from their own experiences and grow from where they are, thus previous knowledge structures would be reorganised to allow for the new learning to be used effectively. Vygotsky also believed that until learners had developed the necessary prerequisite skills, they could not build upon this foundation. Users of Vygotsky's (1962) philosophy would argue against the traditional method of teaching mathematics, but rather would stress using a child's developmental level to determine instruction as well as advocating teaching that does not strictly involve drills of facts and procedures. The social nature of learning is an important aspect according to social constructivists and provides support for using professional learning groups in teacher learning.

Within the discussion of a child's developmental level, Vygotsky further explored the concept of the zone of proximal development (Vygotsky, 1962) as the ideal area for student learning. He believed that students' learning needed to be targeted to where the students were within this zone and that this would give students the best benefits. Vygotsky (1978) believed children build knowledge by working with others with more experience in the subject. Ball and Bass (2000) second this philosophy as being a source of learning in mathematics classrooms specifically. Furthermore, according to Vygotsky's theory of development (1962), a child would be able to use certain knowledge aspects, for example using the word "because" correctly, before he or she would be able to define what the word means due to their social experiences. To Vygotsky, this was an important tenet for schooling because children need to gain this knowledge before they would be able to deliberately use certain concepts. Again, reform-based pedagogy fits within this framework as students use their own foundational concepts in their learning and this is a springboard for further knowledge. This paradigm also supported my program of research as teachers would need foundational knowledge of mathematics before being able to implement effective pedagogy.

Vygotsky provided the foundational ideas used by social constructivists, but it has been noted that Vygotsky solely focused on instruction as being teacher to student and leaves out the possibilities in student to student relationships for learning (Forman & Cazden, 1985). Student learning in schools has been expanded to acknowledge that students can learn from their interactions with each other in addition to their interactions with the teacher. Teachers can also learn views of mathematics from students in the classroom, as well as from each other, but above all it is the social interactions and shared meanings that are important in social constructivism. Social constructivists stress a need for socialisation in order for learning to be effective which is important in analysing knowledge creation (e.g. Fuller, 2007; Lynch, 1998). Social constructivists stress how the self cannot be separated from the social context of learning (Cole, 1985; Moll & Whitmore, 1993; Palincsar, 1998). There is a relationship built between both the social context and the individual, where the learning is influenced in both directions (O'Donnell et al., 2008). Social situations change through the interactions of the individuals as shared meanings are created between the participants.

The basic ideals of a professional learning group allows for groups of teachers to meet and discuss ideas in order to further develop their teaching practices. My research was based in the idea that teachers learn from each other through their interactions and the activities they engage in together. Similarly, children learn meanings of behaviour from the reactions of others and this shared knowledge allows people to relate to each other (Bredo, 2000). Social constructivists believe that each person within the interaction plays off the responses of the other person(s), and these reactions can be built on misinterpreting a response (Berger & Luckmann, 1966). This concept of responding to an interaction was important within my research in the context of which teachers were in constant conversation with each other. The potential for misinterpretation was possible in these interactions and added a dimension to be considered in research into professional learning groups. It is the relationships of the knowledge created among the teachers that are fostered through their interactions in the professional learning group that were most important to my research. In my research it was impossible to separate the social context of the professional learning group from the individual people. Although my research focused on the group interactions, there was an element of how the individual teachers would interpret the same conversations when examining the narratives.

Providing for the Emotional Needs of Teachers

DiPardo and Potter (2003) expanded the work of Vygotsky about the social nature of emotions to include the difficulties faced by teachers in their profession. According to DiPardo and Potter, Vygotsky "condemned the tendency to separate intellect and affect into distinct fields of study, believing that this separation had created the false illusion that thinking is somehow segregated from the fullness of life and from the needs and interests of the thinker" (p. 318). They provide evidence for the emotional nature of the teaching profession and advocate for the need to provide supportive, social contexts for teachers in dealing with stress encountered while teaching. Not only would support come from the social group, but according to DiPardo and Potter, as well as Gergen (1995), emotions are being constructed in social contexts as well. DiPardo and Potter base their argument on the conclusion that "what we usually think of as the intellectual aspects of teaching cannot ultimately be separated from the emotional charge that attends them" (p. 235). As such, we need to both support academic growth of teachers, but also provide for their emotional well-being. The authors provide a description of two teachers who faced stressful situations: one with a supportive colleague situation and the other with more casual acquaintances. In the end, the teacher without support left the teaching profession due to what DiPardo and Potter called “burnout” (p. 327). This work strongly informed my research in which the very nature of a professional learning group could provide the emotional support needed by creating a core group of colleagues who support and challenge each other. In a very real sense, professional learning groups have the potential to prevent teacher burnout by fostering a community environment where teachers can talk about and work to solve their difficulties. As DiPardo and Potter attest, reform policies need to account for not just academic growth, but also the emotional aspects of teaching and teacher change.

Symbolic Interactionism

Although both the foundational work of Vygotsky and the work on emotional support helped to ground my research, there is also the nature of the group dynamics that needed to be more fully explored. The theory of symbolic interactionism added to the framework informing my work with teachers. Bredo (2000) and Prawat (1996) introduce the work of Blumer on symbolic interactionism as being part of social constructivism, yet Richardson (2003) cites it as being a root of psychological constructivism. Perhaps this disagreement is further evidence of how the two sides are not as oppositional as they were once considered. In examining the literature, I feel the importance placed on the social aspects is integral to the definition of symbolic interactionism. Through examining the work of Mead, Blumer (1969, 2004) discusses how people engage in conversations by responding to each other’s gestures. There are two types of response that can occur: reflexive and symbolic interactions. Symbolic interaction occurs when individuals consider the response by interpreting the gestures and consider the follow-up response in the interaction (Blumer, 2004). Blumer concludes that conversation only occurs when participants in the interactions are actively trying to figure out the other person's desired response to their stimulus. According to Prawat (1996), this approach gives equal weight to the idea that group dynamics and individuals act upon each other to shape knowledge. The idea is that the group dynamics assume that each individual is taking the same idea, yet each individual acts on the knowledge and creates a different construction from the interactions, but there are certain norms and procedures of the group dynamic that must be maintained (Prawat, 1996). Furthermore, each group member would interpret and react to the other members of the group and that is shaped by both the individual and the group in equal measure. For my research, this was important because each teacher interpreted the conversations and activities in their own ways through their own lens. As such, I considered the beliefs of the teachers and their individual interpretations in order to get a complete picture in my research.

Based on the concept of knowledge being built on previous learning and experiences while being influenced through social contexts and individual responses to interactions, my research sought to explore how the interactive context of a professional learning group can be used effectively in mathematics education. It was with these structural ideas of social constructivism, and particularly symbolic interactionism, that I answered "What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching and learning practices?"

In the next sections, I now discuss the current literature that provides a description of professional learning groups in mathematics. Current research studies on professional learning groups in mathematics centre around how they adhere to the tenets of a professional learning community and the effects they have on teachers, with little or no discussion about what the actual conversations look like (e.g., Brahier & Schäffner, 2004; Hofman & Dijkstra, 2010).

Mathematics teaching is a field that requires a specialised knowledge of mathematics (e.g., Silverman & Thompson, 2008), and in order to support teacher growth, conversations need to be deeper than simply discussing surface topics, such as which worksheets or problems to assign the next day.

Defining Professional Learning Groups

When examining the existing literature, it is clear there are a multitude of terms that are used to describe the phenomenon of teachers' collaboration, including networks (Lieberman, 2000), study groups (Arbaugh, 2003), professional learning communities (Hall & Hord, 2006), and professional learning groups, yet the defining factors are usually remarkably similar. The main focus of these groups of teachers is that they are created with the intention of exploring the everyday struggles and triumphs of teaching in order to accomplish a goal set by the group itself. As Arbaugh (2003) summarises, study groups are "*a group of educators who come together on a regular basis to support each other as they work collaboratively to both develop professionally and to change their practice*" (p. 141, italics in original). One definition of a professional learning community is "a group of people with a shared interest in the knowledge, application, and improvement of professional education standards" (Loughridge & Tarantino, 2005, p. 76) by giving "job-embedded opportunities for staff to engage in professional conversations around classroom instruction, assessment, and student learning" (p. 74). Comparing the two definitions, the focus of the collaborative effort is strikingly similar except for the choice of descriptor for "group".

The Ontario Ministry of Education (2007) has advocated the use of professional learning groups in schools. Although the Ministry notes there are many definitions for this type of collaboration, they note that "a professional learning community is always a group of people who are motivated by a vision of learning and who support one another toward that end" (p. 1). For the purposes of my research, I used the term "professional learning group" because this was what the group with whom I worked had decided to use.

Characteristics of a Professional Learning Group

Professional learning groups are generally well known among educators and yet the defining characteristics of the individual groups are often varied (Hord & Sommers, 2008). The five typical characteristics that define most effective collaborations are "shared beliefs, values, and vision", "shared and supportive leadership", "collective learning and its application", "supportive conditions", and "shared personal practice" (Hord & Sommers, 2008, p. 9). In order to inform the research question "In what ways does the group adhere to or deviate from the characteristics of a professional learning group as defined in the literature", the following discusses each of these characteristics in more detail.

Shared beliefs, values, and vision.

In order for a professional learning group to be successful, the members of the group need to have a shared understanding of their goal for both the school and classroom. The beliefs, values, vision, and goals of the group need to be set up in the beginning in order to serve as the foundation and guiding principles for the professional learning group discussions (DuFour & Eaker, 1998; Eaker et al., 2002b). All of the discussions would therefore be guided by this mission in order to make the activities productive towards accomplishing the end goal. The vision guiding the group needs to be based on research into best practices for teaching and not just the opinions of the group members (Eaker, 2002) as well as embedded in the daily lives of the teachers (DuFour & Eaker, 1998). By addressing the concerns of the individuals involved in the professional learning group, the discussions take on greater meaning as teachers see how they can be implemented into their own practices. Since the group goals unify the plan of action (Loughridge & Tarantino, 2005), it is important that the views of all the members of the community are taken into account and that the mission is not simply imposed on the teachers (Huffman, 2003). All of the members within the professional learning group need to buy into the mission in order for the group to be successful and the discussions to be productive. Since the group is defined by common goals (Allen, 2006; DuFour, 2002; Eaker et al., 2002b; Hord & Sommers, 2008; Huffman, 2003; Huffman, 2000; Loughridge & Tarantino, 2005; Mullen, 2009; Schmoker, 2006), the goals need to respond to the needs of the group (Lieberman, 2000) and therefore may change over time as needs or concerns shift (Hord & Sommers, 2008). The goals of the group need to remain flexible so that they can be altered as the situations within the classroom and school environment change. The focus needs to be responsive to the participants, the changes that naturally occur within the classroom, or as the goals are met by the professional learning group.

In order for the changes to be successful within the school environment, the whole community needs to be involved (Andrews & Lewis, 2002; Ball & Cohen, 1999; DuFour & Eaker, 1998; Huffman, 2000; Loughridge & Tarantino, 2005). According to researchers (e.g. DuFour & Eaker, 1998), the community includes not only all of the teachers and administration, but also the parents and community members that support the school. Without the participation and support of all the community, successfully meeting the goals of the professional learning group would not be as likely. The Ontario Ministry of Education (2007), on the other hand, suggests that a professional learning group is teachers and principals working together and then separate “networks” are formed when parents and support staff are included.

In defining the mission of the group, the focus needs to be on student learning and not on teachers specifically (Eaker, 2002; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008; Schmoker, 2006). These researchers indicate that the focus needs to be on what can be done to improve student achievement based on the goals of the group and not simply pointing out ineffective teacher practices. Hiebert, Morris, and Glass (2003) note that in order for teachers to learn from each other, they need to create common goals for the students. The teachers would then be able to work together to develop practices that would help their students achieve this collective goal. Reeves (2010) counters this idea by noting that the focus should not be solely on student achievement but teacher practices as well and states that the professional learning group needs to enter the classroom to see what teachers are actually doing with the students. Regardless of whether the focus is on teaching practices or student learning, it is clear that the heart of the professional learning group is the success of students. Research in mathematics professional development specifically notes that there needs to be a focus on teachers' thinking as well as student thinking and learning (Cwikla, 2004). The changes made as a result of the professional learning group would be made with the desire to improve student experiences and learning (Gojmerac & Cherubini, 2012) in order to meet the needs of the students (Hall & Hord, 2006) with a focus on improving student abilities (Hofman & Dijkstra, 2010).

The ability of the group to choose their own goals and create their own learning trajectories fits into the constructivist paradigm. Constructivists believe that learning is accomplished by engaging in activities and adjusting prior schemas (Vygotsky, 1962). In the same way, professional learning groups allow teachers to experience similar activities based on their prior understandings of students and build on this knowledge. They would learn from and work with their peers to gain knowledge, something social constructivists emphasise is essential for learning. Teachers should have the autonomy to set their own goals and build their learning through their discussions with colleagues. The group would need a shared focus or vision in order to create supports that would allow them to learn from each other.

Shared and supportive leadership.

Evidence gathered by other researchers supports the conclusion that a top-down model of administration no longer works for schools (Buckner & McDowelle, 2000; DuFour & Eaker, 1998), so a more democratic policy of working together is sought (Carr, 1997; Hord & Sommers, 2008; Mullen, 2009). These researchers discuss how having administration tell teachers what changes to make causes difficulties through teachers not applying the changes as they were intended. Instead teachers should be given the authority and respect to help decide what changes to make and be involved in the discussion. This model would involve teachers working together and researching changes that would impact their practices as well as reflecting on the changes with each other in order to keep teachers as part of the process. This collaborative, equal power structure is required in professional learning groups in order to maximise the benefits of the professional development.

In a professional learning group, the administration would encourage their teachers to be leaders (Birky, Shelton, & Headley, 2006; Buckner & McDowelle, 2000; Patterson & Patterson, 2004) by empowering the teachers to make their own changes (Blegen & Kennedy, 2000; Caine & Caine, 2000; Ontario Ministry of Education, 2007; Patterson & Patterson, 2004; Schmoker, 2006). Administration would encourage teachers to decide upon desired changes instead of simply enforcing what strategies the teachers need to use in classrooms. Alternatively, teachers would be encouraged to share the leadership role as they work within their schools to accomplish the goals set out by the mission statement of the group. Even though the members should share this role, there still needs to be strong leadership within the school in order for changes to happen (Eaker et al., 2002b; Huffman, 2000; Wixson & Yochum, 2004). No one person would give directives to the school personnel to make changes, but all would work together to decide upon and enact the changes.

> The norm of collaboration and democratic participation in decision making, as well as sharing power and authority, contribute to a culture in which the staff grows in professionalism and efficacy. This efficacy instills a confidence that each faculty member is influential in the learning process of his or her students, persuading faculty that each student can learn with the appropriate material and strategies. (Hall & Hord, 2006, p. 25-6)

In order to fit into these new leadership roles, schools need to develop new identities and create an environment different from the traditional model of schools (Andrews & Lewis, 2002; Caine & Caine, 2000). If the leadership is to be jointly held (Hall & Hord, 2006), then the climate of the schools needs to move beyond the paradigm of administration being in charge and forcing directives on the staff.

Since teachers are the ones who will ultimately enact the changes (Blegen & Kennedy, 2000; Hall & Hord, 2006), they need to be given the power and autonomy to make the decisions that will directly affect their classrooms. As Stigler and Hiebert (2004) note, “All reform efforts to improve teaching and learning must pass through a final common pathway: the classroom. Most reforms get stopped short at the classroom door; all available evidence suggests that classroom practice has changed little in the past 100 years” (p. 12). This is ultimately important in mathematics teaching where research has shown that traditional methods are not as effective as more constructivist based approaches (e.g., Askey, 1999) and teachers are being asked to teach in ways that may be different from their own classroom experiences (McNeal & Simon, 2000). By giving teachers the power to influence changes and have a voice in what changes will be carried out in their daily practices, teachers will be more likely to make adjustments and maintain the strategies in their own classrooms as they see the benefits for student success. Sykes (1999) states that in effective teacher professional development, changes are made in schools by dedicated people actively seeking knowledge and questioning practices. Sykes (1999) further notes that if changes are to be made, reliance needs to be on teacher knowledge and action. Teachers will be the ones carrying out the strategies, so they need to be given support and guidance to pursue difficulties important in their own practices and make the necessary modifications. Simply telling teachers what to do and expecting them to carry out the directives with the same intent of the action would not be as effective. Support can be garnered by leaders emerging within the group to provide guidance in an area of expertise (Ball & Cohen, 1999) or master teachers providing support for less experienced teachers (Buckner & McDowelle, 2000). In research on mathematics professional learning groups, Hofman and Dijkstra (2010) determined that the group with a stronger moderator had more significant success in making changes. The moderator served to keep the discussions on track and ensure conversations were moving in a productive direction. If in mathematics teachers are being asked to teach in new ways, I questioned how the knowledge and leadership can be found and shared within the group.

Collective learning and its application.

In a professional learning group, the mission of the group is determined by the group members and their own situations, and the discussions and activities of the group must also follow this direction. The basis of a professional learning group is founded in the teachers' own experiences and practices (Bednarz, Maheux, & Barry, 2007; Gojmerac & Cherubini, 2012;

Lieberman, 2000; Lieberman & Wood, 2002), therefore teachers are examining their own current problems and needs to seek solutions (DuFour & Eaker, 1998; Linder, Post, & Calabrese, 2012; Loughridge & Tarantino, 2005; West & Curcio, 2004). Since teachers would all be able to learn from each other and give their ideas (DuFour & Eaker, 1998; Gojmerac & Cherubini, 2012; Lieberman & Wood, 2002), the professional learning group's knowledge comes from within the community instead of an outsider who is no longer around to provide support (Brahier & Schäffner, 2004; Schmoker, 2006). The support becomes ultimately important in mathematics research when teachers will be enacting new strategies where they may encounter difficulties or questions. By having the teachers in the professional learning group offer support to one another, they can discuss and encourage each other as questions arise. In other models, teachers may spend days in a workshop, but when they encounter struggles in the daily details, there is no one on hand to help. Since the time spent in professional development directly relates to how much a teacher implements reform-based lessons (Cohen & Hill, 2000), the professional learning group provides a unique opportunity because the professional development is held within the community; therefore they could potentially increase time spent in developing their skills.

Learning in professional development should focus on improving teaching methods and not just improving individuals (Hiebert, Morris, & Glass, 2003). Since a professional learning group is designed for individuals to work together, the teachers can provide support and advice to each other as they attempt strategies. The Ontario Ministry of Education (2007) notes that in a professional learning group, teachers would become concerned with other students in their division as they work together instead of solely focusing on their own students. The focus in developing mathematics teachers should focus on mathematics knowledge and not just creating lesson plans (Allen, 2006). Bray (2011) notes that mathematics professional development needs to focus on student errors and how to deal with them in classroom situations in order to make teachers more effective. Following the implementation of new strategies, it is important for teachers to reflect on their practices (Arbaugh, 2003; DuFour & Eaker, 1998; Eaker et al., 2002b; Hofman & Dijkstra, 2010; Males, Otten, & Herbel-Eisenmann, 2010; Turner, Warzon, & Christensen, 2011). In mathematics research, the format suggested for changing practices begins with discussing new strategies, followed by the teachers attempting them in their classrooms, and then reflecting on the lessons (Brahier & Schäffner, 2004; Linder et al., 2012; Turner et al., 2011; West & Curcio, 2004). Ball and Cohen (1999) state that professional development needs to be embedded in a teacher's own practices and include reflection in order to consider new possibilities. The structure of a professional learning group allows for members to discuss concerns from their own classrooms, locate and attempt strategies that address these needs, and finally allows time for the group to reflect on the strategies.

In order for professional development to be effective in mathematics, the teachers need experiences with reform because teachers need concrete examples of reform practices in order to implement them in their classrooms (Brahier & Schäffner, 2004; Carnegie Corporation of New York, Institute for Advanced Study Commission on Mathematics and Science Education, 2009). These experiences would need to be embedded in a professional learning group in order for teachers to begin implementing reform strategies. Linder, Post, and Calabrese (2012) stated that teachers in their study on professional development, the teachers found the concrete examples to be essential to their development. Since the basis of a professional learning group is the idea that the tools to make changes lies within the school (Eaker, 2002; Schmoker, 2006), this could present a problem in mathematics education since the teachers would already need knowledge about reform-based strategies to share them.

In order for professional learning group conversations about shared personal practice to be effective in creating change, researchers detail different aspects that should be included. As stated previously, the purpose of a professional learning group should be to address student needs in classroom situations (Hall & Hord, 2006). As a result, Hord (2009) and Sowder (2007) suggest a focus on examining student work to ensure students stay in the foreground of the conversations. The focus on the needs of students should be central to all the conversations, so by examining student work, the teachers in the professional learning group ensure that practical assessments of students and their learning are discussed. In mathematics, this focus on students must extend to a group focus on student thinking and learning (Cwikla, 2004; Sowder, 2007). When discussing students' mathematics work, discourse would centre on what students may be thinking and what supports could be put in place to address any misconceptions or stumbling blocks.

Discussions should also examine future needs and teacher growth, not just immediate student learning, because research indicates that this aids in teacher development (Hofman & Dijkstra, 2010). The artefacts that the professional learning group focuses on should consider looking at the bigger picture and not simply changes to be made to help a single lesson or student at the immediate point in time. Van Driel and Berry (2012) suggest there needs to be a focus on developing pedagogical content knowledge in order for professional development to be successful. Huang, Li, and He (2010) stress that professional development in mathematics needs to focus on helping less experienced teachers develop deeper content knowledge. Since mathematics research notes that there is a specialised knowledge of mathematics that is needed by teachers, a focus on this knowledge is important for teacher development.

Finally, a professional learning group should include collaboration with all of the members. The act of building knowledge in a professional learning group structure is based in the tenets of social constructivism where social influences work to build knowledge in individuals. “Human social interaction consists of people acting and reacting to one another” (Blumer, 2004, p. 32). It is precisely these interactions that would stimulate the conversations necessary for rich learning and also difficulties in problem solving classrooms. Research shows that the act of teachers collaborating leads to changes in practices (Gojmerac & Cherubini, 2012; Wixson & Yochum, 2004), so the professional learning group structure is positioned to help teachers gain the support they need in order to make changes in their professions.

Supportive conditions.

Supportive conditions refer to the environmental factors that impact the effectiveness of a professional learning group. Hawley and Valli (1999) discussed that in order for professional development to be effective in changing schools, a supportive environment is necessary. Since collaboration is at the core of a professional learning group, a climate that encourages teachers to lead and work together while supporting changes is required (Blegen & Kennedy, 2000). In a professional learning group, change is the group responsibility to decide upon and enact, not just up to an individual (Opfer & Pedder, 2011). Not only are individuals learning from each other but supporting and encouraging changes, while holding each other accountable for trying the strategies. The supportive environment also includes the support of the larger community, not just the teachers (Coleman, Gallagher, & Job, 2012; Huffman, 2000). Parents, students, and the larger community would be a part of the team in a professional learning group. Administrative support is also vital for the success of professional learning groups (Blegen & Kennedy, 2000; Buckner & McDowelle, 2000; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008;

Huffman, 2000; Ontario Ministry of Education, 2007; Patterson & Patterson, 2004). Administration would make sure to enforce the mission, beliefs, values, and goals as well as promote collaboration in the school (Eaker et al., 2002b). DuFour and Eaker (1998) suggest that the administration would also be responsible to ensure participation of the teachers in the collaboration. Caine and Caine (2000), on the other hand, suggest that participants in a professional learning group must be volunteers since no one can force someone to change. In mathematics research specifically, a lack of administration support including proper professional development and materials has been linked to impeding changes in schools (Handal & Herrington, 2003). Clarke (1997) noted that the school environment, including the administration, were reasons for teachers not adopting reform strategies. In order for professional learning groups to be effective, proper support for teachers would need to be given to encourage group members to work together and make changes.

In order to set up the proper conditions for an effective environment for professional learning groups to flourish, there are several parameters research indicates should be considered. First, the environment needs to be built on trust and cooperation while encouraging growth (DuFour & Eaker, 1998). The teachers need to feel safe to discuss their practices and take risks in trying new strategies with their students without concern for negative consequences when expressing issues. This community of collaboration needs to be built into the school culture for it to be most effective (Coleman et al., 2012; Eaker, 2002). Battey and Franke (2008) found that before their professional development efforts could make changes in teacher practice, they had to acknowledge the existing school culture and work within it. Since the school determines the push to participate in reform and the amount of support given (Opfer & Pedder, 2011), a single individual should not work alone to try and make changes. Next, time to meet needs to be focused (Anderson, 2005; Eaker et al., 2002b; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008) and embedded in the daily lives of the professionals involved (DuFour & Eaker, 1998; Eaker, 2002; Schmoker, 2006). With the number of responsibilities for teachers, time needs to be set aside so that they can meet together, and this time needs to be given value. In order to show the commitment to having teachers work together, Hord (2009) suggests restructuring the school day to allow for more collaboration. Anderson (2005) notes that schools need better resources. In order to make changes, teachers need to have access to resources and materials needed to accomplish the agreed upon mission. Mathematics reform, in particular, requires support and the proper resources to begin or continue changes in curriculum (Boyd, 1994). Finally the number of members in a professional learning group needs to be given consideration. There needs to be a number great enough to have diversity of ideas and yet small enough that all the members can be heard (Arbaugh, 2003; Hofman & Dijkstra, 2010). The professional learning group is built on the ideas of the whole group, so the members need to feel free to share yet also have the time to hear all the ideas.

Shared personal practice.

Hord and Sommers (2008) define professional learning groups as continuous learning, and research into professional development shows learning must be continuous for growth to occur (Ball & Cohen, 1999; Opfer & Pedder, 2011). Professional learning groups allow for sharing practices and resources among group members within the community. Sharing among teachers leads to better teaching practices (Blegen & Kennedy, 2000) because others learn from the experiences of the individuals in the group (Hiebert et al., 2003). In the professional learning group model, teachers would share their experiences and challenges to create a shared knowledge base. All the teachers in the group would have value (Hall & Hord, 2006), and their own “knowledgeabilities” to contribute (Lave, 2008). The term “knowledgeabilities” refers to the knowledge within an individual and how the transmission of ideas is not necessarily just experienced to novice—all have their own expertise. Lieberman (2000) adds that a professional learning group should use the knowledge of the group members but would balance it with research, demands, or professionals outside the group to give more expertise. Although the members of the group bring their own knowledge to the discourse, using knowledge produced by other professionals can keep practices progressing and allow for deeper conversations. The Ontario Ministry of Education (2007) requests that teachers in a professional learning group examine research and attempt to align the practices of a grade or division by sharing teaching practices and beliefs about teaching and learning. Linder et al. (2012) point to the need for a knowledgeable party to support the professional learning group discussions and Anderson (2005) and Heirdsfield et al. (2010) echo this sentiment in their research into mathematics teaching. Mathematics is a field where teachers are learning new strategies, so it may be impossible for a group to be completely self-sufficient for teachers to show growth.

The foundations of social constructivist theory give support for teachers creating shared personal practices. In examining peer learning, collaboration is defined as being “a mutual task in which the partners work together to produce something that neither could have produced alone” (Forman & Cazden, 1985, p. 329). Teachers would be able to work together to create a culture where they can potentially make changes that would not occur for the individual teachers without the group. By creating a collaborative effort, teachers are sharing ideas to learn from each other in order to transform their teaching through the social connections.

Feedback and support.

Professional learning group structures are based in teachers working and learning together in order to address difficulties in their own classrooms. DuFour (2002) argues that feeling connected is a personal need that is violated in traditional schools. Professional learning groups help to correct this deficit by encouraging collaboration (Linder et al., 2012) and reducing the isolation felt by teachers (Bruce & Ross, 2008; Buckner & McDowelle, 2000; DuFour & Eaker, 1998; Eaker, 2002; Eaker et al., 2002b; Hall & Hord, 2006; Hord & Sommers, 2008; Lieberman, 2000; Schmoker, 2006). Collaboration is necessary for changes in teaching (Hiebert et al., 2003), so schools need to create a supportive, caring environment to allow relationships built on trust, respect, and open communication to flourish (Hall & Hord, 2006). Research into mathematics professional development has highlighted the importance of creating these relationships in order to provide support as teachers make changes (Bray, 2011). Teacher collaboration is cited as an important feature in mathematics professional development (Cwikla, 2004) and a lack of collaboration is noted as the reason for not adopting reform (Clarke, 1997). Not only would professional learning groups provide an environment allowing for collaboration, it could also provide moral support for teachers (Arbaugh, 2003). The professional learning group model values the knowledge and experiences of all the teachers (Lieberman & Wood, 2002) as they are brought to the discussions. All the teachers would contribute and support each other as the work through the difficulties of their daily practices as stated in the mission statement.

In mathematics education, teachers are learning about and trying to implement reform strategies in their classrooms. In mathematics specifically, the need for support has been highlighted repeatedly in order to make changes if we hope to encourage the use of reform-based strategies in classrooms (Ball & Cohen, 1999; Brahier & Schäffner, 2004; Cohen, 1990; West & Curcio, 2004). Towers (2012) discovered that support was important to keep new teachers from returning to direct instruction approaches when they encountered difficulties or stress. If reform requires more support for teachers, the professional learning group could address this deficit by providing this supportive element embedded in a teacher's daily environment. "When a teacher receives positive and constructive feedback from a respected peer, there is even greater potential for enhanced goal setting, motivation to take risks, and implementation of challenging teaching strategies" (Bruce & Ross, 2008, p. 348). This relates to social constructivism and the social aspects of emotions that were explored by DiPardo and Potter (2003), where the authors explored the critical nature of teachers needing supportive peers in order to successfully navigate the stressful nature of teaching and to prevent burnout. Isolation has also been cited elsewhere as a problem with the teaching profession (DuFour & Eaker, 1998; Schmoker, 2006), so creating supporting environments that allow for collaboration can bring positive changes to the teaching profession.

Why Professional Learning Groups?

Professional learning groups have the potential to influence dramatic changes in the teaching profession partially because its structure fits into the learning theory of social constructivism. According to O'Donnell, D'Amico, Schmid, Reeve, and Smith (2008), "social constructivist and sociocultural theories of human learning emphasise (a) social participation, (b) authentic tasks in which learning is embedded, and (c) tools to support learning. Both theories place special emphasis on social participation" (p. 264). By examining the social constructivist theory, it is clear that professional learning groups have the potential to address all three of these characteristics of learning. The nature of the group of teachers gathering together to collaborate necessitates social participation for members to learn from the group. The learning of the group is based on school needs and the personal environments of the teachers, so learning is potentially based in authentic tasks that relate to the individual situations. Finally, the teachers could be working together to create tools to support student learning in order to address the needs of the group, as well as engaging in activities to support their own learning. The potential strength of the professional learning group system rises from the structure fitting in the social constructivist theory of learning.

Professional learning group research acknowledges that the teachers meet with a focus on student learning (DuFour & Eaker, 1998; Hall & Hord, 2006; Loughridge & Tarantino, 2005) by discussing individual teaching contexts. This allows teachers to explore and diagnose difficulties within their own environments with the goal of increasing student success. Research has supported the notion that having teachers work together has an impact on student achievement (Hofman & Dijkstra, 2010) and on the classroom (Andrews & Lewis, 2002). The Ontario Ministry of Education (2007) notes that a professional group is one where teachers "engage in processes of inquiry and learning focused on improving student achievement. Through classroom, school, and large-scale assessments, members identify the strengths and needs of a group of students and determine the knowledge and skills required to close the achievement gap" (p. 2). Professional learning groups also have benefits for the teachers involved. Research has shown that professional learning groups can lead to increased confidence for the professionals involved (Arbaugh, 2003). In another study, teachers responded positively about the impact the community changes were having on their practices (Eaker, DuFour, & Burnette, 2002a).

Many studies reviewed herein have examined how effective professional learning groups are in terms of addressing student needs. However, professional learning groups are not always implemented as intended (Hall & Hord, 2006; Lave, 2008). Some hold the power to encourage changes (Andrews & Lewis, 2002; Brahier & Schäffner, 2004; DuFour & Eaker, 1998; Eaker et al., 2002b; Hord & Sommers, 2008), yet all of the studies mention how essential it is that more research is done on the effectiveness of these types of groups. Opfer and Pedder (2011) question how some groups have the characteristics of effective professional learning and no changes happen, and other do not have the characteristics, yet growth is shown. In order to address the perceived need for more research studies, I specifically examined “What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching practices?”

DuFour and Eaker (1998) describe additional characteristics of a professional learning group that I believed were critical in differentiating the effectiveness of mathematics professional learning groups and would help to answer my research question. The three additional characteristics identified by DuFour and Eaker (1998) are “action orientation and experimentation”, “continuous improvement”, and “results orientation” (p. 27-29). First, teachers would learn from both positive and negative experiences and seek growth (DuFour & Eaker, 1998). Second, by identifying professional learning groups as ones that are continuously striving for improvement, DuFour and Eaker (1998) characterise these groups as ones that utilise tenets of action research within their communities. “Action research aims to design inquiry and build knowledge for use in the service of action to solve practical problems” (Punch, 2009, p. 136). As such, these groups would be based in testing and reflecting on research-based strategies (Eaker, 2002; Eaker et al., 2002b; Hord & Sommers, 2008; Gojmerac & Cherubini, 2012). Teachers would be constantly looking for ways to improve their practices (DuFour & Eaker, 1998). The collaboration should combine teacher needs and beliefs with the theories of researchers (Bednarz et al., 2007). This search would take mathematics teachers beyond creating new worksheets for their students and encourage them to attempt new practices that would hold benefits for their students' learning. Finally, teachers in the professional learning group would be focused on results in order to encourage growth and changes (DuFour, 2002; Schmoker, 2006). "School effectiveness should be assessed on the basis of results rather than intentions" (DuFour, 2002, p. 43). In discussing dissemination of educational research, Saha (2009) advocates for schools to create an action research culture in order for new strategies based in research to make their way into classrooms. The Ontario Ministry of Education (2007) maintains that by focusing on results, learning continues to be at the centre, and that teachers must become reflective of their teaching and consider student achievement when making instructional decisions. Examining tangible results of student outcomes ensures teachers keep students in the forefront of the discourse.

Foundational Concepts for *Standards*-Based Education

Standards-based education is firmly rooted in constructivism. There is disagreement on whether reform-based pedagogy falls under social constructivism (Palincsar, 1998) or psychological constructivism (Richardson, 2003). For me, the very basis of mathematics reform is the social relationships that are created in the classroom in order to support student learning. Reform-based classrooms are constructivist in how students work with peers and teachers to develop concepts and new ideas through explorations. The stress of creating meaning is based on the experiences in social situations (Brown & Palincsar, 1989). "If pupils discuss with others what new ideas mean to them, further thinking is generated with more complex links between ideas afforded" (Bartlett & Burton, 2007, p. 126-7). In this way, "students co-construct their knowledge through collaboration on meaningful tasks" (Harkness, 2009, p. 248). In a classroom based in the tenets of constructivism, teachers would provide meaningful experiences for students to work together and discuss strategies. Vygotsky (1962) argues against a traditional method of teaching where the teacher would directly impart the knowledge to the students or enforce drills. As such, reform-based pedagogy focuses on having students explore mathematics in order to push thinking as opposed to having them memorise procedures and facts given by their teachers.

Vygotsky (1962) notes that students have a zone of proximal development and cautions that education should be aimed at pushing students forward in their learning and not simply targeting where they currently are. The use of reform-based education would also fulfill this tenet of constructivism by having students struggle with complex mathematical ideas to determine a solution within their own frame of reference. Vygotsky furthers acknowledges the need for scaffolding in order to support students as they continue to grow and learn. It is the combination of the concrete foundation and the strong supportive structure that allows new learning to be built because constructivism focuses on students building knowledge by engaging with activities or concepts (O'Donnell et al., 2008). Brown and Palincsar (1989) suggest "reciprocal teaching" (p. 394) to be used in a mathematics classroom in order to improve student learning. To them, this method of teaching combines ideas of group work in order to create a learning environment that capitalises on constructivism as well as the social aspects necessary for learning. Creating a classroom community and problem-based learning are teaching strategies in keeping with the ideals of social constructivism (O'Donnell et al., 2008). To social constructivists, student learning is enhanced by this social community which is an important tenet in *Standards*-based education. With the tenets of constructivism serving as a foundation for reform-based pedagogy, the works of Krutetskii (1976), Skemp (1986), and Papert (1993) build their visions of mathematics classrooms within this framework. To follow, the specifics of their works will be expanded to illustrate the evolution of mathematics education into a constructivist pedagogy for teaching and learning.

Mathematics can be found in the nature surrounding students and can open a beautiful world of numbers and patterns to those who can see it. School mathematics on the other hand is said to not pay tribute to the beauty and depth of ‘real’ mathematics, instead focusing on sums and calculations to learn before handling more complicated tasks (Papert, 1993). As such, students have developed “math phobias” that cause them to assume that they are incapable of doing mathematics because of their inability to do long lists of sums that are often required in schools (Papert, 1993; Skemp, 1986). As Papert (1993) points out, oftentimes student learning is “severely hampered by entrenched negative beliefs about their [own] capacities” (p. 42). Skemp (1986) goes further by providing hope that effective teaching, especially at early ages, can reduce this student anxiety and foster an enjoyment for mathematics that will carry students through life. “Children begin their lives as eager and competent learners. They have to *learn* to have trouble with learning in general and mathematics in particular” (Papert, 1993, p. 40). Starting students off with a firm foundation and enjoyment of mathematics can have long-reaching effects, quite the opposite of which can be caused through inappropriate early experiences. In exploring some major works that have helped shape ideas on mathematics, it is clear that many changes still need to occur in school settings, yet these works lay the foundation for today’s standards-based ideals. It is these foundational ideas that also laid the groundwork for determining my research question “In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?”

Krutetskii (1976) studied gifted schoolchildren and what is needed for them to successfully learn mathematics. In his work, Krutetskii differentiates between school mathematics and ‘real’ mathematics, as done by mathematicians, as two separate entities with intersecting regions. He also pointed out the need for positive experiences with good teachers to foster strong inclinations in inspiring students to study mathematics. Krutetskii does believe everyone is capable of learning mathematics, so experiences with capable teachers are crucial to allow for student success. Krutetskii believed that “the emotions a person feels are an important factor in the development of abilities in any activity including mathematics” (p. 347), so positive experiences in school mathematics are vital for supporting students to increase their mathematical abilities. He notes a cycle between students who have positive inclinations towards mathematics causes them to use the abilities, which leads to greater achievement and in turn causes students to use the abilities more often to gain more successes.

According to Skemp (1986), the important ideal in mathematics learning is that it is a scaffolded process: students cannot continue building higher structures on a faulty foundation, which also adheres to the work on child development of Vygotsky (1962). As mentioned previously, Vygotsky advocates against drilling students to memorise facts and procedures. Memorisation of generalised rules is also not advocated as being sufficient for an understanding of mathematics. Skemp (1986) identifies these differences as teaching students a “short cut” versus learning a “meaningful method” (p. 55). In discussing classroom learning, Skemp (1986) talks about “two kinds of learning which we may call habit learning, or rote-memorising, and learning involving understanding, which is to say intelligent learning” (Skemp, 1986, p. 15). It is “intelligent learning” that is promoted and should be stressed in mathematics in school situations. When students are simply asked to memorise information, “it is a *dissociated* model” because the subject matter is not given any value (Papert, 1993, p. 47, emphasis in original). Not only is the material not meaningful to students, but it is cautioned that “unconnected rules are much harder to remember than an integrated conceptual structure" (Skemp, 1986, p. 29-30). Students should learn the reasoning behind the rules while linking new information to previously learned knowledge structures instead of just formulas that are only a means to an answer.

New concepts are introduced with examples and not simply definitions (Skemp, 1986) as is sometimes the case in mathematics textbooks. Teachers would need to have an understanding of mathematics that would allow them to choose the appropriate examples for students (Skemp, 1986). Elementary processes first need to be understood in order to eventually gain automaticity instead of simply being able to memorise and "regurgitate" something given. Automaticity reduces the strain on working memory by keeping mathematical ideas in long-term memory to be recalled when needed to perform tasks (Skemp, 1986). This idea links back to mathematics being a scaffold: the base of the structure must be secure and "learned" prior to building more difficult concepts on top. Given the importance of experiences in mathematics, the emphasis should be on employing teachers who both understand and enjoy mathematics to inspire students to think about mathematics in a way that is markedly different from those presented by some of today's elementary classrooms.

Skemp (1986) further believes in the beauty of mathematics and proposes that school mathematics does not support this ideal. Skemp deals with encouraging the creation of schemas for understanding mathematical concepts as opposed to simply memorising procedures and skills in order for students to recall and use information as needed. Schemas created by students need to be rebuilt and cannot just be thrown away. Since this is a very difficult task, it is important that students are guided to create appropriate schemas from the beginning. Students are given the tools to fit their own solution methods to their created schemas thus giving their new knowledge strength. Skemp further advocates that the role of the mathematics classroom is not just to teach mathematics but to teach students how to “learn mathematics” (p. 50). This in itself is a reconceptualization of a traditional mathematics classroom.

Proposed changes in mathematics classrooms have been advocated for by mathematics researchers in order to facilitate understanding and not rote learning in classrooms (e.g., NCTM, 2000; Sawyer, 2004). Papert (1993) proposes a reconceptualization of what school mathematics really is and suggests that with the advent of calculators and personal computers, the need for learning long division and other such calculations has become obsolete. He argues that simply looking at the old curriculum and adapting it to make way for new ideas is simply “a commitment to preserving the traditional system” (p. 44). Through the use of the LOGO program, Papert demonstrates how students who have never been particularly successful at mathematics can succeed and truly enjoy the subject. His model hinges on the idea that school should be a shared learning environment with teachers no longer being the sole carrier of knowledge.

I believe the work of past mathematics researchers addresses the question: should school mathematics really be so markedly different from ‘real’ mathematics if mathematics is a part of daily life? If Krutetskii (1976) is truly accurate in that every student can learn mathematics, would it not make more sense for school mathematics to fall more in line with ‘real’ mathematics? According to Papert (1993), “a dignified mathematics for children cannot be something we permit to inflict on children, like unpleasant medicine, although we see no reason to take it ourselves” (p. 54). These ideas serve as the foundation for proposed changes in the mathematics classroom. Although ideas such as these have been around for decades, evidence shows they still have not been put fully into practice (Boaler, 2000; McNeal & Simon, 2000; Stigler & Hiebert, 2004). The reform movement advocates a restructuring of the mathematics curriculum to provide a mathematics classroom that is not "inflicted" on students, so professional development must be considered in order to make changes. Teachers need to be educated in negotiating classrooms that allow for greater flexibility and more freedom, yet they need the mathematics knowledge and skills to allow them to see student misconceptions. Students require a mathematics classroom that will best fit their needs and help keep them from encountering 'math phobias' that develop from years of learning that they just cannot succeed at "school mathematics". Based in constructivism, the ideals of Krutetskii (1976), Skemp (1986), and Papert (1993) formed the foundation for what is now being termed *Standards*-based or reform-oriented curriculum.

Defining *Standards*-Based Education

Principles and Standards for School Mathematics (NCTM, 2000) reinforces the idea of using problem solving by emphasizing the use of exploration in the classroom and is the basis of what is being used today to describe what has been referred to as "standards-based" mathematics, a constructivist approach to teaching mathematics. Prior to the release of *Principles and Standards*, the NCTM released three other documents that were the initial attempt to reform the mathematics curriculum. The first was *Curriculum and Evaluation Standards for School Mathematics* (Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989). This document presented a set of standards that should be addressed in curriculum and were divided by grade level clusters. *Curriculum and Evaluation Standards* was based on the "consensus that all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised" (p. 1). The goal of the Working Group of the Commission on Standards was to create standards that would protect students from bad teaching practices as well as represent the current societal shift towards a technology focus. This focus was on ensuring students gained "mathematical power" in their classroom experiences: "an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonrountine problems" (p. 5).

In 1991, the Working Group of the Commission on Standards produced *Professional Standards for Teaching Mathematics* which was used to further define and clarify the 1989 publication. "This document spells out what teachers need to know to teach toward new goals for mathematics education and how teaching should be evaluated for the purpose of improvement" (p. vii). The publication further defines the shifts in mathematics teaching set out in *Curriculum and Evaluation Standards* but focuses on aspects of teacher pedagogy and professional development. *Professional Standards* laid out the five shifts in the environment that were set up in the previous publication:

> toward classroom as mathematical communities—away from classrooms as simply a collection of individuals; toward logic and mathematical evidence as verification—away from the teacher as sole authority for right answers; toward mathematical reasoning—away from merely memorizing procedures; toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding; toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolating concepts and procedures. (p. 3)

The purpose of *Professional Standards* was to illustrate standards for teaching mathematics that would move toward effective teaching practices as well as evaluating those practices.

The final document was *Assessment Standards for School Mathematics* (Assessment Standards Working Groups of the National Council of Teachers of Mathematics, 1995), which was also meant to further define reform practices in mathematics and be used in conjunction with the previous documents. This document focused on assessment practices that would move away from ranking students based on numeric grades in order to align with the practices laid out in *Curriculum and Evaluation Standards. Assessment Standards* defines assessment as "*the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and the making of inferences from that evidence for a variety of purposes*" (p. 3, italics in original). The publication was meant to be used as a "guide" for changing assessment not a "how-to" (p. 3) and sets out six assessment standards: mathematics, learning, equity, openness, inferences, and coherence. These three publications all produced by the NCTM, and reviewed by multiple classroom teachers, mathematics educators, and other stakeholders, were the initial attempts to define standards-based education and revolutionise mathematics classrooms.

The basis of the standards-based curriculum is problem solving, yet using problem solving in a mathematics classroom has evoked many different definitions of how to effectively implement problem solving lessons. While some teachers support the vision of problem solving espoused by the NCTM (2000) as "engaging in a task for which the solution method is not known in advance" (p. 51), others view it as something to be done *after* students are *taught* and *only if there is time* (Holm, 2009; Kajander & Mason, 2007). Effective problem solving, though, has come to be regarded as the learning ideal of exploration embraced by the NCTM. A classroom in which problem solving is used effectively allows students to explore a problem or task without the teacher first supplying students with the method required to solve the problem (Hiebert & Wearne, 1993). Furthermore, the mathematics of the problem allows students an individualised solution method. The students in this classroom change "from passive listener to active explainer, collaborator, or problem solver and [encourages] multiple approaches to problems" (Turner et al., 2011, p. 740) which is the basis of the constructivist theory of learning. This approach differs from traditional uses of problem solving where a teacher presents a problem, shows the students a method to solve the problem, and then allows the students to practice this specified method on several sample problems. In other words, effective problem solving should be more than having students solve a problem using formulas or methods the teacher has previously shown. Not giving teacher-generated formulas or methods is the most effective way to use problem solving in mathematics classrooms (Bay-Williams & Meyer, 2005; Boaler & Humphreys, 2005; Buschman, 2004), and in general it is a more effective method for teaching mathematics than traditional methods (Askey, 1999; Lobato, Clarke, & Ellis, 2005; NCTM, 2000; Riordan & Noyce, 2001; Van de Walle & Lovin, 2006; Wilson, Peterson, Ball, & Cohen, 1996). Furthermore these classrooms would encourage students to engage in discussions about their solution methods (Kazemi & Stipek, 2001; Schleppenbach, Perry, Miller, Sims, & Fang, 2007; Whitenack & Yackel, 2002). Providing a problem to students is not enough by itself, and students must engage in discourse about the process and examine each other's methods. This discourse can help identify gaps in understanding (Schleppenbach et al., 2007) and consider new methods while working to generalize understandings.

Typically, the reform-based classroom has students developing skills in mathematics through the use of manipulatives and other tools. The reform-based mathematics classroom is also referred to as "*inquiry mathematics*" (McNeal & Simon, 2000, p. 475) because students are exploring and creating their own understandings or algorithms in the mathematics classroom. Students would be engaged in discussions with each other to solve problems by learning from their peers. Ball and Bass (2000) maintain that in mathematics classrooms, the learning is not social or individual, but a mathematics specific form of constructivism. Students would be constructing understandings through engaging with the activities and interacting with others which supports how social constructivists believe knowledge is constructed (Cobb, 2005). Mathematical terms would also be constructed and not just memorised because a teacher has told students to remember it (Ball & Bass, 2000). These ideas would represent a shift in the role of the teacher from direct instructor and sole owner of the knowledge to that of facilitator and coach. Teachers would shift from asking “known answer” questions to those that illicit knowledge and understanding based on the discoveries students make during the lesson (Sawyer, 2004, p. 14). Inoue (2011) found that although the studied teachers struggled with this shift in roles, they found it to be beneficial for their students. Since teachers were the concern of my research, how the participants handle this shift could be important. Research indicates that teachers should take the time to focus on what the students understand about the topic by exploring their solutions and addressing student errors (Harkness, 2009). In order to advance the use of inquiry mathematics, I pursued the answer to “In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?”

In order for teachers to allow students to explore problem solving, a classroom teacher needs to believe that students can find the solution methods for themselves, as well as have the knowledge needed to support students mathematically. Teaching in a constructivist classroom requires a deep and strong knowledge of the content area (Hill, 2010; Richardson, 2003), as well as a knowledge of pedagogy (Steele, 2005). Questions have been raised about whether or not elementary teachers, who are generalists, would be able to gain the needed content knowledge in all subject areas (Richardson, 2003; Wu, 2009). Ball et al. (2005) note that in order to implement the reform curriculum as it is intended, teachers need to understand not only mathematics but also the curriculum. Knowledge of mathematics has an effect on pedagogical choices, so next I discuss beliefs and knowledge specific to mathematics and how these factors interact in making classroom decisions.

Defining Beliefs and Knowledge

Philipp (2007) notes that "beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). As such, beliefs are "situated" and specific to the teacher and student interactions (p. 274). Philipp comments that teachers can also hold beliefs that run contrary to the methods they employ within their classrooms. If teachers have experience only with traditional mathematics education and hold true to that vision, somehow they need to become open to more reform-based methodologies and new pedagogies vastly different from their own experiences (McNeal & Simon, 2000). Researchers suggest that past efforts to change teacher practices have failed partially because of not accounting for the beliefs of the teachers affected (Grant et al., 1994; Handal & Herrington, 2003). To address the importance of beliefs in determining the success of a reform effort, I examined "What are the beliefs about mathematics teaching and learning of the individual teachers in the group? How are these beliefs dealt with in the discussions of the group?"

Within every classroom, norms and standard practices govern the acceptable behaviour of students, and these norms are drawn from what the teacher in the classroom believes is important for learning, as well as the priorities of the school. For example, a teacher who believes students learn best when sitting at their desks working quietly would likely place value on students being able to complete tasks quietly and independently. Reform-based mathematics practices, which rely on students participating in activities, would be unlikely to occur in this type of teacher's classroom. On the other hand, a teacher supporting reform-based pedagogy would allow for discussion and exploration, believing that students learn more by experiencing mathematics and talking about their ideas. Students in this type of classroom might be moving around the room and would be encouraged to be vocal in discussions within groups and with the entire class.

Also to be considered is the beliefs of others in the school environment and how conducive they are to these changes. Handal and Herrington (2003) state that "in the reality of today's school climate, students resist unfamiliar approaches, administrators do not provide adequate support either in professional training or in resource materials and they dislike less-orderly classrooms" (p. 63). Clarke (1997) agrees with this concern by identifying a lack of support of administration and the larger community as reasons for teachers not adopting reform-based pedagogy. Gresalfi and Cobb (2011) found there could be a tension between what a teacher believes is best practices and following the school mandate simply because it is what the teacher feels is what must be done. Gresalfi and Cobb felt that this conflict needs to be considered in professional development experiences. Since the entire community would be working together and supporting one another to address difficulties, creating a professional learning group could help to mitigate these factors. While using reform-based mathematics pedagogy is supported by both the NCTM (2000; Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989), as well as the Ontario Ministry of Education (2005; Ministry of Education and Training, 1997), that does not mean that they are universally taken up; teachers must *believe* that these methods work in order to appropriately enact them in their own classrooms, and they must also have sufficient knowledge of mathematics to support student explorations.

Knowledge of mathematical content for teachers goes beyond simply being able to solve mathematical problems. The mathematics knowledge needed by teachers has been termed many things but the foundational ideas of Shulman (1986) in discussing pedagogical content knowledge serves as the basis for more modern terms. In today's research, this mathematical knowledge has been termed mathematical knowledge for teaching (Ball et al., 2008; Silverman & Thompson, 2008), pedagogical content knowledge (Baumert et al., 2010), and profound understanding of fundamental mathematics (Ma, 1999), to name a few. Although researchers may use different names for this essential knowledge, the underpinnings of the ideas are similar.

Teachers need to deeply understand mathematics, see connections among mathematical ideas, have knowledge of mathematical pedagogy, and be able to break down all the concepts with students in order to teach mathematics (Ma, 1999). As such mathematics for teaching goes beyond simply knowing the procedures and curriculum areas in mathematics (Ball, Hill, & Bass, 2005). Baumert et al. (2010) discuss three aspects of pedagogical content knowledge: knowledge of mathematics instruction; knowledge of students' understandings, prior knowledge, and experiences; and being able to connect mathematics and construct multiple solution paths. Baumert et al. further espouse that pedagogical content knowledge is an extension of content knowledge of mathematics and is absolutely essential to effective mathematics teaching. Chamberlin, Farmer, and Novak (2008) argue that "some of this specialized knowledge is mathematical in nature, including knowing alternative algorithms for solving problems, being able to illustrate and model mathematical ideas with diagrams and manipulatives, and knowing why and how mathematical rules work in addition to being able to apply them correctly" (p. 441). Hill (2010) adds that this knowledge includes analysing errors in student work as well as being able to explain definitions in mathematics in a grade appropriate manner. Bray (2011) discovered that teachers with weaker knowledge of mathematics had difficulty identifying student errors when they were presented. The content a teacher needs is something that goes beyond what any non-teacher studying mathematics would need (Ball et al., 2005; Baumert et al., 2010; Hill, 2010; Ma, 1999). The content knowledge needed by teachers also differs from the knowledge a student would need or gain during a typical classroom learning experience (Chamberlin et al., 2008; Davis & Simmt, 2006; Kajander, 2010).

Recent research in the field has been exploring how there are two views on mathematics knowledge for teaching in the field (Lee & Shin, 2012). Ball, Thames, and Phelps (2008) addresses mathematical knowledge needed for teaching as conceptual mathematics that teachers use as they are supporting students, which is separate from knowledge of the curriculum and pedagogy. On the other hand, Silverman and Thompson (2008) claim that mathematics knowledge for teaching only becomes the knowledge needed by teachers when understandings of content are linked to pedagogical knowledge. An example of this intersection with pedagogical knowledge is "understanding why a student may arrive at a particular answer or knowing different instructional approaches for demonstrating a mathematical concept" (Chamberlin et al., 2008, p. 441). Teachers would also need to be able to identify how to base mathematical lessons on the knowledge students already possess in order to bring students towards the lesson goals (Baumert et al., 2010; Silverman & Thompson, 2008). As such, the understandings needed by teachers are complex and varied. Teachers would need mathematical understanding and reasoning to facilitate discussions (Ball & Bass, 2000). Teachers would also need to be able to examine student work and respond to both correct and incorrect solutions (Ball et al., 2005). In order to understand the effects knowledge had on discussions of reform, in my research I examined "In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?" My research was supported by the views taken by Silverman and Thompson (2008) that the specialised content needed for effective teachers is an intertwined knowledge of mathematics *and* teaching. As such, I felt it was important to my research to examine both mathematics concepts discussed as well as knowledge about teaching as a field because they are so intertwined and have an impact on exploring mathematics teaching.

Categories of Teacher Knowledge

Knowledge-Practice Relationship	Knowledge-*for*-practice	Knowledge-*in*-practice	Knowledge-*of*-practice
Images of knowledge			
Images of teachers, teaching, and professional practice			
Images of teacher learning and teachers' roles in educational change			
Current initiatives in teacher education, professional development, and/or teacher assessment			

Figure 2. Framework of teacher learning proposed by Cochran-Smith and Lytle (1998).

Cochran-Smith and Lytle (1998) determined that there are three "conceptions of teacher learning": "knowledge-*for*-practice", "knowledge-*in*-practice", and "knowledge-*of*-practice" (p. 250). Knowledge-for-practice consists of "formal knowledge and theory…*for* teachers to use in order to improve practice" (p. 250, emphasis in original). Knowledge-in-practice is the knowledge that "is embedded in practice and in teachers' reflections on practice" (p. 250).

Knowledge-of-practice "is assumed that the knowledge teachers need to teach well is generated when teachers treat their own classrooms and schools as sites for intentional investigation at the same time that they treat the knowledge and theory produced by others as generative material for interrogation and interpretation" (p. 250). Using the three types of teacher knowledge and the different areas of teaching, Cochran-Smith and Lytle (1998) have generated a framework (see Figure 2) that can be used to examine the knowledge of teaching presented within the discussions to ensure a depth of discussion that is important in professional learning groups.

Addressing Beliefs in Practice

Recent research suggests that a teacher's beliefs have the strongest influence on a teacher's practice (Wilkins, 2008). Since a teacher must believe in the effectiveness of methods being used in the classroom, it is necessary to examine how beliefs determine the chosen pedagogy. Cross (2009) observed teachers in their own classrooms in order to examine both their beliefs about mathematics and how they taught mathematics. Her findings clearly showed examples of how teachers chose lessons and how they dealt with student misunderstandings based on the teachers' beliefs about what is effective in mathematics learning. She discovered that some teachers in her study, when confronted with a lack of student understanding, determined that the best course of action was for the teacher to give better explanations to the students. Simply giving students better explanations would be problematic because it would take the challenge out of the exploration by giving them the answers and telling them what to do. This would also contradict the spirit of the reform-based pedagogy. Teachers who valued student exploration, on the other hand, might treat student misunderstandings as a chance for more exploration and discussion. Grant et al. (1994) determined that those who believed in more procedurally oriented mathematics believed that the teacher is solely in charge of how and what is taught. They also discovered that those who believed that mathematics should focus more on the big ideas than procedures, also believed in environments that the teachers set up to allow the students time to explore.

Teachers who believe in using reform-based methods use these methods more in their classrooms (Wilkins, 2008). Bruce and Ross (2008) determined that beliefs about the influence of teaching practices on students also affected the strategies attempted by teachers. Beswick (2012) found that a teacher who had not realigned beliefs about mathematics experienced difficulties when attempting to use a problem solving approach, when the method contrasted with previous mathematics experiences. Again this highlights the importance of not only encouraging teachers to use new strategies for teaching mathematics, but also to confront their own beliefs about teaching. Bray (2011) discusses how mathematics teachers need time during professional development "to critically examine traditional mathematics teaching practices and assumptions about student learning in order to inspire recognition of the need for alternative mathematics teaching practices and to initiate changes in beliefs" (p. 35). Baumert et al. (2010) discuss using "cognitively activating tasks" (p. 145) in classrooms and how the treatment of them, based on teacher beliefs can make them less cognitively stimulating. For example, they talk about the teacher just giving validation for a solution versus "encouraging students to evaluate the validity of their solutions for themselves or to try out multiple solution paths" (p. 145). External conflicts also play a part in determining a teacher's beliefs. Turner et al. (2011) note that a desire to increase students' test scores had an impact on the changes implemented by the teachers.

Since beliefs impact practice (Beswick, 2012; Potari & Georgiadou-Kabourdis, 2009), professional development needs to include a focus on teacher beliefs, allowing teachers to examine their own beliefs if there are to be changes in classroom practices (Cross, 2009; Wilkins, 2008). Through examining teacher conversations, my research purpose was to identify *how* this may be achieved during professional learning groups. “When practicing teachers have opportunities to reflect upon innovative reform-oriented curricula they are using, upon their own students’ mathematical thinking, or upon other aspects of their practices, their beliefs and practices change” (Philipp, 2007, p. 309). Grant et al. (1994) determined that a teacher’s beliefs impacted how he or she even viewed learning about new instructional strategies. Battey and Franke (2008) extended this idea to professional development, finding that teachers would fit the new learning into their existing practices and make determinations about what would or would not work based on their existing values.

Shulman and Shulman (2004) found that the community a teacher is involved in has an impact on both the teacher’s beliefs and practices; therefore I asserted that the professional learning group could have an important role in changes. The assertion in the literature is that in order to change teaching practices, beliefs about effective teaching must first be confronted (Cross, 2009). To be effective in promoting teacher growth and development, discussions about beliefs in mathematics practice must occur during the professional learning group, since “teachers’ beliefs about the teaching and learning [of] mathematics are critical in determining the pace of curriculum reform” (Handal & Herrington, 2003, p. 59). Battey and Franke (2008) found that a teacher who held traditional beliefs about teaching mathematics that these beliefs had an impact on the use of problem solving in the classroom by still stressing a single correct solution path. This plays a role in professional development because beliefs need to be explored since they have an effect on the way new ideas will be implemented.

Grootenboer (2008) examined beliefs of prospective teachers during their pre-service program in order to make conclusions about how resistant to change beliefs are. He found that prospective teachers fell into three categories: those who did not really engage in the reflection of their beliefs but tended to react as they felt they were supposed to react, those who came away believing that there is a difference between university mathematics and that which they would use in a classroom, and those who were beginning to change their beliefs after careful consideration. The study shows clearly that although all the pre-service teachers were exposed to the same university curriculum, their beliefs about mathematics changed in different ways, thus showing the difficulty of encouraging changes to classroom implementation in mathematics. Grootenboer (2008) notes, "It was clear through the study that the participants' beliefs were a significant factor in their developing understanding of mathematics teaching and learning" (p. 493). If beliefs have an influence on the understanding teachers gain about mathematics, it is necessary for effective professional development to have teachers confront their own beliefs in order to enact influence in changing mathematical understandings. Opfer and Pedder (2011) suggest that professional learning is not as cause and effect as some research seems, so research requires looking at beliefs and how they impact teacher learning. Teachers' beliefs about mathematics affect their understandings of mathematics (Grootenboer, 2008; Wilkins, 2008), and teacher understandings, in turn, affect pedagogical choices made in a classroom.

Intersections of Beliefs and Knowledge

In conducting a large-scale study of teachers, Wilkins (2008) found that teachers with higher levels of mathematical content knowledge tended to use less inquiry-based methods in their classroom. To Wilkins (2008), this led back to teacher beliefs: the idea that because the old method worked for them, they did not see the point in trying something new. If Wilkins' findings hold true, then teachers with weaker understandings of mathematics might in fact be the ones who would be more open to attempting to teach in a more reform-based manner. However, teachers with weaker understandings of mathematics might face major challenges as they strive to provide effective support for students. If facilitating student understanding is dependent on a teacher's understanding of mathematics (Ma, 1999), then supporting students in mathematical explorations necessitates having a teacher deeply understand mathematics in order to provide the most benefit for students. In addition, "a teacher's actions during mathematics instruction are simultaneously shaped by her [sic] knowledge and beliefs, with varying weight being given to particular types of knowledge or beliefs in different situations" (Bray, 2011, p. 4). Thus, there is interdependency between teacher knowledge and beliefs. Research indicates that teacher learning must focus on developing teacher beliefs as well as their knowledge in order to make it effective (Beswick, 2012; Wilkins, 2008). Thus difficulties potentially arise in focusing on either beliefs or knowledge to the exclusion of the other in attempting to effect changes in teaching.

Since mathematics knowledge for teaching is not "just" a knowledge of subject matter (Baumert et al., 2010; Kajander, 2010; Silverman & Thompson, 2008), it is important that teacher knowledge includes a knowledge of students and of teaching mathematics. As teachers develop in their profession, this knowledge of students and teaching improves and grows (Potari & Georgiadou-Kabouridis, 2009). As Potari and Georgiadou-Kabouridis discovered, although a teacher may believe in more reform-based or exploratory ways of learning, a lack of knowledge impedes being able to fully implement the strategies. While reflection may at times help deepen understanding, McDuffie (2004) noted, in studying pre-service teachers, that "limits in pedagogical content knowledge and lack of confidence impede the pre-service teachers' reflection while in the act of teaching" (p. 33). In studying two of her students, McDuffie (2004) found that the teacher without strong mathematics knowledge was unable to make changes during the lesson when encountering misconceptions, often reverting to just telling students what to do. Battey and Franke (2008) echoed this concern by finding that even though a teacher believed in allowing students to find multiple solutions paths, could not fully discuss the solutions with students due to the teacher's own lack of knowledge.

Some studies concentrate on the difficulties that can arise while addressing the mathematics knowledge needed by teachers. Chamberlin et al. (2008) examined in-service teachers participating in professional development who had to be assessed and evaluated on mathematics knowledge because of legislation mandates. They noted the limited early experiences of their studied teachers in mathematics and mentioned the teachers were "somewhat cautious about learning mathematics" (p. 436). These early experiences then had an effect on how the teachers felt entering the professional development and how they responded to assessments of their knowledge within the program. The researchers wanted teachers to reflect both on their own experiences in learning mathematics and how it might benefit their practice as well as deepen their knowledge. Simply ignoring how the teachers felt about mathematics (based on their past experiences) would not have made the professional development as effective, further illustrating how interconnected knowledge about mathematics and beliefs are. Using professional learning groups in mathematics has been suggested to address both teacher content knowledge and beliefs, yet prior research has not amply discussed how this can be enacted.

Student Achievement

Although not a specific outcome of my research, the effects of knowledge and reform-based strategies on student learning have been documented in the literature. The goal of a professional learning group is to improve student achievement (Loughridge & Tarantino, 2005), so student success is indirectly important to my research. Hofman and Dijkstra (2010) found that teachers working together had a positive impact on student achievement. Stigler and Hiebert (2004) have noted that improving teaching methods is linked to increased student learning. Ball and Rowan (2004) note that “it is increasingly clear that instructional quality affects what students learn in school and how they grow over time” (p. 3). As such professional development efforts must focus on improving the quality of teaching methods in order for greater student success. In mathematics specifically, using more reform-based methods is more effective than traditional forms of instruction (Askey, 1999; NCTM, 2000; Van de Walle & Lovin, 2006; Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989), so changing teaching pedagogy would increase student success in mathematics. Riordan and Noyce (2001) determined that none of the traditional classrooms outperformed the reform-based classroom programs in their study, and they saw greater achievement in the classrooms who had used the strategies for longer. As Baumert et al. (2010) determined, “PCK [pedagogical content knowledge] largely determines the cognitive structure of mathematical learning opportunities” (p. 166). They further note that simply having content knowledge of mathematics is not as strong of an indicator. According to Baumert et al. (2010), “higher teacher *qualifications* tend to be associated with better student performance at secondary level, particularly in mathematics” (p. 137). These findings show that teachers with a deeper knowledge of mathematics had higher levels of student achievement.

In analyzing the student achievement based on the School Achievement Indicators Program, Anderson et al. (2006) attempt to determine the variables that affect student performance in schools. Although they note that variables outside of the school control do have an impact, they found a weak positive correlation between instructional practices and student success. Rogers et al. (2006) examined Alberta's achievement tests and discovered that students who spent more time working alone or in small groups outperformed students who encountered direct instruction in their classrooms. This finding further supports the use of reform-based teaching practices in order to improve student success. Rogers et al. also discovered that student confidence in their mathematics ability was linked to higher achievement scores. Kozlow (2012) seconded this finding, discovering that students who were confident in their abilities as well as enjoyed mathematics were more likely to meet the Ministry standard. Both studies support the conclusions drawn by Krutetskii (1976) about positive experiences in mathematics having an impact on their performance in mathematics. Research into mathematics classrooms indicates that positive feelings toward mathematics, as well as changes in those classrooms, has an impact on student performance, so conclusions can be drawn that professional learning groups supporting those changes would have an effect on student success.

Previous Research on Mathematics Professional Learning Groups

Arbaugh (2003) begins by highlighting the need to move away from the one shot professional development model into something more effective. The "study group" examined by Arbaugh consisted of a group of teachers with the researcher serving as facilitator and an active member of the group. The researcher mentioned that the end of each meeting consisted of setting up the schedule for the next meeting so that the researcher and the department head could organise and plan activities. This deviates from the shared and supportive leadership (Hord & Sommers, 2008) characteristic of professional learning groups. Arbaugh (2003) discusses that each meeting consisted of two to four discussion topics. The group felt that "it provided the opportunity to build community and relationships with the other teachers in the group" and one teacher felt she received "a good deal of moral support" (p. 147). The group also "helped them deepen connections between theory, their beliefs, and their practice" (p. 150). Although Arbaugh (2003) begins discussions on what went on in the group meetings, what is evident is the deviation of this group from the characteristics of a professional learning group. This brought me back to my question "In what ways does the group adhere to or deviate from the characteristics of a professional learning group as defined in the literature?"

The research of Brahier and Schäffner (2004) gives an illuminating picture of the effects of study groups on mathematics teachers. Through analyzing the survey data they collected, they determined five themes that accounted for the majority of variance in pre- and post-test data. The five themes they discerned are: "(a) confidence and comfort level of teaching mathematics, (b) inquiry-based teaching methodology and focus on students, (c) collegiality, (d) knowledge of current research in mathematics teaching, and (e) technology use and assessment" (p. 172). Although their study does point to a statistically significant change in the scores from the pre-test to the post-test in beliefs, knowledge, and teaching practice, the specifics about the discussions that were held within the professional development are lacking. Teachers who showed the most significant changes were the ones who had been teaching between 11 and 25 years. Brahier and Schäffner comment that more studies need to be conducted on study groups in mathematics to enrich our understanding of the effects of the groups.

Desimone, Smith, and Ueno (2006) points to a possibly unfortunate trend in mathematics professional development. They studied the amount of mathematics experience and comfort with subject matter of grade 8 teachers and compared that to the length of professional development in which the teachers engaged. Although their study was limited by the data collected from the NAEP testing, they were able to demonstrate a link between teachers with greater comfort in mathematics and their choosing to take high-quality development in mathematics. A problematic area with their study was that their definition of “high-quality” development was any that is longer than sixteen hours in length. This is problematic because workshops that are long do not necessarily mean they are effective professional development. The researchers conclude that it is the teachers who most need the professional development who are not getting the necessary assistance. It could potentially be reassuring to note that over half (53.3%) of the teachers studied had taken professional development in mathematics that was over sixteen hours in duration, which was the definition of “high-quality” in this study. The results of the study lend to the idea that perhaps more resources should be allocated in allowing teachers to create professional learning groups within their own schools in order to more effectively shape practices and reach the teachers who most need it.

Recently, Slavit and Nelson (2010) examined evidence to link collaboration of teachers to changes in instructional practices, including teacher relationships and classroom activities. They frame their study around a “collaboration inquiry cycle” (p. 202) and the impacts using this cycle has on student achievement. Using interviews and discussions, the researchers examined group dynamics and information relating to student successes. They discovered that teacher discussions often centred on generalities of student work and which strategies would work best as opposed to attempting to analyze the work students produced. They indicate promising links between collaboration and student achievement, but believe more research needs to be done to make generalisations about the effects.

Hierdsfield et al. (2010) specifically investigate two mathematics teachers in a professional learning community in Queensland, which is an Australian state. Although they base their findings on the experiences of the two teachers as being similar, they only quote one of the two teachers for the majority of the article. One concern of the study was that the group was called a professional learning community even though there are only two teachers and the researcher involved. Literature points to needing more members for it to be classified as an effective professional learning community (such as Hofman & Dijkstra, 2010), which again illustrates the need to explore which characteristics of a professional learning group are essential to be effective for mathematics teaching. Hierdsfield et al. (2010) point out that "given that many elementary teachers are predominantly generalist teachers with little specialist expertise in mathematics education, there is a need to support teachers to develop their mathematics teaching skills" (p. 94) which has been raised as a concern other researchers (Richardson, 2003; Wu, 2009). The two teachers met with the researcher as the outside mathematics specialist in order to work on improving their techniques in using mental computations in the classroom. Hierdsfield et al. (2010) base their research on the work of "Millett et al. (2004)" and the "Zones of Enactment—time, talk, expertise, and motivation" (p. 100) as being necessary for the PLC to be effective professional development. They stress the need for teacher experts, but perhaps this is because there were only two teacher members, so not enough differing perspectives were able to be incorporated. They conclude that it is imperative for teachers to have "time, talk, expertise, and motivation" in order for the professional learning community to be effective. For this group, "external expertise was deemed to be essential to support teachers' learning with respect to new content knowledge and pedagogical content knowledge" (p. 107), yet the professional learning group characteristics discussed earlier rely on internal expertise. The findings of this research call into question whether an external expert is another characteristic that should be considered for effective mathematics professional learning groups.

Kajander and Mason (2007) studied mathematics professional learning groups in an attempt to define success in such an environment. This work is the only one I have discovered so far that explicitly discusses the conversations of the teachers within a professional learning group. They describe two professional learning groups and the interactions and outcomes of the group to “[characterise] success for evaluating professional learning processes” (p. 434). Kajander and Mason determine that to give meaning to success in a professional learning group, the members of the group need to be given importance in deciding the definition of success. The work of DuFour and Eaker (1998) and the characteristics of action research they propose would separate the two groups studied by Kajander and Mason (2007), defining one as not being a professional learning group. One group simply created new tests to “help” their students, while the other group employed action research characteristics attempting to implement and test new strategies. In the literature, there seems to be an overall promising indication that collaborative groups of teachers are having an impact on teacher learning and teacher practice.

To summarise, the way a teacher chooses to organize a mathematics class is influenced by an intricate combination of a teacher’s beliefs and knowledge. For changes to be effective, professional development or support programs need to address both these issues. As McNeal and Simon (2000) argue, “norms and practices do not change simply by virtue of the teacher using his [sic] authority to assert the new set of rules accompanied by student compliance” (p. 506). Instead teachers need experiences that have them analyze or question their own beliefs (Grant et al., 1994). Simply requiring teachers to use reform-based methods is not likely to have lasting effects on teaching practice. Teachers need professional development that allows their knowledge of mathematics to grow (Ma, 1999), and the knowledge needs to be combined with experiences that are designed to confront and challenge beliefs teachers have about teaching mathematics. Examining the conversations in professional learning groups could shed light on how teachers wrestle with both their knowledge difficulties and beliefs about teaching mathematics. Only when the beliefs and knowledge of the teacher are both considered, can changes in mathematics teaching have real and lasting effects on future generations of students.

I began the chapter by exploring the theoretical framework of social constructivism to provide a structure for building my research study. I continued this structure by examining the research literature by addressing where my study would fit in the existing body of research. I first looked at how the definitions of professional learning groups could define the group explored in my study, and then specifically examined literature related mathematics education in order to provide a context for my research. After examining the foundations and definitions of reform-oriented mathematics teaching, I examined the influence of beliefs and knowledge on both teaching mathematics and making changes in the classroom. I concluded this chapter with a summary of current research studies in mathematics professional learning groups in order to highlight how my research will add to the field. My next chapter addresses the methodology and methods used for data collection in my research in order to answer my questions.

CHAPTER THREE: RESEARCH DESIGN

This chapter describes my chosen research methodology and design of the case study and narrative used to explore a professional learning group. Following the description of the research methodology, I describe the exact methods that I used in conducting my study. I then break the data collection down to examine how I answered each of my research questions individually. This chapter concludes with a look at the ethical implications that are inherent in my study.

Research Methodology

My research was a qualitative, narrative case study that focused on the discussions of, as well as the benefits of or problems with, a mathematics professional learning group in northwestern Ontario. My research examined the multiple facets of the professional learning group and attempted to create a whole picture of the phenomenon, much as qualitative research attempts to encompass differing perspectives (Denzin & Lincoln, 2005) to create descriptions of the participants and research phenomenon (Lichtman, 2010; Merriam, 1998). As Creswell (2008) notes, a qualitative research methodology is used when the research problem needs to be investigated and the exact framework is not known in advance. Prior to conducting my research, I did not know the exact themes that would emerge through observing the professional group that defined its successes or difficulties. My focus question for this research was “What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching practices?” To begin to answer this question, my goal was to record the plurality of voices of the different members of the professional learning group in order to go beyond simply giving a quantitative measure, or numerical account, of the amount of growth of the teachers. Since “the province of qualitative research, accordingly, is the world of lived experience, for this is where individual belief and action intersect with culture” (Denzin &

Lincoln, 2005, p. 8), my research highlighted the journey of a single professional learning group in their activities, discussions, setting, and interactions to paint a picture of the experiences of this group of teachers wrestling with issues in mathematics education. My observational focus was on the social aspects of the research participants and I sought to tell their stories while creating a relationship between researcher and participant (Denzin & Lincoln, 2005; Merriam, 1998). As such, I, as the researcher, participated in the professional learning group as appropriate and was personally involved in the discussions only when invited by the members of the group.

Due to the very personal nature of goal setting in the professional learning group, narrative inquiry was used to illustrate the stories of the members of the group. Kajander and Mason (2007) discuss how the researchers had to set aside their "agenda" in evaluating the effectiveness of a professional learning group because of the teacher-driven nature that is a characteristic of professional learning groups. They note that "the PLG [professional learning group] approach is organized to value the autonomy of teachers and to trust in their capacity to be self-directed and purposeful…research on PLG process should grant to participants' conceptions of success a privileged position in its design" (p. 436). As such, my research expanded upon the conversations of the teachers by moving away from only "defining success" from a research perspective to exploring the stories of the teachers who engaged in the professional learning group. Narrative inquiry posits that the stories of the participants are the focus of the research and should be merged with the story of the researcher as they interact in the social situations of the research (Chase, 2005; Clandinin & Connelly, 2000). In order to accept the personal nature of the professional learning group, narrative inquiry was used to relay the stories of the participants as they navigated within the group. As is important for a narrative researcher (Chase, 2005), I attempted to respect the stories and the journeys of each of the teachers as they functioned within the professional learning group not just discover themes that link the stories of the individuals together. As such my research uncovered "What are the experiences of the individual teachers within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching?" Clandinin and Connelly (2000) emphasize the lack of theoretical framework at the beginning of narrative inquiry in order to allow the stories of the participants to frame the research or "experiencing the experience" (p. 80). This was especially suited to examining the stories of the members of the professional learning group because "success" was to be partially self-defined by the individual participants and could potentially mean different things to different people.

In order to capture the themes in a professional learning group that may potentially generalise to other groups, a case study was used to record the themes within the group discussions. A case study was appropriate for this portion of my research because "qualitative case study is characterized by researchers spending extended time on site, personally in contact with activities and operations of the case, reflecting, and revising descriptions and meanings of what is going on" (Stake, 2005, p. 450). In order to remain true to the case study research format, before examining the data for my research questions, I explore the themes that were presented in the case study. Since the professional learning group itself was a bounded case with clear limits, this group allowed for a "holistic description and explanation" (Merriam, 1998, p. 29) of the group interactions. I looked at the group in its entirety to determine "In what ways does the group adhere to or deviate from the characteristics of the learning group as defined in the literature?" As I was collecting data, I was aware of paying attention to the defining details of the group, as well as the experiences within the meetings (Stake, 2005), in order to gain an understanding of professional learning groups. These activities determined "In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?" My research was directed by the professional learning group as the themes and variables emerged through examining the data. These themes answered "What are the beliefs about mathematics teaching and learning of the individual teachers in the group? How are these dealt with in the discussions of the group?", as well as, "In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?"

Research Methods

This research study focused on a professional learning group of intermediate mathematics teachers. This group was composed of both elementary (grades 6 to 8) and secondary (grades 9 and 10) teachers. Field notes, meeting recordings, and interviews comprised the data collection for the study. Although all the teachers were observed and contacted for an interview, only five of the teachers were focused on for the narrative study. Below I detail the exact process that I went through as I explored my research questions within the group context. See Figure 3 for a diagram of the data collection process.

In order to evaluate the value of studying this specific professional learning group, an observational pilot study was conducted during the 2009-2010 school year as part of another research project[2]. During the year, the meetings were attended and field notes were written about the group discussions. As part of the monthly meetings, the professional learning group conducted two classroom observations in order to continue to ground their own discussions. I collected field notes of the classroom observations as well as the resulting group discussions. The pilot study of the group during this school year identified several areas in which the group

---

[2] This research was part of the CRYSTAL project funded by NSERC entitled *Teachers' Evolving Mathematical understandings*.

differed from previous personal observations of professional learning groups. In particular, the group spent a significant period of time discussing mathematical topics and how to use this knowledge within a classroom setting. The initial phase of data collection identified this professional learning group as a data rich subject to be further explored.

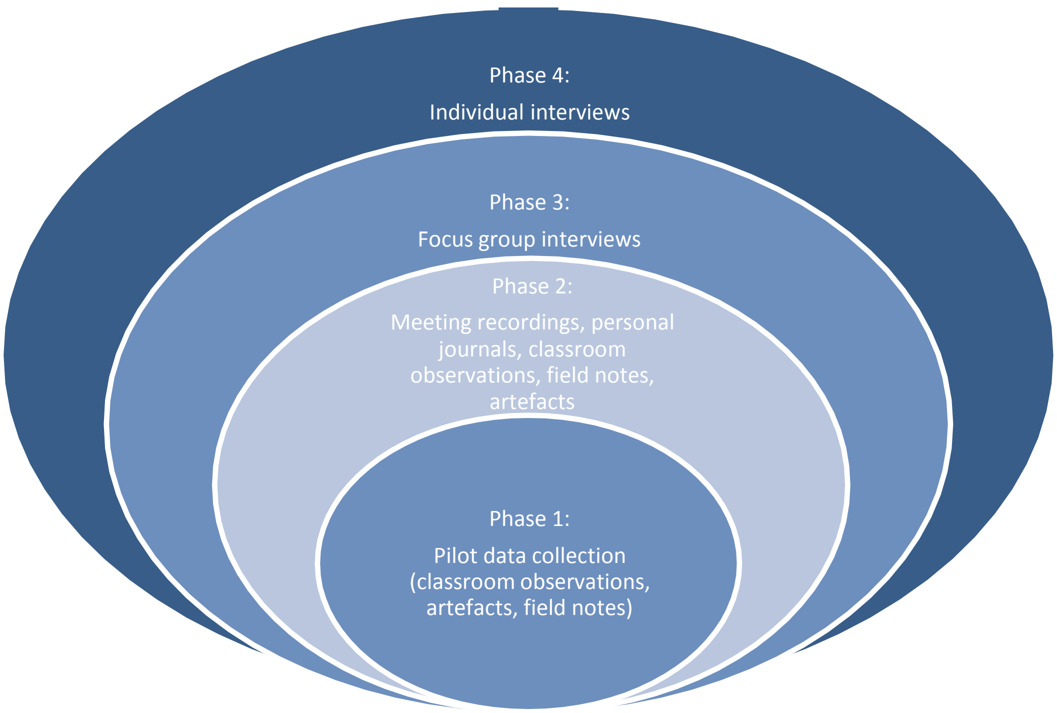


Figure 3. Data collection summary.
Phase 1 was part of another project and used to test the viability of the group for further study. This informed the original drafting of my proposal and research study. Each subsequent phase informed how the data was collected in the next phase.

During the next two school years, I again attended all of the group meetings but, at this time, the meetings were all tape recorded to allow for transcribing and analysis as a data source. Data collection consisted of audio recordings and field notes from each of the monthly group meetings and classroom observations of two of these educators teaching mathematics. Due to the importance of beliefs and knowledge in decision making in mathematics classrooms, the intricacies of the group discussions were recorded in their entirety to give a complete picture of the necessary components of an effective mathematics professional learning group and to look for the themes that arise (Creswell, 2003). These recordings were partially transcribed in order to be analysed. I listened to the recordings in their entirety in order to record times that each of the five participants used in the narrative spoke. General notes about the conversations and the times of occurrence were also collected during this initial listen through the tapes. By recording times and general topics of conversation, the specific recordings could be accessed to gain more information during data analysis. Following the creation of partial transcripts for each meeting, the entire body of work was read and coded based on the five questions that I set out at the beginning of my research. These codes were then able to be explored in further detail as each question was being examined. The recordings were then listened to in their entirety to identify themes in the conversations. Field notes of activities within the meetings were collected as well as a collection of the artefacts from the meetings. Personal journals were kept following each of the meetings in the third year in an attempt to avoid what Arbaugh (2003) noted as the struggle with the research agenda versus allowing professional learning groups to be self-directed. I recorded my own personal comments and thoughts about the meeting in order to highlight areas that I personally felt were successful or unsuccessful.

During the three years, each of the group members could volunteer to have his or her classroom observed during the monthly meetings. During planning for the second year, the group members identified this as a significant and crucial part of the group in order to further their learning and discussions. All of the group members attended the lesson taught by the volunteer, observed the teacher, and interacted with the students. Following the classroom observation, the group reconvened to discuss the lesson that had just been observed. The purpose of the observation was to ground further conversations on mathematics by meeting before to discuss background on the lesson and after the classroom observation to explore what occurred during the class. The first year there were two monthly meetings that included a classroom observation, one at the elementary level and one at the secondary level. During the second year, only a secondary observation was conducted due to scheduling issues. During the third year, only an elementary observation was conducted. I kept notes during the observations about the lessons, student reactions, and classroom details in order to evaluate the level of conformity of a lesson to the tenets of reform-based practices.

In order to allow for the personal definition of success to be determined by the group, my research further focused on the stories told by the 10-15 elementary and secondary teachers who participated in the specified local professional learning group using two types of interviews with the teachers: focus group and semi-structured. I conducted a focus group with the entire group to get their perspectives and stories about what the participation in the professional learning group has meant to each of the teachers. This type of interview “provides opportunities for members of a group to interact with each other and stimulate each other’s thinking” (Lichtman, 2010, p. 89). The focus group interview was conducted first to allow the teachers to consider the effects of the professional learning group and hopefully stimulated greater discussion in the follow-up interviews. The focus group was recorded and later fully transcribed for analysis. The questions or prompts used in the focus group were based in observations during the meetings to gain a greater depth of detail or clarification on discussions raised.

One-on-one interviews were requested of each of the participants in order to give a complete picture of the individual teachers involved in the professional learning group. I conducted nine interviews with the participants of the learning group: four secondary and five elementary. The interviews sought to find out the perspectives of the teachers about the professional learning group experiences and how they felt the meetings were impacting their teaching. The interviews also served as a chance for teachers to discuss personal parts of their story either missed or not shared with the focus group. These interviews were recorded and later fully transcribed for analysis. Interviews were conducted near the end of the school year in 2012. Starting questions from the interview are found in Appendix I. These questions were based in my observations of the professional learning group. Follow-up questions asked during the interviews were based both on participant responses and issues brought up during the focus group interview.

The different forms of data collected, namely, meeting transcripts, interviews, classroom observations, and personal journals, minimized the problems of relying only on self-reporting and allowed me to make "an attempt to secure an in-depth understanding of the phenomenon in question" (Denzin & Lincoln, 2005, p. 5).

Data Analysis

Focus question: What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching practices?

The multiple analyses below were used to gather the types of conversations and the perceived benefits to further define an effective professional learning group in mathematics. The goal of my research was to clarify what features of the group discussions support an effective professional learning group and why. To follow, I have organised the data analysis to address each sub-question (see Table 1 for summary).

1. In what ways does the group adhere to or deviate from the characteristics of a professional learning group as defined in the literature?

Since a case study does set up a theoretical framework prior to beginning the research (Merriam, 1998), examinations of the professional learning group meetings were guided by the following frameworks. First, the overall group characteristics were assessed to explore the degree to which the group adhered to the five characteristics of a professional learning community as defined by Hord and Sommers (2008), namely “shared beliefs, values, and vision”, “shared and supportive leadership”, “collective learning and its application”, “supportive conditions”, and “shared personal practice” (p. 9). The initial field notes of the meetings indicated that all five of the characteristics were present to some degree within this group, and as such, further analysis was essential to demonstrate an explanation of why this group was so effective to its members. The final three characteristics proposed by DuFour and Eaker (1998) “action orientation and experimentation”, “continuous improvement” and “results orientation” (p. 27-29) were also examined in order to identify possible additional characteristics that are essential in mathematic groups. The partial transcripts and meeting artefacts were examined for codes where professional learning group characteristics were indicated for analysis. In order to get the exact details of the conversations, the recordings were again played at the times indicated in my notes.

2. In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?

The teachers were observed and interviewed to determine the level to which they practice and believe in reform-based teaching practices in mathematics through using the guidelines of practices proposed by the NCTM (2000). The classroom observations recorded using field notes about the lesson design, methods of instruction, classroom set-up, and student-teacher interactions. The lesson notes were analysed in order to note the adherence to reform-based strategies, as well as ground individual teacher discussions during the meetings. Again the partial transcript of the meeting was referred to in order to examine the areas that were coded as changes in the different participants. I again listened to the recordings for specifics about the conversations during the writing of the results.

3. What are the beliefs about mathematics teaching and learning of the individual teachers in the group? How are these beliefs dealt with in the discussions of the group?

Examining the existing beliefs of the teachers was important in order to further explore the discussions of the professional learning group. If none of the teachers believed in the reform-based mathematics methods for teaching, then discussions would more than likely take a different tone than in a group which contains teachers who do subscribe to these pedagogical methods. Given the importance of a teacher's beliefs in choosing mathematical lessons, this was an important consideration in evaluating the discussions of the teachers in the professional learning group. For my study, I was concerned with two aspects of teachers' beliefs: "(a) what kind of mathematics is important for students to learn, and (b) how this mathematics should be taught" (Grant, Hiebert, & Wearne, 1994, p. 9). Any artefacts, field notes, or transcripts coded as showing examples of teacher beliefs were examined in order to inform the discussion on the beliefs of the individual teachers. Times from the meeting recordings that were indicated in the transcripts were again referred to in order to get the exact information or quotations from the participants.

4. In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?

Finally, the conversations of the group were analysed using the knowledge practice relationships defined by Cochran-Smith and Lytle (1998). According to Cochran-Smith and Lytle (1998), there are three "conceptions of teacher learning": "knowledge-*for*-practice", “knowledge-*in*-practice”, and “knowledge-*of*-practice” (p. 250). As discussed earlier, the authors believe that these three types of knowledge present themselves differently in different situations. I also examined discussions for evidence of teachers’ struggling with the specialised knowledge of mathematics needed for teachers (Baumert et al., 2010; Ma, 1999; Silverman & Thompson, 2008). Conversations that were coded as being knowledge discussions were analysed by examining the transcripts, notes, and recordings.

5. What are the experiences of the individual teachers within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching?

The interviews themselves as narrative allowed the stories of the participants to be told, so contact with the participants as the writing occurred allowed them to mould their own stories within the research text (Clandinin & Connelly, 2000). Following the transcription of the interviews, the transcripts were returned to the participants in order to allow them to make changes and read over their initial responses. Only three of the participants returned the transcripts with comments. At this time, all of the participants were asked if they would like to continue to read the materials that were being written, and none of the members responded that they wanted to continue being part of the process. The completed data collected about the group was also used to add dimension and richness to the stories, but no framework existed at the onset for analysing the stories. The research came from the stories constructed and relayed by the participants through their interactions. For the purposes of my research, the stories of only five of the teachers within the professional learning group were told in greater detail. The number was chosen based on the data collected during the pilot study in order to give a wealth of differing perspectives of the group. In creating the stories, the partial transcripts for the meetings were referred to in order to get the times that the members spoke during each meeting. Each of the five members was treated as a separate case, and I listened to all of their comments in their entirety for the purposes of writing the narratives.

Table 1

Data Analysis Summary by Research Question

Question	Data collected/ Framework
In what ways does the group adhere to deviate from the characteristics of a professional learning group as defined in the literature?	Pilot study conducted to assess if Hord & Sommers characteristics observed DuFour & Eaker, 1998 used to examine partial transcripts and meeting notes
In what ways is the professional learning group supporting teachers to make changes in their own teaching to improve their teaching to enhance the learning of their students?	NCTM, 2000 definition of reform-based instruction to examine classroom observations, partial transcripts of meetings, and interviews
What are the beliefs about mathematics teacher and learning of the individual teachers in the group? How are these beliefs dealt with in the discussions of the group?	Grant et al., 1994 used to examine artefacts, field notes, and transcripts
In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?	Cochran-Smith & Lytle (1998) framework Silverman & Thompson (2008) used to examine transcripts, notes, and recordings
What are the experiences of the individual teachers within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching?	5 teachers—each case examined separately using field notes, interviews, meetings, and classroom observations

Ethical Considerations

Pseudonyms were used for all members of the professional learning group, and the teachers were given the opportunity to examine potential publications for any identifying information or misrepresentation in their stories. Information about the specifics of the group were and will continue to be kept confidential in order not to inform the rest of the participating school board which group of the four similar professional learning groups also in the participating school board was being observed and researched. Ethical approval has been gained by both Lakehead University and the participating school board for the meeting attendance, observations, and interviews with the group members. (See Appendices 1-3 for appropriate ethics documentation)

As a researcher, my presence in the room necessarily affected the group because it was important to my research to have become involved with the group (deBlois & Sterenberg, 2010). I kept a journal during the meeting times in order to identify my own influences on the professional learning group and attempt to keep track of any personal biases that arose. This also helped me to structure interviews to avoid any personal biases which might have affected the outcome of the discussions. In order to limit inconvenience on the teachers in the group, as well as to minimize how invasive the study was, I did not ask for any additional artefacts to be supplied outside of what was typically part of their professional learning group meetings. My hope was to remain as unobtrusive as possible to allow the group to function as much as they would if I was not there.

Methods of Data Reporting

In order to maintain fidelity to the participants in my research, direct quotations are used whenever possible. Quotations have only been edited to remove any rhetoric that distracts from the meaning, such as “um” and “you know”. Every effort was made to preserve the integrity of the participants’ viewpoints and thoughts.

Although the specifics of the conversations are particular to the professional learning group studied, the general themes of the discussions are meant to generalise to mathematics professional learning groups as a whole. For example, the professional learning group spent time discussing multiplication and division concepts because they were particularly important in their own context. Generalising the conversations to topics of mathematical content and the pedagogy used in instruction should be able to expand to important discussions in all professional learning groups. Although the personal stories of the individual members are particular to this group of participants, the general journey is meant to stimulate discussion in a broader sense. These stories and conversations were used to create a model to be used by the research community in organising mathematics professional learning groups. This dissertation is meant to be read as a story of a group of individuals and the lessons learned from the collective stories of my case study was meant to give direction to the field of professional learning groups as a whole.

To me, my research questions and therefore the results of my research were divided into three distinct parts: the case study, the people, and the organisation. In order to express this, I have divided the results section into three separate chapters (see Table 2). The first chapter (Chapter Four) follows and details the personal aspects of the professional learning group including who the people in the group were and a description of the meetings. This chapter also describes the themes that were presented from examining the case study. The next chapter further details the people in the research by exploring the narratives and beliefs of the members, including linking these aspects to the current literature. The final results chapter (Chapter Six) describes the characteristics inherent in the organization of the professional learning group itself, connecting each section to the relevant research literature. By separating the areas of the case study, narrative, and my research questions, I am able to give attention to the results of both the personal and institutional aspects of the professional learning group.

Table 2

Summary of Sections in the Results Chapters

Chapter	Topic	Sections
Chapter 4	Case Study	Introduction of the participants
		General description of meetings
		Themes in the case study
		Participants' definition of success
Chapter 5	People	Narratives (Q5)
		Beliefs (Q3)
Chapter 6	Organisation	Professional learning group characteristics (Q1)
		Knowledge (Q4)
		Changes (Q2)

Note. Q=question. The number refers to my research sub-question that is specifically addressed in that section.

CHAPTER FOUR: RESULTS—CASE STUDY

This is the first of the three chapters dedicated to presenting the results of my research study. The three parts are the case study, the people, and the organisation (see Table 2). This chapter begins with an introduction of the participants to set the stage for the rest of the descriptions. Next is a brief description of the topics from each of the professional learning group meetings to ground the descriptive information in the rest of the chapters. This chapter continues with a description of the themes in my case study. The chapter concludes with the definition of success for the professional learning group based on the observations of the participants. Chapter Five gives more exploration into the individuals who comprise my study beginning with the narratives of five of the participants. This description is followed by a depiction of the beliefs of the group as a whole, as well as individual participants. The final results chapter, Chapter Six, begins with a description of how the case adheres to or deviates from the characteristics of a professional learning group as defined in the literature. Next, the chapter details the conversations related to the teachers' knowledge of teaching and mathematics. The chapter concludes by chronicling any changes in the individual participants during the course of the study. All of the sections in Chapters Five and Six are followed by a discussion of how the group experiences link or add to the current research literature.

Professional Learning Group Case Study Description

The focal point of my research was the case study itself, so this chapter explores the structure and function of the professional learning group studied. Professional learning groups are teacher-driven professional development, and as such, the members of the group are their most vital aspect. In order to properly respect the importance of the teachers to the professional learning group, I begin by addressing the individual members of the group who are participants in the description. Following the introduction of the members, I summarise each of the meetings that were observed over the three year period. Next, I discuss the themes that were presented through exploring the case study. At the end of this chapter, I note the strengths and weaknesses of the professional learning group based on the individual teacher's descriptions.

Table 3

Members of the Professional Learning Group

Elementary	Secondary
Emma	Ryan
Gabriel	Wesley
Blaine	Owen
April	Noah
Evan	Samuel
Diana	Madison
Claire	Tara

Members[3]

Fourteen different teachers were a part of the professional learning group at different periods throughout the three years. Seven of the members were elementary teachers and seven of the teachers were secondary teachers. Not all of the members were part of the group throughout the entire three years. Each year there was also one member assigned by the participating school

[3] Pseudonyms are used for all of the teachers involved in the professional learning group.

board who would attend the meetings, but who was not a regular member of the professional learning group and was often not a contributor to the meetings when attending. This person would drop in to some meetings, but was not an integral or consistent part of the descriptions. This person will be referred to as the “board liaison” throughout the entire depiction of the group, despite the fact that each year it was someone new. A list of the individual teacher members separated by grade level taught is found in Table 3. In order to help the reader with remembering which grade level each member taught, elementary teachers were given pseudonyms beginning with the letters A to G, and secondary members were given pseudonyms beginning with M to W.

Elementary teachers.

Only four of the seven elementary teachers were involved in the group for the entire three years: Emma, Gabriel, April, and Claire. Members were suggested by the group, but the decision was approved by the participating school board as to who would be included. Evan was only a member for the first two years, and Blaine was only a member for the last two years. During the second year, the board initially made the decision to include Diana in a different professional learning group, so she only attended part of the second year. Since the professional learning group focused on the elementary schools that all fed into the single secondary school, four of the local elementary schools were represented by the members of the group. Both Diana and Claire were the only teachers from their schools. Since Diana’s school actually feeds into multiple secondary schools, the board initially tried to send her to a different professional learning group. Two of the four elementary schools had multiple members: Emma, Gabriel, and Blaine from one, April and Evan both attended from the other. All of the elementary school teachers taught mathematics in grade 7 or 8 or a 7/8 split classroom except for Blaine who taught grade 6 as well as the other grades. Table 4 gives more information about each of the elementary teachers: the length of time holding an Ontario teaching certificate, degrees held other than a Bachelor of Education, and any mathematics related qualifications.

Table 4

Elementary Teachers in the Professional Learning Group

Name	Years	Degree	Qualifications
Emma	20-24	Bachelor of Fine Arts	Mathematics Education (P/J)
		Master of Fine Arts	Parts 1, 2, and Specialist
Gabriel	20-24	Bachelor of Administration	Mathematics Education (P/J)
		Bachelor of Arts	Parts 1, 2, and Specialist
Blaine	15-19	Bachelor of Arts	
		Bachelor of Physical and Health Education	
April	10-14	Bachelor of Arts	
Evan	10-14	Honours Bachelor of Outdoor Recreation	
Diana	10-14	Honours Bachelor of Kinesiology	
Claire	15-19	Bachelor of Arts	I/S Mathematics
		Master of Education	

Note. Years= range of years holding an Ontario teaching certificate. Degree=any degrees held other than a Bachelor of Education. Qualifications=any mathematics related qualifications. All information was provided by the Ontario College of Teachers website (https://www.oct.ca/findateacher)

Secondary teachers.

Table 5

Secondary Teachers in the Professional Learning Group

Name	Years	Degree	Qualifications
Ryan	10-14	Bachelor of Engineering	I/S Mathematics
Wesley	30+	Bachelor of Mathematics	Interim HSA Type A, Mathematics
			I/S Mathematics
Owen	10-14	Bachelor of Arts	I/S Mathematics
Noah	15-19	Bachelor of Science	I/S Mathematics
Samuel	0-9	Bachelor of Arts	I/S Mathematics
Madison	20-24	Bachelor of Science	I/S Mathematics

Note. Years= range of years holding an Ontario teaching certificate. Degree=any degrees held other than a Bachelor of Education. Qualifications=any mathematics related qualifications. All information was provided by the Ontario College of Teachers website (https://www.oct.ca/findateacher)

All of the secondary teachers were teachers at the same secondary school and taught grade 9 or 10 mathematics during the times they were members of the professional learning group. Ryan, Owen, and Noah were members of the group during all three years. Samuel attended two of the years of meetings, and withdrew for one of the years due to an overwhelming teaching and extra-curricular schedule. Madison was only an official member for the first year and attended meetings during the second year and first half of the third when she had preparation periods during the meeting times. Wesley joined the group in the second year, and Tara was a member for only the first semester of the third year. Table 5 gives additional data about the secondary teachers included in the professional learning group who attended more than one meeting.

Meetings

There were seventeen meeting dates spread out over the three years, sixteen of which I observed. During the first year, there were six half-day meetings. In the second year, the group met six times: five half-days and one full day. The third year had the group meeting five times: four half-days and one full day. Next I briefly describe the activities for each of the meetings.

First school year.

The first school year was part of my initial pilot study of the professional learning group, so only field notes were collected during the meetings. Most field notes were just mentions of topics discussed without attaching the significance to certain people or full explanations, but did include impressions or particular quotes that stuck out in the observations.

In the first meeting of the year (Meeting #1), the group established their governing philosophy and set up meeting dates for the remainder of the year based on the funding received from the participating school board. The group also discussed possible topics for the upcoming year including discussions of multiplication and algebra. There was also a discussion about streaming students into secondary school and the different pathways available to the students entering grade 9 (see Figure 1).

In Meeting #2, Evan presented the Math Matrix Approach to Multiplication by David Langford (see Appendix J), which was subsequently discussed in order to address the group's concern over students' lack of basic multiplication facts. This model has students learning their multiplication facts by completing a table. The five categories the students would complete are "I have heard of it", "I can tell someone how to do it", "I can say them in less then [sic] 25 seconds", "I can break my record", and "I can say them in random order". Students complete the entire column for all of the facts (1-12) before they move on to the next category. For example, students must be able to check off that they can do all of the facts in less than 25 seconds before they can work on trying to break their records. Other models, manipulatives, and teaching practices were discussed by the group for addressing multiplication in their classrooms. A copy of the "Pathways" specific to the secondary school was distributed to the group by email following the meeting.

At the start of the New Year, Meeting #3 had discussions about board initiatives being supported in the individual schools, including Blended Learning and the introduction of mathematics coaches. The teachers also discussed whether or not students had been properly streamed into their grade 9 classrooms from elementary school. The group members examined other models for multiplication and methods of multiplying fractions and decimals. Algebra was introduced to the group as well as the use of prime factors and factor trees. The meeting ended with exploring strategies for teaching division.

Following Meeting #4, each group member received a SMART notebook file of "handy pages" of manipulatives and mathematics tools for use on the SMART Board that were discussed during the meeting. The group members also set up the next two meetings each of which was to include a classroom visit. The board liaison brought in a diagnostic to be shared and used by the grade 9 mathematics teachers. The diagnostic was being piloted by the curriculum company Nelson and was to be used this first year to give the grade 9 teachers information about their students in the different curriculum areas. The group again discussed division techniques including division using fractional quantities. The discussion was further expanded to share how division techniques fit into a discussion about algebra. The meeting ended with examining final grades and class test scores of students in grade 9 in order to discuss placements from elementary school.

Meeting #5 began at the secondary school with an overview of the first lesson the group would observe on linear relations. The group then watched Ryan teach a grade 9 Academic math class, following which they met as a group to discuss the lesson and teaching in secondary school more generally. The discussions related to how elementary curriculum expectations supported the secondary lesson, comments about how different teachers worked with the topic in their own classrooms, and where there are gaps in the current curriculum. The group then attended another lesson in Ryan's classroom, this time a grade 10 Applied class on algebra. Following the observation, the group again met together to discuss the lesson and have conversations about teaching algebra. Both observations were very traditional lessons with the teacher directing the classroom in activities and then the students completing an assigned list of problems from the textbook. The group discussed where gaps in the curriculum lay within the topic of algebra. EQAO scoring and data management topics were also examined by the group members.

The last meeting of the first year was Meeting #6, and the meeting was supposed to be an observation in Evan's classroom, but it was switched to Emma's classroom prior to the meeting date. At the start of the meeting, the group met to discuss what they would be observing in the grade 8 classroom on algebra. Emma also shared some information about websites and other materials related to algebra as well as her lesson goals for the observation. The teaching in Emma's class demonstrated many tenets of reform-based pedagogy. Following the observation, the group again met to discuss what they had observed as well as gaps in the curriculum in the algebra strand. The meeting ended with a discussion about possible topics for the upcoming year provided the professional learning groups were again funded by the school board.

Second school year.

During the second year, Ryan continued to organize the meetings, send out email reminders, and take care of any paperwork needed from the group for the school board. All the elementary participants continued this year with Blaine joining the group. There were some difficulties with Diana being a member of the group based on some school board decisions to send her to a different professional learning group because her grade 8 students had a choice of secondary schools. She rejoined the group midyear once the board liaison facilitated securing her transfer. Madison gave up her official spot in the professional learning group to allow Wesley to join the group meetings. She attended whenever the meeting corresponded with a preparation period in her teaching schedule.

The school year began with Meeting #7 and the group again went over their norms for the group meetings and chose dates for the upcoming year. Based on the discussions of the previous year, the group attempted to decide upon topics for the upcoming meetings. Emma ended the meeting sharing materials she used in her classroom including the exploration activities and rubrics.

Meeting #8 focused on discussions about algebra and consisted of group members sharing lessons that they used in their own classrooms. The group also talked about rubrics and EQAO style questions using algebra expectations.

Based on the previous meeting, Meeting #9 focused on using a rubric brought in by Emma to grade samples she also brought in to share from her students. The teachers had a discussion about marking using rubrics and why they had assigned certain grades. The mathematics strand focused on in the meeting was algebra. The meeting concluded with Ryan and Owen sharing lessons that they both used in their grade 9 classrooms.

For Meeting #10, the board liaison described the process used for grading the EQAO in grade 9. Sample questions were examined by teachers, including student samples from the EQAO website. The group again discussed the topic of algebra, and Ryan shared with the elementary teachers how to make spontaneous clicker quizzes.

The only full day meeting of the year, Meeting #11 began at the local university where the group attended a talk by Dr. Florence Glanfield about teaching mathematics and Aboriginal students. Following lunch, the group observed Owen's grade 9 Academic class on linear relations. The group then convened in the meeting room to discuss both the morning talk and the observation. Discussions included racism in schools and questions about linear relations terminology.

Meeting #12 was again planned to be an observation of Evan's classroom, but he had to cancel at last minute. The group instead met at the secondary school where they discussed how they would teach a sampling of mathematics questions that were created and brought in by Ryan. One example of the questions provided is shown in Figure 4 (Appendix K shows all the questions discussed). The teachers worked in either pairs or trios to solve the questions and share how they would teach the question to their students. Each pair or trio contained at least one elementary and one secondary teacher in order to encourage cross panel sharing and give the differing perspectives. The teachers also worked on creating a list of vocabulary used when they are teaching algebra, equations, area/perimeter/volume, angles, and relationships and graphing.

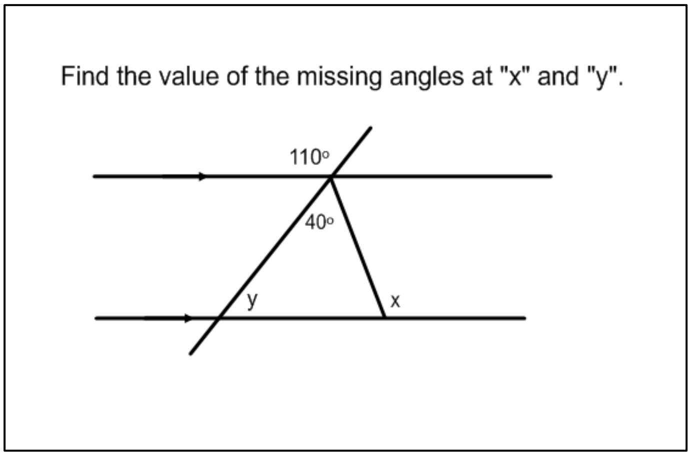


Figure 4. Example of question created by Ryan.
The questions were used to encourage discussions with the teachers about how to teach specific mathematical concepts.

Third school year.

Ryan continued to organize the group meetings during this year as well. Elementary members remained the same, with the exception of Evan withdrawing from the group. In the secondary panel, Madison attended meetings when she had preparation periods during the first semester, and Samuel withdrew from the group during this year. Tara attended a single observed meeting during the first semester.

To begin the third year, Meeting #13 again established the norms of the group and set up the meeting dates and possible topics for the upcoming year. The board liaison brought in the previous year's grade 9 EQAO scores for the school board, so the teachers examined questions that students had particular difficulties with from the previous test. Ryan also brought in his grade 9 quizzes on linear relations for the group to discuss.

For Meeting #14, I was unable to attend the meeting. According to the emails about the meeting, the group was supposed to focus on justification type problems. When I apologized for missing the meeting, Ryan mentioned "it wasn't one of our better ones" (personal email correspondence), and I noted that one of the following meetings again focused on justification.

Meeting #15 was the only full day meeting for the school year, and the group attended a talk at the university for the morning. The talk was given by David Stocker and focused on using social justice type problems in a mathematics classroom. Following lunch, the group convened at the secondary school to discuss the morning talk and made plans to incorporate some of the ideas into their mathematics classrooms before the end of the school year. The group also discussed specific grade 9 students and again examined the streaming of the students from elementary school into secondary school.

For Meeting #16, the teachers were all asked to bring level 2 samples with a justification focus from their classrooms to discuss. Very few of the teachers brought in samples. Three of the teachers discussed how they had implemented ideas about social justice from the previous meeting into their own classrooms. Ryan once again created some mathematics questions that the groups discussed.

The final meeting, Meeting #17, took place at Gabriel's elementary school for a classroom observation in grade 7. The group watched Gabriel navigate an algebra lesson with his class and then met together to discuss the observation. Following the observation, the group also discussed a news report that Ryan had seen about mathematics teaching. The board liaison brought in materials that could be used in a classroom, including a diagnostic the group decided to use in grade 8 in order to help with streaming for grade 9.

Case Study Themes

In order to properly understand the case study of the professional learning group, I listened to all of the recorded meetings and identified the topics being discussed. My notes consisted of times and a brief description of the conversations. I then went through these partial transcripts of the meetings to identify the themes of each of the meetings. These themes were later grouped by subject in order to categorise a list of themes that thoroughly described the discussions of the professional learning group. In the end, there were seven themes that occurred in the meeting conversations: factors outside of control, topics related to the group structure, classroom strategies, student-related conversations, mathematics-specific conversations, learning trajectories, and program differences. Each of these themes are now explained in greater detail to give a complete picture of the case study.

Factors outside of teacher control.

During the meetings, the group members often talked about areas that they could not control, such as board initiatives and issues related to funding the group. For example, the group often talked about the board mandate to include literacy in other subject areas and would explore how literacy could be implemented within their mathematics curriculum. The secondary teachers were just beginning to have to include more literacy into their courses and often asked the advice of their elementary counterparts in doing this. The group also discussed their access to technology that was provided by the school board and whether it was working or not. In terms of funding, the group was always careful to use all of their allotted days each year, and expressed concerns over the continued funding of the professional learning groups each year. Whenever possible, they asked the board liaison to find other pockets of money to fund their attendance to allow them to maximize their time and efforts. For example, the board liaison was able to fund two secondary teachers from each professional learning group and at least one elementary from each group from another funding source so that they could attend the meeting without losing one of their professional learning group days. One conversation with the board liaison pointed to the fact that the group that I observed was over their quota of members for funded release time, so the secondary teachers made arrangements for new members by having Madison only attend meetings during her preparation periods (Meeting # 10 transcript).

The elementary teachers especially worried about the length of their class time in order to be able to get everything accomplished. Emma in particular was concerned about getting the most out of her fifty minute mathematics class, so she made instructional decisions that would maximize her students being able to explore the mathematical concepts each class (Meeting # 8 transcript). The teachers also discussed concerns related to fitting the entire assigned curriculum into the school year and had discussions about areas to spend less time on, based on future expectations for students. For example, during Meeting # 12, the teachers discussed angle theories and how this topic fed into the secondary curriculum. As a result of the conversation, it was suggested that the elementary teachers spend time exploring the Pythagorean theorem to better support the secondary curriculum instead of learning the names for the angle theories. The secondary teachers felt it was only important for students to be able to apply the angle theory to practice instead of knowing the name.

The group members also spent some time discussing the home lives of students and factors that would influence their involvement in school. Following one of the meetings at the university, the group especially spent time talking about Aboriginal perspectives and how sometimes parental support was a deciding factor for student engagement (Meeting # 11 transcript). They also discussed how the home lives of all of their students contributed to racism in their classrooms and schools. Although they acknowledged that these could be contributing factors to their classrooms, Emma noted how she could not control these factors and chose to instead concentrate on the areas that she could change in order to make a difference for her students.

Topics related to the group structure.

The group members spent time debating topics that they wished to discuss during their time together, as well as reviewing previous topics. At the start of each new year, the group members were given a certain number of days that the school board would fund, and then they worked together to choose dates and times that would work within their schedules. During this meeting, Ryan also organised the group meetings for the year by brainstorming topics for discussion. Each meeting, Ryan would review the topics for the meeting at the start and then at the end of the meeting would repeat what they had decided for the next meeting. The topics were changed as new learning opportunities came up or if the discussions were not fully explored during the current meeting. By taking the time to set a schedule of ideas, the group made sure to maximize the use of their meetings times. At the beginning of each year, the group also set a clear purpose for the group in order to encourage all members to participate. Each year the group set the theme of “no busybody work” in order to ensure that the meetings were beneficial to their individual practices (i.e. Meeting #1 transcript). The group also made sure to point out how important it was for the meetings to be a safe space where student issues were explored and teachers were not being judged.

Classroom strategies.

The teachers in the group spent a fairly substantial time during their meetings discussing specific classroom strategies that were used in their mathematics teaching practices. Classroom strategies included discussions of manipulatives, specific lessons, planning, graphic organizers, and assessment. Manipulatives and models that could be used to increase student understanding were explored during the group meetings. The group discussed how integer chips, fraction bars, and algebra tiles in particular were used by their students and to enhance their practices. One model that was particularly discussed was what April referred to as the “magic box” (Meeting # 12 transcript) or area model. This model was explored in relation to its uses with algebra tiles. Ryan and Owen ended Meeting #9 by sharing different lessons they used during the algebra unit. This discussion showed the group different activities used with grade 9 students and allowed the group to access what I called “teacher talk”. I used this term to refer to conversations that were purposefully exploring decisions made in relation to their teaching practices. Many of the classroom strategy conversations accessed the knowledge that they had gained about both students and their teaching during the course of the discussion. For example, when Ryan and Owen shared their lessons, Ryan also talked about using a hybrid of the area model with algebra (see Figure 5). The purpose of using the model was so that students would not need FOIL and its associated arrows, but rather would be able to identify the number of terms needed using the model. Ryan wanted his students to make sense of why each portion is multiplied by the others instead of just relying on the procedure. During the meetings the group also spoke about planning tools they used and how they structured a lesson in their classrooms.

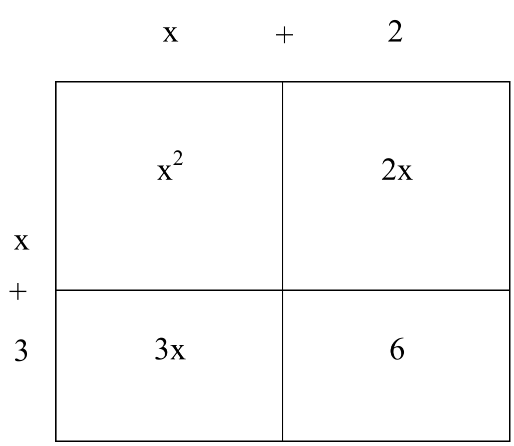


Figure 5. Example of “hybrid” area model for (x+2)(x+3) used by Ryan in his classroom.

Five meetings during the second and third year focused on discussions about testing practices and grading. The teachers spent time exploring the EQAO test in particular in order to adopt strategies in their classrooms to support their students while taking this test. During Meeting # 10, the group spent the first half of the meeting looking at questions and then discussing the grading practices on the EQAO in order to gain a better understanding of the assessment. Gabriel in particular mentioned how it would help examining the EQAO in order to allow them to make instructional decisions in grade 7 and 8 to support student achievement on the test (Meeting # 7 transcript). The secondary teachers mentioned how much their students were struggling with multiple choice questions in particular, and as a result Blaine and Emma mentioned that they were adding more multiple choice questions in their classrooms (Meeting # 10 transcript).

The group also focused on using rubrics to assess mathematics, a practice that was new to the secondary teachers. Emma brought in her assessments and a sample rubric for one of the meetings in order to encourage a discussion of using a rubric to holistically grade an assessment (Meeting # 9 transcript). Ryan also brought in some of his secondary assessments in order to encourage discussions. For example, he discussed a particular question about pizza that he had used in previous years and how he had changed the question to better support his students (Meeting # 10 transcript). The original question stated that "you have five-eighths of a pizza and you give a quarter to your buddy", and he changed the question to "you have five-eighths of a whole pizza and you give a quarter to your buddy" due to the number of differing answers due to the student interpretations of the question. While Emma and Ryan shared, they also discussed classroom decisions that they made to contribute to the assessment as well as wording they changed based on their observations of students.

Gabriel also gave an example of "teacher talk" as he shared the assessments that he used in his grade 7 classroom and discussed the placement of the checklist or rubric at the top of his assignments (Meeting # 8 transcript). He discussed how he used a checkmark, “s”, or “x” to let students know how they are progressing in the content areas. Both Emma and Gabriel noted that they then used this information to group their students in future classes. Gabriel expressed that the decisions he made while creating the checklist helped him focus his assessment on the specific topics.

Student-related conversations.

Every meeting during the three years had a theme of student-related conversations during the course of the discussion. This was one of the most prevalent themes because at the heart of the meetings, the group members were concerned about increasing their student achievement. The conversations that related to students included ways to support their students, misconceptions and concerns they were having, and what students were learning about mathematics. One particular area the teachers spent a lot of time discussing was how to support their students in streaming them into secondary school (i.e. Meeting # 9 transcript). The grade 8 teachers were concerned about making proper recommendations for their students entering secondary school, so asked a lot of questions about what determinants they should use for deciding the correct pathway. The group felt it was important for student success to have students placed into the correct pathway when entering secondary school.

Emma expressed concern that sometimes elementary questions in algebra lead students to believe that the answer needs to be a “nice” whole number and that something was wrong if the answer was a decimal or fraction (Meeting # 9 transcript). Madison brought up that even grade 12 students had difficulties with negative numbers, so the group discussed how to better support student understanding in integers (Meeting # 8 transcript). One notable example of a student misconception that was referred to on multiple occasions was students mistakenly believing that a number divided by itself equaled zero (i.e. Meeting # 8 transcript). In discussing algebra, the teachers were concerned about how this misconception impacted their ability to solve algebraic expressions. Student misconceptions that were discussed were also related to students' not understanding vocabulary that was not specifically related to mathematics. For example, Gabriel mentioned that students questioned the term "baseboards" on his assessment (Meeting # 13 transcript), and Diana shared a story of students being confused about the term "Persian"[4] on the EQAO referring to a cat (Meeting # 10 transcript). While discussing the assessments and lessons in their meetings, the teachers also discussed what students would be learning mathematically during those lessons. Following Gabriel's classroom observation in Meeting # 17, Gabriel began the discussion by explaining algebra lessons that the students had completed and the knowledge that they gained from those lessons. He then discussed his students' current understandings of variables based on the observed lesson, and the next steps he would take with his students to increase their learning.

Mathematics-specific conversations.

Although mathematics was, to some extent, the theme for all of the meetings, some of the conversations were highly mathematics-specific. At times, the group would discuss different mathematics conventions or processes used in their classrooms. For example, the group focused on justification in their mathematics and what constituted effectively justifying a solution. They also discussed how a student should properly communicate their ideas mathematically, such as only putting one equal sign on a line (Meeting #9 transcript). One example of a mathematics-specific conversation was when the teachers discussed what -5^2 would equal: 25 or -25 (Meeting #16 transcript). When this question was presented to the group, the mathematics was equated to $-x^2$, so the square would be attached to the 5, not the negative 5. Ryan identified the question as

[4] Students would be more familiar with the term Persian referring to a local donut-like pastry.

being $-1 \times 5 \times 5$ to help visualise the correct solution. They also discussed that different calculators would give either answer depending on the calculator model. Vocabulary that is specific to mathematics was also explored during the course of the discussions. April expressed her desire for students to learn the correct "mathematics language" now that they were in grade 7 (interview transcript). During Meeting #12, the group built lists of terms that they used in different units such as algebra and geometric area in order for the teachers to use a common set of vocabulary with their students. The secondary teachers mentioned making sure students did not use the incorrect term "cross multiply" or "cancel out" when describing division of fractions.

Learning trajectories.

The teachers engaged in discussions about the curriculum of their grade levels and the interrelation of the expectations in different grades. They also discussed where students would end up in terms of the curriculum, as well as where they came from in order to inform their practices. For example, the teachers discussed the use of "n" or "x" as the variable in algebra, and how eventually students need to be using "x" in order to use the format $y = mx + b$ in graphing where each letter has a specific purpose. Madison also frequently mentioned how different concepts in mathematics related to the secondary science curriculum. The teachers identified gaps in the curriculum that would cause problems for grade 9 students if they were not addressed. The two major gaps the group focused on dealt with the content areas of fractions and algebra (Meeting #2 transcript). The secondary teachers noted that fractions were not in the grade 9 curriculum, so they would expect incoming students to be fluent with fraction operations. They were surprised to learn that in grade 7, students were introduced to addition and subtraction of fractions, and grade 8, they were introduced to multiplication and division, so the majority of students would not have mastered the skills by grade 9. As a result, the grade 9 teachers spent time addressing the deficits before moving on to more complicated concepts involving fractions. Through their discussions, the teachers became aware that the grade 8 curriculum focuses on solving algebra by inspection as being an adequate strategy, yet students who could only do this would struggle in grade 9. In order to narrow this gap, the grade 7 and 8 teachers moved their algebra discussions further to help better prepare students for grade 9.

Program differences.

Two main program differences were discussed during their meetings: elementary and secondary differences; and Academic and Applied differences. The one major example of a difference between elementary and secondary that the teachers spent a significant amount of time discussing was the grading policies. Elementary teachers used rubrics and levels to describe their students' performance; whereas, the secondary teachers used a point system to gain a percentage grade. The secondary teachers expressed an interest in learning more about rubrics. Another example of a difference was the use of manipulatives. The elementary teachers described using manipulatives throughout the lessons with their students; whereas the secondary teachers used them only in the beginning to review the concept with students before moving on. The secondary teachers had a definite lean toward ensuring procedural fluency with their students. One teacher described the manipulatives as "chips for dummies", which horrified Emma who stated that her students never feel that way about the manipulatives because it is such a part of her classroom environment (Meeting #7 transcript). The elementary teachers discussed using factor trees to determine least common multiples and greatest common factors, which the secondary teachers had not used in the same capacity.

The secondary teachers spent time discussing the differences between their expectations for Academic versus Applied students. For example, the secondary teachers felt that Academic students should be more responsible for taking notes than the students in the Applied pathway. During Owen's classroom observation, they brought up the use of imperial measurements in his lesson (Meeting #11 transcript). Two of the secondary teachers commented on how they would provide more support for their Applied students to know these measurements, but they would expect their Academic classes to either know the conversions or be able to look it up for themselves. Another difference within the pathways dealt with the vocabulary that was used in the courses. Ryan noted that he would not use the term "common denominator" with his Applied students because they were just "two big words" for those students (Meeting #12 transcript). Following the classroom observations, the entire group engaged in a conversation about the difference in the student attitude towards mathematics as well. The students in the Academic stream appeared to care more about the mathematics and were much more willing to get involved in answering questions or discussing. The Applied class needed a lot of support, structure, and encouragement to get working.

Strengths and Weaknesses of Professional Learning Groups

Each of the teachers interviewed were asked to provide strengths and weaknesses they felt about the professional learning groups. Five of the teachers interviewed pointed to sharing of resources and ideas being one of the strengths of the professional learning group. April commented on how the professional learning group meetings allowed her to target areas she felt were needed in her own practice. Three of the teachers commented on the support provided by the group and how they were no longer isolated in their practices. For two of the teachers, the strength was also in the two divisions of elementary and secondary working together. Diana noted in the focus group that she was "more excited about teaching math" following the professional learning group meetings and her students noticed it.

Only one of the teachers, Blaine, could not think of a weakness of the professional learning group meetings. Two of the teachers talked about how the meetings might be intimidating for some of the teachers, especially if someone new joined the group. Ryan was especially concerned about they had no feedback about how the information in the meetings was translating into practice for the elementary teachers. Claire saw how few and far between the meetings were as a weakness because the professional development was so beneficial. Owen felt the group often got off topic and thought that particularly the conversations about how the grade 8 students were handling grade 9 was ineffective. For Wesley, the individuals who did not talk because others overpowered the conversations was a real weakness in the group. Finally, Gabriel thought that new members need to be introduced to the group in order to keep growing and gaining new ideas and perspectives.

By starting with an overview of the members of the group, the individuals remained at the front of my description. In order to give some context to the reader, a general description of the meetings followed. Next, I discussed the themes that were presented during an exploration of my case study. This chapter concluded with a discussion of the strengths and weaknesses of a professional learning group as defined the participants.

Since the introductions of the members and the themes that were presented in the professional learning group have been explored, the next chapters address the evidence used to answer my research questions. The next chapter focuses on the individual teachers of the professional learning group and addresses the research questions related to exploring both the narrative of the professional learning group and the beliefs of the teachers.

CHAPTER FIVE: RESULTS—INDIVIDUAL NARRATIVES

In this chapter, five of the teachers are described in detail. I define their personally held beliefs about teaching mathematics, and how the professional learning groups fit into their practices. Next, I describe the beliefs about teaching mathematics presented during the meetings more generally to provide a more complete picture. Both sections are followed by a discussion linking the description to the relevant research literature, as well as some additional analysis of the observations.

Narrative Descriptions

Five of the members were chosen for the narrative description in order to encompass the range of teachers within the group. These teachers were members who attended the majority of the meetings during the years they were involved. The chosen members are Emma, Blaine, April, Wesley, and Owen.

Emma

Emma taught in the same elementary school as Blaine and Gabriel. She taught a grade 7/8 split class for the first two years of the meetings and a straight grade 8 class during the last year. Emma had taught elementary school for over 15 years and expressed her comfort with teaching grade 8 students. “I think that I’ve in many ways have had the luxury of being able to work with that same age group and can kind of recognise when I get to hit some of those ideas again” (interview transcript). She was very versed in the curriculum for these two grades and would often cite it in discussions.

Emma mentioned that “instructional time is a precious commodity” (Meeting #9 transcript), and she strove to use every minute wisely. As such she tried to set up effective lessons for her students and create rubrics to help with assessing her students’ abilities. As she planned her lessons, she found she was “trying to think about in advance, what is it I’m really trying to get kids to do and is this really going to help me figure that out” (Meeting #8 transcript). In order to teach mathematics in the method Emma felt was best for her students, she spent a lot of time creating her own lessons. “That’s the problem with the textbook, right, all they have to do is turn the page and get the punch line. I don’t need to think, I’m just going to turn the page and it’ll tell me what to do versus if you just separate it from the text, it never occurs to them they might actually look in the text and find it” (Meeting #8 transcript). As such, she often pulled good problems out and put them on a separate page or as she put it “Frankenstein many of their activities” because “it’s just a textbook to me and it’s just a launching point” (Meeting #17 transcript).

For Emma, the important aspect of teaching mathematics was that her students “conceptually” understand the concepts through exploration and discussion, and she made “sure they understand the value of the numbers” (Meeting #9 transcript). As she often talked about, she felt that “just because it’s been covered doesn’t mean it’s understood” (Meeting #9 transcript). “I think it’s very easy as teachers to fall into the trap that if I stand in the room and I talk about this, I can say we’ve covered it” (Meeting #15 transcript). She also felt that it took students time to be able to use the new ideas and concepts independently and often talked about students “renting the ideas” until they gained ownership of the knowledge (Meeting #10 transcript). Emma firmly believed that “there are many different ways for people to develop an understanding of mathematics and that it is my job to be as versed and as flexible as possible so that I am able to recognise the significance of my students’ thinking and to meet them where they are and to move them forward, and to also try and recognise when my own thinking gets in the way of their thinking” (interview transcript). She illustrated this philosophy during many of the group meetings where she shared work that her students had done. She often mentioned that she never wanted her students to feel like they could not answer a mathematics question simply because they did not do it the way Emma had used. “My job isn’t to tell you what to think, my job is to help you learn how to think better, express yourself more clearly” (Meeting #15 transcript). She told students that “my job is to push you mathematically” (Meeting #8 transcript) through encouraging them to use more efficient or clearer methods as the students became ready. She did note that “while there are going to be more efficient ways to get things done that I never want to take something away from students if this is their understanding and this is what they truly know” (interview transcript). This desire for students to work within their own comfort level and to build off their own understandings was a common theme throughout her discussions. She was frustrated “when people don’t try because they don’t think they have the right way” (Meeting #12 transcript) and strove to make sure her students never felt this way in her classroom because she would “rather they got there somehow than not at all” (Meeting #13 transcript).

Emma commented that students’ mathematical abilities would grow at different rates, but only when there was a need. “I suppose that whole notion of when do we grow mathematically, it’s when we have a reason to” (interview transcript). She purposefully structured lessons with her students so that they would see the need to use more sophisticated mathematics understandings that went beyond her simply telling them to do it. For example, Emma discussed students using more than solving by inspection in algebra only when they could not figure out the value of “n” by simply looking at the equation.

It became very clear through talking with Emma about mathematics, that she was determined for her students to enjoy and feel successful in her classroom. Emma used checklists on her assignments and then discussed where students would fall based on their work so that they would have immediate feedback on how they were working in her class. She also discussed with her students how they had done on their report cards before sending them home to make sure they understood their grades and what they needed to do in order to improve or maintain them. In order to ensure her students were using the mathematics concepts correctly, Emma had students record their thinking and justify their solutions. As she noted "I don't want the shorthand because otherwise it's just something that somebody told them" (Meeting #9 transcript). "I need students to justify because just giving me a number without knowing where it came from that doesn't meet my criteria for communication" (Meeting #12 transcript). For Emma, it was important for students to share how they solved the problem and she told them "your job as a mathematician is to think and to leave a trail so I can follow your thinking" (Meeting #12 transcript). She felt that it was important to model for students the procedures for recording their thinking in a conventional manner.

Emma was also a self-proclaimed algebra lover, and she believed that her students learned to appreciate the content strand through her enthusiasm. She did comment that she found her students' desire to please her a "double edged sword" (interview transcript). She noted that she liked "to see algebra as a much more efficient way" of solving problems through presenting different examples and activities (Meeting #8 transcript). Emma taught algebra more pictorially, and she decided to teach her entire algebra unit for the school year through the theme of zombies and found that many of her students grabbed onto the hook and were able to solve problems at their own levels of sophistication. She also sought to bring other strands into algebra and found that when she changed to teach algebra at the beginning of the year, her classroom "discussions about algebra are more frequent and they're deeper and they're better" (Meeting #17 transcript).

Manipulatives were also a common sight in her mathematics classrooms, and she encouraged students to use the tools that they felt would help them solve the problems. She encouraged her students to continue modeling the concepts if they were not ready to move beyond the concrete into the abstract. She discussed using factor trees to help her students and how she would not take them away simply to have her students memorise a procedure. During her math lessons, she asked her students questions when they asked questions in order to encourage their own thinking because she felt “everyone doesn’t just believe it because I said it” (Meeting #12 transcript). The discussions in Emma’s math class were focused on the thinking of her students. For example, Emma talked about giving the problem to her students about three consecutive numbers and how to figure out how to find what they numbers were if she gave a total amount. She talked about her solution starting with “n” so then “n+1” and “n+2” would be the next terms. She relayed how one of her students started with “n-1”, then “n” and “n+1” so that the ones would cancel each other out. She encouraged her students to discuss the ideas and compare the two solutions, and then another student asked if it would work if they made the last number “n”, and the discussion continued. Her students’ thinking was vital to Emma, and she believed that was more important that simply finding a correct answer. “I rarely use the word formula, I use the word relationship…formula you just follow the steps and stop thinking, and I never want them to stop thinking” (Meeting #16 transcript).

For Emma, the mathematics community she has created in her classroom to allow her students to have a safe place to question and discuss was extremely important. She used humour in her classroom to reach her students and create the learning community that she wanted. For example on a day she was away, she had a picture of herself that would randomly appear on the SMART Board throughout the day. She said that “I can help create this really stable place for you to come to everyday, and to try and do your best work" (interview transcript). She strove to create "a nonthreatening way to get kids to start to talk about math because I can't be the only one talking about it in the classroom, it needs to be them too" (Meeting #7 transcript). Her classroom community was created so as "to build that trust that it's going to be safe and that absolutely screw up, make mistakes because my goodness, I'm going to make tons of them. And I think students learn more from our mistakes sometimes then they do from, not sometimes, all the times from us being right, whether in terms of how we accept our mistakes or what it is that we learn from them" (interview transcript). In terms of student errors, she felt that "the nature of the error gives you some indication of their learning and their understanding" (Meeting #8 transcript). She shared an activity she did with her students to help set up the "safe to be wrong" mentality in her classroom (Meeting #7 transcript). For this activity, she shared clues with her students, revealing one clue at a time, so the initial guesses would be incorrect because they did not have enough information to figure out the correct number until the last clue was given.

Although Emma expressed comfort in her own mathematics knowledge, she recognised that there was more to learn about understanding the work her students produce. Emma noted that whenever the professional learning group began talking about the mathematics that this was where the conversations were most effective. Again she brought this back to her students because as she noted "the math comes out, that's when the discussions generate because you didn't even know that maybe you can talk about that or that you had a different notion about what that meant and how kids might approach it" (interview transcript).

Emma found value in the classroom visits because "things that hadn't come up in our discussions that didn't actually until you were working with kids,…that was quite interesting" (Meeting #7 transcript). She also found the professional learning groups to be very beneficial to her practice in allowing her to see where her students go after leaving the confines of the elementary school. “I wouldn’t mind finding out today kids that have made that transition because it informs me sometimes better as a teacher…to see how they’re coping with it” (Meeting #7 transcript). Emma continued that the stabilising of the jobs in the elementary schools would “start to channel kids better and also have better discussions with the parents” (Meeting #7 transcript). The professional learning group meetings “certainly [have] given each of us a window into each other’s world” (interview transcript). For Emma, the meetings also served as “an incredibly safe place” for the teachers to get together and discuss mathematics (Meeting #7 transcript). Emma believed that she was “a strong contributing member to that group” (interview transcript), which is evidenced by the amount of discussion that she brought to the table in every meeting. Emma also made suggestions of different activities that group could work on during the year, gave advice to the other teachers, and was the member who brought the visiting speakers at the university up with the group. Emma never missed a meeting, and whenever the assignment was to bring something, Emma ensured she had an example to share. She brought her thoughtful discussion and philosophy of teaching mathematics to the group and discussed with the other teachers how important it is to work with students at their own levels. She did express concern over other teachers in the group finding themselves to be intimidated by her strong presence in the meetings.

Blaine

Blaine taught at the same school as Emma and mentioned that a lot of his lessons had come from things that Emma had created. He was invited to work with the professional learning group despite working with grade 6 when he started with the group. Blaine valued ensuring that all of his students were working in mathematics and that no one was being left behind because “I believe that everyone is capable of doing something in math" (interview transcript). He worried about making sure he could address everyone in his class, from the student who typically achieved a level one to the student who typically achieved a level four, during each lesson. He noted that he used the three part lesson in his own teaching and had students work in groups, which he followed up with having students complete practice questions.

Blaine spoke less than fifty times during the two years of meetings and most of his comments were one or two word answers in response to a question or comment being directed at him. As he noted he was "just trying to soak it in" (Meeting #9 transcript). Emma did encourage comments from Blaine by discussing how they both were doing activities in their classrooms, and most often he would just agree. Emma talked about how she and Blaine were working on more multiple choice, and Blaine replied that he was having his students eliminate information that they did not need in order to support his students answering multiple choice questions. He never brought something to share during his two years working with the professional learning group. In only two of the meetings Blaine talked about his own classroom. During Meeting #8, the group was discussing rubrics and what it meant to be a level 4, and Blaine discussed a quiz he had given to his students where his students had to create gift bags. He further explained the range of his students' answers for the problem from using factors to just drawing out the solution, "so to me the kids that are using factors and solving the problem that way that's pushing forward thinking" and to him was the definition of a level 4 response. He also discussed one particular lesson he did with his grade 6/7 class that he had changed from previous years and met with more success. He talked about teaching the volume of rectangular prisms and triangular prisms together and that "I find that grade 6's that if I say length times width times height for rectangular prisms then they're applying that to triangular prisms as well and they're just using a formula and not really understanding what they're doing" (Meeting #16 transcript). He decided to try the formula area of the base times the height in order to get students to start thinking about the process and met with more success.

Whenever the professional learning group meetings focused on having pairs of teachers presenting a question to the group, Blaine's partner always presented the discussions, such as the questions Ryan presented the group in Meeting # 12. He did seem comfortable with the teachers in the group and often joked with the members about topics other than mathematics. "I feel like I'm the parasite in that group because I really feel like I don't offer much to the discussion, but I take a lot from it in terms of certain things just because my knowledge is so limited I think compared with everyone else in that group" (interview transcript). As Blaine explained, "math was never a strong point for me as a student", and he felt that he had a lot more that he had to do in order to prepare for a mathematics lesson compared with his colleagues (interview transcript). His lack of confidence extended to his lessons as well when he told his students that anything good that they get to do in mathematics came from Emma.

Blaine also asked questions during the meetings in order to clarify his own understandings of mathematics. For example, he expressed his confusion about the question $7-5^2$ being discussed because he did not realise the square was attached to the five not a negative five. Later in the meeting he asked for further clarification to ensure his own understanding of the concept. Since mathematics is "just not a natural thing" for him, Blaine found a lot of benefit in going to the professional learning groups and gaining knowledge from his colleagues. He felt that since his "knowledge of teaching mathematics is so shallow…that anything is going to help", and he found that the meetings where they discussed student work would allow him to gain insights into the practices of his peers (interview transcript).

For Blaine, ensuring that the topics were discussed in the context of how they would be used in the classroom was particularly helpful, so the days where the teachers discussed mathematics questions did not help since they were out of context for him. He felt that he would need more time with those concepts before he would actually gain anything from those discussions. According to Blaine "one time is not enough because when Emma talks about renting a concept, I might get that at the moment, but when I walk out of there, that's not going to sit with me the next day in the one workshop thing" (interview transcript).

Blaine expressed how the professional learning groups were a source of focused and positive energy by working with a group of teachers who did not complain about the state of the world but worked together to make strides to improve. He found that he got a lot out of each of the meetings, including finding a focus for his own teaching based on the discussions of the group. Although he noted that he had always done long range planning, the topics of the group meetings would help him set where he would focus between the meetings, for example during the meetings where the focus was on justification. He also noted "I find a lot of value in watching a high school lesson. When I'm talking to the students about what they can expect, I can at least have someone to look at" (Meeting #12 transcript). Overall, although Blaine lacked confidence in his own mathematical abilities as well as his ability to teach the subject, he found that he was gaining more insight, knowledge, and support from attending the professional learning group meetings.

April

April taught mathematics to grade 7 students. For the first two years, one of the other teachers at April's school, Evan, attended the professional learning group meetings, but in the final year, she was the only teacher from her school. For April, the most important job that she had as a mathematics teacher was to ensure that her students understood the mathematics they were learning. April was very concerned that her students were learning the proper terms for mathematics so that they were creating a language of mathematics that would carry them throughout the rest of their lives. She told her students that “this is how you’re going to speak from now on”, and she felt that making this transition was the “biggest obstacle for them” (Meeting #9 transcript).

April also felt that the mathematics they were learning needed to be applicable to their lives as she noted “they have to see themselves as being able to use this or see it somewhere in their world” (interview transcript). For example, in algebra she talked about showing her students their calculators to relate seeing the letters on the calculator to the algebra concepts, as well as the idea of “undoing” operations being equated to “undoing clothes” (Meeting #9 transcript). She further discussed how she found it essential to link each of her lessons back to the lives of her students as well as to make connections to what they had learned previously. She related percentages to easy ones students had learned to calculate in their heads, like ten percent and having students use repeated addition to figure out larger percentages. In algebra, she would relate operations like 3(a+2) to 3(2+2) in an attempt to connect the new concepts they learned. Her mathematics lessons were integrated with different topic areas so that students were learning “mathematics” not individual lessons or units.

She wanted her students to be comfortable during her lessons so she had her students raise their hands at different levels depending on how confident they felt about their own solutions. She enjoyed the ability to use technology in her classroom and shared different ways she incorporated it. For instance she was really fond of using the Centio clickers with her SMART Board for evaluation and found that her students really enjoyed using them and the evaluation was quicker. She would build the quizzes to include more in depth questions where students would have to “provide me the proof” by showing their work prior to punching in an answer (Meeting #8 transcript). She had managed to procure five laptops for use in her classroom and she would have her “reluctant writers” use them to keep them involved (Meeting #11 transcript).

At the start of the school year, April gave her classes a scale of one to five for her students to rate their enjoyment and comfort with mathematics in order to gain some understanding of the students in her classroom. She was concerned about how “in grade 7, they’re already defeated”, worried about how the trend would continue into later years, and wondered what could be done about it (Meeting #10 transcript).

In her mathematics lessons, she used problem solving with her students to encourage their thinking about concepts. April also used more difficult problems with her students in order to see where the students were currently and “where do I have to focus my attention” (Meeting #16 transcript). She did express concern over the difficulty of using word problems with grade 7 students because some students “throw those numbers in the air and then try and see what something’s going to make sense at the end, and that’s not uncommon for us to have to deal with when you give them word problems” (Meeting #16 transcript).

As her other elementary peers, April graded her students using rubrics. She would level the questions on her page in order to help her determine the correct level for the student based on the work. She structured the activity with the more challenging level 4 question at the end so that the lower students would not just get stuck on the more difficult problem. For her a level 4 was “just a little beyond, apply something in a different way than we’ve seen it, and can you still see it” (Meeting #8 transcript). April tried to only put one of each level of question on her assessments. She worried that her students were not getting what they needed in grade 6 and would enter grade 7 and "they cannot do the regular algorithm for multiplication. They've got nothing" (Meeting #9 transcript). She spoke of sharing with her students the "magic box" which was an area model for teaching multiplication. She felt that grading a student could not be derived from a single assessment and included "other things you saw—it's a bigger package than just that piece of paper" (Meeting #9 transcript).

For April, the professional learning groups had an impact on her classroom practices by encouraging her to spend more time on specific units depending on the feedback of the teachers in the secondary school. For example, she started spending more time on algebra and reduced the amount of time in geometry because of the secondary expectations. She also found it important to introduce the language that the secondary teachers would be using in the future with her students so that they would be prepared. During the meetings she asked to know "what they need to know language-wise on the EQAO. I'd like to know what kind of words I should focus on" (Meeting #8 transcript). April also wanted information about how students were achieving on the grade 9 EQAO as well as the scoring practices so that she could properly support her students. She noted that she found benefit in having the shared marking practices with her peers and gained more insight into grading practices. She adopted practices from her elementary school colleagues including Emma's use of checklists on her papers. Gabriel brought in a work sample and had a smaller checklist at the top of the paper, and when April brought in a work sample the following year, she had used the same practice in her own work.

Although April found the groups very beneficial, she noted that she also found them to be very intimidating, yet she did joke with her elementary school colleagues during the meetings. April brought up being an older female in the group and how sometimes it was intimidating to be around the other teachers. She commented in her interview, “What if I’m teaching these kids all wrong, and they’re going to find out that I’ve messed up?” For her, she realised that she had to be confident in what she was doing because sometimes the stronger personalities would run over her comments and not even acknowledge them. As she noted “you have to have a lot of confidence when you bring in your stuff, especially because you’re dealing with high school teachers who have math degrees. I do not have a math degree” (interview transcript). April shared her lack of confidence with her own mathematics knowledge in comparison to the secondary teachers with their mathematics degrees noting that she had a first year university math course and that was all. April became concerned during one of the meetings where the group members discussed mathematics questions because Ryan had mentioned that they were more difficult than the first year, and she noted “now I’m all stressed” (Meeting #16 transcript). She did note that she felt she was a better elementary mathematics teacher because she could relate better to her students by coming “from a place of learning”, but she found that she learned from the other teachers by seeing why different mathematics operations worked (interview transcript).

April’s lack of confidence also came out in different conversations during the meetings. She asked questions of the other elementary teachers, but was quick to note that she used the same strategies in her classroom as they did. When it was suggested in the second year that if there was a problem with visiting Evan’s classroom, they could just visit April’s, her response was “I’m not good at the show” (Meeting #11 transcript) and mentioned “I don’t like presenting” during the following meeting (Meeting #12 transcript). When one of the secondary teachers expressed concern over what she said was “math vomit” on her students’ work, April said “I might have said math is messy, it might be my fault” (Meeting #8 transcript). During the algebra discussion, April asked the secondary teachers “am I okay to stick with n?...If they’re mucked up, this is grade 7, so that’s like three years, that’s fine” and talked about “I messed up that part” when she realised she had not started with using n in grade 7 (Meeting #8 transcript). At one point she related her difficulties with “explaining” the concept of why they are undoing operations with her students and noted “sometimes I just abandon it and just say you want to get rid of it, do the opposite on the other side. That’s when I get desperate at the end” (Meeting #9 transcript).

Despite any concerns over her own abilities to teach mathematics, she was dedicated to helping her students and meeting their needs. She had one particular student she described as a “solid level 2” that she made sure “he’s always the one I target to say how can I say it better to him” in order to help any struggling students who may misunderstand (Meeting #16 transcript). When teaching negatives she noticed “they rent them really, really well though during the unit”, but ultimately realised that her students had not yet become completely proficient with using them and relayed an activity that she did with her students using chairs and movement to help them understand working with integers better (Meeting #10 transcript).

During the year, April was especially inspired by one of the meetings at the university, and determined to try some of David Stocker’s techniques in her classroom. Following the meeting, she talked about how she could see the ideas as being able to weave real-life contexts into many different subjects as well as link it to the mathematics. She found a lot of success in incorporating the real world in her lesson discussing the disparity of wages of different areas of the world. When she discussed the lesson during the professional learning group she noted that she “loved it, that was just so much fun” (Meeting #16 transcript). She would have liked to do more units but she noted the “amount of time I spent preparing for that week and delivering that week, I couldn't do that more than once this year" (interview transcript). She planned to continue building upon the initial lessons in order to do more in future years since the topic really struck a chord with her philosophy of ensuring to integrate mathematics with students being able to see it in their own lives.

In the end, April believed that the professional learning groups were essential to her growth as a teacher, and she strongly believed they should continue. She was greatly concerned that eventually someone would step in and tell them what they needed to be doing as a group, and she worried that this lack of trust would destroy the group. For her, the group allowed her to focus on her students and the needs of those students in order to make changes and grow in her own classroom teaching practices. "It's really important that everyone have that opportunity to grow in the direction they need to, not the prescribed, somebody else said way" (interview transcript).

Wesley

Wesley was an older secondary teacher. He noted he was "too senior minded" and struggled to look at some of the teaching methods from an elementary perspective (Meeting #12 meeting transcript). For Wesley, his job as a teacher was "to be teaching and helping the kids at learning mathematics at their level and whatever they can do beyond" (interview transcript). Like April, Wesley stressed to students that they needed to be "speaking mathematically" (Meeting #12 transcript), so he ensured his students only used "one equal sign per line" when sharing their work (Meeting #13 transcript). He shared his concern that many times his students were placed in the wrong mathematics classroom because they were given the freedom to choose their classes in secondary school. He felt that the grade 9 EQAO scores would be better if the students were properly streamed and counselled parents to listen to the grade 8 teachers' recommendations about placement in grade 9.

The materials that Wesley brought to the group to share showed that he was more traditional in his classroom practices, with mathematics being a set of rules and procedures for students to memorise in order to be successful. He set up his lessons to start with him sharing examples and then having his students practice the skills. Wesley talked about liking "little tricks" he could show to his students so that they would be successful (Meeting #9 transcript). For solving equations, Wesley brought in a list of rules that he displays for his students to help them in his grade 10 Applied classroom. He talked about how he only had two and a half weeks for students to learn the list of rules "so the kids have the tools and then to move on to the course" (Meeting #8 transcript). He went on to note that the paper was not for all the students, but "it's made up for the kids who want to have some kind of structure to follow" and then he would give them repeated practice using all of the strategies in the list. "At this point in time in grade 10, we probably have zero time to go ahead and talk about, bring out the picture of the teeter totter" for students to gain a picture of the process: they just needed to learn the rules. He mentioned how he was quite surprised to see the amount of problem solving in the new textbooks. He noted his comment on seeing the textbooks was "oh gee whiz, I can't do this because there's an awful lot of skills that I have to really cover before I get to these. I find in our textbook we don't have a lot of skills, we have a lot of problem solving, not a lot of skills" (Meeting #8 transcript). He expressed his desire for more problems to practice using the concepts instead of problem solving in the textbook.

In describing his grading practices, Wesley noted that "if the question is more than one mark, you have to show your work" (Meeting #8 transcript), and he struggled with the concept of possibly using a rubric in his own classroom. “I look at rubrics in the publications we get for grade 9 and 10 math, I take a look at it, and I just go gee, I don’t know if I’m going to get around to that. I get the task, and it would be nice if I could see the strategies” (Meeting #8 transcript). He discussed how he could see the value in using a rubric because then students would know what they need to do in order to get their grade, but “I wonder though at times whether my information is falling on deaf ears” meaning the parents are looking for a percentage for their child’s grade (Meeting #8 transcript). Wesley talked about how he ensured his students were “not jumping to the right answer” initially when talking about the least common multiples, so “before I go to multiply by the leftovers, I like it to be a little painful, a little bit of work, in other words, I get them listing out the multiples so they can actually see what they are and use that method” (Meeting #7 transcript).

Wesley did enjoy the amount of technology that he had access to in his school and found it to be very beneficial for his teaching. He even went so far as to post his SMART notebook lessons on his website for the students to review after class. He noted that in future years, he would like to record voiceovers with the lessons in order to give his students a commentary of what they were seeing on the file. He shared his concern that sometimes his students would simply punch numbers into a calculator and not have a concept of what the numbers mean or where they came from. “I kind of explain that that’s pop machine mathematics, which is very useful at times. It’s very useful to put in your dollar twenty-five and get the pop if you want it, but it’s kind of neat to watch it move through the machine” (interview transcript). He said that he would “kind of emphasise that it doesn’t have to be difficult mathematics to be good mathematics or useful” with his classes of students (interview transcript).

Wesley discussed having a higher mathematics degree and his comfort with the subject area of mathematics. He felt that he was “kind of thinking down a lot of the times” (interview transcript) when discussing the mathematics content. He said that he did learn a lot about teaching of mathematics through the group discussions. In discussions where the teachers were questioning the mathematics, Wesley usually entered the discussion about what the mathematical principles were. For example, the group was discussing whether or not it was mathematically necessary to include a break in a graph if you are not starting at zero, and Wesley noted that “it’s just a procedure” and that it would matter in statistics only (Meeting #11 transcript).

Unlike the other members of the group, Wesley felt that the professional learning groups were very prescribed with someone telling the teachers what needed to be focused on and how to do it. He felt that the groups themselves “can use a little revitalisation really” and talked about the excitement of going to the NCTM conference to be around others who were passionate about teaching mathematics (interview transcript). Wesley shared his concern over the number of dominant personalities in the group and that “sometimes people talk too much” (interview transcript). “I think if you’re coming to the group, and you’re going to sit there and say nothing, why bother coming?” (interview transcript). Despite his concerns about the structure, Wesley did feel that the professional learning groups played an important role in keeping a teacher moving forward in their profession because it “keeps you refreshed” (interview transcript).

Throughout the majority of the first meetings, it was obvious that Wesley felt the groups were there to help the elementary teachers support the secondary teachers as a “vehicle” for setting up students for later in their mathematics careers. For example, he was quite adamant that the grade 7 and 8 teachers needed to be showing their students some more advanced concepts for later on, such as with factoring binomials. To Wesley, this was important for the teachers to do because “I don’t think we always have to think of our mathematics as being something that, oh it’s got to serve a purpose now” (interview transcript). “I think it’s an investment of a skill that they’re going to need three years down the road” (Meeting #9 transcript).

Wesley noted that the group meetings also gave him the opportunities to try new things, like algetiles, that he would have skipped over in his textbook before listening to the group talk about them. As he said, “you get stuck in doing the same old, same old” without having discussions with other professionals (interview transcript). One area he wanted to adopt was Emma’s rule of “undo and be fair” when working with algebra that he felt would help his students avoid some of their common errors (Meeting #8 transcript). Wesley also found the CLIPS videos that Emma had shared showing graphs where you could manipulate the multiplier and constant to be effective teaching tools. After the university meeting with David Stocker in the third year, Wesley became inspired to try something else in his classroom and worked on incorporating a theme from the meeting into his mathematics classroom to address concepts of number sense with his students. To Wesley, the message of the speaker was to “leave the kids with something they’re going to remember” and he felt that this use of real-world contexts would leave a longer lasting message so that the concept would not have to be retaught (Meeting #15 transcript). Wesley believed that “instead of cranking out a standard word problem about whatever, how about a word problem that involved tobacco use for kids. You still have your mathematics, but also have this other effective message going too” (Meeting #15 transcript). He was greatly disappointed when the lesson did not work out the way he had hoped which he felt was due to the extremely low achieving students in his classroom. He did share his desire to try again in future years and hoped a different mix of students would produce a different outcome.

During the meetings, Wesley also asked the other teachers for advice if he had a particularly troubling case with a student. For example, he asked for how to deal with a student who is totally engaged in the lesson but refuses to write notes or do the practice problems during the lesson. "He kind of misses out when it's time to sit back and do those questions because he didn't make that connection" (Meeting #11 transcript). Wesley felt that the professional learning groups were important to keep a teacher from going stale, but that there needed to be "a little bit more personal creative freedom" (interview transcript).

Owen

Owen taught secondary school with Wesley. He described himself as using more problem solving and manipulatives in his classroom as a result of his participation with professional learning groups in mathematics, although the lesson that he shared with the group was very traditional. The lesson the group observed consisted of problems for the students to work on teacher-directed whole group, notes for them to copy after the practice problem, and then exercises for them to complete on their own. For Owen, the most troubling characteristic of his classes was that a lot of the students did not want to be at school. Owen believed that his first priority was to create a relationship with his students. "I find the better the relationship is, the more likely they are to buy into what we're doing" (interview transcript). As such, he greeted his Locally Developed class of students by name at the door at the start of every class. He related a story of engaging one of his students who was passionate about drumming and belonged to a group, so Owen shared videos of the student drumming at the end of the classes. He was sure to talk to the student about drumming and he could see "how proud he was" (Meeting #11 transcript). He clarified that with students who wanted to be at school "you can do whatever you want, and they're still going to want to learn" (interview transcript). Owen also found the access to technology in the school to be an effective teaching tool that made his job easier.

In teaching mathematics he wanted to be sure his students shared "what do you do and why do you do it" when completing problems (Meeting #8 transcript). He talked a lot about "showing" students the "shortcut" so that they could complete the questions given to them (Meeting #9 transcript). For example, he talked about how teaching students to use the distributive property with 3(a+2) would be a shortcut to get them to the answer. He talked about using algetiles in his classroom and how he put them on the students' desks so that they could decide to use them because they would not volunteer for the "chips for dummies" if he did not (Meeting #10 transcript). Owen was also concerned that he prepared his students for university, so he talked with them about effective note-taking skills during his lessons.

For Owen, the professional learning groups represented a time for sharing of resources and allowed him to develop a common vocabulary with the elementary teachers in order to reduce confusion with his students. He worried that at times "we might get off topic" in the meetings, citing the conversations about how grade 8 students were fairing in grade 9 as not being overly beneficial to the group: "it was just kind of a check-up thing, so it was good for curiosity probably, but I don't know if it really helped" (interview transcript). Owen himself only participated sporadically, and his participation seemed to be dependent on whether or not Samuel was sitting next to him so that he could engage in off-topic chats. Ultimately Owen believed "I find that maybe I'm getting a few more kids that would have fell [sic] through the cracks before" because of his participation with the professional learning groups (interview transcript). There were times in the conversations where Owen thought "I never would have thought of teaching it that way", so he valued the sharing to help improve what would be most beneficial for his students (interview transcript). Unlike Wesley and April, Owen was a lot more skeptical of the speaker that came in the third year and worried that his students would think his lessons were "fake" and that he did not have the "parental support" needed to broach real-world topics (Meeting #15 transcript). Owen was conscious of the fact that all the teachers in the group were there because "we're all good math students" and he noted how not all of his students would have the same ease with mathematics and may need more than he did as a mathematics students (interview transcript). He noted that although he could just be shown something in mathematics once and get it, his students needed to be shown in different ways to help reach all of them.

Discussion

Due to the personal nature of professional learning groups, I explored what the experiences of the individual teachers were within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching. Five of the teachers were chosen for narratives in order to give a variety of perspectives to illustrate a range of viewpoints of the members within the group. The first, and perhaps most vocal participant of the group was Emma. Emma was a strong mathematics teacher with a love of mathematics and appreciation for the success of her students. She was also very dedicated to the learning of her colleagues and strove to share not only with the other teachers in the group during meetings, but with her peers at her school.

Blaine was a teacher at Emma's school, but participated very little in the discussions of the group, which he attributed to his lack of knowledge about teaching mathematics. Slavit and Nelson (2010) pointed to a potential problem with members who remained quiet during meetings noting that they did not often adhere to the consensus of the group. Blaine on the other hand claimed to have used the meetings to absorb the wisdom of his colleagues and applied this knowledge to his classroom.

April, also an elementary teacher, pointed to a knowledge and gender issue that was raised for her during the meetings. She felt that being a woman and an elementary teacher put her at a disadvantage in dealing with the predominantly male secondary teachers in her group. She saw the disparity between her mathematics knowledge and the knowledge of those with specialist degrees to be intimidating. She did feel that this very difference made her a better elementary teacher because she could relate to her students being in a position of still learning.

Wesley was an example of how a very traditional teacher could be exposed to a professional development opportunity that would influence and greatly change his priorities in teaching. He questioned his beliefs about teaching mathematics as a direct result of being inspired by another educator who helped Wesley experience a new perspective in his teaching. Wesley attempted a new lesson with his students, but when it was unsuccessful because of the academic difficulties of his students, he did not adapt the lesson and attempt it again. My hope is that Wesley will try the lesson again using some of the reflections from the group about its difficulty and meet with success. Research has pointed to teachers needing to confront their own beliefs about teaching mathematics to make changes (Cross, 2009; Wilkins, 2008), and Wesley has definitely begun that journey. Given his expressed excitement over changing his mathematics lessons, hopefully Wesley will continue to grow and make changes in the coming years of his teaching career.

Owen at times seemed to disagree with the beliefs that the elementary teachers brought to the table in the meetings, yet in his interview pointed to using more of the strategies discussed. One notable example of the difference between Owen and his elementary colleagues was when he referred to the manipulatives as “chips for dummies” (Meeting #10 transcript). The varying perspectives of the members led to rich and meaningful discussions about teaching mathematics.

Beliefs

The majority of the secondary teachers mentioned “showing” their students how to get to an answer so that they would learn the curriculum. Elementary teachers talked about being ready to learn the content and making sure their students “understood” what was being taught. The one secondary teacher who seemed more in the middle of the dichotomy was Ryan. He seemed excited about trying out strategies and activities brought in by the elementary teachers, such as the National Library of Virtual Manipulatives, and he talked about students learning the “conceptual versus procedural” (Meeting #9 transcript). One of the secondary teachers shared an acronym they used in their classroom which the other secondary teachers mentioned liking, but Ryan noted “I don’t like acronyms if it doesn’t come with conceptual knowledge” (Meeting #9 transcript). Ryan also noted many times how long he would spend on concepts in order to make sure he was building understanding with his students. At another time Wesley talked about teaching his class SAMDEB as the procedure students would follow when “undoing” the order of operations, and Emma spoke about having the students create the acronym for themselves within the exploration activities.

One notable example of the difference between the secondary and elementary teachers was Wesley’s discussion about factoring binomials. He brought in the discussion, based on a conversation he had with Madison, that the grade 7 and 8 teachers should be showing students how to “get rid of” the fraction by multiplying by the common multiple. Figure 6 shows the example that Wesley used on the board to share this concept with the group. In this case, students should automatically multiply each part of the equation by “12” in order to remove the fractions. Wesley saw this as a way for students to be more prepared for later in school and that it was important to "teach them" this skill. The elementary teachers were quite concerned that by doing this it would simply become a procedure. As Gabriel noted with his students "they don't own it, and maybe they're not ready to own it" (Meeting #9 transcript). Emma echoed the sentiment commenting that "I don't just want them to be procedurally fluent, I want them to have some conceptual understanding" (Meeting #9 transcript). The board liaison attempted to share a strategy that would create a bridge for the students to lead to the task, but again relied on a strictly procedural application of the concept. Another secondary teacher joined into the discussion to share how he would "show it" to students so that they could use it. The other elementary teachers commented on how their students were just gaining some of the knowledge they would need and this would be too much for where they currently were in mathematics.

Figure 6. Exact example used by Wesley to illustrate his point.
Wesley felt students should automatically multiply by the common denominator 12 to get rid of all the fractions as the first step in solving the problem. Note: the example does not work as a valid equation.

In dealing with the mathematics that needed to be learned in secondary school, some of the secondary teachers struggled with the fact that fractions and integers were not part of their curriculum and yet the students had not mastered it. The elementary teachers voiced concerns over how new those concepts were to their students and that they were still "renting" the ideas.

As Samuel noted, he wanted to spend only a short session with his students on subtracting negatives because it should be just a review and asked if his students should have memorised the procedures for working with fractions. Ryan discussed learning that multiplying and dividing fractions was so new, so he spent more time in his class to properly build the concepts before moving on to more complex uses of fractions. Several of the secondary teachers mentioned teaching their students to write notes in math class so that they would have the skill for university. Other than Evan, none of the elementary teachers found this to be a necessary skill. Emma noted that “I have fifty minutes, which isn’t nearly enough time…I don’t have time for you to copy down that graph. I have time for you to play with the idea, and I’m not convinced you’re going to practice this on your own time…so I really feel if it’s important that you have it, that you need it, I’m going to give it to you” (Meeting #11 transcript).

There also seemed to be differing views about the use of manipulatives. Noah for instance talked about showing his students how to properly use the manipulatives and giving them almost a procedure for using the algetiles. Noah talked about how using algetiles made it so much more of a production to teach the concepts and called them a “pain” to use (Meeting #16 transcript). In another meeting, he talked about having used fraction strips for the first time and how they seemed to help, yet his students got sick of them really quickly. Ryan noted that “all of those tools are just to get them in”, and the male secondary teachers agreed with the idea (Meeting #10 transcript, see Appendix O for transcript). Owen called the manipulatives “chips for dummies” and Emma countered that her students would never say or think of them that way. During the third year, the meeting held at the university with David Stocker brought up some differing beliefs about teaching mathematics. Most of the elementary panel, with the exception of Diana, thought that the ideas were worthwhile and should be incorporated into the classroom to bring some more value to their lessons. On the secondary panel, Wesley was most excited about trying the strategies and was very inspired by the talk. Ryan liked the idea of incorporating more real-world applications but worried about diving into something too deep for him to be able to handle in his classroom. Both Owen and Noah were adamant that their school and the parents there would not accept trying any of the strategies and shared concern over their students thinking they were being fake.

Although the secondary and elementary teachers seemed to have beliefs on opposite ends of the spectrum, there seemed to be differing beliefs for the secondary teachers for their two different types of classes. For them, the way they taught mathematics differed depending on whether it was an Applied or Academic class. To highlight the difference, Samuel talked about how the Academic classes are being geared toward the academic road and need to get those concepts; whereas the Applied class needs to spend more time on concepts to really get them because the Academic students would be able to figure out the concepts for themselves. Madison talked about making sure the Academic classes took more notes so that they were more prepared for later on. The idea came up several times that the Applied classes needed more hands on and manipulatives to help them with the concepts and as Noah noted he gave them more structure and a template to work with. Ryan talked about explaining concepts to Applied students and avoiding using terms like “the common denominator” because it would be too big words for them (Meeting #12 transcript).

The elementary teachers who conversed openly in the professional learning group meetings all seemed to believe in the reform methods of teaching mathematics and focused on their students understanding what they had learned. In the beginning, Noah, Owen, Wesley, and Samuel all seemed focused on ensuring their students could answer their questions and would pass the exams they were giving. Ryan seemed to fall in the middle of the two groups, valuing the understanding of his students, but still teaching in a more traditional method. It was unclear from listening to the discussions Madison had during the sporadic meetings she was involved in where she fell on the spectrum.

Discussion

Due to the importance of a teacher's beliefs in making decisions in the classroom (Potari & Georgiadou-Kabourdis, 2009), I examined what the beliefs about mathematics teaching and learning were of the individual teachers in the group and how these beliefs were dealt with in the discussions of the group. Specifically for my study, I was concerned with two aspects of teachers' beliefs: "(a) what kind of mathematics is important for students to learn, and (b) how this mathematics should be taught" (Grant, Hiebert, & Wearne, 1994, p. 9). In examining how mathematics should be taught, there was a dichotomy that arose between the secondary teachers and elementary teachers. The elementary teachers spoke about supporting a more reform-based approach to teaching mathematics as espoused by the NCTM (2000; Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics, 1989). The elementary teachers talked about making sure their students understood the mathematics they were learning and did not just have procedures to solve the problems given to them. Manipulatives were commonly discussed by these teachers as being used to allow students to work with the mathematical concepts being introduced in their classrooms. A common theme that arose with the secondary teachers was "showing" their students how to solve mathematics problems. Skemp (1986) discussed in his research about the need for teachers to have students learn methods that have meaning as opposed to simply create a short cut to the answer. The secondary teachers were very concerned with the scores that their students were getting on exams and whether or not they were able to answer the questions.

Ryan seemed to lie somewhere in the middle with both valuing that students understood the mathematics and not liking to use acronyms or “shortcuts” that did not have a conceptual basis, yet his classroom lessons that were observed followed a very traditional format. In fact, both the secondary teachers observed were traditional in their teaching methods, while both elementary teachers followed a more reform-based approach to teaching mathematics. To match their beliefs about teaching mathematics using more exploration, the elementary teachers used rubrics to assess the process while the secondary teachers used a point system of right and wrong answers. Manipulatives when used in the secondary classrooms were talked about as having “shown” the students how to use them to answer a question or a “hook” to get them into the ideas; whereas, the elementary teachers took a more exploratory approach to using them. Since beliefs influence the decisions made in a classroom, it is necessary for some of the teachers in the group to believe in reform-based methods in order to influence the conversations. In this group, it was clear that the most outspoken elementary teachers strongly believed in giving students problems to solve and the freedom to choose their solution methods.

The point of separating the results section into three distinct chapters was to ensure the participants in the professional learning group were given a special place in my research. For this chapter, an in-depth description of five of the members of the group was provided in order to provide a range of the different personalities within the group. The chapter concluded with a description of the beliefs of the group members and how there were differences between the beliefs of elementary and secondary teachers.

The next chapter concludes the results section of my research with a description of the internal organizational features of the professional learning group. The chapter begins with an examination of the professional learning group characteristics central to the group's function. A description of the strengths and weaknesses of a professional learning group as provided by the individuals in the group follows. Next is a description of the knowledge of teaching evident in the conversations of the teachers. The chapter concludes with the areas of mathematics knowledge that were discussed by the members of the group.

CHAPTER SIX: RESULTS—PROFESSIONAL LEARNING GROUP DYNAMICS

In this chapter, I first describe to what extent this professional learning group adhered to the characteristics used in the research literature to define a professional learning group. Next, I illustrate how the teachers' mathematical knowledge was addressed through the discussions in the group meetings. Finally, I highlight ways in which the design of the professional learning group supported the teacher members to make changes in their own classrooms. Following each of the sections is a discussion of how the resulting examples from the case study link to current research.

Professional Learning Group Characteristics

In order to answer my first research question, I looked for evidence of the characteristics defined by Hord and Sommers (2008) and DuFour and Eaker (1998). Below is a description of how each of the nine characteristics is illustrated through examining the interactions in the group. Each is followed by a discussion of how the observations adhered to or deviated from the current research literature.

Shared Beliefs, Values and Vision

According to individual conversations with members of the professional learning group discussed in my research, as well as examining the Student Success initiative, the participating school board created and funded the groups in order to facilitate the transition of students moving from grade 8 into grade 9. The group members focused their group on improving the learning of all of their students, not just the grade 8's and 9's. Through their attendance and input in the conversations, all of the members of the group were committed to helping their students achieve success in the transition as well as be successful in mathematics. Participants were very clear that even when examining data from student testing, they were attempting to find ways to help the students, not judge the teachers, and they encouraged each other to ask questions (Meeting #8 transcript). Emma noted that "it's not been a finger pointing place" (interview transcript), so the group was able to maintain a professional examination of the data. The members discussed alternative ways of teaching in order to provide support for students entering secondary school, as well as ways to assist students once they were in grade 9. As Wesley commented, "I think it's a good idea to get a bridge going of what's happening in elementary and what's needed in secondary" (interview transcript), and the group maintained this focus as they worked for the benefit of the students. Although all the teachers were focused on improving student learning, it was apparent that the teachers approached helping students from different perspectives related to their own beliefs about teaching mathematics. For example, some teachers felt that using more manipulatives and taking time to unpack the concepts was most beneficial and others found new acronyms to help the students remember the rules.

The group began the first meeting in the first year with the quote "no busybody work" and on the first meeting of the next two years, they maintained this mantra. To explain this important mission statement, Ryan pointed out, "We're not going to do things just to waste people's time" (Meeting #7 transcript). In order to make the professional learning group successful for all, the teachers maintained that they would not create any new materials simply for the group meetings, nor would they waste their time on paperwork or activities just for the sake of doing something. Members of the group adhered to making sure that the activities in the group meetings would help their practice and items brought in to share with the group were lessons that they were actually using in their own classrooms. Although the teachers approached the meetings from a variety of different beliefs about how to teach mathematics, the core focus of the meetings was the success and learning of their students in mathematics.

Discussion.

In examining the professional learning group during their meetings, I found evidence that suggested that the group sought to accomplish a common goal for students (Hiebert et al., 2003) in ensuring student success in secondary school. Although the research points to needing a shared beliefs, values, and vision (e.g. Hord & Sommers, 2008), this group functioned well with just sharing the common goal of improving student learning, despite differing in their beliefs about how this goal could or should be accomplished. The focus on student learning is highlighted in the research as being vital to the beliefs, values, and vision of a successful professional learning group (Eaker, 2002; Hall & Hord, 2006; Hofman & Dijkstra, 2010; Hord, 2009; Hord & Sommers, 2008; Reeves, 2010; Schmoker, 2006). Research also suggests not imposing a vision on the teachers (Huffman, 2003), and all members were given opportunities to share and encouraged to make changes in practices without being forced to do anything. Research suggests constructing a focus to serve as the guide for the discussions and meetings (DuFour & Eaker, 1998; Eaker et al., 2002b), and the group did use their mantra of "no busybody work" and supporting student success as the guiding principles. The group members may have changed their practices in different ways, but they were all committed to ensuring the success of their students in mathematics.

Shared and Supportive Leadership

Each of the meetings was kept on task by a single member of the group, Ryan. Throughout the three years, he kept the minutes, sent out reminders about the dates, and took charge of any paperwork required by the school board. Although in his interview and during one of the meetings, Ryan mentioned feeling like he was in command, only one other member mentioned this as well. Other than Wesley, all the other members interviewed commented on how the meetings were run by all of them and that their ideas and wishes were valued. The members felt as though the group was part of all of them, despite Ryan and Wesley feeling somewhat differently. Throughout the meetings, whether planning topics or setting dates, Ryan was constantly asking for input and ensured that he took into account the voices and feelings of the other members of the group. Ryan even made sure to point out that the definition of a professional learning group required shared leadership. At the end of the final year, Ryan did comment that he would be taking a leave of absence the next school year, so someone else was going to have to take charge of keeping the minutes and dealing with the administrative duties.

From observing the meetings, there was evidence that it was necessary for someone to take control and keep the group on task as some of the members were prone to wandering off topic or resorting to complaining about something not entirely relevant to the meetings. Anytime that Ryan would leave the room to make copies or get materials for the group, some of the other teachers would start talking about other topics instead of maintaining the flow of the mathematics related conversations. Ryan also stopped members from interrupting each other, and at times would bring the conversation back to the topic that was on the agenda to be discussed. For example, during one meeting several of the teachers began talking about caloric intake and applications for their phones that would help them monitor their calories. The topic of the meeting was supposed to be discussing a mathematics lesson, so Ryan had to bring the off- topic conversation to an end to get back to the discussion. Ryan also stayed cognisant of the time to ensure that everyone had a chance to share or talk but that all items on the agenda were discussed.

One concern that came up in several of the meetings and in three of the interviews was the idea that the school board would take over the meetings. The group members felt that the reason their professional learning group was successful was because the teachers were in control of the events, topics, and discussions and they expressed concern over losing this in favour of a board-mandated plan. As Emma noted, “Sometimes our language plgs [professional learning groups] feel like it’s something being done to us, like they haven’t trusted us to figure out what [it] is that we need to do” (interview transcript). April seconded the notion that the board had “to have the faith and trust in that process that we will go there” (interview transcript). Ryan mentioned that the board had so far left the group alone because it was successful and they did not need to mess with it (focus group interview). Wesley also mentioned the board controlling professional learning groups and how he felt that was unnecessary, but unlike the others, Wesley thought that the math professional learning groups were “very prescribed” and they lacked “creativity” (interview transcript).

Discussion.

Research into professional learning groups supports the need for shared leadership (e.g. Hord & Sommers, 2008). In this case, one member, Ryan, strove to keep the group on track, yet the group was given autonomy to choose their meeting dates and topics. Different members would speak up at different times when they felt the meetings were getting off the purpose or if shared practices were seen as inappropriate or ineffective. Research about effective professional development has documented the need for strong leadership within a group (Eaker et al., 2002b; Hofman & Dijkstra, 2010; Huffman, 2000; Wixson & Yochum, 2004), and having Ryan keep the meetings on course was important for the success of this group. The close camaraderie of the group members often led to off topic conversations that needed to be pulled back to focus and Ryan assumed this role in the meetings.

The group members' concern over having the board step in and impose changes is supported in the literature as being an ineffective leadership model (Buckner & McDowelle, 2000; DuFour & Eaker, 1998). The autonomy of the group was important to the group members who were concerned that someone would step in and moderate the discussions or force the group direction. In the end, the teachers were the ones who would enact the changes in their classrooms and therefore should be given the authority to make decisions about what aspects of their teaching need to be examined (Blegen & Kennedy, 2000; Hall & Hord, 2006). The teachers were supportive of a more democratic model (Carr, 1997; Hord & Sommers, 2008; Mullen, 2009) that was currently being employed in their meetings. Although the professional learning groups were enforced by the administration, the teachers would able to work within the mandate of meeting and make the group their own. By working around this top down model of professional development, the teachers had more freedom in their choices of discussions and focus. The model employed encourages teachers to make changes in their classrooms (Blegen & Kennedy, 2000; Hall & Hord, 2006), which is important since ultimately it is up to the individual teachers what happens in their own classrooms (Blegen & Kennedy, 2000; Caine & Caine, 2000; Patterson & Patterson, 2004; Schmoker, 2006). By giving the teachers the power to control the direction of the meetings, the professional learning group was positioned to support the teachers in making changes based on areas they felt were important to their practices.

Collective Learning and Its Application

From looking at the interviews and listening to the comments of the members during meetings and the focus group, all the elementary and secondary teachers were taking something from the group and applying it to their classrooms. Ryan noted that the goal of the group was "try to keep everything relevant, pertinent to what we're doing in our classroom" (Meeting #7 transcript). Each of the interviewed participants shared something that had an impact on their practice. Ryan described keeping the "different strategies and different terminology we talk about as being consistent through the seven, eight, nine, ten programs" (interview transcript) and talked about seeing the secondary school teachers doing the same thing. Gabriel also described how "the way that I teach, and the words that I use, and the emphasis like really focusing on algebra" was all attributed to the professional learning group meetings and discussions (interview transcript). Blaine talked about how the meetings helped give him a focus of what he was going to stress in his own classroom between the meetings (interview transcript). Owen mentioned "the sharing of resources" and "common practice" were important (interview transcript).

During all of the group meetings, different members asked questions of each other in order to clarify topics and make changes. Eight out of sixteen of the meetings discussed the topic of algebra. The teachers felt the curriculum did not provide enough support for elementary students to be successful in the topic in grade 9. Three of the elementary teachers mentioned how much they have stressed algebra, taking their students further over the past years, based on the discussions of the meetings.

The teachers all worked together in an attempt to create common language used in mathematics from elementary to secondary school. For example in a discussion about factoring, the elementary teachers shared that they used the terms "prime factorization" and "prime numbers" (Meeting #7 transcript) and this encouraged the secondary teachers to use the same language to help with transitions. Another common term that in the beginning came up from the elementary school teachers and was used by the secondary teachers in the next year was the idea of "undo and be fair" (Meeting #1 field notes and #12 transcript). In discussing how students would solve an algebraic equation, the elementary teachers explored with their students making sure to keep the equation balanced by using the opposite operation or "undoing", and then having to do the same thing to the other side, or "be fair" in order to maintain the correct solution. April expressed concern over lower elementary school teachers using "kid friendly" phrases with their students (interview transcript). As she commented, "We are, especially in grade seven I find, going from some language that we don't ever use again to language that we're going to use forever" (interview transcript). She felt it was important for her students to hear the actual mathematics language that would be used in secondary school and throughout the remainder of their lives (Meeting #9 transcript, interview transcript). Wesley reiterated this feeling of ensuring proper mathematics language was used, in his discussion of division noting "never say cancel because cancel means it goes away" (Meeting #9 transcript). At the end of Meeting #12, the teachers compiled a list of terms they used in their own classrooms during algebra, equations, area/perimeter/volume, angles, and relationships and graphing lessons, in order to aid in coordinating vocabulary between the grade levels. Language became a central theme in the discussions in order to ensure students' needs were being met for not only the intermediate years, but for their lifetimes as mathematicians.

During the first year, the teachers engaged in a discussion about using rubrics to grade mathematics since it was common practice in elementary school classrooms. The secondary teachers had questions about how the elementary teachers would define levels and calculate grades for their students. The secondary teachers attempted to coordinate their own grading practices of assigning a percentage and "how many points" would be assigned to each question with using a rubric, and a discussion occurred regarding how different the two processes were (see Appendix L for transcript of the conversation). The secondary teachers expressed a desire to be more comfortable with using a rubric to grade, and the elementary teachers were curious how other elementary teachers used rubrics. Questions by the secondary teachers occurred in a few more meetings, so as a result in Meeting #9, the group members engaged in a moderated marking task. The secondary teachers asked many questions about using rubrics during the meeting, and the elementary teachers also clarified for each other how they defined the levels in their own classrooms. For example, Wesley talked about how he defined “some understanding” as meaning 50% or less is incorrect. The board liaison mentioned how a level one would mean that only 25% of the work would be correct. Several of the elementary teachers argued that the levels on a rubric are not defined like that and cautioned that the levels were not to be attached to percentages in such a literal sense. Gabriel noted that it was not like when the teachers in the group were children, and a certain level did not mean a student had less than 50% understanding. Emma further clarified that a level 2 is defined as “less than par” (Meeting #9 transcript). Samuel commented that elementary teachers gave “a mark for every step” in mathematics, and the elementary teachers clarified that a rubric had them looking holistically at the piece of work, not just giving a mark for a step (Meeting #9 transcript).

For four of the meetings, all of the teachers visited the classroom of one of the teachers in the group. As Gabriel mentioned, the observations created a common shared experience that was very valuable for encouraging discussion (Meeting #13 transcript). Diana found the observations valuable because her lack of experience in teaching mathematics was improved by seeing a more experienced teacher navigate a mathematics lesson. Claire seconded the sentiment in talking about seeing Emma’s classroom the first year and how helpful it was to her own teaching practices.

Discussion.

The group shared their learning about student thinking through their discussions about how to best support students as they transitioned into secondary school. This focus on teacher and student thinking is supported in the literature as being a feature in effective professional development (Cwikla, 2004). The group focused on how algebra was a vital part of success in the secondary curriculum, so it became necessary for the elementary teachers to make changes in their own classrooms to support the students. It was through the questioning of their own practices (Sykes, 1999) that the teachers made decisions about ways to better support their students. By recognising the difficulties students were having with algebra in secondary school, the teachers used their own classroom problems to find solutions (DuFour & Eaker, 1998; Linder et al., 2012; Loughridge & Tarantino, 2005; West & Curcio, 2004) to narrow the gap. The group also sought to share practices in order to encourage consistency of terminology and practices between elementary and secondary school. By creating consistent practices, there was a group focus on improving teaching methods (Hiebert, Morris, & Glass, 2003) as well as mathematics knowledge (Allen, 2006). Furthermore all of the discussions were based in their own classroom practices as has been suggested in research as being helpful (Bednarz et al., 2007; Gojmerac & Cherubini, 2012; Lieberman, 2000; Lieberman & Wood, 2002).

Since change is the group responsibility (Opfer & Pedder, 2011), all of the teachers participated in the conversations they felt they could contribute to and made suggestions. As Gojmerac and Cherubini (2012) and Wixson and Yochum (2004) note, collaboration leads to changes, so having the teachers work together is essential to their professional development. The teachers in the group also reflected on their own practices and the practices of each other, which is important in making improvements to teaching (Arbaugh, 2003; DuFour & Eaker, 1998; Eaker et al., 2002b; Hofman & Dijkstra, 2010; Turner et al., 2011). The group members also visited each other's classrooms, which Reeves (2010) indicates is important to see what is happening in the classroom. This also allows for reflection to be embedded in the actual practices of the teachers (Ball & Cohen, 1999). By keeping the conversations focused on students' learning and remaining critical of their teaching practices, the group members were able to make changes in order to better support their students.

As a note, I found the conversations that the teachers engaged in about rubrics to be interesting. Other than Wesley, none of the secondary teachers mentioned seeing rubrics in their materials, and even Wesley commented that he just did not have time to use them and wondered about whether or not they would be useful (see Appendix L for transcript). In examining the secondary curriculum (Ontario Ministry of Education, 2005b), which all of the teachers are required to use, actually contains a rubric for use with the mathematics curriculum (see Appendix M for rubric). As the Ontario Ministry of Education (2005b) notes, "It enables teachers to make judgements about student work that are based on clear performance standards and on a body of evidence collected over time" (p. 18). The document then explains the different portions of the rubric, not once mentioning how the aspects of the rubric should be equated to a percentage. In comparing the achievement chart to the one found in the elementary curriculum (Ontario Ministry of Education, 2005a; see Appendix N), the information provided in the two charts is identical. The only change is that in the secondary rubric, there are percentages listed above the levels. For example, Level 1 is listed as 50-59%, not 25% as was indicated by the board liaison in the meeting.

Supportive Conditions

The teachers in the meetings were very open about how important it was for the board to set aside funding to allow the meetings to continue (e.g. Meeting #3 field notes). Two of the meetings attended by the group were held at the university in order for the teachers to gain new perspectives about teaching mathematics. During the first year, the board liaison was able to secure additional funding for the teachers to attend the meeting by pulling resources from another pool of money in order not to use the professional learning group budget for the event. Meeting attendance for those who were invited to the professional learning group was declared to be mandatory by the school board in the second and third years. Near the end of the first year of meetings, the teachers were encouraged to hear that the school board's focus for the following school year was to be assessment, and they hoped this would mean additional funding for mathematics (Meeting #6 field notes). The teachers had struggled with the literacy focus of the school board and wanted to make sure they had enough time for teaching mathematics in the manner they felt was best for students.

Time during the school day became one of the common concerns of the teachers. They found having the time set aside to meet was important to their teaching, but struggled finding time to teach mathematics in the ways that they felt were most beneficial to students. As Emma commented, "instruction time is a precious commodity" and noted there was about 50 minutes for mathematics instruction a day when teaching on a rotary schedule (Meeting #8 transcript). The challenge of finding time to teach mathematics was compounded by the board-mandated literacy block as a result of the board's focus on literacy. Owen and Ryan echoed the struggle with fitting all the mathematics they needed to teach into the schedule set out in the secondary curriculum and how difficult they found returning to concepts that were vital for their students (Meeting #11 transcript).

Most of the teachers interviewed mentioned how comfortable and supportive they found the group meetings. Laughter was commonly heard in the meetings as the teachers made jokes with each other and often volunteered each other for tasks that were uncomfortable. It was obvious that the group meetings were designed to be very welcoming and supportive. The teachers were also willing to put in extra time to attend the meetings as the conflict between the different timing schedules for the elementary and secondary schools weighed on their decisions about meeting times. Although the majority of the elementary teachers would have been finished their school day for at least twenty minutes already, the group members attended the meetings and stayed until 3:00.

From examining the proposed activities for each meeting, it is clear that the group did not always stick with their own pre-set agendas. Instead, as group discussions continued and the members felt the value of them, other topics were tabled until the conversation was concluded; alternately if new ideas were more timely they were discussed instead. As Gabriel pointed out, the amount of time taken to talk about a particular subject did not really matter as long as the discussions were valuable (Meeting #9 transcript).

The size of the group was a concern to Wesley who felt that the dominant personalities over ran some of the other members. As he noted, “the talkers of the group tend to overshadow the non-talkers of the group, and I think there was [sic] a couple of members who particularly kept silent, and I think they fell into their silence” (interview transcript). In their interviews, both Gabriel and Emma also expressed concerns about the strong personalities possibly intimidating or overshadowing other members of the group. Wesley commented that the classroom observations might be more beneficial if the groups were smaller so that more voices could be heard and that the teachers might get more out of a slightly different format (Meeting #13 transcript). On the other hand, Ryan, the board liaison, and Gabriel all mentioned that it would be nice to include more new people into the group to help spread the ideas.

Discussion.

The members of the professional learning group worked to create a supportive environment that was vital to their successes (Blegen & Kennedy, 2000), allowing the individual members to make both jokes and a comfortable working group. Although members were invited by the board, they were not forced to join the professional learning group. In personal communication with the members in the group, I discovered that other professional learning groups in the area had trouble with membership, but in the group I observed, the members willingly attended the meetings. Caine and Caine (2000) pointed to the necessity of ensuring that collaborative communities are volunteer based since no one can force individuals to change, and this group thrived by allowing the members to choose to be part of the group. Although some members expressed concern over some of the stronger personalities in the group, the group members were comfortable with attending the meetings and sharing ideas. The group members appreciated having time set aside by the board to meet, and research indicates that having the time to meet is important (Anderson, 2005; Eaker et al., 2002b; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008; Schmoker, 2006). Some of the group members raised concerns over the number of individuals in the group. The literature echoes the need for enough members for diversity of opinions, but not so many that all cannot share their ideas (Arbaugh, 2003; Hofman & Dijkstra, 2010).

Shared Personal Practice

With the exception of three members (Blaine, Claire and Diana), all of the group members entered into the discussions of every meeting. The two of these members that were interviewed commented on how valuable the meetings were despite their lack of participation and Diana mentioned a number of times (e.g. focus group transcript) that she was new to teaching so was gaining a lot from the meetings. Ryan noted, “Our PLG attendance and participation has been excellent. Not all [family of schools] are in the same situation” (Meeting #3 minutes). The majority of the members did attend most of the meetings (see Table 6).

Table 6

Attendance of Group Members

Elementary Member	Meetings Attended	Secondary Member	Meetings Attended
Diana	8/13	Ryan	15/15
Gabriel	14/15	Owen	14/15
Emma	15/15	Noah	13/15
Blaine	10/10	Wesley	8/10
Claire	9/15	Tara	1/1
April	13/15	Samuel	5/14
Evan	10/11	Madison	10/11

Note. Attendance was only recorded for 15 out of the 17 meetings of the three years. Not all members were part of the group for the entire three years. Attendance is recorded as number of meetings attended out of the total number of meetings that the person was a member of the group.

Although the attendance of the meetings was relatively high for all the members, not all of the members participated in the activities. For five of the meetings, teachers were requested to bring something from their own classrooms to discuss. The first meeting requiring a teacher work product was Meeting #8. Teachers were asked to bring in a current assessment and only Ryan, Gabriel, and Emma brought in assessments. Wesley brought in a rule sheet he used in his classroom when teaching algebra. In Meeting #9, for which teachers were asked to bring in a sample lesson plan in the area of algebra, Owen, Evan, and Emma shared their lessons, and Wesley brought in an idea he felt the elementary teachers could employ to support his teaching. For Meeting #16, teachers were asked to bring in a level two sample of student work with a focus on the process of justification. As a note, the teachers were asked to bring a sample of how they used justification in their mathematics classes to Meeting #14, which I missed, and I was told did not go very well. For Meeting #16, only April, Owen, Gabriel, Ryan, and Emma brought in samples.

Following the David Stocker event at the university on the theme of social justice, all of the teachers agreed to teach a lesson using social justice or at least using more realistic numbers in their mathematics lessons and gave a list of ideas to Ryan to put in the minutes. The teachers were all asked to share their practices during the final two meetings of the year. Over the next two meetings, only Wesley, April, and Gabriel met the goal and shared the challenges and successes of their lessons.

Teachers talked during the meetings about how different practices they picked up from individuals during the meetings were spreading amongst the group. Emma, April, and Gabriel brought in lessons to share and several of the other teachers mentioned wanting copies of the files. Whenever Emma would describe one of her lessons during the meetings, she would mention how Blaine was also using it. Emma mentioned in her interview that everything she did, she shared with Blaine. During one of the meetings, Owen and Ryan shared some lessons that they had used and commented on how they had been using the same lessons in order to better align their grade 9 classes (Meeting #9 transcript). Claire mentioned how she adopted the checklist that Emma had previously shared (Meeting #8 transcript). Other resources were also shared among the teachers including a disk of math CLIPS to use in a SMART Board lesson, a binder of graphic organizers, and a file of "handy pages" containing different mathematics manipulatives for SMART notebook. The teachers also discussed websites that helped in their own classrooms including the National Library of Virtual Manipulatives, the EduGains website, twiddla.com to create a virtual whiteboard, and SMART exchange for lessons created for the SMART Board.

Discussion.

All of the meetings showed the group members' commitment to creating a shared personal practice. Although not all of the individuals brought in work samples or items, they did participate in conversations where they felt comfortable or had something to add. Questions asked by the group members showed the importance that was placed on continuous learning, another essential characteristic of professional development (Ball & Cohen, 1999; Hord & Sommers, 2008; Opfer & Pedder, 2011). The group strove to make the meetings useful and related to their own practices of teaching mathematics, which is also supported by the professional learning group literature (Bednarz et al., 2007; Gojmerac & Cherubini, 2012; Lieberman, 2000; Lieberman & Wood, 2002). The group members used the meetings to focus on the future of their teaching practices (Hofman & Dijkstra, 2010) in order to facilitate the transition to secondary school. Blegen and Kennedy (2000) indicated that teachers who engaged in sharing their practices, tended to better their teaching, and the group meetings allowed members the forum to share. This sharing supported continuous learning (Ball & Cohen, 1999;

Opfer & Pedder, 2011) as the teachers adjusted their practices based on the discussions about supporting student learning. Through the common mission of supporting students, the teachers were able to share their own “knowledgeabilities” (Lave, 2008) and learn from each other (Hiebert et al., 2003).

The conversation and direction of the group changed as the needs and concerns of the group members shifted (Hord & Sommers, 2008), allowing the professional learning group to respond to the needs of the individual teachers and the collective group (Lieberman, 2000). As research indicates is important for successful professional learning groups, this group functioned based on trust and cooperation (DuFour & Eaker, 1998).

Feedback and Support

The group members worked to create a safe environment for the teachers to share. In Meeting #7, they explicitly stated that the purpose of the meetings was not to judge the teachers but to build practices that would lead to student success. The teachers felt comfortable in the meetings, and therefore shared without fear of harsh criticism and made jokes with each other. Gabriel mentioned how easy it was to teach in front of the group because by the third year, he knew the “characters” in the group and was not as stressed about sharing his classroom with the group (Meeting #17 transcript). The teachers were complementary, yet critical friends, and as Ryan noted that the group was “all pretty comfortable in math”, and they knew what they wanted to do in their classes and had ideas on how to get their students to that place (Meeting #17 transcript). After hearing the frustration of the secondary teachers with using rubrics, Gabriel commented that just because the elementary teachers had more experience with rubrics does not mean they are better at them (Meeting #8 transcript).

Two of the teachers in rural schools (Diana and Claire) commented on how important it was for them to attend the meetings because of being the only intermediate math teachers at their schools (focus group transcript). The feelings of isolation were helped by attending the meetings to get support from other teachers in a similar teaching situation because they lacked that support in their own schools. The elementary school teachers spoke about having the support of the secondary teachers in making recommendations for a grade 9 pathway. They felt that having met with the secondary teachers gave them more weight in being able to help parents choose the correct stream (Academic, Applied, or Locally Developed) for grade 9. For the elementary teachers, they also appreciated seeing how their students were succeeding in secondary school, which is unusual since after students leave grade 8, they frequently have no more contact with them.

For the secondary teachers, the support of the elementary school teachers for preparing students in the transition from grade 8 to grade 9 was paramount to student success. One of the issues discussed by the group was the grade 9 EQAO that is given to all students in Applied or Academic classes across the province. Due to the timing of the test, the teachers felt that a lot of the concepts that were being tested depended on the grade 8 teachers. During each of the three years, at least one meeting a year was devoted to examining the EQAO scores or talking about questions from the test and how the group could best support the students. In a discussion about EQAO scores increasing between the grade 6 and grade 9 test, Madison noted that it was the grade 7 and 8 teachers that were making the difference (Meeting #10 transcript).

In order to give feedback to the elementary teachers, the marks from the grade 9 students were returned to the individual schools so that they could see how their students were fairing. Several meetings also had discussions about tests given in grade 9 and how specific students were doing. Emma mentioned that she found this particularly helpful in knowing that she had helped prepare her students and suggested the correct grade 9 math classes for them. April and Claire talked about how they felt that having talked with the secondary teachers gave them more credibility for better preparing their students for grade 9.

Discussion.

The group provided feedback and support to one another, which is essential in mathematics education where a lack of collaboration has been linked to not adopting reform practices (Clarke, 1997). Isolation has been cited as a problem in teaching (DuFour & Eaker, 1998; Schmoker, 2006) and the professional learning group studied strongly reduced these feelings of isolation for the group members (Buckner & McDowelle, 2000; Eaker, 2002; Eaker et al., 2002b; Hall & Hord, 2006; Hord & Sommers, 2008; Lieberman, 2000). This was especially important for Diana and Claire who were the only intermediate mathematics teachers in their schools. DiPardo and Potter (2003) illustrated that the teaching profession is wrought with emotions that need to be examined and supported in order to keep teachers from leaving the profession. Connection is a personal need, which is violated by traditional school paradigms (DuFour, 2002). By creating the professional learning group, the members were able to make connections which is important to fulfil the need for connection as well as provide moral support (Arbaugh, 2003). In mathematics evidence suggests that in order to make changes to more reform-oriented practices, having support is vital (Ball & Cohen, 1999; Brahier & Schäffner, 2004; Cohen, 1990; West & Curcio, 2004). Bruce and Ross (2008) note, “When a teacher receives positive and constructive feedback from a respected peer, there is even greater potential for enhanced goal setting, motivation to take risks, and implementation of challenging teaching strategies" (p. 348). The setup of the professional learning group allowed for the teachers to gain this valuable feedback and gave them the support needed to make more significant changes.

The grade 8 teachers felt they were being supported by the secondary teachers in their recommendations for their students' placements in grade 9 as well as advising them in making decisions about the placements. The grade 9 teachers shared their concerns regarding the EQAO results, and the elementary teachers supported them by providing assistance in lower grade levels for difficult areas that needed it and could be targeted earlier.

The group meetings allowed the members time for collaboration, another important characteristic, (Buckner & McDowelle, 2000; DuFour & Eaker, 1998; Eaker, 2002; Eaker et al., 2002b; Hall & Hord, 2006; Hord & Sommers, 2008; Lieberman, 2000; Schmoker, 2006) on areas needing to be targeted, through the environment of trust, respect, and open communication (Hall & Hord, 2006). By providing an environment where respectful colleagues could support each other while offering suggestions for improvement, the group provided the collaboration necessary to make potential changes to the classrooms of those involved (Hiebert et al., 2003).

Action Orientation and Experimentation

Each year the group spent time looking at the grade 9 scores in both the secondary courses and on the EQAO to make decisions about how to better support their students. During Meeting #10, the board liaison brought in some samples of EQAO materials and discussed with the teachers the grading practices used. As a result the teachers adopted new practices that they would reinforce in the upcoming year with their students to build skills for being successful on the EQAO. For example, the teachers discovered that simply writing the numbers from an open response question would give the student a level 10, the lowest actual score, on the question. Figure 7 shows a rubric from the EQAO used to evaluate the question discussed. During Meeting

#13, the board liaison brought in scores from the previous year's EQAO to discuss and examine specific questions. The questions that students in the district were found to have particularly struggled with were put up on a screen for the teachers to analyse. For example, a problem asking students to find the perimeter of an ice rink was given as an example (see Figure 8). The teachers felt this was a difficult question because of the rounded edges of the rink, so a student would have to correctly apply several formulas to find the correct answer.

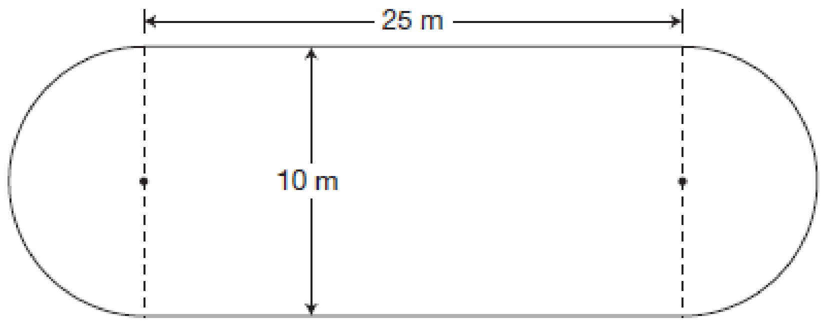
Part-Time Job

Code	Descriptor
B	Blank: nothing written or drawn in response to the question
I	- Illegible: cannot be read; completely crossed out/erased; not written in English; - Irrelevant content: does not attempt assigned question (e.g., comment on the task, drawings, "?", "!", "I don't know"); - Off topic: no relationship of written work to the question.
10	Application of knowledge and skills of percent to determine total weekly sales shows limited effectiveness due to • misunderstanding of concepts; • incorrect selection or misuse of procedures.
20	Application of knowledge and skills of percent to determine total weekly sales shows some effectiveness due to • partial understanding of the concepts; • errors and/or omissions in the application of the procedures.
30	Application of knowledge and skills of percent to determine total weekly sales shows considerable effectiveness due to • an understanding of most of the concepts; • minor errors and/or omissions in the application of the procedures.
40	Application of knowledge and skills of percent to determine total weekly sales shows a high degree of effectiveness due to • a thorough understanding of the concepts; • an accurate application of the procedures (any minor errors and/or omissions do not detract from the demonstration of a thorough understanding)

Figure 7. Scoring guide used for a question on the Grade 9 Academic EQAO (2010). Retrieved from http://www.eqao.com/pdf_e/10/9e_Acad_2010_Web.pdf

30 **Building an Ice Rink**

Jake builds an ice rink as shown below.



Determine the perimeter of the rink.

Show your work.

Figure 8. Sample question from the Grade 9 Academic EQAO (2011).
Retrieved from http://www.eqao.com/pdf_e/11/9e_Acad_0611_web.pdf

The conversations about using rubrics for grading mathematics allowed the teachers to consider new possibilities that extended beyond simply marking an answer as right or wrong. In order for the secondary teachers to gain a different perspective, a rubric was shared during two of the meetings, and the teachers went around the room to discuss how many “points” they would have assigned the questions. The secondary teachers found it interesting how aligned their own practices had become in grading, and the elementary teachers gained a perspective of what their students would be facing as they moved into secondary school (see Appendix L for transcript). One of the meetings was specifically set aside for the secondary teachers to gain experience using rubrics since they had been encouraged to use them more in their own practice and rubrics were used for grading on the EQAO (Meeting #9 transcript). During the meeting at which Emma provided samples of student work with a rubric, the group went around the room discussing what they would give each student and why.

The group members did express concern over the number of new initiatives that they were being asked to implement at any given time. One member said “shut up all researchers” (Meeting #3 field notes) so that one initiative could be given the time to be developed in their classrooms before they attempted something new. Their dedication to improving the mathematics experiences of their students extended to a frustration with schools “chasing the money” and hoped that they would continue funding projects for several years in order to allow time for the teachers to see if the project actually worked with students (Meeting #3 field notes). Other than the professional learning group itself, one of the initiatives the teachers discussed was coaching. At the end of each year, the teachers would comment on whether or not they would again have funding for professional learning groups or if there would be a new initiative that would be supported despite how beneficial they found the meetings, and the lack of certainty was of concern

During the last meeting of the third year, the board liaison brought in a diagnostic tool that was being used in another professional learning group. In order to better support their students, the professional learning group I observed decided to give the diagnostic to their grade 8 students. These diagnostics would then track with the students to grade 9. The teachers hoped seeing the work would allow the grade 9 teachers to better assess if the students had chosen the right courses and to allow them to have better discussions with parents earlier in the school year. A common theme in the discussions within the group was whether or not students were placed correctly and what to do if parents made a decision that went against the teacher recommendation about the students’ abilities in mathematics. Since student success was important to the group, they decided that having this diagnostic in the student file when they got to grade 9 would help with reaching students before they failed in secondary school.

Discussion.

Previous research into mathematics professional learning groups had pointed to a disparity in how successful groups were in changing mathematics teaching (Kajander & Mason, 2007). I added the three additional characteristics of professional learning groups by DuFour and Eaker (1998) to my research framework in order to discover a difference in this group by comparing it to tenets of action research. As DuFour and Eaker (1998) define, this group was focused on trying new strategies to make adjustments in their classrooms and examining the effects of these practices. The group members focused on grade 9 EQAO scores and how to improve the results by using new pedagogy in their classrooms. To these teachers, student success was paramount, so they focused on tangible results to make decisions about their teaching. Stigler and Hiebert (2004) point out that little has changed in mathematics classrooms as a whole, so having a focus on making changes is important for improving teaching practices. The idea of a professional learning group was to learn from both the positive and negative aspects of their practices to seek growth in their teaching (DuFour & Eaker, 1998), and such learning was present in this group. Their classroom practices were based on trying and reflecting on the experiences as suggested by research (Eaker, 2002; Eaker et al., 2002b; Hord & Sommers, 2008; Gojmerac & Cherubini, 2012). Although dedicated to improving, the teachers shared their frustration with too many initiatives being pushed on them at once and hoped they would be given time to find out if something worked before administration mandated moving on to something new.

Continuous Improvement

All of the meetings focused on what the teachers could do to support their students to be successful in mathematics. Gabriel and Claire suggested creating resources that could be used in their classrooms in order to align teacher practices and support the teachers (Meeting #7 transcript). As a result, any resources brought in by the teachers were shared among the different members of the group to the benefit of the entire group. In the latter part of Meeting #7, Emma brought in several activities for teaching algebra which Ryan suggested would be beneficial for his grade 9 class, so he took copies of the resources. Different pieces of technology were also shared and adopted by other teachers, such as using the National Library of Virtual Manipulatives (Meeting #7 transcript) and creating spontaneous clicker quizzes (Meeting #10 transcript).

During the final two years, the group members made the decision to reschedule one of their professional learning group meetings each year to coincide with a researcher who was brought in by the local university. The teachers hoped to get new ideas and information to take back and use in their own classrooms. After each of the meetings at the university, the group met together so that they could discuss the presentation and the ideas to use in their own professional practices. Following the meeting in the second year, both Wesley and April shared lessons that were inspired by the talk. Although Wesley described his lesson as a “total flop” (Meeting #16 transcript), he was excited to try it again the following year. April’s lesson met with more success, and she shared how she combined algebra with a lesson talking about wages in poor countries compared to the millions of dollars actors or sports stars earn a year.

The group also arranged for five classroom observations, which the teachers followed with gathering together and discussing the lesson they watched. Suggestions were made by the group in order to improve the lesson or to better support the students. Following watching Owen teach, Madison noted that the graph at the start of the lesson would have been better for explaining the line of best fit because it would have been more clear (Meeting #11 transcript). In one of the graphs in his lesson, Owen required students to graph height, so they had to break the axis between 5 feet and 6 feet in equal increments. Several of the teachers commented on how the students may not have had the knowledge for dividing the graph because they would be unfamiliar with how many inches are in a foot. When resources were shared with the group, the teachers would also reflect on how it could be improved using a different perspective. For example, when Gabriel shared one of his assessments, he commented on how he would change the organisation so that the rubric was closer to each of the sections. He felt that this would help with his grading technique by not having to constantly flip to the front page (Meeting #8 transcript). Ryan commented that rearranging one of the questions might help the student define a final algebraic expression more accurately.

Discussion.

As DuFour and Eaker (1998) noted, teachers in a professional learning group need to be constantly looking to improve their practices. With this end in mind, the group attended presentations at the university as part of their professional development to gain new knowledge about mathematics teaching and then reflected together as a group. One of the university meetings in particular gave concrete examples of reform-based practices, which research has indicated is important for growth or changes in mathematics pedagogy (Brahier & Schäffner, 2004; Carnegie Corporation of New York, Institute for Advanced Study Commission on Mathematics and Science Education, 2009). The university meetings were aligned with questions the teachers had and needs they felt were important to their practices, allowing for spaces for growth to occur. Bednarz, Maheux, and Barry (2007) determined that changes should combine the needs and beliefs of the teachers with research practices. In defining professional learning groups, the Ontario Ministry of Education (2007) states that the groups should be focused on research in order to maintain growth in teacher practices. In order to continue this dedication to improvement, they observed each other's practices and reflected on the observations. By focusing on best practices in teaching through the discussions and observations (Eaker, 2002), the teachers were able to improve their teaching to support their students. Through their activities, the group members balanced their own knowledge of teaching mathematics, with the ideas of other members, as well as research about best practices, which is important in making changes to improve their teaching methods (Lieberman, 2000).

Results Orientation

The teachers were conscious of making sure their practices were making a difference for their students. One way they examined the outcomes of their practices was by sharing grade 9 scores to see students' achievement in secondary school (Meeting #3 field notes). During the second year, the board liaison noted that the scores were going up between the grade 6 and 9 EQAO tests. Emma commented that knowing the results in grade 9 helped the grade 8 teachers recommend a correct pathway for future students (Meeting #15 transcript). The group also examined personal diagnostics to discuss the effects their teaching practices were having on students (Meeting #8 transcript). The teachers also took the time to discuss next steps in order to continue growing.

During the first year, the board liaison brought in a diagnostic that was being piloted by Nelson Education to use in grade 9 in order to analyse students' performance (Meeting #4 field notes). According to the liaison the purpose of the diagnostic was to inform instruction, examine gaps, and help parents make informed decisions about streaming their students. The following year April asked how the diagnostic worked and if it had helped. Ryan explained that it was "terrible" and too "heavy" for use, so it was discontinued (Meeting #7 transcript). He noted that they tried one unit “and it was so over the top”. April agreed and commented that when she had seen the algebra section, she was concerned it was way above where her grade 7 students would be when they reached grade 9.

The teachers talked about algebra being an indicator of success in secondary school (Meeting #2 field notes), and Ryan noted that students stronger in algebra tend to be stronger overall (Meeting #13 transcript). As a result, the elementary teachers made the decision to include more algebra in their elementary classes despite the lack of emphasis in their own curriculum (Meeting #8 transcript).

Discussion.

The group members had a focus on the results of their work (DuFour, 2002; Schmoker, 2006) such as by examining test scores and student work. A focus on student work is expressed as important in the literature (Hord, 2009; Sowder, 2007). The teachers in the group also saw student work as important because as Russ noted in his interview, it goes beyond simply figuring out a grade because “there’s always good talk about why it [the grade] should be that way and how it was taught and what the students should be thinking.” The group members also had a strong focus on student success in secondary school by examining the grade 9 test scores from classroom assessments in addition to the EQAO. By examining student samples and assessment, the group members kept a focus on student thinking and learning in their discussions (Cwikla, 2004; Sowder, 2007). The Ontario Ministry of Education (2007) asserts that keeping student learning as the focus in a professional learning group ensures positive results. Ryan’s suggestion that students stronger in algebra do better in secondary school is also echoed in the literature (Rasmussen et al., 2011). Kozlow (2012) notes that students who achieve the standard in lower grades are more likely to succeed in secondary school, so building the bridge with elementary teachers ensures best practices continue.

Having teachers work together has been shown to have an impact on student achievement (Hofman & Dijkstra, 2010) and classroom practices (Andrews & Lewis, 2002). In modelling an action research approach, the professional learning group members showed their dedication to continuous learning and improvement of their practices, through their reliance on concrete data and discussions of actual student work samples.

Addressing Teacher Knowledge

During the group meeting discussions and interviews, there was evidence of the teachers addressing the knowledge needed for understanding mathematics as well as effectively teaching mathematics. Since a focus on teachers' thinking and student learning is important in mathematics (Cwikla, 2004), I wanted to discover how the teachers addressed mathematics knowledge discussions in the professional learning group. In her interview, Emma noted that the conversations in the professional learning group that focused on mathematics specifically were the times during which the interesting discussions happened. I also noticed that these discussions were definitely the most spirited about what students should know as well as how to impart this knowledge. Research about teacher development discusses teacher knowledge as being vital and specialised for effective teaching (Baumert et al., 2010; Cochran-Smith & Lytle, 1998; Shulman, 1986; Sykes, 1999). In this section, I discuss how knowledge was presented by the teachers during their meetings. First I look at the evidence supporting the knowledge-practice relationship framework discussed by Cochran-Smith and Lytle (1998) and then I discuss the conceptual knowledge needed for teaching mathematics discussed by the group in order to illustrate "In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?” Following each section, I link the discussions and activities to the pertinent literature for each area of knowledge.

Knowledge-*for*-practice

Knowledge-for-practice consisted of discussions by the teachers on the knowledge needed for the profession of teaching and specifically for improving the teaching of mathematics.

Images of knowledge.

The elementary school teachers in the professional learning group discussed how they used the three part lesson in teaching mathematics, and the secondary teachers also talked about the format that they used to structure lessons sharing their definitions of best practices for mathematics teaching. For example, Ryan discussed using a review to start each lesson or “seeding” because he was “always trying to implant ideas before we get to the main idea” so that “the actual challenging stuff isn’t so terrible” (interview transcript). April talked about making sure all of her lessons integrated other subjects or content strands so that students could see the connections between the topics.

Another area where teachers showed their knowledge of best practices in mathematics education was their discussions of using manipulatives and models in teaching. The most frequently used manipulatives discussed in the group were algebra tiles and integer chips. These two manipulatives were used by both secondary and elementary teachers even though some of the teachers had different feelings on the purpose of the tools. One discussion of best practices the teachers engaged in was a discussion about how to prove to students why -2×3 is not the same as 2×3 (Meeting #11 transcript). Ryan struggled with helping his students understand why multiplying a negative by a positive made the answer negative. Emma and Madison both suggested using patterning, and Ryan said he had used the technique but wanted to find a way to justify this phenomenon better to his students. During two meetings (Meeting #12 and #16 transcripts), Ryan created questions for the teachers to share their mathematical understanding of topics used in their teaching as well as show ways they would teach the topic in their classrooms. One of the small groups in the first meeting were tasked with 3 (a+2) as their question. In the small group discussion, the teachers discussed models used in their classroom including repeated addition and April talked about the “Magic Box”, or area model. Many other examples of discussions about models and manipulatives used in the classroom occurred throughout the three years.

Images of teachers, teaching, and professional practice.

Several of the teachers brought in lessons they used in their own classrooms and discussed how they implemented them. For example, Emma brought in a lesson she used to help students learn the order of operations (Meeting #7 transcript). The lesson featured students trying to decide if Excel would use order of operations in solving equations. She shared both the lesson she used with her students as well as the rubric she used to assess her students during the exploration. During the same meeting, Emma shared a lesson she used with students to help them gain understanding of scientific notation. She shared that she used this lesson to give her students time to learn what is needed to effectively “justify” their solutions and again shared a rubric she used to evaluate this lesson.

Discussions about using rubrics occurred in several of the meetings due to the discomfort the secondary teachers were feeling with using them and their desire to learn more. In Meeting #9, Emma shared a rubric she used with her students during her algebra unit and the other teachers asked about how she evaluated her students using it. One discussion related to how Emma defined “clarity and precision” with her students, and Emma replied that she made sure the students lined up their equal signs, worked down the page, showed the reverse of the operation, and verified one step per line. While examining students' work from the lesson, the teachers discussed how properly verifying a solution would have helped one of the students catch mistakes made during the assignment. Emma also described how some of her "rubrics" were more like checklists at the top of assignments to give students immediate feedback (Meeting #8 transcript). Gabriel, April, and Claire all commented on how they had adopted the practice as well. Emma and Gabriel spoke about how the checklist not only helped focus the assignment, but they also used it to group students during a follow up lesson.

5 What is the value of the expression $\frac{-12}{-6+3}$?

a $\frac{1}{4}$

b $\frac{4}{3}$

c 4

d 5

Figure 9. Multiple choice question from the Grade 9 Applied EQAO (2011).
Retrieved from http://www.eqao.com/pdf_e/11/9e_App_0611_web.pdf

Another example of a focus on knowledge-for-practice was during discussions about EQAO testing. In Meeting #13, the board liaison specifically brought in questions that students had struggled with on the test in the previous year. The teachers discussed ways the questions may have misled students and how they could better support their students in future years. They also discussed difficulties built into the questions and their concern about whether the questions were actually testing mathematics or the students' ability to decode a question. One such question (see Figure 9), Wesley felt tested more their ability to be able to take a test rather than really addressing integers. Both the board liaison and Wesley pointed out how students would need to know about the “implied bracket” on the denominator of the question because simply punching it into a calculator would get an incorrect solution (choice d). In a different question, Noah mentioned how students who struggle with reading have difficulties when they are confronted with problems that are all words with no digits (Meeting #8 transcript). A problem that was continually addressed was how much trouble students had with multiple choice questions and how to provide support for success. According to the secondary teachers, the difficulty with multiple choice was compounded when the question contained “not” (Meeting #10 transcript). Although the teachers felt it was important for students to learn how to attempt multiple choice questions, they shared their concerns over how hard it is to know a student’s thinking based on a multiple choice question (Meeting #8 transcript). Emma commented that she had students turn in their work pages so that the students were held more accountable for their work (Meeting #13 transcript).

Images of teacher learning and teachers’ roles in educational change.

The professional learning group members discussed how the participating school board had been concerned about increasing literacy scores in schools throughout the board. One of the directives the elementary teachers needed to follow was to combine literacy skills with other subject areas. Although increasing literacy scores was not the mandate of the professional learning group, the teachers found ways to integrate literacy into their mathematics lessons in order to address these board mandates. Techniques described included using word problems and ensuring students justified their solutions while they solved mathematics questions. Although originally designed to help literacy, the teachers discussed how these skills were valuable in mathematics and would in fact be tested on the EQAO. This joint discussion of mathematics and literacy fit in with the group's concerns over increasing scores on the Grade 9 EQAO.

The teachers showed their dedication to enhancing their own professional learning by deciding that the topic of each meeting would be both the conceptual development of a mathematics strand as well as different ways to approach teaching the topic. Three of the meetings focused on the mathematical knowledge needed to teach multiplication and division as well as the strategies that could be used by teachers to teach these topics (eg. Meeting #2 field notes). During these meetings, the teachers shared different models including area models, factor trees, and repeated subtraction. They also discussed using pictures of multiplication operations to examine if number order mattered, and they examined the digital roots of the numbers. The discussions in these meetings extended to multiplying and dividing fractions and the difference for students with tackling fractions versus whole numbers, including what mathematical understanding would be necessary for a student to be successful in working with fractions. By focusing on a specific topic area, the group was able to increase their own knowledge of the content as well as share practices for teaching.

As mentioned, another area the teachers felt needed to be discussed for the benefit of their teaching practices and increasing student learning was algebra. Eight of the meetings featured a discussion on algebra as well as assessment tools and teaching strategies related to teaching algebra in different grade levels. They discussed the difference between an equation and expression and how to build understanding with students, as well as using algebra tiles to build student knowledge. They described how students would need to know that in an expression they could not solve for "n" because they did not know what it was worth (Meeting #6 field notes). The teachers also talked about helping students understand that $5n \div 5$ would not be 0 and that the opposite of a multiplication operation, for example “5n”, would be division. By examining student misconceptions, they explored which teaching practices could help students to gain the necessary knowledge of algebra.

Current initiatives in teacher education, professional development, and/or teacher assessment.

The professional learning group itself is a current initiative in professional development and all of the teachers took part in defining the group dynamics as well as sharing ideas. During each of the meetings, the teachers discussed new practices to use in their own classrooms in order to further develop their practices. For example in the meeting during which Gabriel described how he used a number line with his students when teaching about operations using integers (Meeting #12 transcript), a secondary teacher mentioned that using a number line was new to secondary since they had always used either integer chips or the rules. Gabriel noted how the number line was very familiar to his grade 7 students, so this was a more natural progression for his students. He shared with the group how he guided his students to draw their own number lines and used the directionality of the numbers for adding and subtracting. Samuel, who presented how he would teach the question 9-(-2), had shared that he used either the integer chips or told his students that “a negative and a negative is positive”. Emma cautioned how students should be able to work with the numbers 9 and -2 and still have a concept of the numbers and argued against just using the rule.

The teachers discussed the fact that their students in all the grade levels struggle with basic multiplication and division facts. Keeping with the current research trends, the teachers discussed moving away from straight rote practice of the facts and talked about wanting “automaticity” (Meeting #1 field notes). According to the teachers, students should know strategies to help them remember the facts instead of just being able to spit them out during a drill.

Discussion.

Since, according to the literature (Andrews & Lewis, 2002; Brahier & Schäffner, 2004; DuFour & Eaker, 1998; Eaker et al., 2002b; Hord & Sommers, 2008), some professional learning groups have the capacity to support changes in classroom practices, the discussions about teacher knowledge are important to examine. The teachers discussed knowledge-for-practice in examining lesson structure and content of different lessons. The elementary teachers discussed using the three part lesson plan, which is supported by research as being effective for teaching because it requires students to engage in discussions to show their understanding (Shulman, 2000). Research supports using more exploration as being more effective than traditional approaches to teaching mathematics (Askey, 1999; NCTM, 2000; Riordan & Noyce, 2001; Van de Walle & Lovin, 2006; Wilson et al., 1996). Discussing different ways of approaching topics allows teachers to broaden their understanding of ways to teach mathematics (Turner et al., 2011). The teachers also discussed using authentic tasks, which research has linked to engaging higher order thinking (Volante, 2006). Shulman (1986) adds that teachers need to know multiple ways of representing topics in order to reach all of the learners in a single classroom. Evidence of all of these efforts was found in my data.

The teachers also discussed specific questions on the EQAO and how being aware of potential questions influenced their teaching. The conversations about using EQAO questions is important to practices, because, as Volante (2006) notes, allowing students to become familiar with tests reduces their anxiety. By exploring different ways to teach mathematics, as well as discussing ways to support students, the teachers strove to make a difference in EQAO test scores. In researching standardised tests in Canada, Anderson et al. (2006) discovered that 80% of the variation in scores occurred at the class and student level. In another Canadian study, Rogers et al. (2006) determined that student level was where most of the difference occurred. By working to standardise practices in different classes while working on multiple representations to support individual students, the teachers created an environment with the potential to make changes to their test scores. The discussions also focused on pedagogical content knowledge, which Van Driel and Berry (2012) noted was important for professional development to consider for best supporting students.

The teachers discussed concrete ways of discussing mathematics topics in their classrooms, and as Emma noted, explored ways to ensure students did not lose the value of the numbers when working with them. Skemp (1986) noted that when ideas are not connected, it is more difficult to remember. The teachers worked to make connections between topics and the previous knowledge students had gained about mathematics. Research supports moving away from drill practices (Sawyer, 2004; Skemp, 1986; Vygotsky, 1962; Volante, 2006) toward automaticity with an understanding of the process, and evidence of such a goal was found in my data.

Knowledge-*in*-practice

Although there were only five classroom visits during the professional learning group meetings, there were times where teachers discussed how they used their knowledge in their classrooms through reflections and presentations of their practice.

Images of knowledge.

Following each of the classroom observation sessions, the teachers would get together to discuss and reflect on the lesson. Oftentimes there were suggestions from the other teachers about different ways to make the lesson stronger or comments on how to help increase support for students moving from grade 8 to 9. After Owen's lesson (Meeting #11), the teachers discussed how misleading using the imperial system in one question might have been to students since they had not been exposed to it yet that year. For example, when they were graphing heights, the middle between five feet and six feet would have been five feet six inches instead of 5.5 which teachers thought most students would have assumed it to be. This was important since Owen had collected the heights being graphed from the students, and so had to graph what they gave as their height, such as five feet four inches. Gabriel discussed how the students would have been unfamiliar with a "number system using 12 as its base" and how many struggle with it (Meeting #11 transcript). A discussion also came up about drawing misleading graphs. Emma questioned why one of the graphs had started at "5", and there was no break on the scale to denote that it did not start at zero. The elementary teachers commented on how this was part of their curriculum and discussed how it led to misleading graphs and a misrepresentation of data, so they wondered if it was something that would be talked about later in the unit. Owen noted that he did not use the break because it is not something you could do on the SMART Board, but he would talk about it in the future. Ryan noted that he never used the break because he found that too many students were using it incorrectly and that mathematically there was no reason for using it at all.

Following the observation in Gabriel's class (Meeting #17), the teachers discussed some of the student responses and how some of the students ignored typical mathematical conventions. Gabriel's lesson was about learning more about algebra, and felt that cleaning up students' expressions was something for another day once they had acquired the foundational concepts. According to the board liaison, who also observed the lesson, it was obvious that Gabriel's students had a clear understanding of variables but were having difficulties with expressing themselves formally.

When teachers brought in work samples, they would discuss changes they might make to their own lessons, but the other teachers in the room also addressed how they might change the activity to make it stronger. During Meeting #10, Ryan brought in one of his assessments for the teachers to examine for student understanding. The teachers took turns describing some of the errors and misconceptions Ryan's students had in fractions, integers, and order of operations based on the completed assessment. They ended up also discussing some of the questions on the exam and how the questions could be changed to eliminate some of the misunderstandings in the student responses.

Images of teachers, teaching, and professional practice.

During the classroom visits, most of the teachers walked around and interacted with the students in the classes. During the secondary visits, the elementary teachers were very excited to interact with their former students and see their achievement and engagement in the secondary lessons. Secondary teachers mentioned getting to know where their future students were coming from through watching an elementary lesson. One notable example of a teacher working with students was during the visit to Gabriel's classroom. April worked with one particular student and helped her feel confident in her thinking as she was exploring the algebra task that Gabriel had given the class. Gabriel mentioned afterwards that this particular student does not normally share because of the lack of confidence, but after working with April, she volunteered to show her solution method to the entire class.

Another way the teachers showed their knowledge of students and teaching mathematics was through examining student work. Whenever a piece of work was brought into the group meetings, the teachers would talk about where they saw strengths or weaknesses and what the potential next steps for the student might be. In the second year, the teachers examined student samples taken from the EQAO test in order to enrich discussions about supporting students during testing. In one example (see Figure 10), the teachers looked at four different exemplars of student work and discussed where they would place them on the four-level rubric used to assess such a question. The teachers were told that the four samples each fit as an exemplar for one of the levels on the rubric. The discussions among the teachers focused on the concept of percentage being the difficult part for the students to work with and analysed the amount of mathematical knowledge or misunderstandings evident for each student shown in the samples. During the same meeting, Ryan also brought in samples from his students on a linear relations test. The focus was on examining the samples for errors that they saw in the students' mathematical thinking.

Part-Time Job

Ezre works part-time at a clothing store. He earns $80 per week plus 6% of the value of his weekly sales.

This week Ezre earns $119.

What is the total value of his sales this week?

Show your work.

Figure 10. Sample question from Grade 9 Academic EQAO (2010).
Retrieved from http://www.eqao.com/pdf_e/10/9e_Acad_2010_Web.pdf. Teachers were given exemplar student solutions of the problem to analyse and score using the rubric (see Figure 7).

Teachers shared their knowledge of students in practice by discussing particular students they had in their classrooms. Madison brought up a student she was struggling with so that she could get some input on new suggestions to try (Meeting #11 transcript). The student was struggling with the concepts associated with operations of negatives and was disagreeing with the way that Madison explained the concept. According to Madison, the student was having trouble seeing that subtracting a positive was just subtraction now that negatives had been introduced. Madison shared that when she would write x-x+1-3 that the student wanted it changed to x-x+1-(+3). Emma suggested bringing the student to simpler examples such as 4-3 and 4+(-3), but Madison said the student saw those as two different concepts. It was also discussed that if the student was modeling those two questions with integer chips that it would actually be different operations, even though the answer is the same. Gabriel suggested the use of a number line to help the student with visualising the concept to see if Madison could get her past the difficulty.

During the meetings, the secondary teachers shared their difficulties with students who would not take notes in their classes, which several of them considered vital to student success. As Owen mentioned, he felt it was important in preparing the students for university that they know how to take notes quickly and only record the important parts (Meeting #11 transcript). Gabriel and Emma both worried that their students would not have gotten much practice in note taking in elementary school, but Evan noted that his class had been engaging in taking more notes. Emma spoke about creating a student file from her SMART Board lessons so that students could have partially created notes from her lessons. She felt that leaving a few blanks on the sheets given to the students would help them stay focused on the lesson to fill in the missing information, but they would not have to spend the entire time writing notes. Owen mentioned that he prints the SMART Board lessons for some of his students so that they can review them. Wesley spoke in his interview about posting his lessons on his website so that students could review the material before an exam. He also discussed a particular student in class who would not take notes, and the group made suggestions of having that student write on the SMART board or use some other piece of technology to encourage him to record during a math lesson.

Images of teacher learning and teachers' roles in educational change.

As a result of the meetings, the elementary teachers discussed how they were using more algebra in their classes to help better prepare students for secondary school. The teachers also discussed strategies that they have used with their students to encourage correct mathematical conventions in algebra. On several occasions the teachers discussed the benefits of using "x", "n" or a different letter in algebraic expressions. Most of the elementary teachers liked using "n" because it could be used to stand for number, but stressed that they were flexible with what their students chose to use themselves. They did make sure to let their students know that in secondary school they would be using "x" because it would transition into the format $y=mx+b$ for graphing (Meeting #8 transcript).

In Meeting #8, Emma talked about ensuring that grade 8 students were solving algebraic expressions using methods beyond simply using inspection. She stressed that students would isolate the variable and learn the conventions for it only when they needed to and could not answer the question by simply looking at it. Madison noted the importance of students needing to do more than solve by inspection so that they fully understand the process.

The discussions also turned to how to express multiplication in algebra. In elementary school, the teachers talked about using the traditional symbol of × in multiplication and how they tried guiding students away from this in algebra so that $3\times x$ would not confuse students. Secondary teachers talked about using a $\cdot$ in their expressions to denote multiplication but only $3x$ in algebra (Meeting #3 field notes).

Emma also wanted to make sure that her students understood the iterative nature of algebra and not just the recursive nature that would allow them to finish a given pattern. Emma noted that her students would naturally see how much is being added to get the next number or the recursive nature of the pattern. Instead, she wanted them focusing on what they needed to do to the picture number to get the total number of the pattern because that would be the same thing done to any picture number, illustrating the iterative nature of algebra. In Meeting #12, Emma noted she changed her beginning algebra unit to the start of the year, and as a result has found that algebra was being discussed throughout the year and in much deeper and more meaningful ways.

Current initiatives in teacher education, professional development, and/or teacher assessment.

During the first couple of meetings, the secondary teachers talked about having math coaches at the secondary level. For the secondary teachers, the math coach was someone who would come in and work with their class and support changes in teaching practices. The teachers were clear on how the coaches were not there because they were doing something wrong, but instead to support and build practices together. The elementary teachers expressed a desire to have a support like that at their level, although they did discuss the previous concept of coaching was very different in that, at the elementary level, the “coach” came in, did a lesson with students, and then left (Meeting #3 field notes). The level of support for the coaching model as described by the secondary teachers was missing in the elementary model.

Discussion.

In order to address knowledge-in-practice, the teachers participated in observations and reflections of each other’s classrooms. As Shulman and Shulman (2004) note, teachers need to reflect on their practices in order for changes to be effective, and evidence of this process was found in my data. They would also discuss and reflect on different lessons they brought to share. During the meetings, the group members also discussed different examples of completed student work. The Ontario Ministry of Education (2007) encourages teachers in professional learning groups to consider student achievement in their discussions as well as reflection on practices. Bruce and Ross (2008) suggest that reflecting on practices and assessments allows teachers to see problems and where they are dissatisfied with student performance, allowing the teachers to be in a position for making changes. The teachers discussed using peer coaches in secondary and the desire to have them in elementary school, which is supported as an effective practice (Bruce & Ross, 2008).

Teachers need to know possible misconceptions students may have about a topic in order to help students create new structures of understanding in order to correct those misconceptions (Shulman, 1986). Through their discussions of student knowledge, the teachers worked to create a concrete foundation with strong support in order to engage students in building new understandings as is recommended (O’Donnell et al., 2008). Although they wanted their students to learn their facts as well as more advanced skills, most of the teachers had a focus on having students learn a “meaningful method” as advocated by Skemp (1986) and not just a “shortcut” (p. 55). The dedication to exploring student understandings as well as building on their foundations of knowledge allowed the teachers to make changes that would support further student learning.

The concerns expressed by the teachers over their students’ dislike of mathematics is also a concern shared in the research literature as something needing to be addressed in classrooms. Anderson et al. (2006) found that students who had positive feelings about mathematics had greater achievement scores. Kozlow (2012) examined EQAO scores and determined that students who liked mathematics and felt they did well were more likely to meet the Ministry standard in both Academic and Applied classes. Kozlow (2012) also found that those in the Academic stream were more likely to say they enjoyed mathematics and have positive views of their own abilities than those in the Applied stream. Confidence also plays a part in student test scores, as Rogers et al. (2006) determined that students who were confident in their abilities in mathematics had higher achievement scores. To address this concern, the teachers in the professional learning group discussed of creating a community of learners (Shulman, 2000; Shulman & Shulman, 2004) which supports academic growth and may potentially increase students' positive feelings about mathematics.

In examining student work, the teachers discussed the mathematics knowledge they saw in the students' solutions and how to improve the work. By examining ways to scaffold students learning (Skemp, 1986; Vygotsky, 1962), the teachers worked on supporting students as well as continuously improving their practices (DuFour & Eaker, 1998). The Ontario Ministry of Education (2007) discusses how sharing and examining student work is important to keep the group focus. One aspect of teacher knowledge important for teachers that was also discussed by the group was that of curricular knowledge (Shulman, 1986). Shulman notes that this includes knowledge of the many resources available to a teacher and being able to choose the resource most suitable for the topic.

Two particular items that came out of the group discussions bears further note here. First, when Madison brought up a student she struggled with who saw $4 - 3$ as $4 - (+3)$, and how this was a cause of concern for Madison. The student does have a point though because the two operations would be modelled in the same way. Having the teachers unpack why Madison was trying to get the student away from writing it in this manner would have been interesting for the teachers to pursue. The second was Emma's discussion about algebra and wanting her students to see the iterative and recursive nature of algebra. Typically discussions about the iterative nature of algebra actually refers to the recursive solution and not what Emma was trying to convey where students would figure out an algebraic expression that involved the picture number in the pattern.

Knowledge-*of*-practice

The teachers spent time during the meetings discussing and investigating their teaching practices in order to improve their practices.

Images of knowledge.

The teachers in the group discussed gaps in student understanding and how these gaps might have been caused by problems in the design of the curriculum. The first area that the teachers addressed was the problem with fractions and their operations. The elementary teachers noted that students began learning about adding and subtracting fractions in grade 7 and that they were still unsure about it in grade 8 since this was a relatively new concept. Students were then expected to learn about multiplying and dividing fractions in grade 8, so the teachers were concerned that students had not yet mastered these skills before leaving elementary school. In their discussions, the teachers learned that fractions are not in the grade 9 curriculum at all. After discussing this gap in the curriculum that was leading to so many difficulties with fractions in secondary school, the grade 9 teachers decided to start working with fractions also in grade 9 to help strengthen the foundations started in grades 7 and 8.

The teachers also discussed how after grade 9, geometry had been pulled out of the secondary curriculum with the new curriculum changes, so spending an exorbitant amount of time on these topics was not as vital to future success as other areas. For example, some of the secondary teachers mentioned that when they were discussing angle theories in their classes that they did not enforce using the proper names for the theories because it was knowledge that would not be used again after grade 9 (Meeting #12 transcript). All the teachers agreed that the students needed to develop an understanding of what the different theories were and how to apply them, but they were divided on whether or not students needed to remember the exact names. Evan mentioned that he stressed to his students that they would be responsible for knowing the names in secondary school, which sparked the debate on whether it was necessary (Meeting #12 transcript). Alternatively Madison noted that the Pythagorean theorem would be more beneficial for the students to spend time on because of trigonometry, so angle theory could be given less time.

As mentioned previously, a final major area of difficulty in the curriculum was the algebra strand. The teachers quickly discovered that there was not enough algebra in the grade 8 curriculum to sufficiently prepare students for being successful in grade 9 or later on at the secondary level. As the teachers noted in Meeting #6, the leap from elementary school where they mainly talked about expressions in algebra, to creating equations and graphing in secondary school was too large for the majority of the students. As a result more algebra was implemented by the group teachers not just in grade 8, but also grade 7, and in one school grade 6, to help support students better.

In addressing the curriculum progression of students, the teachers also discussed how different areas of mathematics fit into other grade levels and even other subject areas. For example, the secondary teachers were talking about determining correct units in physics, such as, velocity being m/s because the formula requires dividing distance (m) by time (s) (Meeting #16 transcript). The teachers also linked the discussions about linear relationships to needing to know the dependent and independent variables in science (Meeting #11 transcript).

The teachers engaged in discussions about the textbooks they used and how to supplement such lessons in order to provide the students with a meaningful mathematics curriculum. One example was the circle unit in the grade 8 textbook. As Emma noted the text has circumference, area, surface area of spheres and volume all combined together. She realised that teaching it all as a single unit was very overwhelming for students and has learned to break it apart. It was noted by one of the other teachers that the reason she knew to do this was because of her experience and knowledge as a mathematics educator. Emma also discussed how the textbook does not go far enough in algebra for the students to be successful in secondary school (Meeting #8 transcript).

Images of teachers, teaching, and professional practice.

During a discussion about multiplication, the teachers in all the grade levels commented on how weak their students were as a whole in remembering their multiplication facts. The teachers did not feel that simply forcing their students to memorise the facts through drills was appropriate. Instead, Evan shared the Math Matrix Approach to Multiplication by David Langford that he used in his own classroom (Meeting #2 field notes, see Appendix J). He found that having the students spend time learning patterns and working with each of the facts was helping students with their automaticity. As a result, Ryan began using the Math Matrix in his grade 9 Applied class in order to support the students (Meeting #4 field notes).

Wesley brought in a topic that he felt would help in calculus and with other advanced concepts if it was brought to elementary students, so he “wanted to encourage this process” (Meeting #9 transcript). He began with the example of $12 \times \frac{3}{4}$ and how he wanted students to divide the 12 by 4, leaving the student with 3 multiplied by 3 in the final step. He contrasted this with have students calculating 12 × ¾ as / and then getting 9 as the solution. He then extended the process to an example with dividing polynomials and where students commonly make mistakes (see Figure 9). He felt that the investment of having students learn earlier to find a common denominator and use this to eliminate the fraction in solving algebraic equations would be most beneficial. The elementary teachers maintained that the students had not learned enough about using fractions by this point for them to be able to understand the concept and that it would become solely a procedural operation. The board liaison tried to find a way to bridge this gap of a completely procedural method by building on previous understandings of the students. In his final interview, Wesley noted that he was going to again bring it up because he strongly felt that this would be better started in elementary school "because it will lay the ground work" for working with "rational expressions in grade 12".

Figure 11. Polynomial example given by Wesley in discussion.
The example on the left is what he hopes students will learn to do so that they remove the when dividing, but the example on the right is what he typically sees students doing where they simply remove the and .

Another issue that the teachers discussed was creating a mathematics community in their school. During the meetings, the teachers spent time discussing the overwhelming number of students who claim that they "hate" mathematics and how to help the issue. Both Ryan and April gave surveys to their students to get a feel for how many students were in the dislike or hate category and were both disappointed to find that the majority of their students were. To Gabriel this dislike stemmed from over-testing in a subject area and how students who are not doing well tend not to like the subject. The board liaison noted that sometimes it is the feelings and attitudes of other teachers on staff who have an impact on students' perceptions of mathematics because of "their own anxiety in math" (interview transcript). Diana thought the dislike came from not being ready for where the curriculum said they were supposed to be in math. As a result, Gabriel noted that as teachers sometimes it is difficult to avoid "helicopter parenting" and just jumping in to save a student instead of watching them struggle (Meeting #13 transcript). The teachers discussed ways to keep the mathematics classroom appropriately challenging, yet a safe place to make mistakes and where students want to come to learn. The desired community would have the mantra that everyone who tries, gets appreciated (Meeting #2 field notes).

The other concern of the teachers was what the students were bringing from home about school and mathematics. One of the issues that the teachers discussed was the minority student who did not see his or herself reflected in the educational system and how hard it would be to ascribe to the ideology of the importance of school (Meeting #11 transcript). Ryan talked about making sure to get the students to buy in to feel a part of the community in the classroom in an attempt to combat outside forces. In an attempt to create a community between the elementary schools, secondary schools, and parents to support students, the teachers discussed proper placement of students in streams. During the second year, the secondary teachers commented on how much better the year had started with fewer students needing to change classrooms because they were not properly streamed. In the third year, they did share concerns that because fewer parents attended the meetings with the secondary teachers, they worried that not having the opportunity to reiterate the recommendations of the elementary teachers would have an impact on grade 9 placements the following year (Meeting #9 transcript). Several secondary teachers pointed out the difficulties for students when parents chose to put their children into a stream they were not academically ready to handle (e.g. Meeting #13 transcript). According to the teachers, there needed to be support from the elementary and secondary teachers as well as the parents for success in grade 9.

Images of teacher learning and teachers' roles in educational change.

During the professional learning group Meeting #9, the teachers engaged in a moderated marking exercise in an attempt to better align their evaluation practices. The elementary teachers and secondary teachers alike engaged in using a rubric to evaluate work samples and then the teachers discussed their grades (see Appendix L for transcript). The teachers also talked about the mathematics in the student work in order to support changes in each other's practices and gain new insights on student work.

In some of the meetings, the teachers focused on mathematical concepts that they focus on in their classrooms and how they would build the lesson with their students. Teachers engaged in not only mathematical talk, but ways in which to change their own professional practices in order to better support students. For example, a discussion of solving algebraic equations using algebra tiles was shared and then the group talked about ways in which the lesson laid the foundation for further concepts (Meeting #9 transcript). Samuel shared an acronym he used with his students to procedurally solve equations in the question that the teachers further discussed. Some of the teachers expressed concern that the acronym was not based in conceptual understanding of the operation, but in simply remembering a procedure. Even Samuel himself raised concerns about it not really being built on mathematical understanding.

In order to better support their students, the teachers in the professional learning group discussed vocabulary changes to help maintain a common language among the grade levels.

April maintained that mathematics language needs to be taught to students especially in elementary school so that it could become a lifelong language. She talked about making sure that the language she used in her classroom was the correct terminology that would follow the students through their lives as mathematicians. At several points, teachers mentioned how sometimes students are given mistaken impressions in the early grades that set them up for failure in later grades. Emma mentioned how she has to reinforce to students that a decimal can actually be a correct answer to a problem and avoid students thinking that if the answer is not just a small, positive whole number that they must have done something wrong (Meeting #9 transcript).

Current initiatives in teacher education, professional development, and/or teacher assessment.

During the most recent two years of the study, the professional learning group teachers made the commitment to furthering their own knowledge about mathematics teaching by attending two events held at the local university. The first event was Dr. Florence Glanfield who spoke to the teachers about teaching mathematics with an Aboriginal perspective, which is something that the teachers felt was pertinent to their own classrooms. Following the talk, they discussed the implications to their own teaching that were based on ideas shared by the professor. Although Wesley noted that he did not see racism as a part of his school, the majority of the other teachers, including two in his school, shared stories that they had witnessed with students. The discussion turned to talking about how students attend school with different perspectives that even the teacher sometimes cannot imagine. This again supported the teachers' desire to create a community to help support their students because sometimes the outside influences are much stronger and more prejudicial than can be imagined.

The second speaker brought in by the university was David Stocker who taught elementary mathematics with the theme of social justice. The teachers in the group attended the presentation by the speaker and then again met to discuss the morning activities. Madison noted after the meeting that it was not necessarily about incorporating social justice into the mathematics, but making sure that the math lessons are authentic and based in something real and meaningful for students (Meeting #15 transcript). She suggested the idea of discussing rate of change using a canoe if students in the classroom were interested in that instead of always using a bicycle. Ryan talked about real topics not just something from the textbook. All of the teachers in the meeting committed to trying at least one lesson that took a more authentic approach to teaching mathematics before the end of the school year. Wesley said, he felt the morning “was absolutely true professional development” (Meeting #15 transcript) and was very interested in trying his ideas with his students.

During the meetings, teachers also discussed articles and news about mathematics teaching. Emma brought in an article from the NCTM about mathematical reasoning (Meeting #15 transcript). Ryan discussed a news segment he had seen by Rex Murphy, including an article in *MacCleans* about mathematics education and how ineffective it has become (Meeting #17 transcript). The news segment also debated the quality of the textbook series used by the elementary teachers, and the teachers debated the merits of the opinions shared.

Discussion.

Evidence of discussions of knowledge-of-practice was shown when the teachers explored how the Ontario curriculum developed through grade levels and how they could adapt their classrooms to close gaps left by the curriculum in order to support their students. As in the example of Emma, both the conversations and her knowledge and experience with mathematics teaching allowed for meaningful changes to be made to support her students (Potari & Georgiadou-Kabouridis, 2009). Their conversations about the curriculum led to enlightening realisations about gaps resulting for their students. Shulman (1986) identifies two types of curricular knowledge important for teachers: lateral and vertical. Lateral curricular knowledge refers to knowing what students are studying in any other subjects at the given time. In their conversations, the teachers discussed how the topics they were teaching in mathematics impacted what they were learning in other areas, especially science. Through examining the vertical curricular knowledge, or where students are going or have been in a subject area, the teachers determined places that they needed to fill holes created by the curriculum. Had they not engaged in the discussions about the curriculum, the teachers would not have been aware of particularly necessary places to support their students. They also discussed how certain mathematics topics had cross-curricular relationships and how they might work with students to see these relationships. In examining their classroom practices the teachers also engaged in discussions to make transparent their strategic knowledge, or the form of knowledge employed by teachers when two ideas about teaching conflict and where professional judgment is needed to solve the dilemma (Shulman, 1986). In summary, during all of their discussions, the group members were focused on the learning of students.

Conceptual Knowledge

Not only did the teachers engage in discussions about teaching mathematics, they also discussed the specialised mathematics needed for teaching, which includes conceptual knowledge. When the interviewed teachers were asked about whether or not they felt their mathematics knowledge had improved as a result of the meetings, there was a mix of answers. Of the nine interviews conducted, only five of the teachers said that they felt they had deepened their knowledge of mathematics as a result of the discussions. These teachers all felt that their knowledge had improved through seeing different approaches to working with mathematics. The four who said no their knowledge of mathematics was not deepened did note that they felt their knowledge of teaching mathematics was strengthened through the discussions.

Table 7

Responses to Question about Deepened Mathematics Knowledge

Teacher	Knowledge increased?	Interview Quote
Elementary:		
April	Yes	"It was really understanding why something worked the way it did."
Blaine	No	"I don't know if my knowledge of mathematics has, but my knowledge of teaching mathematics is certainly improved."
Emma	Yes	"I also feel like I'm picking up content knowledge from them, little bits here and there."
Gabriel	Yes	"I think there were things that I don't think that I understood, or that I only saw in one particular way; whereas, now I can see it in a different way or ways that I had not even anticipated on seeing it."
Claire	No	"I've got a better understanding of how other people teach it, and I think that helps me reach everyone in here."
Secondary:		
Ryan	No	"I would say my knowledge of mathematics probably hasn't changed much. I would say my teaching of mathematics may have changed a little bit, if that's a subtle distinction."
Wesley	No	"I think teaching of mathematics has been deepened, but not my actual knowledge of mathematics, no."
Owen	Yes	"They're all math backgrounds, you're bound to pick up some things, right, so there's definitely a few things that I never would have thought of teaching it that way."
Board liaison	Yes	"We know certain things as specialists in mathematics, but understanding sometimes the root of where it really comes from and how they [elementary teachers] approach it, allows us to make a big difference for especially the nine Applied kids."

Note. Table contains responses to the question "Do you feel your knowledge of mathematics itself has been deepened through the discussions?"

Table 7 shows a breakdown of which teachers said yes or no regarding whether their knowledge increased and how they defined the answers. Although the teachers' ideas about whether or not they had gained mathematics knowledge differed, my data indicated there were definitely discussions that focused on mathematics knowledge needed for teaching. I found that their discussions typically centered around two types of conceptual knowledge: the knowledge needed by teachers and students' misconceptions in mathematics.

Knowledge needed by teachers.

In Meeting #4, the teachers engaged in a discussion about division and what is necessary for teachers to know as they teach division. They discussed the difference between quotative and partitive interpretations and created a chart of words that describe each model of division. The teachers also created word problems that illustrate the difference between the definitions. They spent time discussing the division definitions because, as they noted, if students do not understand the quotative or measurement method of division, they will be unable to understand division by a fraction.

During Meeting #16, the teachers discussed the question -5^2 and what the correct answer would be. As the teachers noted, depending on the calculator, there are two different possible answers that a calculator would give: 25 or -25. Even some of the teachers were divided about what the correct solution would be to the question. In the debate, the teachers talked about the mathematics inherent in the question and how the exponent is attached to the five and not a negative five, so the answer would be -25. They used the example of $–x^2$ to illustrate, and one of the teachers suggested looking at it as $-1\times5\times5$.

During the same meeting, the teachers were looking at a question about the volume of a rectangular prism and began a discussion about formulas. The elementary teachers maintained that they use area of base times the height, while the secondary teachers talked about using l×w×h. Emma noted that she does not use the word "formula", but wants her students to recognise that it is an algebraic expression that can be generalised and gives students an understanding of the volume. She noted that a formula often is just something into which you plug numbers. Gabriel noted that he often has students that get mixed up over where the length and width are and just stop working when they cannot figure it out. As one of the teachers was solving the problem and explaining a solution method, Noah pointed out that there are different interpretations of the problem since only the measurements were given. The problem asked teachers to figure out the percentage of empty space in the box based on the measurements. If a student had designed the box in a different way, then they would have gotten a different solution. Also the teachers talked about the conceptual understanding behind why the answer would be listed in cm^3 and not cm^2. According to the teachers, the students sometimes lose the reason for it being cm^3 (because it is cm×cm×cm) since they just put the units in at the end. Wesley brought up examples of where in physics it is really important to see the units in the problem in order to see the correct unit at the end and wondered if the teachers were doing a disservice to students by only requiring units at the end. Emma and Gabriel both noted that their students will often include units with all the numbers, even when not necessary, such as after the pi in the circle area formula. For example, students would incorrectly write $3.14\text{ cm} \times (6\text{ cm})^2$ when calculating the area of a circle with a radius of 6 centimeters.

Student misconceptions.

The teachers also spent their time discussing students' misconceptions and how they affect their mathematical understandings. One of the biggest misunderstandings seen by the teachers, which was consequently spoken about in many of the meetings, was that of the use of negative numbers. The teachers talked about different strategies to help their students with negative numbers and gave examples of issues. One such misunderstanding was introduced by Gabriel who talked about students reversing the number line and having negative and positive numbers on the wrong sides. He also mentioned students who, when drawing a four quadrant graph, will start with the largest negative number beside zero and then count down. The teachers talked about how students often lose their understanding of the numbers once negatives are introduced. Ryan and Madison both noted that in EQAO testing, as soon as a negative number is introduced, students have a “significant decrease” in correct solutions (Meeting #10 transcript).

Fractions were another area about which the teachers noted students had a lot of misconceptions. Emma again noted that students have lost their feel for the numbers once they start using fractions. Secondary teachers talked about having to reteach fractions even though it is not in their curriculum because students did not have the understandings needed to advance (Meeting #10 transcript). The elementary teachers talked many times about how the students are still “renting” the information and do not yet “own” it because of how little exposure they have had to fractions and their operations (Meeting #4 field notes). Teachers discussed how students struggle with beginning models of fractions and suggested spending the time to have students make the fractional parts out of strips of paper. At two different meetings, teachers noted that student difficulties in fractions can often be traced to faulty models when drawing circles (Meeting #4 field notes and #12 transcript). Extending this discussion of fractions, elementary teachers commented on how students are not ready to conceive of division being presented as a fraction (Meeting #13 transcript). The teachers discussed how students should be able to recognise that – —, and are all the same thing in order to extend student conceptions of fractions beyond the pictures and to give them a foundation for algebra.

Another area of misconception was that of algebra and dealing with the use of variables in mathematics. As Gabriel pointed out, seeing a letter in mathematics sometimes gave students the “deer in the headlights” look where they believed they “can’t do math anymore” (Meeting #17 transcript). Owen commented on how some of his students would add the ones to the “x” when working with an expression like 3x+4 to come up with 7x, and linked the discussion to how the pieces are different sizes and colours in the algebra tiles (Meeting #9 transcript). April, Emma, and Owen all talked about how when dividing 3x by 3, students will come up with “0” as the solution. Owen talked about how he made sure not to use the words “cross multiply or cancel out” so that students do not get the mistaken impression that something is disappearing (Meeting #9 transcript). Noah noted that it was important for students to understand that 3x was really 3 of x so that they would associate it with multiplication (Meeting #12 meeting transcript). He also mentioned that some students struggled with seeing the expression as an operation so they would be unsure of the opposite operation needed.

Discussion.

Research has shown that there is a specialised body of knowledge that is particular to mathematics (Ball et al., 2008; Chamberlin et al., 2008; Davis & Simmt, 2006; Kajander, 2010; Ma, 1999; Shulman, 1986; Silverman & Thompson, 2008) and that teaching in a constructivist classroom needs a deep and strong knowledge of the subject matter (Richardson, 2003). In order for professional learning groups to be effective for mathematics teachers, this essential body of knowledge needs to be addressed by the group. A strong and flexible understanding of mathematics is needed in order to be an effective mathematics teacher, but in my own data I observed that it also had an impact on participation in group discussions. In particular, Blaine noted that his lack of knowledge of teaching mathematics discouraged his participation in the professional learning group. Research points to issues with elementary teachers' mathematics knowledge because they are generalists in all subject areas (Richardson, 2003; Wixson & Yochum, 2004), so ensuring ways of increasing this essential knowledge is critical. Battey and Franke (2008) pointed to another area of concern when they found that a teacher, who said little during the meetings because of the lack knowledge, yet attempted the strategies in the classroom. They found that the teacher's lack of knowledge made it difficult to engage students in sense making and discussing the multiple solutions. In my research specifically, Blaine expressed concern over his being able to discuss anything during the meetings, but he did use the strategies and the lessons in his classroom. His lack of knowledge could therefore have an impact on how these strategies were implemented in his classroom, but this was not included in my research. Slavit and Nelson (2010) noted that non-participating group members in their research did not actually use the strategies being discussed. This was not the case in my research with any of the three teachers (Blaine, Diana, and Claire) who did not participate in conversations, who all claimed to be using the strategies discussed. My research supported the idea that these participants simply did not have anything to add to the conversations, yet gained a lot of new information or strategies for their own practices. As in the case of Blaine, a lack of knowledge inhibited his participation, and both Claire and Diana mentioned being new to teaching mathematics, so a lack of knowledge of teaching mathematics could have decreased their participation.

In examining the teacher's background, it is worthwhile to note that none of the elementary teachers had a degree related to mathematics (see Table 4). Of the secondary teachers, one had a degree in mathematics, Wesley, and Ryan had a degree in engineering (see Table 5). All of the secondary teachers had qualifications in teaching intermediate/secondary mathematics (grades 7-12). For the elementary panel, Claire had an intermediate/secondary qualification in mathematics, and Emma and Gabriel had primary/junior qualifications in mathematics (grades 1-6). The rest of the elementary teachers had no mathematics related qualifications. The concerns raised by researchers about elementary teachers being generalists (Richardson, 2003; Wu, 2009), as well as the need to have a strong mathematics knowledge to teach mathematics (Ball, et al., 2008), raises concerns of the impact of elementary teachers in mathematics instruction with no special qualifications.

The members of the professional learning group made distinctions between mathematics knowledge and mathematics teaching knowledge. When asked about whether or not their knowledge of "mathematics" increased, four of the group members said no, it had not been increased, but their knowledge of "teaching mathematics" had been. Silverman and Thompson (2008) believe that mathematics for teaching is only specialised once it has been combined with pedagogical knowledge. Based on this definition, all the examples of what these members claimed to have learned from the professional learning group falls under the category of the specialised knowledge for teaching mathematics. For example, Blaine discussed how seeing student work and analysing their responses helped improve his knowledge of teaching. Some research into mathematics teaching does not point to a direct distinction between the knowledge of teaching and knowledge of mathematics (e.g., Silverman & Thompson, 2008), and it was apparent this intertwined relationship was pointed to by this group. Teachers need a deep understanding of the subject matter in order to choose appropriate examples to use in teaching the topic (Skemp, 1986).

During the meetings, the group members discussed the knowledge needed by teachers for teaching mathematics and identifying common student misconceptions. Research points to these two areas being part of this umbrella of specialised content knowledge for mathematics teaching (Baumert et al., 2008). Ball, Hill, and Bass (2005) indicate that included in this specialised knowledge is discussing student work and responding to both correct and incorrect solutions. During the meetings, the teachers examined student samples and then discussed the student responses and what the solutions meant in terms of the student's mathematics knowledge. Facilitating student understanding is dependent on a teachers' deep understanding (Ma, 1999), so discussing student work or misconceptions is also important for building a stronger understanding of mathematics to support students. Hence I found evidence in my data that all participants felt they had gained the specialised knowledge related to mathematics and teaching.

Research into professional learning groups suggests that a knowledgeable outside party is necessary for a group to operate successfully (Anderson, 2005; Heirdsfield, Lamb, & Spry, 2010; Linder, Post, & Calabrese, 2012). In mathematics this could be especially important with teachers using methods that are potentially different from what they have experienced themselves when learning (McNeal & Simon, 2000). In examining the discussions in the professional learning group in my study, many of the teachers in the group already possessed significant mathematical knowledge. It was also clear in the discussions and classroom observations that some of the teachers had experience with teaching in a more reform-based methodology. Therefore in my research it was not necessary for the group to have a "knowledgeable outside party" participate since some of the group members already held this necessary knowledge. If this knowledge of mathematics and reform methods is not held by members within the group, it would need to be gained from an outside source. In summary, this group was able to have thoughtful discussions that could potentially lead to changes because the basic understandings of reform strategies as well as significant knowledge of mathematics were already within the group.

Making Changes

As mentioned earlier, the stated purpose of the professional learning group was to support students in the transition from elementary to secondary school. One of the reasons that the teachers focused on changing their practices in order to support students was, as Ryan mentioned, to narrow the gap for students (Meeting #13 transcript). He continued that the additional freedom in secondary caused some students to fall through the cracks and expressed a desire to even make “environmental” changes if it would help students. The board liaison stressed this importance because secondary students need to have more accountability and can now fail in mathematics classrooms. In examining the meeting transcripts and interviews, the teachers talked about changing vocabulary and strategies to make them more aligned. The elementary teachers talked about changing focus in their curriculum to help support students, and several of the secondary teachers mentioned using manipulatives and different models as a result of the discussions.

In his interview, Gabriel noted, “It [the professional learning group] helps you take on something that you might not be comfortable [with] because you know you have some colleagues that you can turn to and say, look I tried this, this is what came out, this is what I was anticipating to get from it, what can I do to change it?” The board liaison seconded this mentality of feeling secure to try new ideas “because it’s a supporting group that I could come back to and talk to” (interview transcript). The professional learning group itself allowed for an environment to encourage changes simply by providing a safe place to discuss new practices with encouragement and support. Although changes were discussed within the previous section, using the categories of curriculum, pedagogy, and vocabulary or common practices, I now discuss in more detail some of the changes described by the teachers. This is followed by a connection to the relevant literature in the discussion.

Curriculum

April felt her teaching changed through seeing where her students needed to move in future years. From the discussions of the secondary teachers, the elementary teachers realised that there was not enough algebra specified in the elementary curriculum to prepare their students for grade 9. As Emma mentioned in her interview, “I have pushed kids further in grade 8 than I ever had before because seeing where they wanted them to be in grade 9…if you leave grade 8 and all you can do is solve by inspection, you’re not going to be able to survive what they ask you to do in grade 9” (interview transcript). April noted that “I’ve always spent a lot of time on algebra, but what I downsized was the geometry” based on the discussions with the secondary teachers on what was needed in grade 9 and beyond (interview transcript). As she said, “The things they spend more time on, I spend more time on” (interview transcript). Claire noted from talking to the other elementary teachers that she would have to change the order of her units in order to address the correct strands for a reporting period (interview transcript).

One of the discussions of the group focused around EQAO testing in grade 9 and how to properly support the students so that they would be successful on the assessment. In particular, students really struggled with answering the multiple choice questions. Also, Madison expressed concern over questions containing “not”. Other concerns included multiple step questions and the fact that common errors were listed as possible solutions for the questions. Emma, for one, talked about adding multiple choice questions to her grade 8 class activities so that she could support students’ future capacity for testing. She mentioned that she also spends times talking about mistakes made in multiple choice questions to further support her students' growth. Although the grade 9 EQAO was not necessarily an elementary expectation, the teachers wanted to give students support to help them be more successful in secondary school.

Pedagogy

Blaine talked about changes in teaching coming from discussions about student work. To him, this set the stage for discussions about how the lessons were taught and gave him a context for his learning. He commented that the observations also provided a rich context for learning by witnessing how a teacher handled situations within their own classroom. Gabriel noted, "at least this [the professional learning group meetings] is sustained amount of time, and we're focused on one topic, we're focused on one need, so it does change the way that you present your lessons, the way that you do your things in your classroom" (interview transcript). Several of the secondary teachers discussed using manipulatives as a result of the discussions, which they would not have used prior to the meetings. As Wesley noted in his interview, when he first saw algetiles mentioned in the textbook, he would have skipped over those lessons. As a result of the discussions with the group, he talked about keeping "an open mind" and trying something new (interview transcript). Owen mentioned how he had not used a lot of manipulatives prior to the professional learning groups and noted that now he was using fraction strips or circles and algetiles more.

For Owen, one of the important aspects of the professional learning groups was just sharing resources that could support creating a common practice within the different grade levels. He specifically talked about using "factoring trees" in his classroom now after having heard the elementary teachers discuss how they had used them. He noted that prior to the meeting discussions, he would not have thought to use them because that was not something he would have used to solve the problem himself. Owen even spoke of using more investigations in his classroom as a result of the discussions instead of simply giving students the formula. For Emma, the group discussions helped her to “recognise there’s so many different ways to get at it [the mathematics], and it’s just trying to find one that works” (interview transcript). Gabriel talked about how the conversations “change the way that you present your lessons, the way that you do your things in your classroom” (interview transcript).

Wesley discussed how the morning at the university set him up for changes in his classroom. As he noted, the talk gave him ideas to get “a little more power out of what I’m doing right now in my mathematics” (Meeting #15 transcript). He talked about using the ideas to do more substantial mathematics with his students instead of just common textbook problems that are not necessarily based in real-life ideas. He described in his interview being very disappointed about his students not understanding the lesson he attempted, but expressed the desire to try again with a class with more number sense. April also found the same talk inspirational because “I love it when we go and see somebody and then we get to talk about it later or work together” (interview transcript). She discussed “putting what’s really important in the world in my lessons” noting that “textbooks don’t often do that” (interview transcript). April’s lesson met with more success than Wesley’s, and she was determined to try more even though she noted how much work the one lesson was to put together. Gabriel discussed in his interview having seen David Stocker one time before and how uncomfortable he was with the ideas that were discussed about using social justice. He noted that going with the group was a completely different experience and wondered if discussing the ideas following the talk had helped with his comfort level. Gabriel further noted that he had incorporated the ideas into one of his lessons this year.

Developing Common Practices and Vocabulary

In his interview, Owen talked about the importance of the common vocabulary among the different grade levels saying, "terminology I was using where kids would be confused, and really it was the exact same thing they were already doing, it was just called something else" (interview transcript). Ryan noted in his interview that through observing his fellow secondary teachers he could see how the language had changed to incorporate more of the terminology that was being used with the students in elementary school. April talked about how she incorporated what the secondary teachers would call a concept or how they would teach a concept to prepare her students for secondary school.

Emma made the change to starting her n-chart at 0 after Ryan mentioned it during the observation in her classroom. She found it useful during her grade 8 class because in an input/output chart of a linear pattern, the constant is the number paired with the zero making a more direct link to the pictorial representation (Meeting #12 transcript). Discussions with the group shared that starting at the zero also linked to graphing in secondary school where the number paired with zero is the y-intercept.

Discussion

Professional development research stresses the need for constant growth and development to be part of teaching (Ball & Cohen, 1999; Opfer & Pedder, 2011). As such, I sought to discover in what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students. Sykes (1999) addresses the need to rely on teacher knowledge and actions if there are to be changes in the profession. The discussions in the group were all based in participants' own practice related to topics with which they had concerns in terms of supporting their own students.

Research points to the importance of teachers choosing their own topics for discussion and meeting with others (Linder et al., 2012). In my data, the main benefit discussed by members of the group was that they were able to work together and discuss their own classrooms, which aligns with the literature. As April stated, “not having somebody come in and tell us or report on us, gives us the freedom to explore. …We will meet, and we will get something accomplished” (interview transcript). Based on their statements in interviews, two of the teachers, Diana and Claire, noted they were the only intermediate mathematics teachers in their schools, and they felt that without the professional learning group they would have had no peers to reflect with and discuss teaching practices. Linder et al. (2012) also stress the need for autonomy within the group, and I noted that members of the group studied here shared concerns about this remaining a characteristic of their own professional learning in order to continue supporting their personal development.

In their research into mathematics professional development, West and Curcio (2004) maintain that support after professional development makes changes more likely. The structure of the professional learning group meetings in this study provided the members support to try new ideas in their classrooms and then discuss them with the other teachers. When interviewed, the professional learning group members noted how the meetings gave them new ideas that they could apply to their classrooms to improve their practices. Gabriel specifically mentioned how the group meetings allowed him the confidence to try new strategies because the others would be there to discuss what worked or did not work after attempting the ideas. The support of their peers gave the teachers a sounding board for new ideas and difficulties encountered.

Brahier and Schäffner (2004) pointed out that in mathematics, teachers need concrete examples of reform strategies, a time to try the strategies, and then reflection for changes to actually occur in the classroom. The group attended presentations at the local university to learn new strategies for teaching mathematics. The group would then meet together and reflect on the new learning and how it could be applied to their personal classroom situations. During the meetings following the presentations, the group members would discuss strategies to try in the coming months so that they could reflect together on the effects. One of these presentations proved to be a strong catalyst for three of the individuals especially: Wesley, Gabriel, and April. Wesley in particular embraced the new ideas which were a vast change from his usual mathematics teaching. At this particular presentation, the teachers were actually trying the new strategies for teaching mathematics for themselves, which research has pointed out is necessary before teachers will begin to implement new strategies in their classrooms more generally (Carnegie Corporation of New York, Institute for Advanced Study Commission on Mathematics and Science Education, 2009). From having the experiences for themselves and hearing how another educator was using the strategies in his teaching practice, the teachers attempted something new in their own classrooms.

In conclusion, the professional learning group adhered in many ways to the characteristics used in the research literature to define a professional learning group. The group also used action research characteristics in their discussions in order to continue moving forward in their practices. I examined how a teachers' knowledge of teaching was addressed through the discussions in the group meetings. By examining both conversations that discussed teachers' knowledge and conversations that were specific to mathematics, I was able to highlight how the professional learning group was dedicated to improving teaching practices. Lastly, I examined changes made in the teachers' practices as well as areas inherent in the professional learning group that supported those changes. Each of the sections was also linked to the relevant literature that arose from the discussions.

In the next chapter, I begin by summarising the answers to my research questions based on the data collected from the professional learning group. A new model of an effective mathematics professional learning group is then developed using the information from the case. The model is then compared to two existing professional learning group models. Finally the model is used to re-examine the professional learning groups explored in the work of Kajander and Mason (2007) in order to add to the literature on effective professional learning groups in mathematics.

CHAPTER SEVEN: SUMMARY AND MODEL

Through my research, the overarching focus question I sought to answer was: What are the conditions of a professional learning group of intermediate mathematics educators that improve their teaching practices? By examining the characteristics of the professional learning group that were evident in the meetings, the beliefs and knowledge of the individual teachers, and the stories of the group members, I have provided data explaining how the professional learning group I was part of was able to make change in their own mathematics classrooms. The group members engaged in thoughtful discussions about their mathematics teaching and knowledge. In terms of the group's success in impacting their teaching practices, it is clear that it had a strong impact for the majority of the members. Through focusing on student work, test results, and specific classroom lessons, the group members were able to discuss their beliefs about teaching mathematics and knowledge of mathematics. As stated earlier, research has shown these two aspects have a profound effect on the teaching of mathematics, so these conversations needed to be at the forefront of the group discussions. The group also sought to expand their knowledge base by attending presentations of visiting speakers. I now summarise the data which addressed each of the sub-questions which informed my focus research question.

Research Questions

1. In what ways does the group adhere to or deviate from the characteristics of a professional learning group as defined in the literature?

After defining each of the characteristics of an effective professional learning group, it became clear that the studied group adhered to each of those characteristics in some way. The most problematic characteristic was that of shared beliefs, values, and vision. Although the entire group appeared to focus on increasing student success in secondary school, they did not all believe that it would be accomplished in the same way, nor share a common vision of mathematics classrooms. I believe this goes back to the influence of teacher beliefs on mathematics teaching and a similar situation would be present in other mathematics professional learning groups. Although the group members did work together to set a common agenda and all contributed their ideas, there appeared to be characteristics of the group that did not fit the literature's definition of "shared leadership". For one, Ryan organised the meetings and kept the discussions on track and moving forward. Also there was a small group of dominant personalities who tended to push the conversations toward more reform-based strategies. The professional learning group did follow the research in exhibiting collective learning, supportive conditions, and shared personal practice.

In order to better define how this group was potentially moving their teaching of mathematics forward, I added three characteristics from DuFour and Eaker (1998) when examining the professional learning group: action orientation and experimentation, continuous improvement, and results orientation. It was clear that the teachers were focused on trying strategies and seeing the impact it had on their students' work. This action research focus may be the reason the group was able to really make changes and have deep discussions about teaching mathematics. One aspect that could be beneficial for the group to consider in terms of future growth is strengthening this area of experimentation in what Van Driel and Berry (2012) describe regarding how the group would follow through on trying new strategies and then reflecting on them. During the one meeting where they did talk about trying new strategies based on David Stocker's presentation, only three of the members actually discussed trying the strategies as the group had agreed. Nevertheless, from an overall research standpoint based on the entirety of the research criteria, the professional learning group was successful and potentially had a lasting impact on the individual practices of the teachers.

2. In what ways is the professional learning group supporting teachers to make changes in their own teaching in order to improve their teaching to enhance the learning of their students?

Examining the discussions of the group as well as talking with participants, it was apparent to me that the teachers' practices were changing. Some of the changes may have seemed small, such as changing vocabulary, but based on the teachers' evaluations, it appeared to be having an effect on their students' achievement in mathematics. None of the teachers made radical changes in their mathematics teaching, but all of those who spent a significant time in the meetings were altering their practices. In the case of Wesley, and his very traditional practices, it was a step in a forward direction for him to just consider using a new lesson that was more exploratory. The fact that he did try a new strategy was a push toward more reform-oriented teaching and it is hoped that this is just a beginning for his growth. The use of more manipulatives in the secondary school was also a promising change that would only help students in the future. The elementary teachers adopted more of each other's practices, allowing those with less experience with reform-strategies to gain more skills. Since all of the teachers began the journey at different places along the continuum of using reform-based practices, it would be appropriate for them all to end at different spots. It was encouraging to see how all of them seemed to be continuing to move more toward this type of instruction, despite some needing to move a lot further than others.

3. What are the beliefs about mathematics teaching and learning of the individual teachers in the group? How are these beliefs dealt with in the discussions of the group?

Since beliefs in teaching mathematics play such an important role in determining pedagogy (Wilkins, 2008), all my research results needed to be viewed with this in mind. I chose to focus on two aspects of teachers' beliefs: "(a) what kind of mathematics is important for students to learn, and (b) how this mathematics should be taught" (Grant et al., 1994). It was clear that there was a dichotomy that developed, with the majority of the secondary teachers being on one end, and the elementary teachers on the other. This dichotomy also illustrated how tied these two aspects were in that the teachers who felt students should be able to answer questions on an exam, also felt that direct instruction with a focus on memorising rules was the best way to accomplish this. On the other hand, those who felt that students should deeply understand mathematics, also believed that it was best for students to engage with mathematics through problem solving and exploration. Ryan really did not fit into either category both believing students should get correct answers on exam and deeply understand concepts, yet used more traditional lessons and was a big advocate for the use of manipulatives with students. There was evidence that all of the secondary teachers used more manipulatives in their classrooms, and perhaps more reform-based strategies as a result of the professional development. Since I did not actually observe all of their classrooms, my results are tied to their reporting which items they used. Research has indicated that it is possible for manipulatives to still be used in a traditional method without being considered a constructivist approach to teaching (Windschitl, 2002), yet I believe simply incorporating those tools could indicate a shift in beliefs.

4. In what ways are the teachers' mathematics knowledge addressed in the discussions of the professional learning group?

To teach mathematics effective, teachers need a deep and flexible knowledge of mathematics (Silverman & Thompson, 2008). As such, professional development in mathematics should include discussions about knowledge. This group definitely focused on discussing both knowledge of teaching and knowledge of mathematics. The teachers discussed strategies they used for teaching, shared resources from their classrooms, and offered advice on each other's practices. They openly shared student work and test scores in order to have honest and critical conversations about their practices. The teachers also engaged in conversations about the mathematics used in student work. The professional learning group participants also discussed mathematical models and manipulatives used in their teaching and how to use them to better support students in developing their mathematical knowledge. The conversations also included discussions about both mathematical curriculum and other subject areas in order to determine gaps in the curriculum design as well as areas the mathematics supported in other subjects. As a result of the conversations, the teachers all claimed that their own knowledge of mathematics or teaching mathematics increased from engaging in the professional learning group meetings. By including discussions in areas of mathematical knowledge and teaching strategies, the teachers were able to make changes to their practices in order to better support their students.

5. What are the experiences of the individual teachers within the professional learning group in relation to their participation in the group and the impact on their personal mathematics teaching?

By further exploring the stories of Emma, Blaine, April, Wesley, and Owen, I was able to give a picture of who the individuals in my research were and how a professional learning group could impact teachers. Although all of the stories were different, these stories gave a more complete picture of who was involved in this process during my research. All of the teachers in the narratives began at different spots in the journey and got different things out of the experience. For example, Emma was very active in the conversations and ensured the discussions moved the teachers toward a more reform-oriented approach to teaching. Among other things, she felt she gained more ideas about all the different students could approach mathematics from listening to all the other teachers in the group. On the other hand, Blaine did not participate in conversations, instead listened to the discussions and absorbed new knowledge about teaching mathematics and trying the strategies in his classroom. Considering the people involved in the professional learning group is essential in order to get a complete picture of the experience and the impact it has on individuals.

As part of the narrative of my research, I wanted to ensure that the teachers were able to define the success of the professional development. As a researcher, it was clear that the group was successful in moving the teachers forward in their practices from wherever they began. Although all of the teachers got different knowledge from the meetings, they were clearly improving and changing their own practices. As I mentioned, since the nature of a professional learning group is very personal and teacher driven, the definition of success as defined by the members is as equally important to be considered in research as an external perspective. All the teachers interviewed in the final year of the study noted that the professional learning group was a positive, worthwhile professional development experience. In the focus group, Diana noted she would return to her classroom “more excited about teaching math” after the group meetings. Claire credited the professional learning group as giving her power to talk to her students about experiences in the secondary school and said it was “by far the best pd [professional development]” that she got (interview transcript). Gabriel talked about the group being really positive, and “I do come out of there with something” whenever they would meet (interview transcript). Blaine found them helpful in supporting his self-defined lack of knowledge about teaching mathematics, and Emma found it interesting to see how her students succeeded after they left her classroom. April said, “I really see a lot of benefit” (interview transcript). The teachers in the secondary panel also saw the benefit of attending the professional learning group. Owen commented on how helpful they were to influencing his practice in creating a common terminology as well as sharing practices. Wesley noted that professional development “has to be done” and found a benefit to getting “a bridge going of what’s happening in elementary and what’s needed in secondary” (interview transcript). For Ryan, the information from the professional learning group was added to his “mulcher” to be combined with other ideas he was gaining that were influencing his teaching. Overwhelmingly, the teachers found the professional learning group to be beneficial and important to their practices.

Professional Learning Group Models

As research described a variety of different approaches to professional learning groups in mathematics (e.g. Brahier & Schäffner, 2004; Hierdsfield, Lamb, & Spry, 2010), but not all such groups appear to have the same impact on the teachers involved (Kajander & Mason, 2007). If moving to a more reform-oriented pedagogy is indeed beneficial for student learning (Askey, 1999), then effective professional development in mathematics is needed. Now that I have explored the answers to my research questions, as well as defining “success” based on the participants’ viewpoints, I return to my initial queries about how some professional learning groups are successful, as this one was, and how some are not. Since my research focused on using professional learning groups to address this need in mathematics education, and as part of this work, I created a model of mathematics professional learning groups that came from my research data. The purpose of the model is to describe the characteristics of a professional learning group to be effective in supporting teachers as they learn and refine new strategies for teaching mathematics.

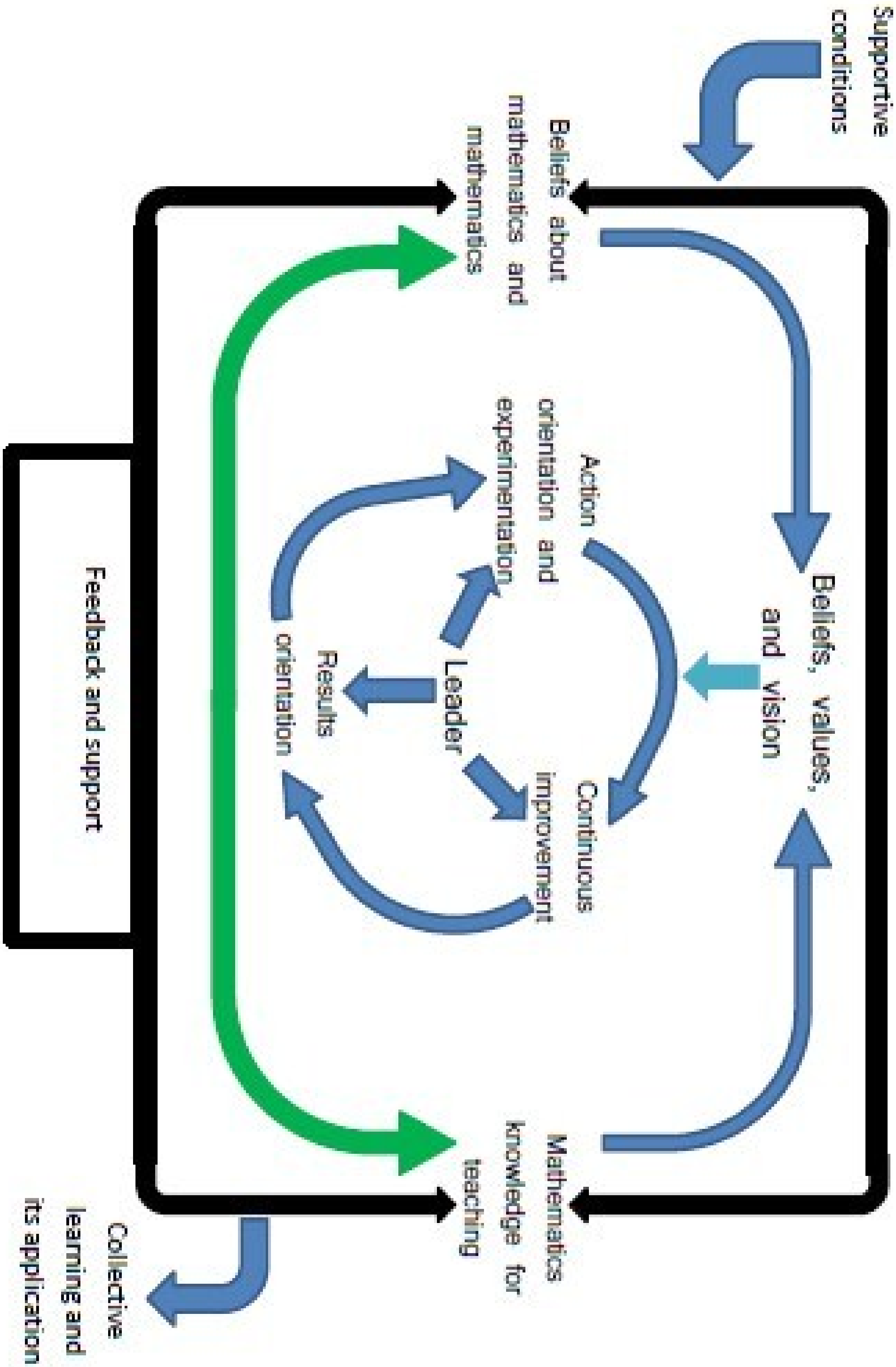


Figure 12. Proposed model for mathematics professional learning groups based on my research.

My model uses elements of professional learning group research, research in mathematics teaching and learning, and observations of the professional learning group I attended. This chapter presents an illustration of the model (see Figure 12) as well as a description of each of the parts and why they were included. I also detail how these characteristics resemble or differ from previous research studies. The goal is to present how each section fits into the overall scheme of ensuring a professional learning group can be used to support teachers in moving toward a more reform oriented approach to teaching mathematics. Following the discussion of the model I created, I discuss two current models for professional learning groups that are currently in research. I discuss how the model in my research addresses gaps in the current models. To conclude the chapter, I revisit the discussions of the professional learning groups in Kajander and Mason (2007) and examine how my model could be used to further illuminate what makes an effective professional learning group.

Proposed Model of Professional Learning Groups

The proposed model begins with ensuring that supportive conditions are being met. As research into professional learning groups has indicated, this includes having the focused time to meet as a group (Anderson, 2005; Eaker, DuFour, & Burnette, 2002b; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008). The teachers in the group I observed expressed at the end of each year that they were never sure if there would be funds available from the board for the following year in order to allow them to continue to meet. Without this necessary investment, I feel it is less likely that the teachers would have been able to make changes in their practices. Supportive conditions also include making sure the teachers have the support of their administration to make the changes to their classrooms (Blegen & Kennedy, 2000; Buckner & McDowelle, 2000; Hall & Hord, 2006; Hord, 2009; Hord & Sommers, 2008; Huffman, 2000;

Patterson & Patterson, 2004). Reform-oriented classrooms potentially have more noise as students are working together and engaging in discussion, so it would be necessary for the administration at the school to value this type of work since it differs from a quiet, traditional classroom. Past reform efforts in mathematics have been said to fail due to unsupportive administration (Clarke, 1997; Handal & Herrington, 2003). Supportive conditions also include allowing the teachers to make their own decisions about what topics to discuss instead of having a pre-made format for their agenda. Several of the teachers discussed how frustrated they were by professional learning communities where decisions were made for them about what to discuss and when. As April said, “Not having somebody come in and tell us or report on us, gives us the freedom to explore. …We will meet, and we will get something accomplished” and that she wanted administration “to have the faith and trust in that process that we will go there” (interview transcript).

When the supportive conditions are met, the model for the actual professional learning group can be put into motion. The entire cycle of professional learning groups is framed by both the teachers’ beliefs about mathematics and mathematics teaching, as well as the teachers’ specialised knowledge of mathematics. As research has indicated, the knowledge a teacher has about mathematics influences their beliefs about mathematics, and vice versa (Holm & Kajander, 2012). The beliefs a teacher holds about mathematics are going to influence their pedagogical choices (Cross, 2009; Potari & Georgiadou-Kabourdis, 2009; Wilkins, 2008) as well as how they interpret the conversations of the group and how they are enacted in their classroom (Grant et al., 1994). A teacher holding traditional beliefs is going to have to confront their own beliefs before being open to something new. The knowledge a teacher has of mathematics also affects their ability to implement reform-based strategies (Potari & Georgiadou-Kabouridis, 2009). Most of the discussions within the professional learning group had a content focus. Knowledge of teaching mathematics needs to be taken into consideration because what is gained from conversations would change if teachers did not understand what is being discussed. Research into specialised knowledge of mathematics for teaching links content knowledge with pedagogy (Silverman & Thompson, 2008). Discussions of the professional learning group would also need to include how the mathematics is used in the classroom with students. A lack of teacher knowledge would also impede being able to fully implement the strategies and engage in discussions within the classroom. Both beliefs and knowledge influence discussions of a professional learning group and how those discussions are interpreted and implemented in a classroom. Personal beliefs and knowledge are also important for framing a professional learning group in mathematics because of the influence on the beliefs, values, and vision of the group.

In contrast to previous research of professional learning groups, for my model, I changed the characteristic of a "shared" beliefs, value and vision for mathematics professional learning groups because it did not hold true in my study. It became apparent in the discussions of the groups, that not all the members held the same beliefs about either mathematics or what they would get out of the professional learning groups. There was a common vision of improving student success in secondary school that guided the group; as Wesley stated, "I think it's a good idea to get a bridge going of what's happening in elementary and what's needed in secondary" (interview transcript). It was apparent that the teachers had differing opinions about what this meant and how it could be accomplished in the classroom. As long as some of the members have a reform-oriented vision for teaching mathematics, then the group could function in making changes. Not all of the members needed to share this belief as long as they are exposed to the strategies that can affect their practices and give them a chance to examine their own beliefs about mathematics teaching. Wesley was a perfect example of someone who saw the professional learning group as being important for his practice, but in personal communication he expressed that the purpose of the meetings was for the elementary teachers to make changes to better support him in what he was doing in secondary school. Anything Wesley shared initially showed a very traditional belief of teaching mathematics where students memorised and applied procedures to the concepts. His beliefs about teaching mathematics began to change after the meeting with David Stocker where he was confronted with a radically different approach to teaching mathematics. Although he did not initially share the vision of reform-oriented teaching that some of the others held throughout the meetings, he was pushed in that direction from attending the group meetings. The beliefs, values, visions of the group though would influence the direction of learning discussed during the professional learning group cycle that is found in the center of my model.

The professional learning group cycle of my model references the work of DuFour and Eaker (1998) by including elements of action research: action orientation and experimentation, continuous improvement, and results orientation. It was this cycle of continuous improvement with a focus on results and trying out new methods that seemed to keep pushing the professional learning group to work with and refine more reform-oriented strategies in their classroom. By focusing on this cycle, each part of it would lead to a shared personal practice that would include new strategies that would benefit their students.

Also influencing the cycle of the professional learning group shown in my proposed model is what I have called the “leader”. My interpretation differs from the shared leadership model of past research in professional learning groups (e.g. DuFour & Eaker, 1998; Hord & Sommers, 2008), and this aspect is not necessarily a person who convenes or runs the meetings, rather it represents something more fluid. Here, the leader could be an individual, group of individuals, or piece of research that would help drive the cycle of the professional learning group toward the goal of making and supporting changes in mathematics classrooms. An example of this “leader” would be Emma and Gabriel during the meeting at which Wesley brought in his idea for having elementary students learn more rules that would support his work in grade 10 in factoring polynomials. Emma and Gabriel were able to push the direction of the meeting away from simply having elementary students memorise a procedure, and towards a more reform-oriented approach. Her belief in having students explore and learn about mathematics in a more conceptual manner was challenged by Wesley’s insistence on teaching students to memorise this procedure. Other members of the group joined into the discussion on how to address Wesley’s concern but still keep the idea conceptually accessible to the elementary students. The group could have just implemented this inappropriate practice in their classrooms or ignored the comment. Instead, Emma took a leadership role, and as an individual who believed in making sure students were learning in a conceptual manner, pushed the conversation toward somewhere it would be more effective for the students involved. Other examples included attending the meetings at the university where other professionals with a more reform-oriented mindset worked with the group to make changes in their practices. This becomes important in mathematics to allow the cycle to move the group forward in making reform-oriented changes, and not simply making better procedures or new worksheets for their classrooms.

The entire professional learning group model rests on a base of feedback and support for the teachers involved in the discussions. It is this characteristic that is the backbone of the entire process. As Diana stated, “I am more excited about teaching math” based on the discussions of the group meetings (focus group transcript). In terms of support, Gabriel noted that the group "helps you take on something that you might not be comfortable [with] because you know you have some colleagues that you can turn to and say, look I tried this, this is what came out, this is what I was anticipating to get from it, what can I do to change it?" (interview transcript).

Finally if the model is followed as shown and described, then the outcome would be collective learning that would prepare teachers to use more reform-oriented tasks in their classrooms, and its application would be increasing the mathematics learning of their students. As Gabriel noted, "At least this is [a] sustained amount of time, and we're focused on one topic, we're focused on one need, so it does change the way you present your lessons, the way that you do things in your classroom" (interview transcript).

In order to assess its usefulness, the model needs to be used with research about other teacher groups to explore if it represents teacher professional learning group development more generally. To begin this discussion, I now examine two existing models for professional learning groups and compare them to the model which I proposed.

Current Models of Professional Learning Groups

The first model I examine (see Figure 13) was created as part of the MathGAINS website for use with professional learning groups. The model focuses on having teachers plan out their actions for the meetings, use them in their classrooms, and then reflect on the strategies. The model here nicely sets out the activities in the center of the model I proposed. I believe that the Classroom Dynamics model fits with the work begun by DuFour and Eaker (1998) on using action research tenets in a professional learning group. What is missing in this model is the mathematics specific piece of the impact of beliefs and knowledge on mathematics teaching.

Classroom Dynamics within a Professionallearning Cycle

Plan

1. After _ectllll a duS1B of Cl.I'rio.ILWn upe<tldonsfor • L.W lt of swcfy a:unine d.:rta/«'111 .-to mtklpate .-ofs&J'*'t need nc-dlnJ'--tlln5 Stlls and wort Habits:
   - acnlew!mcmcbta (e-a.• pmr assessment. f:Q,\0)
   - leo m ,. L!s le.grc:spordll"v, coll;d:aor.rtionl
   - "' Jc;,m le-a... tme on tasl(. ecb help...ne_n Mtl:e<led)
   - student ben31110Ur'S dult 111! c:onducwe to — ermronment lor powerfullurnu- c
2. llclmlne daQ/eW!Sia!to determine todler rwoeds lltld lnUn!SBrd:lted to classroom dynamks:
   - tellchælf-n'flecuon
   - atdalfncnd/peer lwC!d re'IK:IOn of cbssmom dynamiCs
   - ldlntoiicatal of teKher tr.zcrem ln d&sroom dv lc§ Iwould 1-.to wa manip'Uiat:M:s 'liON often but am CCII'lCI!f1leO llbout clauroom m:u apmem;I would lite to ho!lp my s!Uclents worlt more cffectl\<lelv on grotr S; I ""-ould. b to change tho ysiiCIII:J¥OUt of mv 11»1n quktlv to crate erwfronme.nts wh«e I cll1 hdp mys:udents lc!am Ikxlblegr ings)
3. Seleda classroom dynan.lcs lumlJ1C locus(lor a 2..w.tmocfl.lle/unlt) th:rt ..d.circ:sses the ..:a of student need,••a of te;xn.nced/inten:st lltld thiJt wpports c!Jr-ealy tho kaml"'of thes.tecud duster of curriculum t:lq)OCtortlcns:
   - ciC'tcrmlM the d<lS.lr00IT> c!vnamla foals by lllcnt.. lnitheteKI= need($}/mteresr(s) that SUllPOtU d.rr.dJy the S1uaent ncedl l. (e.g,. "Many nude rs ln"'Y emsseen to need ""P"J"ernen& inthe ·collaboration" r...ntnc U J _ My seJ r.!llealon-.!lntcnsts ln dassl00r'l aynanwcs anct me to leo rntc more aboote «tMt grotrllworic. therefore, I wWI.select ef'ecu.e ll)"O.Jpjngs :u my clawoom dyn.amlcs foau.1
4. Oetermlne teacher lamlnc I la., wtat and how) required to addnss the student nllctd lltld d;usroom dynamia focus (e.c.. r11vlcw cumana lnstruoctlonal pqctlaa ;and raurc:h to dasr00111lfyn•mlcs focus)
   - dassraom tcc:t.JGAINS-bs:.te.01 A!SOU CEIQea gues
5. Decide liONt nfd.ence wtl lncfialte tbe lewis of student :Khiewrnl!'llt frK selected ciL.lstler of currlalu vowth ln tbelnmlrlc skills f00I!SCtd on wlltt tbc dassroom dynamla str.ltqY Implemented:
   - tellcher-d!M!Ioped C'aluabon toub,.SCXM"lng toob or tr.dlrc tools
   - teacher.d./orsc.ia=cn1ofleatnS skills
   - rv;obon checldists
   - studerca ;ortlfxts IN:hatl"lCf\gU.I;emena ln lmmln5(.• exit canis)
   - athel'lncflators (e,g., task ClCMT1llletlon rates, umeon tas <l

Act

6. Implementewd.,.,_based stnt-Ps and xdons
   - Adjust classToom gr.u:tJCZs and rauunes asn ea based on onsoiini assessment fw Ing:
     - –lfT111111Yt tr.nuo'tlOnS from one lrouoi"C to anacn. bv Pirc cxpllat Instructions and re blll:y to studenasfolng desks moreq- kily
     - al!lust norms fwgba.sed or on_....,bans
   - lnAeln pror.wlonallcllfTW1i 1 .co tmchellceT obselv.rtton, coxhing/'MIltaril>g) to build a coll«tMt undemilndl"'of the cbssroomc .s slr.l!egy
   - Aali!SS pratesslonal lt:arnlf1i A!SOUra!5 [,release trne, class COWr.lge, lumlf1i matcNls .aboUt Clll120Dm G'fNmiCS , on ltle resoura:s such as eauGAINS.-.!clawoom clynamla trowi ablecolllngue:s)

Observe

7. Monitor st\ld.ellll ;and t:Ddler leamlng:
   - SJQre and u.amine e\'ldcnce of swdentlaa.mlr1g-lndude student 6eedbac.i., ti!CDid USN tr.lChng chart/d01ta lllr.lll, deo.tse next steps
   - t:Dmtne ewi Hffom cb=oom ct,narnocs tnldlshcc1, assassmen:a/learn ,.s etc.
   - Analvze taadler;ond student self asscssmerrt of lean.:ls .nd qapment ln lumlng
   - SJQre tnmuctlam lQractJCc, ciJ:o.ln lnstructor lltmeli, find .solruans far challtnges. de:ermlne nens1rQS for adler Nfl'WI8
   - Ident."v trte students w'lo;are, .nd ate not. responding pauMiy to1M cb=txlm ayTI.amlcs st;mqy being uuh:ed.
   - Adjust thestr:rtqr aproi!lrt,ote to eng: ge those who are not re , oodmg.

R,..flt'Ct

8. wmlne. lliU.Ivze andCYalu;ata results:
   - CoIM!Iaao:s:udenort. sn;ares!Uclent fl:ed d:.dresults, (ea...: alvre ..,deabpor\8oF cbsuoom, AMewsUJacnt rcfl«tMt lCKlfNls,comp; te pre, mod and post tcxn..aself aw:ssment ofami,.!.allis enmine ercncil IndiWlOOm dynarnocs tr.lcbns tool) and student
   - Oecde, b®ea aniNidena>, the extent to whch the–oFstudent need 1\asbeen ..d.d.reslltd
   - ltc:flect on teacher lc;unng regardirld;:wmom dynamics (le., das.sroom mOJNI_gement, cburoom mlJll, and relat:.onship aldil18 indas.sroom dynamics
   - Octade whenelse toapplv this stntegy to Improve s!Uclcnt ..___,t.

Adopred /fOnt Swdb'lt SU<ces.s/I.Nifllflg 18 Imp/emMtO[JOf\ TtOirlittg and E...aJuaUon Brar"ch

A group of teachers could follow the model provided by MathGAINS and still never change their practices toward more reform-based strategies. For example, a group of teachers could decide to increase student algebra knowledge by creating new worksheets for the students. This could meet all of the “planning” criteria in the model if they focused on direct instruction and testing as the classroom behaviour they wished to implement. These teachers would be able to use these resources, observe students or teachers, and reflect on the use of the strategies. Where this model is lacking is acknowledging the beliefs and knowledge of the teachers and how this impacts the strategies chosen by the teachers. Also, the addition of the “leader” in my model pushes the focus toward trying reform-based strategies. Although the Classroom Dynamics model clearly specifies actions for teachers to enact in a professional learning group, it ignores the difficulties in mathematics education in encouraging the use of more reform-based strategies.

Figure 14. Model for professional learning groups Blankstein (2012)

The second model (see Figure 14) is found in the work of Blankstein (2012) and is supposed to be focused on “the thoughtful, smart use of resources for the greatest impact and the best possible outcomes” (p. 22). The model indicates six principles to guide professional learning group organisation. As I indicated earlier, Principle 1, the “common mission, vision, values, and goals” is potentially problematic in mathematics education where the teachers could hold differing beliefs about mathematics teaching and learning. Again this model does not account for how beliefs and knowledge impact the discussions of a professional learning group.

I believe my model deals with the difficulties that specifically face mathematics education professional learning groups. In order to examine my proposed model further, I apply it to the data reported on in Kajander and Mason (2007) since they specifically address the conversations of the professional learning group.

Putting Proposed Model into Action

Kajander and Mason (2007) presented descriptions of two different professional learning groups in mathematics: Pine and Maple. In examining the discussion of both professional learning groups and applying my proposed model, it is apparent to me where the Maple group succeeded and the Pine group did not. The vast majority of the Pine teachers strongly believed that what was needed for their mathematics teachers was more worksheets for their students to complete. Although one member, Kevin, attempted to serve the role of “leader” from my model, he was unable to make a difference because of how strongly held the other teachers’ beliefs were. Perhaps with more support (as in the professional learning group I observed) or a strong catalyst event, such as the meeting at the university, this may have been different for the Pine group. The researchers noted that there was a large number of teachers in the Pine group, so perhaps the number of participants impacted the group’s ability to really focus on their teaching practices. Another potential issue with the Pine group was the center of my proposed model of action orientation, continuous improvement, and results orientation. The researchers noted “quick changes of topic” and “no attempt was made to investigate or resolve the problem” (Kajander & Mason, 2007, p. 425). This lack of focus on making changes and examining the impact of those changes, left the group unable to properly reflect on new strategies, so instead they made new worksheets to include in their classrooms.

On the other hand, the Maple group contained multiple members who were open to try new problem strategies in their classrooms. This group was also much smaller with only four members. The teachers were able to share their strategies and discuss the results of the changes on their students and practices. The teachers used written materials and their own experiences with new strategies to “lead” the discussions. Unlike the Pine group, this professional learning group did enact the center of my proposed model by focusing on sharing new strategies, trying them in their classrooms, and then examining the results of the changes on their students’ achievement. The discussions also included examining models and manipulatives to unpack the mathematics inherent in the models. As a result of the focus on mathematics knowledge and an action research stance, the teachers were poised to make positive changes in their classrooms, moving their teaching toward more reform-oriented strategies.

In the next chapter, I discuss implications of my research for teacher practice. I continue with a description of future research needed. The chapter ends with concluding thoughts on the entire study.

CHAPTER EIGHT: CONCLUSIONS AND IMPLICATIONS

This chapter suggestions implications for effective teacher practice inherent in my research study. Next I discuss future research needed in professional learning groups. The chapter concludes with a final discussion of my study including its place in the current research literature.

Implications

An initial implication for effective teacher practice is based on a concern Ryan brought to my attention. He shared his concern that the group members did not really understand the professional learning group model, so he was worried that the group was not as effective as it could be. He expressed that the elementary teachers had more training on the ideas but it was new to the secondary teachers. In terms of future implications of using a professional learning group model, there is a need for professional development for the teachers on using the model effectively. If teachers are meant to enact the professional learning group model appropriately on their own, then support in developing a picture of the model could eliminate a stumbling block to success. Ryan's main concern was the leadership characteristic not being shared, but in this case, the group definitely needed someone to keep them on track as well as deal with the paperwork. As shown in my model for professional learning groups, a mathematics group also needs someone who is comfortable with reform-oriented strategies to take a leadership role and guide discussions.

With strong mathematics knowledge being held by some group members, the professional learning group I observed was able to make changes or suggestions for each other's practices. The group members chose to focus on student learning and reflect on ideas for how to make their students more successful in future years. They did not simply focus on creating new assessments or have discussions about creating particular lessons that would not yield long term changes, as has been reported in other studies (e.g., Kajander & Mason, 2007). This group gives an example of how productive a professional learning group can be when the knowledge and beliefs for teaching mathematics effectively are found within the group. It does point to the need for providing a structure to give support to groups of teachers when those ideas and beliefs are not already present. Since each member took their own ideas from the group meetings and applied them to their practices, the reform-based strategies for teaching mathematics would need to be explored at some point with the group members.

Another aspect of knowledge needed by the teachers was knowledge about students and curriculum. One case in point about knowing students would be Wesley's discussion about working with binomials, and how inappropriate it would have been to introduce to the lower grade levels. The elementary panel needed those convictions and comfort with their own grade levels in order to note that this skill was too advanced for the current level of students and would simply become a procedure. Encouraging only a procedural understanding was starkly against the beliefs of the elementary teachers, and they needed to have the strength to stand up for their convictions. Having a deep knowledge of the Ontario curriculum led to conversations about where gaps in the curriculum occur. The secondary teachers expressed surprise over how late dividing and multiplying fractions fell in the elementary curriculum, and learned that the students could not possibly have mastered a skill that was introduced in grade 8 for the first time. This led to more time spent on fractions in grade 9 even though it is not a specific expectation in the curriculum. The elementary teachers learned how much emphasis is placed on algebra for success in secondary school and realized that their curriculum expectations did not adequately prepare their students for where they needed to be. This idea points to the fact that teachers need to know not only their own curriculum, but where students are heading or have come from in their mathematics journeys. Shulman (1986) describes this as vertical curricular knowledge and supports its importance in effective teaching. The identified gaps also suggest a need to revisit the curriculum or provide materials for teachers which share gaps between elementary and secondary curriculum expectations.

The group discussions were focused on the results of implementing changes with the elementary panel and how this resulted in performance in secondary school as well as subsequent changes in the secondary school to better support students. The group wanted to maintain a focus on tangible results. By focusing on the results, the teachers examined test scores that they had access to within their schools. This focus on results supports the action research stance being integrated into the professional learning group. Focusing on making changes and seeing the results of those changes allowed the group to make long lasting effective changes to their classroom practices.

Funding for professional development was another area of discussion. In a discussion about the grade 6 mathematics scores, Gabriel noted that the elementary teachers were struggling with their mathematics scores falling in comparison to the literacy scores. "We spend a lot of time money and energy on literacy on the elementary side, but we haven't really spent a lot of time on math so results really went down" (Meeting # 7 transcript), pointing to the need to spend money on mathematics professional development to work with these students. Claire pointed out that the professional learning group meetings were the only professional development she was given in mathematics. Clarke (1997) as well as Handal and Herrington (2003) point to the difficulty of implementing mathematics reform strategies when there is not enough support for the teachers. The professional learning groups were a great benefit to the teachers observed who were dedicated to improving mathematics teaching for their students, and yet the teachers were in constant fear that the board would remove funding for the following year and cancel the meetings. Their concern arose from the fear that the board would transfer funding to a different initiative. Cwikla (2004) points to the difficulties of too many changes occurring all at once, and the teachers in the group struggled with many initiatives being added to their plates at one time and not having the time with any single one to see it through. Overall, the professional learning group was extremely effective professional development for the teachers, who need the support in order to allow them to continue to develop their practices.

Future Research

In terms of future research, I believe it would be beneficial to compare my findings to research about other professional learning groups in order to create a more generalizable description of optimal characteristics. Using a similar framework, as well as my proposed model, to examine multiple groups would be the next step in order to complete the description of effective professional learning groups in mathematics. I believe it would also be beneficial to examine a professional learning group in which the information about mathematics is not held by the members of the group. Such a broadened perspective would be helpful in order to get a better picture of how to support these groups in making effective changes and teacher growth.

Conclusion

In summary, professional development and support are essential for both keeping and developing effective teachers. Gojmerac and Cherubini (2012) showed that high-quality professional development is needed in order for this to happen. Research into professional development shows that assigning teachers to one day workshops and then enforcing that they make "changes" is not working to create lasting growth within the profession (Arbaugh, 2003;

Ball & Cohen, 1999; Brahier & Schäffner, 2004; Gojmerac & Cherubini, 2012; Hawley & Valli, 1999; Hofman & Dijkstra, 2010; Lieberman, 2000; Schmoker, 2006; West & Curcio, 2004; Wetzel, 2001). Professional learning groups on the other hand show promise in effectively supporting teachers to make changes and growth within their own teaching practices. My research sought to define the conditions that were essential in creating an effective professional learning group in mathematics.

In looking at the characteristics of an effective professional learning group, it was clear that all the characteristics were upheld by the group members in some fashion. In their desire to support students in their mathematics classrooms, the teachers created a shared vision for the professional learning group meetings. The differing beliefs the teachers held about how best to support their students led to an inability to create a shared belief or vision for their classrooms. Although Ryan tended to keep the meetings on track and take care of paperwork, the group members were given autonomy and encouraged to voice their opinions about the directions of the meetings. There was also the need for members of the group to push the conversations to more reform-based strategies for teaching. The group meetings focused on teacher learning about mathematics teaching and were supportive for all the group members. The members of the group also worked to give each other feedback on their teaching practices and materials. The group maintained an action research focus in that they examined test results and student work to assess the impact of their discussions on the students. I felt that the one area where the group could further develop was to implement more new strategies in the classroom and then make the concerted effort to continue to use and reflect upon them.

The participants relied on a variety of methods in order to encourage changes and support each other in growing as mathematics teachers. During the meetings, the teachers focused on student work by examining tests or other materials from their classrooms and discussing them. They also observed the teaching of several members of the group and reflected on these classroom experiences. For two of their meetings, the teachers chose to partake in learning experiences at the local university in order to gain new knowledge and skills for teaching mathematics. They also focused on discussing mathematics questions within the group to work on their specialised knowledge of mathematics.

Through examining the discussions of the teachers in the professional learning group, a dichotomy arose between the elementary and secondary teachers. The elementary teachers focused on teaching for understanding and having students explore mathematics concepts. In line with this focus, the elementary teachers used rubrics to assess their work, allowing for more descriptive feedback. The secondary teachers often referred to showing students how to get answers through using procedures and grading their exams on a point scale. Ryan fell somewhere in the middle with both valuing test scores and wanting students to understand the concepts being taught. The elementary teachers expressed support of more reform-based strategies in the classroom and being unwilling to teach concepts as only a procedural skill.

Knowledge of mathematics was held at the core of the discussions of the group, and I felt this was a significant contributor to the group's effectiveness. In the discussions of student work and mathematical concepts, the teachers were able to keep refining their skills and supporting their students through making use of this increased knowledge. Although the teachers themselves separated knowledge of teaching mathematics from mathematics knowledge itself, it is clear that a focus on both topics was important to increase the effectiveness of the teachers because of their intertwined nature. This group was able to accomplish more than simply creating new worksheets or tests for their students; instead they tackled big issues in their classrooms largely through examining the underlying mathematical knowledge needed for teaching. The teachers also made sure to share their beliefs about teaching mathematics, and those who held more reform-oriented beliefs about teaching mathematics attempted to sway the others in this direction.

The stories of the five teachers helped to add richness to the discussion by highlighting how individuals took the same activities and discussions and made something different in their classrooms. Emma strongly felt her students should understand mathematics and that they would achieve this by being given the freedom to explore the concepts and discover their own solution methods. She viewed the discussions in the professional learning group meetings as helping her see more alternative solutions that her students could discover. Blaine shared very little in the meetings but claimed to have gained a lot from the discussions to take back to his classroom. His feelings of lack of knowledge may have impeded his discourse but did allow him to see areas where he could grow. April's lack of confidence was evident in some of her discussions in the larger group which she attributed to her intimidation caused by the strong mathematical understandings of some of the other teachers. In her interview however, April shared her increased confidence in her teaching abilities and what she was doing for her students. She felt she was able to take the knowledge she gained from the meetings and apply it to her classroom to further support her students in mathematics. Wesley staunchly believed that students should be shown procedures at any age and that this would support them in mathematics. He was inspired to try new strategies from a meeting at the university, but when his idea was stopped by his students' lack of mathematical understanding; he gave up on the lesson completely for the year. Owen still seemed traditional in his teaching beliefs but admitted to using more explorations in his room, yet his classroom observation still followed a very traditional pedagogy.

As professional development, professional learning groups have the potential to move teachers forward in developing their mathematics teaching pedagogy. One caveat is that professional learning groups are a process, and any changes must be considered based on the initial beliefs and knowledge of the teachers. Since not all teachers would begin in the same position in terms of their teaching pedagogy and beliefs, they would not all end with the same views and knowledge of teaching. Any changes are relative to initial capacity.

Although in the beginning, I set out to define the conditions that would make a successful professional learning group, I realized that this would be a challenging task because of the very personal nature of a professional learning group. There would need to be emphasis on the definition of success created by the members themselves. In examining the professional learning group, it was clear that the conversations about mathematical knowledge and teaching, while confronting the beliefs about teaching mathematics, added together to make the group successful. In speaking with the members of the group, they agreed that the group was successful and cited specific reasons that aligned with current research in professional learning groups. For instance, the group made sure to specifically examine and discuss student work, which research supports as being important in professional learning groups (Hord, 2009; Sowder, 2007), and the teachers themselves cited this as being something essential in their discussions. The teachers also stated that sharing practices and supporting each other were reasons that their group was successful.

Since professional development is so valuable to the teaching community, yet very expensive, research is important in determining where funds could be spent to be most beneficial. This professional learning group painted an excellent picture of what a professional learning group in mathematics could look like in order to support teachers in making changes in their professions. By providing teachers with the support to create a community in which individuals who seemed dedicated to their profession work together, the teachers were able to explore changes that would benefit their students in mathematics learning. It was unfortunate that the future of this professional learning group was uncertain as funding for the group was continuously in question. Research into professional development is in favour of what is being termed “sustained and significant” learning opportunities (Brahier & Schäffner, 2004), and this professional learning group gave an excellent example of how teachers could create this environment when given the time, funding, and opportunity.

References

Allen, D. S. (2006). The push to excellence: Teachers focus on professional learning to lift student achievement. *Journal of Staff Development, 27*(1), 56-60.

Anderson, J. (2005). Implementing problem solving in mathematics classrooms: What support do teachers want? In P. Clarkson, A. Dowton, D. Gronn, M. Home, A. McDonough, R. Pierce, & A. Roche (Eds.), *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australia, Melbourne* (pp. 89-96). Sydney: MERGA. Retrieved from http://www.merga.net.au/documents/RP42005.pdf

Anderson, J. O., Rogers, W. T., Klinger, D. A., Ungerleider, C., Glickman, V., & Anderson, B. (2006). Student and school correlates of mathematics achievement: Models of school performance based on pan-Canadian student assessment. *Canadian Journal of Education, 29*(3), 706-730.

Andrews, D., & Lewis, M. (2002). The experience of a professional community: Teachers developing a new image of themselves and their workplace. *Educational Research, 44*(3), 237-254.

Arbaugh, F. (2003). Study groups as a form of professional development for secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*, 139-163.

Askey, R. (1999). Knowing and teaching elementary mathematics. *American Educator*. Retrieved January 25, 2007, from http://www.aft.org/pubs-reports/american_educator/fall99/amed1.pdf

Assessment Standards Working Groups of the National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Ball, D. L. & Bass, H. (2000). Making believe: The collective construction of public mathematical knowledge in the elementary classroom. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 193-224). Chicago, IL: National Society for the Study of Education.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 3-32). San Francisco, CA: Jossey-Bass Publishers.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(1), 14, 16-17, 20-22, 43-46.

Ball, D. L., & Rowan, B. (2004). Introduction: Measuring instruction. *The Elementary School Journal, 105*(1), 3-10.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Bartlett, S., & Burton, D. (2007). *Introduction to education studies* (2nd ed.). London: SAGE Publications Ltd.

Battey, D., & Franke, M. L. (2008, Summer). Transforming identities: Understanding teachers across professional development and classroom practice. *Teacher Education Quarterly*, 127-149.

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A. Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal, 47*(1), 133-180.

Bay-Williams, J. M., & Meyer, M. R. (2005). Why not just tell students how to solve the problem? *Mathematics Teaching in the Middle School, 10,* 340-341.

Bednarz, N., Maheux, J., & Barry, S. (2007). Context-based professional development in mathematics education: Analysis of a collaborative research process aimed at supporting it. In T. Lamberg, & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (pp. 788-795). Stateline, NV: University of Nevada, Reno.

Bell, C. A., Wilson, S., Higgins, T., & McCoach, D. B. (2010). Measuring the effects of professional development on teacher knowledge: The case of developing mathematical ideas. *Journal for Research in Mathematics Education, 41*(5), 479-512.

Berger, P. L., & Luckmann, T. (1966). *The social construction of reality: A treatise in the sociology of knowledge*. New York: Anchor Books.

Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*, 127-147.

Birky, V. D., Shelton, M., & Headley, S. (2006). An administrator's challenge: Encouraging teachers to be leaders. *NASSP Bulletin, 90*(2), 87-101.

Blankstein, A. (2012). We're a learning community! Now what? In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations in mathematics teaching and learning: Seeking success for all* (pp. 19-29). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Blegen, M. B., & Kennedy, C. (2000). Principals and teachers, leading together. *NASSP Bulletin, 84*(616), 1-6.

Blumer, H. (1969). *Symbolic interactionism: Perspective and method.* Englewood Cliffs, NJ: Prentice-Hall, Inc.

Blumer, H. (2004). *George Herbert Mead and human conduct*. (T. J. Morrione, Ed.). Walnut Creek, CA: AltaMira Press.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior, 18*(4), 379-397.

Boaler, J. & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*, Portsmouth, NH: Heinemann.

Boyd, P. C. (1994). Professional school reform and public school renewal: Portrait of a partnership. *Journal of Teacher Education, 45*(2), 132-139.

Boylan, M. (2010). 'It's getting me thinking and I'm an old cynic': Exploring the relational dynamics of mathematics teacher change. *Journal of Mathematics Teacher Education, 13*(5), 383-395.

Brahier, D. J., & Schäffner, M. (2004). The effects of a study-group process on the implementation of reform in mathematics education. *School Science and Mathematics, 104*(4), 170-178.

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38.

Bredo, E. (2000). Reconsidering social constructivism: The relevance of George Herbert Mead's interactionism. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 19-40). Chicago, IL: National Society for the Study of Education.

Brown, A. L., & Palincsar, A. S. (1989). Guided, cooperative learning and individual knowledge acquisition. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 393-451). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Brown, L., & Coles, A. (2010). Mathematics teacher and mathematics teacher educator change—insight through theoretical perspectives. *Journal of Mathematics Teacher Education, 13*(5), 375-382.

Brown, R. E., Vissa, J. M., & Mossgrove, J. L. (2012). The importance of collaboration to new teacher development: Two central features of an induction fellowship. In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations in mathematics teaching and learning: Seeking success for all* (pp. 103-115). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Brown, S. A., Pitvorec, K., Ditto, C., & Kelso, C. R. (2009). Reconceiving fidelity of implementation: An investigation of elementary whole-number lessons. *Journal for Research in Mathematics Education, 40*(4), 363-395.

Bruce, C. D., & Ross, J. A. (2008). A model for increasing reform implementation and teacher efficacy: Teacher peer coaching in grades 3 and 6 mathematics. *Canadian Journal of Education, 31*(2), 346-370.

Bruner, J. (1985). Vygotsky: A historical and conceptual perspective. In J. W. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 21-34). Cambridge: Cambridge University Press.

Buckner, K. G., & McDowelle, J. O. (2000). Developing teacher leaders: Providing encouragement, opportunities, and support. *NASSP Bulletin, 84*(616), 35-41.

Burbules, N. C. (2000). Moving beyond the impasse. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 308-330). Chicago, IL: National Society for the Study of Education.

Buschman, L. (2004). Teaching problem solving in mathematics. *Teaching Children Mathematics, 10*(6), 302-309.

Caine, G., & Caine, R. N. (2000). Principals and teachers, leading together. *NASSP Bulletin, 84*(616), 7-14.

Carnegie Corporation of New York, Institute for Advanced Study Commission on Mathematics and Science Education. (2009). The opportunity equation: Transforming mathematics and science education for citizenship and the global economy. *Report of the Carnegie Corporation of New York—Institute for Advanced Study Commission on Mathematics* *and Science Education* (Executive Summary). Retrieved from http://opportunityequation.org/report

Carr, D. A. (1997). Collegial leaders: Teachers who want more than just a job. *The Clearing House, 70*, 240-242.

Chamberlin, M. T., Farmer, J. D., & Novak, J. D. (2008). Teachers' perceptions of assessments of their mathematical knowledge in a professional development course. *Journal of Mathematics Teacher Education, 11*, 435-457.

Chapman, O., & Heater, B. (2010). Understanding change through a high school mathematics teacher's journey to inquiry-based teaching. *Journal of Mathematics Teacher Education, 13*(6), 445-458.

Chase, S. E. (2005). Narrative inquiry: Multiple lenses, approaches, voices. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3rd ed., pp. 651-679). Thousand Oaks, CA: SAGE Publications.

Clandinin, D. J., & Connelly, F. M. (2000). *Narrative inquiry: Experience and story in qualitative research.* San Francisco, CA: Jossey-Bass.

Clarke, D. M. (1997). The changing role of the mathematics teacher. *Journal for Research in Mathematics Education, 28*(3), 278-308.

Clements, D. H. (2007). Curriculum research: Toward a framework for "research-based curricula". *Journal for Research in Mathematics Education, 38*(1), 35-70.

Cobb, P. (2005). Where is the mind? A coordination of sociocultural and cognitive constructivist perspectives. In C. T. Fosnot (Ed.), *Constructivism: Theory, perspectives, and practice* (2nd ed., pp. 39-57). New York: Teachers College Press.

Cobb, P., Wood, T., & Yackel, E. (1991). A constructivist approach to second grade mathematics. In E. von Glaserfeld (Ed.), *Radical constructivism in mathematics education* (pp. 157-176). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3/4), 175-190.

Cochran-Smith, M., & Lytle, S. L. (1998). Relationships of knowledge and practice: Teacher learning in communities. *Review of Research in Education, 24*, 249-305.

The Cognition and Technology Group at Vanderbilt. (1995). From visual word problems to learning communities: Changing conceptions of cognitive research. In K. McGilly (Ed.), *Classroom lessons: Integrating cognitive theory and classroom practice* (pp. 157-200). Cambridge, Mass?: The MIT Press.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*(3), 327-345.

Cohen, D. K., & Hill, H. C. (2000). Instructional policy and classroom performance: The mathematics reform in California. *Teachers College Record, 102*, 294-343.

Cole, M. (1985). The zone of proximal development: Where culture and cognition create each other. In J. W. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 146-161). Cambridge: Cambridge University Press.

Coleman, M. R., Gallagher, J. J., & Job, J. (2012). Developing and sustaining professionalism within gifted education. *Gifted Child Today, 35*(1), 27-36.

Copsey-Haydey, D., Zakaluk, B. L., & Straw, S. (2010). The changing face of content area teaching. *Journal of Applied Research on Learning, 3*, Article 3, 1-29.

Crespo, S., Oslund, J., & Parks, A. (2007). Studying elementary preservice teachers' learning of mathematics teaching: Preliminary insights. In T. Lamberg & L. R. Wiest (Eds.), *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 975-982). Stateline, NV: University of Nevado, Reno.

Crespo, S., & Sinclair, N. (2008). What makes a problem mathematically interesting? Inviting prospective teachers to pose better problems. *Journal of Mathematics Teacher Education, 11*, 395-415.

Creswell, J. W. (2003). *Research design: Qualitative, quantitative, and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: SAGE Publications.

Creswell, J. W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (3rd ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education, 12*, 325-346.

Cwikla, J. (2004). Show me the evidence: Mathematics professional development for elementary teachers. *Teaching Children Mathematics, 10*(6), 321-326.

Davis, B. A. (1994). Mathematics teaching: Moving from telling to listening. *Journal of Curriculum and Supervision, 9*(3), 267-283.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*, 293-319.

deBlois, L., & Sterenberg, G. (2010). Contemporary and emergent research methodologies in mathematics education. In P. Liljedahl, S. Oesterle, & V. Abu-Bakare (Eds.), *Proceedings of the 2009 annual meeting of the Canadian Mathematics Education Study Group* (pp. 39- 48). Burnaby, BC: CMESG.

Delaney, P. F., Ericsson, K. A., & Knowles, M. E. (2004). Immediate and sustained effects of planning in a problem-solving task. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 30*(6), 1219-1234.

Denzin, N. K., & Lincoln, Y. S. (2005). Introduction: The discipline and practice of qualitative research. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3rd ed., pp. 1-32). Thousand Oaks, CA: SAGE Publications.

Desbiens, B. L. (2011). *In the know with EQAO: Student testing: Why Ontario's on the right track.* Retrieved from http://www.eqao.com/Research/pdf/E/EQAO_StudentTesting_OntarioOnTrack.pdf

Desimone, L. M., Smith, T. M., & Ueno, K. (2006). Are teachers who need sustained, content-focused professional development getting it? An administrator's dilemma. *Educational Administration Quarterly, 42*(2), 179-215.

DiPardo, A., & Potter, C. (2003). Beyond cognition: A Vygotskian perspective on emotionality and teachers' professional lives. In A. Kozulin, B. Gindis, V. S. Ageyev, & S. M. Miller (Eds.), *Vygotsky's educational theory in cultural context* (pp. 317-345). Cambridge: Cambridge University Press.

DuFour, R. (2002). Time, Perspective, and Priorities. In R. Eaker, R. DuFour, & R. Burnette (Eds.), *Getting started: Reculturing schools to become professional learning communities* (pp. 31-56). Bloomington, IN: National Educational Service.

DuFour, R., & Eaker, R. (1998). *Professional learning communities at work: Best practices for enhancing student achievement.* Bloomington, IN: National Educational Service.

Eaker, R. (2002). Cultural shifts: Transforming schools into professional learning communities. In R. Eaker, R. DuFour, & R. Burnette (Eds.), *Getting started: Reculturing schools to become professional learning communities* (pp. 9-29). Bloomington, IN: National Educational Service.

Eaker, R., DuFour, R., & Burnette, R. (2002a). A call for action. In R. Eaker, R. DuFour, & R. Burnette (Eds.), *Getting started: Reculturing schools to become professional learning communities* (pp. 31-56). Bloomington, IN: National Educational Service.

Eaker, R., DuFour, R., & Burnette, R. (2002b). Getting started: A conceptual framework for creating a professional learning community. In R. Eaker, R. DuFour, & R. Burnette (Eds.), *Getting started: Reculturing schools to become professional learning communities* (pp. 1-8). Bloomington, IN: National Educational Service.

Eaker, R., & Keating, J. (2012). Improving mathematics achievement: The power of professional learning communities. In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations in mathematics teaching and learning: Seeking success for all* (pp. 3-18). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Education Quality and Accountability Office. (2011). *EQAO tests in secondary school: A guide for parents and students.* Toronto: Queen's Printer for Ontario.

Ellis, M. W., Malloy, C. E., Meece, J. L., & Sylvester, P. R. (2007). Convergence of observer ratings and student perceptions of reform practices in sixth-grade mathematics classrooms. *Learning Environments Research, 10*(1), 1-15.

Felton, M. D. (2012). “After all, math was only numbers, right?” Transitions in teachers’ beliefs about equity. In L. R. Van Zoest, J. J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 503-6). Kalamazoo, MI: Western Michigan University.

Forman, E. A., & Cazden, C. B. (1985). Exploring Vygotskian perspectives in education: The cognitive value of peer interaction. In J. W. Wertsch (Ed.), *Culture, communication, and cognition: Vygotskian perspectives* (pp. 323-347). Cambridge: Cambridge University Press.

Fuller, S. (1994). The reflexive politics of constructivism. *History of the Human Sciences, 7*, 87-93.

Fuller, S. (2007). *The knowledge book: Key concepts in philosophy, science and culture.* Montreal: McGill-Queen’s University Press.

Gadanidis, G., & Namukasa, I. K. (2007). Mathematics-for-teachers (and students). *Journal of Teaching and Learning, 5*(1), 13-22.

Geelan, D. (2006). *Undead theories: Constructivism, eclecticism, and research in education.* Rotterdam, The Netherlands: Sense Publishers.

Gergen, K. J. (1995). Metaphor and monophony in the 20th-century psychology of emotions. *History of the Human Sciences, 8*(2), 1-23.

Gojmerac, A., & Cherubini, L. (2012). Connecting for greater purpose: An examination of a district professional learning community. *Journal of Teaching and Learning, 8*(1), 43-58.

Grant, T. J., Hiebert, J., & Wearne, D. (1994). Teachers’ beliefs and their responses to reform-minded instruction in elementary mathematics. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA. Retrieved from ERIC database online. (No. ED376170)

Gresalfi, M. S., & Cobb, P. (2011). Negotiating identities for mathematics teaching in the context of professional development. *Journal for Research in Mathematics Education, 42*(3), 270-304.

Grodsky, E., & Gamoran, A. (2003). The relationship between professional development and professional community in American schools. *School Effectiveness and School Improvement, 14*(1), 1-29.

Grootenboer, P. (2008). Mathematical belief change in prospective primary teachers. *Journal of Mathematics Teacher Education, 11*, 479-497.

Guiney, E. (2001). Coaching isn't just for athletes: The role of teacher leaders. *Phi Delta Kappan, 82*(10), 740-743.

Hall, G. E., & Hord, S. M. (2006). *Implementing change: Patterns, principles, and potholes.* Boston, MA: Pearson Education, Inc.

Handal, B., & Herrington, A. (2003). Mathematics teachers' beliefs and curriculum reform. *Mathematics Education Research Journal, 15*(1), 59-69.

Harkness, S. S. (2009). Social constructivism and the *Believing Game*: A mathematics teacher's practice and its implications. *Educational Studies in Mathematics, 70*, 243-258.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 127-150). San Francisco, CA: Jossey-Bass Publishers.

Heirdsfield, A., Lamb, J., & Spry, G. (2010). Leading learning with a PLC: Implementing new mathematics content. *The Montana Mathematics Enthusiast, 7*(1), 93-111.

Hiebert, J., Morris, A. K., & Glass, B. (2003). Learning to learn to teach: An "experiment" model for teaching and teacher preparation in mathematics. *Journal of Mathematics Teacher Education, 6*, 201-222.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393-425.

Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 41*(5), 513-545.

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H. C., Rowan, B., & Ball, D. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, P. T., & Harvey, J. (2004). A fresh assessment: Why reform initiatives fail. In P. T. Hill & J. Harvey (Eds.), *Making school reform work: New partnerships for real change* (pp. 1-7). Washington, D.C.: The Brookings Institution.

Hinchey, P. H., Mamana, L., & Steele, B. (1997). Professors and teachers as collaborators in classroom change. *The Clearing House, 70*, 243-245.

Hofman, R. H., & Dijkstra, B. J. (2010). Effective teacher professionalization in networks? *Teaching and Teacher Education, 26*, 1031-1040.

Holm, J. (2009). Perceptions of 'Problem Solving' in Elementary Curriculum. *Delta-K, 47*(1), 9-15.

Holm, J., & Kajander, A. (2012). Interconnections of knowledge and beliefs in teaching mathematics. *Canadian Journal of Science, Mathematics, and Technology Education*, *12*(1), 7-21.

Hord, S. M. (2009). Professional learning communities: Educators work together toward a shared purpose—improved student learning. *Journal of Staff Development, 30*(1), 40-43.

Hord, S. M., & Sommers, W. A. (2008). *Leading professional learning communities: Voices from research and practice.* Thousand Oaks, CA: Corwin Press.

Howe, K. R., & Berv, J. (2000). Constructing constructivism, epistemological and pedagogical. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 19-40). Chicago, IL: National Society for the Study of Education.

Huang, R., Li, Y., & He, X. (2010). What constitutes effective mathematics instruction: A comparison of Chinese expert and novice teachers' views. *Canadian Journal of Science, Mathematics and Technology Education, 10*(4), 293-306.

Huffman, J. (2003). The role of shared values and vision in creating professional learning communities. *NASSP Bulletin, 87*(637), 21-34.

Huffman, J. B. (2000). One school's experience as a professional learning community. *Planning & Changing, 31*(1/2), 84-94.

Inoue, N. (2011). Zen and the art of *neriage*: Facilitating consensus building in mathematics inquiry lessons through lesson study. *Journal of Mathematics Teacher Education, 14*(1), 5-23.

Johnson, Jr., B. L. (2009). Understanding schools as organizations: Implications for realizing professional learning communities. In C. A. Mullen (Ed.), *The handbook of leadership and professional learning communities* (pp. 17-28). New York, NY: Palgrave Macmillan.

Kajander, A. (2007). Unpacking mathematics for teaching: A study of preservice elementary teachers' evolving mathematical understandings and beliefs. *Journal of Teaching and Learning, 5*(1), 33-54.

Kajander. A. (2010). Elementary mathematics teacher preparation in an era of reform: The development and assessment of mathematics for teaching. *Canadian Journal of Education, 33*(1), 228-255.

Kajander, E. A. (1989). *Possible components of mathematical ability: A study of mathematical creativity in a deductive geometry curriculum unit.* (Doctoral thesis, University of Toronto, Toronto, Ontario).

Kajander, A., & Mason, R. (2007). Examining teacher growth in professional learning groups for in-service teachers of mathematics. *Canadian Journal of Science, Mathematics, and Technology Education, 7*(4), 417-438.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal, 102*(1), 59-80.

Kamii, C. (2004). *Children continue to reinvent arithmetic, 2nd grade.* New York: Teachers College Press.

Killion, J. (2008). Are you coaching heavy or light? *T3: Teachers Teaching Teachers, 3*(8), 1-4.

King, N., & Horrocks, C. (2010). *Interviews in qualitative research*. Thousand Oaks, CA: SAGE Publications, Inc.

King, A. J. C., Warren, W. K., Boyer, J. C., & Chin, P. (2005). Double cohort study: Phase 4 report submitted to the Ontario Ministry of Education. Retrieved from http://www.tcu.gov.on.ca/fre/document/reports/phase4/report4.pdf

Knight, J. (2009). Coaching: The key to translating research into practice lies in continuous, job-embedded learning with ongoing support. *Journal of Staff Development, 30*(1), 18-22.

Kozlow, M. (2012). Factors that are related to student achievement on the EQAO grade 9 assessment of mathematics. *Education Quality and Accountability Office Research Bulletin #11.* Toronto: Queen's Printer for Ontario. Retrieved from http://www.eqao.com/Research/pdf/E/ResearchBulletin11_en.pdf

Krupa, E. E. (2012). Effect of professional development on teachers' implementation of a reform oriented curriculum. In L. R. Van Zoest, J. J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 92-5). Kalamazoo, MI: Western Michigan University.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. (Translated from Russian by J. Teller.) J. Kilpatrick & I. Wirszup (Eds.) Chicago: The University of Chicago Press.

Kvale, S. (1996). *InterViews: An introduction to qualitative research interviewing*. Thousand Oaks, CA: SAGE Publications, Inc.

Lachance, A., & Confrey, J. (2003). Interconnecting content and community: A qualitative study of secondary mathematics teachers. *Journal of Mathematics Teacher Education, 6*, 107-137.

Laufgraben, J. L., Shapiro, N. S., & Associates. (2004). *Sustaining and improving learning communities*. San Francisco, CA: Jossey-Bass.

Lave, J. (2008). Epilogue: *Situated learning* and changing practice. In A. Amin & J. Roberts (Eds.), *Community, economic creativity, and organization* (pp. 283-296). Oxford: Oxford University Press.

LDCC Project. (2005). *Locally developed compulsory credit courses, grades 9 and 10: Mathematics.* Retrieved from http://www.edugains.ca/resources/CurriculumDocuments/LDCC_Math_9_10.pdf

Lee, S. J., & Shin, J. (2012). Teachers' key developmental understandings for measurement fraction divisions. In L. R. Van Zoest, J. J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 527-30). Kalamazoo, MI: Western Michigan University.

Lichtman, M. (2010). *Qualitative research in education: A user's guide* (2nd ed.). Thousand Oaks, CA: SAGE Publications.

Lieberman, A. (2000). Networks as learning communities: Shaping the future of teacher development. *Journal of Teacher Education, 51*(3), 221-227.

Lieberman, A., & Wood, D. R. (2002). From network learning to classroom teaching. *Journal of Educational Change, 3*, 315-337.

Linder, R. A., Post, G., & Calabrese, K. (2012). Professional learning communities: Practices for successful implementation. *The Delta Kappa Gamma Bulletin, Spring Professional Development*, 13-22.

Lobato, J., Clarke, D., & Ellis, A. B. (2005). Initiating and eliciting in teaching: A reformulation of telling. *Journal for Research in Mathematics Education, 36*(2), 101–136.

Loughridge, M., & Tarantino, L. R. (2005). *Leading effective secondary school reform: Your guide to strategies that work.* Thousand Oaks, CA: Corwin Press.

Lynch, M. (1998). Towards a constructivist genealogy of social constructivism. In I. Velody, & R. Williams (Eds.), *The politics of constructionism* (pp. 13-32). London: SAGE Publications.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Lawrence Erlbaum Associates.

Ma, X., & McIntyre, L. J. (2005). Exploring differential effects of mathematics courses on mathematics achievement. *Canadian Journal of Education, 28*(4), 827-852.

Males, L. M., Otten, S., & Herbel-Eisenmann, B. A. (2010). Challenges of critical colleagueship: Examining and reflecting on mathematics teacher study group interactions. *Journal of Mathematics Teacher Education, 13*(6), 459-471.

Matthews, M. R. (2000). Appraising constructivism in science and mathematics education. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 161-192). Chicago, IL: National Society for the Study of Education.

McCarty, L. P., & Schwandt, T. A. (2000). Seductive illusions: Von Glaserfeld and Gergen on epistemology and education. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 41-85). Chicago, IL: National Society for the Study of Education.

McDuffie, A. R. (2004). Mathematics teaching as a deliberate practice: An investigation of elementary pre-service teachers' reflective thinking during student teaching. *Journal of Mathematics Teacher Education, 7*, 33-61.

McNeal, B., & Simon, M. A. (2000). Mathematics culture clash: Negotiating new classroom norms with prospective teachers. *Journal of Mathematical Behavior, 18*(4), 475-509.

Merriam, S. B. (1998). *Qualitative research and case study applications in education: Revised and expanded from case study research in education.* San Francisco, CA: Jossey-Bass Publishers.

Ministry of Education and Training. (1997). *The Ontario curriculum: Grades 1-8 mathematics.* Ontario: Queen's Printer for Ontario.

Moll, L. C., & Whitmore, K. F. (1993). Vygotsky in classroom practice: Moving from individual transmission to social transaction. In E. A. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 19-42). New York: Oxford University Press.

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn? *Journal for Research in Mathematics Education, 40*(5), 491-529.

Mosenthal, J. H., & Ball, D. L. (1992). Constructing new forms of teaching: Subject matter knowledge in inservice teacher education. *Journal of Teacher Education, 43*(5), 347-356.

Moss, J., Messina, R., Morley, E., & Tepylo, D. (2012). Building and sustaining professional collaborations: Using Japanese lesson study to improve the teaching and learning of mathematics. In J. M. Bay-Williams & W. R. Speer (Eds.), *Professional collaborations* *in mathematics teaching and learning: Seeking success for all* (pp. 297-309). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Mullen, C. A. (2009). Introducing collaborative communities with edge and vitality. In C. A. Mullen (Ed.), *The handbook of leadership and professional learning communities* (pp. 1-9). New York, NY: Palgrave Macmillan.

Murray, S., Ma, X., & Mazur, J. (2009). Effects of peer coaching on teachers' collaborative interactions and students' mathematics achievement. *The Journal of Educational Research, 102*, 203-212.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

Nielsen, W. S., Triggs, V., Clarke, A., & Collins, J. (2010). *The teacher education conversation*: A network of cooperating teachers. *Canadian Journal of Education, 33*(4), 837-868.

Nipper, K., Ricks, T., Kilpatrick, J., Mayhew, L., Thomas, S., Kwon, N. Y., Klerlein, J. T., & Hembree, D. (2011). Teacher tensions: Expectations in a professional development institute. *Journal of Mathematics Teacher Education, 14*(5), 375-392.

O'Donnell, A. M., D'Amico, M., Schmid, R. F., Reeve, J., & Smith, J. K. (2008). *Educational psychology: Reflection for action* (Canadian ed.). Mississauga, ON: John Wiley & Sons Canada, Ltd.

Ontario Ministry of Education. (2005a). *The Ontario curriculum grades 1-8: Mathematics* (Rev. ed.). Ontario: Queen's Printer.

Ontario Ministry of Education. (2005b). *The Ontario curriculum, grades 9 and 10: Mathematics* (Rev. ed.). Ontario: Queen's Printer.

Ontario Ministry of Education. (2007). *Professional learning communities: A model for Ontario schools (Secretariat: Special Edition #3)*. Toronto, ON: Ontario Ministry of Education. Retrieved from http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/PLC.pdf

Opfer, V. D., & Pedder, D. (2011). Conceptualizing teacher professional learning. *Review of Educational Research, 81*(3), 376-407.

Oslund, J. A. (2012). Stories of mathematical problem solving in professional learning communities. In L. R. Van Zoest, J. J. Lo, & J. L. Kratky (Eds.), *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 543-6). Kalamazoo, MI: Western Michigan University.

Palincsar, A. S. (1998). Social constructivist perspectives on teaching and learning. *Annual Review of Psychology, 49*, 345-375.

Papert, S. (1993). *Mindstorms: Children, computers, and powerful ideas* (2nd ed.). New York, NY: Basic Books.

Patterson, J., & Patterson, J. (2004). Sharing the lead. *Educational Leadership, 61*(7), 74-78.

Philipp, R. A. (2007) Mathematics teachers' beliefs and affect. In F. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-315). Charlotte, NC: Information Age Publishing.

Phillips, D. C. (2000). An opinionated account of the constructivist landscape. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 1-16). Chicago, IL: National Society for the Study of Education.

Potari, D., & Georgiadou-Kabouridis, B. (2009). A primary teacher's mathematics teaching: The development of beliefs and practice in different "supportive" contexts. *Journal of Mathematics Teacher Education, 12*, 7-25.

Potari, D., Sakonidis, H., Chatzigoula, R., & Manaridis, A. (2010). Teachers' and researchers' collaboration in analysing mathematics teaching: A context for professional reflection and development. *Journal of Mathematics Teacher Education, 13*(6), 473-485.

Prawat, R. S. (1996). Constructivisms, modern and postmodern. *Educational Psychologist, 31*(3/4), 215-225.

Punch, K. F. (2009). *Introduction to research methods in education.* Thousand Oaks, CA: SAGE Publications, Inc.

Rasmussen, C., Lambdin, D. V., Heck, D. J., Baltzley, P. C., Tarr, J. E., Quander, J. R., Knuth, E., Barnes, D., & White, D. Y. (2011). Trends and issues in high school mathematics: Research insights and needs. *Journal for Research in Mathematics Education, 42*(3), 204-219.

Reeves, D. B. (2010). *Transforming professional development into student results.* Alexandria, VA: ASCD.

Resnick, L. B. (1989). Introduction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 1-24). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Richardson, V. (2003). Constructivist pedagogy. *Teachers College Record, 105*(9), 1623-1640.

Riordan, J. E., & Noyce, P. E. (2001). The impact of two standards-based mathematics curricula on student achievement in Massachusetts. *Journal for Research in Mathematics Education, 32*(4), 368-398.

Riveros, A., Newton, P., & Burgess, D. (2012). A situated account of teacher agency and learning: Critical reflections on professional learning communities. *Canadian Journal of Education, 35*(1), 202-216.

Rockoff, J. E., Jacob, B. A., Kane, T. J., & Staiger, D. O. (2008). *NBER working paper series: No.14485. Can you recognize an effective teacher when you recruit one?* Retrieved from National Bureau of Economic Research website: http://www.nber.org/papers/w14485.pdf

Rogers, W. T., Ma, X., Klinger, D. A., Dawber, T., Hellsten, L., Nowicki, D., & Tomkowicz, J. (2006). Examination of the influence of selected factors on performance on Alberta learning achievement tests. *Canadian Journal of Education, 29*(3), 731-756.

Ross, J. A., Ford, J., & Bruce, C. D. (2007). Needs assessment for the development of learning objects. *Alberta Journal of Educational Research, 53*(4), 430-433.

Ross, J. A., McDougall, D., Hogaboam-Gray, A., & LeSage, A. (2003). A survey measuring elementary teachers' implementation of standards-based mathematics teaching. *Journal for Research in Mathematics Education, 34*(4), 344-363.

Saha, L. J. (2009). The dissemination of knowledge about research on teachers, to the teachers. In L. J. Saha, & A. G. Dworkin (Eds.), *International Handbook of Research on Teachers and Teaching* (pp. 71-79). New York: Springer.

Sawyer, R. K. (2004). Creative teaching: Collaborative discussion as disciplined improvisation. *Educational Researcher, 33*(2), 12-20.

Schifter, D. (2005). A constructivist perspective on teaching and learning mathematics. In C. T. Fosnot (Ed.), *Constructivism: Theory, perspectives, and practice* (2nd ed., pp. 80-98). New York: Teachers College Press.

Schleppenbach, M., Perry, M., Miller, K. F., Sims, L., & Fang, G. (2007). The answer is only the beginning: Extended discourse in Chinese and U.S. mathematics classrooms. *Journal of Educational Psychology, 99*(2), 380-396.

Schmoker, M. (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning*. Alexandria, VA: Association for Supervision and Curriculum Development.

Shapiro, B. L. (2011). Towards a transforming constructivism: Understanding learners' meanings and the messages of learning environments. *The Journal of Educational Thought, 45*(2), 165-201.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher,15*(2), 4-14.

Shulman, L. S. (2000). Teacher development: Roles of domain expertise and pedagogical knowledge. *Journal of Applied Developmental Psychology, 21*(1), 129-135.

Shulman, L. S., & Shulman, J. H. (2004). How and what teachers learn: A shifting perspective. *Journal of Curriculum Studies, 36*(2), 257-271.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*, 499-511.

Silva, D. Y., Gimbert, B., & Nolan, J. (2000). Sliding the doors: Locking and unlocking possibilities for teacher leadership. *Teachers College Record, 102*(4), 779-804.

Skemp, R. R. (1986). *Psychology of learning mathematics* (2nd ed.). London, England: Penguin Group.

Slavit, D., & Nelson, T. H. (2010). Collaborative teacher inquiry as a tool for building theory on the development and use of rich mathematical tasks. *Journal of Mathematics Teacher Education, 13*, 201-221.

Slezak, P. (2000). A critique of radical social constructivism. In D. C. Phillips (Ed.), *Constructivism in education: Opinions and second opinions on controversial issues: Ninety-ninth yearbook of the National Society for the Study of Education* (pp. 91-126). Chicago, IL: National Society for the Study of Education.

Sowder, J. (2007). The mathematical education and development of teachers. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 157-223). Charlotte, NC: Information Age Publishing.

Stake, R. E. (2005). Qualitative case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *The SAGE handbook of qualitative research* (3rd ed., pp. 443-466). Thousand Oaks, CA: SAGE Publications.

Steele, M. D. (2005). Comparing knowledge bases and reasoning structures in discussions of mathematics and pedagogy. *Journal of Mathematics Teacher Education, 8*, 291-328.

Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12-17.

Stipek, D. J., Givven, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213-226.

Strahan, D. (2003). General patterns and particular pictures: Lessons learned from reports from "beating the odds" schools. *Journal of Curriculum and Supervision, 18*(4), 296-305.

Stylianides, A. J., & Ball, D. L. (2008). Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education, 11*, 307-332.

Sykes, G. (1999). Teacher and student learning: Strengthening their connection. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession* (pp. 151-179). San Francisco, CA: Jossey-Bass Publishers.

Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*, 253-279.

Towers, J. (2012). Administrative supports and curricular challenges: New teachers enacting and sustaining inquiry in schools. *Canadian Journal of Education, 35*(1), 259-278.

Turner, J. C., Warzon, K. B., & Christensen, A. (2011). Motivating mathematics learning: Changes in teachers' practices and beliefs during a nine-month collaboration. *American Educational Research Journal, 48*(3), 718-762.

Turner, S. (1998). The limits of social constructionism. In I. Velody & R. Williams, *The politics of constructionism* (pp. 109-120). Thousand Oaks, CA: SAGE Publications.

Van de Walle, J. A., & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades 3-5*. Boston: Allyn and Bacon.

Van Driel, J. H., & Berry, A. (2012). Teacher professional development focusing on pedagogical content knowledge. *Educational Researcher, 41*(1), 26-28.

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education: An International Journal of Research and Studies, 24*(1), 80-91.

Volante, L. (2006). Toward appropriate preparation for standardized achievement testing. *The Journal of Educational Thought, 40*(2), 129-144.

von Glaserfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. London, Eng: RoutledgeFalmer.

Vygotsky, L. S. (1962). *Thought and language* (E. Hanfmann & G. Vakar, Trans.). New York: John Wiley & Sons, Inc.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Cambridge, MA: Harvard University Press.

West, L., & Curcio, F. R. (2004). Collaboration sites: Teacher-centered professional development in mathematics. *Teaching Children Mathematics, 10*(5), 268-73.

Wetzel, K. (2001). Preparing teacher leaders: A partnership that works, part 2. *Learning and Leading with Technology, 29*(3), 50-53.

Whitenack, J., & Yackel, E. (2002). Making mathematical arguments in the primary grades: The importance of explaining and justifying ideas. *Teaching Children Mathematics, 8*(9), 524-527.

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*, 139-164.

Wilson, S. M., Peterson, P. L., Ball, D. L., & Cohen, D. K. (1996). Learning by all. *Phi Delta Kappan, 77*(7), 468-476.

Windschitl, M. (2002). Framing constructivism in practice as the negotiation of dilemmas: An analysis of the conceptual, pedagogical, cultural, and political challenges facing teachers. *Review of Educational Research, 72*(2), 131-170.

Wixson, K. K., & Yochum, N. (2004). Research on literacy policy and professional development: National, State, District, and teacher contexts. *The Elementary School Journal, 105*(2), 219-242.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. *American Educational Research Journal, 28*(3), 587-616.

Wolhuter, K. A., & Quintero, E. (2003). Integrating mathematics and literacy in early childhood teacher education: Lessons learned. *Teacher Education Quarterly, 30*(4), 27-38.

Working Group of the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

Working Groups of the Commission on Teaching Standards for School Mathematics of the National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

Wu, H-H. (2009, Fall). What's sophisticated about elementary mathematics? Plenty—that's why elementary schools need math teachers. *American Educator*, 4-14. Retrieved from http://www.aft.org/pdfs/americaneducator/fall2009/wu.pdf

Yoon, K. S., Duncan, T., Lee, S. W. Y., Scarloss, B., & Shapley, K. L. (2007). *Reviewing the evidence on how teacher professional development affects student achievement* (Issues & Answers Report, REL2007-No. 033). Retrieved from US Department of Education, Institute of Education Sciences, National Center for Education and Evaluation and Regional Assistance website: http://ies.ed.gov/ncee/edlabs/regions/southwest/pdf/rel_2007033.pdf

APPENDICES

Appendix A
Pilot Study Ethics Approval

Lakehead
UNIVERSITY

Office of Research

Tel (807) 343-8283
Fax (807) 346-7749

August 12, 2005

Dr. Ann Kajander
Faculty of Education
Lakehead University
955 Oliver Road
Thunder Bay, Ontario P7B 5E1

Dear Dr. Kajander:

Re: REB Project #: 124 04-05
Granting Agency name: University of Manitoba CRYSTAL grant (NSERC)
Granting Agency Project #: 19389

Based on the recommendation of the Research Ethics Board, I am pleased to grant ethical approval to your research project entitled, "Teachers' Evolving Mathematical Understandings".

The Research Ethics Board requests an annual progress report and a final report for your study in order to be in compliance with Tri-Council Guidelines. This annual review will help ensure that the highest ethical and scientific standards are applied to studies being undertaken at Lakehead University.

Completed reports may be forwarded to:

Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
FAX: 807-346-7749

Best wishes for a successful research project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/len

cc: B.L. Crutchley, University of Manitoba
Margot Ross, Finance, Lakehead University
Research Office, Lakehead University

1965 • 2005 CELEBRATING 40 YEARS

955 Oliver Road Thunder Bay Ontario Canada P7B 5E1 www.lakeheadu.ca

Appendix B

Pilot Study Ethics Renewal

Lakehead
UNIVERSITY

Office of Research

Tel (807) 343-8283
Fax (807) 346-7749

August 10, 2009

Principal Investigator: Dr. Ann Kajander
Faculty of Education
Lakehead University
955 Oliver Road
Thunder Bay, Ontario P7B 5E1

Dear Dr. Kajander:

Re: REB Project #: 124 04-05
Granting Agency name: NSERC (Sub-grant from University of Manitoba)
Granting Agency Project #: N/A

On behalf of the Research Ethics Board, I am pleased to grant renewal of ethical approval to your research project entitled, "Teachers' Evolving Mathematical Understandings".

Ethics approval is valid until **August 10, 2010**. Please submit a Request for Renewal form to the Office of Research by July 10, 2010 if your research involving human subjects will continue for longer than one year. A Final Report must be submitted promptly upon completion of the project. Research Ethics Board forms are available at:

http://bolt.lakeheadu.ca/~researchwww/internalforms.html

During the course of the study, any modifications to the protocol or forms must not be initiated without prior written approval from the REB. You must promptly notify the REB of any adverse events that may occur.

Completed reports and correspondence may be directed to:

Research Ethics Board
c/o Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
Fax: (807) 346-7749

Best wishes for a successful research project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/len

cc: Office of Financial Services
Office of Research

Appendix C

Pilot Study Consent Form

Information for Participating Teachers at Lakehead Public Schools

Research Project Title: ***Teachers' Evolving Mathematical Understandings***

Researcher(s): Ann Kajander, Ralph Mason

Sponsor (if applicable): NSERC (CRYSTAL), University of Manitoba

This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

Research Study by Dr. Ann Kajander Faculty of Education, Lakehead University, email ann.kajander@lakeheadu.ca, phone (807) 343-8127

The purpose of this research is to examine mathematics beliefs and knowledge of teachers, and to help you study your own abilities, values, and growth areas. Participation may give you a better idea of your own level of mathematical understanding at the conceptual level, as well as a better understanding of your values in the teaching and learning of mathematics.

Participation in this study is strictly voluntary, and individual results will not be communicated to Lakehead Public Schools. Submissions will be numbered, and confidentiality maintained - at no time will your name be used in reporting any research results.

Teachers who volunteer to participate will be asked to complete the Perceptions of Mathematics Survey which contains mathematical questions as well as questions about your beliefs about mathematics. Completing the survey should take less than an hour and all answers are acceptable. You may also be asked for comments about how well you feel the Survey characterizes your values and understanding in mathematics, and how the Survey might be improved. You may also be asked if you wish to voluntarily participate in several brief interviews. Any data collected will be recorded by participant number and will be kept completely confidential. At no point will names of participants be made public.

Final analysis of results will be made public and participants will be made aware of how they can see the results. Participation is voluntary and participants may withdraw at any time from the Study with no repercussions. Data will be securely stored at Lakehead University for seven years.

If you are willing to participate, please sign the attached Consent Form and submit it with your Survey. Thank you for your interest in this project!

Ann Kajander

Consent Form (Lakehead Public Schools)

Your signature on this form indicates that you have understood to your satisfaction the information regarding participation in the research project and agree to participate as a subject. In no way does this waive your legal rights nor release the researchers, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from the study at any time, and /or refrain from answering any questions you prefer to omit, without prejudice or consequence. Your continued participation should be as informed as your initial consent, so you should feel free to ask for clarification or new information throughout your participation. Feel free to contact

Dr. Ann Kajander ann.kajander@lakeheadu.ca (807)343-8127

This research has been approved by the University of Manitoba Research Ethics Board as well as Lakehead University Research Ethics Board. If you have any concerns or complaints about this project you may contact any of the above-named persons or the Human Ethics Secretariat at 204-474-7122. A copy of this consent form has been given to you to keep for your records and reference.

My signature on this sheet indicates I agree to participate in a study by Dr. Ann Kajander, of Lakehead University on *Teachers' Evolving Mathematical Understandings* and it also indicates that I understand the following:

1. I am a volunteer and can withdraw at any time from the study.

2. There is no apparent risk of physical or psychological harm.

3. The data I provide will be confidential and data will be securely stored at Lakehead University for 7 years.

4. I will receive a summary of the project, upon request, following the completion of the project.

I have received explanations about the nature of the study, its purpose, and procedures. I am willing to answer a written survey and I am also aware I may be asked to participate in related interviews, from which I may also withdraw at any time

Participant's Signature ____________________ Printed Name: ______________

Researcher Signature ____________________ Date ______________

Appendix D

Research Study Ethics Approval

Lakehead
UNIVERSITY

Office of the Vice-President (Research)

Tel (807) 343-8201
Fax (807) 766-7105

September 5, 2010

Principal Investigator: Dr. Ann Kajander
Student Investigator: Jennifer Holm
Faculty of Education
Lakehead University
955 Oliver Rd
Thunder Bay ON P7B 5E1

Dear Dr. Kajander and Ms Holm:

Re: REB Project #: 137 09-10/ROMEO #1461286
Granting Agency name: N/A
Granting Agency Project #: N/A

On behalf of the Research Ethics Board, I am pleased to grant ethical approval to your research project entitled, "Improving Mathematics Teaching Through Professional Learning Groups".

Ethics approval is valid until **September 5, 2011**. Please submit a Request for Renewal form to the Office of Research by August 5, 2011 if your research involving human subjects will continue for longer than one year. **A Final Report must be submitted promptly upon completion of the project**. Request for Renewal and Final Report forms are available at:

http://research.lakeheadu.ca/ethics_resources.html

During the course of the study, any modifications to the protocol or forms must not be initiated without prior written approval from the REB. You must promptly notify the REB of any adverse events that may occur.

Completed reports and correspondence may be directed to:

Research Ethics Board
c/o Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
Fax: (807) 346-7749

Best wishes for a successful research project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/sw

cc: Office of Research
Faculty of Graduate Studies

955 Oliver Road Thunder Bay Ontario Canada P7B 5E1 www.lakeheadu.ca

Appendix E

Research Study Consent Form

Lakehead
UNIVERSITY

Faculty of Education

Information for Potential Participants

Research Project Title: Improving Mathematics Teaching Through Professional Learning Groups

Researcher: Jennifer Holm, jholm@lakeheadu.ca

Supervisor: Ann Kajander, Lakehead University, akajande@lakeheadu.ca, 343-8127

Research Ethics and Administration Officer: Susan Wright, Lakehead University, 343-8283

This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

The purpose of this research is to examine conversations about mathematics teaching within professional learning groups. The research will seek to answer questions about what effective mathematics discussions look like and how to best support teachers as they grow as mathematics teachers.

Participation in this study is strictly voluntary. Submissions will be coded, and confidentiality maintained - at no time will your name be used in reporting any research results.

If you volunteer to participate, you are agreeing to allow me to sit and observe your professional learning group meetings. You may be asked to participate in interviews. The purpose of the interviews would be to ask your opinions about participating in professional learning groups and what is most supportive for you in mathematics teaching. These interviews will be audio-recorded in order to be transcribed at a later time to ensure that quotations are accurate. You may also be asked if the researcher can sit in during your teaching when it occurs during the meetings. The purpose will be to allow for the researcher to fully observe any and all meetings of the group. Any data collected will be recorded with pseudonyms and will be kept completely confidential. At no point will names of participants be made public. Also, all participation in interviews and observations is completely voluntary.

Final analysis of results will be part of my doctoral dissertation and may be made public. Participants will be made aware of how they can see the results of the study. Participation is voluntary and participants may withdraw at any time from the study with no repercussions. There are no benefits for participation in the study. Data will be securely stored with Ann Kajander at Lakehead University for five years.

If you are willing to participate, please sign and return the attached Consent Form. Thank you for your interest in this project!

Jennifer Holm

955 Oliver Road Thunder Bay Ontario Canada P7B 5E1 www.lakeheadu.ca

Lakehead
UNIVERSITY

Faculty of Education

Consent Form

Your signature on this form indicates that you have understood to your satisfaction the information regarding participation in the research project and agree to participate as a subject. In no way does this waive your legal rights nor release the researchers, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from the study at any time, and /or refrain from answering any questions you prefer to omit, without prejudice or consequence. Your continued participation should be as informed as your initial consent, so you should feel free to ask for clarification or new information throughout your participation. Feel free to contact Jennifer Holm, jholm@lakeheadu.ca or Dr. Ann Kajander, ann.kajander@lakeheadu.ca.
This research has been approved by Lakehead University Research Ethics Board, as well as by the Lakehead District School Board. If you have any concerns or complaints about this project you may contact any of the above-named persons or Susan Wright, Lakehead University's Research Ethics and Administration Officer, 343-8283. A copy of this consent form has been given to you to keep for your records and reference.

My signature on this sheet indicates I agree to participate in a study by Jennifer Holm, of Lakehead University *on Improving Mathematics Teaching Through Professional Learning Groups* and it also indicates that I understand the following:

1. I am a volunteer and can withdraw at any time from the study.

2. There is no apparent risk of physical or psychological harm.

3. There are no specific benefits for participation in the study.

4. The data, including field notes and audiotapes, I provide will be confidential and data will be securely stored at Lakehead University for 5 years.

5. Data will be used by Jennifer Holm in the creation of her doctoral dissertation and will be reviewed by her supervisor and committee, if needed.

6. I will receive a summary of the project, upon request, following the completion of the project.

I have received explanations about the nature of the study, its purpose, and procedures.

Participant's Signature ____________________ Name (printed) __________________

Researcher and/or Delegate's Signature ___________________ Date Sept __ 2010

Appendix F

School Board Ethics Approval

Lakehead Public Schools

2135 Sills Street
THUNDER BAY, ON P7E 5T2
Telephone 625-5100
Fax 623-5833
www.lhbe.edu.on.ca

Director of Education: Catherine Siemieniuk

July 3, 2013

Ms. Jennifer Holm
Faculty of Education
Lakehead University
955 Oliver Rd.
Thunder Bay, ON P7B 5E1
Via email : "Jennifer Holm" <jholm@lakeheadu.ca>

Dear Jennifer:

On behalf of the Lakehead District School Board, I am pleased to grant you permission to carry out your research entitled, *Improving Mathematics Teaching Through Professional Learning Groups.*

The Lakehead District School Board looks forward to cooperating with you and to receiving your final report. Please ensure the Principals at the schools planned for your research are aware of the project and approve of their staff's participation.

I will return the signed application package to you by mail.

Sincerely,

Charles Bishop
Education Officer
Lakehead District School Board

Appendix G

Research Study Ethics Renewal

Lakehead
UNIVERSITY

Office of Research

Tel 807-343-8934
Fax 807-346-7749

September 9, 2011

Principal Investigator: Dr. Elizabeth (Ann) Kajander
Student Investigator: Jennifer Holm
Faculty of Education
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1

Dear Dr. Kajander and Ms Holm:

Re: REB Project #: 137 09-10 / Romeo File No: 1461286
Granting Agency: N/A
Granting Agency Project#: N/A

On behalf of the Research Ethics Board, I am pleased to grant renewal of ethical approval to your research project entitled, "Improving Mathematics Teaching Through Professional Learning Groups".

Ethics approval is valid until **September 9, 2012.** Please submit a Request for Renewal form to the Office of Research by August 9, 2012 if your research involving human subjects will continue for longer than one year. A Final Report must be submitted promptly upon completion of the project. Research Ethics Board forms are available at:

http://research.lakeheadu.ca/ethics_resources.html

During the course of the study, any modifications to the protocol or forms must not be initiated without prior written approval from the REB. You must promptly notify the REB of any adverse events that may occur.

Completed reports and correspondence may be directed to:

Research Ethics Board
c/o Office of Research
Lakehead University
955 Oliver Road
Thunder Bay, ON P7B 5E1
Fax: (807) 346-7749

Best wishes for a successful research project.

Sincerely,

CShahi

Dr. Chander Shahi
Chair, Research Ethics Board

/scw

Lakehead Research...CREATING THE FUTURE NOW

955 Oliver Road Thunder Bay Ontario Canada P7B 5E1 www.lakeheadu.ca

Appendix H

Research Study Ethics Amendment

Lakehead
UNIVERSITY

Office of Research

(807) 343-8283
(807) 346-7749

MEMORANDUM

Date: May 11, 2012

To: Dr. Ann Kajander, Ms Jennifer Holm

From: Dr. Richard Maundrell

Subject: Amendment for REB Project #137 09-10 / ROMEO #1461286

Thank you for your request for amendment to your project entitled, "Improving Mathematics Teaching Through Professional Learning Groups".

Your request to add a focus group interview of the participants to take place during the meetings and the revised information and consent letters provided are acceptable to the Research Ethics Board.

Please continue to advise us of any future changes to this project.

Sincerely,

Dr. Richard Maundrell
Chair, Research Ethics Board

/scw

Information for Potential Participants

Research Project Title: Improving Mathematics Teaching Through Professional Learning Groups

Researcher: Jennifer Holm, jholm@lakeheadu.ca

Supervisor: Ann Kajander, Lakehead University, akajande@lakeheadu.ca, 343-8127

Research Ethics and Administration Officer: Susan Wright, Lakehead University, 343-8283

This consent form, a copy of which will be left with you for your records and reference, is only part of the process of informed consent. It should give you the basic idea of what the research is about and what your participation will involve. If you would like more detail about something mentioned here, or information not included here, you should feel free to ask. Please take the time to read this carefully and to understand any accompanying information.

The purpose of this research is to examine conversations about mathematics teaching within professional learning groups. The research will seek to answer questions about what effective mathematics discussions look like and how to best support teachers as they grow as mathematics teachers.

Participation in this study is strictly voluntary. Submissions will be coded, and confidentiality maintained - at no time will your name be used in reporting any research results.

If you volunteer to participate, you are agreeing to allow me to sit and observe your professional learning group meetings. You may be asked to participate in individual interviews and a focus group during the learning group meetings. The purpose of the individual interviews would be to ask your opinions about participating in professional learning groups and what is most supportive for you in mathematics teaching. These interviews will be audio-recorded in order to be transcribed at a later time to ensure that quotations are accurate. Confidentiality from other members of the group will be limited when participating in the focus group. You may also be asked if the researcher can sit in during your teaching when it occurs during the meetings. The purpose will be to allow for the researcher to fully observe any and all meetings of the group. Any data collected will be recorded with pseudonyms and will be kept completely confidential. At no point will names of participants be made public. Also, all participation in individual interviews, focus groups, and observations is completely voluntary.

Final analysis of results will be part of my doctoral dissertation and may be made public. Participants will be made aware of how they can see the results of the study. Participation is voluntary and participants may withdraw at any time from the study with no repercussions. There are no benefits for participation in the study. Data will be securely stored with Ann Kajander at Lakehead University for five years.

If you are willing to participate, please sign and return the attached Consent Form. Thank you for your interest in this project!

Jennifer Holm

Lakehead
UNIVERSITY

Faculty of Education

Consent Form

Your signature on this form indicates that you have understood to your satisfaction the information regarding participation in the research project and agree to participate as a subject. In no way does this waive your legal rights nor release the researchers, sponsors, or involved institutions from their legal and professional responsibilities. You are free to withdraw from the study at any time, and /or refrain from answering any questions you prefer to omit, without prejudice or consequence. Your continued participation should be as informed as your initial consent, so you should feel free to ask for clarification or new information throughout your participation. Feel free to contact Jennifer Holm, jholm@lakeheadu.ca or Dr. Ann Kajander, ann.kajander@lakeheadu.ca.
This research has been approved by Lakehead University Research Ethics Board, as well as by the Lakehead District School Board. If you have any concerns or complaints about this project you may contact any of the above-named persons or Susan Wright, Lakehead University's Research Ethics and Administration Officer, 343-8283. A copy of this consent form has been given to you to keep for your records and reference.

My signature on this sheet indicates I agree to participate in a study by Jennifer Holm, of Lakehead University *on Improving Mathematics Teaching Through Professional Learning Groups* and it also indicates that I understand the following:

1. I am a volunteer and can withdraw at any time from the study.

2. There is no apparent risk of physical or psychological harm.

3. There are no specific benefits for participation in the study.

4. The data, including field notes and audiotapes, I provide will be confidential and data will be securely stored at Lakehead University for 5 years.

5. Data will be used by Jennifer Holm in the creation of her doctoral dissertation and will be reviewed by her supervisor and committee, if needed.

6. I will receive a summary of the project, upon request, following the completion of the project.

I have received explanations about the nature of the study, its purpose, and procedures.

Participant's Signature ____________________ Name (printed) __________________

Researcher and/or Delegate's Signature ___________________ Date Sept __ 2010

Appendix I

Interview Questions

1. What is your personal philosophy on teaching mathematics?

   -Describe a mathematics lesson that you teach that you feel is particularly successful.
2. What is your opinion about attending the professional learning group in mathematics, and why?
   -If negative, how they might be improved to be more beneficial?

   -What are examples of specific meetings or discussions?
3. What do you feel are the strengths of the group meetings? What are the weaknesses?

   -What are examples of specific meetings or discussions?
4. In what ways has your teaching been influenced through attending the professional learning group meetings?

   -Do you feel that your philosophy of teaching mathematics has changed as a result of the meetings? How so?
   -Give an example of a lesson you've taught differently as a result of the professional learning groups. How was it different?
5. Has your knowledge of mathematics been deepened through the discussions of mathematics? How so? Can you give an example?

6. Do you have anything else to add?

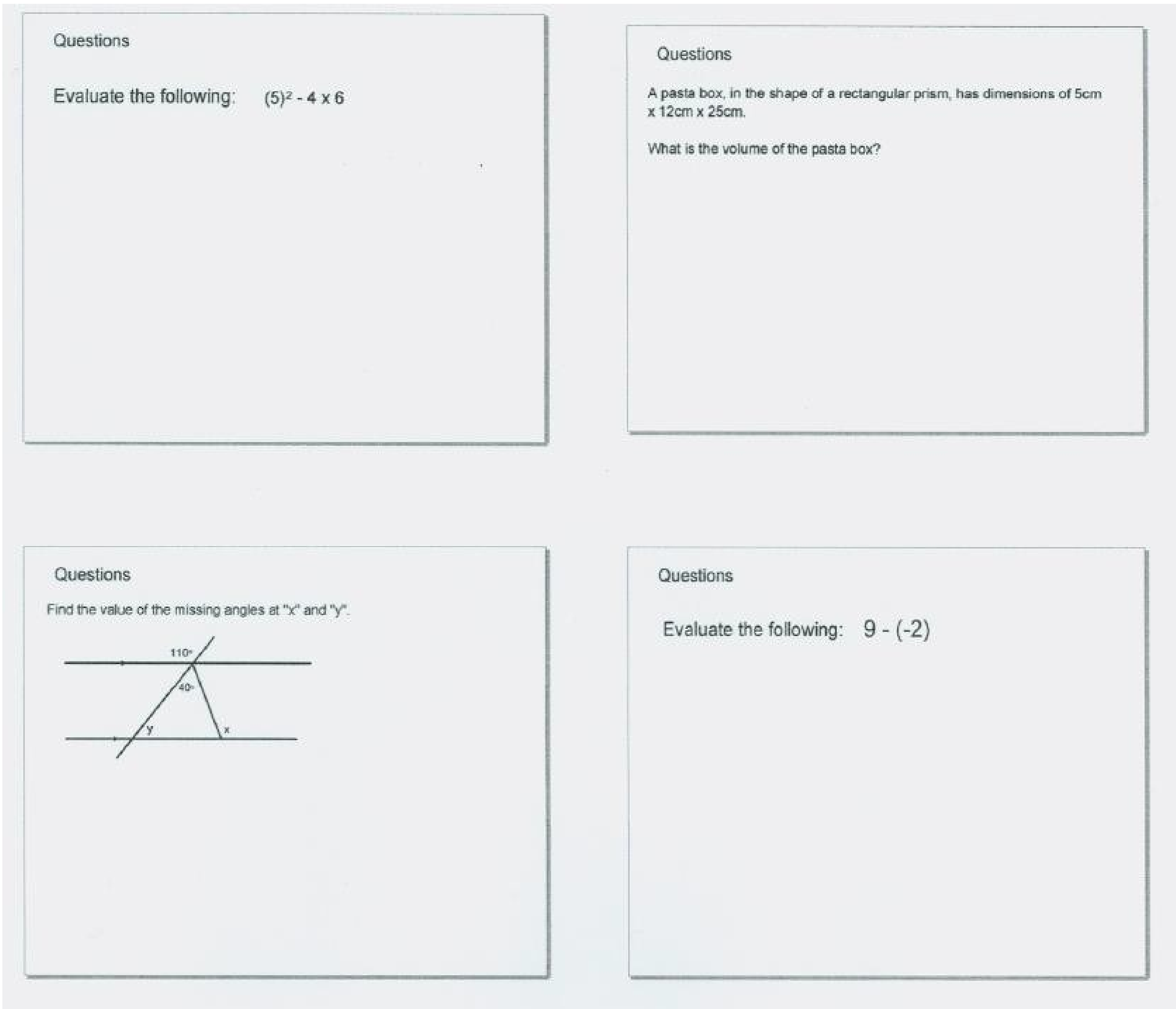
Appendix J

Math Matrix

Tables Matrix Name: ______________________				I have heard of it	I can tell someone how to do it	I can say them in less then 25 seconds	I can break my record	I can say them in random order
Aim	Capacity	Capacity Breakdown						
To understand numbers	Numbers/tables	1 times tables	1					
		2 times tables	2					
		3 times tables	3					
		4 times tables	4					
		5 times tables	5					
		6 times tables	6					
		7 times tables	7					
		8 times tables	8					
		9 times tables	9					
		10 times tables	10					
		11 times tables	11					
		12 times tables	12					
		20 times tables	13					
		100 times tables	14					

Appendix K Questions from

Meeting #12



Questions

Simplify the following: 3 (a+2)

Questions

Solve the following: 2x - 3 = 9

Questions

Evaluate the following: $\frac{1}{2}+\frac{3}{4}-\frac{1}{8}$

Questions

Jim recorded his earnings for the number of hours he worked in the table below but he spilt some water on it and now has to work backwards to determine what the missing numbers were. Can you help Jim determine the missing numbers?

Time (hrs)	Wage ($)
0	10
1	
	14
4	18
5	
	22

Appendix L

Meeting #8 Partial Transcript

Ryan: How many marks is the entire question? Just get it into your head. How many marks did you assign to question number two. In general we tend…

Wesley: Oh for all three parts?

Madison: All three parts?

Ryan: For the entire number two, what is all of number two worth? I'm going to, I've got to show Emma my answer. (pause) Okay, you've got it in your head?

Madison: Yeah.

Ryan: Owen, how many?

Owen: Six.

Ryan: Wesley?

Wesley: Six, maybe nine, depending on…

Ryan: Madison?

Madison: Nine for grade 9 Applied, six for grade 10.

Emma: Wait, including verification?

Wesley: Oh pardon me. (Muttering/no's from rest of the secondary teachers, multiples oh's). Pardon me, pardon me, pardon me, nine for sure.

Board liaison: Yeah, nine.

Owen: Yeah, at least.

Ryan: Okay, Noah?

Noah: Um, maybe twelve.

Owen: Yeah.

Madison: Yeah.

Ryan: Yep. Okay, without verification, we're all pretty close to six.

Wesley: Six or seven.

Madison: Yeah.

Ryan: We're predictable. Just curious.

Emma: I think, using a rubric is…

Ryan: I wrote six by the way.

Owen: I did too.

Emma: It's an answer, but you're just using numbers to modify their performance, and we're trying to use descriptors. Well not we, the Ministry, the government, whatever.

Madison: Yeah.

Emma: But you're just using words to moderate that, so whether you're using the veneer of objectivity that comes with numbers, or the subjectivity that obviously comes with words. But it's just trying to describe their performance, but I think, I don't know, even if you just mark a test with numbers, how many times do you go back and think gee I should really rethink whether or not this is worth three. To me, that's normal, and I have to mark the first ten wrong before I went.

Wesley: I have to ask a question. How, how do you feel about that marking subjectivity as opposed to objectivity? How…

Emma: I'm going to have to, I'm not sure how much objectivity there is really because I think you're always making judgments and you're always making choices. It's what do you value and what is important…

Wesley: Yeah.

Emma: and I think that just because you decided to value verification, or accuracy, or you decided to value the process, and the way you're showing that is you're subjectivity awarding marks or not, right?

Wesley: Yeah.

Emma: So I don't necessarily see that as being too diametric ways of looking at things, I just, a different way of describing performance.

Wesley: If, if someone asks me that question, the way I would answer it is, is, I, I, I like the way of marking with a rubric, as a teacher. I wonder though at times whether my information is falling on deaf ears.

Emma: Are you talking about kids? Parents? Other teachers?

Wesley: Both, yeah. I wonder, and, and that's the question I'm asking. Have you been finding, because I'm kind of wondering how much of that information is being sent home with the child and the parent can be, actually being digested. When you know, I mean, Parent's Night, I, I get constantly, oh, give me a percent.

Emma: We, I don't, we don't get that as much because the weird thing about our report card is they get levels, they get levels, they get levels, and when it's report card time, those levels turn into, turn into percents, right?

Madison: Yeah.

Emma: And, and it's this total morph, but it's supposed to be the most consistent, most recent performance, right, so ideally we could do ten things, but it's the most recent performance that's supposed to carry the heaviest weight, which in some ways, I mean you can struggle with it, you make mistakes, you can learn, but ideally at the end this is what you have pulled together.

Appendix M

Achievement Chart Ontario 9/10 Curriculum

Achievement Chart – Mathematics, Grades 9–12

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Knowledge and Understanding *Subject-specific content acquired in each course (knowledge), and the comprehension of its meaning and significance (understanding)*				
	The student:			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
Thinking *The use of critical and creative thinking skills and/or processes**				
	The student:			
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Communication *The conveying of meaning through various forms*				
	The student:			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application *The use of knowledge and skills to make connections within and between various contexts*				
	The student:			
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	– applies knowledge and skills in familiar contexts with considerable effectiveness	– applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	– transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

Note: A student whose achievement is below 50% at the end of a course will not obtain a credit for the course.

(Ontario Ministry of Education, 2005b)

Appendix N

Achievement Chart Ontario 1-8 Curriculum

Achievement Chart – Mathematics, Grades 1–8

Categories	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding *Subject-specific content acquired in each grade (knowledge), and the comprehension of its meaning and significance (understanding)*				
	The student:			
Knowledge of content (e.g., facts, terms, procedural skills, use of tools)	– demonstrates limited knowledge of content	– demonstrates some knowledge of content	– demonstrates considerable knowledge of content	– demonstrates thorough knowledge of content
Understanding of mathematical concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
Thinking *The use of critical and creative thinking skills and/or processes**				
	The student:			
Use of planning skills – understanding the problem (e.g., formulating and interpreting the problem, making conjectures) – making a plan for solving the problem	– uses planning skills with limited effectiveness	– uses planning skills with some effectiveness	– uses planning skills with considerable effectiveness	– uses planning skills with a high degree of effectiveness
Use of processing skills* – carrying out a plan (e.g., collecting data, questioning, testing, revising, modelling, solving, inferring, forming conclusions) – looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)	– uses processing skills with limited effectiveness	– uses processing skills with some effectiveness	– uses processing skills with considerable effectiveness	– uses processing skills with a high degree of effectiveness
Use of critical/creative thinking processes* (e.g., problem solving, inquiry)	– uses critical/creative thinking processes with limited effectiveness	– uses critical/creative thinking processes with some effectiveness	– uses critical/creative thinking processes with considerable effectiveness	– uses critical/creative thinking processes with a high degree of effectiveness

Categories	Level 1	Level 2	Level 3	Level 4
Communication *The conveying of meaning through various forms*				
	The student:			
Expression and organization of ideas and mathematical thinking (e.g., clarity of expression, logical organization), using oral, visual, and written forms (e.g., pictorial, graphic, dynamic, numeric, algebraic forms; concrete materials)	– expresses and organizes mathematical thinking with limited effectiveness	– expresses and organizes mathematical thinking with some effectiveness	– expresses and organizes mathematical thinking with considerable effectiveness	– expresses and organizes mathematical thinking with a high degree of effectiveness
Communication for different audiences (e.g., peers, teachers) and purposes (e.g., to present data, justify a solution, express a mathematical argument) in oral, visual, and written forms	– communicates for different audiences and purposes with limited effectiveness	– communicates for different audiences and purposes with some effectiveness	– communicates for different audiences and purposes with considerable effectiveness	– communicates for different audiences and purposes with a high degree of effectiveness
Use of conventions, vocabulary, and terminology of the discipline (e.g., terms, symbols) in oral, visual, and written forms	– uses conventions, vocabulary, and terminology of the discipline with limited effectiveness	– uses conventions, vocabulary, and terminology of the discipline with some effectiveness	– uses conventions, vocabulary, and terminology of the discipline with considerable effectiveness	– uses conventions, vocabulary, and terminology of the discipline with a high degree of effectiveness
Application *The use of knowledge and skills to make connections within and between various contexts*				
	The student:			
Application of knowledge and skills in familiar contexts	– applies knowledge and skills in familiar contexts with limited effectiveness	– applies knowledge and skills in familiar contexts with some effectiveness	– applies knowledge and skills in familiar contexts with considerable effectiveness	– applies knowledge and skills in familiar contexts with a high degree of effectiveness
Transfer of knowledge and skills to new contexts	– transfers knowledge and skills to new contexts with limited effectiveness	– transfers knowledge and skills to new contexts with some effectiveness	– transfers knowledge and skills to new contexts with considerable effectiveness	– transfers knowledge and skills to new contexts with a high degree of effectiveness
Making connections within and between various contexts (e.g., connections between concepts, representations, and forms within mathematics; connections involving use of prior knowledge and experience; connections between mathematics, other disciplines, and the real world)	– makes connections within and between various contexts with limited effectiveness	– makes connections within and between various contexts with some effectiveness	– makes connections within and between various contexts with considerable effectiveness	– makes connections within and between various contexts with a high degree of effectiveness

(Ontario Ministry of Education, 2005a)

Appendix O

Transcript of Manipulatives Discussion Meeting #10

Madison: The fractions on the back, I don't know if you flipped over yet.

Ryan: Yeah, they can't add or subtract fractions.

Madison: *Brutal*. And I think our Academics are fine, I mean, and you'll see that, I mean it's Applied kids and that's probably where the dividing line becomes, right? The kids who are Academic and can handle fractions, I mean…

Owen: Some of my Academics are great, but…

Samuel: But do they even handle the fractions or do they just learn method? They, they have the skills to learn the method?

Madison: Yeah.

Samuel: I mean memorise the method so they can them, but the kids that can't memorise that many methods or that many things to do, and don't understand the fraction part. Don't understand the concepts on the fraction strips or whatever, so they're just.

Noah: I actually used the fraction strips there, for the first time. And I thought they helped, but then they got real sick of them quick.

Owen: But, you know, that's okay, for about four or five kids it's like, oh, they've got to be the same colour.

Noah: Nice connection…

Owen: Get them all to be the same colour.

Samuel: And then add how many of the colours you have.

Noah: A nice connection with them is to algebra in that, it's the same that. You can add a half to a half, you can add an x to an x, and that's what I like about them.

Ryan: Then again, all of those tools are just to get them in.

Owen: Yeah.

Ryan: That's all it is, right?

Noah: Yeah.

Ryan: So, some kids…

Emma: Well you can still add a half to a third, it's just what you call it when you get there.

Noah: I know, but they understand, like, that they have to be the same size.