ACOUSTOELECTRIC EFFECT IN SEMICONDUCTORS

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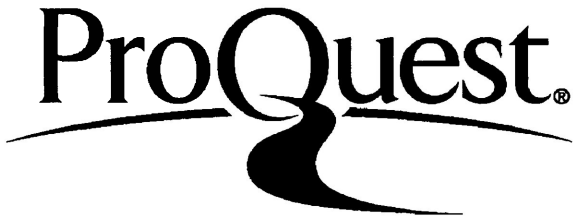

Abstract

The possibility of electron temperature decreasing below the lattice temperature in the presence of an external electric field (electron cooling) has been pointed out by Paranjape and Ambrose (1964). In the present work, we show that under suitable conditions an external sound wave may produce the phenomenon of electron cooling in a semiconductor. We have shown that a decrease in electron temperature may occur when (1) the electrons are predominantly scattered by optical polar or non-polar optical modes of the lattice vibrations and (2) when the incident sound wave energy flux W is greater than a certain critical value W_0 (which depends on the type of semiconductor). Chapter I consists of a description of the model and a brief outline of the calculations. In Chapter II, using a displaced Maxwellian function, we have calculated the rates of energy and momentum transfer from the electrons to the lattice for acoustical, optical polar and non-polar optical types of scattering. The rates of energy and momentum transfer from the sound wave to the electrons are calculated in Chapter II, Section (2.b.1). Using these rates in conservation conditions (1.11) and (1.12), we obtain the expression for the electron temperature T as a function of the energy flux W (Eqn. (3.6)). Inequality conditions (3.7) and (3.8) are the main results of our calculations. Condition (3.7) is equivalent to the electron cooling condition obtained by Paranjape and de Alba (1965) in the case of an electric field, while (3.8) gives the minimum sound wave energy flux W_0 required to produce electron cooling. In non-polar and polar

substances, the required predominance of optical scattering over acoustical scattering is expressed by the ratios μ_{on}/μ_{ac} and μ_{op}/μ_{ac} in Eqns. (3.24) and (3.30), respectively. In Sections (3.b.1) and (3.b.2), we have obtained the expressions for the sound absorption coefficient α and acoustoelectric current J_{ac} in terms of the energy flux W.

Acknowledgements

The following work is partly based on the paper:

"ACOUSTOELECTRIC EFFECT IN POLAR SEMICONDUCTORS", Paranjape, V.V. and Joshi, S.B., *Physical Review*, Vol. 174, No. 3, pp. 919-920, October 15, 1968.

The calculations for the non-polar materials, sound absorption coefficient and acoustoelectric current have been added to the original contents of the paper.

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TABLE OF CONTENTS

TABLE OF CONTENTS (*continued*)

INTRODUCTION

The energy-momentum distribution of conduction electrons in a non-degenerate semiconductor is given by the Maxwell-Boltzmann distribution function. In thermal equilibrium

$$f(\underset{\sim}{p}) = 4\pi n_{el} \left(\frac{\hbar^2}{2\pi m \kappa T_0} \right)^{3/2} \exp(-\hbar^2 p^2 / 2m\kappa T_0) \qquad (1)$$

where $f(\underset{\sim}{p})d\underset{\sim}{p}$ represents the number of electrons with the wave vector between $\underset{\sim}{p}$ and $\underset{\sim}{p}+d\underset{\sim}{p}$, T_0 is the lattice temperature, n_{el} is the conduction electron density and m is the isotropic effective mass of an electron.

In the presence of an external electric field, the electron distribution deviates from its thermal equilibrium value. In the calculations of the transport properties of the material, the field induced distribution function is conveniently developed along the field direction as

$$f(\underset{\sim}{p}) = \sum_{n=0}^{\infty} f_n(p) P_n(\cos\theta) \qquad (2)$$

where θ is the angle between $\underset{\sim}{p}$ and the applied field $\underset{\sim}{F}$ and $P_n(\cos\theta)$ is the Legendre polynomial of degree n. In a weak field, the expansion is truncated after the second term, so that

$$f(\underset{\sim}{p}) = f_0^e(p) + f_1(p)\cos\theta \qquad (3)$$

where $f_0^e(\underset{\sim}{p})$ is the unperturbed distribution given by Eqn. (1) and $f_1(\underset{\sim}{p})$

is the anisotropic part of the function $f(\underset{\sim}{p})$. This approximation is the usual starting point in the discussion of linear transport theories of solids (e.g. Ohm's Law) and is valid in sufficiently weak electric fields.

In strong fields, however, non-linear conduction phenomena may become significant and the distribution function $f(\underset{\sim}{p})$ cannot be adequately described by Eqn. (3). It is possible to deal with the high field case by taking into account more terms from the right-hand side of Eqn. (2). Their computation, however, is highly involved, which limits the usefulness of this approach.

To avoid the inclusion of higher order terms and the difficult calculations associated with it, Fröhlich (1947) introduced the concept of electron temperature. He pointed out that the distribution function in high electric fields can be conveniently expressed by using this concept. The theory of electron temperature is based on the assumption that the electrons and the lattice can be treated as two separate systems which are connected by electron-lattice interaction. In thermal equilibrium, (i.e. in the absence of an external field), there is no net transfer of energy from one system to another. The electric field gives energy and momentum to the electrons, but it has no direct effect on the lattice and, as a result, the equilibrium between the two systems is disturbed. A stationary state will, however, be reached when the rates at which electrons receive momentum and energy from the field equal the rates at which they transfer momentum and energy to the lattice.

It will be shown later (APPENDIX III) that in the stationary state the average energy of the electrons is larger than its zero-field

value (i.e. $3/2\ \kappa T_0$). According to Fröhlich, if the electron system reaches a quasi-equilibrium through interelectronic collisions, then the average energy $\overline{E}$ may be equated to $3/2\ \kappa T$ where T is the temperature of the electron gas (the interelectronic collisions randomize the energy and momentum over the electron gas). Fröhlich and Paranjape (1956) proposed that the distribution function $f(\underset{\sim}{p})$, in the presence of an electric field, under suitable conditions, can be represented by a Maxwellian function displaced in momentum space and characterized by the electron temperature (different from the lattice temperature).

$$f(\underset{\sim}{p}) = A \exp \{ - \frac{\hbar^2 (\underset{\sim}{p}-\underset{\sim}{p}_0)^2}{2m\kappa T} \} \qquad (4)$$

$$A = 4\pi n_{el} \left(\frac{\hbar^2}{2\pi m \kappa T} \right)^{3/2} \qquad (5)$$

where $\underset{\sim}{p}_0$ is the average wave vector of an electron in the direction of the applied field. The values of $\underset{\sim}{p}_0$ and T depend on the strength of the applied field and the types of lattice scattering relevant to the crystal. It is important to note that the function given in Eqn. (4) is valid for high fields, as well as for zero-field. Hence, $F \to 0$, $\underset{\sim}{p}_0 \to 0$ and $T \to T_0$.

Until recently, it has been assumed that an increase in the average energy due to electric fields always leads to an increase in the electron temperature, $T > T_0$, which is known as the "hot electron" phenomenon. Contrary to this, it has been theoretically demonstrated by Paranjape and Ambrose (1964) that the phenomenon may not always

occur; in fact, under suitable conditions, the electron temperature may, on application of the field, decrease below the lattice temperature, T_0. This may be described as the "cooled electron" phenomenon and was predicted when the electrons were predominantly scattered by the phonons, whose energy is greater than the average electron energy. The theoretical possibility of electron cooling was further investigated by de Alba and Warman (1965), Blötekjaer (1967) and Baynham *et al.* (1968). So far, there is no experimental verification of this phenomenon - the difficulties are immense. The present work proposes an alternative method of producing cooled electrons in semiconductors. We show in this work that the cooled electrons can be produced by passing a sound wave through a suitable semiconductor.

An external acoustic wave of energy flux W transfers energy and momentum to the conduction electrons as it propagates through the crystal; and, as a result, the electrons acquire drift velocity in the direction of the sound wave. Under suitable conditions, given by Fröhlich and Paranjape (1956), the steady state distribution can be represented by a "displaced Maxwellian" function given in Eqn. (4). The values of $\underset{\sim}{p}_0$ and T, in this case, will depend on the energy flux W. Using a procedure similar to the one used by Paranjape and Ambrose (1964) and Paranjape and de Alba (1965) for the case of an electric field, we obtain conditions under which electron cooling may occur. The conditions show that, if the incident energy flux W is greater than a critical value W_0 (which depends on the types of lattice scattering effective in the semiconductor), electron cooling is possible;

and, if it is less than W_0, electron heating would take place. We discuss the electron cooling conditions for various types of lattice scattering. The calculations for the polar and non-polar semiconductors show that electron cooling is possible only if optical polar or optical non-polar scattering dominates over acoustical scattering. Finally, the acoustoelectric current and the sound absorption coefficient are calculated as a function of W.

CHAPTER I

§1.a.1. Model

We consider a high frequency sound wave incident at a crystal of an n-type (polar or non-polar) semiconductor. The crystal is kept in contact with a heat bath and its two ends along the direction of the sound wave are connected by a suitable conductor.

We wish to investigate the effect of the sound wave on the energy distribution of the conduction electrons under the following assumptions:

(1) The sound wave while passing through the crystal interacts only with the electrons. The assumption is valid at sufficiently low temperatures, because for temperatures higher than 20°K the sound absorption due to other sources (e.g. lattice vibrations) dominates the absorption due to electrons.

(2) The temperature of the lattice T_0 is maintained constant by the heat bath and the lattice (phonon) energy distribution is given by Planck's distribution

$$n_k = \left[\exp(\hbar\omega/\kappa T_0)-1\right]^{-1}$$
$$n_k + 1 = n_k \exp(\hbar\omega/\kappa T_0) \qquad (1.1)$$

where n_k is the excitation number of phonons with wave vector $\underset{\sim}{k}$.

(3) Our calculations assume a free electron model. Thus an electron with a wave vector $\underset{\sim}{p}$ will have energy $E(\underset{\sim}{p})$, given by

$$E(\underset{\sim}{p}) = \hbar^2 p^2/2m \qquad (1.2)$$

where m is the effective mass of an electron. The assumption is not usually valid due to non-parabolicity in the band structure at high temperatures. However, for the range of temperatures in which we are interested (less than 20°K), the assumption is reasonable.

Throughout this work, the energy momentum distribution of the conduction electrons, in the presence of a sound wave, is represented by a displaced Maxwellian function. Fröhlich and Paranjape (1956) have discussed the conditions under which this representation is valid (see APPENDIX I). They pointed out that for electron densities higher than $10^{14}/cm^2$ the electron-electron interaction is stronger than the electron-lattice interaction. Under this condition, it is reasonable to assume the electron distribution given by the function in Eqn. (4).

§1.a.2. Mathematical Outline of the Calculations

A steady state Boltzmann equation, in the presence of a sound wave, can be written as

$$\left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{s\text{-}el} + \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}el} + \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}la} = 0 \qquad (1.3)$$

where the first, second and third terms represent the rates of change

of $f(\underset{\sim}{p})$ due to electron-sound wave interaction, electron-electron scattering and electron-lattice scattering, respectively. The solution of the above equation would represent a steady state energy distribution of the electrons under the influence of an external sound wave. It is, however, well known that the electron-electron collision term in Eqn. (1.3) is difficult to evaluate and its exact solution cannot be easily obtained. We know that the interelectronic collisions conserve total momentum and total energy of the electron system. This fact can be utilized to avoid the evaluation of $df(\underset{\sim}{p})/dt\big|_{el-el}$ term by transforming Boltzmann's equation into momentum and energy conservation conditions.

The momentum conservation follows since

$$\sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{s-el} + \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-el} + \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-la} \right\} = 0 \qquad (1.4)$$

and the energy conservation because

$$\sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{s-el} + \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-el} + \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-la} \right\} = 0 \qquad (1.5)$$

However, since

$$\sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-el} \right\} = 0 \qquad (1.6)$$

and

$$\sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \frac{\partial f(\underset{\sim}{p})}{\partial t}\bigg|_{el-el} \right\} = 0 \qquad (1.7)$$

it follows that

$$\sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{s\text{-}el} \right\} = - \sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}la} \right\} \qquad (1.8)$$

and

$$\sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{s\text{-}el} \right\} = - \sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}la} \right\} \qquad (1.9)$$

Using the following simplifying notations

$$\left.\begin{aligned}
\sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{s\text{-}el} \right\} &= \left.\frac{d\mathbf{P}}{dt}\right|_{s\text{-}el} \\
\sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{s\text{-}el} \right\} &= \left.\frac{d\mathbf{E}}{dt}\right|_{s\text{-}el} \\
-\sum_{\underset{\sim}{p}} \hbar \underset{\sim}{p} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}la} \right\} &= \left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la}
\end{aligned}\right\} \qquad (1.10)$$

and

$$-\sum_{\underset{\sim}{p}} \frac{\hbar^2 p^2}{2m} \left\{ \left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el\text{-}la} \right\} = \left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la}$$

equations (1.8) and (1.9) become

$$\left.\frac{d\mathbf{P}}{dt}\right|_{s\text{-}el} = \left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la} \qquad (1.11)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{s\text{-}el} = \left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la} \qquad (1.12)$$

These two equations simply show that in a stationary state the rates at which momentum and energy are absorbed from the sound wave equal the rates at which these quantities are transferred to the lattice. We shall use these conditions to determine the steady state distribution of the electrons.

Boltzmann's equation, given in Eqn. (1.3), can be solved in two extreme cases. In the first case, electron-electron interaction is entirely neglected and the distribution function is developed in the first few terms. This corresponds to the case of low electron density and weak electric field. In the following work, we shall follow the other case and assume the dominance of the $df(p)/dt\big|_{el\text{-}el}$ term. The assumption allows us to represent the solution of Eqn. (1.3) by a displaced Maxwellian function containing parameters $\underset{\sim}{p}_0$ and T. Using this distribution, we evaluate the four rates given in Eqns. (1.11) and (1.12). We shall then solve the momentum and energy conservation conditions and obtain the parameters. The electron cooling conditions will then follow from the expression for the electron temperature.

CHAPTER II

§2.a.1. Calculations Concerning Momentum and Energy Transfer From the Electrons to the Lattice: General Theory

To evaluate the momentum and energy transferred from the electrons to the lattice, we must first calculate $df(\underset{\sim}{p})/dt\Big|_{el\text{-}la}$. This can be done by considering the scattering process which involves the absorption or emission of a quantum of lattice vibrations (phonon) by an electron. Let $\underset{\sim}{q}$ be the wave vector of the phonon and $\hbar\omega(\underset{\sim}{q})$ be the energy. Momentum conservation requires that electronic transitions from a state $\underset{\sim}{p}$ (momentum $\hbar\underset{\sim}{p}$) are only possible to the state $\underset{\sim}{p}+\underset{\sim}{q}$ in the case of absorption and to the state $\underset{\sim}{p}-\underset{\sim}{q}$ in the case of emission of such a quantum. Clearly we can write

$$\left.\frac{\partial f(\underset{\sim}{p})}{dt}\right|_{el\text{-}la} = \sum_{\underset{\sim}{q}} \{f(\underset{\sim}{p})\rho_a(\underset{\sim}{p}\to\underset{\sim}{p}+\underset{\sim}{q}) - f(\underset{\sim}{p}+\underset{\sim}{q})\rho_e(\underset{\sim}{p}+\underset{\sim}{q}\to\underset{\sim}{p})$$

$$- f(\underset{\sim}{p})\rho_e(\underset{\sim}{p}\to\underset{\sim}{p}-\underset{\sim}{q}) + f(\underset{\sim}{p}-\underset{\sim}{q})\rho_a(\underset{\sim}{p}-\underset{\sim}{q}\to\underset{\sim}{p}) \qquad (2.1)$$

where $\rho_a(\underset{\sim}{p}\to\underset{\sim}{p}+\underset{\sim}{q})$ and $\rho_e(\underset{\sim}{p}\to\underset{\sim}{p}-\underset{\sim}{q})$ are the probabilities per unit time, for the transition from state $\underset{\sim}{p}$ to $\underset{\sim}{p}+\underset{\sim}{q}$ and $\underset{\sim}{p}$ to $\underset{\sim}{p}-\underset{\sim}{q}$, respectively. These probabilities can be calculated using standard quantum mechanical procedures leading to

$$\begin{aligned} \rho_a &= \frac{2\pi}{\hbar} B(q) n_q \delta(y_a) \\ \rho_e &= \frac{2\pi}{\hbar} B(q) n_q \delta(y_e) \\ y_a &= E_i - E_f + \hbar\omega(q) \\ y_e &= E_i - E_f - \hbar\omega(q) \end{aligned} \tag{2.2}$$

where $B(q)$ is the square of the interaction element, δ is Dirac's delta function, E_i and E_f are the energies of the initial and the final electronic states and n_q is the phonon excitation number with a wave vector $\underset{\sim}{q}$ given by Planck's distribution. The distribution functions occurring in Eqn. (2.1) are displaced Maxwellian and, for the sake of convenience, we develop them in powers of p_0, i.e.

$$f(p) \simeq f_0(p)\{1 + \frac{\hbar^2 p p_0}{mkT}\cos\theta - \frac{\hbar^2 {p_0}^2}{mkT}\left[1 + \frac{\hbar^2 p^2}{mkT}\cos^2\theta\right]\} \tag{2.4}$$

and

$$f(\underset{\sim}{p}\pm\underset{\sim}{q}) \simeq f_0(\underset{\sim}{p}\pm\underset{\sim}{q})\{1 + \frac{\hbar^2 p_0}{mkT}\left[p\cos\theta \pm q\cos(\underset{\sim}{p}_0\hat{\underset{\sim}{q}})\right]$$

$$- \frac{\hbar^2 {p_0}^2}{mkT}\left[1 + \frac{\hbar^2}{mkT}(p\cos\theta \pm q\cos(\underset{\sim}{p}_0\hat{\underset{\sim}{q}}))^2\right] \tag{2.5}$$

where

$$f_0(p) = A \exp\{-\frac{\hbar^2 p^2}{2mkT}\} \tag{2.6}$$

$$f_0(p\pm q) = A \exp\{-\frac{\hbar^2 (\underset{\sim}{p}\pm\underset{\sim}{q})^2}{2mkT}\} \tag{2.7}$$

The angle between $\underset{\sim}{p}_0$ and $\underset{\sim}{q}$ is denoted by $(\hat{\underset{\sim}{p}_0\underset{\sim}{q}})$ and that between $\underset{\sim}{p}_0$ and $\underset{\sim}{p}$ is denoted by θ. It is important to note that even though the succeeding terms in these expansions decrease rapidly, retention of second order terms in p_0 is necessary. In fact, it will be shown later that the prediction of electron cooling depends entirely on the inclusion of these second order terms in the calculations.

Changing the summation in Eqn. (2.1) to integration over polar coordinates

$$\sum_{q} \longrightarrow \frac{V}{8\pi^3} \int q^2 dq \sin\theta' d\theta' d\phi' \qquad (2.8)$$

and then using the integration over θ' to eliminate the δ functions (which ensure the energy conservation) we get the condition

$$\frac{\hbar^2 |\underset{\sim}{p} \pm \underset{\sim}{q}|^2}{2m} = \frac{\hbar^2 p^2}{2m} \pm \hbar\omega(q) \qquad (2.9)$$

which implies

$$\cos(\hat{\underset{\sim}{p}\underset{\sim}{q}})_{\pm} = \mp \frac{q}{2p} + \frac{m\hbar\omega(q)}{\hbar^2 pq} \qquad (2.10)$$

where the suffix + or − corresponds to the case of absorption or emission of a phonon, respectively.

We also note that

$$\int_0^{\pi} \cos(\hat{\underset{\sim}{p}}\underset{\sim}{q})_{\pm} d\theta' = \int \{\cos\theta\cos(\hat{\underset{\sim}{p}}\underset{\sim}{q}) + \sin\theta\sin(\hat{\underset{\sim}{p}}\underset{\sim}{q})\cos\theta'\}d\theta'$$

$$= 2\pi \{ \mp \frac{q}{2p} + \frac{m\hbar\omega(q)}{\hbar^2 pq} \} \cos\theta \qquad (2.11)$$

Substitution of Eqns. (2.2) to (2.11) in Eqn. (2.1) gives

$$\left.\frac{\partial f(\underset{\sim}{p})}{\partial t}\right|_{el-la} = g_0(p) + p_0 g_1(p) + p_0{}^2 g_2(p) \qquad (2.12)$$

where

$$g_0(p) = \frac{V}{4\pi\hbar^3}\frac{m}{p} f_0(p) \int qdqB(q)n_q \times \{[1 - \exp\left(\frac{\hbar\omega(q)}{\kappa T_0} - \frac{\hbar\omega(q)}{\kappa T}\right)]_+$$

$$+ [\exp\left(\frac{\hbar\omega(q)}{\kappa T_0}\right) - \exp\left(\frac{\hbar\omega(q)}{\kappa T_0}\right)]_-\} \qquad (2.13)$$

$$g_1(p) = \frac{V}{4\pi\hbar} f_0(p) \frac{\cos\theta}{\kappa T} \int qdqB(q)n_q \times \{[1 - \exp\left(\frac{\hbar\omega(q)}{\kappa T_0} - \frac{\hbar\omega(q)}{\kappa T}\right)$$

$$(1 + \frac{q}{p}\cos(\hat{\underset{\sim}{p}}\underset{\sim}{q}))]_+ + [(\exp\left(\frac{\hbar\omega(q)}{\kappa T_0}\right) - \exp\left(\frac{\hbar\omega(q)}{\kappa T}\right))(1 - \frac{q}{p}\cos(\hat{\underset{\sim}{p}}\underset{\sim}{q}))]_- \qquad (2.14)$$

and

$$g_2(p) = \frac{V}{4\pi\hbar p}\frac{1}{\kappa T}\int qB(q)n_q dq$$

$$\times \{ \left(\frac{\hbar^2 p^2\cos^2\theta}{m\kappa T} - 1\right)[\{1 - \exp\left(\frac{\hbar\omega(q)}{\kappa T_0} - \frac{\hbar\omega(q)}{\kappa T}\right)\}_+$$

$$+ \{ \exp\left(\frac{\hbar\omega(q)}{\kappa T_0}\right) - \exp\left(\frac{\hbar\omega(q)}{\kappa T_0}\right)\}_-]$$

$$- [\exp\left(\frac{\hbar\omega(q)}{\kappa T_0} - \frac{\hbar\omega(q)}{\kappa T}\right) \{ \frac{\hbar^2 q^2\cos^2\theta\cos^2(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{m\kappa T}$$

$$+ \frac{\hbar^2 q^2\sin^2\theta\sin^2(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{2m\kappa T} + \frac{2\hbar^2 pq\cos^2\theta\cos(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{m\kappa T} \}_+]$$

$$- [\exp\left(\frac{\hbar\omega(q)}{\kappa T}\right) \{ \frac{\hbar^2 q^2\cos^2\theta\cos^2(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{m\kappa T}$$

$$+ \frac{\hbar^2 q^2\sin^2\theta\sin^2(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{2m\kappa T} - \frac{2\hbar^2 pq\cos^2\theta\cos(\hat{\underset{\sim}{p}\underset{\sim}{q}})}{m\kappa T} \}_-]\} \qquad (2.15)$$

The integration limits are partially determined by the fact that $|\cos(\hat{\underset{\sim}{p}\underset{\sim}{q}})_\pm|$, as given by Eqn. (2.10), can never exceed unity and this imposes different upper and lower limits on q for two cases denoted by + and -. It now follows from Eqn. (1.5) that

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la} = \int \underset{\sim}{p}\left(\left.\frac{\partial f(p)}{dt}\right|_{el\text{-}la}\right) d^3p = \frac{\hbar^2 p_0}{3}\int \underset{\sim}{p} g_1(p) d^3p \qquad (2.16)$$

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la} = \int \frac{\hbar^2 p^2}{2m} \left(\left.\frac{\partial f(p)}{\partial t}\right|_{el\text{-}la} \right) d^3p$$

$$= \frac{\hbar^2}{2m} \int p^2 \{ g_0(p) + p_0{}^2 g_2(p) \} d^3p \qquad (2.17)$$

To carry out the final integration over q in g_0, g_1 and g_2, it is first necessary to define $\omega(q)$ and $B(q)$. The dependence of these quantities on q is determined by the type of lattice modes involved in the process of electron scattering. We, therefore, separate the cases of acoustical, optical polar and optical non-polar scattering and evaluate for each the integrals over q and p in Eqns. (2.16) and (2.17).

§2.a.2 Acoustical Scattering

For acoustical scattering, we have

$$\hbar\omega(q) = \hbar s q$$

and (2.18)

$$B(q) = \frac{2C^2 q}{9MsN_{io}}$$

where C is a constant of energy of the order of 1 ev and N_{io} is the number of lattice points in the crystal. If the large values of p are

unimportant, then the limits of integration on q are determined from Eqn. (2.10) and are

$$\begin{aligned} &\text{Lower Limit} \quad q(\pm) = 0 \\ &\text{Upper Limit} \quad q(\pm) = 2p \pm \frac{2ms}{\hbar} \end{aligned} \tag{2.19}$$

Further, we consider the case in which the temperature is sufficiently high such that

$$\kappa T_0 >> ms^2$$

and (2.20)

$$n_q \simeq \frac{\kappa T_0}{\hbar s q}$$

Combining Eqns. (2.12) to (2.20), we are able to write

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la}^{ac} = \frac{32 N_{el} e \hbar p_0}{9\pi \mu_{ac} m} \left(\frac{T}{T_0}\right)^{1/2} \tag{2.21}$$

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la}^{ac} = \frac{32 N_{el} s^2 e}{3\pi \mu_{ac}} \left(\frac{T}{T_0}\right)^2 \left\{ \frac{\Delta T}{T_0} \left(1 + \frac{\hbar^2 p_0^2}{2m\kappa T}\right) + \frac{\hbar^2 p_0^2}{3m\kappa T} \right\} \tag{2.22}$$

where

$$\Delta T = (T - T_0) \tag{2.23}$$

and

$$\mu_{ac} = \frac{3\sqrt{\pi}s^2\hbar^4 M N_{io} e}{(2m\kappa T_0)^{1/2}\kappa T_0 m^2 C^2 V} \tag{2.24}$$

§2.a.3. Optical Non-Polar Scattering

We assume $\omega(q)$ independent of q, i.e.

$$\omega(q) = \omega \tag{2.25}$$

and the limits of integration over q thus become

$$\begin{aligned} \text{Lower Limit} \quad q(\pm) &= \pm \left\{ \frac{\hbar^2 p^2 \pm 2m\hbar\omega}{\hbar} \right\}^{1/2} \mp p \\ \text{Upper Limit} \quad q(\pm) &= p + \left\{ \frac{\hbar^2 p^2 \pm 2m\hbar\omega}{\hbar} \right\}^{1/2} \end{aligned} \tag{2.26}$$

Following Seitz (1948), we write

$$B(q) = \frac{\hbar^2 D^2 \boldsymbol{\kappa}^2}{2N_{io} M \hbar\omega} \tag{2.27}$$

where D is a constant of energy and $\boldsymbol{\kappa}$ is the reciprocal lattice distance. Now, combining Eqns. (2.25) to (2.27) and (2.12) to (2.17), we get

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la}^{on} = \frac{16}{9\pi}\left(\frac{n_q}{2n_q+1}\right)\frac{N_{el}e\hbar p_0}{m\mu_{on}}\left(\frac{T}{T_0}\right)^{1/2}$$

$$\times\left[\left\{\exp\left(\frac{\hbar\omega}{\kappa T_0}-\frac{\hbar\omega}{\kappa T}\right)+1\right\}I_1+\frac{\hbar\omega}{\kappa T}\exp\left(\frac{\hbar\omega}{\kappa T_0}-\frac{\hbar\omega}{\kappa T}\right)I_0\right] \quad (2.28)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la}^{on} = \frac{8}{3\pi}\left(\frac{n_q}{1+2n_q}\right)\frac{N_{el}e\hbar p_0}{m\mu_{on}}\left(\frac{T}{T_0}\right)^{1/2}$$

$$\times\left[\left\{\exp\left(\frac{\hbar\omega}{\kappa T_0}-\frac{\hbar\omega}{\kappa T}\right)-1\right\}I_0+\frac{\hbar^2 p_0^{\,2}}{3m\kappa T}\frac{\hbar\omega}{\kappa T}\right.$$

$$\times\left\{\exp\left(\frac{\hbar\omega}{\kappa T_0}-\frac{\hbar\omega}{\kappa T}\right)\right\}I_0+\frac{\hbar^2 p_0^{\,2}}{2m\kappa T}\left\{\exp\left(\frac{\hbar\omega}{\kappa T_0}-\frac{\hbar\omega}{\kappa T}-1\right)\right\}$$

$$\left.\left(\frac{2}{3}I_1-I_0\right)\right. \quad (2.29)$$

where

$$\mu_{on} = \frac{8\sqrt{\pi}e\hbar^2 MN_{io}\hbar\omega}{3(2m\kappa T)^{1/2}m^2D^2\kappa^2(2n_q+1)V} \quad (2.30)$$

$$I_0 = \int_0^\infty \exp(-y)y^{1/2}\left(y+\frac{\hbar\omega}{\kappa T}\right)^{1/2}dy$$

and

$$I_1 = \int_0^\infty \exp(-y) y^{1/2} \left(y + \frac{\hbar\omega}{\kappa T}\right)^{1/2} dy \qquad (2.31)$$

Paranjape and de Alba (1965) have evaluated the integrals for the case $\hbar\omega/\kappa T_0 > 1$ and they are

$$I_0 \simeq \left(\frac{\pi\hbar\omega}{4\kappa T_0}\right)^{1/2}$$

$$I_1 \simeq \left(\frac{9\pi\hbar\omega}{4\kappa T_0}\right)^{1/2} \qquad (2.32)$$

§2.a.4. Optical Polar Scattering

Following Fröhlich and Paranjape (1956), we define $\omega(q)$ independent of q and

$$B(q) = \frac{2\pi e^2 \hbar^3 \omega}{V q^2} \left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right) \qquad (2.33)$$

where ε_∞ and ε_0 are, respectively, the high frequency and static dielectric constants of the crystal. Now Eqns. (2.26), (2.33) and (2.13) to (2.18) lead to

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la}^{op} = \frac{N_{el} e}{2m\mu_{op}} \left(\frac{\hbar p_0}{J_0}\right) \times \left[\frac{\hbar\omega}{2\kappa T}\left\{\exp\left(\frac{\hbar\omega}{\kappa T} - \frac{\hbar\omega}{\kappa T}\right) - 1\right\} J_0 \right.$$

$$\left. + \left\{\exp\left(\frac{\hbar\omega}{\kappa T_0} - \frac{\hbar\omega}{\kappa T}\right) + 1\right\} I_0\right] \qquad (2.34)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el}^{la} = \frac{3(\hbar\omega)N_e l^e}{4m\mu_{op}\kappa T_0}\frac{1}{J_0}$$

$$\times \left[\left\{ \exp\left(\frac{\hbar\omega}{\kappa T_0} - \frac{\hbar\omega}{\kappa T} \right) - 1 \right\} J_0 + \frac{\hbar^2 p_0^2}{2m\kappa T}\frac{\hbar\omega}{\kappa T} J_0 \right.$$

$$\left. + \left\{ \exp\left(\frac{\hbar\omega}{\kappa T_0} - \frac{\hbar\omega}{\kappa T} \right) - 1 \right\} \left(J_1 + \frac{\hbar\omega}{\kappa T} J_0 - I_0\right) \right] \qquad (2.35)$$

where

$$J_0 = \int_0^\infty \exp(-x)\log\left\{ \frac{(x + \hbar\omega/\kappa T)^{1/2} + x^{1/2})}{(x + \hbar\omega/\kappa T)^{1/2} - x^{1/2})} \right\} dx \qquad (2.36)$$

$$J_1 = \int_0^\infty \exp(-x)\log\left\{ \frac{(x + \hbar\omega/\kappa T)^{1/2} + x^{1/2})}{(x + \hbar\omega/\kappa T)^{1/2} - x^{1/2})} \right\} x dx \qquad (2.37)$$

and

$$\mu_{op} = \frac{3\sqrt{\pi}}{8em\omega}\left(\frac{2\hbar^3\omega}{m}\right)^{1/2}\left(\frac{\kappa T_0}{\hbar\omega}\right)^{1/2}\frac{1}{J_0}$$

$$\times \left\{ \exp\left(\frac{\hbar\omega}{\kappa T_0} \right) - 1 \right\} \left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)^{-1} \qquad (2.38)$$

So far in this chapter, we have calculated the rates of momentum and energy transfer from the electrons to the lattice for three different scattering processes. These rates were calculated by using the "displaced Maxwellian" distribution function and, there-

fore, they contain the parameters p_0 and T. The difference between the rates calculated by Fröhlich and Paranjape (1956) and those calculated above (originally calculated by Paranjape and de Alba (1965)) is that in the former calculations the functions in Eqns. (2.4) and (2.5) were developed only to the first order in p_0; while in the latter, the terms proportional to p_0^2 were included. Paranjape and Ambrose (1964) have shown that the presence of these additional terms leads to the possibility of electron cooling when the semiconductor is under the influence of an electric field (see APPENDIX II).

§2.b.1. Momentum and Energy Transfer From the Sound Wave to the Electrons

The problem of interaction between the conduction electrons and the sound wave has been treated by many workers using macroscopic models. Their calculations are based on the assumption $\omega\tau<<1$ where $2\pi\omega$ is the frequency of the applied sound wave and τ is the mean collision time for the electrons. However, when ω is large, so that the wave length of the sound wave is comparable to the mean free path of the electrons, these macroscopic considerations break down and a more sophisticated quantum mechanical treatment becomes necessary.

Van den Beukel (1956) and Mikoshiba (1959) have treated the problem quantum mechanically with the assumption

$$\omega\tau>1 \ . \qquad (2.39)$$

They, however, neglected the electron heating produced by the sound wave.

Paranjape (1964) has considered the effect of electron heating in the calculations by using a Maxwellian (undisplaced) distribution function. Epshtein (1966) has taken into account the asymmetric part of the distribution function, but his calculations are for the case of weak electron-electron interactions.

We consider this problem by using a displaced Maxwellian distribution function (which requires the dominance of electron-electron interaction). Under the inequality condition, Eqn. (2.39), a sound wave in a crystal can be considered as a phonon flux. (At 20°K, $\tau \simeq 10^{-10}$ sec., then to satisfy the inequality, Eqn. (2.39), the value of ω required is of the order of 10^9 cy/sec.) That is, the intensity of a high frequency sound wave, at a certain point in the crystal, can be expressed by a phonon excitation number. This phonon number, in general, will be a function of position in the crystal because the sound wave intensity is reduced as it progresses away from the incident face. For the sake of simplicity, however, we shall confine our calculations to a small region in the crystal in which the variation in this number is negligible.

We assume that the applied acoustical wave is monocromatic and hence can be represented as

$$N_k = N_q \delta(\underset{\sim}{k}-\underset{\sim}{q}) d\underset{\sim}{k} \qquad (2.40)$$

where N_q is the total number of phonons excited by the sound wave. At any instant, in the small volume under consideration, the total phonon excitation number N_k^s of wave vector k is the sum of n_k internal or thermally excited phonons and N_k external or sound wave-excited phonons. These phonons are absorbed or emitted during their inelastic collisions with

the lattice. When only internal phonons are present in the thermal equilibrium, there is no overall transfer of momentum and energy between electrons and lattice. However, the addition of acoustical phonons to the lattice results in a net absorption of these phonons (and hence momentum and energy from the sound wave) by the electrons. We have

$$N_k^s = N_k + n_k \tag{2.41}$$

and

$$\frac{dn_k}{dt} = 0$$

Hence

$$\frac{dN_k^s}{dt} = \frac{dN_k}{dt} \tag{2.42}$$

The rates of momentum and energy gained by the electrons from the sound wave may now be defined as

$$\left.\frac{d\mathbf{P}}{dt}\right|_{s-el} = \sum_k (\hbar \underset{\sim}{k}) \frac{dN_k}{dt}$$

$$= \mu \int \hbar \underset{\sim}{k} \frac{dN_q}{dt} \delta (\underset{\sim}{k}-\underset{\sim}{q}) d^3k$$

$$= \mu (\hbar q) \frac{dN_q}{dt} \tag{2.43}$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{s\text{-}el} = \sum_k \hbar s k \frac{dN_k}{dt}$$

$$= \mu \int (\hbar s k) \frac{dN_q}{dt} \delta (\underset{\sim}{k}-\underset{\sim}{q}) \, d^3k$$

$$= \mu (\hbar s q) \frac{dN_q}{dt} \qquad (2.44)$$

where μ is the density of the lattice vibrational modes with wave vector q in the volume element under consideration. To evaluate dN_q/dt in the Eqns. (2.43) and (2.44), we follow Paranjape (1962) and Conwell (1964) and write

$$\frac{dN_q}{dt} = \int \frac{B(q)}{\hbar} \{(N_q+1)f(p+q) - N_q f(p)\}$$

$$\times \delta \left(\frac{\hbar^2 (\underset{\sim}{p}+\underset{\sim}{q})^2}{2m} - \frac{\hbar^2 p^2}{2m} - \hbar s q \right) d^3p \qquad (2.45)$$

where the interaction is through acoustical scattering. Putting the values of the distribution functions in Eqn. (2.45) from Eqn. (4) and assuming that $N_q >> 1$, we get

$$\frac{dN_q}{dt} = \{\Phi(p_0,q) - \Psi(p_0,q)\} \qquad (2.46)$$

where

$$\Phi(p_0,q) = A' \int \exp\left\{\frac{-\hbar^2}{2m\kappa T}(\underset{\sim}{p}-\underset{\sim}{p}_0+\underset{\sim}{q})^2\right\}$$

$$\times \delta\left(\frac{\hbar^2(\underset{\sim}{p}+\underset{\sim}{q})^2}{2m} - \frac{\hbar^2 p^2}{2m} - \hbar sq\right)d^3p \qquad (2.47)$$

$$\Psi(p_0,q) = A' \int \exp\left\{\frac{-\hbar^2}{2m\kappa T}(\underset{\sim}{p}-\underset{\sim}{p}_0)\right\}$$

$$\times \delta\left(\frac{\hbar^2(\underset{\sim}{p}+\underset{\sim}{q})^2}{2m} - \frac{\hbar^2 p^2}{2m} - \hbar sq\right)d^3p \qquad (2.48)$$

and

$$A' = \frac{C^2 q\hbar^2 N_{el}}{MsN_{io}(2\pi m\kappa T)^{1/2}} N_q \qquad (2.49)$$

Since the direction of the incident phonon flux and the drift velocity is the same

$$\cos(\underset{\sim}{p}_0\hat{\underset{\sim}{q}}) = 1 \qquad (2.50)$$

and recalling that

$$\cos(\underset{\sim}{p}\hat{\underset{\sim}{p}}_0) = \cos\theta$$

we have (2.51)

$$\cos(\underset{\sim}{p}\hat{\underset{\sim}{q}}) = \cos\theta$$

Now integrating over θ in Eqns. (2.47) and (2.48) to eliminate the delta functions, we get the condition

$$\cos\theta = \left[\frac{2m\hbar sq - \hbar^2 q^2}{2\hbar^2 pq} \right] \tag{2.52}$$

Since $\cos\theta \leq 1$, it follows that

$$p \geq \frac{ms}{\hbar} - \frac{q}{2} \simeq -\frac{q}{2} \tag{2.53}$$

Using this limit and carrying out the integration over p in Eqns. (2.47) and (2.48), we get

$$\Phi(p_0, q) = \frac{2\pi^2 m^2 \kappa T_0}{q\hbar^3} A' \exp\left\{ -\left(\hbar p_0 - \frac{\hbar q}{2} \right)^2 + \frac{2m\hbar s(q - p_0)}{2m\kappa T} \right\} \tag{2.54}$$

and

$$\Psi(p_0, q) = \frac{2\pi^2 m^2 \kappa T_0}{q\hbar^3} A' \exp\left\{ -\frac{\left(\hbar p_0 - \frac{\hbar q}{2}\right)^2 + 2ms\hbar p_0}{2m\kappa T} \right\} \tag{2.55}$$

Developing the expressions in Eqns. (2.54) and (2.55) in the powers of p_0 and substituting in Eqn. (2.46), we get

$$\frac{dN_q}{dt} = -\pi \left(\frac{mN_{el}}{MN_{io}}\right) \frac{C^2}{s\hbar(2\pi m\kappa T_0)^{1/2}} N_q \left\{ 1 - \frac{\hbar sq}{2\kappa T_0} - \frac{\hbar p_0}{ms} \right.$$

$$\left. \left(1 - \frac{ms^2}{\kappa T_0} - \frac{\hbar sq}{2\kappa T_0} \right) - \frac{3}{2} \frac{\hbar^2 {p_0}^2}{m\kappa T_0} + \frac{\hbar^3 {p_0}^3}{2m\kappa T_0 ms} + \cdots \right\}$$

$$\simeq -\pi \left(\frac{mN_{el}}{MN_{io}}\right) \frac{C^2}{s\hbar(2\pi m\kappa T_0)^{1/2}} N_q \left[1 - \frac{\hbar p_0}{ms} \right] \qquad (2.56)$$

In these calculations, we have assumed the relative orders of magnitude for various quantities involved, which can be expressed as

$$\frac{\hbar^2 {p_0}^2}{m} \leqslant ms^2 < \hbar sq << \kappa T_0 \qquad (2.57)$$

If the first order terms in $\hbar p_0$ of the right-hand side in the equation are neglected, then Eqn. (2.56) coincides with the value of dN_q/dt obtained by Paranjape (1964). However, for arbitrarily large values of sound wave flux, the drift velocity can become comparable to sound velocity and then it is incorrect to neglect the term $\hbar p_0/ms$ in comparison with unity. Finally, substituting the value of dN_q/dt from Eqn. (2.52) in Eqns. (2.43) and (2.44), we get

$$\left.\frac{d\mathbf{P}}{dt}\right|_{s\text{-}el} = \mu(\hbar q)\, \pi \left(\frac{mN_{el}}{MN_{io}}\right) N_q$$

$$\frac{C^2}{s\hbar(2\pi m\kappa T_0)^{3/2}} \left[1 - \frac{\hbar p_0}{ms} \right] \qquad (2.58)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{s\text{-}el} = \mu(\hbar sq)\,\pi\left(\frac{mN_{el}}{MN_{io}}\right)N_q$$

$$\frac{C^2}{s\hbar(2\pi mkT_0)^{3/2}}\left[1-\frac{\hbar p_0}{ms}\right] \qquad (2.59)$$

If W is the sound wave energy flux reaching the element of volume, then it can be defined as

$$W \equiv \mu\int(\hbar sq)N_q\delta(\underset{\sim}{k}-\underset{\sim}{q})d^3k$$

$$= \mu(\hbar sq)N_q \qquad (2.60)$$

With this definition of W, the above rates can be expressed in the following way

$$\left.\frac{d\mathbf{P}}{dt}\right|_{s\text{-}el} = W\beta\left[1-\frac{\hbar p_0}{ms}\right] \qquad (2.61)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{s\text{-}el} = W\beta s\left[1-\frac{\hbar p}{ms}\right] \qquad (2.62)$$

where

$$\beta = \frac{\pi^2\hbar^4 qN_{el}e}{4kT_0\mu_{ac}(mkT_0)^2} \qquad (2.63)$$

These are the rates of momentum and energy transferred from the sound wave to the electrons.

CHAPTER III

§3.a.1. Electron Cooling Conditions: General Discussion

We have so far calculated the rates of momentum and energy transfer from the sound wave to the electrons: Eqns. (2.61) and (2.62) and from the electrons to the lattice: Sec. (2.a.2), (2.a.3) and (2.a.4). Using these rates in the momentum and energy conservation equations (1.11) and (1.12) and solving those equations, we shall obtain the values of the parameters p_0 and T of the displaced Maxwellian distribution. The electron cooling conditions would then follow from the value of T. If we develop the expressions for $d\mathbf{P}/dt\big|_{el-la}$ and $d\mathbf{E}/dt\big|_{el-la}$, given in Sec. (2.a.2) (2.a.3) and (2.a.4), in the powers of p_0 and ΔT, then these rates for three different scattering mechanisms can be expressed by two general equations:

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el-la} = \gamma_\varepsilon \rho_\varepsilon \hbar p_0 \qquad (3.1)$$

and

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el-la} = \gamma_\varepsilon \theta_\varepsilon \left[\frac{\Delta T}{T_0} + \frac{\hbar^2 {p_0}^2}{3mkT_0}\right] \qquad (3.2)$$

where γ_ε, θ_ε and ρ_ε are the constants characterizing the scattering mechanism. Note that in the right-hand side of Eqns. (3.1) and (3.2) the terms containing $\hbar^2 {p_0}^2/2mkT_0$ are neglected in comparison with unity; however, the term in ${p_0}^2$ of Eqn. (3.2) must be retained, since it is of

the same order of magnitude as $\Delta T/T_0$.

Equating these rates with the corresponding rates of momentum and energy transfer from the sound wave to the electrons, we get

$$W\beta \left[1 - \frac{\hbar p_0}{ms}\right] = \gamma_\varepsilon \rho_\varepsilon \hbar p_0 \tag{3.3}$$

and

$$W\beta s \left[1 - \frac{\hbar p_0}{ms}\right] = \gamma_\varepsilon \theta_\varepsilon \left[\frac{\Delta T}{T_0} + \frac{\hbar^2 {p_0}^2}{3mkT_0}\right] \tag{3.4}$$

Solving these equations for $\hbar p_0$ and T, we get

$$\hbar p_0 = ms \left[1 + \frac{\gamma_\varepsilon \rho_\varepsilon ms}{W\beta}\right]^{-1} \tag{3.5}$$

and

$$T = T_0 \left\{ 1 + \frac{\rho_\varepsilon ms^2}{\theta_\varepsilon} \left[1 + \frac{\gamma_\varepsilon \rho_\varepsilon}{W\beta} ms\right]^{-1} \right. \tag{3.6}$$

$$\left. - \frac{ms^2}{3kT_0} \left[1 + \frac{\gamma_\varepsilon \rho_\varepsilon}{W} ms\right]^{-2} \right\}$$

It is easy to see that $(T-T_0) = \Delta T$ is negative only when

$$3kT_0 \frac{\rho_\varepsilon}{\theta_\varepsilon} < 1 \tag{3.7}$$

In addition to this

$$W > \frac{\gamma_\varepsilon \rho_\varepsilon ms}{\beta} \left(\frac{\theta_\varepsilon}{3\rho_\varepsilon kT_0} - 1\right)^{-1} \tag{3.8}$$

is also a necessary condition for electron cooling. Inequality condition (3.7) corresponds to the electron cooling conditions obtained in the case of an electric field by Paranjape and de Alba (1965).

We shall discuss this phenomenon in the cases of three different scattering processes.

§3.a.2. Acoustical Scattering

The values of the constants γ_ε, ρ_ε and θ_ε can be obtained from Sec. (2.a.2) and they are

$$\begin{aligned} \gamma_1 &= \frac{32 n_e l^e}{3\pi \mu_{ac}} \\ \rho_1 &= \frac{1}{3m} \\ \theta_1 &= s^2 \end{aligned} \qquad (3.9)$$

Putting these constants in condition (3.7), we see that for electron cooling we must have

$$\frac{\kappa T_0}{ms^2} < 1 \qquad (3.10)$$

The inequality is contradictory to the assumption made in Eqn. (2.20) and hence when the electron-lattice scattering is acoustical carrier temperature never decreases below the lattice temperature.

§3.a.3. Optical Non-Polar Scattering

The constants in this case are

$$\gamma_2 = \frac{2N_e l^e}{3\pi\mu_{on}} \left(\frac{n_q}{1+2n_q} \right)$$

$$\rho_2 = \frac{2}{3m} \left[2I_1 + \frac{\hbar\omega}{\kappa T_0} I_0 \right] \tag{3.11}$$

$$\theta_2 = \frac{\hbar\omega}{m} \left(\frac{\hbar\omega}{\kappa T_0} \right) I_0$$

The electron cooling conditions then become

$$\left(\frac{\hbar\omega}{\kappa T_0} \right) > \left[1 + \left(1 + \frac{4I_1}{I_0} \right)^{1\ 2} \right] \tag{3.12}$$

and

$$W > \frac{16}{9\pi^3} \left(\frac{n_q}{1+2n_q} \right) \left(\frac{\kappa T_0 s}{\hbar^4 q} \right) \left(\frac{\mu_{ac}}{\mu_{on}} \right) (m\kappa T_0)^2$$

$$\times \left[2I_1 + \left(\frac{\hbar\omega}{\kappa T_0} \right) I_0 \right]$$

$$\times \left\{ \left(\frac{\hbar\omega}{\kappa T_0} \right)^2 \frac{I_0}{2 \left[2I_1 + \left(\frac{\hbar\omega}{\kappa T_0} \right) I_0 \right]} \right. \tag{3.13}$$

If $\hbar\omega > \kappa T_0$, then I_0, I_1 take approximate values given in Eqn. (2.32).

Hence, the above conditions become

$$\left(\frac{\hbar\omega}{\kappa T_0} \right) > 3.646 \tag{3.14}$$

and

$$W > \frac{16}{\pi^4} \left(\frac{n_q}{1+2n_q} \right) \left(\frac{\kappa T_0 s}{\hbar^4 q} \right) \left(\frac{\mu_{ac}}{\mu_{on}} \right) (m\kappa T_0)^2$$

$$\times \left\{ 1 + \frac{\hbar\omega}{3\kappa T_0} \right\}^2 \left(\frac{\pi\kappa T_0}{\hbar\omega} \right)^{3/2} \tag{3.15}$$

§3.a.4. Optical Polar Scattering

The constants in this case are

$$\gamma_3 = \frac{N_{el} e}{\mu_{op}}$$

$$\rho_3 = \frac{1}{m} \left(\frac{I_0}{J_0} \right) \tag{3.16}$$

$$\theta_3 = \frac{3}{4} \left[\frac{(\hbar\omega)^2}{m\kappa T_0} \right]$$

Using these values of constants in conditions (3.7) and (3.8) we see that the cooling of electrons can occur if

$$\left(\frac{\hbar\omega}{\kappa T_0} \right) > 2 \left(\frac{I_0}{J_0} \right)^{1/2} \tag{3.17}$$

and

$$W > \frac{4}{\pi^2} \left(\frac{\mu_{ac}}{\mu_{op}} \right) \left(\frac{I_0}{J_0} \right) \frac{(m\kappa T_0)^2}{\hbar^4 q} s\kappa T_0 \tag{3.18}$$

§3.a.5. Non-Polar Substances

In non-polar substances, such as silicon, germanium, etc., if the temperature is not too low, then the important scattering mechanisms are both acoustical and non-polar optical. Thus, the total energy and momentum transferred to the lattice will be the sum of these quantities transferred via acoustical modes and optical non-polar modes. We may, therefore, write

$$\left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la} = (\gamma_1\rho_1 + \gamma_2\rho_2)\,\hbar p_0 \tag{3.19}$$

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la} = (\gamma_1\theta_1 + \gamma_2\theta_2)\left[\frac{\Delta T}{T_0}\quad\frac{\hbar^2 {p_0}^2}{2m\kappa T_0}\right] \tag{3.20}$$

Eqns. (3.19) and (3.20) can be expressed conveniently in terms of the constants

$$\gamma_4 = \frac{2N_{el}e}{3\pi\mu_{ac}}$$

$$\rho_4 = \frac{16}{3m} + \frac{2}{3m}\left(\frac{\mu_{ac}}{\mu_{on}}\right)\left(\frac{nq}{1+2nq}\right)\left[2I_1 + \left(\frac{\hbar\omega}{\kappa T_0}\right) I_0\right] \tag{3.21}$$

$$\theta_4 = 16s^2 + \frac{\hbar\omega}{m} I_0 \left(\frac{nq}{1+2nq}\right)\left(\frac{\hbar\omega}{\kappa T_0}\right)\left(\frac{\mu_{ac}}{\mu_{on}}\right)$$

where

$$\gamma_1\rho_1 + \gamma_2\rho_2 = \gamma_4\rho_4$$

and (3.22)

$$\gamma_1\theta_1 + \gamma_2\theta_2 = \gamma_4\theta_4$$

The electron cooling conditions in this case become

$$\left(\frac{\hbar\omega}{\kappa T_0} \right) = l^2 > 3.646 \qquad (3.23)$$

$$\left(\frac{\mu_{on}}{\mu_{ac}} \right) < \frac{\sqrt{\pi}}{4} l^2 \, Y(l) e^{-l^2} \qquad (3.24)$$

where

$$Y(l) = \left[l^4 - 2l^2 - 6 \right] \qquad (3.25)$$

and

$$W > \frac{8}{\pi^3} \frac{(m\kappa T_0)^2}{\hbar^4 q} (\kappa T_0)^2 \, ms$$

$$\times \left\{ \rho_4 \left[3\kappa T_0 + \frac{\theta_4}{\rho_4} \right]^{-1} \right\} \qquad (3.26)$$

§3.a.6. Polar Substances

In polar substances like pbs, CdSe, InSb, etc., the important scattering mechanisms are optical polar and acoustical. Hence, in this case, we define

$$\gamma_5 = \frac{N_e l^e}{m\mu_{ac}}$$

$$\rho_5 = \frac{32}{m} + \frac{I_0}{mJ_0} \left(\frac{\mu_{ac}}{\mu_{op}} \right) \qquad (3.27)$$

$$\theta_5 = \frac{32s^2}{3\pi} + \frac{3}{4} \frac{(\hbar\omega)^2}{m\kappa T_0} \left(\frac{\mu_{ac}}{\mu_{op}} \right)$$

where

$$\gamma_1\rho_1 + \gamma_3\rho_3 = \gamma_5\rho_5$$

and (3.28)

$$\gamma_1\theta_1 + \gamma_3\theta_3 = \gamma_5\theta_5$$

The electron cooling conditions in this case become

$$\left(\frac{\hbar\omega}{\kappa T_0}\right)\frac{J_0}{4I_0} > 1 \tag{3.29}$$

$$\left(\frac{\mu_{op}}{\mu_{ac}}\right) < \frac{9\pi}{32}\left\{\left(\frac{\hbar\omega}{\kappa T_0}\right)^2 \frac{J_1}{4I_0} - 1\right\} \tag{3.30}$$

and

$$W > \frac{24}{\pi^3}\frac{(m\kappa T_0)}{\hbar^4 q}(\kappa T_0 s)\left\{\rho_5\left[3\kappa T_0 + \frac{\theta_5}{\rho_5}\right]^{-1}\right\} \tag{3.31}$$

Eqns. (3.14), (3.15), (3.17), (3.18), (3.24), (3.26), (3.30) and (3.31) express the conditions under which the phenomenon of electron cooling may occur in different substances.

§3.b.

Attenuation of the sound wave and generation of acoustoelectric current are the two physical aspects of the electron-sound wave interaction. In the following section, we shall calculate sound absorption coefficient α and the acoustoelectric current J_{ac} with the help of the distribution function established in Chapter III.

§3.b.1. Sound Absorption Coefficient

From Eqn. (2.56), we observe that the rate at which the phonons are absorbed by the electrons from the sound wave is proportional to the number of phonons reaching the volume element under consideration. Thus,

$$- \left.\frac{dN_q}{dt}\right|_{s\text{-}el} \propto N_q \tag{3.32}$$

and if the sound wave is travelling along the x-direction, with the velocity s, then

$$- \left.\frac{dN_q}{dt}\right|_{s\text{-}el} = - \frac{1}{s} \left.\frac{dN_q}{dx}\right|_{s\text{-}el} \tag{3.33}$$

It now follows that $N_q(x)$, the number of phonons at a distance x from the incident face of the elements, can be written as

$$N_q(x) = N_q(0) \exp\{-\alpha x\} \tag{3.34}$$

where α is called the coefficient of sound absorption and can be defined as

$$\alpha \equiv \frac{1}{sN_q(0)} \left.\frac{dN_q}{dt}\right|_{s\text{-}el} \tag{3.35}$$

Substituting the value of $\left.dN_q/dt\right|_{s\text{-}el}$ from Eqn. (2.56), we get

$$\alpha = \pi \left(\frac{mN_{el}}{MN_{io}}\right) \frac{C^2}{s^2\hbar(2\pi m\kappa T_0)^{1/2}} \left[1 - \frac{\hbar p_0}{ms}\right] \tag{3.36}$$

Finally, using Eqn. (3.5), we obtain

$$\alpha = \pi \left(\frac{mN_{el}}{MN_{io}} \right) \frac{C^2}{s^2 \hbar (2\pi m \kappa T_0)^{1/2}}$$

$$\times \{ 1 - [1 + \frac{\gamma_\varepsilon \rho_\varepsilon}{W\beta} ms]^{-1} \} \qquad (3.37)$$

for the sound absorption coefficient.

3.b.2. Acoustoelectric Current

The sound wave travelling through the crystal transfers momentum to the electrons and, as a result, the electron gas acquires drift velocity in the direction of sound wave propagation. The electron drift causes a flow of charge across the crystal and, if the two ends of the specimen are connected by a suitable conductor, then a current flows through the closed circuit. This current, induced by the sound wave, is called acoustoelectric current.

If there are no positive charge carriers (or electron holes) in the crystal, then the current is due to the conduction electrons only and can be calculated from the distribution function established in Section (3.a.1). The acoustoelectric current J_{ac} is given by

$$J_{ac} = e \int f(\underset{\sim}{p}) \{ \frac{\hbar \underset{\sim}{p}}{m} \} d^3p \qquad (3.38)$$

Substituting the value of $f(p)$ from Eqn. (2), we get

$$J_{ac} = e \frac{\hbar p_0}{m} n_{el} \qquad (3.39)$$

and hence

$$J_{ac} = n_e l_{es} \left[1 + \frac{\gamma_\varepsilon \rho_\varepsilon m_s}{W\beta} \right]^{-1} \qquad (3.40)$$

Discussion

The main results of our calculations are the electron cooling conditions obtained in Section (3.a.1).

(I) $$\frac{\theta_\varepsilon}{3\kappa T_0 \rho_\varepsilon} > 1$$

and (II) $$W > \frac{\gamma_\varepsilon \rho_\varepsilon}{\beta} ms \left(\frac{\theta_\varepsilon}{3\rho_\varepsilon \kappa T_0} - 1 \right)^{-1}$$

The first inequality is equivalent to the electron cooling condition obtained by Paranjape and de Alba (1965) in the case of an electric field. The second inequality defines the minimum value of sound wave energy flux W_0 required to produce electron cooling. In non-polar substances, the condition

$$\frac{\hbar\omega}{\kappa T_0} > 3.646$$

determines the maximum value of lattice temperature at which we can expect to achieve the effect of electron cooling. The condition

$$\frac{\mu_{on}}{\mu_{ac}} < \frac{\sqrt{\pi}}{4} l^2 Y(l) e^{-l^2}$$

shows that electron cooling can occur only if the non-polar optical scattering dominates over the acoustical scattering.

In polar substances, the condition (3.29)

$$\frac{\hbar\omega}{\kappa T_0} > \left(\frac{4I_0}{J_0} \right)^{1/2}$$

gives the maximum value of T_0. The integrals I_0 and J_0 are evaluated

by Paranjape and de Alba (1965) by expressing them in terms of Bessel functions from which the minimum value of $\hbar\omega/\kappa T_0$ is $\simeq 5$.

The condition

$$\left(\frac{\mu_{op}}{\mu_{ac}}\right) < \frac{9\pi}{32}\left\{\left(\frac{\hbar\omega}{\kappa T_0}\right)^2 \frac{J_0}{4I_0} - 1\right\}$$

shows that the electron cooling in polar substances can occur only if optical polar scattering predominates over acoustical scattering.

Eqns. (3.5) and (3.6) give the values of the parameters p_0 and T as a function of incident energy flux W. We observe that as $W\to 0$, $T\to T_0$ and $p_0\to 0$ and the displaced Maxwellian function becomes simple Maxwellian, as expected. In this limit, the sound absorption coefficient tends to a value α_0, i.e.

$$W\to 0, \alpha\to\alpha_0 = \pi\left(\frac{mN_{el}}{MN_{io}}\right)\frac{C^2}{2\hbar(2\pi m\kappa T_0)^{1/2}}$$

This result is in agreement with the results of Paranjape (1964) and Epshtein (1966), except for small numerical factors. We also see that

$$W \to 0 \ , \ J_{ac} \to 0$$

On the other hand, for infinitely large values of W, we get

$$p_0 \to \frac{ms}{\hbar} \quad \text{and} \quad J_{ac} \to n_{el}es$$

i.e. the drift velocity $\hbar p_0/m$ tends to the sound velocity. For all finite values of W, however, the drift velocity is less than the sound velocity. In other words, the sound wave cannot drive the electrons

faster than its own speed.

$$W \to \infty \quad , \quad \alpha \to 0$$

show that the electrons drifting with the velocity of sound remain unaffected by the sound wave.

For very large but finite values of energy flux, we get

$$W >> \frac{\gamma_\varepsilon \rho_\varepsilon}{\beta} ms$$

$$\alpha \to \alpha_0 \left(\frac{\gamma_\varepsilon \rho_\varepsilon ms}{\beta} \right) \left(\frac{1}{W} \right)$$

This result is in agreement with the result obtained by Epshtein (1966), i.e. for large values of W the absorption coefficient varies as the inverse of the energy flux.

APPENDIX I

The electrons and the lattice can be considered as two separate systems when the density of conduction electrons is so high that the rate of energy exchange between electrons, through mutual collisions, is large compared with the rate of energy exchange between electrons and lattice. The electron density required to make interelectronic collisions more effective than the electron-lattice collisions can be estimated as follows:

(1) The rate of energy exchange between lattice and electrons is given by

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la} = \frac{\overline{E}}{\kappa T_0}\,\frac{ms^2}{\tau(\overline{E},T_0)} \qquad \text{(I)}$$

where s is the speed of sound and $\tau(\overline{E},T_0)$ is the average time between two electron-lattice collisions.

(2) The rate of energy exchange of an electron due to collisions with other electrons, Pines (1953), is of the order

$$\left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}el} = \frac{4\pi n_{el} e^{*4}}{\overline{P}} \log\gamma_p \qquad \text{(II)}$$

where e^* is an effective charge, $\overline{P}$ is the r.m.s. momentum of an electron and $\log\gamma_p$ is a factor weakly depending on the

electron density and is of the order of unity.

(3) From Eqns. (I) and (II), it follows that the electron density n_{el}^0, at which these two rates of energy transfer are equal, is of the order

$$n_{el}^0 \simeq \frac{1}{4\pi} \frac{\overline{E}^{3/2} m^{3/2} s^2}{\kappa T_0 e^{*4} \tau(E_0, T_0)} \qquad \text{(III)}$$

If $\tau \simeq 10^{-14}$ sec., $e^{*4}/e^4 \simeq 10$ and $m/m^* = 1$, then n_{el}^0 is of the order $10^{14}/cm^3$. When the condition $n_{el} > n_{el}^0$ holds, electron-electron collisions dominate and the concept of electron temperature is valid. Equidistribution of momentum is realized only when the rate of momentum exchange between mutual collisions of the electrons is large compared to the rate of momentum exchange in electron-lattice collisions. To achieve this condition, higher electron densities than n_{el}^0 are required.

APPENDIX II

Electron Cooling Conditions in an Electric Field

If we denote in this case

$$\int \hbar p \left(\left. \frac{\partial f(p)}{\partial t} \right|_{\text{Field}} \right) d^3p = \left. \frac{d\mathbf{P}}{dt} \right|_{\text{F-el}} \tag{I}$$

and

$$\int \frac{\hbar^2 p^2}{2m} \left(\left. \frac{\partial f(p)}{\partial t} \right|_{\text{Field}} \right) d^3p = \left. \frac{d\mathbf{E}}{dt} \right|_{\text{F-el}}$$

then the momentum and energy balance equations (similar to Eqns. (1.11) and (1.12)) become

$$\left. \frac{d\mathbf{P}}{dt} \right|_{\text{F-el}} = \left. \frac{d\mathbf{P}}{dt} \right|_{\text{el-la}} \tag{II}$$

and

$$\left. \frac{d\mathbf{E}}{dt} \right|_{\text{F-el}} = \left. \frac{d\mathbf{E}}{dt} \right|_{\text{el-la}} \tag{III}$$

However, Fröhlich and Paranjape (1956) have shown that

$$\int \hbar \underset{\sim}{p} \left(\frac{\partial f(p)}{\partial t}\bigg|_F \right) d^3p = e\underset{\sim}{F} n_{el} \qquad \text{(IV)}$$

and

$$\int \frac{\hbar^2 p^2}{2m} \left(\frac{\partial f(p)}{\partial t}\bigg|_F \right) d^3p = \frac{e n_{el}}{m} F \hbar p_0 \qquad \text{(V)}$$

where $\underset{\sim}{F}$ is the applied field.

Thus, using Eqns. (2.16), (2.17), (I), (II) and (III), we get

$$\frac{\hbar p_0}{3} \int \hbar p g_1(p) d^3p = e\underset{\sim}{F} n_{el} \qquad \text{(VI)}$$

and

$$\tfrac{1}{2} \int \frac{\hbar^2 p^2}{m} \{g_0(p) + p_0{}^2 g_2(p)\} d^3p = \frac{eF n_{el} \hbar p_0}{m} \qquad \text{(VII)}$$

In the case of acoustical scattering, using the rates from Section (2.a.2.), we get

$$e\underset{\sim}{F} n_{el} = \frac{d\mathbf{P}}{dt}\bigg|_{el-la}^{ac} \qquad \text{(VIII)}$$

and

$$\frac{eF n_{el} \hbar p_0}{m} = \frac{d\mathbf{E}}{dt}\bigg|_{el-la}^{ac} \qquad \text{(IX)}$$

Now eliminating p_0 between Eqns. (VIII) and (IX) and assuming $\Delta T/T_0 << 1$, we get

$$\frac{\Delta T}{T_0} = 3 \left(\frac{3\pi}{32} \right)^2 \frac{F^2 \mu_{ac}}{s^2} \left(1 - \frac{ms^2}{\kappa T_0} \right) \qquad \text{(X)}$$

From Eqn. (X) we observe that $\Delta T < 0$ would require $ms^2 > \kappa T_0$ which is not permissible in view of the assumption, Eqn. (2.20). Thus, we can conclude that even the inclusion of second order terms in p_0 in the calculations does not lead to the possibility of electron cooling if the scattering mechanism is acoustical.

In optical non-polar scattering, we have

$$e\underset{\sim}{F} n_{el} = \left. \frac{d\mathbf{P}}{dt} \right|_{el-la}^{on} \qquad \text{(XI)}$$

and

$$\frac{eF n_{el} \hbar p_0}{m} \qquad \left. \frac{d\mathbf{E}}{dt} \right|_{el-la}^{on} \qquad \text{(XII)}$$

which on elimination of p_0 gives

$$\frac{\Delta T}{T_0} = \frac{F^2 \mu_{on}^2 m}{\hbar p} \left(\frac{2n_q + 1}{n_q} \right)^2 \frac{27\pi^2}{128} \frac{\kappa T_0}{h\omega}$$

$$\times \left[I_0 \{ 2I_1 + I_0 \left(\frac{\hbar\omega}{\kappa T_0} \right) \} \right]^{-1}$$

$$\{ 1 - \left(\frac{\hbar\omega}{\kappa T_0} \right)^2 \frac{I_0}{4I_1 + 2I_0 (\hbar\omega/\kappa T_0)} \} \qquad \text{(XIII)}$$

Hence, $\Delta T<0$ would require

$$\left(\frac{\hbar\omega}{\kappa T_0}\right)\frac{I_0}{4I_1 + 2I_0(\hbar\omega/\kappa T_0)} > 1 \qquad \text{(XIV)}$$

Putting the values of I_0 and I_1 from Eqn. (2.32) in Eqn. (XIV) we obtain the condition for electron cooling in the case of optical non-polar scattering as

$$\frac{\hbar\omega}{\kappa T_0} > 1 + 7^{1/2} \simeq 3.646 \qquad \text{(XV)}$$

In optical polar scattering, we have

$$e\underset{\sim}{F}n_{el} = \left.\frac{d\mathbf{P}}{dt}\right|_{el\text{-}la}^{op} \qquad \text{(XVI)}$$

and

$$\frac{eFn_{el}\hbar p_0}{m} = \left.\frac{d\mathbf{E}}{dt}\right|_{el\text{-}la}^{op} \qquad \text{(XVII)}$$

in which case we get

$$\frac{\Delta T}{T_0} = \frac{\hbar^2 p_0^{\,2}}{3m\kappa T_0}\left\{\frac{J_0}{4I_0}\left(\frac{\hbar\omega}{\kappa T_0}\right)^2 - 1\right\} \qquad \text{(XVIII)}$$

Thus, the electron cooling condition in the case of optical polar scattering becomes

$$\left(\frac{\hbar\omega}{\kappa T_0}\right) > 2\left(\frac{I_0}{J_0}\right)^{1/2} \qquad \text{(XIX)}$$

In addition, Paranjape and de Alba have derived the electron cooling conditions in a polar and non-polar crystal where optical polar and acoustical or optical non-polar and acoustical types of electron scatterings are simultaneously in effect. The conditions were

(1) For the polar crystals in a weak field, for electron cooling, we must have

$$\frac{\mu_{op}}{\mu_{ac}} < \frac{9\pi}{32} \left\{ \left(\frac{\hbar\omega}{\kappa T_0} \right)^2 \frac{J_0}{4I_0} - 1 \right\} \qquad \text{(XX)}$$

(2) For non-polar crystals, we require

$$\frac{\mu_{on}}{\mu_{ac}} < \frac{1}{4} \left(\frac{n_q}{1+2n_q} \right) \left\{ \left(\frac{\hbar\omega}{\kappa T_0} \right)^2 I_0 \right. \qquad \text{(XXI)}$$

$$\left. - 2(2I_1 + \left(\frac{\hbar\omega}{\kappa T_0} \right) I_0 \right\}$$

It should be noted that these electron cooling conditions arise only because the second order terms are included in the calculations. Butcher *et al.* (1968) have calculated these conditions by taking higher order terms than p_0^2 and their results are basically the same as above, except that the discontinuous behaviour of the electron temperature in high fields is removed; in fact, they have shown that for large fields electrons are heated.

APPENDIX III

We point out that the decrease in electron temperature does not lead to a decrease in the average energy $\overline{E}$ of an electron.

$$\overline{E} = \int \frac{\hbar^2 p^2}{2m} f(\underset{\sim}{p}) dp$$

$$= \frac{3}{2} \kappa T + \frac{\hbar^2 p_0^2}{2m}$$

and not just $\frac{3}{2} \kappa T$ as assumed by Fröhlich (1948). Putting the values of p_0 and T, we get

$$\overline{E} = \frac{3}{2} \kappa T_0 + \frac{3}{2} \kappa T_0 ms^2 \frac{\rho_\varepsilon}{\theta_\varepsilon} \left[1 + \frac{\gamma_\varepsilon \rho_\varepsilon}{W\beta} ms \right]^{-1}$$

which shows that for any finite value of W, $\overline{E}$ is always greater than $\frac{3}{2} \kappa T_0$.

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List of Symbols

$2\pi\hbar = h$	.....	Planck's Constant
m^*	.....	Mass of an Electron
m	.....	Effective Mass of an Electron
e	.....	Electronic Charge
M	.....	Mass of an Ion
κ	.....	Boltzmann's Constant
V	.....	Volume of the Crystal
$\underset{\sim}{p}_0$	.....	Average Drift Wave Vector of an Electron in the Presence of an External Electric Field
T_0	.....	The Lattice Temperature
T	.....	Electron Temperature
n_q	.....	Thermal Excitation Number of Phonons With Wave Vector $\underset{\sim}{q}$
N_{el}	.....	Total Number of Electrons in the Conduction Band of a Solid
n_{el}	.....	Electron Density in the Conduction Band
N_{io}	.....	Number of Lattice Points