SITE QUALITY EVALUATION OF JACK PINE IN NORTHERN ONTARIO USING SITE-INDEX CURVES

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fulfillment of the requirements for the Degree of
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ABSTRACT

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Key words: Site-index curves, stem analysis, polymorphism, non-linear regresssion.

Jack pine stem analysis data from 383 fully stocked, mature, undisturbed plots were collected from studies located in four Ontario Ministry of Natural Resources regions (Northeast, Northern, North Central, Northwestern) and four broad landforms (lacustrine, glaciofluvial, morainal, shallow depth to bedrock). Comparisons of height-growth patterns in the regions and in the landforms were made using covariate analysis for nonlinear equations.

Several different height-growth models and site-index prediction models were fitted to stem analysis data from 323 plots; the remaining 60 plots were used as verification plots. Results show that height growth was best described by a model developed by Ek (1971) and later modified by Newnham (1988). The 95% prediction interval for differences (observed - predicted) were within +/- 1.39 m and +/- 1.59 m for the computation and verification data sets respectively. A linear model developed by Monserud predicted site index better than an exponential or difference equation. But site-index predictions made indirectly from Newnham's height dependent model were as accurate as using the linear Monserud model. Early growth before 20 years breast-height age (BHA) was highly variable and resulted in poor prediction of site index at 50 years. Site index prediction intervals for data older than 20 years BHA were within +/- 1.69 m.

Jack pine height-growth patterns were similar among regions, but some significant differences were found among landforms. Jack pine growing on good sites on all landforms had similar height-growth patterns. But significant differences in height-growth patterns were found on poor quality sites; the upper asymptote flattens more sharply on shallow soils when compared to glaciofluvial soils. Height-growth patterns were similar to patterns found for other studies in North Central Region (Lenthall 1986, Carmean and Lenthall 1989, Goelz and Burk 1992). Results show that the anamorphic curves developed by Plonski (1974) slowed more rapidly after index age as site quality decreased. Jack pine height-growth patterns in Ontario are similar to published curves from other areas in Canada with the exception of more rapid early growth on poor quality sites and a flatter upper asymptote on good sites.

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INTRODUCTION

The need for accurate methods for estimating the capability of forest land to grow trees increases with more intensive forest land management, with increased costs of forest management, and with greater demands and higher prices for wood fiber. The most intensive forest management should be concentrated on the most productive sites where rapid growth and greater yields warrant the application of intensive silvicultural practices. Therefore, forest managers must have precise tools for identifying productive sites so that intensive management can be concentrated on these productive sites. Identifying productive sites requires: (a) the use of direct methods of site-quality evaluation based on tree measurements from suitable stands and trees, and (b) indirect methods based on soil and topographic features for areas where suitable stands and site trees are lacking.

A site-quality evaluation program began at Lakehead University in the early 1980's that involved both the direct and indirect methods listed in Table 1.1. This program uses direct estimates of height growth for dominant and codominant trees growing in fully stocked, evenaged, undisturbed forest stands. This site-index method has long been the most commonly accepted method for site-quality evaluation in North America and Europe (Frothingham 1918, 1921; Carmean 1975; Carmean et al.

1989; Hagglund 1981). Height is favoured because it is independent of density or stocking (Rudolph 1951, Ralston 1953, Ware and Stahelin 1948, Lanner 1985), is easy to measure, and can provide site-index comparisons among tree species . Height is closely related to volume in fully stocked evenaged stands and as well to the capacity of a given site to produce wood (Carmean 1975, Spurr and Barnes 1980).

Table 1.1 Methods used for estimating site index (Carmean 1987,1994).

A.	DIRECT ESTIMATION OF SITE INDEX FROM FOREST TREES
	1 Site-index curves -harmonised site-index curves -polymorphic site-index curves 2 Growth intercepts 3 Site index comparisons between species
B.	INDIRECT ESTIMATION OF SITE INDEX FROM SOIL, VEGETATION AND TOPOGRAPHY
	1 Soil-site evaluation 2 Soil types 3 Habitat, ecosystem, and physiographic types

Accurate direct estimation of site quality uses tree height as a basis for estimating site index. Thus accurately modelling and predicting height-growth patterns of forest trees is important for estimating site index as well as for planning silvicultural investments and for determining future yields. The earliest work at Lakehead University (Wiltshire 1982) was an undergraduate thesis involving the development of preliminary site-index curves for jack pine (Pinus banksiana Lamb.). This species was selected because jack pine is one of the most abundant tree species in northern

Ontario, and thus is important to the forest industry in northern Ontario (Campbell 1990).

The Wiltshire (1982) thesis was based only on 34 plots located near Thunder Bay. Accordingly, his thesis was expanded into a M.Sc. Forestry thesis by Lenthall (1986) using 141 plots covering a wider area of north central Ontario. Conclusions from this study (Lenthall 1986, Carmean and Lenthall 1989, Carmean 1994) were:

a) site-index curves based on breast-height age are more precise than curves based on total age;

b) average height-growth patterns are similar for jack pine growing on four glacial landforms - shallow to bedrock, morainal, glaciofluvial, and lacustrine soils;

c) polymorphic height-growth patterns are related to the level of site index becoming more curvilinear as site index increases; and;

d) height-growth patterns before 50 years are similar to Plonski's (1974) jack pine site-class curves, but after 50 years height growth for all sites was somewhat better than predicted by Plonski.

The inference space of Lenthall's (1986) study was extended by Jackman (1990) to northwestern Ontario to determine the applicability of the Plonski and Lenthall height-growth curves. Jackman concluded:

a) the Lenthall height-growth curves are applicable to northwestern Ontario;

b) height-growth on shallow to bedrock, morainal, and glaciofluvial soils are similar to height-growth patterns expressed by the Lenthall curves; and

c) height growth on lacustrine soil differs slightly from other landforms because growth before 50 years was somewhat faster, and after 50 years height growth appeared slower than that expressed by the Lenthall curves.

The primary objective of my thesis is to develop height-growth curves for jack pine that are applicable to all regions and all major soils of northern Ontario. This objective will be attained using the following methods:

1) Stem-analysis data will be collected from all Regions and from soils on all major landforms of northern Ontario.
2) Comparisons of height-growth patterns will be made to determine if separate height-growth curves are needed for jack pine in the four northern Administrative Regions recognized by the Ontario Ministry of Natural Resources prior to 1992:
   a) North Central Region;
   b) Northwestern Region;
   c) Northeastern Region; and
   d) Northern Region.
3) Comparisons of height-growth patterns also will be made to determine if separate height-growth curves are needed for jack pine growing on:
   a) soils that are shallow to bedrock (< 1 m of soil);
   b) deep moraine soils (> 1 m of soil);
   c) glaciofluvial soils; and
   d) lacustrine soils (silt and clay soil profiles).

Additional objectives of this study are to determine if polymorphic height-growth patterns are associated with different levels of site quality, and to compare height-growth patterns for northern Ontario with patterns shown for jack pine by studies in other regions of Canada.

The final product of this thesis will be height-growth curves and site index prediction equations that are applicable to jack pine growing on all major glacial landforms throughout northern Ontario.

2

LITERATURE REVIEW

2.1 Historical Perspective

Three major methods for evaluating the productive capacity of forest land (site quality) are: 1) volume growth (Bates 1918), 2) forest site types (Cajander 1926) and, 3) height growth (Graves 1906; Roth 1916; Watson 1917; Frothingham 1918, 1921; Sterrett 1921). The evolution of methods for site-quality evaluation have been reviewed by Coile (1952), Mader (1963), Ralston (1964), Jones (1969), Carmean (1975), Pritchett and Fisher (1987), Rennie (1962), Rowe (1962), and Daniel et al. (1979) for the United States, Burger (1972) for Canada, Spurr and Barnes (1980) for North America and Europe, and Hagglund (1981) for Europe.

Problems are encountered in using volume growth and site types for site-quality evaluation, hence the most widely accepted method for evaluating site quality is by determining total tree height at a particular index age (site index) for a specific species (Appendix I). The age most commonly accepted for tree species in eastern North America is 50 years; for young plantations an index age of 25 years or less is often used, and breast-height age is commonly used for species having slow and erratic early height growth (Carmean 1994, Carmean et al. 1989). Site index can only be accurately estimated when fully stocked, evenaged, undisturbed older stands are available that have free growing, uninjured, dominant, and/or dominant and codominant trees. Site index cannot be directly estimated for areas where suitable stands and site trees are lacking such as for cutover areas, or for very young, poorly stocked and unevenaged stands. Accordingly, a variety of direct and indirect methods have been developed (Table 1.1) for estimating site index depending upon stand conditions. Figure 2.1 is a key for use in determining which site-index estimation method to use for areas which have different forest stand conditions.

2.2 Direct Estimation of Site Index

Table 1.1 and Figure 2.1 show how measurements of total height and tree age from forest trees can be used for directly estimating site index. Site index curves can be used for older stands, while growth intercepts can be used for certain young conifer stands such as red pine (Pinus resinosa Ait.), white pine (Pinus strobus L), and white spruce (Picea glauca Moench (Voss)) that have recognizable nodes marking annual height growth. Some evenaged stands may lack suitable trees of the species for which site index is desired, thus site-index comparison methods can be used. Growth intercepts and species site-index comparisons have been decribed elsewhere (Carmean 1975, 1994) and these methods will only be briefly described here. This literature review will concentrate on site-index curves because this is the major concern of this thesis.

2.2.1 Site-Index Curves

The use of height/age as an index of site was introduced to North America at the turn of the century (Frothingham 1918, Graves 1906, Watson 1917, Vincent 1961). The concept originated in Europe nearly a century earlier (Huber 1824) for use with evenaged, uniform stands. Curves were originally hand drawn and later the points on the curve were expressed using mathematical equations. More recent work in Europe and North America has focused on mathematical models that are more flexible, reflecting the changes in height-growth patterns with changes in the level of site quality. Some models incorporate variables to reflect site specific height-growth patterns. The sections that follow describe recent developments in site-quality research and studies that have been made in North America examining height-growth of jack pine and sites that influence height-growth patterns.

2.2.1.1 Harmonized Site-Index Curves

Harmonized site-index curves are a family of height-age curves used for directly estimating total height of trees at a specified index age. Historically, total height and total age measurements from dominant and/or codominant trees within an evenaged stand were used for constructing harmonized curves. These height and age measurements were used for calculating a single average guiding curve. Then curves for a range of good and poor sites were fitted proportionally to this average curve. A key point is that these early harmonized site-index curves were not based on actual height-growth observations. Instead they were usually based only on total height and total age measurements from growth and yield plots scattered throughout a particular forest region.

Two basic procedures used to develop early guide curves were Baur's method and the anamorphic curve method. Baur's or the limiting curve method involved drawing two curves based on the upper and lower limits of the data (Gray 1945); intermediate curves then are drawn based on the shape of the upper and lower limit curves. The "guide curve" method involves fitting an average curve to a range of total heights and ages from dominant, and/or dominant and codominant trees over a range of sites. Curves are then drawn for each of the site classes proportional to the average guide curve (Carmean 1975).

Average curves drawn by hand in pioneering work have since been replaced by mathematical models and regression analysis. Models have progressed from simple linear models to more complicated parameter prediction models and algebraic difference equations (Inions

1992; Goelz and Burk 1992; Deschamps 1991).

Guide curves are based on two assumptions: a) the guide curve represents the average top height growth of an average site; and b) a given site index maintains a constant position in the distribution of heights over time (Inions 1992). Both a) and b) were found to be untenable in studies by Monserud (1985), Smith (1984), Carmean and Lenthall (1989) that showed site index often was abnormally related to stand age, that is, certain young or old age classes may have an abnormal distribution of good or poor site plots instead of a normal distribution of site indices. Monserud (1985) found that average site index for Douglas-fir (Pseudotsuga menziesii (Mirb.) France) declined from 20 m at 50 years to 17 m at 200 years thus indicating that older ages were represented by more poor site index plots. But a non-uniform distribution of plots results in a flattening off of the average guide curve for older ages in situations where most of the older plots are on poor sites; consequently weighted regression techniques are used (Goelz and Burk 1992).

Recent studies show tree height-growth patterns vary depending on level of site quality as suggested by information which includes:

(1) comparisons of different sets of harmonized site index curves for species that range over large forest regions;

(2) soil-site studies;

(3) periodic height-growth measurements from permanent growth study plots; and

(4) newer site-index curves based on stem analysis (Carmean 1975).

Curve shape also is influenced by past history of a site (Inions 1992; Carmean 1994), by soil groups (Carmean 1970, Payandeh 1991, Hamlin and Leary 1988), habitat types (Monserud 1984, 1985), competition

(Hamilton and Krause 1985) or location in different portions of a forest region (Carmean 1970). The above factors are often confounded and result in polymorphic height-growth patterns

2.2.1.2 Polymorphic Site-Index Curves

Intensive forest management requires more precise site-index curves for each tree species than were possible using the harmonizing technique. Long-term height-growth records from permanent growth and yield plots could be used if sufficient number of plots were available covering the wide range of site quality, and the wide range of soil and topography found in most forest regions. Usually such long-term growth and yield plot data are not available and stem-analysis methods are now the favoured method for developing more accurate site-index curves (Carmean 1975).

Gertner (1985) and Borders et al. (1988) suggest the effect of autocorrelation from using time series data will vary depending on functional form and measurement interval used and recommend maximum likelihood regression be used. Monserud (1984) and Goelz and Burk (1992) did not find autocorrelation to be a limiting factor in developing their height-growth models.

Height-growth functions are usually formulated using stem analysis data involving the removal of discs at known intervals along the length of the tree bole. On small plots (0.08-0.04 ha), 3 to 5 trees are generally sampled. Height/age data for each disk is recorded for use in developing height-growth curves. Age is often determined from time of seed which is estimated from the number of rings at height of the disk taken at stump height. Height-growth below breast height (1.3 m) is often erratic and has little bearing on site index and future height patterns (Thrower 1986; Carmean 1994). Consequently, many models avoid early erratic growth by fixing age zero at 1.3 m, commonly refered to as breast-height age (BHA) (Carmean 1994, 1972; Thrower 1986; Carmean and Lenthall 1989).

Polymorphic site-index curves express different curve shapes at different levels of site quality. Many polymorphic curves resemble a reparameterization of the von Bertalanffy's (1941) anamorphic growth equation generalized by the nonlinear Richards (1959) function [Eq. 1].

$$H_t = \beta1 * [1 - EXP(\beta2 * Age_t)]^{\beta3} + \varepsilon \qquad [1]$$

where H_t = total stand or tree height at Age_t
AGE_t = total stand or tree age
$\beta1$ to $\beta3$ = model parameters to be estimated
ε = error of model

Richard's nonlinear growth function (1959) can be expanded to incorporate site index as a variable (Deschamps 1991, Goelz and Burk 1992). Equation [2], a commonly used modified version of the Richards function, was developed by Ek (1971).

$$H_t = \beta1 * SI_t^{\beta2} * [1 - EXP(\beta3 * Age_t)]^{\beta4 * SI_t^{\beta5}} + \varepsilon \qquad [2]$$

where: SI_t = site index (total height at reference age)

A similar version of the Richards function was used by Beck (1971) to model height growth of stem analysis data from white pine in the southern Appalachians. Ker and Bowling (1991) fitted the model to jack pine stem-analysis data in New Brunswick and found an improvement using equation [3] over equation [2].

$$H_t = 1.3 + (\beta o + \beta 1 * SI_b) * (1 - \exp(-\beta 2 * BHA))^{\beta 3 * SI_b^{\beta 4}} + \varepsilon \qquad [3]$$

where SI_b = site index (total height at reference age measured from breast height)

Monserud (1984) successfully used a logistic function [Eq. 4] to model height-growth of Douglas-fir. Monserud's logistic equation has since been used to predict height growth of species such as black spruce in northeastern Ontario (Payandeh 1991), trembling aspen (Populus tremuloides Michx.) in north central Ontario (Deschamps 1991) and white spruce, lodgepole pine (Pinus contorta var. latifolia Engelm.), and trembling aspen in Alberta (Dempster and Associates 1983).

$$H_t = 1.3 + \left[\frac{\beta_1 SI_b^{\beta_2}}{1 + EXP(\beta_3 + \beta_4 \ln(BHA) + \beta_5 \ln(SI_b))}\right] + \varepsilon \qquad [4]$$

Inions (1992) and Clutter et al. (1983) described the disadvantages of such nonlinear functions:

a) Parameters for height are expressed as a function of site index and a change in the index age results in a different pattern of height-growth development;

b) For many models, height predicted at index age does not match the specified site-index value used to predict height;

c) It is difficult to solve height dependent models for site index given height and age. Site index values must be obtained graphically or by using iterative procedures; and

d) Convergence using iterative approaches such as Gauss-Newton, or Marquardt or secant methods for estimating parameters is a problem particularly for the asymptotic parameter (Biging 1985; Grey 1989).

Algebraic difference equations address many of the problems mentioned above. Most difference equations do not use site index as an independent variable and such height-growth models are termed base-age invariant. This does not mean difference equations are not polymorphic (Inions 1992). Goelz and Burk (1992) found Eq. [5] predicted polymorphic height-growth almost as well as Eq. [2] using jack pine data furnished by the Lenthall (1986) study for north central Ontario. Difference equations have two advantages: 1) a previous estimate of site index is not required to predict height, and 2) any age can be used as an index age.

$$H_2 = 1.3 + (H_1 - 1.3)\left[\frac{1 - EXP(-\beta_1 ((H_1 - 1.3)/A_1)^{\beta_2} * A_1^{\beta_3} * A_2)}{1 - EXP(-\beta_1 ((H_1 - 1.3)/A_1)^{\beta_2} * A_1^{\beta_3} * A_1)}\right]^{\beta_4} + \varepsilon \qquad [5]$$

where: H_2 = total tree height at remeasurement
H_1 = initial total tree height
A_1 = BHA at H_1
A_2 = BHA at H_2

In response to the problem of height observed at index age not matching the predicted height, Newnham (1988) modified the Ek model [Eq. 2] to pass through a specified height at index age [Eq. 6] by solving for $\beta 3$.

$$H_t = 1.3 + \beta 1 * SI_b^{\beta 2}\left(1 - K^{\frac{BHA}{BHA_0}}\right)^{\beta 4 * SI_b^{\beta 5}} + \varepsilon \qquad [6]$$

where: $$K = 1 - \left[\frac{SI_b}{\beta 1 * SI_b^{\beta 2}}\right]^{\frac{1}{\beta 4 * SI_b^{\beta 5}}}$$

BHA_0 = breast height base index age

Newnham (1988) also constrained the Ek (1971) model [Eq. 2] to pass through a specified height using a general technique described by Clutter et al. (1983). The resulting formulation [Eq. 7] performed poorly and has not been used widely (Deschamps 1991).

$$H_t = SI_t \left[(1 - EXP(-\beta_3 * AGE_t)) / (1 - \beta_3^{Age_{t0}}) \right]^{\beta_4 SI_t^{\beta_5}} + \varepsilon \qquad [7]$$

where: Age_{t0} equals index age of 50 years

Ker and Bowling (1991) constrained equation [3] using similar methods to pass through total height at a specified age [Eq. 8]. Base age 50 years BHA was used when modelling mature jack pine stem-analysis data in New Brunswick.

$$H_t = 1.3 + (S - 1.3) * (1 - Exp(-\beta 2 * 50))^{\beta 3 * SI_b^{\beta 4}} * (1 - Exp(-\beta 2 * BHA))^{\beta 3 * SI_b^{\beta 4}} + \varepsilon \qquad [8]$$

As well, Dempster and Associates (1983) constrained Monserud's model [Eq. 4] to pass through a specified index age [Eq. 9] by applying the same technique used by Burkhart and Tennent (1977). They fitted the equation to stem-analysis data collected in Alberta for trembling aspen, jack pine, lodgepole pine and white spruce.

$$H_t = 1.3 + SI_b * \left[\frac{1 + EXP(\beta 1 + \beta 2 * \ln(BHA_0) + \beta 3 * \ln(SI_b))}{1 + EXP(\beta 1 + \beta 2 * \ln(BHA) + \beta 3 * \ln(SI_b))} \right] + \varepsilon \qquad [9]$$

Since height dependent nonlinear growth models are difficult to solve for site index, new models using site index as the dependent variable were developed. Monserud (1984) developed a linear model

[Eq. 10] for predicting site index of Douglas-fir to complement predicting height with Equation [4]:

$$SI_b = \beta o + \beta 1 * H_t + \beta 2 * \ln(H_t) + \beta 3 * \ln(BHA) + \beta 4 * \ln(BHA)^2 + + \beta 5 * \left(\frac{H_t}{BHA}\right) + \beta 6 * \ln(H_t) * BHA + \varepsilon \quad [10]$$

Payandeh (1974) developed an analogue of the Ek (1971) model in which height and site index were exchanged in equation [2] to predict site index for major tree species in Ontario [Eq. 11]. This Payandeh analogue has been used by Hahn and Carmean (1982) and Carmean et al. (1989) for developing site-index prediction equations for various tree species in the United States.

$$SI_t = \beta 1 * H_t^{\beta 2} * [1 - EXP(\beta 3 * Age_t)]^{\beta 4 * H_t^{\beta 5}} + \varepsilon \quad [11]$$

Various other approaches have examined splining equations at different ages thus creating polymorphic models with desirable properties (Borders et al. 1984). Attributes of the ideal site-index equation are described by Devan and Burkhart (1982), Borders et al. (1984), and Goelz and Burk (1992).

Zeide (1978) developed a method using two points at different ages to predict height-growth patterns but it has received little attention. Smith (1984), and Smith and Kozak (1984) developed a structural model for predicting both height growth and site index that examined stochastic errors. The structural method is based on linear regression but has several disadvantages and was debated by Smith (1988) and Payandeh (1988). Payandeh did not recommended the structural method (Payandeh

1983).

A functional height-growth model was developed by Cieszewski and Bella (1989) for lodgepole pine in Alberta. Deschamps (1991) used a model similar to the Cieszewski and Bella model for describing height-growth of trembling aspen in north central Ontario. The functional model performed well, but the Monserud model [Eq. 9] performed even better for the trembling aspen data.

The equations shown above are simply a small portion of the many models available for predicting height growth. In general, all models tend to predict height growth with very high coefficients of determination and have standard errors ranging from 0.5 to 1.3 m and occassionally greater depending on the data set. All models have problems with trees that have variable early or late height-growth patterns, when predicted curves pass through the same height at 50 years. Attempts have been made to explain this early and late height-growth variation, and ecological or soils classification systems have been used to stratify sites that appear to have different height-growth patterns.

Monserud modified equation [4] by expanding $\beta 2$ to include dummy variables representing habitat types in Idaho and relating them to height growth differences in Douglas-fir. Habitat-specific dummy variables reduced the unweighted standard error significantly from 1.33 m to 1.11 m. He found differences due to habitat type were trivial at young ages and only 70 years after BHA was reached did habitat effects become significantly different.

Payandeh (1991) modified equation [3] in a manner similar to that of equation [2] to form equation [12]. He used FEC Clay Belt vegetative units in northeastern Ontario to explain differences in height-growth patterns. He observed that some vegetative units contained more variability than others, and so he grouped a number of units that appeared highly variable together and fit the model separately. For the more consistent units, he used dummy variables to represent FEC units for several parameters.

$$SI_b = 1.3 + \left[\frac{\beta_1 H_t^{(\beta_{21} + \beta_{22} * D_1 + \beta_{23} * D_2)}}{1 + EXP\,(\beta_3 + \beta_4 \ln(BHA) + \beta_5 \ln(H_t))} \right] + \varepsilon \qquad [12]$$

where: D1 and D2 are dummy variables

The standard error of the estimate (SEE) of the site-index prediction equation [Eq. 12] was 0.279 m. When H_t was used as the dependent variable, stratification of the data reduced the SEE of the more consistent FEC units (±0.843 m) from the pooled data set (±1.024 m) without the use of dummy variables.

Larocque (1991) made a similar study of black spruce in northeastern Ontario using FEC-vegetative units. He used permanent sample plot remeasurement data, ranging from 1 to 12 remeasurements of total-height of 100 trees/ha. Larocque used a modified Weibull function (Yang et al. 1978) for predicting site-index at 50 years total age for each plot. He found different height-growth patterns for certain FEC units, but wide variations in height-growth remained especially in the first 50 years of total age.

Goelz and Burk (1992), Deschamps (1991) and Inions (1992) provide detailed reviews on reparameterization methods used in site quality research.

2.2.1.3 Site Tree Selection

Trees selected for site index measurement are important because these trees provide the standard for estimating forest site quality. Thus site trees should not be affected by suppression, top breakage, or injuries that prevent free and uninterrupted height growth. Most studies in eastern North America sample 3 to 5 trees in the dominant and/or codominant crown classes that do not show signs of damaged crowns or overtopping; frequently trees are located on 0.08 ha plots in well-stocked, undisturbed, evenaged forest stands (Carmean 1975, 1994, Carmean et al. 1989).

Differences exist between many studies about what trees and how many trees should be selected for site-index estimations. Dahms (1966) and Zeide and Zakrzewski (1993) reviewed these differences in detail. Dahms (1966) points out how the type of height measurement differs between studies: selection of the tallest trees in Australia and the Netherlands (Gray 1945, Braathe 1957), selection of the 100 largest diameter trees per hectare in much of Europe (Braathe 1957) and western North America (British Columbia Ministry of Forests 1990) In northern Ontario, Plonski (1974) measured the tree of mean basal area among the dominant trees on the plot. Rennolls (1978) points out how a standard method for selecting site trees is needed because, since trees are not selected at random, site index is affected by sample and plot sizes. Zeide and Zakrzewski (1993) emphasize that one should use the same quantity and quality of site trees used to develop the site-index curves to obtain unbiased results since the trees are not randomly chosen.

Dahms (1966) examined height-growth of lodgepole pine on 67 0.08 ha plots and 27 0.04 ha plots in the northwestern United States. The following four measures of height were related to both gross periodic annual volume increment and stand volume estimates for lodgepole pine: a) average height of the tallest trees selected, b) average height of the largest diameter trees, c) average height of dominant trees, d) average height of dominant and codominant trees. He found that heights of the tallest, and heights of the dominant trees provided height estimates that were most closely related to volume. Increased subsampling resulted in greater precision, although the gain was small. Dahms suggested sampling only the tallest tree per plot because greater gains were made by increasing the number of plot locations in a stand than by increased subsampling.

Heights of trees selected for site-index estimation varies thus studies have considered the precision associated with measuring different numbers of site trees. Johnson and Carmean (1953) studied sampling errors associated with measurements using different numbers of dominant and codominant site trees. They found that sampling errors for estimating site index were quite large when only a few site trees were measured.

Lloyd (1981) generated sample-size tables for measuring tree heights at various ages to attain specific accuracy levels for various stand ages and site-index class intervals (index age 50). He found the standard error to be very large prior to 20 years and the sample size to meet accuracy levels became extremely high. However, after 20 years, 2 to 4 trees were sufficient for predicting site index within a 6 m class for loblolly pine in South Carolina. Lloyd defined a plot as an area of sufficiently small size to be deemed uniform in productivity; the plots were large enough to contain 30 to 35 mature dominant trees. McQuilkin and Rogers (1978) found that confidence intervals for predicting white oak site index were reduced 50 % when the number of sample trees was increased from one to five.

Ziede and Zakrzewski (1993) determined a selection method for site trees to incorporate stand density and mortality. They proposed developing site-index curves using a sampling intensity of the 100 largest diameter trees per hectare. In the process of establishing the plots, it is necessary to sum the total number of living trees within each plot. A fixed proportion is computed by dividing the number of site trees sampled by the total number of live trees in the sample plots. Field estimates of site index would be computed as:

$$h_c(t) = \frac{h_n(t) + h_p(t)}{2} \qquad [13]$$

where: $h_c(t)$ is the combined estimate of top height at any given age t
$h_n(t)$ and $h_p(t)$ are the estimates of top height at age t obtained from the fixed number and the fixed proportion methods, respectively

They were developing a methodology that could be reproducable from a plot tally sheet and would provide unbiased estimates of both height and diameter sampled over time. They examined remeasurement data from 11 permanent sample plots (0.08 ha plot size) in northern Ontario. Plots were 56 to 102 years old and were measured 5 to 7 times over periods ranging from 21 to 34 years. Their study tracked the diameter of the 8 largest trees at the final remeasurement and compared the diameter of these trees to the diameter of the actual largest 8 trees in each remeasurement period for the 11 plots. The bias was similarly computed using the fixed proportion method. They found that average diameter was constantly overestimated using largest diameter trees, and that diameter was underestimated using the fixed proportion methodology. Since the bias in diameter estimates cancelled each other, Zeide and Zakrzewski concluded that an average using both methods would give the most accurate estimate of diameters and heights.

Murchison and Kavanagh (1989) recommended that when conducting stem-analysis studies in mature jack pine, two stage unequal probability samples be used to obtain volume estimates. A minimum of 2 to 3 plots of 20 to 25 trees each should be established per study area, and a minimum of 2 to 4 trees subsampled per plot for stem analysis. Future research may find similar sampling intensities are sufficient for determining site index.

2.2.1.4 Site-Index Curves for Jack Pine

A list of jack pine site-index curves is given in Table 2.1 This table shows a trend from older harmonized to more recent polymorphic curves. Few studies, except for Carmean and Lenthall (1989) and Jackman (1990), state a level of accuracy and confirm the accuracy using an independent validation data set.

The earliest study (Bedell and MacLean 1952) used total heights and ages of dominant jack pine trees in both pure and mixed stands across Forest Section B.9 in north central Ontario. A linear regression line was fitted to tree heights to create an average guide curve. Dominant heights were then read from the lines for specific ages. Jack pine was sampled across a range of six site types (Hills 1950) to create yield curves for each site type. Bedell and MacLean assumed that the height-growth patterns would be the same for each site type based on work by Gaiser (1950) with loblolly pine (Pinus taeda L.) in the coastal region of Virginia and the Carolinas.

Other early site-index curves, such as those by Gevorkiantz (1956) and Kabzems and Kirby (1956), also used a guide curve procedure for developing harmonized site-index curves. These curves assume the same proportional pattern of height growth for all regions, soils and levels of site quality.

Table 2.1 Summary of site-index curves for jack pine.

Reference	Area Sampled	Type of Curves
Bedell and MacLean (1952)	North Central Ontario	Harmonized
Gevorkiantz (1956)	Saskatchewan	Harmonized
Kabzems and Kirby (1956)	Saskatchewan	Harmonized
Plonski (1956; 1974)	Ontario	Harmonized
Chrosciewicz (1963)	Northern Ontario	Harmonized
Jameson (1963)	Saskatchewan	Harmonized
Heger (1968)	Saskatchewan Manitoba Ontario	Harmonized
Bella (1968)	Manitoba	Harmonized
Shea (1973)	Northern Ontario	Harmonized
Carmean and Lenthall (1989)	North Central Ontario	Polymorphic
Jackman (1990)	Northwestern Ontario	Polymorphic
Ker and Bowling (1991)	New Brunswick	Polymorphic
Millar and Woods (1989)	Northern Ontario	Polymorphic
Zakrzewski (1990)	Northern Ontario	Polymorphic
Goelz and Burk (1992)	North Central Ontario	Polymorphic

Jameson (1963) studied height growth on six ecologically defined sites in Saskatchewan (Figure 2.2). Stem-analysis data were taken from two dominant trees on 28 0.08 ha plots; trees used were those that most closely approximated the computed mean height. Three to six plots were established on each of six site types for a total of 28 plots and 55 trees used for stem analysis. The stands averaged 70 years and ages ranged from 64 to 73 years. Height-age curves were drawn for each tree; age zero was set at stump-height (0.3 m) and heights were read at 10 year intervals starting at 3 to 5 m. Average curves were computed for each site type using a linear regression model [Eq. 14].

$$\log H_t = a + b\,(\log Age_t) + \varepsilon \qquad [14]$$

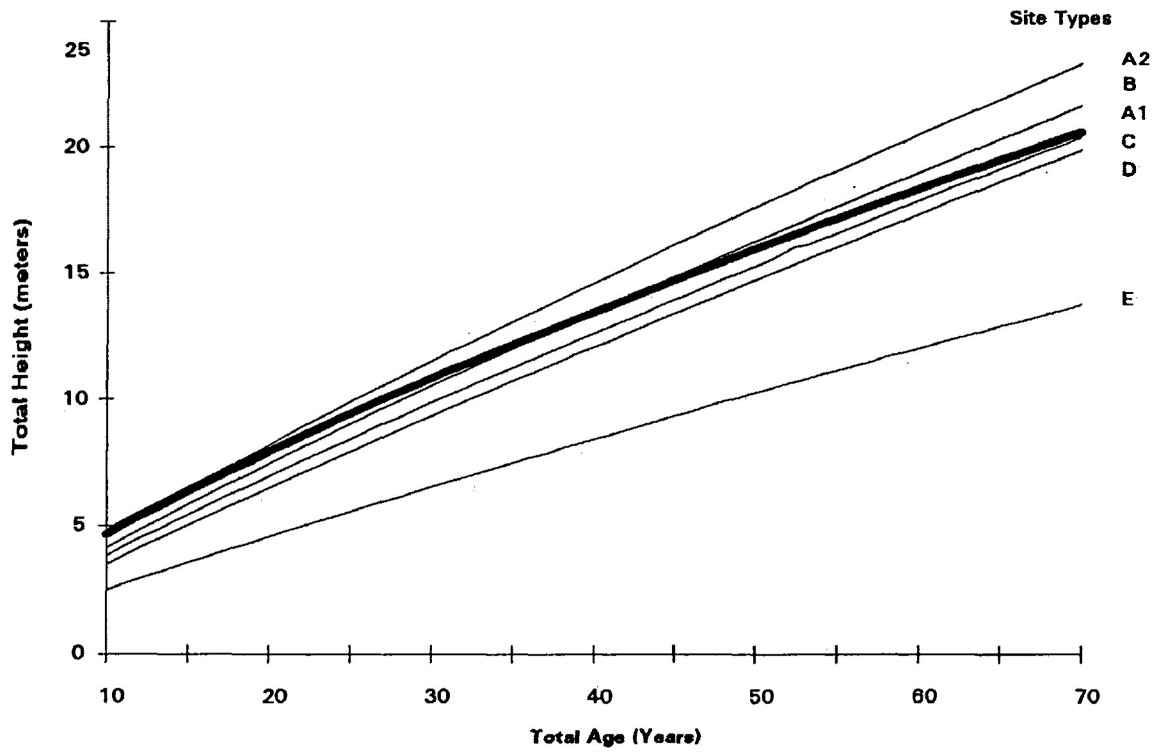

Figure 2.2 Height-age curves for jack pine in relation to site type. The height-growth curve for the sandy clay loam soil group (Site Type A1) is high-lighted to show a pattern that differs from the other site types (heights are converted from feet to metres) (Jameson 1963).

High correlation coefficients (0.98-0.99) and small standard-errors of the regression coefficient (+/- 0.0155 to +/-0.0232) for slope (b) suggest consistent linear height-growth patterns between the logarithums of height and age occurred in each site type (Figure 2.2).

The slope of the curve for Site Type A1 was significantly different from the other site types; A1 was a stratified soil with sandy loam surface horizons over a sandy clay loam C horizon. In contrast, the other site types had sandy and sandy-loam C horizons. The clay Site Type A1 had greater heights before 25 years but had lower heights after 50 years.

It is not surprising that Jameson (1963) showed a strong linear relationship between the logarithums of height and age because early growth was eliminated by modelling height starting at 10 years of age when trees were 3 to 5 m tall. Secondly, polymorphic differences tend to be more pronounced at older ages and his oldest plots were only 64 to 73 years. Jameson's study involved six site types having medium to poor site quality (site index: 15.95, 17.62, 16.27, 15.30, 14.74, 10.25 m) and polymorphic height growth due to site quality would be minimized for such a relatively narrow range of site quality. A t-test was used to compare slope coefficients.

Jameson concluded that three interrelated factors (soil moisture, soil nutrients, soil texture) probably have a greater influence on height growth than other factors. Trends shown in Table 2.2 suggest higher site indices are related to increased cation exchange capacity (C.E.C), increased silt content and greater moisture regimes. Also on sites with similar sandy soil profiles (Figure 2.3 and Table 2.2), higher site indices were observed when moisture regime increased.

Jameson (1963) concluded that the curves developed by Kabzems and Kirby (1956) could not be considered valid in his study area. He suggested that curves for the ecologically defined sites would be more appropriate because his observed height-growth curves did not flatten as rapidly as predicted by the Kabzems and Kirby curves at 60 to 70 years total age.

Table 2.2 Site index and soil relations for six ecological sites (Jameson 1963).

Site Type	Site Index	Soil Moisture Regime	C.E.C. Average of Horizons	Soil Texture Class
A1	15.95	Fresh	36.83	SL / SCL
A2	17.62	Fresh	23.61	SL
C	15.30	Fresh	10.05	S
B	16.27	Moist	9.52	S
C	15.30	Fresh	10.05	S
D	14.74	Dry	8.62	S
E	10.25	Very Dry	5.43	S

*SCL sandy clay loam * S sand
*SL sandy loam

Bella (1968) developed yield tables for jack pine in southern Manitoba according to five ecological site types (Figure 2.3).

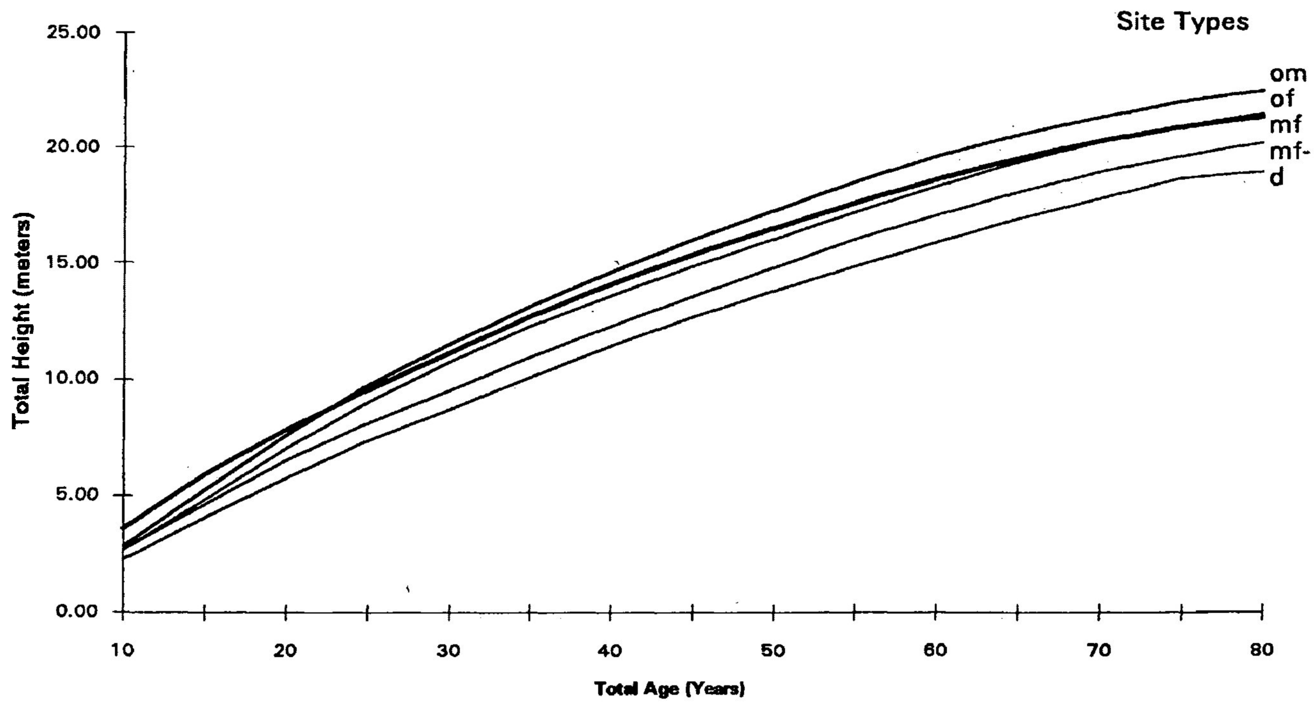

Figure 2.3 Height-age curves for jack pine in southern Manitoba. The height-growth curve for the Mesotrophic Fresh (mf) site type (stratified soil) is high-lighted to show the change in pattern compared to other more uniform site types (Bella 1968).

Bella sampled the following five site types as described by Mueller-Dombois (1964): Oligotrophic Moist type (om); Oligotrophic (nutritionally poor) Fresh type (of); Mesotrophic (nutritionally intermediate) Fresh type (mf); a drier subtype (mf-), and; Dry type (d).

Bella collected stem-analysis data on 10 to 14 dominant, mature (age 50 to 80 years) jack pines on each site type, but did not document the sampling intensity and plot size from which the trees were selected. An average curve was then plotted based on the average height value observed at 5 year intervals for each site type. Bella used co-variance analysis and a simple parabolic function to compare the site types and found significant differences among all site types (Figure 2.3). The most apparent difference is that of the mf site type. This site type has a sand cap approximately 30 to 90 cm deep that overlies gravelly beach deposits; this sand cap is also found on slopes of recessional moraines, on ground moraines of bouldery loamy sand and on outwash sand over clay loam. The ranking of site quality shown in Figure 2.3 follows the order of increasing moisture regime.

Heger (1968) developed site-index curves for jack pine based on stem-analysis data collected in Saskatchewan, Manitoba and Ontario. Data were collected between 1920 and 1949 by various forest researchers in different forest regions of Canada. The sampling intensity and plot size were not documented. Heger used 206 trees including 123 dominants, 65 codominants, and 18 unclassified trees; the youngest plot was 50 years BHA. Stem-analysis intervals varied, consequently, total heights at 5-year BHA intervals were estimated using linear interpolation.

Heger's study was different from most other studies because at each 5 year age interval, from 5 to 45 years, he fitted equation [15] to predict total height given site index.

$$HT = a1 + b1*(SI) + \varepsilon \qquad [15]$$

The correlation coefficients at each 5 year interval showed that early growth was variable making it difficult to predict site index of jack pine. In addition, Chi-square tests for normality showed heights were not normally distributed at ages 10, 20, and 30 years.

Unlike other studies, he determined accuracy by comparing the average of the ten best and ten worst trees to the computed group average value. Heger found that values were within 0.45 m of the average across each five year interval.

Ker and Bowling (1991) sampled 114 plots across a range of site types in New Brunswick. Their jack pine plots ranged in age from 51 to 175 years breast-height age and covered a range of site index from 7.2 m to 21.0 m. They fitted a generalized Chapman-Richards model [Eq. 3]. The coeficients from equation [3] were then used in equation [8]. The sample intensity and plot size used for selecting free growing dominant and codominant mature jack pine trees for stem analysis was not documented. Ker and Bowling compared estimated heights predicted by their equation to heights predicted by the Carmean and Lenthall (1989) equation for site index 21 m and 80 years breast-height age. Results showed that their model predicted a height of 25.24 m which was within about 1 % of the height predicted by Carmean and Lenthall.

Chrosciewicz (1963) sampled a narrow range of site conditions in northern Ontario; sampling was restricted to almost pure jack pine stands on deep uniformly sorted, sandy soils of aeolian, fluvial, and glaciofluvial origin. Areas sampled included five soil moisture regime classes (Hills 1952, 1954), three soil texture classes and two soil petrography classes. Within each petrography class sampling was replicated in Hill's (1959) site regions 4E, 4S, and 3W. Stem analyses were made using three to six dominant trees per combination of soil moisture, texture, and petrography classes in stands ranging from 43 to 97 years.

Chrosiewicz found that height growth at 50 years total age varied with soil moisture class, soil texture class and soil petrography in each of the regions and between the regions. He recommended extending the height-growth study over a much wider range of site conditions; he also recommended sampling the range of site productivity in the three categories described above and, further, a study of the effects of site and stand density on jack pine height-growth.

Chrosiewicz plotted the stem-analysis data for each sectioned tree. He made average height-growth curves for each treatment combination using readings made at 5-year intervals from the tree height-growth curves. Chrosiewicz did not formulate his curves. Comparisons were based on site-index values interpolated from the average height-growth curve for each treatment unit. He did not make statistical comparisons.

Plonski (1956, 1974) located 181 plots in jack pine stands across Ontario; stem analyses were made on each plot using at least one tree of average basal area among the dominants on each plot. Measurements from each plot were used to develop yield tables and also to estimate current annual increment based on stem analysis from trees of average basal area. Height-age curves were stratified into provisional site classes...e.g. site classes 1, 2 and 3. Plonski graphically smoothed irregularities in the height-age curves by smoothing irregularities in the current periodic height increment (c.p.i.) curve. He plotted height-age curves for six site-class categories, examined trends, and then developed three site-class curves. The resulting anamorphic curves were adjusted or "balanced" based upon the position of the average curve so that curves passed through desired end points that were predetermined by Plonski; this ensured 45% of the data were in site class 2, 27.5% of the data were in site class 1, and 27.5% of the data were in site class 3. The 20 year age category that had the widest range of heights was used to examine the distribution of plots and thus determine the desired end points of the average guide curve. Plonski did not document the distribution of plots based on site class, age class, or site description.

Payandeh (1974) formulated Plonski's site-class curves using Ek's (1971) height-growth model [Eq. 2] which expresses polymorphic growth; Payandeh also formulated a site-index prediction equation [Eq. 11]. However, Plonski's original anamorphic curves were harmonized curves, therefore, Payandeh's formulations could not express polymorphism and can only be considered an alternative to the original Plonski curves for graphical interpretations.

The first polymorphic site-index curves for jack pine in north central Ontario were developed by Wiltshire (1982). Stem-analysis data from three to five dominant and codominant trees per plot were collected from 34 plots; his preliminary curves used the Ek [Eq. 2] model fitted to breast-height age data.

The data set was expanded by Lenthall (1986) and Carmean and Lenthall (1989) using the same sampling procedures; a wider area covering a greater range of soil and site conditions was sampled resulting in a total of 141 plots in north central Ontario. Lenthall sampled the following landforms: 1) shallow to bedrock morainal soils (<1.0 m); 2) deep morainal soils (>1.0 m); 3) outwash glacial sands; and 4) lacustrine silts and clays. Comparisons showed that breast-height age curves were more precise than height-age curves based on total age. Lenthall fitted Ek's (1971) model [Eq. 2] to data for each landform; landform curves were then compared at SI = 16 m, and the conclusion was reached that there were no significant differences in height-growth patterns for the four landforms. Lenthall came to the same conclusion when he compared the 95% confidence intervals for the individual parameter estimates for the four average landform curves and found overlap in all instances.

Lenthall (1986) developed polymorphic site-index curves using Ek's (1971) model [Eq. 2] and a computation data set of 109 plots. He found that future estimates of height growth using site index and breast-height age should lie within +/-1.03 m of the true height (α=0.05). Lenthall also fitted Payandeh's site-index prediction model [Eq. 11] to all breast-height age data; this model also was fitted to data older than 20 years BHA. He concluded that site-index prediction using data before age 20 was too variable, and that the site-index prediction equation should only be based on data older than 20 years BHA. The 95% prediction interval for the final site-index prediction model was +/- 1.17 m for data >20 years breast-height age. Lenthall used 32 verification plots to confirm the precision of his height-growth formulations and site-index prediction equation formulations.

Lenthall (1986) tested for the presence of polymorphic height-growth in jack pine by separately fitting the non-polymorphic Richards 3-coefficient model [Eq. 1] to data representing individual 2 m site-index classes. He found distinct polymorphic height-growth patterns. Poor sites had almost linear height-growth, and as site index increased he found that height growth became more curvilinear. Lenthall compared his polymorphic site-index curves to the Plonski's (1974) curves; he found that Plonski's curves underestimated height growth by about 0.75 m at 80 years, and slightly over estimated height growth before 50 years.

Jackman (1990) tested Lenthall's site-index curves to determine if they were applicable to the northwestern region of Ontario. Jackman used stem analyses on a single well-formed dominant tree from 61 plots; almost pure stands were sampled with more than 70% of the basal area in jack pine. His results showed that height-growth patterns on moraines and glaciofluvial soils in the northwestern region were similar to those observed by Lenthall in the north central region. But he found that height-growth patterns on lacustrine soils had slightly more rapid early height growth, and somewhat slower growth after 50 years than predicted by the Lenthall curves for north central Ontario. Jackman's results were similar to the trends observed by Jameson (1963) who also found that trees on soils having silt, or clay textured C horizons had somewhat slower height growth after 50 years (Figure 2.3).

Goelz and Burk (1992) fitted a base-age invariant model [Eq. 5] to data collected by Lenthall (1986) and Carmean and Lenthall (1989). Their model did not predict height at base age 50 years better than the Ek model used by Lenthall (1986) and Carmean and Lenthall (1989). The advantage of their base-age invariant model is that it minimizes residual squared error for all possible base ages and can predict height at any age without first having to determine a site-index value. The Goelz and Burk model [Eq. 5] eliminates the step of having first to determine site index before predicting height. Goelz and Burk found that the Ek model

[Eq. 2] performed better before index age, and their Goelz and Burk model was better after index age. Both models had very similar mean deviation, similar mean absolute deviations, and similar root-mean-square-error when compared using the same validation data set. Goelz and Burk (1992) concluded the base age invariant model [Eq. 5] captured the true shape of the polymorphic pattern better than equation [2].

Zakrzewski (1990) fitted a functional model to stem analysis of 130 dominant jack pine trees from 36 permanent sample plots located in northwestern and north central Ontario. The same plots were used by Millar and Woods (1989). He fitted his functional model using total age.

2.2.2 Growth Intercepts

Growth intercept methods are generally used with uninodal species such as red and white pines, white spruce and Douglas-fir where internodes are easily recognizable. The method is generally used for young stands not suitable for predicting site index directly from site-index prediction equations. Growth intercept refers to the average length (or total length) of several internodes measured above breast height as a measure of site index (Carmean 1994). Thrower (1986) studied young white spruce and red pine plantations and found that a minimum of three internodes measured directly above 2.0 m for white spruce and 1.5 m for red pine resulted in the best predictions of site index at base ages 15 and 20 years breast-height age, respectively.

Multinodal species such as jack pine usually are not suitable for growth intercept studies due to difficulties in identifying limb whorls marking annual height growth. Thus there are no reported growth-intercept equations for jack pine. A disadvantage of the growth-intercept method is that early height growth of a tree is not always representative of height growth in later years (Carmean 1975). The advantage of using growth intercepts are that internodes are quickly and easily measured without damage to trees; errors associated with counting annual rings or estimating total height are avoided, and measuring internodes above breast height eliminates slow and erratic growth which occurs below breast height.

2.2.3 Site-Index Comparisons Between Species

Forest managers need to be able to determine which species is the most productive for a specific site. This is important because tree species have different growth rates on different site conditions and thus differ in site index and in volume and value of yield Accordingly, for each area site index estimates are needed for all tree species that might be considered for management on that area.

Site-index comparison graphs show the site-index relationship between two species that may occur on the same site (Figure 2.4). In order to develop such graphs fully-stocked, evenaged, undisturbed mixed stands are required which contain suitable dominant and codominant trees of two or more tree species. Regressions expressing site index of one species in relation to another species are then computed.

Figure 2.4A, 2.4B, 2.4C based on regression equations, show site-index differences for the various species commonly associated with jack pine in mixed conifer and hardwood stands. For example, trembling aspen has higher site index values than jack pine on good sites (Figure 2.4A), but for poorer sites ($SI < SI_{BH15}$) black spruce has higher site index values than jack pine (Figure 2.4C).

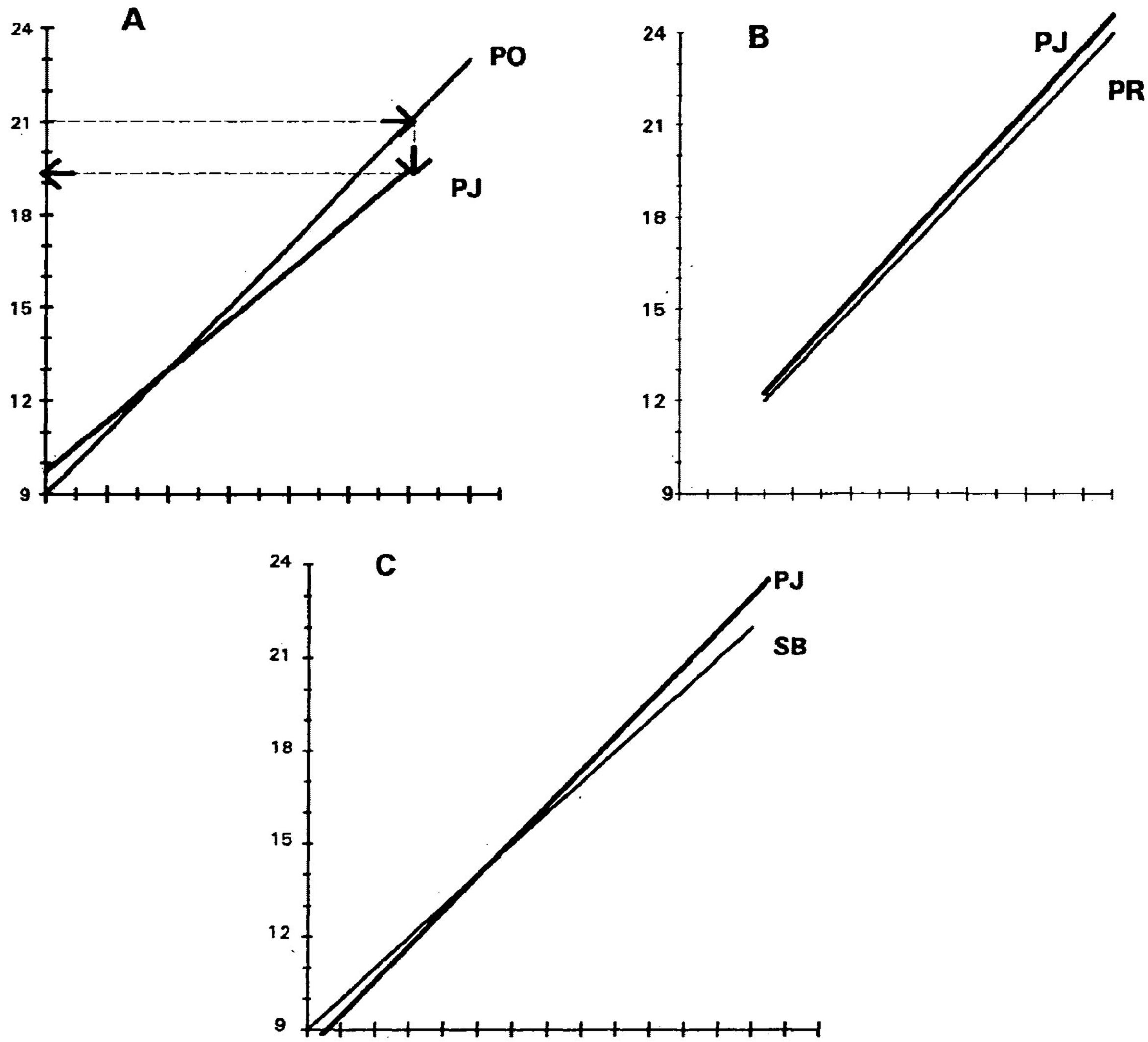

Figure 2.4 Site-index comparison curves between jack pine and trembling aspen (A), red pine (B), and black spruce (C). For example, measured site index of aspen is read across to the aspen line then down to the jack pine line and back to the Y axis for an estimation of jack pine site index.

Figure 2.4B shows that jack pine has somewhat higher site index values than red pine at total age 50 years. Not apparent in Figure 2.4B is that jack pine has more rapid early growth than red pine; however, red pine maintains height growth in later years and is, after 50 years, usually taller than jack pine (Wilde et al. 1964). Neither is it apparent that red pine stands support higher basal areas than jack pine and would have higher volumes even though site indices may be similar (Alban 1978).

2.3 Indirect Estimation of Site Index

As shown in Figure 2.1, tree measurements can not be used to directly estimate site index for stands that have been clearcut or partially cut, where stands are uneven in age or poorly stocked, or where stands are very young. However, site index can be indirectly predicted by using soil and site characteristics that are known to be closely related to site-index. Also site index can be estimated in the event soil types or ecosystem types are defined using soil and topographic features closely related to site index (Figure 2.1).

Soil-site evaluation studies involve locating plots in stands which represent the range of site quality, soil characteristics (physical and chemical), topographic and climatic features in a study area. On each plot site index is determined from stem analysis, then multiple regression methods are used to identify soil and topographic features that are closely correlated to site index. The output is a regression equation which predicts site index using a combination of soil, topographic, and climatic features closely related to site index. Most often the variables of importance are those soil properties "... which influence the quality and quantity of growing space for tree roots" (Coile 1952).

Tree species have different physiological responses to soil and topography. There are numerous soil-site studies relating site index of jack pine to soil, topographic, and climatic features (Table 2.3) (Coile, 1952; Carmean, 1975, 1994; Kayahara, 1989; Schmidt 1986; Schmidt and

Carmean 1988; LeBlanc 1994). These studies indicate that site features of importance differ according to tree species, study areas, variables tested, and methods of statistical analysis. The jack pine soil-site studies listed in Table 2.3 cover a wide range of soils, topography and climate spread across eastern and central Canada as well as in Michigan and Minnesota.

Table 2.3 Soil variables related to jack pine site index (LeBlanc, 1994).

REFERENCE	AREA	SOIL VARIABLES RELATED TO SITE INDEX	R^2
Pawluk and Arneman (1961)	Minnesota	a) percent very fine sand+silt+clay in A2+B horizons b) soil moisture holding capacity in Ao, A2,B2,B3 c) cation exchange capacity of A2 and B horizons d) very fine sand+silt+clay of A2 horizon	0.69 0.44 0.54 0.69
Chrosciewicz (1963)	Northern Ontario	a) moisture regime , f(porosity, texture) b) texture c) macroclimate (ie. Hills (1959) site regions)	N/A
Shetron (1969)	Wisconsin & Michigan	a) depth to fine sand or finer textured soil horizons b) levels of P, K and Ca in A and B horizons	
Hannah and Zahner (1970)	Wisconsin & Michigan	a) depth to fine textured lenses	N/A
Schmidt and Carmean (1988)	North Central Ontario	Stratified by Landforms a) Shallow to bedrock: depth to bedrock & coarse fragments in A horizon b) Moraines: depth to root restricting layer, percent coarse fragments in the C horizon, and percent clay in the A horizon c) Glacialfluvial: depth to root restricting layer, percent slope d) Lacustrine: thickness of A horizon, and pH of BC horizon	 0.83 0.65 0.66 0.75
PLANTATIONS: site variables related to current annual height increment			
Wilde et al. (1964)	Wisconsin	a) percent organic matter of soil+available phosphorus and silt + clay content	0.59
Shetron (1972)	Wisconsin & Michigan	a) depth to maximum fine sand b) depth to max. f. sand and thickness of B horizon	0.79 0.80
Hamilton and Krause (1985)	New Brunswick	a) soil drainage class+depth Ae horizon+depth rooting+occupancy of Kalmia and Vaccinium b) same as (a) except substituting total nitrogen for rooting depth	0.78 0.86

Different results occur for some studies where site index is based on total age instead of site index at breast-height age. Also for some studies site-index estimates were based on older harmonized curves instead of on stem analysis.

Features found to be closely related to site index depend on the soil and topographic conditions of the study area. For example, aspect, slope position and topographic features are usually important in mountainous or hilly terrain instead of gently undulating topography. Soil texture and drainage tend to be important site factors in areas which have a wide range in texture and drainage (Carmean 1975). This is apparent in studies by both Chrosciewicz (1963) in northern Ontario and Hamilton and Krause (1985) in New Brunswick. Both study areas have glacial soils which differ widely in texture, and both have areas of rapidly drained to poorly drained soils.

Study areas with mostly coarse-textured soils usually show increases in site index with increased silt and clay (Carmean 1975, Pawluk and Arneman 1961, Hannah and Zahner 1970, Wilde et al. 1964, Zahner and Hedrich 1966, Shetron 1969, Shetron 1972 , Van Eck and Whiteside 1958, Jameson 1963, Chrosciewicz 1963). All studies found that features highly correlated with site index were usually those features in the A and B horizons (approximately 45 cm) where the majority of jack pine root systems are found (Cheney 1932). Greater amounts of fine particles is usually associated with increased water-holding capacity (Ralston, 1964) and with higher cation exchange capacity (Carmean 1975).

Schmidt and Carmean (1988) and LeBlanc (1994) found only weak correlations when all study plots were analyzed together, but much stronger relationships were found when plots were stratified by landform.

Soil properties found to be strongly correlated to site index differed for each landform; however, all the properties listed in Table 2.3 affect the quantitiy and quality of soil available for root growth.

The influence of physical and chemical soil properties on tree growth are discussed by Carmean (1975), Spurr and Barnes (1980), and Pritchett and Fisher (1987). All of the studies, except for Schmidt and Carmean (1988), found no relationship between site productivity and C horizon characteristics which is the basis for defining soil categories of the FEC (Forest Ecological Classification System) used in northwestern Ontario (Sims et al. 1990).

Shea (1973) commented in a study of 14 jack pine plots that variation in height-growth patterns (polymorphism) was due to soil properties. He noted from stem analyses that a soil profile with fine surface soil over a coarse textured horizon resulted in rapid early growth which abruptly stopped at ages between 35 and 40 years. He also noted that trees on soils with deep root penetration characteristically had slow initial growth but that growth was sustained longer than on other soils.

Site quality was classified by Bakuzis (1959) by ranking areas according to synecological coordinates based on nutrient and moisture regimes. He constructed an ordination with a nutrient regime on the vertical axis and moisture regime on the horizontal axis. Both nutrient and moisture regime values were based on estimated cumulative needs of the species present within a stand. The moisture and nutrient regime values were estimated by creating a species list on a particular site and ranking the nutrient or moisture competitiveness of a particular species found on that site. The problem with this approach is that very little is known about the nutrient and moisture requirements of many understory shrubs and herbaceous plants; consequently, moisture and nutrient needs of many species were simply estimated. Such coordinates show weak relationships with various moisture and nutrient characteristics such as level of silts and sands in the upper A and B horizons (Pluth and Arneman, 1963). Poor relationships between site index and synecological coordinates for jack pine occur because of weak relationships between soil characteristics and synecological moisture and nutrient regimes (Frissel and Hansen 1965, Pluth and Arneman 1963).

Coile's (1952) original concept was that site quality is largely a function of the quantity and quality of growing space for tree roots. Many of the studies listed in Table 2.3 found various soil depth and soil profile properties highly correlated with growth of jack pine and thus confirm Coile's ideas.

2.4 Factors Influencing Height-Growth of Jack Pine

Height growth may be influenced to varying degrees by biological (biotic) factors such as stand density, genetic variation, competing vegetation, diseases, insects, and past history of a stand (Coile 1952, Ralston 1964, Vincent 1961, Carmean 1975, Pritchett and Fisher 1987, Inions 1992, Burns and Honkala 1990). Abiotic factors, related to the height growth of trees, include:

1)	Climatic variables:	temperature,photoperiod, precipitation
2)	Physiographic variables: (topographic)	altitude, aspect, slope gradient, topographic position categories ie. crest versus bottom.

3)	Soils variables:	parent material, soil depth, water table, soil moisture, soil aeration and soil nutrients

Many of the abiotic factors (ie. climate, topography, soils) listed above have been quantitatively expressed and included in soil-site multiple regression equations (Table 2.3). Soil-site studies can minimize the effect of biotic factors (ie. density, competetion, insect and disease damage) by not placing sample trees or placing plots in forest stands that have these problems.

2.4.1 Density

Height growth is usually considered to be independent of stand density. However, in less dense stands height growth has been reported to be less than in fully stocked stands for certain species (Carmean 1975). Conversely, trees in overly dense stands may have stagnanted height growth that is less than trees in normally stocked natural stands (Holmes and Tackle 1962).

Holmes and Tackle (1962) reviewed the literature examining the relationship between density and height growth. They concluded that no generalization is applicable to all species, age class and sites. Their review included a discussion of the effect of density on jack pine.

Initial stand densities of managed plantations do not appear to affect early height-growth development of jack pine (Bella 1986). Bella found 15 year old jack pine planted at initial spacings of 1.2 m, 1.8 m and 2.4 m had similar average stand heights of 8.0 m, 8.3 m, and 8.2 m, respectively. Buckman (1964) examined jack pine plantations at operational spacings and did not observe significant differences in height-growth of jack pine on good sites as a result of different planting densities. But Buckman observed that crown differentiation occurred more rapidly on good sites than on poor sites.

Reduced height growth can occur at wide spacing (Bella 1986). Bella (1986) found jack pine, planted at 3.0 m spacing, had an average height of 6.9 m which was significantly shorter than heights recorded above for managed plantations on the same site. Reduced height growth may be a result of poor apical dominace of jack pine at wide spacings which results in greater biomass placed in lateral branches.

Jack pine planted at very high densities did not appear shorter than pine planted at managed densities. Guilkey and Westing (1956) found no significant differences in the dominant height of jack pine planted at 0.5 m, 0.9 m, 1.5 m, 2.1 m, 2.7 m spacing 15 years after planting. Zavitkovski and Dawson (1978) found no significant differences in height growth of jack pine 7 years after planting at initial spacings of 0.2 m, 0.3 m, and 0.6 m, which represents 189,036, 111,111, and 26,880 trees per hectare, respectively. They found that, although the 0.2 m spacing was not significantly different from the other spacings, jack pine did not appear to be expressing dominance.

Cayford (1961) studied natural jack pine stands spaced at age 18 years on a poor site that had an inital densities of 10,000 trees per hectare at the time trees were spaced. Trees spaced at 0.9 m and 1.8 m were not significantly different in total height at 50 years. Similarly, a stand spaced at 10 years, where initial densities were 28,000 stems per ha, showed no significant height differences at 50 years when spaced to 1.1 m, 1.4 m and 2.0 m spacings.

2.4.2 Genetic Variation

All soil-site studies have some amount of unexplained variation not associated with site features which may be a result of genetic variation. Across local geographic areas, genetic variation in height growth is considered to be relatively insignificant (Carmean 1975).

Phenotypic characteristics of jack pine such as various crown, bark, wood, and cone characteristics varied from northern Ontario and the Lake States west toward Saskatchewan (Schoenike 1962, 1976). Schoenike found a distinct change from the Nipigon area west to Manitoba. Recently Maley (1990) also found that cone and needle characteristics had a distinct gradient that might be useful in the development of future jack pine site-index models. Monserud and Refeldt (1990) studied Douglas-fir in Montana and Idaho and found large changes in a genetic index were correlated to site index at age 50 years.

Results from jack pine provenance trials at 5 to 20 years suggest 10 to 15 percent gains in early height growth are possible through careful selection of seed sources (King 1973, Yeatman 1974, Rudolph and Yeatman 1982). Jeffers and Jensen (1980) examined provenances from 17 locations across Minnesota, Wisconsin, Michigan and one from Ontario. Using height measurements taken at 5 year intervals until 20 years, Jeffers and Jensen found that jack pine maintained height positions over time and they identified a reliable age at which differences among provenances could be evaluated. They compared their results to those of Yeatman (1974) for provenances across Ontario and found that their correlations were slightly higher than those of Rudolph and Yeatman. They concluded that 10 to 15 years are better for estimating future height. Jeffers and Jensen also found that more northern areas

(Minnesota/Ontario border areas) were colder sites which had delayed differentiation in height growth.

Site quality work by Lenthall (1986) in north central Ontario found that reliable site-index predictions were not possible for stands younger than 20 years BHA because of the amount of variation in early height growth. Genetic variation might explain a portion of the variance in early heightgrowth.

3

METHODS

3.1 The Data

3.1.1 Study Area

The area sampled across northern Ontario is composed of two major forest types: 1) Boreal forest, and 2) Great Lakes St. Lawrence forest. The boreal forest has historically regenerated from extensive fire disturbances. Jack pine is a pioneer species and is often associated with black spruce, aspen, paper birch and white spruce. In the Great Lakes St. Lawrence area, jack pine tends to occupy very dry sites because of the competitive pressure from other species. It is often associated with red pine on drier sites and with white pine on more mesic sites (Rowe, 1972).

Historic weather information for northern Ontario was compiled by MacIver and Whitewood (1991) and Chapman and Thomas (1968). These reports show that the mean annual length of the growing season is similar for all regions (140 to 180 days) (Chapman and Thomas 1968). The mean annual precipitation ranges from 500 mm to 700 mm in the Northwestern Region, increases eastward from 650 mm to 900 mm in the North Central Region and increases further to 750 to 1000 in the Northeastern and Northern Regions (MacIver and Whitewood 1991). Mean daily temperature for the year decreases from the southern portion to the northern portion of the study area (5 to -1 °C) (Chapman and Thomas 1968). Monthly mean July temperatures follow a pattern similar to mean growing season length (MacIver and Whitewood 1991). There is a cool band of air that extends downward from the north to Lake Nipigon, Beardmore, and Kapuskasing.

The natural range of jack pine in Ontario (Figure 3.1) covers the heavily glaciated Precambrian Shield that was ice covered during the Wisconsin stage, 7,000 to 10,000 years ago. Land formations resulting from glaciation include: 1) glaciofluvial deposits, 2) glaciolacustrine deposits, and 3) glacial moraines (Boissonneau 1966, 1968; Zoltai 1961, 1965, 1967). A large portion of northern Ontario has very shallow soils formed from ground moraines deposited by receding glaciers overlying bedrock as shown by NOEGTS (Northern Ontario Engineering Geology Terrain Study) maps (Mollard and Mollard 1980 to 1983).

Moraines were separately classified into shallow (<1.0 m to bedrock) moraine deposits, and deep (> 1.0 m to bedrock) moraine deposits. The shallow to bedrock soils also are recognized in the FEC (Forest Ecological Classification) for northwestern Ontario (Sims et al. 1990) as SS1 to SS9 soil types. Soils containing material that is well sorted with rounded coarse fragments were identified as glaciofluvial (GFL); lacustrine sands also were included. Glaciolacustrine deposits (LAC) were fine-textured soils deposited in glacial lake beds with a C horizon texture ranging from silty clay to clay. Soils on alluvial plains also were included in LAC because they are predominantly fine textured. Moraine (MOR) soils were

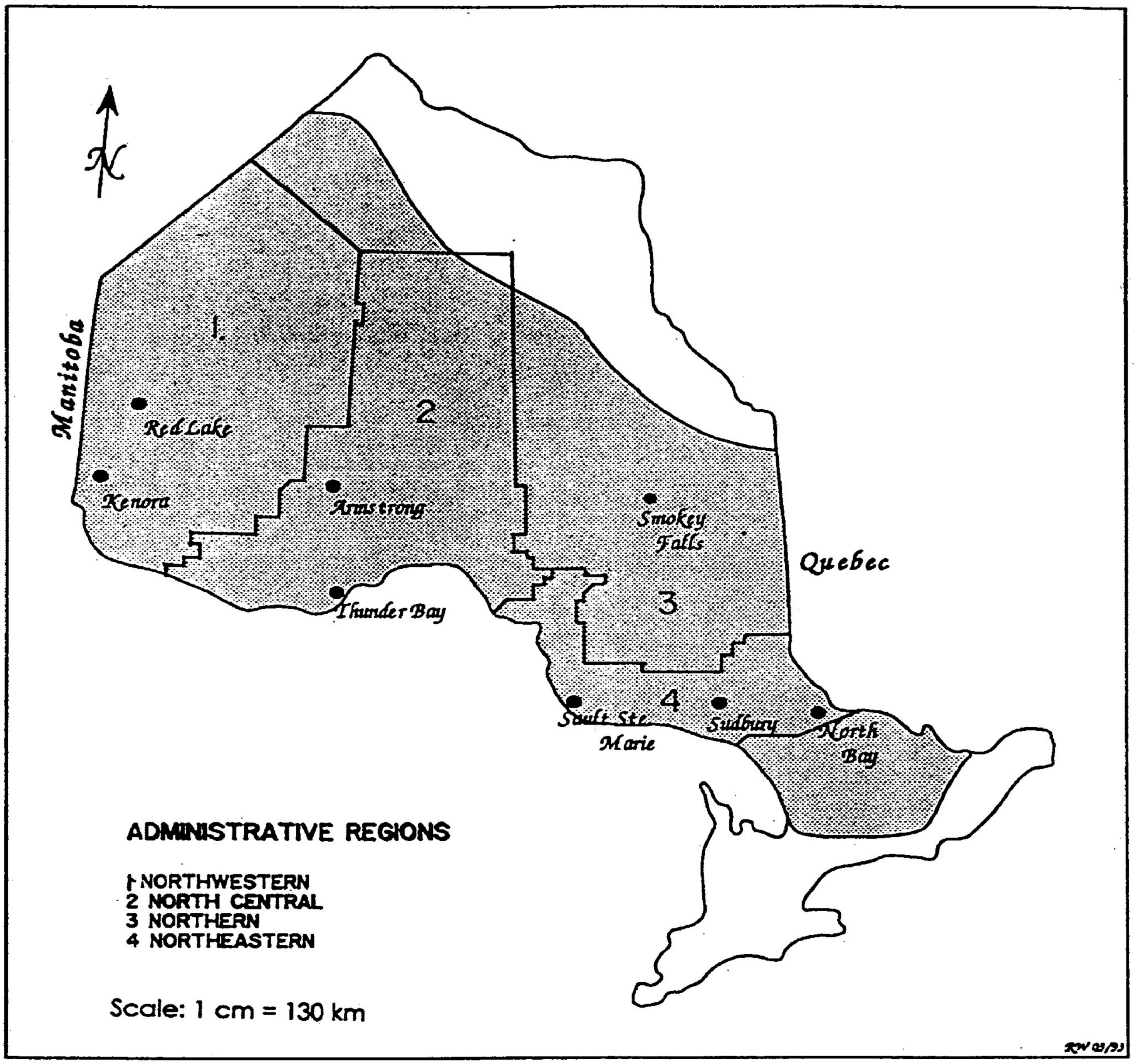

Figure 3.1 The natural range of jack pine in Ontario (Hosie 1979).

unconsolidated deposits greater than 1 m in depth and containing coarse fragments; shallow moraine deposits were called bedrock deposits (BED) and were defined by the FEC as soil profiles with less than 1 m depth to bedrock.

Plots established during the development of the FEC program in the North Central Region were summarized by broad texture, moisture and drainage (Table 3.1) categories based on C-horizon texture and on Hill's (1952, 1954) moisture and drainage regimes.

Table 3.1 Description of major landforms based upon C-horizon texture (Wickware 1989).

Glacial - Landform	Complex	Soil Texture	Moisture Regime	Drainage Class
Glacio-fluvial	Outwash plain Eskers Kames	G.sandy C.Loamy-F.loamy/silty G.Sandy	Fresh	Rapid/Well
Glacio-lacustrine	Beach Delta Plain	G.sandy G.sandy F.loamy/silty	Fresh/Moist	Well/Imperfect
Glacial Moraines	Ground Moraine	C.loamy	Fresh	Rapid/Well

The FEC program (Sims <u>et al.</u> 1990) grouped beach and delta deposits with glaciofluvial deposits resulting in a grouping of mostly sandy soils. Eskers and kames are minor in area thus were not heavily sampled resulting in few glaciofluvial plots having fine textured C-horizons. Shallow moraine material over bedrock (<1.0 m) sites have soil profile characteristics similar to the deeper morainal soils; occassional depressions in bedrock may act as a restrictive layer often resulting in raised water tables and imperfect drainage.

3.1.2 Data Collection

Data used in this study were gathered from various independent studies conducted across northern Ontario (Table 3.2). Many of the studies had different purposes for the data collected and it was necessary to go over study description, old tally sheets and decide if the information was of value to this study.

Stem-analysis data were available from a total of 383 plots obtained from seven independent studies (Table 3.2). As indicated, stem-analysis data were collected from 95 plots established by LeBlanc and Niznowski; data from the other 288 plots were obtained from the various sources listed in Table 3.2

Plots obtained from the seven sources covered an area extending from the Ontario-Quebec border (80° W) to the Manitoba border (95° W). Data were collected as far south as Sudbury and Sault St. Marie (46° N) and north to Red Lake (51° N), Armstrong (50° N) and Smokey Falls (49° N) (Figure 3.1).

The 383 plots were grouped (Table 3.3) by OMNR administrative regions, by glacial landforms, and by site-index classes described below:

Region The four OMNR administrative regions, prior to 1992, were Northwestern (NWR), North Central (NCR), Northern (NOR), and Northeastern (NER) (Figure 3.1).

Landform The four broad landforms were glaciofluvial (GFL), shallow to bedrock (BED), moraines (MOR), and lacustrine (LAC).

Site-index classes Site-index classes were LOW (< 15.0 m), MEDIUM (15.0 m to 19.0 m), and HIGH (>19.0 m).

Table 3.2 Sources of data used in the study.[1]

CONTRIBUTOR	NO. OF PLOTS SAMPLED	SAMPLING METHOD	HEIGHT CORRECTION METHOD	REGION SAMPLED
Leblanc and Niznowski (1989)	95	3 dominant and/or co-dominant trees, 0.05 ha plot	Carmean (1972)	NWR, NER, NOR
NWRS&T[a]	6	" " "	Carmean (1972)	NCR, NWR
Lenthall (1986)	142	" " "	Carmean (1972)	NCR
Jackman (1990)	61	1 dominant 0.05 ha plot	Carmean (1972)	NWR
NEST[b]	24	leading trees selected 10 m X 10 m plot	Carmean (1972)	NOR
Millar and Woods (1989)[c]	45	" " "	Millar (1991)[e]	NCR, NWR
Millar[d]	10	" " "	Millar (1991)[e]	NER
TOTAL	383			

[1]Data were provided by the OMNR from the following sources:

a) NWRS&T — Mark Roddick of North West Region Science and Technology provided six plots from the Boreal Road and Raith area.

b) NEST — Neil Maurer of North East Science and Technology provided stem analysis and soil descriptions for plots used by Murchison (1990) of which 24 plots were used in this study.

c) Millar and Woods (1989) — Stem analysis data from 54 jack pine permanent sample plots were provided by Roj Millar, Ontario Forest Research Institute, Saulte St. Marie, Ontario. A total of 45 plots were used in my analysis; the other 9 plots were discarded because of variable soils on broken bedrock upon review of soils information provided by M. Roddick.

d) Millar — Roj Millar, OFRI, provided stem analysis and soils descriptions for 10 plots collected in the Saulte St. Marie area.

e) Roj Millar — Personal communication, OMNR, May, 1990.

Plots were located in stands classed as jack pine working group (most of the basal area was jack pine). In most cases, these jack pine stands had few black spruce, trembling aspen, or paper birch. All plots were located in fully stocked, evenaged stands that were at least 50 years of age or older. Plots were located in areas having even microtopography and uniform soil profile characteristics. Complete soil profile descriptions (Ontario Institute of Pedology 1985) were made at each plot for the purpose of LeBlanc's (1994) soil-site study.

Table 3.3 Distribution of sample plots according to glacial landforms and OMNR administrative regions, and site-index classes.

LAND-FORM	SITE INDEX[1]	REGION NWR	NCR	NER	NOR	TOTALS
		— Number of Plots —				
Bedrock (BED)	Low	16	17	6	3	42
	Medium	5	12	6	1	24
	High	0	1	2	3	6
Morainal (MOR)	Low	9	6	4	1	20
	Medium	10	28	11	4	53
	High	0	18	5	3	26
Glacial fluvial - (GFL)	Low	14	10	2	3	29
	Medium	22	36	13	25	96
	High	1	20	5	6	32
Lacustrine (LAC)	Low	6	2	1	0	9
	Medium	11	10	1	2	24
	High	3	8	2	0	13
None[2]	Low	0	1	0	0	1
	Medium	0	3	0	1	4
	High	0	1	0	3	4
TOTALS		97	172	58	55	383

[1] Low:<14.99 m; Medium: 15.00 m - 18.99 m; High: >19.00 m

[2] Plots without descriptions for glacial landform

3.1.2.1 Stem Analysis

Three to five free-growing dominant and codominant jack pine trees were selected for stem analysis on each of the 142 plots established by Lenthall (1986), and on the 95 plots established by LeBlanc and Niznowski (Table 3.2). All trees were free from suppression and injuries such as broken tops, forks, or excessive crooks. Trees were sectioned at stump height, 0.75 m, 1.3 m, 2.0 m, then at 1 m intervals from 2.0 m to 13.0 m and at 0.5 m intervals thereafter until the top of the tree.

Jackman (1990) revisited 61 FEC sample plots located in the NWR (Table 3.2) where jack pine had at least 70% of the plot basal area. A single dominant or codominant tree was selected for stem analysis near the edge of each FEC plot. The tree had no sign of stem deformities and was within 0.5 m of the average height of the three to five dominant and/or codominant trees in the FEC plot. He used a soil auger hole to confirm that the soil profile beneath the sample tree was similar to the soil profile description made within the FEC plot.

The 24 plots obtained from NEST and 10 of the plots obtained from Millar had stem analysis of leading trees growing in 10 m by 10 m plots (Table 3.2). Trees from each plot were graphed and the tallest 3 to 5 trees free of defects were selected as site trees.

Millar and Woods (1989) studied how long jack pine could be stored on the stump by examining amount of cull using stem analysis data from 45 permanent sample plots established by a) Kimberly Clark Limited near Longlac, and b) Boise Cascade Limited throughout the northwestern region (Table 3.2). The trees were tallied by 2 cm dbh classes and 2 m height classes; at least one tree was randomly sampled from each diameter class for stem analysis. Stem analysis of trees from the largest diameter classes were graphed and, 3 to 5 site trees were selected that were free of defects.

3.1.2.2 Correction Of Stem Analysis Bias

Ring counts on each section were made using a microscope attached to a digimicrometer for all data sets in Table 3.2 with the exception of Lenthall (1986), who counted all rings using a hand lense. All the data sets in this study were corrected for stem analysis bias using Carmean's procedure (1972) with the exception of 55 plots from Millar and Millar and Woods (1989) (Table 3.2).

Section points along the bole usually fall between two internodes, resulting in a bias that underestimates the actual height that the tree attained at that particular age. This bias can be compensated using procedures described by Carmean (1972) when data sets are available for paired ring counts and section heights. Carmean's methodology [Eq. 16] makes two assumptions: (a) the section point will lie in the middle of an annual leader, and (b) annual height growth between section points is constant. The correction involves increasing height at each section point by half the estimated annual leader length. Dyer and Bailey (1987) recommend Carmean's procedure [Eq. 16] over six other alternative techniques.

$$Ht_{ij}=ht_i+\frac{ht_{(i+1)}-ht_i}{2(age_i-age_{(i+1)})}+(j-1)*\frac{ht_{(i+1)}-ht_i}{(age_i-age_{(i+1)})} \qquad [16]$$

where

- i = section number
- j = ring number above age_i between sections i and i+1
- ht_i = height at the i th cross-section (i.e., the sum of all bolt lengths below the i th cross-section),

age_i = number of growth rings at the i th cross-section,
Ht_{ij} = estimated total tree height at age_{ij} (Dyer and Bailey 1987)

Corrections for bias due to sectioning height on ring counts for the Millar and Millar and Woods (1989) plots (Table 3.2) were made using a program called Tree Ring Increment Measurement (TRIM) system developed by the OMNR (MacIver et al. 1985). The software on the Apple 2E and MSDOS system that records the measurements from the digimicrometer used a random number generator to estimate the location of the annual buds between stem sections. The TRIM program makes the assumption that internode lengths are random and all annual internodes are randomly located between section points[2]. In some cases, the random number generator used by the TRIM system to assign annual heights for the stem analysis data gave extremely large internodes when rapid early growth was observed on certain plots. Carmean's (1972) procedure was preferred whereever it was possible to obtain original data.

3.1.2.3 Tree Screening

After correcting for this bias, ages were then converted from total age to breast-height age (BHA). Age zero began at 1.3 m instead of stump-height level and then paired height and age values below 1.3 m were discarded. Paired height and BHA data were plotted for each tree and then individual height/age curves were inspected for irregular height-growth patterns. Erratic patterns that might indicate suppression, injury or erroneous ring counts were corrected. For example, excessively erratic

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2 Roj Millar, Personal communication, OMNR, May, 1990

growth was occassionally observed at the very top of the height-growth curve, that might be due to top breakage as sometimes indicated by a forked top; such erratic top portions of curves were eliminated. A few trees were also eliminated when excessively erratic growth was observed lower on the bole.

The LeBlanc and Niznowski plots and the Lenthall plots used a standard plot size (0.08 ha) and the standard Carmean method of correcting bias. But the other studies listed in Table 3.2 used different plot sizes for selection of sample trees, correction methods for stem analysis bias, and the number and type of soil descriptions varied. Accordingly, data were closely screened in an attempt to resolve differences that could complicate statistical analyses. Nine plots did not have soils information and were used as part of the verification data set.

3.1.2.4 Computation of Average Plot Curves

After eliminating erratic trees, a minimum of three and a maximum of five trees were available for computing an average height-growth curve for each of the 383 plots. For each plot, an arithmetic average of tree heights was computed for each year up to the age of the youngest tree on the plot. Average height-growth data at five-year intervals then was used in the analysis. Site index (SI_{BH50}) for each plot at 50 years BHA was observed from the average height-growth curve.

3.1.2.5 Computation and Verification Data Sets

Data were separated into computation and verification sets. The verification data set consisted of 60 plots randomly selected for estimating the accuracy of the computation models (Snee 1977) based on the 323 plots in the computation data set (Table 3.4). The verification set included the nine plots that were without landform designation (Tables 3.3 and 3.5); an additional 51 plots were randomly selected from the remaining plots listed in Table 3.4. The computation and verification data sets appeared to be similar based on the range of (a) BHA of the plots sampled, (b) site index, and (c) total height at time of sampling (Tables 3.4 and 3.5).

Table 3.4 Description of range of data in the computation and verification data sets.

Data Set	BHAGE (Years) Mean	Min.	Max	s^2	SITE INDEX (m) Mean	Min.	Max	s^2	TOTAL HEIGHT (m) Mean	Min.	Max	s^2
Computation	75	50	157	20	16.62	7.99	22.42	2.62	19.50	10.53	26.33	2.80
Verification	72	50	152	23	16.35	7.58	22.33	3.31	19.12	10.12	25.15	3.16

Table 3.5 Description of plots by region and landform used in the computation and verification data sets.

Computation Set						Verification Set					
Landform	NW	NCR	NER	NOR	Totals	Landform	NW	NCR	NER	NOR	Totals
	— Number of Plots —						— Number of Plots —				
BED	18	27	11	6	62	BED	3	3	3	1	10
GFL	32	53	18	31	134	GFL	5	13	2	3	23
LAC	19	16	4	1	40	LAC	1	4	-	1	6
MOR	17	45	18	7	87	MOR	2	7	2	1	12
						No Landform	-	5	-	4	9
Sub Totals	86	141	51	45	323	Sub Totals	11	32	7	10	60

3.2 Analysis

3.2.1 Height-Growth Curves

Three polymorphic nonlinear equations (Ek 1971 [Eq. 2], Ker and Bowling 1991 [Eq. 3] and Monserud 1984 [Eq. 4]) were fitted using paired height and age values taken from the average height-growth curves for each of the 323 computation plots.

The Nonlinear Regression (NLR) command from SPSS-X Inc. (1988) (V.

2.0 on VMS) was used to fit nonlinear regression equations to the data. The command uses Marquardt's iterative method of least squares (Marquardt 1963) to compute final parameter estimates for the models. In some cases, the Constrained NLR (CNLR) command was used because bounds were necessary to prevent parameter estimates exceeding realistic limits and thus stalling the program. Starting coefficients for these equations were taken from published papers (Lenthall 1986, Deschamps 1991, Ker and Bowling 1991), respectively.

The assumption made using least squares regression, $\varepsilon_i \sim \mathrm{NID}(0, \sigma^2)$, $i = 1, 2 \ldots n$, was examined by (1) visual examination of normal probability plotting, (2) examination of standardized residuals (actual versus predicted heights) for heteroscedastic tendencies, and (3) an examination of residuals by comparing Studentized deleted residual values to Bonferonni-t values (Weisberg 1985).

3.2.2 Height-Growth Patterns

One-way analysis of covariance for nonlinear equations (Hinds and Milliken 1988) was used to detect possible differences in height-growth patterns between: (a) the four OMNR regions (NWR, NCR, NOR, NER), (b) the four glacial landforms (GFL, LAC, MOR, BED), and (c) combinations of regions and glacial landforms at three levels of site index. The methodology refered to by Hinds and Milliken (1988) as covariance analysis is what many statisticians refer to as "full versus reduced model analysis" (Weisberg 1983). This approach is commonly used to compare linear models (Weisberg 1983) and is approximate for nonlinear models (Ratkowsky 1983). The approximation will be reasonably accurate provided the nonlinear model does not exhibit excessive curvature near the solution (Ratkowsky 1983).

The confidence bounds about the differences in predicted values were used to locate where differences were likely to occur. The comparison uses the pooled degrees of freedom rather than the number of observations at time t_i. This may give the perception that differences at old ages appear significant although one group may be poorly represented at old ages. Also, this procedure assumes that a common model equally represents height-growth patterns in each data set.

Accordingly, a precision level of +/- 1 m was selected because this approximates the precision of hypsometers in field conditions. Under favourable conditions, errors larger than +/- 0.5 m are not uncommon, nor are errors of 1 to 1.5 m uncommon under unfavourable conditions (Romesburg and Mohai 1990, Bruce and Schumacher 1942). Errors are inevitable because the base of a tree is a poorly defined target and an imperceptible lean will cause considerable error.

3.2.2.1 One-way Analysis of Covariance

One-way analysis of covariance was used to determine if a common model for a pooled data set represented the height-growth pattern better than individual models for the regional, landform, and site index categories being compared. This method involves fitting a nonlinear model to each treatment (T_i, i=1,2..t). An assumption was made that the height-growth pattern for each treatment was adequately described by the model fitted to each treatment. The model fitted to each data set was the height-growth curve previously found which explained the most variation in height-growth of jack pine in northern

Ontario. A summary of the analysis of variance is shown in Table 3.6.

Table 3.6 One-way analysis of covariance table for nonlinear equations (Hinds and Milliken 1988).

Source of Variation	Degrees Of Freedom	Sums of Squares	F-Ratio
Treatment	(t-1)p	SSHo	Fc
Error	N-tp	SSRes (Pooled)	

*p = number of parameters in model
t = number of treatments (regions or landforms)
N = total number of height-age data combinations

The sums of squares were obtained by adding the sums of squares of residuals from each of the treatments, as:

$$SSRES(POOLED) = SSRES(T_1) + SSRES(T_2) + .. + SSRES(T_t)$$

One model was then fitted to the combined data for all treatments to obtain the combined sumd of squares of residuals, SSRES(COMBINED). The sums of squares to test the null hypothesis were computed, as:

$$SHo = SSRES(Combined) - SSRES(Pooled).$$

The test statistic (Fc) was

$$Fc = \frac{SSHo \; / \; df \; (SSHo)}{SSRES\,(\,Pooled\,) \; / \; df \; (\,SSRES\,(\,Pooled\,))}$$

If differences in height-growth patterns were found among the treatments at $\alpha = 0.05$, individual pairs of models were then compared by plotting the confidence bands about the differences between the two predicted values over time (Hinds and Milliken 1988).

3.2.2.2 Construction of Confidence Bands About Two Models

Confidence bands from BHA 0 to 150 years were constructed about the differences of predicted values of any two models being compared as a result of a significant Fc value. The models were considered to be significantly different when the difference between the two predicted values exceeded the confidence interval.

In general, computation of the confidence intervals involved computing the standard errors of the differences between the two models being compared at each five year interval:

$$SE(DIFF) = \sqrt{SE^2(\text{Model } 1) + SE^2(\text{Model } 2)}$$

The confidence intervals were computed as:

$$(f_{1_i} - f_{2_i}) \pm t_{\alpha/2\,(v)} * SE$$

where f_{1_i}, f_{2_i} = predicted heights from models 1 and 2 at time $\forall$ i=5...150,

$t_{\alpha/2\,(v)}$ = t-value for v degrees of freedom from the pooled data set at $\alpha = 0.05$.

In greater detail, the standard errors of a model or function ($SE_{(f)}$) were approximated by expanding the model in a first order Taylor's Series (Hinds and Milliken 1988). An estimate of the $SE_{(f)}$ was computed using first order partial derivatives of the model for each parameter. In addition, standard errors of each parameter and the correlation matrix between the parameters were obtained from the statistical output from SPSS-X (1988) when the model was fitted to the data set.

For the purpose of describing the first order Taylor Series using the five parameter Ek model [Eq. 2], let the first order partial derivative of each parameter be expressed as:

$$d_{b1} = \frac{\partial f}{\partial b1},\ d_{b2} = \frac{\partial f}{\partial b2},\ d_{b3} = \frac{\partial f}{\partial b3},\ d_{b4} = \frac{\partial f}{\partial b4},\ d_{b5} = \frac{\partial f}{\partial b5}.$$

The partial derivatives were computed by hand and verified using Maple V.5, Waterloo Maple Software on PC (Appendix II). The standard errors of the parameters were denoted respectively by s_{b1}, s_{b2}, s_{b3}, s_{b4}, s_{b5} and the correlation's by r_{b1b2}, r_{b1b3}, r_{b1b4}, r_{b1b5}, r_{b2b3}, r_{b2b4}, r_{b2b5}, r_{b3b4}, r_{b3b5}, r_{b4b5}.

The estimated standard errors of the model were evaluated at a given time using the partial derivatives which were determined by the least squares estimate of each the parameter.

$$SE_{(f)}=\begin{bmatrix} d^2b_1{*}s^2b_1 & + & d^2b_2{*}s^2b_2 & + & d^2b_3{*}s^2b_3 & + \\ d^2b_4{*}s^2b_4 & + & d^2b_5{*}s^2b_5 & + & 2db_1{*}db_2{*}r_{b1b2}{*}s_{b1}{*}s_{b2} & + \\ 2db_1{*}db_3{*}r_{b1b3}{*}s_{b1}{*}s_{b3} & + & 2db_1{*}db_4{*}r_{b1b4}{*}s_{b1}{*}s_{b4} & + & 2db_1{*}db_5{*}r_{b1b5}{*}s_{b1}{*}s_{b5} & + \\ 2db_2{*}db_3{*}r_{b2b3}{*}s_{b2}{*}s_{b3} & + & 2db_2{*}db_4{*}r_{b2b4}{*}s_{b2}{*}s_{b4} & + & 2db_2{*}db_5{*}r_{b2b5}{*}s_{b2}{*}s_{b5} & + \\ 2db_3{*}db_4{*}r_{b3b4}{*}s_{b3}{*}s_{b4} & + & 2db_3{*}db_5{*}r_{b3b5}{*}s_{b3}{*}s_{b5} & + & 2db_4{*}db_5{*}r_{b4b5}{*}s_{b4}{*}s_{b5} & + \end{bmatrix}^{1/2}$$

When data from several treatments were used, such as comparing the four landforms, the four standard errors for each landform were recomputed using the pooled estimate of the error variance. For example, the adjusted standard error for parameter b1 ($s_{b1(Adj)}$) of the nonlinear model fitted to lacustrine data was:

$$s_{b1(Adj.)} = s_{b1} {*} \sqrt{\frac{MSE(Pooled)}{MSE}}$$

where: MSE(Pooled) = the pooled mean square error from all four landforms being compared

MSE = is the MSE of the lacustrine data

s_{b1} = is the standard error of b_1 when the nonlinear model was fitted only to the lacustrine data.

3.2.2.3 Regions

The average height-growth curves for each plot within each of the four regions are shown in Figure 3.2. This figure shows the range of site index and breast-height age observed in each region. Hinds and Milliken's (1988) method of covariate analysis was used to test for differences between height-growth patterns in each region, to determine where differences occurred and to see if polymorphic patterns were present.

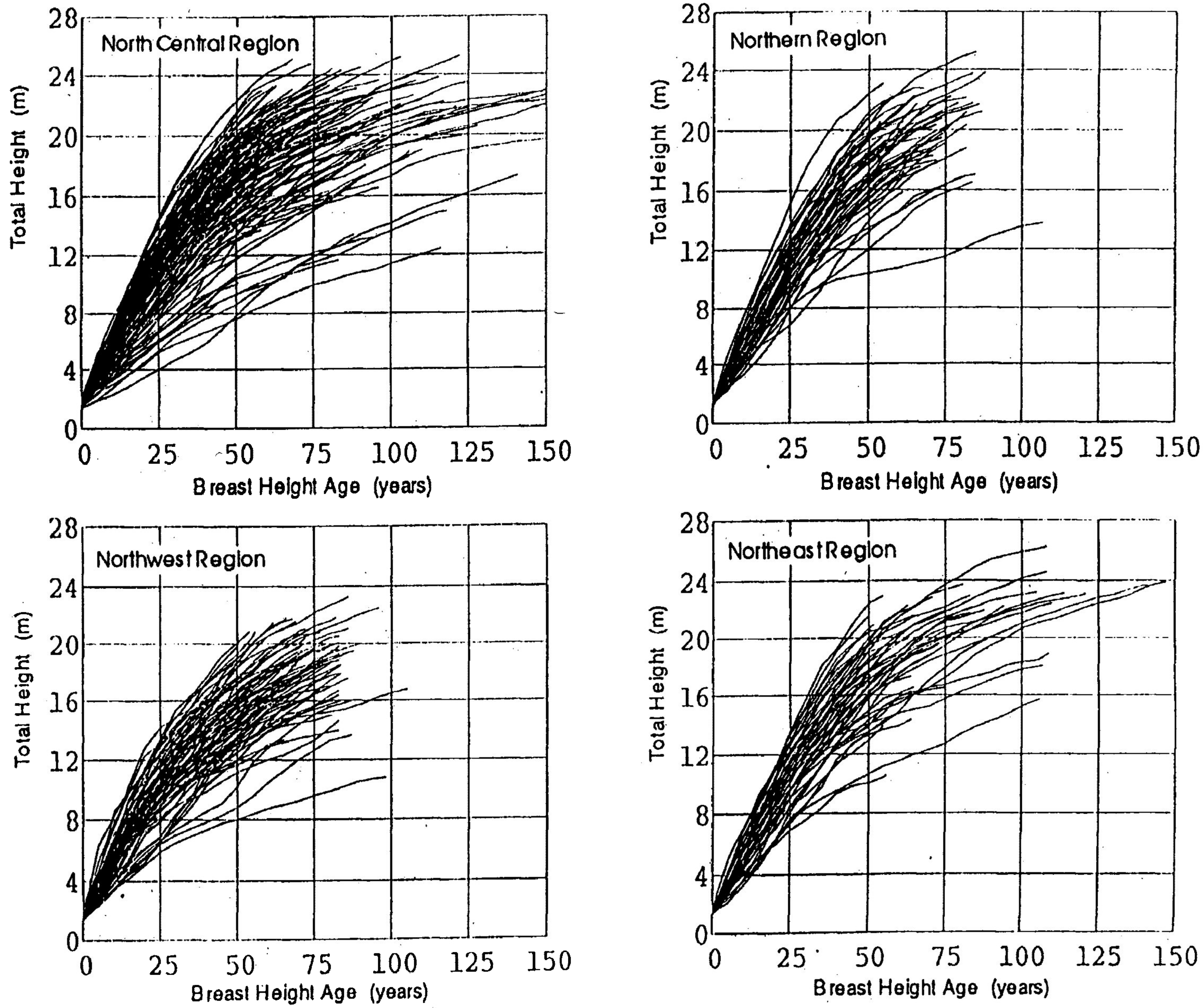

Figure 3.2 Average height-growth curves for individual plots within each of the four regions.

Based on the range of site quality sampled in each region (Table 3.7), a level of site quality within the range of all regions was chosen to represent height growth on poor, medium and good sites.

Table 3.7 Range of site index (SI_{BH50}) values observed in each regional data set.

Regions	Number Of Plots	Site Index (m) Poor 1/	Medium 1/	Good 1/	Maximum BHA (Yrs.)
NWR	86	7.99	15.20	20.10	105
NCR	141	7.58	16.99	22.33	157
NER	51	10.00	16.94	22.42	147
NOR	45	10.41	17.26	22.39	107

1/
Poor < 15 m
Medium 15 - 19 m
Good > 19 m

The levels of site quality chosen to represent the lower, average and upper limits of height-growth across all four regions was SI_{BH50} = 10 m, SI_{BH50} = 17 m, and SI_{BH50} = 20 m. The poorest site found in the NOR was SI_{BH50} = 10.41 m and was slightly above the lower limit selected. Graphs of the plots in each region (Figure 3.2) were used to account for apparent differences that might only be due to poor representation of plots.

3.2.2.4 Landforms

Hinds and Milliken's (1988) method of covariate analysis was used to compare height-growth patterns within each landform (Figure 3.3), to locate the tree ages where differences in height-growth occurred, and to identify if differences occurred at various levels of site quality.

The levels of site index selected to compare height-growth patterns at good, medium and poor levels of site index were the same as those used for comparing regions. The range of site indices observed among landforms (Table 3.8) was similar to that observed among regions (Table

3.7). These were two exceptions. Shallow to bedrock soils had an average site index about three metres lower than the other landforms, and the poorest site index that lacustrine soils had was much higher than observed for the other three landforms (Table 3.8). Possibly this is due to lacustrine soils having very few poor site quality plots (Figure 3.3).

Table 3.8 Range of site index (SI_{BH50}) values observed in each glacial landform data set.

Glacial Landforms	Number Of Plots	Site Index (m) Poor 1/	Medium 1/	Good 1/	Maximum BHA (Yrs.)
BED	62	7.58	14.35	19.95	128
GFL	134	7.80	17.00	22.42	152
LAC	40	13.98	17.38	20.71	147
MOR	87	10.19	17.09	22.33	157

1/
Poor <15 m
Medium 15 - 19 m
Good >19 m

The bedrock and lacustine soils had few plots that exceed 100 years BHA (Figure 3.3) in comparison to glaciofluvial and morainal soils. Thus, this analysis also will try to identify and describe these differences in range and distribution of site index within each landform.

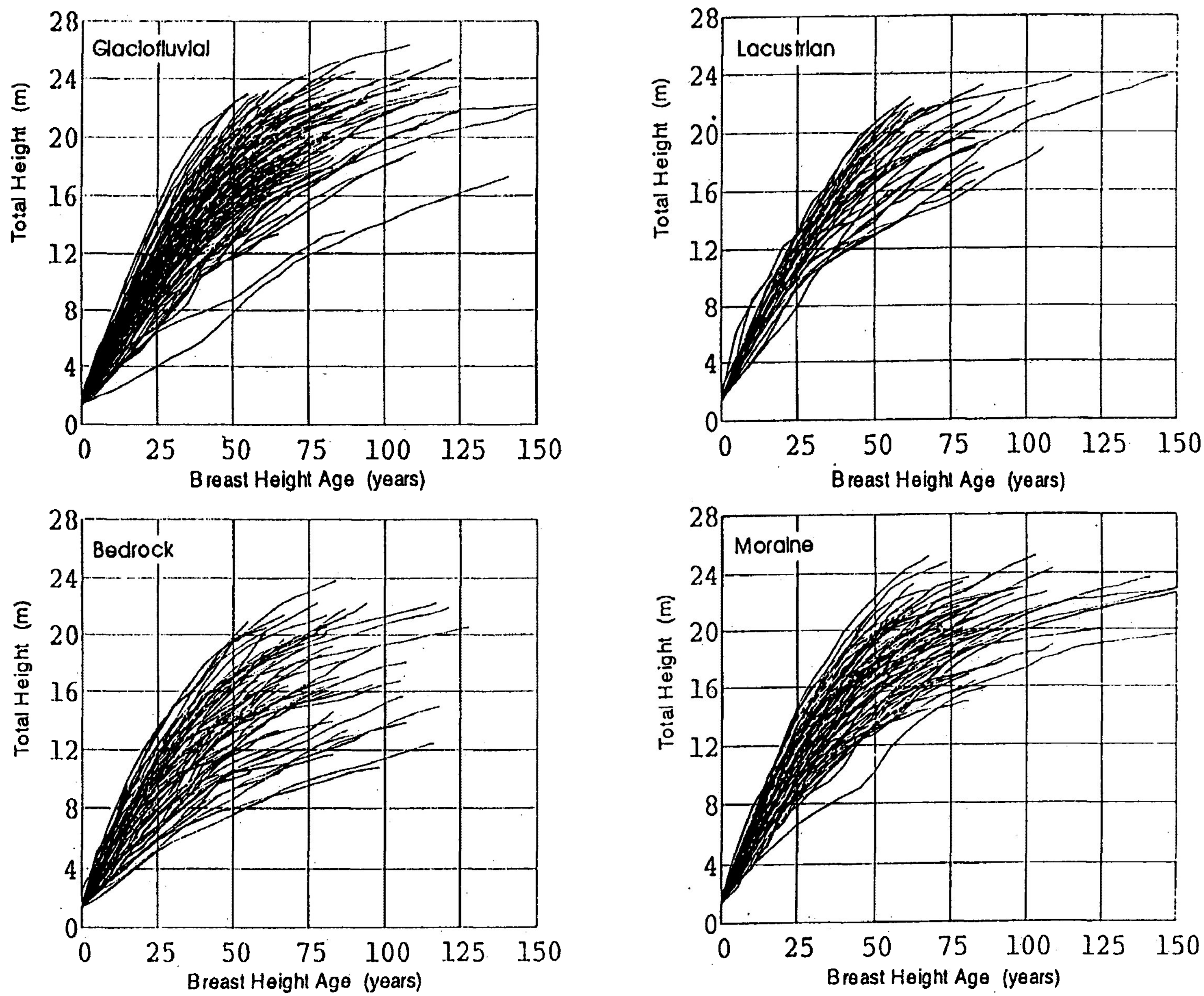

Figure 3.3 Average height-growth curves for individual plots within each of the four landforms.

3.2.2.5 Regions, Landforms and Site Classes

Plots were located across a wide range of site indices in each region (Figure 3.2, Table 3.7), and on each landform (Figure 3.3, Table 3.8). The number of plots by region, landform, and site-index class is shown in Table 3.3. It was therefore possible to compare height-growth patterns at the same level of site index for the four regions and for the four landforms using data representing a narrow range of site quality. The differences in curve shape, which is possibly associated with regional climate or glacial landforms, might reveal biological factors explained in past studies. A similar procedure was used by Carmean and Lenthall (1989) to test for polymorphic height patterns with increasing site quality.

A minimum of 13 plots was used for comparing regression curves for each of the regions, landforms and site-class combinations. At least 13 plots were considered necessary for each category because it was felt that the sacrifice in precision observed at a point in time anywhere along the curve dropped sharply with less than 12 *df* in the t-table. From Table 3.3, four comparisons were examined that met the sample size of 13 plots in each treatment; an additional 7 comparisons were examined that sacrificed precision slightly, but these comparisons were made because they might provide some insight to the broad relationships between height-growth patterns for various regions, landforms and site quality classes. A multitude of individual test were made because of the complexity of using an experimental design with uneven replication. The comparisons listed in Table 3.9 were tested using one-way analysis of covariance for nonlinear equations (Hinds and Milliken 1988).

Table 3.9 Tests to compare individual categories while controlling for region, landform or site class.

Test No.	Comparison	Control Variable	Site Class
1	NCR vs NWR	BED	12
2	NCR vs NWR	GFL	12
3	BED vs GFL	NWR	12
4	BED vs GFL	NCR	12
5	NCR vs NWR	LAC	17
6	NCR, NWR, NER	MOR	17
7	ALL REGIONS	GFL	17
8	MOR vs GFL	NCR	21
9	MOR vs GFL	NER	17
10	MOR vs GFL vs LAC	NWR	17
11	ALL LANDFORMS	NCR	17

3.2.3 Site-Index Curves

Site-index curves are height-growth curves that pass through a specified height at index age. The two constrained forms of the Ek model developed by Newnham [Eq. 6] and Burkhart and Tennent [Eq. 7], the constrained form used by Ker and Bowling model [Eq. 8], and the constrained form of the Monserud model [Eq. 4] by Dempster and Associates [Eq. 9], were fitted to the computation data set to see if any precision was sacrificed by constraining the height-growth curve to pass through the specified height level at index age. The residuals were examined for $\varepsilon_i \sim NID(0,\sigma^2)$, $i = 1, 2 \ldots n$, using the same steps used for fitting height-growth curves. A simple F test was performed to see if there were any significant differences in the mean square error between various constrained and non-constrained models (Weisberg 1985).

3.2.4 Site-Index Prediction Equations

Three site-index prediction equations were fitted to the computation data set: (a) Monserud (1984)[Eq. 10]; (b) Payandeh (1974)

[Eq. 11]; and (c) Goelz and Burk (1992)[Eq. 5]. The residuals were analyzed using the same procedures as described for the height-growth models. The Goelz and Burk (1992) base-age invariant model [Eq. 5] was examined by letting A_2=50 and H_2= SI_b.

The effect of variation in early height-growth on site index prediction was investigated. Equations [5], [10] and [11] were fitted to the computation data set separately on five occassions; each time data were eliminated at 5-year intervals, starting from age 0 until age 20 years BHA. As age approaches base age the standard error of the estimate (SEE) will continue to decrease (Goelz and Burk 1992). The decision not to eliminate data for trees older than 20 years BHA was based on previous studies by Deschamps (1991) and Lenthall (1986) who found that precision of site index estimates were greatly improved when data were eliminated for ages less than 20 years BHA. The SEE for each model was examined to see which model explained the most variation and if the removal of early height-growth resulted in improved precision.

3.2.5 Accuracy and Validation

The model that explained the most variation was selected from each of the three types of models fitted to the computation data set: height-growth curves, site-index curves and site index prediction equations. Since site-index curves can be used to predict height or site-index, the accuracy of site-index curves for predicting height-growth was compared to the accuracy using height-growth curves; the accuracy of site-index curves for predicting site-index was also compared to the accuracy using site index prediction equations.

The final height-growth curves for northern Ontario were compared to other curves by plotting the Plonski (1974), Carmean and Lenthall (1989), and Goelz and Burk (1992) curves on the same graph. Finally all height-growth curves for other provinces were plotted and compared across the entire range of jack pine. Site-index curves for northern Ontario were compared to site-index curves for jack pine in New Brunswick by Ker and Bowling (1991) and site-index curves for jack pine and lodgepole pine in Alberta by Dempster and Associates (1983).

3.2.5.1 Site-Index Curves Versus Height-Growth Curves

The height-growth and site-index equation with the smallest SEE of those models fitted to the computation data set were compared for accuracy. The accuracy of each model fitted to the computation data set was described using tests by Freese (1960) [Eq. 17] and Reynolds (1984) [Eqs. 18, 19, 20]. The accuracy was validated using the independent data set of 60 plots (Tables 3.4 and 3.5). Computations were made at 10 year age intervals.

The Freese (1960) test [Eq. 17] determines if a model adequately describes data given a user specified allowable error limit. The limit used in this study was -/+ 1.5 m, the same as that used by Lenthall (1986) for his jack pine study, and by Deschamps (1991) for his trembling aspen study.

$$\chi^2_{(\alpha_1, n)} = \frac{\sum_{i=1}^{n}(x_i - \mu_i)^2}{\sigma^2} \qquad [17]$$

where: $\sigma^2 = \dfrac{e^2}{Z^2_{(\alpha_1)}}$

Reynolds (1984) computed a critical error (e*) which defined the confidence bounds on the upper 95% quantile of the distribution of residuals under the assumption $\varepsilon_i \sim NID(0, \sigma^2)$, $i = 1\ 2\ \ldots\ n$. He computed e** defining the confidence bounds on the lower 5% quantile of the distribution.

$$e^* = \left[\frac{(Z^2)\left(\sum_{i=1}^{n} (x_i - \mu_i)^2 \right)}{\chi^2_{(\alpha_1, n)}} \right]^{\frac{1}{2}} \qquad [18]$$

$$e^{**} = \left[\frac{(Z^2)\left(\sum_{i=1}^{n} (x_i - \mu_i)^2 \right)}{\chi^2_{(\alpha_2, n)}} \right]^{\frac{1}{2}} \qquad [19]$$

Prediction intervals around future predicted residuals were computed using a methodology developed by Hahn and Nelson (1973). Dechamps (1991) and Lenthall (1986) used this technique in the construction of site-index curves in northwestern Ontario. The formula below computes a $(1-\alpha_1)$ 100 % interval around a future predicted value:

$$\overline{D} \pm \left(1 + \frac{1}{n}\right)^{1/2} * S * t_{(1-\alpha_1/2)} \qquad [20]$$

where:

$\overline{D}$ = mean difference between predicted minus observed values

S = the standard deviation of the residuals

t = the Student's t statistic with n-1 degrees of freedom

3.2.5.2 Site-Index Curves Versus Site-Index Prediction Equations

Constrained height-growth curves pass through a specified height at index age which classifies them as site-index curves. When graphed, the axes are the same as those produced using site-index prediction equations. The accuracy of these site-index curves to predict site-index was compared to the site-index prediction model. To solve the constrained height-growth model for site index, a fortran algorithm was developed which determined what site index value would estimate a height that matched the height observed at that age within a tolerance level of 0.01 m. The verification data set was used to compare the Freese [Eq. 17] and Reynolds [Eqs. 18, 19] and Hahn and Nelson [Eq. 20] statistics, which were computed for both height and site index dependent models. The comparisons were made when early growth from time zero was included and when data were eliminated for early growth less than 20 years BHA. Studies have shown that site index predictions are highly variable using data for ages less than 20 years BHA (Lenthall 1986, Goelz and Burk 1992).

3.2.5.3 Final Site Index Curves for Northern Ontario

The selection of a model chosen to represent jack pine height growth in northern Ontario was based on the accuracy and desirable characteristics of the various models that were tested. The model developed using the 323 computation plots was then fitted to the entire set of 383 plots and the final coefficents and site-index curves were computed. The average deviation (observed minus predicted) was examined using all 383 plots grouped by 2-m site-index classes at 10 year age intervals. Average deviations were computed over time to identify weaknesses in the model and possible areas for future improvements of the model.

3.2.5.4 Comparison to Existing Height-Growth Curves in Ontario

Plonski's (1974) height-growth curves for 10, 16, and 22 m SI_{TOT50} formulated by Lenthall (1986), were converted to breast-height age and then plotted. Equation [21] fitted by Lenthall (1986), equation [22] fitted by Goelz and Burk (1992) and the final equation [Eq. 6] fitted to all 383 study plots were compared to Plonski's curves by plotting the site-index curves through SI_b read from each of Plonski's BHA curves.

$$\hat{H}_t = 1.3 + 2.13762 * SI^{0.82800} * (1 - EXP(-0.02522 * BHA))^{3.61558 * SI^{-0.42555}} \qquad [21]$$

$$\hat{H}_2 = 1.3 + (H_1 - 1.3) \left[\frac{1 - EXP(-0.0185((H_1 - 1.3)/A_1) 1.3382 A_1 + 0.4257 A_2)}{1 - EXP(-0.0185((H_1 - 1.3)/A_1) 1.3382 A_1 + 0.4257 A_1)} \right]^{1.0464} \qquad [22]$$

The average deviation (observed minus predicted) was examined using all 383 plots grouped by 2-m site-index classes at 10 year age intervals for equations [21] and [22]. Deviations over time were computed to verify the accuracy of using published site-index curves for predicting the shape of jack pine height-growth curves in northern Ontario similar to procedures used by Newnham (1988) to compare white spruce site index curves in Saskatchewan.

3.2.5.5 Height-Growth Patterns Across the Range of Jack Pine

Height-growth patterns for jack pine representing the western, central and eastern portions of the species range were compared.

Height-growth patterns for jack pine in the western portion of the range were represented by the Dempster and Associates (1983) model [23] for mixed lodgepole and jack pine stands.

$$\hat{H}_t = 1.3 + (SI_b - 1.3) * \left[\frac{1 + EXP(7.4871 + (-1.2036) * \ln(50) + (-0.9576) * \ln(SI_b - 1.3))}{1 + EXP(7.4871 + (-1.2036) * \ln(BHA) + (-0.9576) * \ln(SI_b - 1.3))} \right] \quad ...[23]$$

Height-growth patterns of jack pine in the eastern portion of the range were represented by the Ker and Bowling (1991) model [24]:

$$\hat{H}_t = 1.3 + (SI_b - 1.3) * (1 - EXP(-0.02862 * 50))^{-5.5393 * (SI_b - 1.3)^{-0.5102}}$$

$$\times (1 - EXP(-0.02862 * BHA))^{5.5393 * (SI_b - 1.3)^{-0.5102}} \quad [24]$$

The final site-index curves developed in this study for northern Ontario will represent the central portion of the species range in Canada. The three models are compared for poor, medium, and good site index levels, SI_{BHA50}= 10, 16, and 20 m respectively as defined by BHA curves of Plonski's site class 1, 2 and 3.

4

RESULTS

4.1 Height-Growth Curves

Exploratory modelling was done using the 323 plots of the computation data set (Table 3.4). The Ek [Eq. 2], Ker and Bowling [Eq. 3] and Monserud [Eq. 4] models were used for expressing height-growth patterns and for determining what model best fitted the data. Table 4.1 shows that the Ek model [Eq. 2] explained more variation than either the Ker and Bowling model [Eq. 3] or Monserud model [Eq. 4] . There only was a small difference between standard error measures for the Ek model [Eq. 2] and the Ker and Bowling model [Eq. 3]; however, the Ek model had the lowest standard error when fitted to the jack pine computation data set and was, therefore, selected for comparing height-growth patterns. Coefficients and summary statistics for models are given in Appendix III. Plots of standardized residuals versus predicted heights appeared random for all models.

Table 4.1 Summary statistics for height-growth models. Comparison of Ek [Eq. 2], Ker and Bowling [Eq. 3], and Monserud [Eq. 4] height-growth models.

Model	*df*	SS	MSE	SEE
Ek [Eq. 2]	5293	2832.5640	0.5352	0.73
Ker and Bowling [Eq. 3]	5293	2850.7455	0.5385	0.73
Monserud [Eq. 4]	5293	3663.0113	0.6921	0.83

The Ek [Eq. 2] model expressed height-growth better than Eq. [3] or Eq. [4]. However, examination of Studentized deleted residuals identified 7 data points belonging to 3 plots as potential outliers (Table 4.2). Examination of the individual tree height-growth patterns for these three plots (Appendix IV) revealed two unusual plots (GP890074 and KO870003) on lacustrine and glaciofluvial deposits, respectively. Late height growth for these two plots increased at a linear rate even though predicted height flattened off at these older ages. Trees on a shallow to bedrock plot (DL860017) had rapid early height growth (Appendix IV) that could not be adequately modelled before 20 years BHA as indicated in Table 4.2.

Table 4.2 Potential outlying data points associated with three plots as shown in the actual computer-input file.

Data Point	Plot Number	Region	Land-form	Breast height age	Observed total height (m)	Predicted height (m)	Observed site index (m)
1	GP890074	NER	LAC	140	23.46	20.50	14.33
2	GP890074	NER	LAC	145	23.79	20.59	14.33
3	GP890074	NER	LAC	147	23.88	20.63	14.33
4	K0870003	NCR	GFL	120	21.50	18.39	12.98
5	K0870003	NCR	GFL	125	21.78	18.53	12.98
6	DL860017	NCR	BED	15	9.44	6.31	14.08
7	DL860017	NCR	BED	20	10.94	7.82	14.08

These three plots were not eliminated even though growth at very old or at very young ages differed considerably from the computed model.

4.2 Height-Growth Patterns

Height-growth patterns were significantly different among regions, landforms, and several levels of site quality. These differences are described separately.

4.2.1 Regions

The Ek model [Eq. 2] was separately fitted to the data set for each region (Figure 3.2) resulting in the formulated height-growth curves of Figure 4.1. Summary statistics were similar and are found in Appendix V.

An analysis of covariance detected significant differences in height-growth patterns at 1 % probability for the four different regions (Table 4.3).

Table 4.3 One-way nonlinear analysis of covariance to detect differences in height-growth patterns of jack pine found in four OMNR administrative regions.

Source Variation	*df*	SS	MS	F	P
Regions	15	289.6036	19.3069	34.84	0.0000
Error	6130	3397.1150	0.5542		

Ages where significant differences in patterns of height growth occurred between regions are shown in Table 4.4. Most differences were at ages older than 85 years except for the comparison of NCR vs. NWR where NWR had faster early growth from 0 to 20 years BHA. At these old ages, data were available from relatively few plots. No significant differences in curve patterns were observed between NER vs. NWR, NOR vs. NWR, and NER vs. NOR.

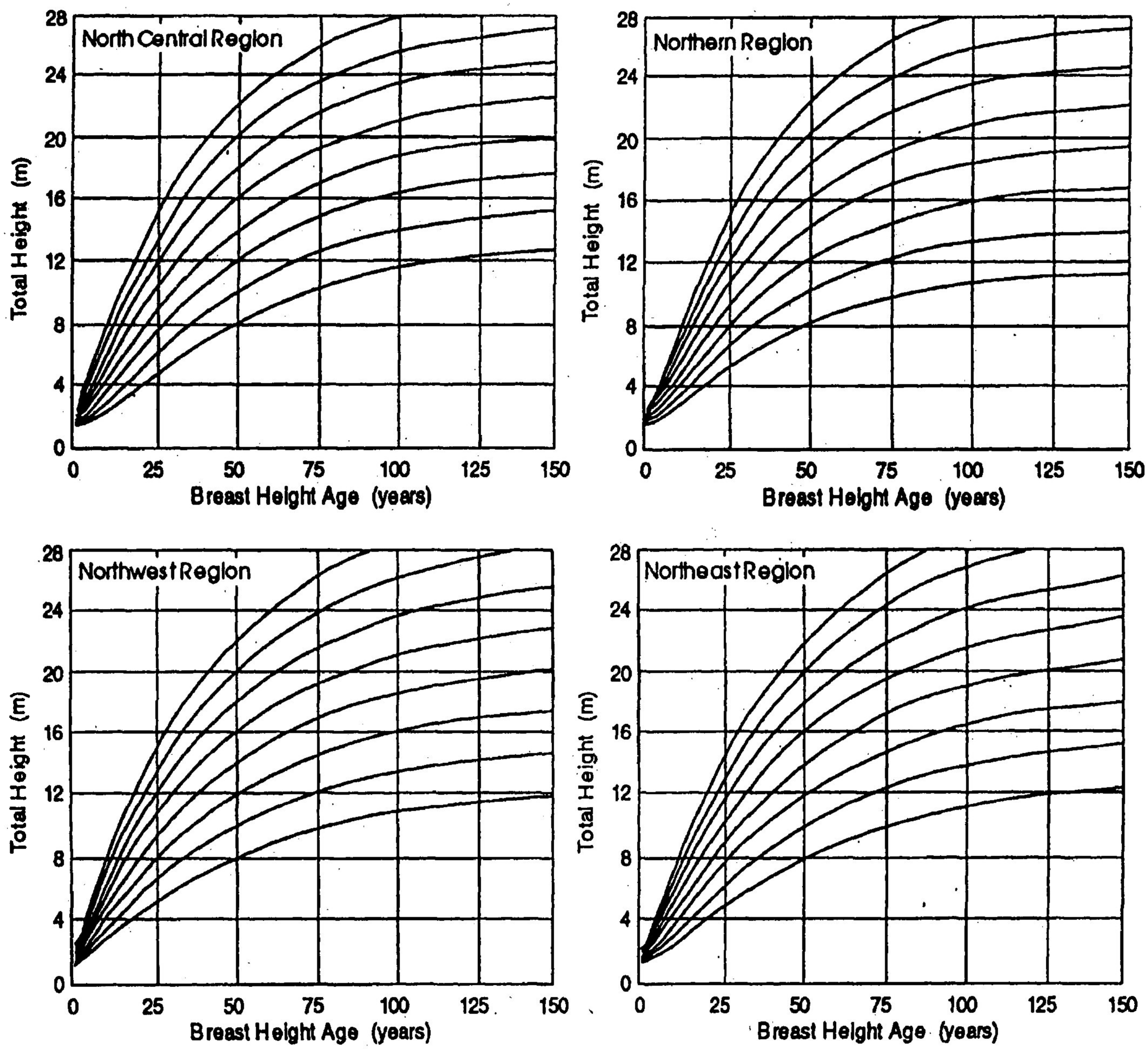

Figure 4.1 Regional height-growth curves.

Table 4.4 Significant differences greater than 1.0 m (α = 0.05) when height-growth curves were compared for region and site class combinations.

Regions Compared	Level of Site Index (m)[1] Poor BHA	Poor Diff.	Medium BHA	Medium Diff.	Good BHA	Good Diff.
NCR vs. NWR	0-20	NWR ⇑	-	None	>90	NWR ⇑
NER vs. NCR	-	None	>135	NCR ⇓	>115	NCR ⇓
NOR vs. NCR	>85	NCR ⇑	-	None	-	None

[1]Poor: <15.0 m; Medium: 15.0 m to 19.0 m; Good: >19.0 m
Note: ⇑ greater in height, ⇓ lower in height
NWR Northwestern Region, NCR North Central Region,
NOR Northern Region, NER Northeastern Region

4.2.2 Landforms

The Ek model [Eq. 2] was separately fitted to the data set for each landform (Figure 3.3) resulting in the formulated height-growth curves of Figure 4.2. Summary statistics were similar and are found in Appendix VI.

An analysis of covariance detected significant differences in height-growth patterns at 1 % probability for the four different glacial landforms (Table 4.5).

Table 4.5 One-way nonlinear analysis of covariance to detect differences among glacial-landforms.

Source of Variation	*df*	SS	MS	F	P
Landforms	15	487.3355	32.4890	58.62	0.0000
Error	6130	3397.1150	0.5542		

Ages where significant differences in patterns of height-growth occurred between landforms are shown in Table 4.6. Most differences were at ages older than 85 years except for GFL vs LAC comparison, where the LAC had faster early growth from 0 to 25 years BHA.

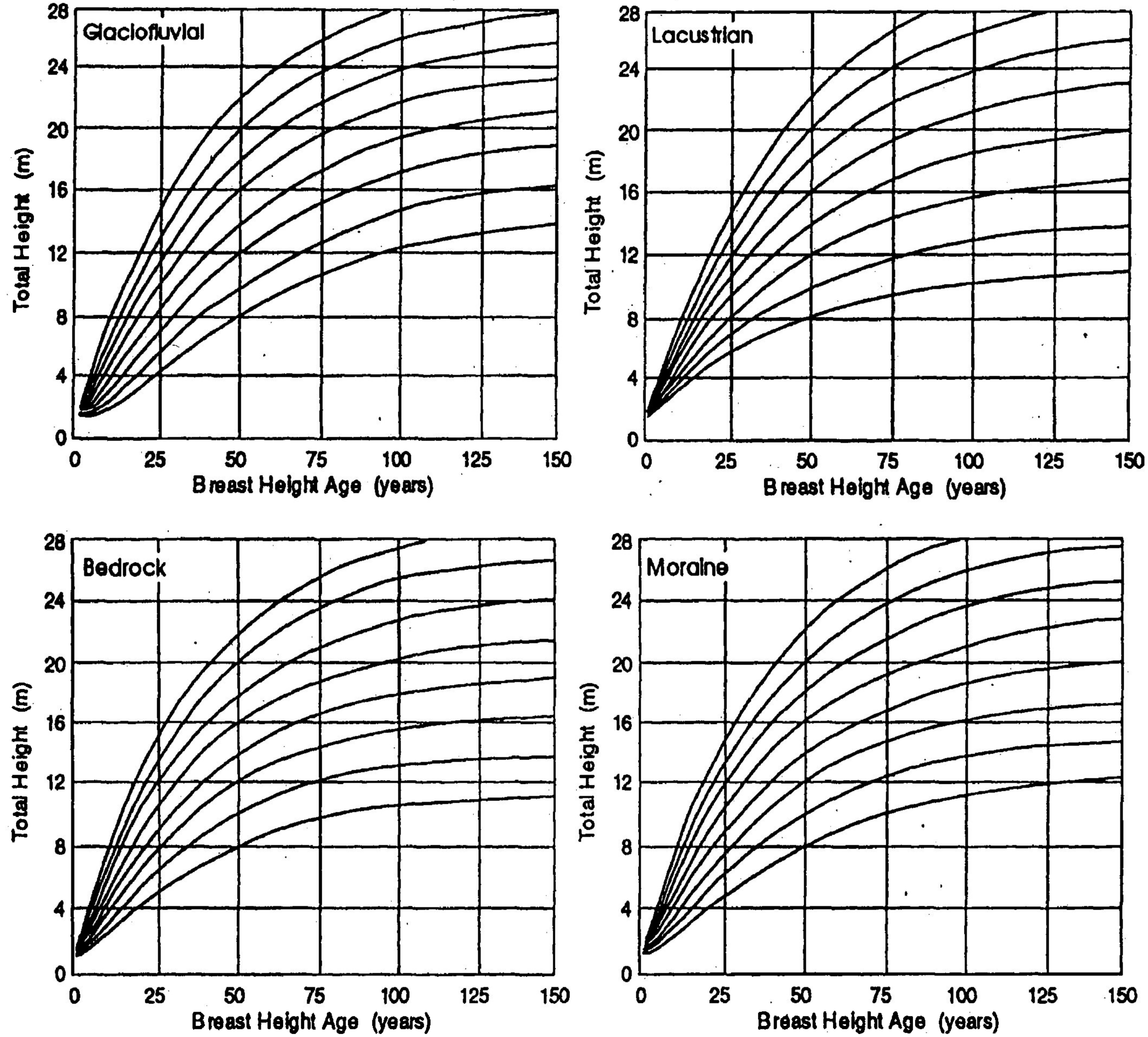

Figure 4.2 Landform height-growth curves.

Differences were most frequent for poor site classes as shown in Table 4.6. These differences were usually at ages of 85 years or older except for BED vs GFL where differences were evident at 65 years BHA. Few differences were evident for medium (15.00 to 18.99 m) and good (+19.00 m) site indices and these were usually at ages exceeding 100 years represented by relatively few plots.

Table 4.6 Differences greater than 1.0 m were significant (α=0.05) when compared for landform-site class combinations.

Landforms Compared	Level of Site Index (m)[1] Poor BHA	Poor Diff.	Medium BHA	Medium Diff.	Good BHA	Good Diff.
BED vs. LAC	>115	LAC ⇑	>100	LAC ⇑	>105	LAC ⇑
GFL vs. LAC	>85	LAC ⇑	-	None	>100	LAC ⇑
	0-25	LAC ⇑				
MOR vs. LAC		None		None	>120	LAC ⇑
BED vs. MOR	>90	MOR ⇑		None	-	None
GFL vs. MOR	>90	MOR ⇓		None	-	None
BED vs. GFL	>65	GFL ⇑	>95	GFL ⇑	-	None

[1]Poor: <15.0 m; Medium: 15.0 m to 19.0 m; Good: >19.0 m

Note: ⇑ greater in height, ⇓ lower in height

LAC Lacustrine, MOR Morainal, GFL Glaciofluvial, BED Bedrock

4.2.3 Regions, Landforms and Site Classes

Eleven tests (Table 3.9) were made to compare height-growth patterns on various combinations of regions, landforms and site classes. These comparisons showed significant differences greater than 1 m for four of the tests (Table 4.7). The lack of significant differences in the other tests is an indication one of two situations; a) the height-growth curves for the groups compared are very uniform in shape and the average curves are similar, or b) the height-growth curves for the groups compared are very irregular and the variance about the average curve was too large to detect a difference.

Test 1 (Table 3.9) and Appendix VII (Figure 1 and 2) indicate that relatively large variations in height-growth patterns occur among the individual plots on poor sites on the shallow to bedrock landform. This is why the shallow to bedrock comparisons usually are not listed among the significant differences in Table 4.7; apparently this relatively large variation among individual curves mean that significant differences due to region or site class are difficult to express. Accordingly, significant differences could not be established on these poor-site bedrock soils because of this wide variation in height growth for individual plots in both the NWR and NCR regions.

Table 4.7 Differences significant at α=0.05 and greater than 1 m for selected combinations of regions by landforms, and by site classes.

	CRITERIA FOR COMPARISON			EARLY GROWTH DIFFERENCES				LATE GROWTH DIFFERENCES			
Test			Site	Age	Height (m)			Age	Height (m)		
No.	Region	Landform	Class	(Years)	Min	Max	Larger	(Years)	Min	Max	Larger
3	NWR	BED vs GFL	Poor					80-90	1.44	1.84	GFL ⇑
6	NCR, NWR, NER	MOR	Medium								
	a) NER vs NWR			0-20	1.11	1.15	NWR ⇑				
7	ALL REGIONS	GFL	Medium								
	a) NOR vs NWR			10	1.04		NWR ⇑				
	b) NCR vs NER							120	1.00		NCR ⇑
11	NCR	ALL LANDFORMS	Medium								
	a)	BED vs GFL						95-130	1.00	1.56	GFL ⇑
	b)	BED vs LAC						105-115	1.00	1.07	LAC ⇑

* Refer to Table 3.9 for number of sample plots representing each category and description of test.

#3: Comparison of site class 12 in the NWR across BED and GFL

#6: Comparison of site class 17 on morainal deposites across NER, NCR, and NWR

#7: Comparison of site class 17 on glacial-fluvial deposites across NER, NOR, NCR, NWR

#11: Comparison of site class 17 in the NCR across GFL, MOR, BED, LAC

Test 2 (Table 3.9) and Appendix VII (Figure 3 and 4) indicates that relatively large variations in height growth occur among individual plots with poor site on the glaciofluvial landform. Accordingly, significant differences also could not be established on these poor sites because of this wide variation in height growth for both the NWR and NCR regions.

Poor sites in the NWR and NCR on BED and GFL landforms had much variation in height-growth patterns as mentioned above for tests 1 and 2, thus significant differences were difficult to establish. But test 3 (Appendix VII - Figure 2 and 4) indicates that comparisons of poor site plots between BED vs GFL in the NWR region showed significant differences in height growth after 80 years BHA; however, at 80 years, only 7 and 4 plots were present for the BED vs GFL comparisons, respectively. An identical comparison in the NCR region (test 4: Table 4.7) for poor site plots showed no significant differences between BED and GFL; significant differences could not be established because of the larger observed variation in height-growth patterns among individual plots on bedrock soils (Appendix VII - Figure 1 and 3).

In contrast to poor sites (tests 1 and 2), results indicate that height-growth curves for individual plots on medium sites on the lacustrine landform (test 5 in Table 3.9, Appendix VII - Figure 8 and 13) are relatively consistent and have little variation in height-growth patterns. Accordingly, significant differences were not observed in test 5 because height-growth patterns are relatively similar on medium sites in both the NWR and NCR regions.

Test 6 (Table 4.7) and Appendix VII (Figure 6, 12 and 14) indicates that individual plots on medium sites on the moraine landforms also have relatively consistent height growth in early years, but height-growth patterns did have significant differences from 0 to 20 years; NWR expressed rapid earlier growth than NER.

Test 7a (Table 4.7) and Appendix VII (Figure 9 and 11) indicates that medium sites for all four regions on the glaciofluvial landform had similar individual height-growth curves in a narrow band. This shows consistent growth in early years for all plots, but the NWR expressed significant height-growth differences at 10 years BHA in comparison to NOR. Test 7b indicates that height growth in NCR and NER appeared different after 120 years BHA, but at 120 years BHA, the NER had only one plot (Appendix VII - Figure 7 and 9).

Tests 8, 9, and 10 (Table 3.9) indicates that good sites and medium sites on morainal, glaciofluvial and lacustrine landforms in the NCR, NER and NWR have little difference in height-growth patterns (Appendix VII - Figure 15, 16, 14, 10, 9, 12, 13). Accordingly, growth on these soils appears similar and no significant differences greater than 1 m were observed.

Test 11 indicates that medium sites on all four landforms in the NCR (Appendix VII - Figure 5, 6, 7 and 8) have some variation in height-growth for ages greater than 95 years. Height-growth on lacustrine sites was very consistent in contrast to bedrock sites where more variable patterns of height growth occur. Furthermore, bedrock landforms only had three very old plots, glaciofluvial landforms only had four very old plots, and the lacustrine landform only had one very old plot. Thus these possible differences at very old ages are based on little evidence.

4.3 Site-Index Curves

Site-index curves pass through a specified height at index age. The previous height-age models do not pass through index age; they may agree closely with specified height for average levels of site index but may differ somewhat for very good or very poor levels of site index (Newnham 1988). Equations [6], [7], [8], [9] are constrained versions of height-growth equations that force the curves through a specified height at index age.

The constrained Ek model by Newnham [Eq. 5] had the smallest standard error of the estimate of the four constrained models examined (Table 4.8).

Table 4.8 Site-index curves fitted using three different constrained height-growth models.

Model	df	SS	MSE	SEE
Ek-Newnham [Eq. 6]	5294	2671.2248	0.5046	0.71
EK-Burkhart & Tennent [Eq. 7]	5295	22262.9150	4.2045	2.05
Ker and Bowling [Eq. 8]	5295	3065.1268	0.5789	0.76
Constrained Monserud [Eq. 9]	5295	2945.9941	0.5564	0.75

Accuracy, was improved (Table 4.8) using the constrained Newnham model [Eq. 6] and the constrained Monserud height-growth model [Eq. 9] in contrast to poorer accuracy using the unconstrained versions (Eq. [2] and Eq. [4]) of these models (Table 4.1). But reduced precision resulted when the constrained version of Ker and Bowling model [Eq. 8] and the Burkhart and Tennent constrained version [Eq. 7] of the Ek model was used (Table 4.8).

The standardized residuals for the Newnham [Eq. 6] constrained model showed no major biases when fitted to the computation data set (Figure 4.3); that is, there were no heteroscedastic trends indicating systematic lack of fit. Summary statistics are found in Appendix III.

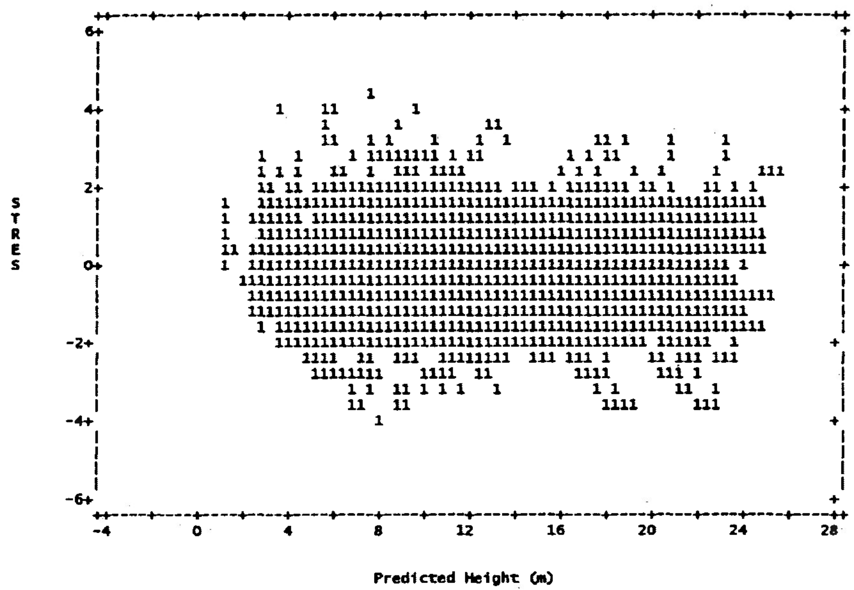

Figure 4.3 Scattergram of standardized residuals versus predicted heights from fitting the Newnham [Eq. 6] model to the computation data set.

4.4 Site-Index Prediction Equations

Height-growth equations [Eqs. 2 to 9], with the exception of equation [5], cannot be solved for site index. Modified analogues of the height-growth equations, suggested by Payandeh (1974), can be used as site-index prediction equations. Accordingly, the Payandeh formulation [Eq. 11], the Monserud linear model [Eq. 10], and the Goelz and Burk difference equation [Eq. 5] were fitted to the computation data (Appendix III). Several transformations of Eq. [10] where fitted to the data and computations showed that Eq. [25] performed the best.

$$SI_b = \beta o + \beta 1 * H_t + \beta 2 * \ln(H_t) + \beta 3 * \ln(BHA) + \beta 4 * \ln(BHA)^2 + \beta 5 * \left(\frac{H_t}{BHA}\right) + \varepsilon \qquad [25]$$

The SEE were very large for all three models when data were included starting from zero BHA. Therefore changes in the SEE were examined when equations were fitted after deleating data for early height growth; 5-year periods of early height-age data were deleated (Table 4.9) until age 20 as recommended by Lenthall (1986). The Monserud model [Eq. 10] consistently showed the lowest SEE from 0 to 20 years BHA. Using data ≥ 20 years BHA, the t-test for β_2 in Eq. [25] was not significant and therefore was dropped and the model re-evaluated. No change in error values occurred using equation [26] thus this model is the recommended site-index prediction formulation using data that are 20 years BHA and older.

$$SI_b = \beta o + \beta 1 * H_t + \beta 2 * \ln(BHA) + \beta 3 * \ln(BHA)^2 + \beta 4 * \left(\frac{H_t}{BHA}\right) + \varepsilon \qquad [26]$$

Table 4.9 Standard error of the estimate of site-index prediction models using data older than 0, 5, 10, 15, and 20 years BHA.

Starting BHA	Standard Error of the Estimate		
	Goelz and Burk [Eq. 5]	Monserud [Eq. 10]	Payandeh [Eq. 11]
	←	m	→
0	1.61	1.36	1.99
5	1.28	1.19	1.31
10	1.10	1.06	1.14
15	0.97	0.95	1.01
20	0.86	0.85	1.03

The scattergram of standardized residuals using data older then 20 years BHA (Figure 4.4) showed no major systematic trends to suggest the model did not accurately fit the data.

Examination of the residuals identified nine data points (Table 4.10) as potential outliers when studentized deleted residuals were compared to the Bonferroni-t (Table 4.10). The individual tree height-age curves for these two plots (KN2, TT6.12) were examined to see if any abnormalities were apparent. This examination showed that the individual tree height-age curves were tightly grouped all plots. Maximum tree ages were 61 and 72 years BHA, thus the height-age curves were located on the relatively straight portion at the middle age of the height-growth curves. Large errors associated with these two plots at age 20 years BHA suggest that height growth at 20 years can be highly variable, resulting in errors in site-index prediction for jack pine.

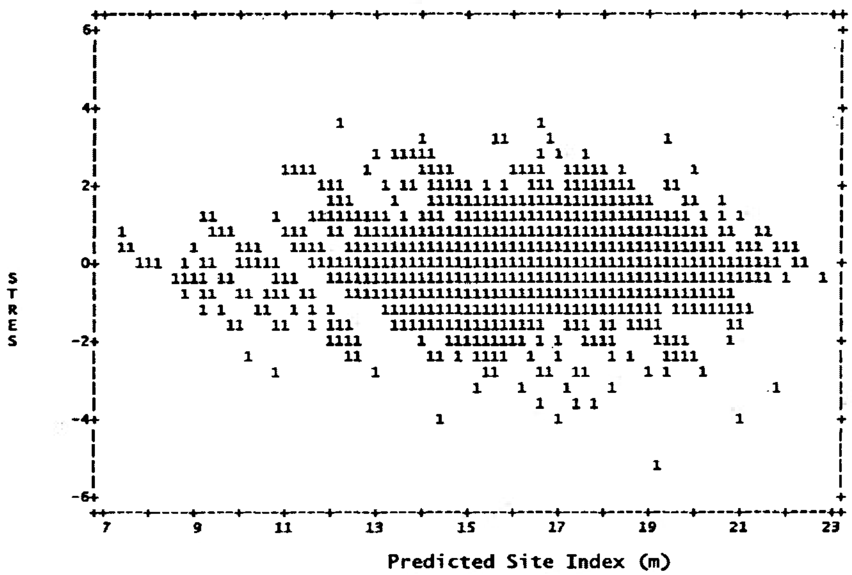

Figure 4.4 Scattergram of standardized residuals versus predicted site index using the modified Monserud model [Eq. 26] with computation data ≥ 20 years BHA.

Table 4.10 Plot characteristics of two data points identified as potential outliers at 20 years BHA using the modified Monserud's site-index prediction equation [Eq. 26] with computation data ≥ 20 years BHA

Plot Number	Region	Landform	BHA (Years)	Site Index (m)	Predicted Site Index (m)
TT6_1223	NO	G	20	5.92	12.15
T__KN211	NO	G	20	9.10	16.61

4.5 Accuracy and Validation

The Ek model [Eq. 2], Newnham model [Eq. 6], and the modified

Monserud linear site-index prediction model [Eq. 26] explained the most variation in their respective categories, height-growth curves, site-index curves, and site-index prediction equations. Accuracy tests between the site-index curves, the height-growth curves, and the site-index prediction equations are described in the preceeding sections. Final curves and comparisons with published jack pine height-growth curves for both Ontario and across the range of jack pine follow.

4.5.1 Height-Growth Curves Versus Site-Index Curves

The accuracy of the final height-growth and site-index curves based on the 323 computation plots was determined using a Chi-squared test to determine if 95 percent of the predicted values were within +/-1.5 m of the observed values (Table 4.11) (Freese, 1960). The residuals of both models fitted to the computation data set appeared to be normally distributed based on the pattern of normal probability plots (Weisberg, 1985). The Chi-squared values in Table 4.11 show that both the Ek [Eq. 2] and Newnham [Eq. 6] models are below the tabulated critical values for 0.05 and 0.95 probability, thus these models predict height within the +/-1.5 m level.

Table 4.11 Freese (1960) accuracy test of the final height-growth models using the computation data set of 323 plots.

Model	D.F	Observed $\chi 2$	Tabulated Chi-squared values $\alpha = 0.05$	$\alpha = 0.95$
Ek [Eq 2]	5297	4836.27	5467.43	5128.85
Newnham [Eq. 6]	5297	4641.65	as above	

Since the residuals appear normally distributed for both the Ek [Eq. 2] and Newnham [Eq. 6] models, the Reynolds (1984) critical errors (e* and e**) provide an estimate of the upper and lower confidence intervals containing 90 % of the residuals 95 % of the time. Almost identical critical error values in Table 4.12 for both the computation data set and the verifiication data set show that Eq. [2] and Eq. [6] are equally accurate. The similar prediction intervals for the two models confirm the confidence intervals suggested by the critical errors (Table 4.12). The differences in Reynolds critical error values and the prediction intervals between the validation set and computation data set (Table 4.12) are a result of the smaller sample size.

Table 4.12 Reynolds (1984) accuracy test of the final height-growth models using the computation data set of 323 plots and validation data set of 60 plots.

Measure of Central Tendency	Computation Data Set		Validation Data Set	
	Ek Model [Eq. 2]	Newnham Model [Eq. 6]	Ek Model [Eq. 2]	Newnham Model [Eq. 6]
N	5297		967	
Critical e*	1.41	1.37	1.54	1.54
Critical e**	1.46	1.41	1.66	1.66
95 % Prediction Interval for $\overline{D}$	0.02 +/- 1.43	0.01 +/- 1.39	0.13 +/- 1.59	0.11 +/- 1.59

Note: N= number of paired observations

4.5.2 Site-Index Curves Versus Site Index Prediction Equation

Newnham's (1988) height dependent model [Eq. 5] was solved for site index and compared to site-index predictions made using the modified version of Monserud's site-index prediction model [Eq. 26]. In addition, computations using all data ≥ 5 years BHA were compared to computations when data were restricted to values 20 years BHA and older. Age 0 was not included because it caused difficulties for the algorithm to converge on site-index using Newnham's height dependent model.

The results in Table 4.13 show that Newnham's height-dependent model [Eq. 6] can be solved for site index as accurately as Monserud's site index dependent model [Eq. 25 or Eq. 26]. Also the inclusion of data younger than 20 years BHA did not bias the comparison, but using this younger data resulted in a greater prediction interval of approximately +/- 0.30 m for both models.

Table 4.13 Accuracy predicting site index using Newnham's [Eq. 6] height dependent model versus linear site-index dependent models [Eq. 25 and Eq. 26] using the validation data set of 60 plots.

Measure of Central Tendency	Data ≥5 Years BHA		ata ≥20 Years BHA	
	Newnham Model [Eq. 5]	Monserud Model [Eq. 24]	Newnham Model [Eq. 5]	Monserud Model [Eq. 25]
N	897		717	
Critical e*	2.18	2.24	1.65	1.66
Critical e**	2.35	2.43	1.80	1.82
95 % Prediction Interval for $\overline{D}$	0.03 +/- 1.92	0.16 +/- 2.33	-0.05 +/- 1.66	0.15 +/- 1.71

Note: Age 0 was not included because of difficulties solving Newnham's equation for site index

4.5.3 Final Height-Growth - Site-Index Curves

The final height-growth/site-index curves for jack pine in northern Ontario were made using the constrained Newnham model [Eq. 6]. The site index prediction equation [Eq. 26] was fitted to all data after 20 years BHA. Both models were recomputed with all 383 computation and validation plots and the final coefficients are are shown in equations [27] and [28]. Summary statistics are in Appendix III. Height-growth/site-index curves for jack pine across northern Ontario are shown in Figure 4.5

$$\hat{H}_t = 1.3 + 8.7405 * (SI_b - 1.3)^{0.3531} \left(1 - K^{\frac{BHA}{50}}\right)^{0.3962 * (SI_b - 1.3)^{0.3799}} \quad [27]$$

where: $$K = 1 - \left[\frac{(SI_b - 1.3)}{8.7485 * (SI_b - 1.3)^{0.3530}}\right]^{\frac{1}{0.3962 * (SI_b - 1.3)^{0.3799}}}$$

The final estimated parameters for Eq. [6] changed slightly from those reported when Eq. [6] was fitted to the computation data set; however, the accuracy remained similar (-0.02 ± 1.41 m where α=0.05)(See Table 4.12).

The final estimated coefficients, shown in Eq. [28], changed slightly from those obtained when Eq. [26] was fitted to the computation data set. The accuracy of Eq. [28] to predict site-index from ages ± 20 years BHA is 0.00 ± 1.67 m where α=0.05.

$$\hat{SI}_b = 26.1832 + 0.7396 * H_t - 8.401658 * \ln(BHA) + 0.5225 * \ln(BHA)^2 + $$

$$+ 13.1841 * \left(\frac{H_t}{BHA}\right) \quad [28]$$

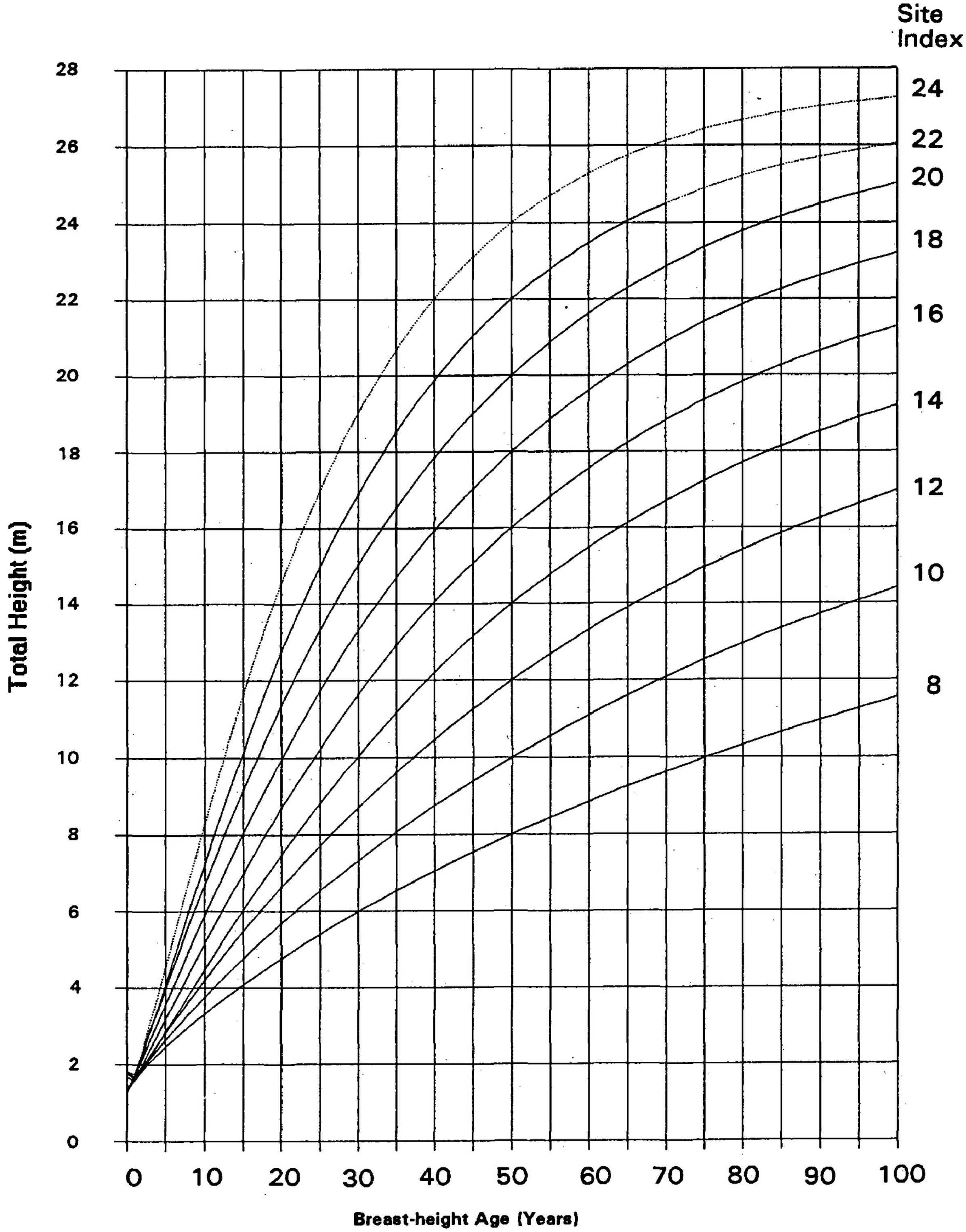

Figure 4.5 Final height-growth/site-index curves for northern Ontario.

The average deviations (observed minus predicted) for the site index curves of northern Ontario are shown in Table 4.14.

Table 4.14 Average deviations of observed height from predicted height by 2 m site-index classes at 10 year age intervals using the Newnham [Eq. 27] height-growth model.

	Number of plots and 2 m Site-Index Classes																Total
BHA	No. Plot	SI 8	No. Plot	SI 10	No. Plots	SI 12	No. Plots	SI 14	No. Plots	SI 16	No. Plots	SI 18	No. Plot	SI 20	No. Plots	SI 22	No. of Plots
0	5	-0.14	11	-0.15	20	-0.20	65	-0.20	97	-0.22	106	-0.26	69	-0.26	10	-0.26	383
10	5	0.34	11	-0.02	20	-0.15	65	-0.34	97	0.02	106	-0.27	69	0.03	10	0.57	383
20	5	0.31	11	-0.02	20	-0.06	65	-0.39	97	0.09	106	-0.14	69	0.35	10	0.85	383
30	5	0.09	11	0.01	20	-0.07	65	-0.26	97	0.09	106	-0.08	69	0.37	10	0.70	383
40	5	0.15	11	-0.02	20	-0.15	65	-0.13	97	-0.02	106	-0.04	69	0.21	10	0.51	383
50	5	0.00	11	0.00	20	0.00	65	0.00	97	0.00	106	0.00	69	0.00	10	0.00	383
60	5	-0.44	7	0.09	17	-0.21	59	0.01	78	-0.01	77	0.03	77	0.03	4	-0.18	324
70	5	-0.80	6	0.08	12	-0.24	39	0.20	57	0.11	46	0.25	23	0.24	1	-0.26	189
80	5	-0.92	6	0.06	11	-0.37	32	0.22	31	0.13	26	0.42	18	0.27			129
90	3	-0.91	4	0.74	4	-0.95	18	0.02	13	0.36	13	0.46	7	0.48			62
100	2	-1.75	3	0.34	4	-1.25	12	-0.04	11	0.46	8	0.14	2	-1.04			42
110	2	-2.02	1	-0.58	3	-1.42	7	-0.82	8	0.67	4	0.46					25
120	1	-3.57			2	-1.70	5	-0.77	6	1.01	2	-0.25					16
130	1	-3.79			1	-1.12	4	-0.49	3	0.51							9
140	1	-4.12			1	-1.20	4	-0.63	3	0.44							9
150					1	-1.57	3	-0.29	1	0.72							5

The average deviations were highest in 8, 12, and 22 m site index classes; the average deviations shown in Table 4.14 suggests height growth on poor sites after 100 years BHA tends to be somewhat underestimated, and height growth on very good sites is overestimated by 51 to 85 cm before the 50-year index age.

Deviations remain higher than 1.0 m after 100 BHA for site-index classes 8 m and 12 m. The sign pattern of the average deviations across all site-index classes except 20 and 22 m suggests that the deviations are not random across age classes. Height plots on poor sites is consistently overestimated especially at ages older than 100 years BHA. However, several site index classes such as 10, 16 and 18 m, show small average deviations.

4.5.4 Comparison to Existing Height-Growth Curves in Ontario

Lenthall (1986) formulated the Plonski site-class curves for 10, 16 and 22 m SI_{50}. In the present study, these total-age curves were converted to SI_{BHA50} in order to compare them to the site (SI_{BHA50}) curves formulated using Newnham's constrained model [Eq. 27] and Lenthall's unconstrained Ek model [Eq. 21]. In addition, H_1 was set at SI_b and A_1 at 50; then H_2 was predicted using Eq. [22] with coefficients from Goelz and Burk (1992).

Before 50 years BHA, the four sets of curves show similar growth for all levels of SI_b. After 50 years BHA, the Plonski curves for poor (SI_{BHA50} = 10.91 m) and medium (SI_{BHA50} = 16.97 m) levels of site index predicted a flattening off of height growth greater than predicted by Goelz and Burk [Eq. 22] and my curves (Figure 4.6). But good (SI_{BHA50} = 23.14m) sites appeared to have similar growth even though the Plonski curves were slightly higher. This flattening off is greater for poorer levels of site index due to the polymorphic nature of jack pine height growth, and the inability of the Plonski anamorphic curves to show this polymorphism. For example, at 100 years BHA, Plonski's curves are lower than my curves on medium and poor sites by 1.69, and 2.34 m, respectively. In contrast, my curves predicted heights of 22.21 m and 15.61 m, and Goelz and Burk predicted height of 22.19 m and 16.29 m respectively, for the same levels of site index. The only difference between the Goelz and Burk [Eq. 22] curves and my curves [Eq. 27] occurs for poor sites at ages 80 to 100 years breast-height age. Lenthall's curves followed the curves of this study on poor and medium sites. On the good sites, Lenthall's curve was above Plonski's curve and passed below site index.

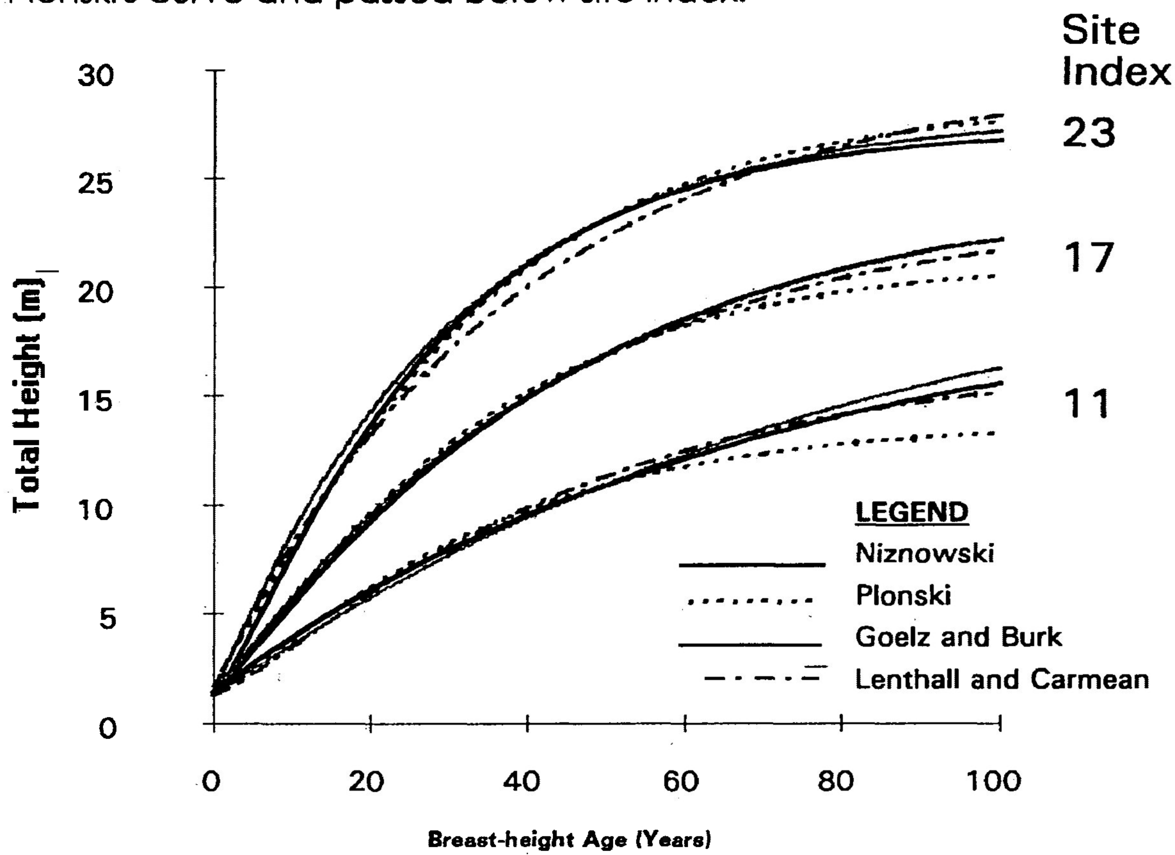

Figure 4.6 Comparison of final jack pine site-index curves with other curves developed in northern Ontario.

A similar comparison of average deviations as shown in Table 4.15 using Eq. [27] was performed for Eq. [21] and Eq. [22] (Appendix VIII). Goelz and Burk model [Eq. 22] was slightly more variable than Eq. [27] before 50 years, and Eq. [22] had smaller deviations beyond 100 years. Both equations overestimated growth on very good sites before index age; however, the Newnham model appeared to reflect the true shape on very good sites better than the Goelz and Burk's model. The size of the average deviations observed using Lenthall's equation [Eq. 21] were larger before and after 50 years BHA than either Eq. [22] or Eq. [27].

4.5.5 Height-Growth Patterns Across the Range Of Jack Pine

Height-growth patterns of jack pine were compared using curves developed in Alberta (Dempster and Associates [Eq. 23]), New Brunswick (Ker and Bowling [Eq. 24]), and Ontario (my curves using Newnham's model [Eq. 27]).

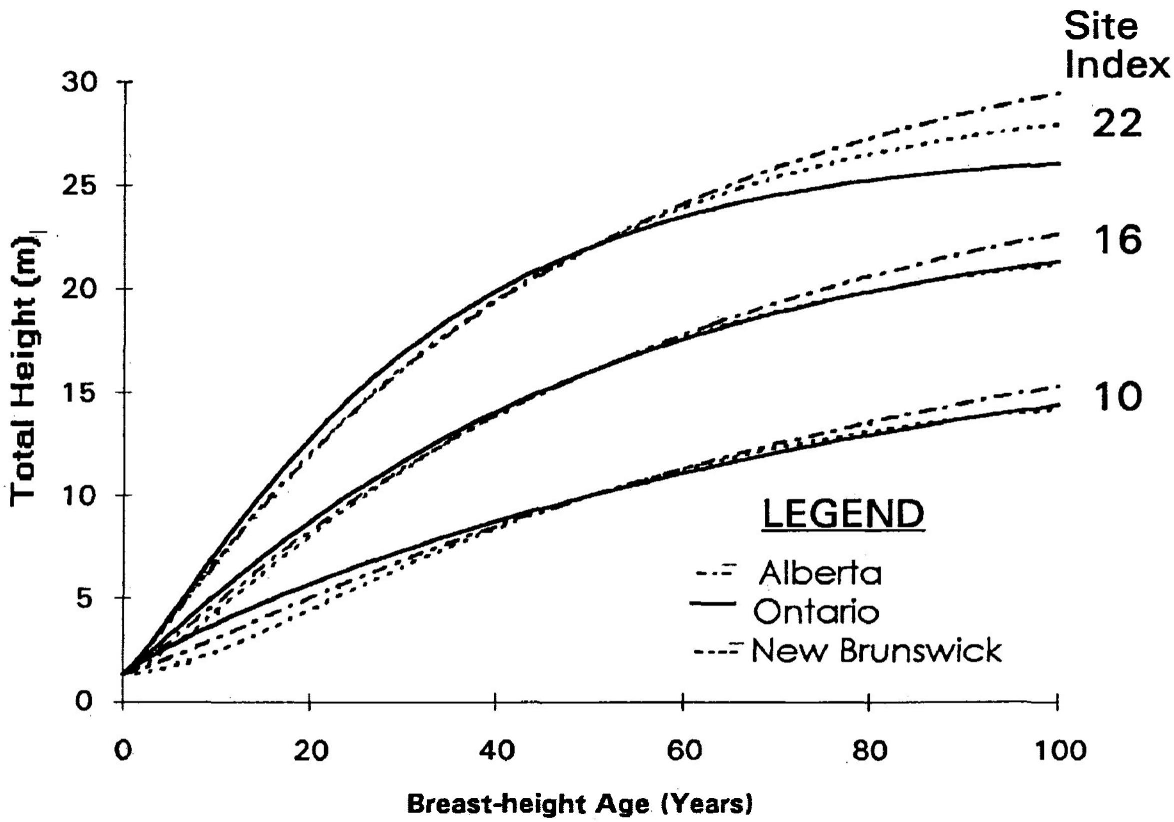

Figure 4.7 Comparison of final jack pine site-index curves with curves from Alberta and New Brunswick.

Height-growth patterns in Alberta indicate that better height growth is maintained longer for good, medium and poor sites than for Ontario and New Brunswick. However, in New Brunswick, growth after 50 years BHA appears to be better than in Ontario on good sites. Height-growth on medium sites in Ontario and New Brunswick appear to be similar. Early growth before 50 years BHA varied on poor sites with growth in New Brunswick appearing to be much slower than in Alberta and Ontario.

5

DISCUSSION

Harmonized site-class curves for jack pine in Ontario have been developed by Plonski (1974) and polymorphic site-index curves for north central Ontario have been developed by Lenthall (1986). Lenthall's curves also have been published by Carmean and Lenthall (1989) and Goelz and Burk (1992). Data for the Lenthall curves were only collected in north central Ontario. We presently do not know if these curves are suitable for use in estimating site quality for jack pine in other regions of northern Ontario. Likewise, we presently do not know if Lenthall's curves are applicable to jack pine growing on all glacial landforms of northern Ontario.

This thesis uses stem analysis data from different landforms in all regions of northern Ontario. We were able to study height-growth patterns for jack pine growing in different geographic regions and on different landforms of northern Ontario. Statistical methods used were proposed by Hinds and Milliken (1988); the procedure involves use of the same height-growth model fitted to each strata of data being compared.

5.1 Height-Growth Curves

Results show that the Ek (1971) model [Eq. 2] was superior to the Monserud (1984) logistic model [Eq. 4] for computing height-growth curves for jack pine (Table 4.1). Lenthall (1986) and Goelz and Burk (1992) also found that the Ek model [Eq. 2] accurately predicted height-growth of jack pine based on computations from a subset of data used in this study. Goelz and Burk (1992) compared the Ek model [Eq. 2] to their base-age invariant model [Eq. 5]. They found that the Ek model [Eq. 2] had the lowest SEE using a base-age of 50 years breast-height age; however, they also found that their base-age invariant model [Eq. 5] better described curve shapes at older ages.

The model used by Ker and Bowling [Eq. 3] was also used by Beck (1971) for white pine site index curves. Beck exchanged $\beta o + SI^{\beta 1}$ for $\beta o * SI^{\beta 1}$. The reparameterization did not result in a signicant difference between the two models. Ker and Bowling found that equation [3] had an R^2 value 1 to 2 % higher than the Ek model when fitted to jack pine data from New Brunswick. The opposite was true in this study; the Ek model was selected for expressing height growth because of the lower SEE and higher R^2.

The selection of the Ek model was primarily because of the lower MSE value, and secondly, that this was obtained using a very large data base that represents the different regions and landforms of northern Ontario. The Ek model had excellent precision for all sites up until 100 years BHA and, therefore, performs well within the normal range of ages for jack pine management.

Deficiencies with the Ek model occurred before 20 years BHA and after 100 years BHA. Poor to medium quality sites may have somewhat more rapid early growth before 20 years than predicted by the Ek model; later growth beyond 120 years on poor to medium sites also may be underestimated by the Ek model by approximately 2 to 3 m. Rapid early growth sometimes occurs on very shallow soils over bedrock when they are located at the base of a slope; these shallow soils often have moisture and nutrient properties better than uplands with shallow soils over bedrock. The lack of deep rooting on these shallow upland soils (i.e. depths < 30 cm) results in a rapid early flattening off of height growth. In contrast, depressions and drainways in moraines sometimes have better site quality due to teluric water carrying disolved nutrients and minerals that provide an initial boost in height-growth (Horton and Lees 1961, Arnup et al. 1988, Johnstone 1977). Shea (1973) also observed differences in jack pine height-growth patterns on stratified soils when a fine textured layer overlays a course textured horizon. Horton (1958) found vertical root development of lodgepole pine in the Hinton region of Alberta was restricted on such stratified soils and similarly on sites with a high water table or impermeable layer. Cochran (1984) reports that when restrictive soil horizons occur, there tends to be reduced growth of lodgepole pine because of less access to water and nutients.

5.2 Height-Growth Patterns

The Ek model [Eq. 2] was used to compare height-growth patterns for various regions, landforms and combinations of regions, landforms and site classes. This study makes the assumption that the Ek model can accurately express variations in patterns of height-growth among the various subsets of data. This may not necessarily be the case in all instances as regression models can be biased and different patterns occuring across large geographic areas (Carmean 1975). This study compares several height-growth models and results show that the Ek model is effective in accurately predicting height growth of jack pine. This study sets precision limits at +/- 1 m and statistical tests were found to be significant at 95% probability.

5.2.1 Regions

Past studies comparing harmonized site-index curves for Douglas-fir, upland oaks, and loblolly pine from different portions of their range have found contrasting height-growth patterns (Carmean 1956, 1970, 1972, 1975). Some of these differences might only be due to differences in quality of original data and to computation methods. But some differences also might be due to actual effects of climate, soil, and site quality.

Results from this study indicate that there are few significant differences between jack pine height-growth curves computed for the different regions of northern Ontario. Each of these regions vary greatly in soils, topography and climate. But this study shows that only at ages older than 100 years do we find significant differences in the height-growth patterns. At these old ages relatively few plots were available to represent old age growth. Also few forest managers in northern Ontario manage jack pine beyond 100 years. Accordingly, in this study, most regional height-growth differences (Table 4.4) occurred beyond ages where jack pine is managed, or differences occurred at extremes of age and levels of site quality where data were limited (Figure 3.2). Thus a single set of site index curves were found to be accurate in estimating site index in all regions of northern Ontario.

Jack pine in the NWR had somewhat greater height growth before 20 years on poor sites than jack pine on similar quality sites in the NCR (Table 4.4). Lenthall (1986) and Carmean and Lenthall (1989) recommend predicting site index using data older than 20 years BHA for jack pine in north central Ontario. Such differences in early years are considered unimportant because research has shown large errors in predicting jack pine site index (SI_{BHA50}) occurred when predictions are made using measurements younger than 20 years BHA (Lenthall 1986, Carmean and Lenthall 1989, Goelz and Burk 1992).

Future studies might examine factors influencing early growth as research with provenances has also shown poor correlation between growth intervals before 20 years total age (Jeffers and Jensen 1980). Land history, soil groups, habitat types, competition, and geographic location may be important for making silvicultural treatments by foresters in the different regions.

This study confirms the height-growth comparisons made by Jackman (1990) that showed height-growth patterns were similar for the North Central and Northwestern regions. It also shows that height-growth patterns in the North Central Region are similar to height-growth patterns for other regions of northern Ontario.

5.2.2 Landforms

This study shows that height-growth patterns were usually similar for landforms. However, significant differences in height-growth patterns were found between poor quality shallow to bedrock soils and glaciolfluvial soils (Table 4.6). Height growth on poor quality shallow to bedrock soils (SI_{BHA50} = 10 m) flattens off significantly at age 65 years when compared to height growth on poor quality glaciolfluvial soils (SI_{BHA50} = 10 m) (Figure 3.3 and Figure 4.2). Projecting height-growth curves on a shallow to bedrock soil (SI_{BHA50} = 10 m) to height at 100 years using the equation for glaciofluvial deposits results in an overestimation of height by approximately 2.5 m. This can have a bearing on determining which poor-site stands on shallow to bedrock soils are commercial or uncommercial sites. The reason is that later height growth on poor quality shallow to bedrock soils may less than height growth on poor sites on other landforms.

Differences also were observed between jack pine growing on poor quality lacustrine soils versus poor glaciofluvial soils before 25 years BHA. But there were few plots on lacustrine soils that were of poor site quality, thus the comparison at SI_{BHA50} = 10 m was beyond the range of the lacustrine data set (Table 3.8). Greater early height growth on lacustrine sites than on glaciofluvial sand deposits has been observed by Jameson (1963) (Figure 2.2) and Bella (1968)(Figure 2.3). Poor site quality on some lacustrine soils might be due to poor or imperfect drainage (Table 3.1). On such soils, nutrients and moisture are abundant in the better drained surface horizons and rapid initial growth may occur, but in later years shallow water tables, and lack of soil structure and poor aeration in subsoils may result in slower later height growth. Applying this height-growth pattern observed on poor site (SI_{BHA50} = 10 m) lacustrine soils (Figure 4.2) to poor site glaciofluvial soils might result in an overestimation of site index by 2 m, and an overestimation of about 4 m in height at 100 years.

5.2.3 Regions, Landforms, and Site Classes

Stratification of region and landform height-growth data by site-index classes permitted testing for height-growth patterns associated with specific site classes in the different regions and on the different landforms.

Using such narrow site-index bands allowed more precise testing for possible differences in pattern of height growth. Such precise testing is possible because grouping plots into narrow bands of site index avoids the influence of polymorphism associated with wide variations in site quality.

Differences in height-growth patterns are often the result of the physiological response of trees to various soil, topographic, or climatic conditions on the site. The quality and quantity of soil available for rooting has been shown to influence height growth as proven in various soil-site studies (Table 2.3, Çoile 1952, Carmean 1975, Kayahara 1989). Soil-site studies for jack pine (Schmidt 1986, Schmidt and Carmean 1988, LeBlanc 1994) also confirm that quality and quantity of soil available for rooting is closely related to site quality (Figure 2.2 and 2.3).

Comparisons revealed that height-growth curves on good quality sites have consistent height growth patterns within plots as well as between plots. Similarly, Alban and Prettyman (1984) and Alban et al. (1987) found no differences in height or diameter growth patterns of red pine growing on sandy well drained soils or on well drained fine-textured soils.

In contrast, poor quality sites usually have trees showing a wide range of height-growth patterns on each plot as well as between plots. The most notable differences occur on poor sites where height growth on shallow to bedrock soils is curvilinear over time as compared to almost linear height growth observed on poor quality glaciofluvial soils.

The growth pattern on the shallow sites might be related to the depth to bedrock. Height-growth in early years is rapid and then at some point the roots become restricted by the bedrock and height growth abruptly decreases. Variablity may be due to the type of rock and surface of the bedrock. Texture differences between glaciofluvial, morainal and lacustrine material may influence the rate of early growth before growth slows due to limitations related to moisture retension and CEC (Jameson 1963, Bella 1968, Chrosciewicz 1963).

Poor quality glaciofluvial soils tend to be deep coarse-medium sands. For such soils, opportunities for improved growth are often associated with the depth to fine textured lenses that are rich in nutrients and moisture (Pawluk and Areman 1961; Shetron 1969, 1972; Hannah and Zahner 1970). Thus, growth appears linear until better growing conditions are found. The variablity in height growth on poor quality GFL sites might be correlated to wet or droughty periods or allelopathic relationships with ericaceous species (Hamilton and Krause 1985).

The Schmidt (1986) soil-site study for jack pine separately analyzed plots by landform resulting in more accurate multiple regression soil-site prediction equations (Table 2.3). The variables for shallow to bedrock soils included depth to bedrock and amount of coarse fragments in the A horizon. It may be possible to incorporate such soil variables into height-growth models using methods similar to those of Hamlin and Leary (1988) who incorporated thickness of A horizon into a differential equation to predict site index of black walnut using data from Carmean (1966).

This study showed early differences in height-growth occur on deep morainal soils between the Northeastern and Northwestern Regions until 20 years BHA. However, present models are imprecise in projecting heights using ages younger than 20 years BHA. Future models might examine possible differences related to site factors known to influence height growth such as competition (Hamilton and Krause 1985; Wilde

1970), soil characteristics (Table 2.3), past history of a site (Carmean 1975), climate (Chrosciewicz 1963) and genetics (Maley 1990). It is possible that early competition is greater on moraines in the NER thus jack pine takes longer to reach free to grow conditions.

Variations in height-growth patterns are evident among regions, landforms and level of site class (Table 4.7). Most models used to generate height-growth or site-index curves use various transformations of height and age and do not incorporate environmental factors into the models. Perhaps future research can use methods sometimes used in soil-site research where tree age is an independent variable together with other soil and topographic variables, and possibly may even include a genetic index.

Height-growth curves developed for each landform using variables identified in soil-site studies might be useful in capturing more variation in height-growth patterns than is possible using a general height-growth equation to represent each landform. Alternative procedures such as those identified by McDill and Amateis (1992) which allow the number of parameters to vary and use both stand level parameters and global parmeters. Zakrzewski (1990) developed a functional base-age invariant model which can be modified to incorporate various site variables or transformations. Equation splining may be used to reflect a change in the variables influencing height growth during early growth stages, or in late growth stages where the greatest variation (polymorphism) in height-growth patterns often occurs.

5.3 Site-Index Curves

A desired trait of height-growth curves is that they pass through a specified height at a specific point in time which usually is index age. However, most regression equations that model height using tree age and site index as independent variables tend to predict biased estimates of site index. Specified height and actual computed height may agree for average site quality, but site index may be slightly underestimated on good sites and slightly overestimated on poor sites. In contrast, site-index curves are height-growth curves that pass though the specified site-index height at index age.

This study found that the Newnham (1988) constrained height-growth model [Eq. 6] performed well as a site-index model, and also explained more variation in height-growth patterns than the other constrained height-growth models (Tables 4.8). Both the Newnham model [Eq. 6] and Dempster and Associates model [Eq. 9] explained more variation than their unconstrained parent models, Eq. [2] and Eq. [4] respectively. Likewise, Newnham (1988) found that the constrained Ek model predicted height growth of white spruce with a minimal decrease in accuracy. Conversely, Deschamps (1991) found that the Dempster and Associates model [Eq. 9] fitted trembling aspen stem-analysis data from northwestern Ontario better than the Newnham model [Eq. 6].

5.4 Site-Index Prediction Equations

The Monserud site-index prediction model [Eq. 26] explained more variation when predicting site index than either Payandeh's model [Eq. 11], or the Goelz and Burk's base-age invariant model [Eq. 5] (Table 4.9). The Monserud model [Eq. 26] consistently predicted site index better than the other models when all data were used; this model showed considerable improvements when growth before 20 years was not used for developing the site-index prediction equation.

Goelz and Burk (1992) found that the structure of the data set influenced the estimation of parameters in equation [5]. They did not obtain their best parameter estimates when only the previous measurement was used to predict height. But they obtained suitable parameter estimates for equation [5] when all possible growth intervals were considered. By predicting across all possible combinations they were able to minimize error over time. Most base-age specific models have a very distinct pattern of residuals over time with a distinct decrease in residual size in the proximity of index age.

The data set used in this study predicted from each previous 5-year increment to site index at 50 years. It did not predict across all possible growth intervals as recommended by Goelz and Burk (1992). Improvements may be made in predicting site index at 50 years breast-height age by fitting equation [5] to all possible growth intervals.

Goelz and Burk (1992) compared Payandeh's model [Eq. 11] to their base-age invariant model [Eq. 5]. Their results showed that the Payandeh model [Eq. 11] explained more variation in site-index prediction than the base-age invariant model [Eq. 5] at 50 years BHA.

Both Lenthall (1986) and Goelz and Burk (1992) found equation [11] and equation [5] were imprecise in predicting site index at 50 years when data younger than 20 years BHA was used. But results show that poor estimates at age 20 might still be possible using the modified linear Monserud model [Eq. 26]. Potential outliers, identified in Table 4.10, occurred at 20 years on medium to good quality sites on glaciofluvial soils in the northern region. These outlier plots had a prolonged slow initial height-growth period which resulted in underestimating site index by approximately 4 m. This unusual height-growth pattern occurs at times on stratified soils when slow root growth probably occurs for surface soils that are coarse textured and dry, but better root and height growth may occur when roots penetrate subsoils that have a higher moisture holding and nutrient holding capacity. This pattern was observed in Figure 4.2 where glaciofluvial deposits showed slower early growth before 20 years BHA than the other landforms.

5.5 Accuracy and Validation

Once a model is developed using a data set from the study population, there is a need to confirm the accuracy of the model using an independent data set. Such an independent confirmation insures that the model developed does reflect the height-growth patterns observed in the population. Confirmation can be accepted when the magnitude of error that is computed using the computation data set is similar to observed error using the verification data set. The 60 plots randomly selected from the 383 plots for the verification data set is larger than most complete data sets that often are to develop jack pine site-index curves (Millar and Woods 1989, Zakrewski 1990). The greatest deficiency in most site-index studies are the verification of the models developed. Of the jack pine site index curves listed in Table 2.1 only Carmean and Lenthall (1989) and Goelz and Burk (1992) had an independent verification data set. There are few reports that quantify the accuracy of other site-index models. For instance, Ker and Bowling (1991) only compared one data point.

The need to have a separate model for height-growth curves, site-index curves and site-index prediction are discussed below. Variation in height-growth curves from Ontario and across the range of jack pine will be discussed.

5.5.1 Site-Index Curves Versus Height-Growth Curves

Newnham (1988) fitted the unconstrained Ek model [Eq. 2] and his constrained version [Eq. 6] using white spruce stem-analysis data from a study conducted by Alemdag (1988). He found that equation [6] improved upon the Ek model by (1) passing through the exact height specified by site index, and (2) it did so with minimal loss of overall accuracy. My study also shows that the constrained model can improve accuracy over the unconstrained version. For jack pine in Ontario, there is no benifit in having separate height-growth curves and site-index curves since the Newnham model [Eq. 6] accomplishes both objectives.

5.5.2 Site-Index Curves Versus Site Index Prediction Equations

A comparison was made of the accuracy of predicting site index using the Newnham [Eq. 6] and the modified linear Monserud site-index prediction equation [Eq. 25 and Eq. 26] (Table 4.13). This comparison showed that the Newnham model was as accurate as the Monserud model even when early growth before 20 years was included. However, the inclusion of growth before 20 years resulted in much wider prediction intervals for both models. Jack pine site-index curves developed from Newnham's model [Eq. 6] can be used to predict height growth and site-index values with accuracy equivalent to individual height or site index dependent models. To solve Newnham's model for site-index requires a computer algorithm to converge on the correct site-index level that will match the current height of the jack pine trees. The predictions using the two models may differ slightly but this difference is insignificant based upon the amount of unexplained variation.

5.5.3 Final Height-Growth - Site-Index Curves

The Newnham [Eq. 6] constrained model showed a number of deficiencies even though it was considered to be the most precise model of those fitted (Table 4.8). Examination of average deviations over 2 m site-index classes at every 10 year interval were used to determine if the model predicted shape well over the range of site-index classes. The average deviations showed that the Newnham model predicted the true shape of each site-index class within +/- 1 m except for older age classes (+100 years BHA) on poor sites.

Newnham's model [Eq. 27] consistently overestimates heights in site-index class 8 m and 12 m. These small overestimations are due to somewhat different height growth on four bedrock and glaciofluvial plots. These differences in height-growth patterns were apparent in early comparisons between landforms on poor sites. There are few plots having old trees on such poor sites for the different landforms to verify accuracy of height-growth predictions for older ages. The other site classes have much more data for ages under 100 years BHA and estimated height agrees closely with actual height for almost all ages under 100 years BHA.

5.5.4 Comparisons to Existing Height-Growth Curves in Ontario

Comparisons showed that the Plonski (1974), Lenthall (1986), Goelz and Burk (1992) and my curves were almost identical on very good sites. This similarity suggests that consistent height growth on good sites is predictable.

Polymorphic growth patterns are apparent on medium and poor sites thus Plonski's anamorphic curves flatten much sooner than do the polymorphic equations. All three polymorphic models tend to be conservative and underestimate height-growth for older trees on poor sites. This underestimation on the part of the polymorphic models suggest the magnitude of error is even greater than predicted when Plonski's harmonized curves are used.

For example, the height of Plonski's Site Class II and III curves when converted to BHA are 13.27 m and 20.52 m at 100 years BHA. This would imply that the Site Class boundry would occur at the half-way point or 16.90 m. My height-growth curve [Eq. 27] for poor sites is 2.34 m above Plonski's curve at 100 years BHA in Figure 4.6 for Site Class III. Using Table 4.14, my curves underestimated height growth for SI_{BHA50}=12 m at 100 years BHA by 1.25 m, thus Plonski's curves would underestimate the height of the average SI_{BHA50}=12 m plot by 3.59 m. The height of a 100 year old jack pine on Plonski's Site Class III curve (13.27 m) when combined with the average deviation (3.59 m) would be at the boundry (16.86 m) of Site Class II and III. When poor site stands are measured before index age using Plonski's curves instead of more recent curves (Lenthall 1986, Carmean and Lenthall 1989), the stands may be better sites than expected and have the potential of growing into the Site Class above and thus producing more growth than predicted. The chance of misclassifying Site Class I and II is less likely to occur than between Site Class II and III because of the polymorphic nature of the height-growth pattern of jack pine.

5.5.5 Height-Growth Patterns Across the Range Of Jack Pine

Height-growth of jack pine in northern Ontario flattens [Eq. 27] more rapidly than height-growth of jack pine in Alberta or in New Brunswick (Figure 4.7). An important factor that determines the shape on good sites is the amount of representation there is on good sites at old ages. This study has no data points for SI_{BH50}=22 m to represent tree growth past 70 years BHA (Table 4.15). The Alberta and New Brunswick studies also may have lacked data for modelling height growth at old ages. Thus, the upper asymptote of height growth on good sites may be an extrapolation and their estimated differences may not be real. Growth differences also may occur due to climatic differences because both Alberta and New Bruinswick have experience moderating climates from the east and west coasts. Precipitation is much higher in New Brunswick than in most of Ontario or Alberta. In addition, the growth curves computed for Alberta are for combined jack pine and lodgepole pine stands. Separate studes for lodgepole pine indicate better and more sustained later height growth than occurs for jack pine on good sites. Accordingly, if the stands used in the Alberta study contain more lodgepole pine than jack pine this might explain the apparent better growth for older trees in Alberta.

Early sigmoid growth patterns are distinct for jack pine on poor sites in New Brunswick and to some degree in Alberta. In New Brunswick, it is not uncommon to find jack pine growing on organic soils. Such sites tend to have large amounts of <u>Kalmia</u> spp. and <u>Vaccinium</u> spp. Work by Hamilton and Krause (1985) found that <u>Kalmia</u> spp. and <u>Vaccinium</u> spp. were important variables in predicting early height-growth increment of jack pine in New Brunswick. These ericacious species are very competitive for moisture and nutrients and also have an allelopathic effect on jack pine, consequently young jack pine may have slower early height growth. On richer sites, ericacious plants are uncommon and the demand for nutrients is less and there is little or no early sigmoid growth on the better sites.

6

CONCLUSIONS

1) This study shows that the site-index curves developed by Lenthall (1986) for north central Ontario have height-growth patterns closely resembling the height-growth patterns for other regions of northern Ontario. Consequently, the site-index curves developed by my study appear to be applicable to jack pine throughout northern Ontario.

2) Height-growth patterns for the different regions and glacial landforms found throughout northern Ontario are similar even though certain combinations of regions and glacial landforms have somewhat different height-growth patterns for very young ages or for very old ages.

3) The Newnham (1988) version [Eq. 27] of Ek's height-growth model constrained to pass through specified height at index age predicts height with greater precision and accuracy than the Ek [Eq. 2] unconstrained model. This model also predicts site index with equal accuracy and precision as a linear site-index prediction model [Eq 28].

4) A linear site-index prediction equation [Eq. 28] is recommended for use in predicting site index given total height and age. When

predicting site index using Eq. [28], predictions should not be made using data from stands younger than 20 years BHA.

5) Plonski's (1974) site-class curves can result in imprecise predictions of site index on poor sites (10 m to 14 m). These poor sites have better height growth in later years than predicted by the Plonski site-class curves.

6) Height-growth patterns for jack pine vary somewhat across the species range. It is not known if these small differences between provincial curves are due to sampling of data or if the height curves compared reflect the adaptive response of jack pine to climate.

7). Final site-index curves based on Eq. [27] and a final site-index prediction Eq. [28] are given for use in northern Ontario.

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APPENDICES

APPENDIX I

GLOSSARY OF TERMS

The following terms used in text were assembled by Carmean (1994).

1.) SITE

A position, place, site, local position of ---; the situation of a growing plant with respect to all the environmental factors (as climate, soil, drainage, other plant and animal life) affecting growth (Gore 1971).

An area considered in terms of its environment, particularly as this determines the type and quality of the vegetation the area can carry (Ford-Robertson 1983).

The sum total of environmental conditions surrounding and available to the plant. The term site (habitat) usually includes both the position in space and the associated environment (Spurr and Barnes 1980).

2.) SITE TYPE

Sites are classified --- by their climate, soil and vegetation into site types (Ford-Robertson 1983).

3.) SITE QUALITY

A loose term denoting the relative productivity of a site for a particular tree species (Ford-Robertson 1983).

The inherent capacity of the site to produce plant growth. Site quality is a function of the physiography, climate, soil and other features of the environment that are not easily altered (Pritchett and Fisher 1987).

The sum total of all of the factors affecting the capacity to produce forests or other vegetation: climatic factors, soil (edaphic) factors and biological factors (Spurr and Barnes 1980).

4.) SITE PRODUCTIVITY

The rate of product growth can be considered to be the sum of site quality plus management input. Productivity is subject to varying degrees of alteration by manipulation of growing stock or modification of the site (Pritchett and Fisher 1987).

5.) SITE CLASS

A measure of the relative productive capacity of a site for the crop or stand under study based e.g. on volume or height (dominant, codominant or mean) or the maximum annual increment that is attained or attainable at a given age (Ford-Robertson 1983).

6.) SITE INDEX

A particular measure of site class based on the height of the dominant trees in a stand at an arbitrarily chosen age (Ford-Robertson 1983).

For most eastern forest species site index is defined as height of dominant and codominant trees at 50 years total age. However, age from breast height is sometimes used for species such as spruces, balsam fir, and red pine that have slow and erratic growth before reaching breast height. Younger index ages are sometimes used for plantations, shortlived species, or species managed on short rotations (Carmean et al. 1989).

7.) TOTAL SITE

Site is the integrated complex of all the features of a prescribed area and, as such, is a specific unit. Both site and environment are here considered to include both the organisms and their physical surroundings (Hills 1952).

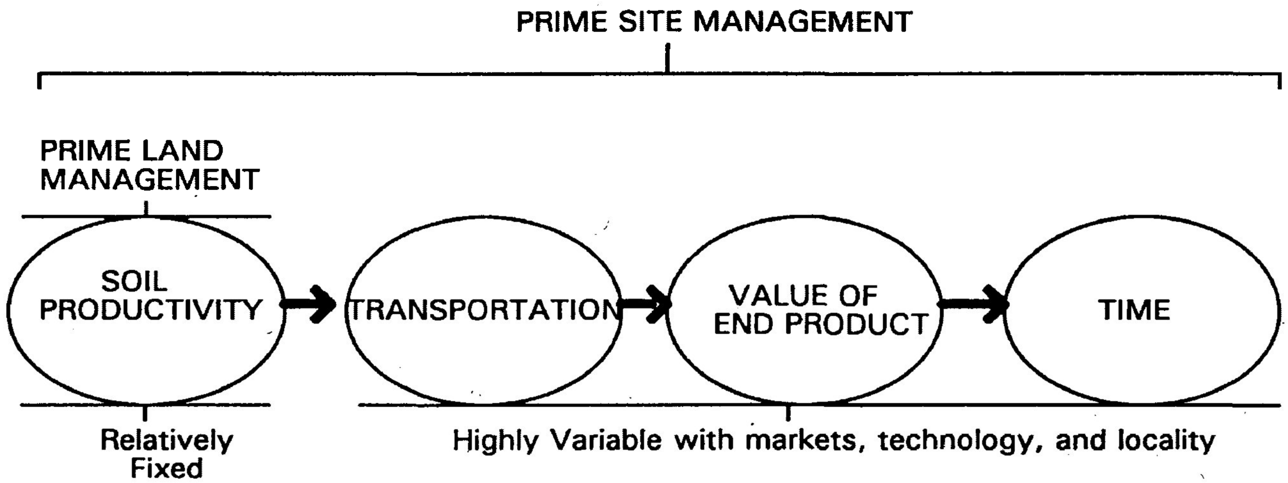
8.) PRIME LAND

Prime land is considered to be land upon which the growth rate of commercial tree species is better than that of other sites, due to the inherent capability of the soil to supply moisture and nutrients for forest growth (Towill and White 1987).

9.) PRIME SITE MANAGEMENT

Prime site management refers to the directing of forest land management decisions in accordance with the highest overall return on investment. Many factors are considered when making these decisions including such things as stand growth, current stand conditions, wood supply requirements, current silvicultural knowledge, constraints, financial resources available, social and economic conditions and proximity to mills (Towill and White 1987).

Differences between prime land management and prime site management have been illustrated by LeBlanc and Towill (1989).



10.) Autocorrelation

The correlation between X_i and X_j being a function of their distance apart $(i - j)$ and diminishing as the distance increases (Cochran 1977).

APPENDIX II

PARTIAL DERIVATIVES FOR THE EK MODEL [EQ. 2]

$$H_t = 1.3 + \beta1 * SI^{\beta2} * \left(1 - e^{\beta3 * BHA}\right)^{\beta4 * SI^{\beta5}} + \varepsilon$$

Partial Derivatives:

$$\frac{d}{d\beta1} = SI^{\beta2} * \left(1 - e^{\beta3 * BHA}\right)^{\beta4 * SI^{\beta5}}$$

$$\frac{d}{d\beta2} = \beta1 * \left(1 - e^{\beta3 * BHA}\right)^{\beta4 * SI^{\beta5}} * SI^{\beta2} * LN(SI)$$

$$\frac{d}{d\beta3} = \beta3 * SI^{\beta2} * \beta4 * SI^{\beta5} * \left(\left(1 - e^{\beta3 * BHA}\right)^{(\beta4 * SI^{\beta5} - 1)}\right) * \left(-e^{\beta3 * BHA} * BHA\right)$$

$$\frac{d}{d\beta4} = \beta1 * SI^{\beta2} * \left(\left(1 - e^{\beta3 * BHA}\right)^{\beta4 * SI^{\beta5}}\right) * LN\left(1 - e^{\beta3 * BHA}\right) * SI^{\beta5}$$

$$\frac{d}{d\beta5} = \beta1 * SI^{\beta2} * \left(1 - e^{\beta3 * BHA}\right)^{\beta4 * SI^{\beta5}} * LN\left(1 - e^{\beta3 * BHA}\right) * \beta4 * SI^{\beta5} * LN(SI)$$

APPENDIX III

SUMMARY STATISTICS AND COEFFICIENTS FOR HEIGHT, SITE-INDEX AND SITE-INDEX PREDICTION MODELS

MODEL: HEIGHT GROWTH EQUATION - EK (1971) [EQ.2]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
β1	2.57916	0.05758	2.46627	2.69205
β2	0.79887	0.00822	0.78275	0.81499
β3	-0.02330	0.00029	-0.02386	-0.02273
β4	2.14610	0.10618	1.93793	2.35426
β5	-0.25467	0.01737	-0.28872	-0.22061

STANDARD ERROR OF THE ESTIMATE	0.7315
MEAN SQUARE ERROR	0.5352
SUM OF SQUARES	2832.5640
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1	-0.9849	0.0385	0.7543	-0.7888
β2	-0.9849	1	0.1247	-0.7993	0.8151
β3	0.0385	0.1247	1	-0.3226	0.1931
β4	0.7543	-0.7993	-0.3226	1	-0.9894
β5	-0.7888	0.8151	0.1931	-0.9894	1

MODEL: HEIGHT GROWTH EQUATION - KER AND BOWLING (1991) [EQ. 3]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β1	4.04726	0.17304	3.70802	4.38650
β2	1.22499	0.01289	1.19971	1.25026
β3	0.02317	0.00029	0.02261	0.02373
β4	2.02547	0.09802	1.83332	2.21763
β5	-0.23441	0.01699	-0.26772	-0.20111

STANDARD ERROR OF THE ESTIMATE	0.7339
MEAN SQUARE ERROR	0.5386
SUM OF SQUARES	2850.7455
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1	-0.8911	0.0315	0.7721	-0.798
β2	-0.8911	1	-0.4547	-0.8054	0.7785
β3	0.0315	-0.4547	1	0.3135	-0.1802
β4	0.7721	-0.8054	0.3135	1	-0.9888
β5	-0.798	0.7785	-0.1802	-0.98888	1

MODEL: HEIGHT GROWTH EQUATION - MONSERUD [EQ.4]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limits Upper	Lower
β1	0.56154	0.03411	0.49468	0.62840
β2	0.64732	0.02219	0.60382	0.69082
β3	0.49229	0.01237	0.46803	0.51654
β4	-1.00349	0.00798	-1.01913	-0.98785
β5	-0.36782	0.04442	-0.45490	-0.28074

STANDARD ERROR OF THE ESTIMATE	0.8319
MEAN SQUARE ERROR	0.6921
SUM OF SQUARES	3663.0113
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1	-0.9874	0.9298	0.0789	-0.9528
β2	-0.9874	1	-0.9617	0.0692	0.9737
β3	0.9298	-0.9617	1	-0.2234	-0.9936
β4	0.0789	0.0692	-0.2234	1	0.1278
β5	-0.9528	0.9737	-0.9936	0.1278	1

MODEL: SITE-INDEX EQUATION - NEWNHAM (1988) [Eq.6]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limits Upper	Lower
β1	4.119389	0.259171	3.611306	4.627472
β2	0.623573	0.023037	0.578412	0.668735
β4	1.400000	0.155256	1.095634	1.704366
β5	-0.080871	0.040486	-0.160240	-0.001503

STANDARD ERROR OF THE ESTIMATE	0.7356
MEAN SQUARE ERROR	0.5411
SUM OF SQUARES	2864.5761
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β4	β5
β1	1	-0.9986	-0.7449	0.7475
β2	-0.9986	1	0.7448	-0.7496
β4	-0.7449	0.7448	1	-0.9984
β5	0.7475	-0.7496	-0.9984	1

MODEL: SITE-INDEX EQUATION - TENNENT AND BURKHART [EQ. 7]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β3	-0.07410	0.00119	-0.07643	-0.07176
β4	0.17462	0.03154	0.11278	0.23646
β5	0.10082	0.64541	-0.02571	0.22735

STANDARD ERROR OF THE ESTIMATE	2.0505
MEAN SQUARE ERROR	4.2045
SUM OF SQUARES	22262.9150
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β3	β4	β5
β3	1.0000	-0.1295	-0.0152
β4	-0.1295	1.0000	-0.9880
β5	-0.0152	-0.9880	1.0000

MODEL: SITE-INDEX EQUATION - KER AND BOWLING (1991) [EQ. 8]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
β2	0.02351	0.00024	0.02304	0.02398
β3	2.54651	0.14532	2.26163	2.83140
β4	-0.30331	0.01956	-0.34166	-0.26496

STANDARD ERROR OF THE ESTIMATE	0.7608
MEAN SQUARE ERROR	0.5789
SUM OF SQUARES	3065.1268
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β2	β3	β4
β2	1	0.3418	-0.2485
β3	0.3418	1	-0.9939
β4	-0.2485	-0.9939	1

MODEL: SITE INDEX EQUATION - DEMPSTER AND ASSOC. [EQ. 9]

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limits Upper	95% Confid. Limits Lower
β1	6.767256372	0.103461002	6.564430171	6.970082574
β2	-1.241212157	0.007008463	-1.254951632	-1.227472682
β3	-0.785814601	0.036244985	-0.856869709	-0.714759494

STANDARD ERROR OF THE ESTIMATE	0.7459
MEAN SQUARE ERROR	0.5564
SUM OF SQUARES	2945.9941
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3
β1	1	-0.3438	-0.9871
β2	-0.3438	1	0.1997
β3	-0.9871	0.1997	1

MODEL: SITE INDEX PRED. EQUATION - GOELZ AND BURK [EQ.5]
(Base-age Invariant Model solved for SI at 50 years BHA fitted to data from time 0 BHA)

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limits Upper	Lower
β1	-0.06160	0.00033	-0.06226	-0.06095
β2	1.04721	0.01152	1.02463	1.06979
β3	-0.01589	0.00421	-0.02416	-0.00763
β4	1.04429	0.00173	1.04089	1.04768

STANDARD ERROR OF THE ESTIMATE	1.6052
MEAN SQUARE ERROR	2.5766
SUM OF SQUARES	13640.5318
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3	β4
β1	1	0.0209	-0.0721	-0.4901
β2	0.0209	1	0.8903	0.0108
β3	-0.0721	0.8903	1	0.0785
β4	-0.4901	0.0108	0.0785	1

MODEL: SITE-INDEX PRED. EQUATION - MONSERUD [EQ.10]
FITTED TO DATA FROM AGE 0 BHA

COEFFICIENTS

Parameter	B	SE B	βETA	T
βO	20.762564	0.14332		144.866
β1	0.591538	0.02096	1.37482	28.221
β2	3.403127	0.16046	1.47699	21.208
β3	-3.350079	0.12847	-2.02737	-26.077
β4	-0.843624	0.01744	-1.69651	-48.384
β5	-0.882597	0.08236	-0.20263	-10.716
β6	0.149425	0.01198	0.72086	12.473

R-SQUARED	0.73308
STANDARD ERROR OF THE ESTIMATE	1.3581
MEAN SQUARE ERROR	1.8443
SUM OF SQUARES	9758.2895
NUMBER OF OBSERVATIONS	5298

MODEL: SITE INDEX PRED. EQUATION - PAYANDEH [EQ.11] FITTED TO DATA FROM AGE 0 BHA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limits Upper	Lower
β1	2.02323	0.76223	0.52895	3.51751
β2	0.42017	0.00859	0.40333	0.43702
β3	-0.00100	0.00133	-0.00360	0.00160
β4	-0.29342	0.00547	-0.30415	-0.28269
β5	0.00737	0.00571	-0.00381	0.01856

STANDARD ERROR OF THE ESTIMATE	1.9937
MEAN SQUARE ERROR	3.9747
SUM OF SQUARES	21038.1456
NUMBER OF OBSERVATIONS	5298

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1	-0.453	-0.9961	-0.3965	0.8669
β2	-0.453	1	0.3799	-0.5202	-0.5566
β3	-0.9961	0.3799	1	0.4755	-0.8655
β4	-0.3965	-0.5202	0.4755	1	-0.3973
β5	0.8669	-0.5566	-0.8655	-0.3973	1

MODEL: FINAL SITE-INDEX EQUATION [Eq. 27]

COEFFICIENTS

Parameter	Calculated Parameter	Std. Error Parameter	95% Confid. Limits Upper	95% Confid. Limits Lower
β1	8.748534	0.495597	7.776995	9.720074
β2	0.353038	0.020481	0.312887	0.393188
β4	0.396246	0.033247	0.331070	0.461422
β5	0.379881	0.030826	0.319451	0.440310

STANDARD ERROR OF THE ESTIMATE	0.7217
MEAN SQUARE ERROR	0.5208
SUM OF SQUARES	3255.7741
NUMBER OF OBSERVATIONS	6255

CORRELATION MATRIX

	β1	β2	β4	β5
β1	1	-0.9985	-0.7237	0.7194
β2	-0.9985	1	0.7289	-0.7273
β4	-0.7237	0.7289	1	-0.9977
β5	0.7194	-0.7273	-0.9977	1

MODEL: FINAL SITE-INDEX PREDICTION EQUATION [EQ. 28]

COEFFICIENTS

Parameter	B	SE B	βETA	T
βO	26.183227	0.88976		29.427
β1	0.739620	0.01127	1.11451	65.654
β2	-8.401658	0.40736	-1.39580	-20.625
β3	0.522488	0.05002	0.66514	10.445
β4	13.184068	0.49636	0.39716	26.562

R-SQUARED	0.90748
STANDARD ERROR OF THE ESTIMATE	0.8515
MEAN SQUARE ERROR	0.7250
SUM OF SQUARES	3420.5558
NUMBER OF OBSERVATIONS	4723

APPENDIX IV

INDIVIDUAL TREE HEIGHT-GROWTH PATTERNS ON THREE PLOTS THAT PRODUCED LARGE RESIDUAL VALUES USING THE EK MODEL [EQ. 2]

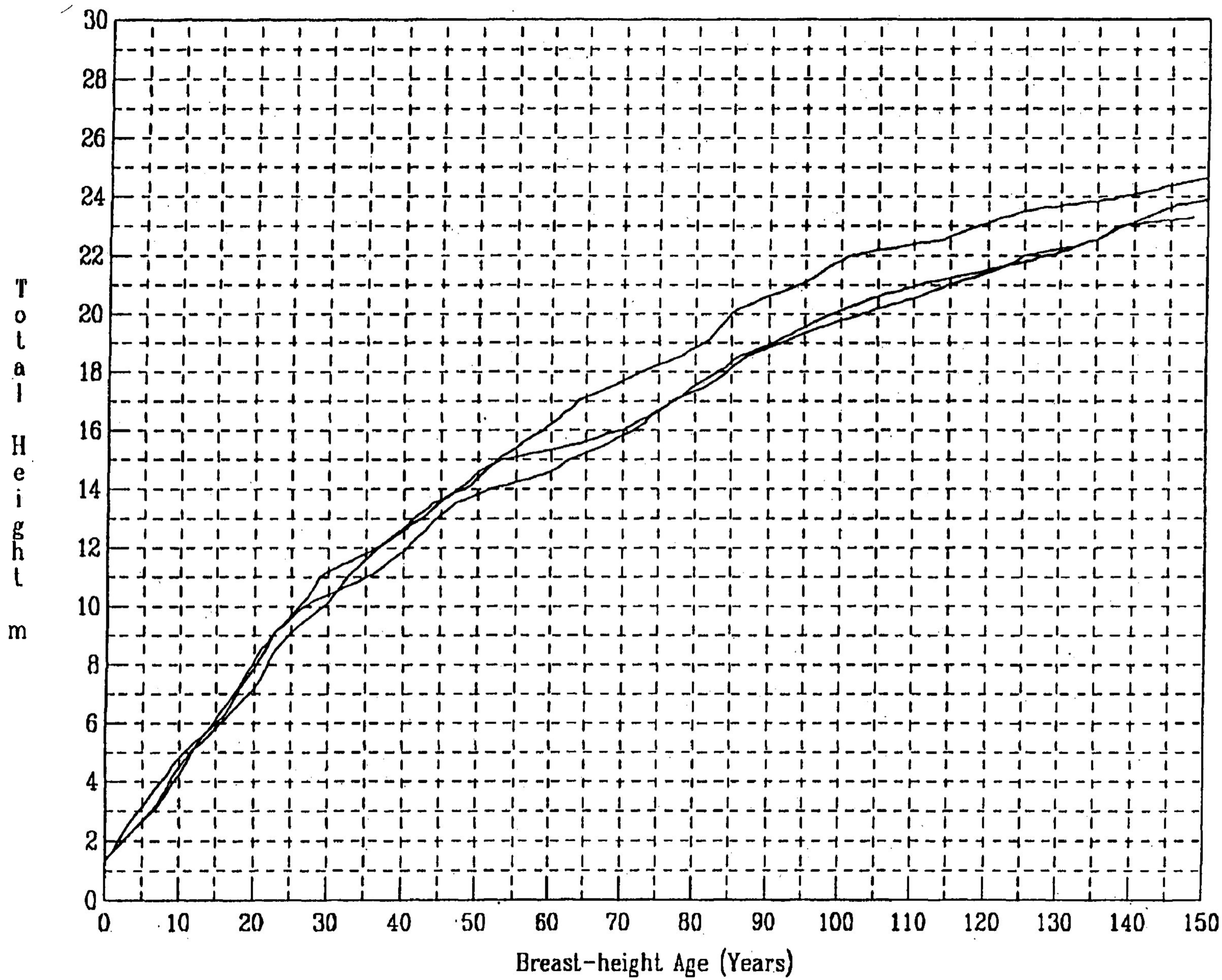


Figure 1. Individual tree height-age information for GP890074.

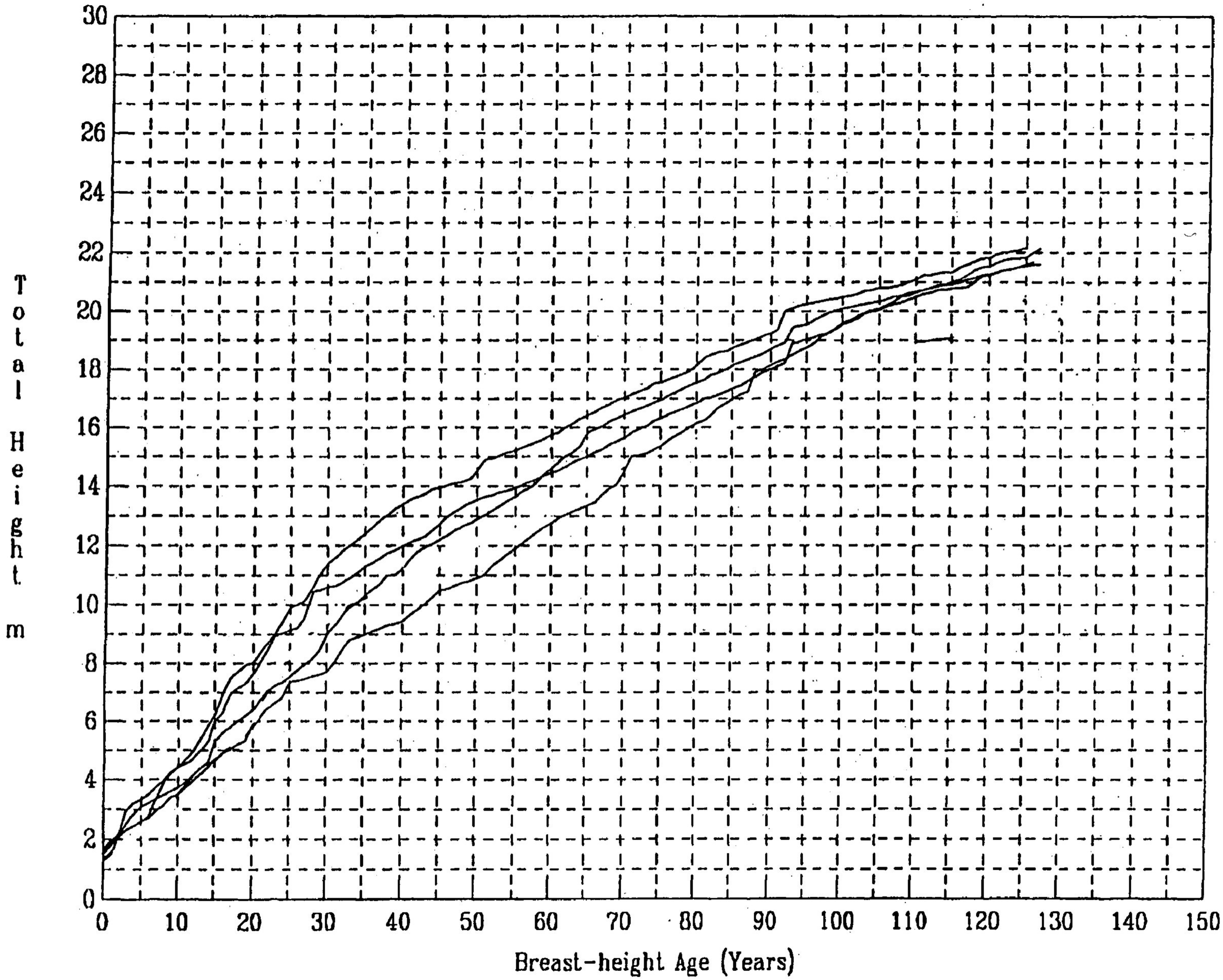


Figure 2. Individual tree height-age information for KO870074.

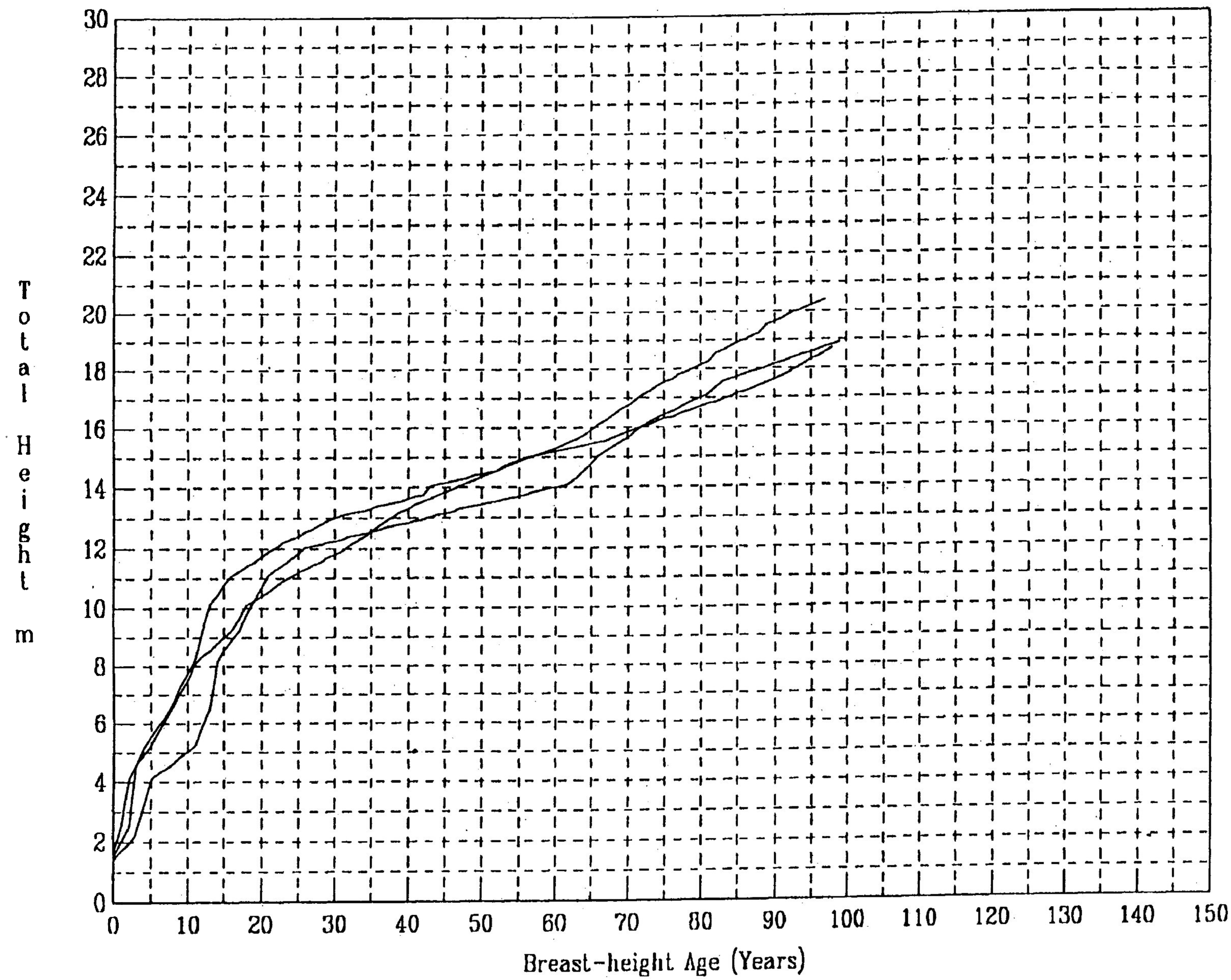


Figure 3. Individual tree height-age information for DL86017.

APPENDIX V

SUMMARY STATISTICS OF THE EK MODEL [EQ. 2] FITTED TO EACH REGIONAL DATA SET

The Ek model [Eq. 2] was to fitted stem analysis data from each of the four OMNR administrative regions.

MODEL: EK (1971) [EQ.2] FITTED TO NWR DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β1	1.92524	0.11084	1.70784	2.14265
β2	0.89281	0.02043	0.85273	0.93288
β3	-0.01987	0.00075	-0.01840	-0.02134
β4	2.00070	0.21686	1.57533	2.42608
β5	-0.26745	0.03835	-0.19222	-0.34268

STANDARD ERROR OF THE ESTIMATE	0.6947
MEAN SQUARE ERROR	0.4827
SUM OF SQUARES	733.1547
NUMBER OF OBSERVATIONS	1524

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9719	-0.2362	0.7405	0.7973
β2	-0.9717	1.0000	0.0062	-0.8247	-0.8527
β3	-0.2362	0.0062	1.0000	0.2725	0.1340
β4	0.7405	-0.8247	0.2725	1.0000	0.9886
β5	0.7973	-0.8527	0.1340	0.9886	1.0000

MODEL: EK (1971) [EQ.2] FITTED TO NCR DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
β1	2.92729	0.08296	2.76462	3.08997
β2	0.72863	0.01018	0.70867	0.74860
β3	-0.02396	0.00038	-0.02322	-0.02470
β4	5.18980	0.36793	4.46837	5.91124
β5	-0.55326	0.02375	-0.50669	-0.59983

STANDARD ERROR OF THE ESTIMATE	0.7616
MEAN SQUARE ERROR	0.5801
SUM OF SQUARES	1655.0467
NUMBER OF OBSERVATIONS	2858

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9867	0.0130	0.7027	0.7398
β2	-0.9867	1.0000	-0.1637	-0.7564	-0.7783
β3	0.0130	-0.1637	1.0000	0.4311	0.3159
β4	0.7027	-0.7564	0.4311	1.0000	0.9907
β5	0.7398	-0.7783	0.3159	0.9907	1.0000

MODEL: EK (1971) [EQ.2] FITTED TO NER DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
β1	2.06102	0.11302	1.83922	2.28281
β2	0.87668	0.01977	0.83788	0.91548
β3	-0.02134	0.00050	-0.02036	-0.02232
β4	2.57968	0.28366	2.02301	3.13635
β5	-0.30332	0.03741	-0.22991	-0.37674

STANDARD ERROR OF THE ESTIMATE	0.6111
MEAN SQUARE ERROR	0.3734
SUM OF SQUARES	352.1509
NUMBER OF OBSERVATIONS	948

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9904	0.0637	0.7897	0.8132
β2	-0.9904	1.0000	-0.1928	-0.8206	-0.8317
β3	0.0637	-0.1928	1.0000	0.3550	0.2492
β4	0.7897	-0.8206	0.3550	1.0000	0.9927
β5	0.8132	-0.8317	0.2492	0.9927	1.0000

MODEL: EK (1971) [EQ.2] FITTED TO NOR DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
β1	1.54241	0.09924	1.34761	1.73721
β2	0.94411	0.02254	0.89987	0.98835
β3	-0.02786	0.00090	-0.02609	-0.02963
β4	2.76867	0.41468	1.95469	3.58265
β5	-0.27959	0.05076	-0.17996	-0.37923

STANDARD ERROR OF THE ESTIMATE	0.6775
MEAN SQUARE ERROR	0.4590
SUM OF SQUARES	367.1594
NUMBER OF OBSERVATIONS	805

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9853	-0.0775	0.7388	0.7716
β2	-0.9853	1.0000	-0.0866	-0.7829	-0.7973
β3	-0.0775	-0.0866	1.0000	0.2838	0.1607
β4	0.7388	-0.7829	0.2838	1.0000	0.9910
β5	0.7716	-0.7973	0.1607	0.9910	1.0000

APPENDIX VI

SUMMARY STATISTICS OF THE EK MODEL [EQ. 2] FITTED TO EACH LANDFORM DATA SET

The Ek model [Eq. 2] was fitted to stem-analysis data from four broad glacial landform groups.

MODEL: EK (1971) [EQ.2] FITTED TO LACUSTRINE DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	95% Confid. Limit Lower
$\beta 1$	1.78026	0.14547	1.49467	2.06586
$\beta 2$	0.92335	0.03021	0.86405	0.98265
$\beta 3$	-0.02060	0.00080	-0.01864	-0.21000
$\beta 4$	0.78773	0.13336	0.52590	1.04955
$\beta 5$	-0.07328	0.05747	-0.18611	-0.03955

STANDARD ERROR OF THE ESTIMATE	0.7274
MEAN SQUARE ERROR	0.5290
SUM OF SQUARES	381.9691
NUMBER OF OBSERVATIONS	727

CORRELATION MATRIX

	$\beta 1$	$\beta 2$	$\beta 3$	$\beta 4$	$\beta 5$
$\beta 1$	1	-0.9883	0.2115	0.8314	0.8393
$\beta 2$	-0.9883	1	-0.3518	-0.8431	-0.8377
$\beta 3$	0.2115	-0.3518	1	0.34	0.2425
$\beta 4$	0.8314	-0.8431	0.34	1	0.9938
$\beta 5$	0.8393	-0.8377	0.2425	0.9938	1

MODEL: EK (1971) [EQ.2] FITTED TO MORAINAL DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β1	2.39900	0.10650	2.19011	2.60790
β2	0.80196	0.01605	0.77049	0.83344
β3	-0.02304	0.00045	-0.02216	-0.02392
β4	3.25415	0.33633	2.59446	3.91383
β5	-0.40156	0.03513	-0.33265	-0.47047

STANDARD ERROR OF THE ESTIMATE	0.6869
MEAN SQUARE ERROR	0.4718
SUM OF SQUARES	778.4002
NUMBER OF OBSERVATIONS	1655

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9915	0.0997	0.7795	0.7997
β2	-0.9915	1.0000	-0.2196	-0.8088	-0.8187
β3	0.0997	-0.2196	1.0000	0.3813	0.2868
β4	0.7795	-0.8088	0.3813	1.0000	0.9940
β5	0.7997	-0.8187	0.2868	-0.9940	1.0000

MODEL: EK (1971) [EQ.2] FITTED TO GLACIOFLUVIAL DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β1	3.86231	0.12582	3.61156	4.10903
β2	0.64483	0.01150	0.62228	0.66738
β3	-0.02314	0.00038	-0.02239	-0.02389
β4	7.25028	0.54390	6.18376	8.31680
β5	-0.65611	0.02530	-0.60649	-0.70573

STANDARD ERROR OF THE ESTIMATE	0.6976
MEAN SQUARE ERROR	0.4867
SUM OF SQUARES	1242.9830
NUMBER OF OBSERVATIONS	2559

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1.0000	-0.9858	-0.0837	0.6903	0.7325
β2	-0.9858	1.0000	-0.0754	-0.7548	-0.7820
β3	-0.0837	-0.0754	1.0000	0.4080	0.2990
β4	0.6903	-0.7548	0.4080	1.0000	0.9920
β5	0.7325	-0.7820	0.2990	0.9920	1.0000

MODEL: EK (1971) [EQ.2] FITTED TO SHALLOW-BEDROCK DATA

COEFFICIENTS

Parameter	Estim. of Parameter	Std. Error Parameter	95% Confid. Limit Upper	Lower
β1	1.60754	0.06047	1.48891	1.72618
β2	0.92201	0.01346	0.89561	0.94841
β3	-0.02517	0.00066	-0.02387	-0.02647
β4	2.43027	0.21085	2.01659	2.84396
β5	-0.30495	0.03076	-0.24459	-0.36530

STANDARD ERROR OF THE ESTIMATE	0.6526
MEAN SQUARE ERROR	0.4259
SUM OF SQUARES	506.4276
NUMBER OF OBSERVATIONS	1194

CORRELATION MATRIX

	β1	β2	β3	β4	β5
β1	1	-0.9783	-0.2348	0.6672	0.7329
β2	-0.9783	1	0.0421	-0.7407	-0.7783
β3	-0.2348	0.0421	1	0.2854	0.1132
β4	0.6672	-0.7407	0.2854	1	0.9819
β5	0.7329	-0.7783	0.1132	0.9819	1

APPENDIX VII

INDIVIDUAL PLOT HEIGHT-GROWTH CURVES USED TO COMPARE REGIONS, LANDFORMS AND SITE CLASS

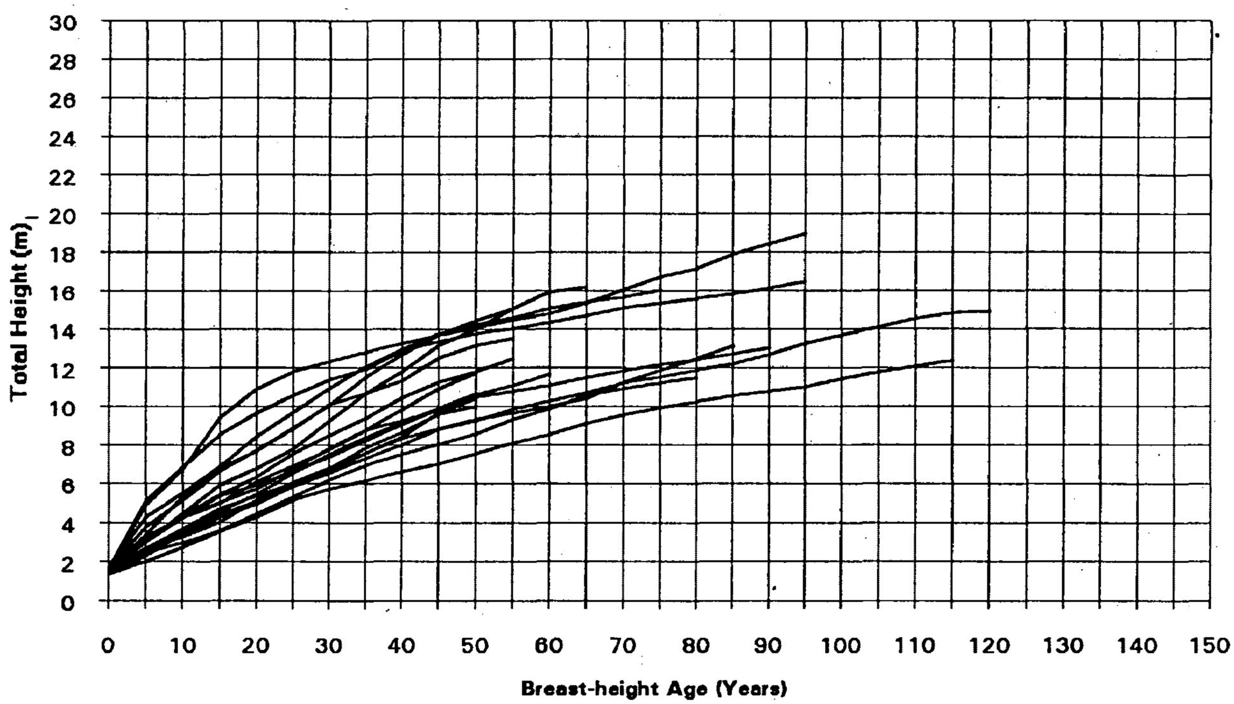


Figure 1. Height-growth curves for plots in NCR on bedrock sites having poor site quality.

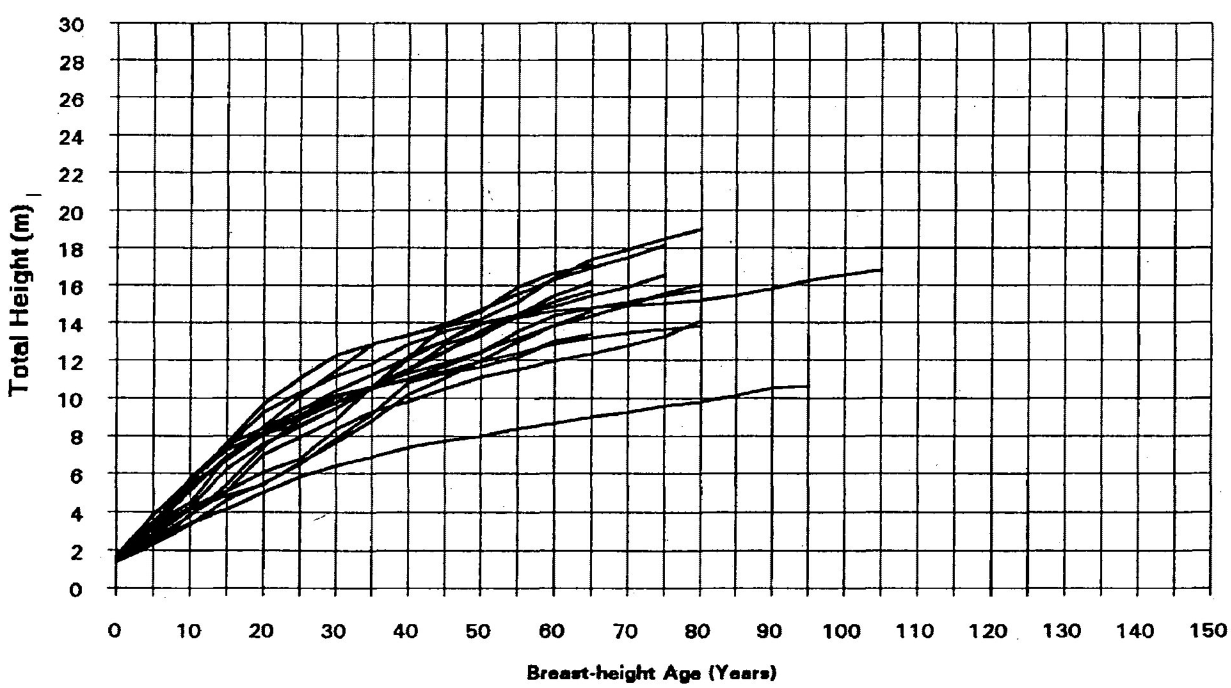


Figure 2. Height-growth curves for plots in NWR on bedrock sites having poor site quality.

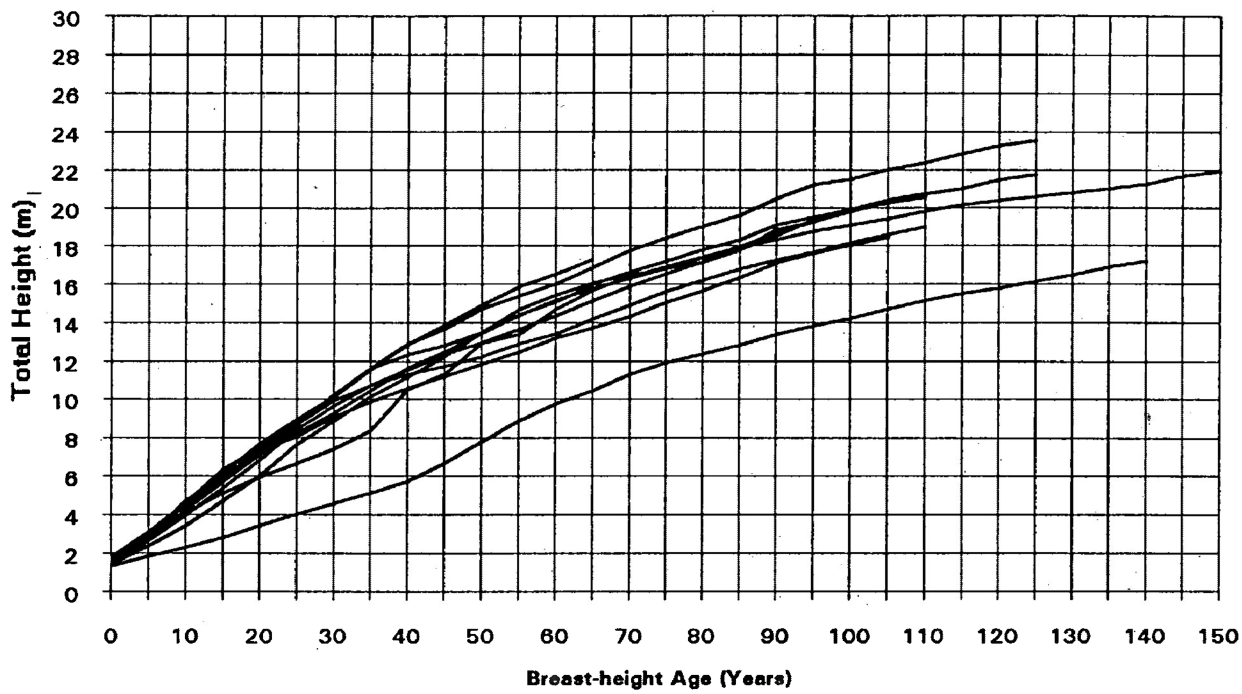


Figure 3. Height-growth curves for plots in NCR on glaciofluvial sites having poor site quality.

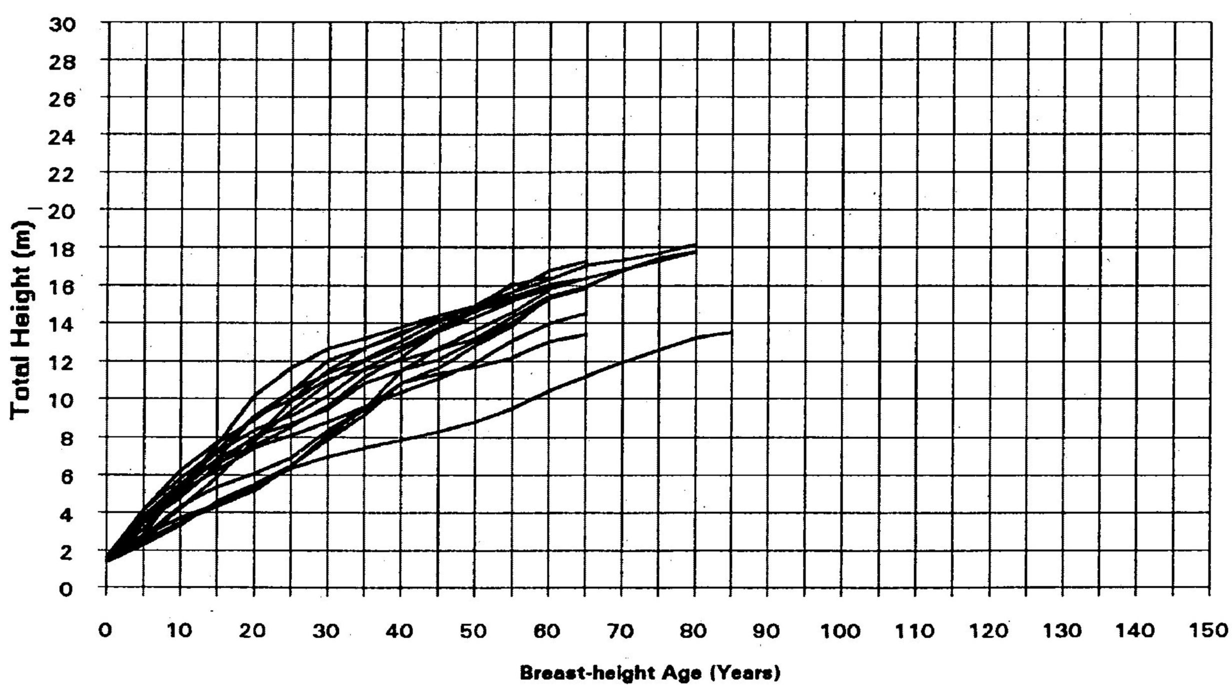


Figure 4. Height-growth curves for plots in NWR on glaciofluvial sites having poor site quality.

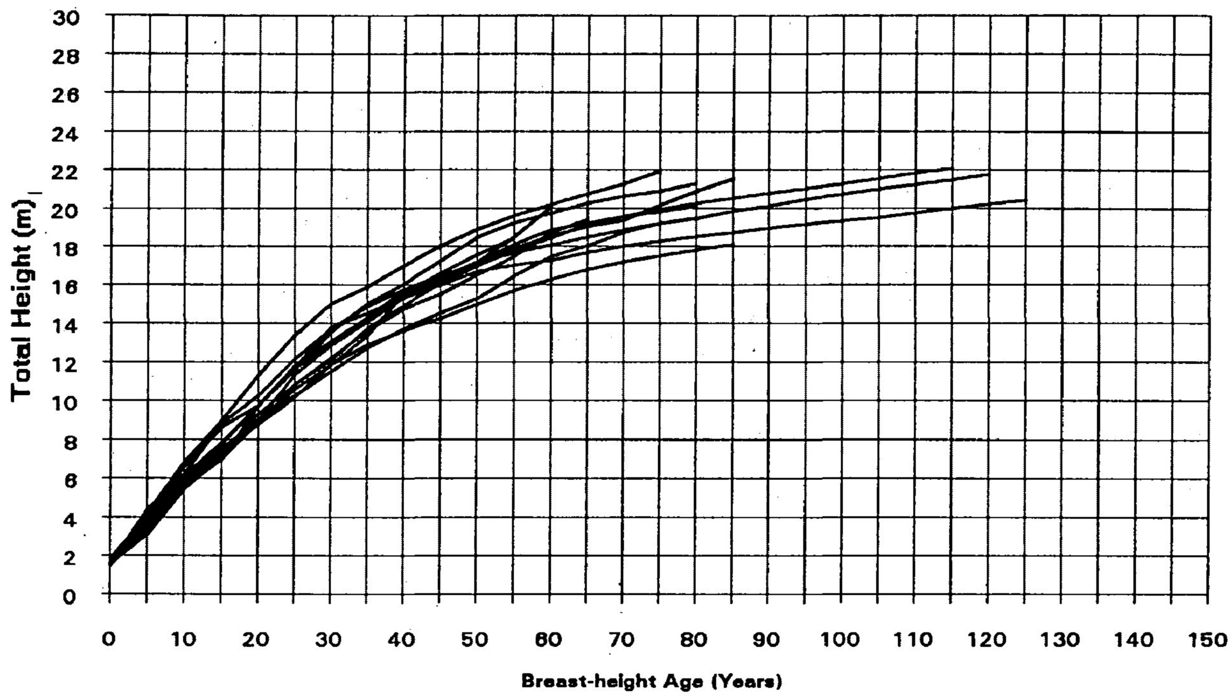


Figure 5. Height-growth curves for plots in NCR on bedrock sites having medium site quality.

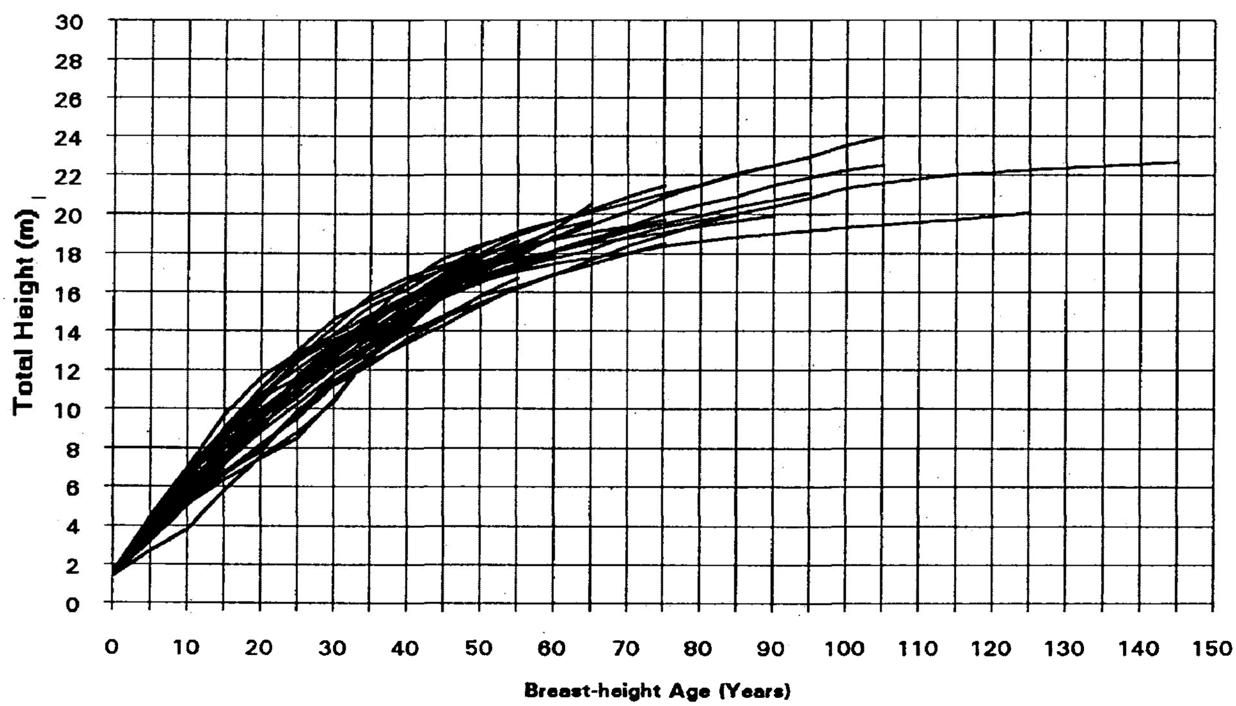


Figure 6. Height-growth curves for plots in NCR on morainal sites having medium site quality.

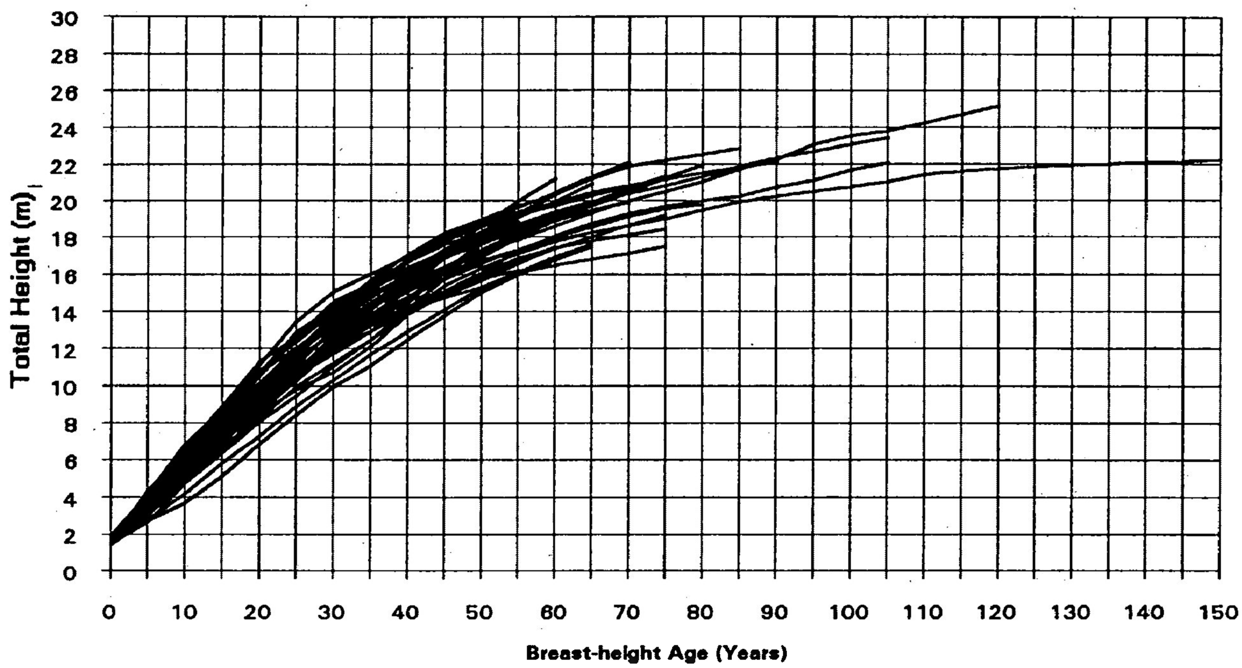


Figure 7. Height-growth curves for plots in NCR on glaciofluvial sites having medium site quality.

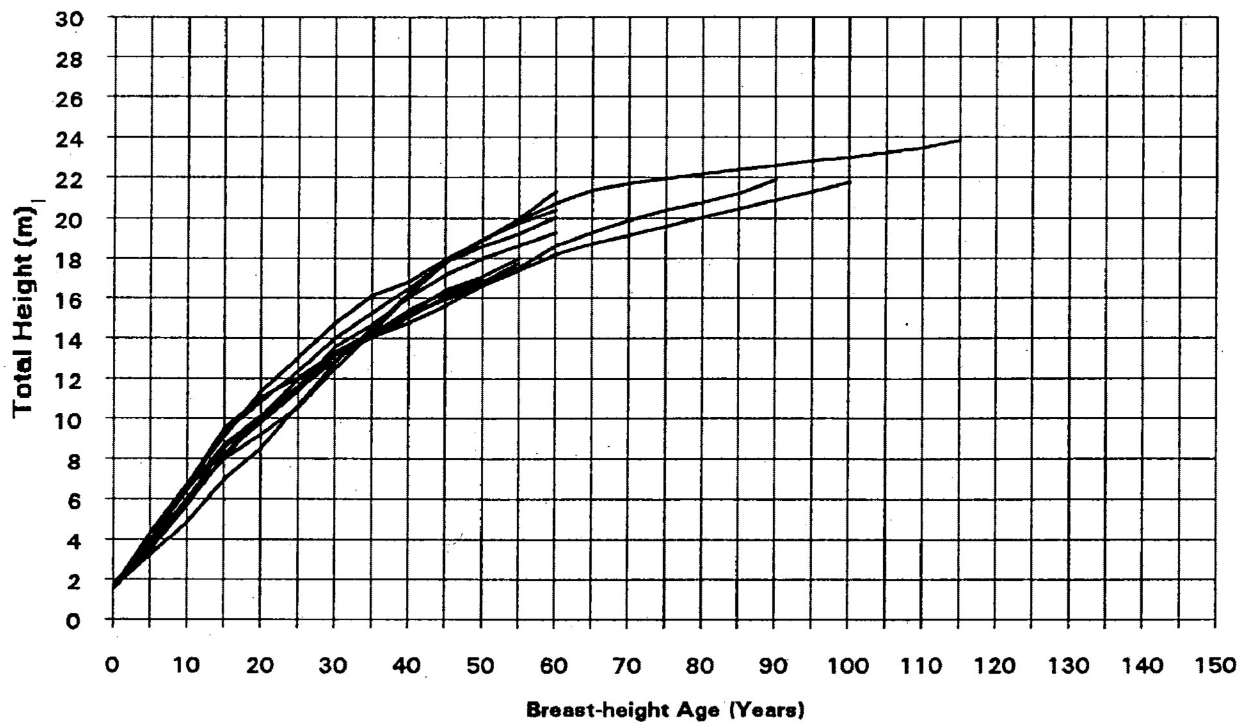


Figure 8. Height-growth curves plots in NCR on lacustrine sites having medium site quality.

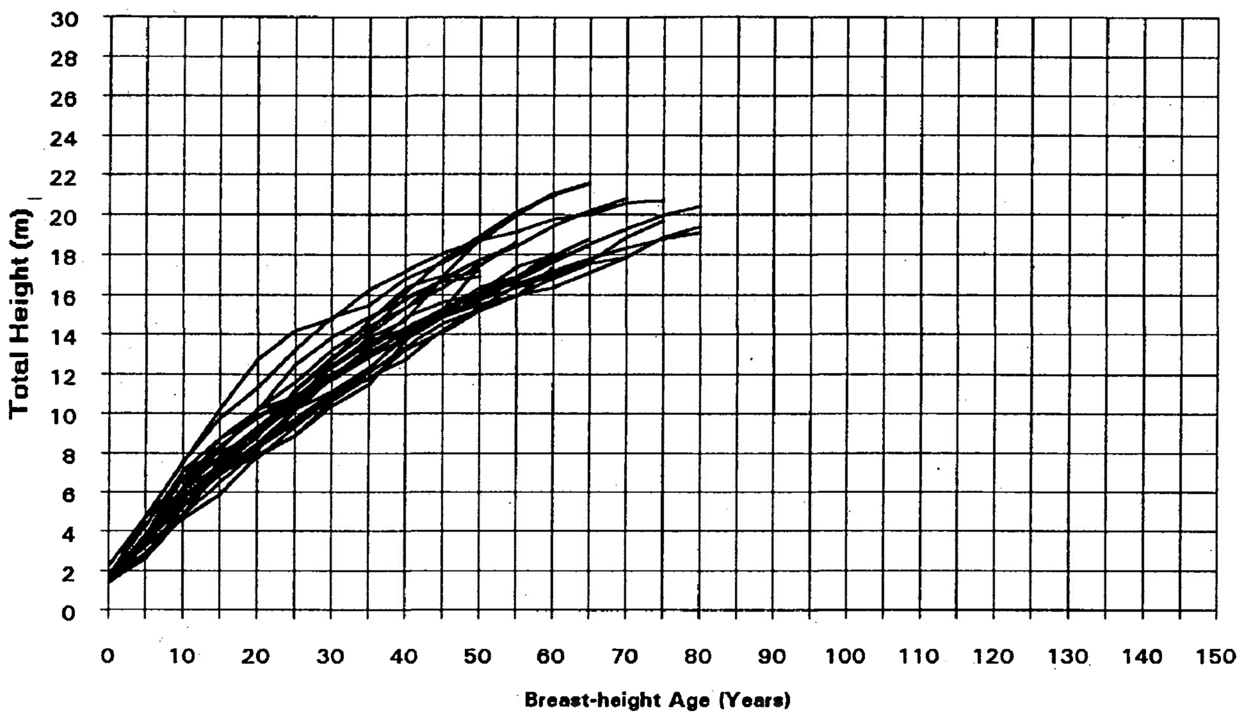


Figure 9. Height-growth curves for plots in NWR on glaciofluvial sites having medium site quality.

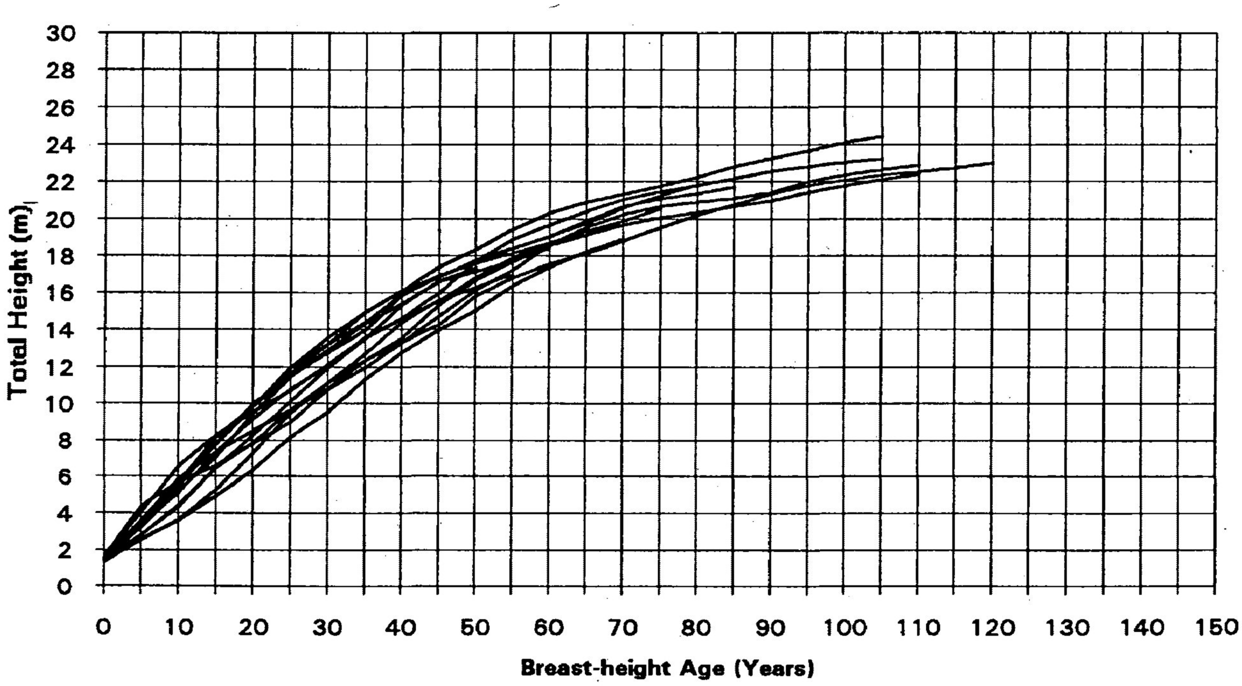


Figure 10. Height-growth curves for plots in NER on glacialfluvial sites having medium quality.

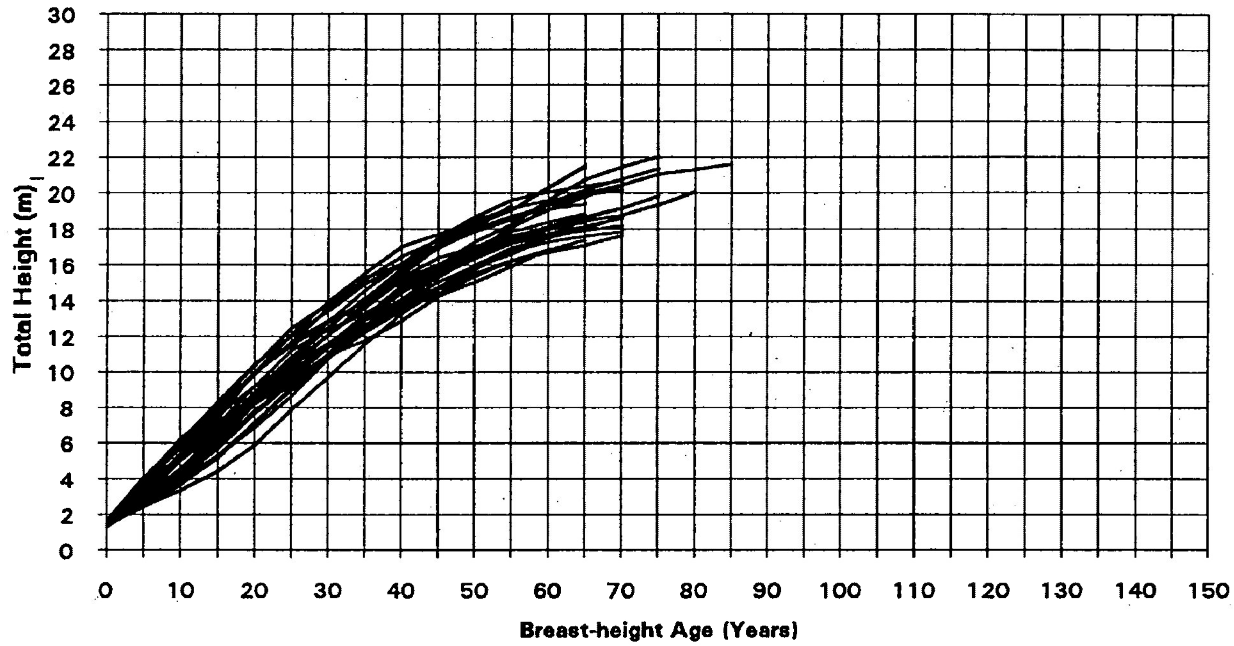


Figure 11. Height-growth curves for plots in NOR on glaciofluvial sites having medium site quality.

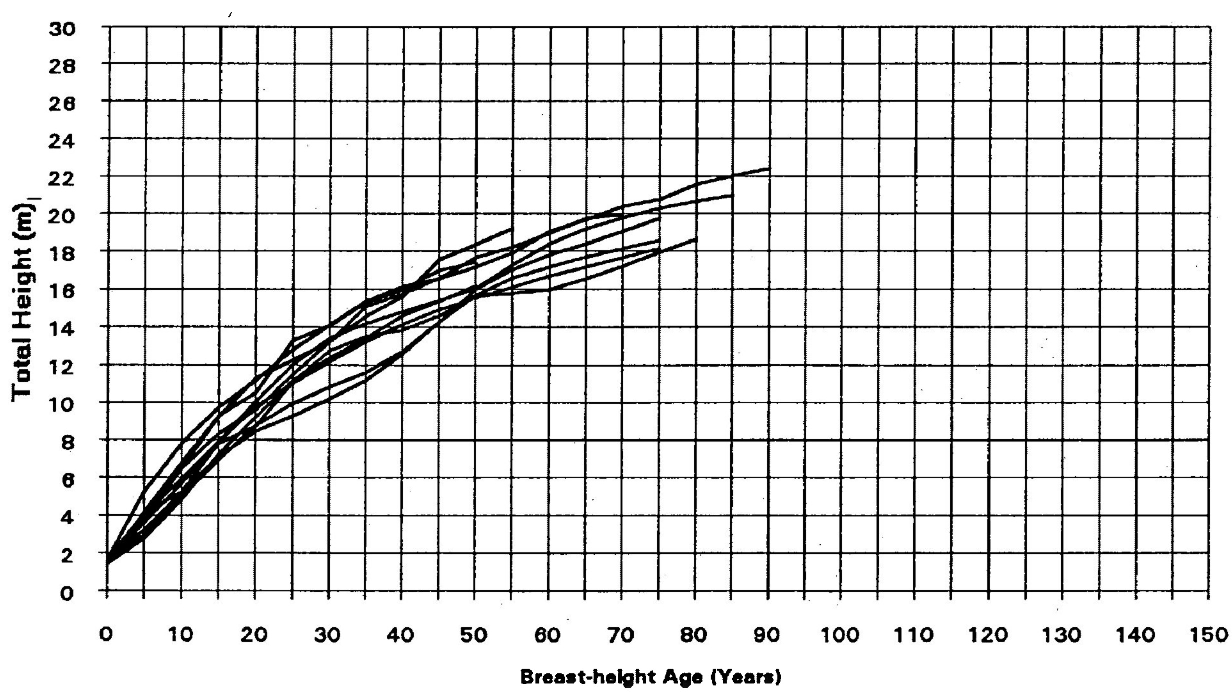


Figure 12. Height-growth curves for plots in NWR on morainal sites having medium site quality.

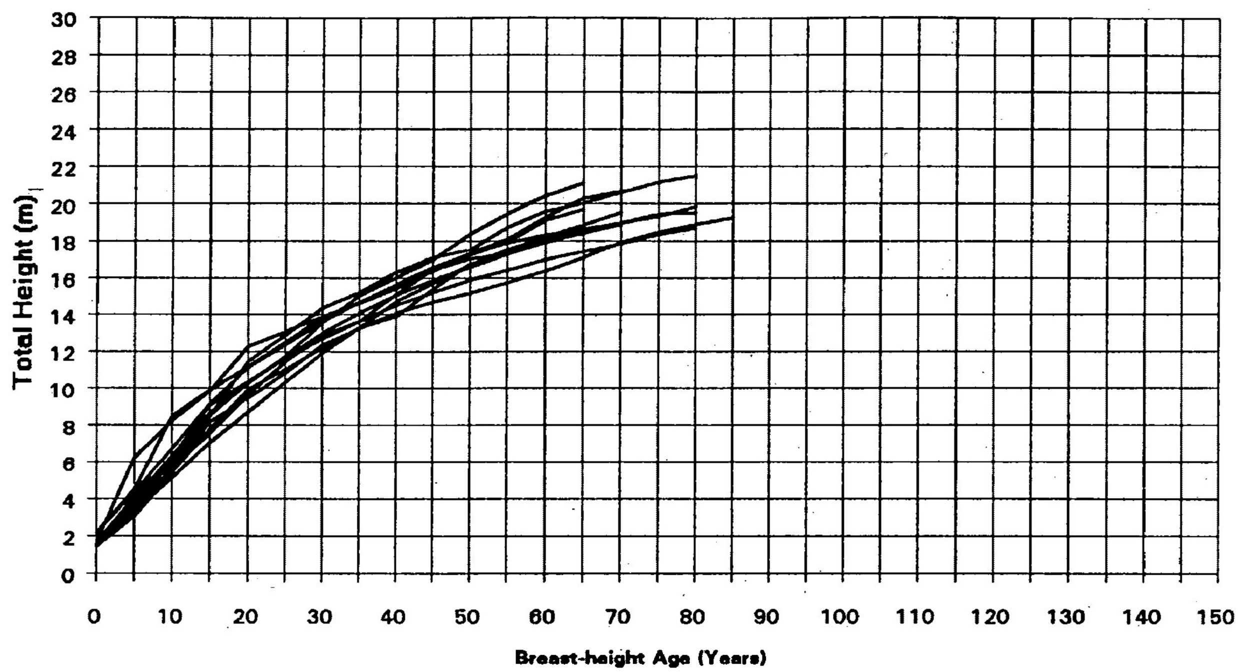


Figure 13. Height-growth curves for plots in NWR on lacustrine sites having medium site quality.

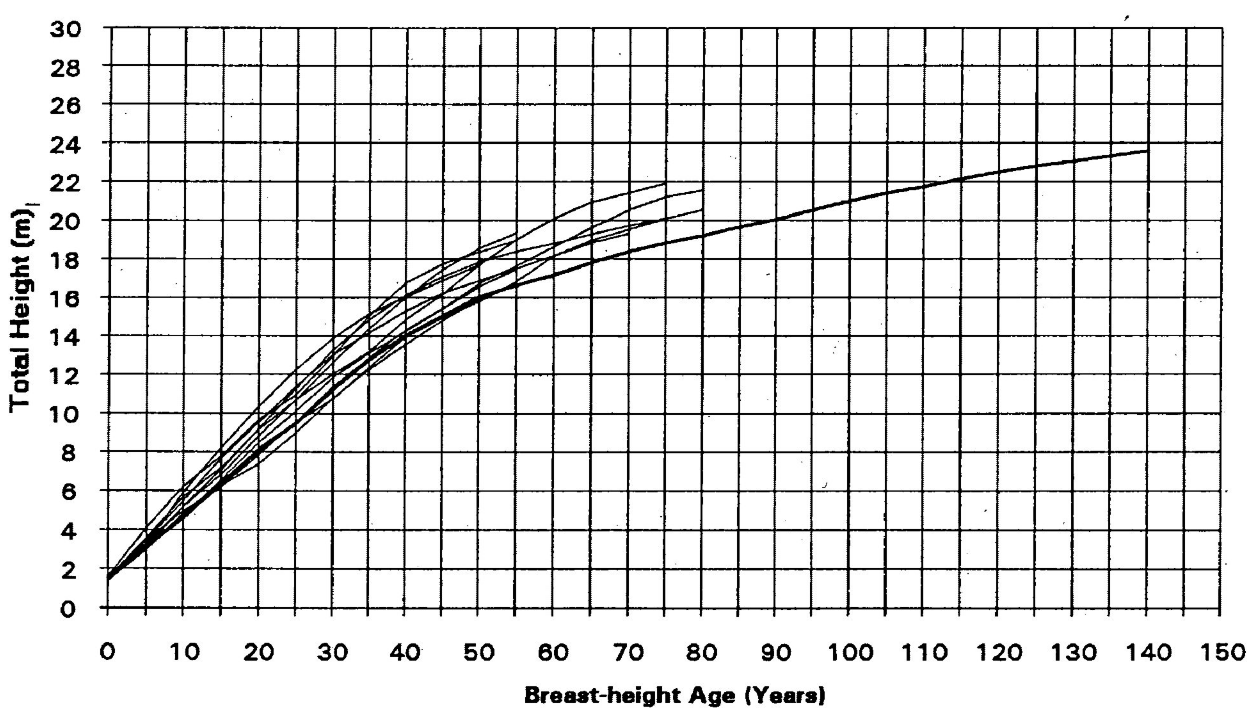


Figure 14. Height-growth curves for plots in NER on morainal sites having medium site quality.

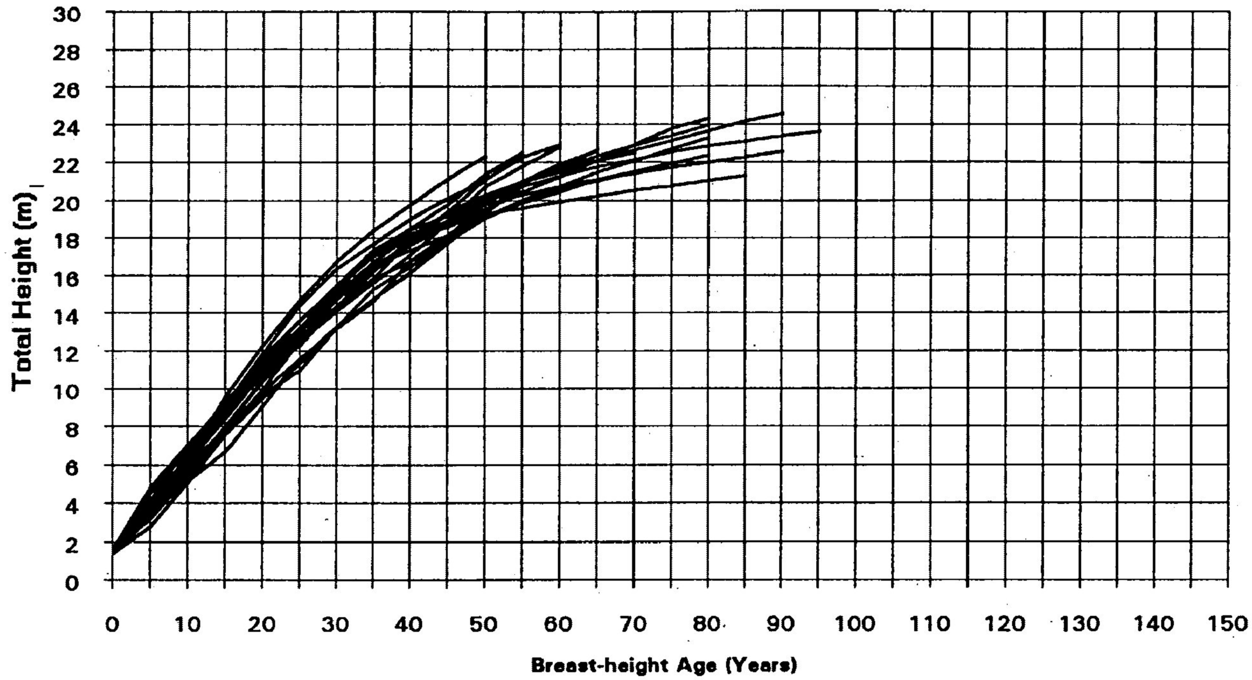


Figure 15. Height-growth curves for plots in NCR on glaciofluvial sites having good site quality.

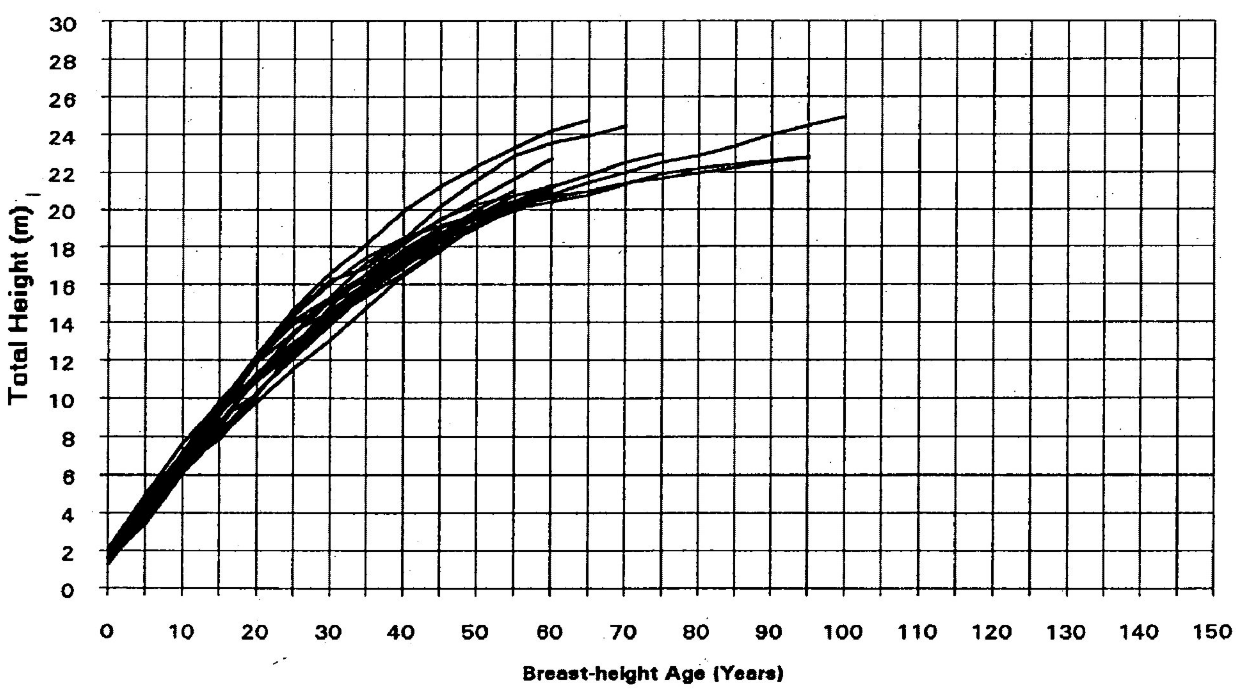


Figure 16. Height-growth curves for plots in NCR on morainal sites having good site quality.

APPENDIX VIII

AVERAGE DEVIATIONS BY 2 M SITE-INDEX CLASSES SITE-INDEX CURVES BY CARMEAN AND LENTHALL (1989) AND GOELZ AND BURK (1992)

Table 1. Deviations of observed height from predicted height by 2 m site-index classes at 10 year age intervals using Lenthall's height-growth model [Eq. 21].

	Number of plots and 2 m Site Index Classes																Total
	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No. of
BHA	Plots	8	Plots	10	Plots	12	Plots	14	Plots	16	Plots	18	Plots	20	Plots	22	Plots
0	5	0.14	11	0.15	20	0.20	65	0.20	97	0.22	106	0.26	69	0.26	10	0.26	383
10	5	0.41	11	0.58	20	0.49	65	0.46	97	-0.15	106	-0.04	69	-0.48	10	-1.06	383
20	5	0.14	11	0.29	20	0.14	65	0.33	97	-0.26	106	-0.05	69	-0.46	10	-0.70	383
30	5	-0.06	11	-0.08	20	-0.09	65	0.08	97	-0.26	106	0.06	69	-0.17	10	-0.11	383
40	5	-0.48	11	-0.30	20	-0.12	65	-0.07	97	-0.03	106	0.16	69	0.17	10	0.23	383
50	5	-0.56	11	-0.44	20	-0.29	65	-0.14	97	0.05	106	0.23	69	0.43	10	0.67	383
60	5	-0.24	7	-0.54	17	0.01	59	-0.03	78	0.18	77	0.27	77	0.27	4	0.64	324
70	5	0.11	6	-0.45	12	0.21	39	-0.05	57	0.20	46	0.10	23	0.09	1	0.48	189
80	5	0.31	6	-0.26	11	0.54	32	0.11	31	0.29	26	-0.04	18	-0.04			129
90	3	0.35	4	-0.70	4	1.37	18	0.51	13	0.17	13	-0.05	7	-0.39			62
100	2	1.34	3	-0.07	4	1.91	12	0.78	11	0.16	8	0.27	2	1.04			42
110	2	1.83	1	0.91	3	2.33	7	1.74	8	0.03	4	-0.01					25
120	1	3.66			2	2.85	5	1.85	6	-0.24	2	0.83					16
130	1	4.14			1	2.48	4	1.75	3	0.31							9
140	1	4.72			1	2.75	4	2.04	3	0.42							9
150					1	3.29	3	1.83	1	1.70							5

Table 2. Deviations of observed height from predicted height by 2 m site-index classes at 10 year age intervals using Goelz and Burk base-age invariant height-growth model [Eq. 22].

	Number of plots and 2 m Site Index Classes																Total
	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No.	SI	No. of
BHA	Plots	8	Plots	10	Plots	12	Plots	14	Plots	16	Plots	18	Plots	20	Plots	22	Plots
0	5	0.14	11	0.15	20	0.20	65	0.20	97	0.22	106	0.26	69	0.26	10	0.26	383
10	5	0.21	11	0.44	20	0.40	65	0.40	97	-0.21	106	-0.15	69	-0.68	10	-1.42	383
20	5	0.24	11	0.45	20	0.32	65	0.46	97	-0.25	106	-0.19	69	-0.85	10	-1.44	383
30	5	0.32	11	0.32	20	0.26	65	0.33	97	-0.25	106	-0.11	69	-0.64	10	-0.99	383
40	5	0.07	11	0.19	20	0.26	65	0.17	97	-0.01	106	-0.05	69	-0.32	10	-0.60	383
50	5	0.00	11	0.00	20	0.00	65	0.00	97	0.00	106	0.00	69	0.00	10	0.00	383
60	5	0.21	7	-0.27	17	0.11	59	-0.06	78	0.02	77	0.01	77	0.01	4	0.20	324
70	5	0.34	6	-0.44	12	0.05	39	-0.29	57	-0.09	46	-0.18	23	-0.17	1	0.26	189
80	5	0.24	6	-0.57	11	0.08	32	-0.37	31	-0.12	26	-0.36	18	-0.21			129
90	3	-0.02	4	-1.39	4	0.59	18	-0.22	13	-0.35	13	-0.40	7	-0.44			62
100	2	0.60	3	-1.15	4	0.82	12	-0.20	11	-0.47	8	-0.10	2	1.05			42
110	2	0.67	1	-0.52	3	0.95	7	0.54	8	-0.70	4	-0.45					25
120	1	2.04			2	1.22	5	0.49	6	-1.06	2	0.23					16
130	1	2.10			1	0.60	4	0.16	3	-0.58							9
140	1	2.26			1	0.63	4	0.27	3	-0.53							9
150					1	0.96	3	-0.11	1	0.60							5