REFORM MATHEMATICS TEACHING AND HOW IT HELPS STUDENTS UNDERSTAND THE CONCEPT OF AREA

by

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Abstract

The focus of this research was to determine the effectiveness of the reform method of teaching in helping Grade 7 students acquire a conceptual understanding of area. A pre-test, intervention, post-test model was used. Both the tests and the lessons were developed to address six areas of difficulty as identified in the literature as causing problems for students in this measurement topic: measuring areas of various objects, static perspective of area, dynamic perspective of area, spatial structuring and covering, estimation, and the area of an amorphous figure. There were four major findings: students had a positive affective response to the reform method of teaching, students adopted a wider range of strategies to solve problems, students were able to communicate and justify their ideas to others and, students developed a conceptual understanding of topics that the research literature stated are normally only superficially understood by Grade 7 students.

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"I do not know if it will be read by everyone, but it is meant for everyone"
– Victor Hugo

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Chapter One: Introduction

Context

Educational change is a slow process. There are many reasons for this including: lack of resources, limited commitment to professional development, and public perceptions of schools and school curriculum (Hiebert, 1999; Smith & Star, 2007; Van de Walle & Folk, 2005). Additionally, teaching is a cultural activity and "is governed by powerful forces that function outside of conscious awareness, forces that change slowly over time—if they change at all" (Stigler & Hiebert, 1999, p. 107). Teachers behave in ways that reflect their upbringing and their schooling and have difficulty making substantive changes in their behaviour. It is therefore not surprising that teachers have had difficulty making the changes that the National Council of Teachers of Mathematics [NCTM] called for almost two decades ago. The NCTM publication *Professional Standards for Teaching Mathematics* (1991), a companion document to the curriculum document *Curriculum and Evaluation Standards for School Mathematics* (1989), reflected the NCTM's position that teaching needs to change in order to improve student learning. The Ontario Ministry of Education [MOE] revised its Grades 1-8 Mathematics Curriculum in 2005, providing content and teaching direction to match the philosophy of the NCTM materials. These curriculum changes, commonly known as *reform* teaching methods, are being promoted by many educators (e.g., Battista, 1999; Baturo & Nason, 1996; Cady, 2006; Fosnot & Dolk, 2001) and are mandated by the Ministry of Education. This study looks at the efficacy of the reform teaching method that is called for in today's curriculum in the teaching of the concept of area to Grade 7 students.

Reform Teaching Method Defined

The call for reform and the literature outlining the tenets of reform have led to the current articulation of the meaning of reform teaching methods. Battista (1999) describes reform in the following way:

> In the classroom environment envisioned by NCTM, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, write, and discuss mathematics; and to formulate and test the validity of personally constructed mathematical ideas so that they can draw their own conclusions. Students use demonstrations, drawings, and real-world objects – as well as formal mathematical and logical arguments – to convince themselves and their peers of the validity of their solutions. (p. 427)

Battista contends that this classroom environment supports the learning of all students and concentrates on teaching for understanding, which results in a stronger conceptual grasp of the topics studied. This proposed classroom environment and process has implications for teaching all mathematical concepts, including the topic of area.

Purpose of the Study

This study was designed to examine one of the key topics in the MOE's Grade 7 Mathematics Curriculum in the measurement strand. The purpose of this research was to study the effectiveness of the reform method of teaching in helping Grade 7 students acquire a conceptual, rather than only procedural, understanding of the concept of area. The research project was designed to be a case study, and not a comparative study. A pre-test was administered as a diagnostic tool to determine the students' understanding at the beginning of the unit. A unit on area was taught using concepts and ideas based on the reform method, and a post-test was administered to determine students' understanding following the unit.

Research Question

Does the reform method of teaching help Grade 7 students move beyond a procedural understanding of area?

Significance of the Study

Reform instruction of mathematics has come under some criticism by researchers and educators alike (Finn, 1993; Haimo, 1998; Halat, 2007; Wu, 1997), especially concerning the lack of formal rigour with respect to mathematical arguments. Nevertheless, the same critics support the pre-proof concepts of making conjectures and looking for alternative solutions to problems. Moreover, the Ontario Curriculum promotes these methods stating, for example, that: "area is to be determined through investigation using a variety of tools" (MOE, 2005, p. 101).

The Standards of Practice for the Teaching Profession, as outlined in the Ontario College of Teachers' [OCT] document *Foundations of Professional Practice*, states that teachers should have "a commitment to students and student learning" (2006, p. 13), and should strive to be "current in their professional knowledge" (p. 13). As reform teaching in mathematics is a prominent area of current education research, it follows that teachers adhering to the Standards of Practice should learn about reform methods and their purported benefits to students. Since this study focuses on the effectiveness of this teaching method, it provides additional information for teachers attempting to improve their practices.

Considerable research pertaining to reform mathematics instruction has been conducted. However, most of the focus has been on students in the primary grades (e.g., Battista, 2003; Casa, Spinelli & Gavin, 2006; Chapin, O'Connor & Anderson, 2003). In addition, the extant research addressed topics such as computation, numeracy, and arithmetic operations (Anghileri, 2001; Fosnot & Dolk, 2001). There is a lack of research at the Intermediate grade level. Searches of the ERIC and CBCA databases, using the keywords "area" and "reform mathematics", found only studies exploring young children's understanding of area (Outhred & Mitchelmore, 2000) and teacher-candidates' (Baturo & Nason, 1996) and teachers' (Ma, 1999) struggles with area and their response to learning area through reform mathematics techniques.

This is an action research project, and thus provides a snap-shot of classroom activity. It is a case study that is specifically related to content and delivery in a Grade 7 mathematics classroom. Durrant and Holden (2006) state: "case studies are a key element of the cumulative body of educational knowledge that can and should be used to inform the profession more widely" (p. 72). This study provides information that is relevant to intermediate mathematics teachers, and as such acts as a valuable tool to inform their pedagogy. In addition, because this research was conducted by a practicing teacher rather than an external agency, it may prove to be more relevant to educators in the classroom.

Limitations of the Study

Students involved in the study came from a variety of mathematical backgrounds. There was little homogeneity with respect to the content and teaching strategies to which these students had been exposed, creating a diverse set of participants. The selected class was in the French Immersion stream. Lazaruk (2007), in his study on the benefits of the French Immersion program in Canada, determined that "French Immersion students enjoy significant linguistic, academic and cognitive benefits" (p. 605). It could be argued, following Lazaruk, that these students could be assumed to be stronger in math and have a deeper understanding of the concept of area than other students – which might lead to reasonably strong pre-test scores, and therefore less latitude for gains. If a class from the English stream were to be studied, pre-test results might have been at a level that provided more opportunities for improvement. It is clear that no single class is representative of all classrooms. Classrooms differ because of socio-economic status, population differences, and location. Replication of this study in other jurisdictions could provide fruitful information. A larger and more comprehensive sample would have increased the generalizability of the findings of the study. In addition, performing a study with a control group taught using the traditional method would have allowed for a comparison of the effectiveness of the two teaching techniques.

The pre-test and post-test contained similar, yet not identical, questions. Two different tests were used to mitigate the practice effect (DeKeyser, 1996). However, the use of these two different tests may have affected the study's results. The question changes may have had an effect on student performance that was not a direct outcome of the teaching method. Nevertheless, the pre-test and post-test were designed to address the same content, and changes that were made in the questions did not affect the level of difficulty of the individual items. The post-test was part of the summative evaluation of this unit. This fact may have motivated students to perform well on this test, as opposed to the pre-test, which was used only for diagnostic purposes. However, the post-test is only a portion of the data that was used to draw the conclusions for this study and therefore the impact of the summative nature of the post-test should have been minimal.

Chapter Two: Literature Review

Introduction

Measurement is a topic that deserves to be closely studied in each mathematics classroom. In fact, many researchers and educators have argued that measurement is the aspect of mathematics that is most frequently used in everyday life (Chappell & Thompson, 1999; MOE, 2005; Pitta-Pantazi & Christou, 2009; Shaw & Cliatt, 1989). According to researchers Baturo and Nason (1996), "measurement is the domain of mathematics that is most closely allied with real-world application" (p. 236). Determining area and volume is often necessary in household applications (Shaw & Cliatt). Area, in particular, is important in such common household activities as: painting, deck building, sewing, flooring, wall-papering, and cake decorating. Given students' everyday experience with the concept of area, one would expect this to be an undemanding topic of instruction for teachers. Surprisingly, this turns out not to be the case.

Problems with Traditional Teaching Methods

Traditionally, the concept of area has been taught by introducing formulae to the students, demonstrating how to use the formulae through examples, and requiring the students to complete practice problems to show their procedural proficiency (Malloy, 1999; Stigler & Hiebert, 1999). However, Battista's (1999) analysis of issues relevant to the reform of mathematics education led him to believe that: "traditional methods of teaching mathematics not only are ineffective but also seriously stunt the growth of students' mathematical reasoning and problem-solving skills" (p. 425). He theorized that: "to genuinely understand new mathematical ideas, students must personally construct meaning for these ideas" (Battista, 2002, p. 333). Traditional instruction typically does not allow students to develop their own methods of solving problems; rather, they are expected to follow and apply prescribed sets of rules. Other researchers have made similar claims about traditional mathematics instruction. For example, in a study on calculus reform and traditionally-instructed students' use of calculus in engineering mechanics courses by Roddick (2003), it was found that students taught using the traditional method produced solutions that were more procedural in character. Anghileri (2001), in her research on students learning division, found that a procedural understanding of standard algorithms led to incorrect memorization of procedures, resulting in inaccurate problem solutions. Concerns similar to these about the quality of teaching and learning of mathematics have led to numerous attempts to improve the curriculum throughout recent history.

The History of Reform

Although earlier reform efforts had been proposed throughout the century, it was not until the 1960's that a serious change was made to the mathematics curriculum. This reform movement, entitled New Math, was spurred by Russian advances in technology and the launching of *Sputnik*, the first satellite to orbit the earth (Brahier, 2005). The fear that the United States was falling behind the rest of the world, technologically speaking, resulted in this movement that focused on "fundamental principles of logical deduction and formal notation" (Van de Walle & Folk, 2005, p. 3). The New Math curriculum shifted the exploration of more difficult concepts to an earlier time in the students' schooling. For example, "topics such as set theory and non-Euclidean geometry" (Brahier, p. 11) were introduced at the secondary level. The goal was to ensure that each student "visited content that required much more rigor than had previously been the case" (p. 11). With the introduction of more difficult topics at an earlier age came the expectation that students be given the opportunity to explore mathematical situations on their own. The writers of *Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics* [CCSM] promoted this exploration. They wrote that:

> It is important for the student to get the feeling that definitions and lines of attack are matters of choice. You first explore the situation, and then pick a particular point of view for its convenience and for its power. (CCSM, 1964, p. 80)

Critics argued that by pressuring students to explore more difficult topics, the curriculum was catering to the needs of future mathematicians (Brahier, 2005). The content created difficulties for many students. At the time, the New Math movement caused leading mathematicians to complain about the proposed changes (Ahlfors et al., 1962). The difficulties arising with the New Math movement were sufficiently public that they were even skewered in the form of a song entitled *New Math* by mathematician and comedic musician Tom Lehrer (Lehrer, 1990). The disillusionment with the New Math resulted in the 1970s movement to the back-to-basics style of teaching. The back-to-basics movement had an "emphasis on rote memorization" (Van de Walle & Folk, 2005, p. 4), and tended to "place a low ceiling on mathematical competence" (NCTM, 1980, p. 6). Since reverting to the basics was a backwards shift with respect to the educational research that had been started during the New Math period, educators soon began to look for other solutions to address the low math scores in schools.

Continued research into classroom instruction led to yet another curriculum shift addressing not only content, but also teaching methods in the 1980s. The newest call for reform by the NCTM (1989) proposed that a focus on problem solving would increase a student's confidence in mathematics, and "is essential if he or she is to be a productive citizen" (p. 6). Another emphasis of this reform movement was that "mathematics should be for *all* students – regardless of gender, race, socioeconomic status, or any other factor that may have caused inequities in the past" (Brahier, 2005, p. 12, emphasis in original). These, in addition to other goals, were first outlined in the NCTM document *An Agenda for Action* (1980). Following this initial document, NCTM released a series of four documents outlining the standards for mathematics education that included: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), *Assessment Standards for School Mathematics* (1995), and *Principles and Standards for School Mathematics* (2000). Other groups also recognized the need for reform and expressed ideas that supported the NCTM standards. In particular, the National Research Council [NRC] reacted to the movement by publishing *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989). Writers of this report recognized the "urgent national need to revitalize mathematics and science education" (NRC, 1989, p. iii) and echoed the NCTM belief that all students should receive high-quality education in mathematics.

This NCTM call for reform reflects a concern about student achievement and classroom practice. In addition, it acknowledges a shift in the way many researchers and educators believe people learn; they theorize that learning is constructed by individuals rather than received as a package from teachers.

The Theory of Constructivism

Understanding the nature of intellectual development is central to improving teaching and learning. One theory that has affected teaching and learning practices is Piaget's theory of constructivism. Sternberg (2002), in his review of Piaget's book *The Psychology of Intelligence,* states: "Piaget's theory remains the single most comprehensive theory of intellectual development to date" (p. 483). Constructivism is a theory describing how learning happens and "not a description of teaching" (Fosnot & Perry, 2005, p. 33). Its premise is that children build new knowledge on knowledge that they have previously constructed. Brahier (2005), an educator who has authored books on assessment and teaching of mathematics at the middle and secondary levels, states that constructivists believe "that knowledge is built up or constructed from within as we have experiences in our lives" (p. 45). Van de Walle (1999) theorizes that the result of constructing new ideas is "a network of meaningful, related, useful ideas" (p. 3). Fosnot and Perry describe constructivism as the "theory of learning and development that is the basis of the current reform movement" (p. 8). Application of this theory to the teaching process has led to new approaches with respect to instructional practice.

Constructivism in Practice

In her article *A Constructivist Perspective on Teaching and Learning Mathematics,* Deborah Schifter (1996) contrasts the traditional approach, whereby the teacher demonstrates the correct procedure and students are asked to replicate this procedure (p. 493), with what she terms *the constructivist approach* of asking students to complete a task without first providing an example. Stigler and Hiebert (1999) identify this approach as being consistent with how Japanese teachers involved in the Third

International Mathematics and Science Study began their classes. They found that Japanese teachers posed a problem at the beginning of the lesson, helped students understand the problem, then allowed students time to work on the solution themselves. These examples support Fosnot and Perry's (2005) contention that: "teachers need to allow learners to raise their own questions, generate their own hypotheses and models as possibilities, test them out for viability, and defend and discuss them in communities of discourse and practice" (p. 34). Cady (2006) also supports the constructivist approach. In her two-year exploration of how to implement reform practices in a middle school classroom, one of her goals was to find tasks that "required students to use their prior knowledge" (p. 462) to solve problems "that had no readily apparent solution" (p. 462). Drawing on a child's prior knowledge is key when developing authentic tasks in a reform classroom. Researchers have drawn on constructivist theory to develop and analyze instructional methods that they believe are more likely to lead to the development of children's conceptual understanding of mathematics topics. Do we have evidence that this is the case?

Teaching Methods That Promote Conceptual Understanding

Hiebert and Grouws (2007) have long been concerned with teaching for conceptual understanding and define it as: "mental connections among mathematical facts, procedures and ideas" (p. 380). Based on their research, they have proposed two main features that they think promote conceptual development in students. The first feature is that teaching needs to attend "explicitly to connections among mathematical facts, procedures, and ideas" (p. 383). They promote discussions about the mathematical meaning of procedures and relationships among mathematical ideas, and purport that:

"reminding students about the main point of the lesson and how this point fits within the current sequence of lessons and ideas" (p. 383) helps to create connections that will build a student's conceptual understanding. Stigler and Hiebert (1999) analyzed the video portion that was part of the Third International Mathematics and Science Study that compared the teaching of eighth-grade mathematics in Germany, Japan, and the United States. They found that Japanese teachers often referred back to the main point of the lesson at the end of the class, and also reminded students of it throughout the lesson. They did not see evidence of any type of summarization of main points in US teachers' lessons.

The second feature that facilitates students' conceptual understanding described by Hiebert and Grouws (2007) is that students must "expend effort to make sense of mathematics, to figure something out that is not immediately apparent" (p. 387). They support this statement by referring to the constructivist theory as espoused by Dewey and other cognitive researchers. Based on this theory, they suggest that the process of struggling to make sense of a subject is connected to gaining a deeper understanding of that subject. Fosnot and Perry (2005) also agree that "disequilibrium facilitates learning" (p. 34), meaning that errors should be embraced as they are a result of a learners conception and offer a fruitful source of discussion and learning. What are the specific components of classroom instruction that embody reform instruction underpinned by a constructivist theory of learning?

Components of Effective Classroom Practice

Teaching through problem solving. A fundamental aspect of reform instruction is teaching *through* problem solving (MOE, 2005, p. 11). This goes beyond teaching the steps of problem solving advocated by Polya (1980, 1988), Krulik and Rudnick (1980), Resnick (1987), and others; instead it is using problems as an entry point to a mathematical topic. There is no pre-determined method, or even a list of possible methods from which to choose, for solving the problem. In Japan, lessons begin with a key problem that "sets the stage for most of the work during the lesson" (Stigler & Hiebert, 1999, p. 79). This idea is supported by research by Taplin & Chan (2001) who determined that "students in general are usually more motivated to acquire knowledge in the context of solving a problem" (p. 287). Teaching through problem solving requires the teacher to devise an open-ended problem that students can solve. The students are then responsible for their own learning when they make an attempt at solving the problem. The open-ended nature of this type of instruction should nonetheless lead to an integrated model of accountability within the classroom.

Accountability in the classroom. According to Hiebert and Grouws (2007), "teaching is influenced by students and has a bidirectional quality" (p. 372). Similar views are presented by Cohen, Raudenbush and Ball (2003) in their analysis of instruction that promotes achievement when they state: "effective teaching encouraged and closely supported what students did in instruction, and students' work helped them to learn, or not" (p. 122). In essence, the teacher responds to the students' ideas and changes the direction of the lesson, if necessary, in order to support their learning. Sherin, Mendez and Louis (2004) espouse the idea that: "for effective learning to occur, the learner must be an active agent in the learning process and must be able to reflect on this learning" (p. 209). All of these informed opinions make both teachers and students responsible for learning. Stigler and Hiebert (1999) further support this dual-accountability by stating that: "teaching is a system" (p. 75), and by likening it to "a machine, with the parts operating together and reinforcing one another" (p. 75). This dual responsibility makes discourse a central aspect of instruction.

Discourse in the classroom. Sherin, Mendez and Louis (2004), in their research on the relationship between fostering a community of learners and the teaching of mathematics, claim that: "mathematics-education reform promotes discourse as a central component of classroom practice" (p. 212). The NCTM (1989) supports this sentiment in its fourth goal as outlined in *Curriculum and Evaluation Standards for School Mathematics*. The goal emphasizes "learning to communicate mathematically" (p. 6) and promotes the discussion of ideas "in which the use of the language of mathematics becomes natural" (p. 6). Communication between and among students is of paramount importance. Who engages in talking, and the nature of the talk, are of prime importance in an effective mathematics classroom.

Student-talk. Students should be encouraged to talk among themselves about mathematically relevant topics (Sherin et. al., 2004). Cobb, Stephan, McClain, and Gravemeijer (2001) performed classroom based research on linear measurement and determined that conceptual discourse resulted in productive mathematical learning. They provided the example of conceptual discourse in a measurement setting as students "giving a backing by explaining how they structured space as they measured" (p. 134). Franke, Kazemi, and Battey (2007) reviewed research on classroom practice in mathematics and found that "increasing the level of discourse in cooperative groups often produces greater student learning" (p. 232). Knuth and Peressini's (2001) interest in the nature of discourse in mathematics classrooms, and their experience as educators, have led them to opine that: "students will acquire a deeper understanding of mathematics when they use their own statements, as well as those of their peers and teacher" (p. 325). They believe that student discourse is essential, as students need "to be able to communicate their mathematical knowledge in a technological society" (p. 321). In student-to-student discourse, students tend to mirror what they have seen from their teachers (Franke et al., 2007, p. 232); thus, it is important that teachers provide examples of effective mathematical discourse.

Teacher-talk. One of the main elements of effective discourse that Franke, Kazemi, and Battey (2007) highlight is the importance of details. Teachers can ensure that they are making instructions explicit, and that both they, and their students, are providing significant details during explanations. "Teachers are only as good as the methods of teaching they use" (Stigler & Hiebert, 1999, p. 175). They need to develop questioning and assisting techniques that elicit appropriate responses and clearly describe the ideas they are trying to promote.

Questioning is also an important part of classroom discourse. Both Schifter (1996) and Cady (2006) highlight the importance of questioning. They believe that the following are important questioning techniques: providing wait-time, asking open-ended questions, and re-phrasing questions. Questions can be used to heighten student curiosity. It is curiosity and the concomitant efforts to develop mathematical intuition and analytical capabilities that characterize classrooms that support mathematical proficiency (Franke et al., 2007).

The previous sections outline the difficulties with traditional instruction and discuss the theory and research on what effective reform instruction should look like. Do the difficulties with traditional instruction and suggestions and research on effective reform methods apply to teaching the concept of area?

Why Don't Students Understand Area?

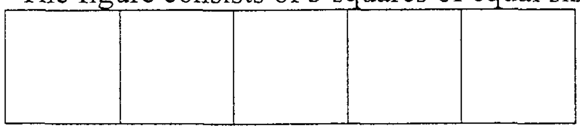
Evidence. Area has long been a topic of concern with respect to student understanding. In the 1963 document *Goals for School Mathematics: The Report of the Cambridge Conference on School Mathematics*, the following concern was expressed:

> It appears to us that the teaching of the use of units of measurement involves some serious problems. Most of these problems are due to the incongruity between the simplicity of the formal operations and the conceptual obscurity of the underlying ideas. (p. 89)

More recently, the Third International Mathematics and Science Study provided evidence that students' understanding of area is incomplete, and that they have difficulties measuring areas of various objects (Baturo & Nason, 1996; Van De Walle & Folk, 2005). One item in particular (Figure 1) caused significant difficulty among Grade 7 and 8 students involved in the study. Only 23% of Grade 8 and 17% of Grade 7 students determined the correct answer to the question involving perimeter (IEA, 1996). Although these results relate to the international average, Robitaille, Taylor, and Orpwood (1996) have determined that Grade 8 students in Canada also have a significant lack of understanding of the topic. The results of Grade 6 testing conducted by the Education Quality and Accountability Office [EQAO] in Ontario have also demonstrated that measurement is a strand in which students experience difficulty. In particular, the report of the 2005/2006 EQAO test indicates that: "Grade 6 students were least successful in the measurement strand" (EQAO, 2006, p. 44).

Figure 1. TIMSS Item

The figure consists of 5 squares of equal size. The area of the whole figure is 405 cm^2.



Find the area of one square.
Answer __________ square centimetres

Find the length of the side of one square.
Answer __________ centimetres

Find the perimeter of the whole figure in centimetres
Answer __________ centimetres

(IEA, 1996, p. 114)

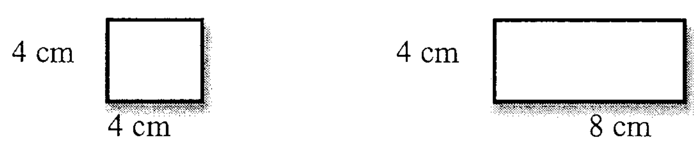
Static and dynamic perspectives. Much research has been conducted to determine whether adult students understand measurement topics. In particular, Baturo and Nason (1996) conducted research that was designed to evaluate first-year teacher education students' understanding of subject matter knowledge in the domain of area. They cite two perspectives from which area should be considered: static and dynamic (p. 238). Their claim is that if students have a limited notion of these two perspectives, they will not have the understanding to calculate area. The authors explain that the static perspective "equates area with an amount of region that is enclosed within a boundary and the notion that this amount of region can be quantified" (p. 238). Static understanding could be demonstrated by students being able to calculate the area of standard figures, given their dimensions, or estimating the areas of amorphous figures. The dynamic perspective "focuses on the relationship between the boundary of a shape and the amount of surface that it encloses so that, as the boundary approaches a line, the area approaches zero" (p. 238). Students with an understanding of the dynamic perspective should be able to discover that a square gives the largest area for a rectangular shape of a given perimeter. An example of a problem of this sort would be: A farmer has 48 metres of fencing and would like to use it to create a rectangular garden that has the largest possible area. What are the dimensions of the largest possible garden? A student solving this problem might create a chart, as seen in Figure 2, and eventually conclude that the dimensions creating the largest area are that of a square with 12 metre sides.

Figure 2. Example Chart

Length (m)	Width (m)	Area (m^2)
2	22	44
3	21	63
4	20	80
6	18	108
8	16	128
10	14	140
12	12	144
13	11	143

The dynamic perspective is often excluded from curriculum expectations. The relationship between perimeter and area, which is essential to understanding the dynamic perspective, is also frequently neglected. Researchers have found that this limits students' understanding of the concept of area. In particular, having a poor understanding of the dynamic perspective can result in misconceptions that "rectangles with the same perimeter have the same area" (Baturo & Nason, 1996, p. 239) and that "doubling the length of the sides of a square doubles its area" (Outhred & Mitchelmore, 2000, p. 145). Ma (1999) has also investigated the dynamic perspective of area. She presented teachers with a 'new theory', and proof (Figure 3) to support the hypothesis that "as the perimeter of a closed figure increases, the area also increases" (p. 84). This is clearly not true, as a 4 cm x 8 cm rectangle has a perimeter of 24 cm and an area of 32 cm^2, but a 16 cm x 2 cm rectangle has a larger perimeter of 36 cm; however, the area remains 32 cm^2. She asked teachers from the United States and China if they agreed with this new theory and proof. The results of her study, involving 23 American teachers and 72 Chinese teachers, showed that 9% of American teachers simply accepted the claim, and only 13% actually performed some type of mathematical investigation to authenticate the claim. The remaining 78% consulted a text or other individual, claiming a lack of knowledge and understanding about area and perimeter prevented them from investigating on their own. Of the Chinese teachers studied, 70% determined that the theory was untrue.

Figure 3. Liping Ma's Problem



Perimeter = 16 cm
Area = 16 square cm

Perimeter = 24 cm
Area = 32 square cm

(Ma, 1999, p. 84)

Spatial structuring. Another reason many students are unable to grasp the concept of area is related to the way in which area calculations are traditionally taught. Students are typically given measurements, provided with a formula, and are expected to calculate the area. This rote memorization of procedure is not linked to the conceptual understanding of area, resulting in errors such as: 6 cm x 4 cm = 24 cm, instead of 24 cm^2 (Baturo & Nason, 1996, p. 239). Outhred and Mitchelmore (2000) cite evidence, based on their research involving young children solving rectangular covering tasks, that visualizing the structure of an array is not intuitive for children, and further explore this idea stating that: "success depends on an operational understanding of the structure of the rectangular array" (p. 146). Battista (2003) also emphasizes the importance of understanding how to meaningfully enumerate arrays of squares, and calls it "the foundation for developing competence with measuring area" (p. 122).

Battista, Clements, Arnoff, Battista, and Van Auken Barrow (1998) conducted an in depth study investigating students' spatial structuring of 2D arrays of squares. They concluded that spatial structuring "is an essential mental process underlying students' quantitative dealings with spatial situations" (p. 503). The authors defined spatial structuring as: "the mental operation of constructing an organization or form for an object or set of objects" (p. 503). In other words, students who possess this mental process are able to create a picture in their minds when looking at an object or objects, and rearrange that picture to best suit their needs. Students with an understanding of spatial structuring could see a regular hexagon as six congruent triangles, or two congruent trapezoids. Outhred and Mitchelmore (2000) investigated the intuitive understanding of the concept of area possessed by children in Grades 1 through 4. Strategies children used to complete the tasks in the research project were classified into operational principles. The authors' second principle that states: "the units must be aligned in an array with the same number of units in each row" (p. 161), implies an understanding of spatial structuring. Baturo and Nason (1996) also support the essentiality of this structuring process with their statement that: "many students only have an understanding of the one-dimension linear representation of multiplication as repeated addition. Thus, they are unable to make sense of the area measurements calculated by the formulae" (p. 239). Spatial structuring is a mental process that requires students to construct new ideas. Van de Walle (1999), a professional educator who has contributed significantly to the teaching of elementary mathematics, believes that, to construct new ideas, "children must be mentally engaged in the act of learning" (p. 1). One way to mentally involve students is to encourage them to use estimation techniques.

The Role of Estimation

When children are first introduced to mathematics much time is spent calculating the exact answers of problems, such as sums and products of numbers. It is in geometry, especially in the area of measurement where one may not be able to determine an exact answer, that estimation is a powerful tool — useful in real life as well as in the classroom. Striving simply to achieve the exact answer to a problem is only part of mathematics (Usiskin, 1986). The National Council of Teachers of Mathematics [NCTM] has an expectation that all students should "use common benchmarks to select appropriate methods for estimating measurements" (NCTM, 2000, p. 204). Morgan, in her 1986 article on teaching measurement estimation, claimed that: "instruction in estimation is likely to enhance the learning of measurement concepts" (p. 204). Adams and Harrell (2003) further discuss the importance of estimation and suggest that the estimation process "offers rich opportunities for students to make judgment and develop opinions that can have rewards for their lifelong measurement tasks" (p. 229). They also suggest that students can use estimation in measurement tasks to "solve problems, validate formal measurements, and make decisions related to measurement" (p. 229).

When asked to find the area of an amorphous figure, students often have difficulty applying traditionally learned formulas (Casa et al., 2006). A better strategy for calculating the area of irregularly shaped polygons is to use centimetre graph paper as a guide (NCTM, 1989). Casa et al. found that by tracing the figure onto the paper, primary students were able to estimate the area by:

- counting individual whole squares
- using arrays within the figure and then adding other whole and partial squares
- using an array outside the figure and then subtracting other whole and partial squares
- piecing together two half squares to make a whole square
- putting together any number of partial squares to make one whole square

(p. 172)

The writers of the Ontario Ministry of Education curriculum documents also viewed estimation as an important skill. The comparison and order of objects relative to their dimensions and size is an Ontario Ministry of Education expectation that begins in Grade 1 (MOE, 2005, p. 35). Therefore estimation of area should be addressed in the classroom. Drawing a rectangle outside the figure and determining its area provides an upper bound on the area of the figure. Similarly the area of a rectangle enclosed by the figure provides a lower bound for the area. The average of these two area measurements can, in some cases, be used to determine an approximate value of the area of the irregular figure. This concept of applying upper and lower bounds is listed as an expectation in the Measurement standard for Grades 9-12 in *Principles and Standards of School Mathematics* (NCTM, 2000, p. 320).

Summary of the Implications of the Literature

This literature suggests that the concept of area is best taught in a classroom in which reform methods, such as problem solving, hands-on activities, group work, effective communication, and estimation, are the main components of instruction. Problems should be the vehicle for instruction. Hands-on opportunities should be used to support children's understanding of the concept. A variety of guided group-work situations allow students time for discourse, which, in turn, introduces them to diverse solution methods. Math discourse within the classroom should be modeled effectively by the teacher (Cobb et al., 2001), and students should be encouraged to discuss concepts and ideas with their peers and their parents. Practice in estimation should be an integral part of the lesson sequence.

The goal of instruction on area, according to Battista (2003), is "for students to develop properly structured mental models that enable them to reason powerfully about these concepts in a wide variety of situations" (p. 135). In order for this goal to be achieved, students will need to be exposed to numerous situations involving area. Professional mathematics educator Marilyn Burns (2000) supports this by stating that: "one experience is not generally sufficient to cement understanding of a relationship that is new to a learner" (p. 25).

Even though professional groups, such as the NCTM, recommend the use of reform teaching methods, and our own curriculum guidelines suggest that the concept of area is most effectively taught using these same methods as outlined above, there is little research on the effectiveness of these techniques at the Intermediate level. This study will determine how effective the reform method of teaching is in helping Grade 7 students acquire understanding of the concept of area.

Chapter Three: Methodology

Research Design

This is a qualitative action research study of the effect of reform mathematics instruction on Grade 7 students' understanding of the concept of area. Action research involves the identification of a problem, an attempt to resolve the problem, and a check to see if the efforts were successful (O'Brien, 2001). The goal of action research is solving real problems, and is thus typically used in real life situations. The components of action research can be repeated as necessary until a solution to the problem is found; however, the results in this project were based on one cycle of the action research components.

A pre-test, intervention, post-test model was used. The intervention included using reform instructional techniques to teach concepts explored in earlier grades, to develop skills to determine area and a deep understanding of the concept, and to ultimately address the Ontario Curriculum expectation for measurement in Grade 7 of determining the area of a trapezoid. I provided the instruction.

Research Sample

The research was conducted with a *convenience sample* of students. Guba and Lincoln (1994) define this type of sample as one "to which the enquirer happens to have access" (p. 128). The project was carried out in a Grade 7 class at a public school in Southern Ontario. The class consisted of 27 students, all of whom were enrolled in the French Immersion stream, however; they received mathematics instruction in English. Of the 27 students, one had an Individual Education Plan (IEP) for accommodations; however, none of the students was on a modified mathematics program. Although some of the students knew me through their participation in extra-curricular activities, and from having seen me within the school, I had not formally taught any of the students.

Procedure

Ethics approval was granted by Lakehead University, the school board, and by the principal of the school where the study was conducted. Since student data were being collected, introductory letters (Appendices A & B) and permission forms (Appendices C & D) were sent home. To protect the identity of the board, school, and students taking part in the study, pseudonyms were used and identifying wording in the appendices was altered.

The teaching took place over a two and a half week period. Students attended class Tuesdays and Wednesdays for 80 minutes, and Fridays for 40 minutes. Lessons began on January 12, 2010 and ended on January 27, 2010. This time period was consistent with the normal time allotted to this topic.

During the initial class, students completed the pre-test. They were instructed to attempt each question to the best of their ability, and were told that the results would be used as a diagnostic tool to assist in future lesson planning. The test was not timed, and students who required additional time outside of class to complete the test came in at lunch that same day.

Following the pre-test, the lessons were taught using the reform method of instruction. The lesson plans were developed to reflect the needs identified in the literature and to take into consideration the implications of the results of the pre-test. With the exception of the pre-test and post-test lessons, students were video-taped to capture a record of individual work, partner activities, and whole group interactions.

A post-test was administered at the end of the unit to determine the progress each child had made.

Design of the pre-test and post-test instruments. The items on the pre-test (Appendix E) and post-test (Appendix F) were designed to assess the major issues of difficulty as cited in the research literature. Table 1 includes a list of each difficulty and the items on the pre-test and post-test that address that difficulty.

Table 1. Design of the Pre-Test and Post-Test Instruments

Letter	**Difficulty cited in literature on understanding of area**	**Citation**	**Instrument Item**	
			Pre-Test	**Post-Test**
A	Measuring areas of various objects.	Baturo & Nason (1996); Van De Walle & Folk, (2005)	5, 10, 12, 13	4, 9, 11, 12
B	Static perspective of area.	Baturo & Nason (1996)	7, 15	6, 14
C	Dynamic perspective of area.	Baturo & Nason (1996)	6, 14	5, 13
D	Spatial structuring and covering.	Battista (2003); Outhred & Mitchelmore (2000)	4, 9	8
E	Estimation.	Adams & Harrell (2003); Morgan (1986)	8	7
F	Area of an amorphous figure.	Casa et al. (2006)	11	10

The Reform Teaching Method

I began each lesson by posing a problem without providing instruction on the particular methods children should use to solve the problem (Appendix G). This is consistent with the technique employed in Japanese schools as discussed in *The Teaching Gap* (Stigler & Hiebert, 1999). Over the course of the instructional period students were given the opportunity to work in homogeneous pairs. By pairing students who were at the same mathematical level, "full participation of both students" (Lawson, 2007, p. 3) was encouraged. Following the partner work, students came together to participate in what Fosnot and Dolk (2001) refer to as a "math congress" (p. 3). During the math congress, student mathematicians "communicate their ideas, solutions, problems, proofs, and conjectures to one another" (p. 29). The learners, not the teacher, are the ones who are responsible for defending their thinking (Fosnot & Dolk, 2001; Fosnot & Perry, 2005). During the congress I facilitated discussions, addressed particular solutions that supported the goal of the lesson, and supported individual learners - all aspects that are consistent with the structure of a math congress as described by Fosnot and Dolk.

Classroom discourse was a consistent element in this reform mathematics teaching environment. Student-led discussions were guided by me, and served to provide the students with an opportunity to evaluate different approaches, recognize problems with their own approach, and gain a deeper understanding of the topics. I elicited responses using the "five productive talk moves" as discussed by Chapin, O'Connor, and Anderson (2003). These included: revoicing, having students restate someone else's reasoning, asking students to apply their own reasoning to someone else's reasoning, prompting students for further participation, and using wait time (pp. 12-15). Student pairs worked through their solution on chart paper, then presented their solutions to the class in the math congress, as described above. Instead of directing questions to me, other students were encouraged to ask the student who was presenting his or her idea, for clarification. When necessary, I stepped in to clarify a point and keep the discussion on track.

Consistent with the concept of reform mathematics, numerous opportunities for the use of manipulatives were provided to the students. Hands-on exploration with the Geoboard in Lesson 3 allowed students to physically change the area of a figure by manipulating the elastic that was stretched over the pegs of the Geoboard. Tactile opportunities allow students to "easily explore and adjust shapes" (Bray, Dixon & Martinez, 2006, p. 132). The particular problem used in Lesson 3 was adapted from the Bray et al. study of Grade 4 children, and modified for Grade 7 students to "stimulate flexible reasoning about area in terms of square units" (p. 132). In addition, in Lesson 4, students created their own polygon using the perimeter obtained from the outline of their foot. They then used this manipulative to explore area and perimeter situations.

Lesson 5 addressed the relationships between the area of squares, rectangles, triangles and parallelograms. Although, according to Ontario Curriculum expectations, students in Grade 7 should have learned these relationships in previous grades, students may not have understood the formulae they had memorized. The hands on exploration and partner work encouraged mathematical discourse and allowed students to work with one another to solve the problem.

Formulae were not presented directly to the students. Although some students recalled the formula for the area of a rectangle, effort was made to help them gain a conceptual understanding of that formula. Students were encouraged to investigate the area relationships among different figures. For example, when determining the area of a triangle, students found that two lines could be added to a triangle to create a parallelogram. Students investigated whether this would work for any triangle, supporting a generalization and their construction of a formula, (that the area of any given triangle is simply half the area of a corresponding parallelogram).

The lessons (Table 2) were intended to address the problem areas that were identified in the literature. They followed a particular sequence designed to provide the students with information that ultimately assisted them in determining, through a variety of methods, the area of a trapezoid.

Table 2. Area Unit Plan Overview

Lesson Sequence	**Topic Covered**	**Difficulty Cited in Table 1**
1	Pre-test – to determine the students' understanding of area before the lessons	All
2	The concept of covering	A, B, D
3	Exploring figures using the geo-board	A, B, C, E
4	Investigating dynamic properties of area (the foot problem)	All
5	Looking at objects in different ways	A, B
6	Estimating areas (upper and lower bounds, average)	D, E, F
7	Determining the area of a trapezoid	A, B, D, E
8	Trapezoid #2	A, B, D, E
9	Synthesis	All
10	Post-test – to determine students' understanding of area following the lessons	All

Data Collection

The pre-test was collected at the end of Lesson 1. The post-test was collected at the end of the unit and was used as part of the students' evaluation. Student work, which was completed on chart paper and note paper, was handed in at the end of each lesson, photocopied, and returned to the students during the next class. Anecdotal notes were kept by the teacher and were reviewed after each lesson. Subsequent lessons were adjusted based on observations and anecdotal notes.

A video camera was used to capture the students' procedures as they worked. The camera was unobtrusively placed on a tripod at the front of the room. Student discussions were recorded using the microphone on the camera. If a close-up view of student work was required, the zoom feature on the camera was activated using a remote control.

Particular effort was made to record the activities of a sample of three students. This sample was chosen to represent a homogeneous level of achievement on the pre-test; each of the students chosen received the mark closest to the median on the test. None of these students was aware that they had been chosen as key video subjects. The video-tape sessions took place three times during the research period to provide a record of student progress and process. Table 3 outlines the schedule of the video sessions and the questions that were addressed as part of the in-class procedures during those sessions.

Table 3. Video Tape Sessions

Video Session	Questions to be answered	General Questions
Lesson 4 – Investigating dynamic property of area	- What is the relationship between area and perimeter?	- What is your understanding of the concept of area?
Lesson 6 – Estimating areas	- What methods can you use to estimate the area of an amorphous figure?	- How would you describe your attitude towards the task?
Lessons 7 & 8 – Determining the area of a trapezoid	- What happens to the area when a figure is reorganized into a new shape? - What is the area of a trapezoid? - Can you develop a general formula for the area of a trapezoid?	- Can you use multiple strategies to solve the problem? What are they?

Data Analysis

To ensure anonymity, numbers were used instead of names to identify each student. Students were not aware of their numbers, and all work, including the pre-test and post-test, was coded with this number after it was submitted. I marked both sets of tests. On the tests, an incorrect response garnered an 'I', while a correct response received a 'C', and if the child responded "I don't know" they received a "U". Both an incorrect response and the response "I don't know" were considered to mean that the student was unable to solve the problem. In questions where an explanation or justification was required, I categorized the responses. The coding categories used were: graph paper, diagram, partitioning, formula, and written explanation. I expected deeper understanding to be demonstrated on the post-test by students attempting more questions, answering more questions correctly, and utilizing a wider range of strategies to solve problems.

Additional data were obtained from the anecdotal journal I maintained, in-class work by students, and videotapes taken of the students solving the problems. My journal entries reflected my personal view of the pros and cons of the lesson taught that day, as well as observed student reactions to the lessons. These observed reactions provided me with insight into how engaged the students were in the lessons. In class work was collected daily and coded. I drew on the research literature, used to create the pre-test and post-test, to formulate my initial set of codes of expected difficulty or of demonstrated understanding of the concept of area. These codes were: measuring areas of various objects, static perspective of area, dynamic perspective of area, spatial structuring and covering, estimation, and area of an amorphous figure.

The purpose of the video was to capture the process the students used when solving problems. During the screening of the video, students' verbal and written responses were analyzed. The video was coded using *Atlas.ti*, a software program designed to assist researchers when analyzing qualitative data. Each video was viewed and coded using the system described in the previous paragraph. The expectation was that, as we progressed through the unit, students would garner an improved conceptual understanding of the material. I anticipated that this improved conceptual understanding would be demonstrated by students' increased sophistication with respect to the handling of the questions and the clarity and comprehensiveness of the communication of their answers to their partners and to the class.

Chapter Four: Results and Analysis

The impact of the reform teaching method was observed and evaluated in a variety of ways. I evaluated pre- and post-tests, gathered student work after each class, noted classroom observations in a journal, and took video-tape records of students at work during each instructional class. Testing was done to determine students' levels of understanding of the topic of area. Progress was also monitored through observation and evaluation of classroom work. Students' attitudes about geometry, and in particular measurement of area, were determined by asking them directly about their preferences with respect to the types of problems they liked to solve, and by observing their performance on oral and written work. Student response to the reform teaching method was assessed through their participation in, and reaction to, the problem solving activities presented in each lesson.

Test Results and Classroom Observations

Items on the pre-test and post-test were developed to address particular areas of difficulty as cited in the literature (Table 4). The pre-test and post-test results were compared by analyzing the students' responses to questions that addressed the same difficulty on each test.

Table 4. Design of the Pre-Test and Post-Test Instruments (reprint of Table 1)

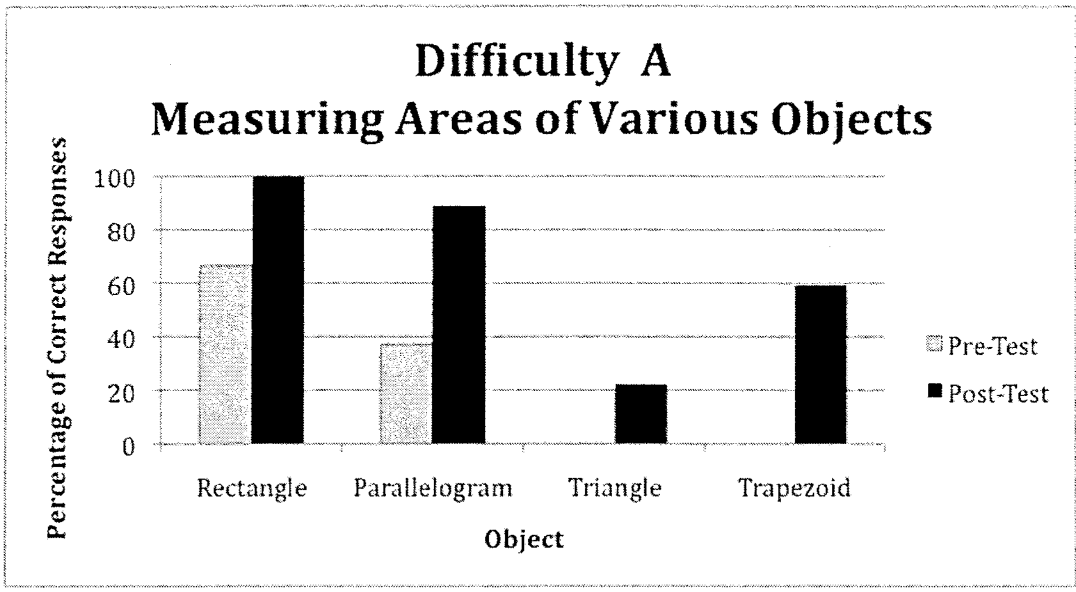
Letter	Difficulty cited in literature on understanding of area	Citation	Instrument Item	
			Pre-Test	Post-Test
A	Measuring areas of various objects.	Baturo & Nason (1996); Van De Walle & Folk, 2005	5, 10, 12, 13	4, 9, 11, 12
B	Static perspective of area.	Baturo & Nason (1996)	7, 15	6, 14
C	Dynamic perspective of area.	Baturo & Nason (1996)	6, 14	5, 13
D	Spatial structuring and covering.	Battista (2003); Outhred & Mitchelmore (2000)	4, 9	8
E	Estimation.	Adams & Harrell (2003); Morgan (1986)	8	7
F	Area of an amorphous figure.	Casa et al. (2006)	11	10

The in-class results were similarly analyzed according to the above cited areas of difficulty. The following is a discussion of the impact of the reform teaching method on the areas of difficulty cited in the literature.

Difficulty A: Measuring areas of various objects - test results. On the post-test, with the exception of one individual, students improved in their ability to measure the area of various objects (Figure 4). Katherine[1], who did not improve, remained consistent in that she was able to measure the area of a rectangle and parallelogram on both the pre-test and post-test, but not the area of a triangle or trapezoid.

[1] Pseudonyms have been used to protect the identity of the participants.

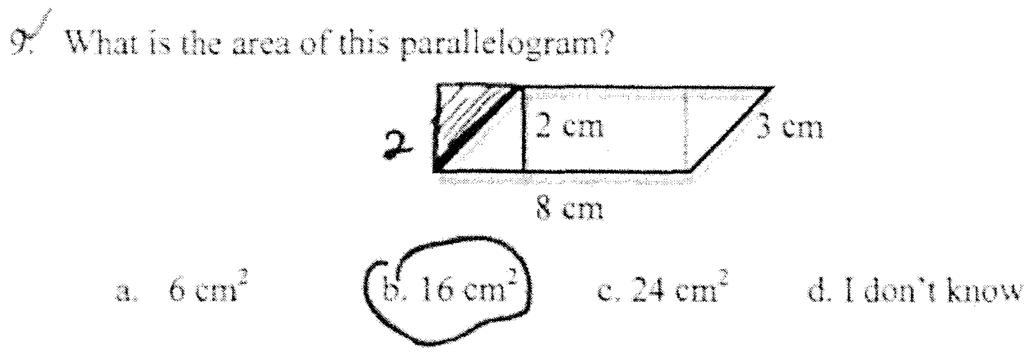
Figure 4. Pre-Test and Post-Test Results for Difficulty A



When asked on the pre-test to determine the area of a rectangle, 18 students were able to provide the correct answer. On the post-test, all 27 students were able to determine the area of a rectangle. As this was a question where dimensions for the length, the width and the diagonal of the rectangle were given, this demonstrates that students were able to select the appropriate measurements and use a suitable method to determine the area.

Ten students were able to determine the area of a given parallelogram on the pre-test, while 24 were able to correctly determine a parallelogram's area on the post-test. On the pre-test, no students demonstrated the use of a technique to decompose the parallelogram into another figure for which the area might be more easily calculated. Although it was difficult to determine the precise technique students used to solve the post-test question, Billy, among others, sectioned off a triangle from the right side of the parallelogram and drew it on the other side to create a rectangle (Figure 5). He had not obtained the correct answer on the pre-test; however, was able to solve the post-test question correctly. His approach to the problem demonstrated that, rather than relying on a memorized formula, he had developed a conceptual understanding of how to obtain the area of a parallelogram.

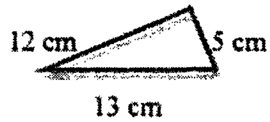
Figure 5. Sectioning Off a Triangle from a Parallelogram to Create a Rectangle (Billy)



On the pre-test, none of the students was able to calculate the area of a triangle or trapezoid. Common mistakes included: using an incorrect algorithm (Figure 6), using incorrect measurements in the correct algorithm (Figure 7), and confusing perimeter with area (Figure 8). Only seven students were able to determine the area of a given triangle on the post-test. This result was surprising, given the success students had demonstrated during the lessons. Although a right-angled triangle was used on the test, the right angle was not clearly indicated on the diagram. Students were required to understand the meaning of the word 'hypotenuse', and that the right angle was directly across from the hypotenuse. This multi-step problem, which was difficult to solve if students did not know where the right angle was located, prevented them from using many of the area-finding strategies discussed in class.

Figure 6. Using an Incorrect Algorithm to Calculate Area (Chris)

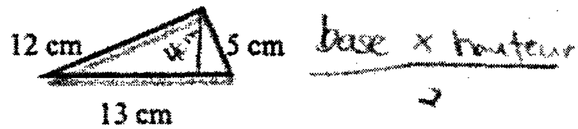
12. Determine the area of a right triangle with legs 5 cm and 12 cm, and hypotenuse 13 cm. Explain how you obtained your answer.



The area of this triangle would be 65²cm. I know this because in order to get the area of a object you must use the length times hight formula and that formula would be 13 X 5 wich is equal to 65.

Figure 7. Using Incorrect Measurements in the Correct Algorithm (Katherine)

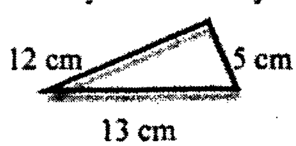
12. Determine the area of a right triangle with legs 5 cm and 12 cm, and hypotenuse 13 cm. Explain how you obtained your answer.



The formula to calcul'at a triangle is base × hight divided by 2: .
13 × 4 = 52 52 ÷ 2 = 26
The area of the triangle is 26 cm²

Figure 8. Confusing Perimeter With Area (Mary)

12. Determine the area of a right triangle with legs 5 cm and 12 cm, and hypotenuse 13 cm. Explain how you obtained your answer.



I think the answer is 30cm. I just added all of the sides.

When asked to calculate the area of a trapezoid on the post-test, 16 students were successful. The Ministry of Education expectation for the Grade 7 Mathematics

Curriculum that states students are to "solve problems involving the calculation of the area of a trapezoid" (MOE, 2005, p. 101) was achieved by 63% of the class on the post-test. Three students who did not obtain the correct answer used correct methods but made calculation errors when multiplying or dividing. Two students did not explain how they arrived at their solution, and the remaining six students attempted the question using an incorrect formula.

Difficulty A: Measuring areas of various objects - classroom observations. Throughout the lessons, I noticed that the students became increasingly confident with respect to their ability to determine the area of an object. Their explanations as to how the area was determined became more detailed and they increased their use of mathematical language. Their ability to describe their processes in detail demonstrated that they held a deeper conceptual understanding of the concept of area.

The first shape for which students were asked to determine a rule to calculate its area was a rectangle. When first asked, a handful of students were able to write down the formula for the area of a rectangle; however, none of them could explain the reasoning for the formula (Video, Lesson 2, January 13, 2010). When working through the problems, students used a variety of methods to determine areas of various rectangles. One strategy, employed by 10 of the 27 students, was to draw a 1 cm x 1 cm grid directly onto a given rectangle and count the squares (Figure 9). A similar strategy that the remainder of the students used involved tracing a rectangle onto 1 cm x 1 cm grid paper and counting the squares (Figure 10). Adrian was very pleased with himself when he determined why the formula to calculate the area of a rectangle is *length x width*. Using a rectangle with a length of 4 cm and a width of 3 cm he explained the following: "The four across the bottom is counting columns, and the three across the side is counting the rows, so it's twelve, and 4 x 3 is 12" (Video, Lesson 2, January 13, 2010). In the math congress, Chris added to this idea explaining that: "you have to count the four columns three times, which is why it's four times three" (Video, Lesson 2, January 13, 2010).

Figure 9. Drawing a Grid Directly Onto a Given Rectangle and Counting Squares (Matt, Lesson 2, January 13, 2010)

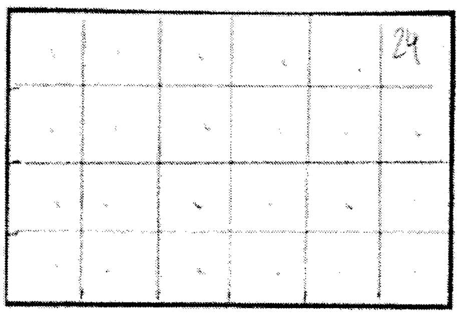
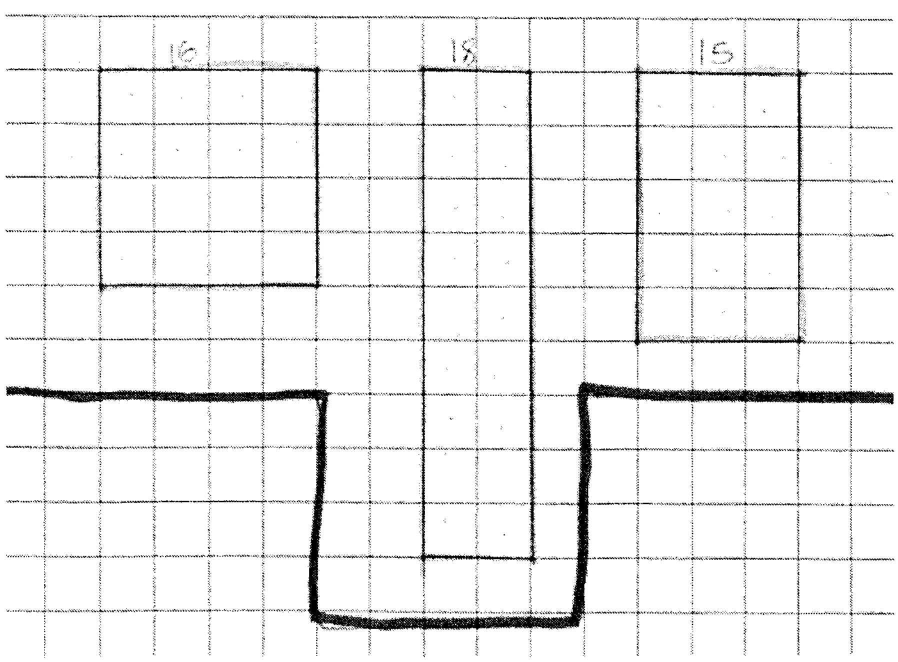


Figure 10. Tracing the Given Quadrilaterals Onto Graph Paper and Counting Squares (Pam, Lesson 2, January 13, 2010)



After determining the area of a rectangle, students were asked to discover rules to calculate the area of a triangle and a parallelogram. In analyzing the relationships between rectangles and triangles, students cut a given rectangle diagonally creating two triangles, and were able to determine that the two triangles were congruent, and that each represented half the area of the rectangle. They were quickly able to justify why the formula for the area of a triangle that had been taught in previous years, (½)*base x height*, was valid (Video, Lesson 5, January 19, 2010).

The most difficult part for the students in this lesson was attempting to create a parallelogram with their two triangles. However, once this was achieved, one student noted that: "Since I can still take my two pieces of triangle that make up my parallelogram and put them back to make my original rectangle, I must be able to use base times height to calculate the area of a parallelogram as well" (Sophie, Video, Lesson 5, January 19, 2010). One group decided to ensure that this logic was correct. The students traced their parallelogram on graph paper, counted the full squares that it enclosed, and determined what partial squares created a whole square, which they added to their count. They then verified that multiplying base and height obtained the correct area (Figure 11). Six other groups of students recognized that they could draw a perpendicular line onto their parallelogram, thus creating a triangle that could be moved to the other side and affixed so as to make a rectangle (Figure 12).

Figure 11. Verification that Base Multiplied by Height Calculates the Area of a Parallelogram. (Mark & Adrian, In-Class Notes, Lesson 5, January 19, 2010)

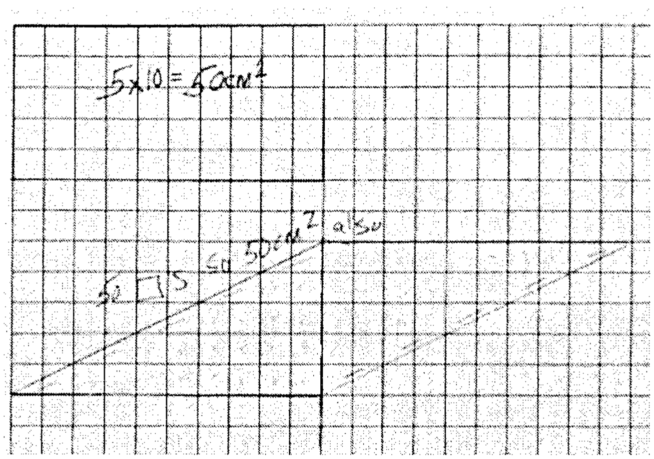
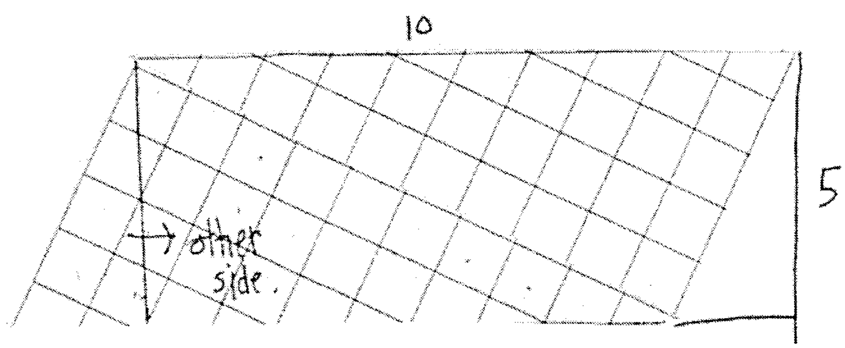


Figure 12. Moving a Triangle from One Side of a Parallelogram to the Other to Create a Rectangle (Gina & Andrew, In-Class Notes, Lesson 5, 2010)



Determining a rule to calculate the area of any given trapezoid proved to be the most difficult. The first trapezoid presented was an isosceles trapezoid. Eight pairs of students were able to use similar strategies as they did with the parallelogram to determine that one can cut off a triangle, move it to the other side and flip it, creating a rectangle, then multiply the base and the height to obtain the area (Figure 13). Another strategy involved sectioning the trapezoid into a rectangle and two congruent triangles, finding the area of each section, and then adding the areas together (Figure 14). Problems arose when students were presented with a non-isosceles trapezoid. I had hoped that one strategy students would devise would be to create a second trapezoid, flip it, and append it to the original figure, thus creating a parallelogram with a base equal to the sum of the top and bottom of the original trapezoid. Following this, students could have multiplied the base and the height and divided by two to obtain the area of the original trapezoid. However, this was not a strategy students used. They did, however, use the familiar strategy of tracing the trapezoid on graph paper and counting the squares it covered. One pair of high-achieving students developed the appropriate formula as seen in Figure 15.

Figure 13. Removing a Triangle from One Side of the Trapezoid (Katherine & Alison, In-Class Notes, Lesson 7, January 22, 2010)

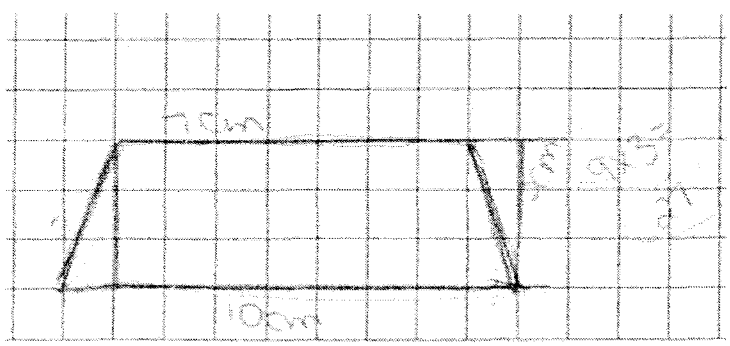


Figure 14. Sectioning the Trapezoid Into Parts (Amy & Pam, In-Class Notes, Lesson 8, January 25, 2010)

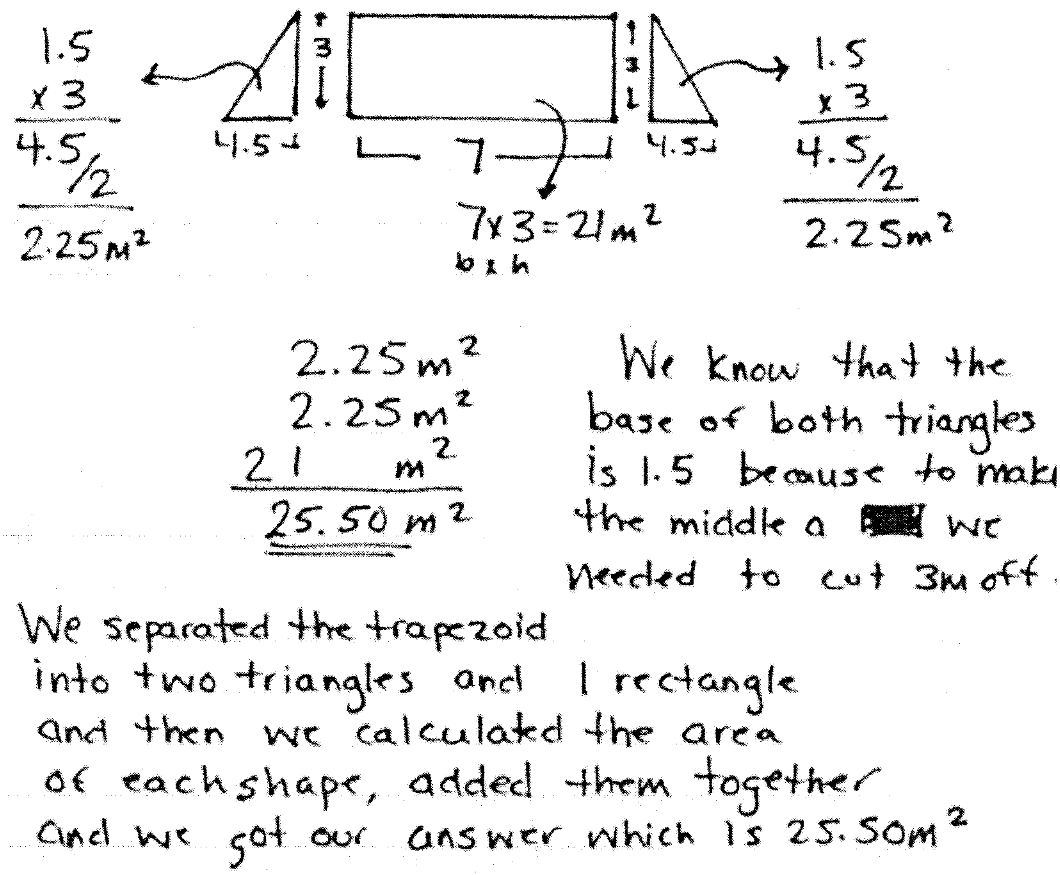
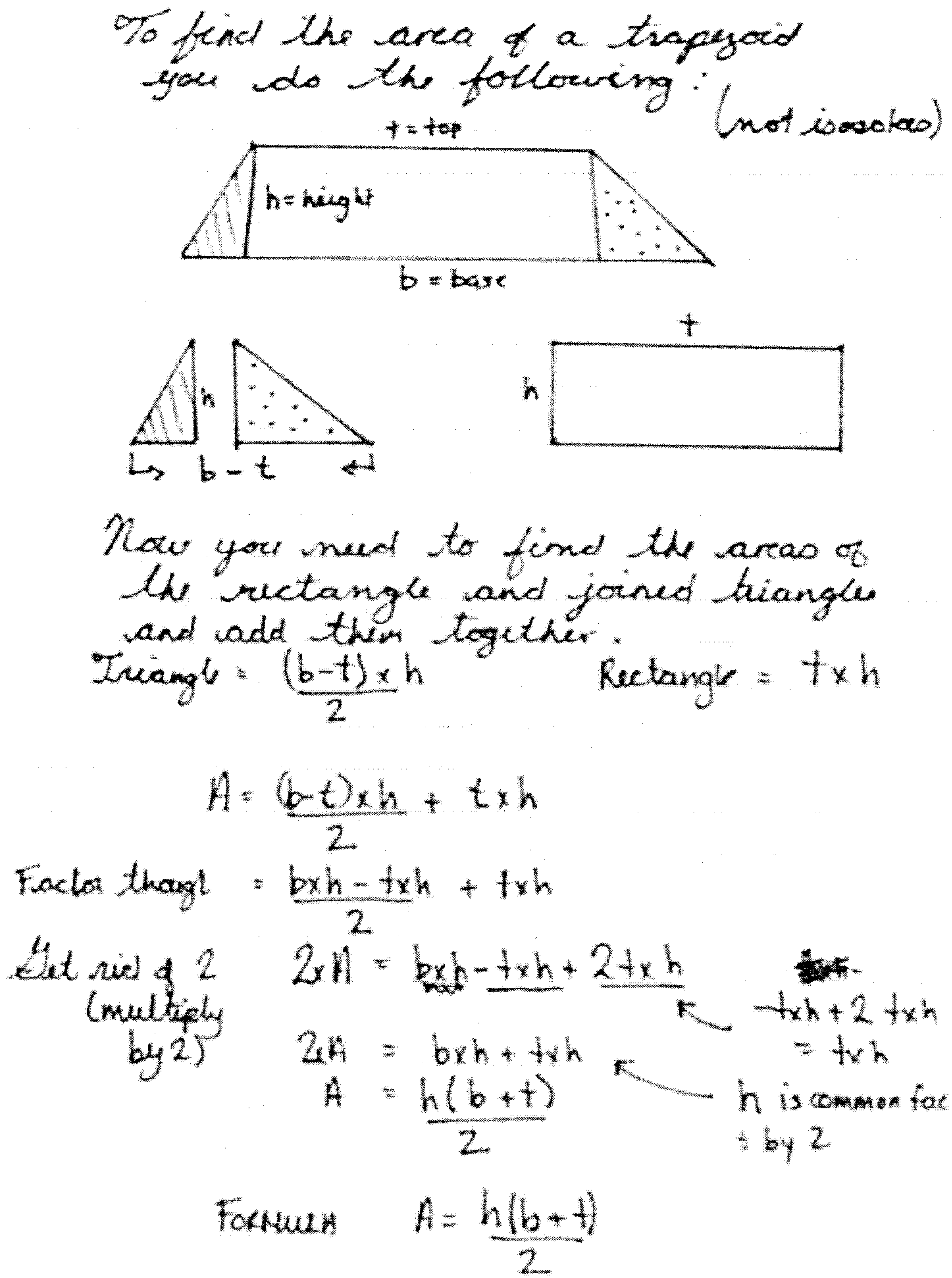


Figure 15. Developing a Formula to Calculate the Area of a Trapezoid (Gina & Andrew, In-Class Notes, Lesson 8, January 25, 2010)



Difficulty B: Static perspective of area - test results. On the post-test, 15 students demonstrated an improved understanding of the static perspective of area by correctly answering questions related to this cited difficulty (Figure 16). On the pre-test, only 11 students answered one question correctly, with the remaining 16 answering both incorrectly. Eleven students maintained the same level of understanding on the post-test as they demonstrated on the pre-test. Four of the students who did not improve sectioned off the appropriate area for this final question on the post-test, however, did not use a "straight line" as the question requested. This demonstrated that they understood the static perspective of area, however, struggled with spatial structuring, or thought typically in whole-numbered units. One student did not attempt the problem, and the other six students miscalculated the area of the original figure by counting the number of squares incorrectly. In only one case did a student have less success on the post-test than on the pre-test. In discussions with the student, she admitted to having guessed on the pre-test, and that she had not understood the question. This last question on each test was adapted from Kamii & Kysh (2006), and asked students to: "Draw a straight line on the figure (b) to show where you would make a straight cut to have exactly the same amount of space as in figure (a)". This is the question that caused the most difficulty on both the pre-test and post-test. On the pre-test, only two students correctly answered this question. An additional eight students experienced success with this question on the post-test. Students who solved this problem demonstrated that they were able to create a figure with a given area. Student examples (Figure 17 & Figure 18) show two ways that this problem was solved.

Further review of the last question has convinced me that it was more challenging than the first question on each test that related to the static perspective of area. Baturo and Nason (1996) contend that the static perspective of area is related to being able to find the area of a given figure. In addition to gauging a students' understanding of the static perspective, this final test question on both the pre-test and post-test addressed the more difficult concept of conservation of area. Kordaki (2003), in her study on intermediate students' understanding of the concept of conservation of area, determined that: "an area may be conserved while the shape of its figure is altered" (p. 179). Altering the second shape in the test question, to produce the area given in the first shape is an example of conserving area. Van de Walle and Folk (2005), as well as Kordaki, cite this concept as being difficult for students to understand. In particular, Kordaki found that: "the possibility of equivalence of an area when it is represented in shapes of different forms" (p. 180) and "the concept of area as the sum of its parts" (p. 180) are ideas with which students struggle, yet are paramount to understanding the concept of conservation of area.

Figure 16 is therefore mis-leading as it contains data from both test questions relating to the static perspective of area. In separating the questions, it is clear that students were more successful on the first item, on both the pre-test and post-test, than on the second item. This can be attributed to the fact that the second item was more complex in that it dealt with an additional concept - the conservation of area.

Figure 16. Pre-Test and Post-Test Results for Difficulty B

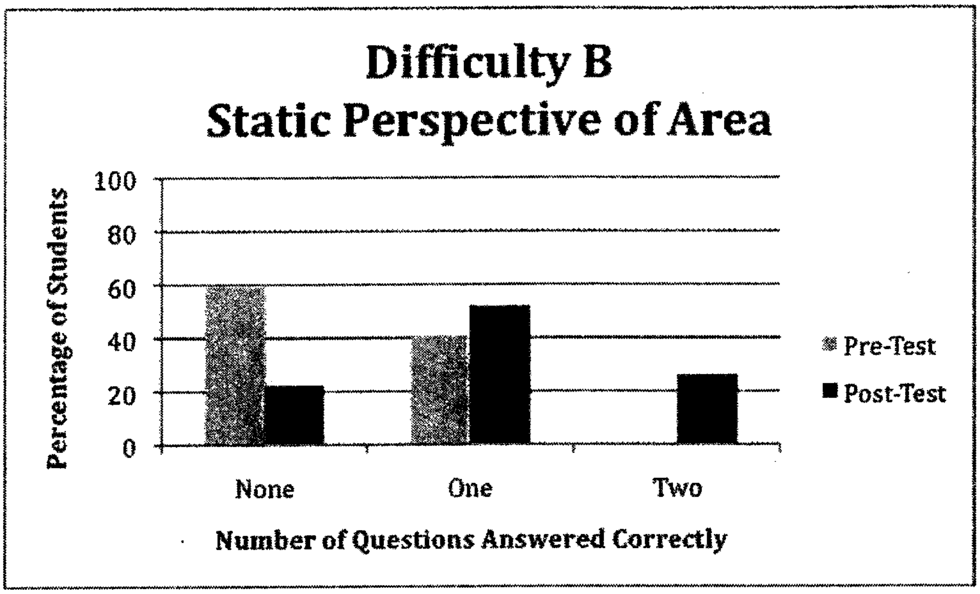


Figure 17. Demonstrating an Understanding of the Static Perspective of Area (Ella)

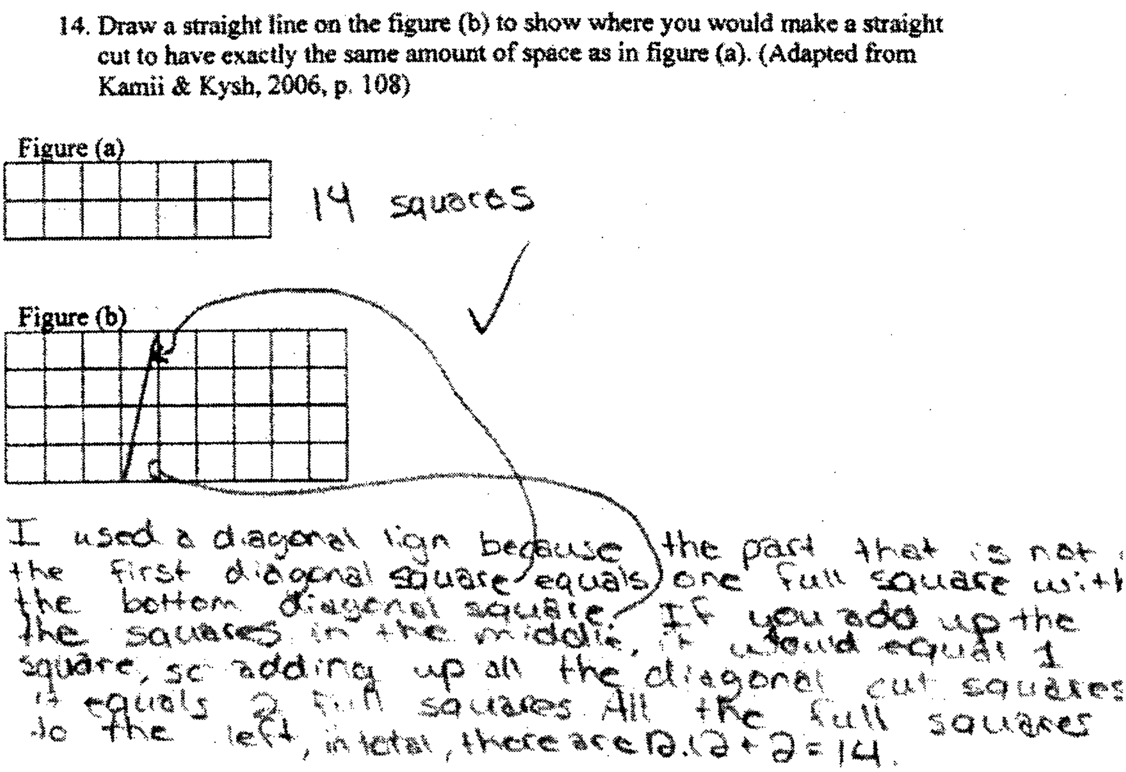
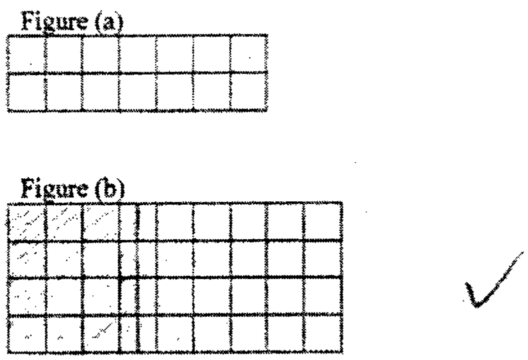


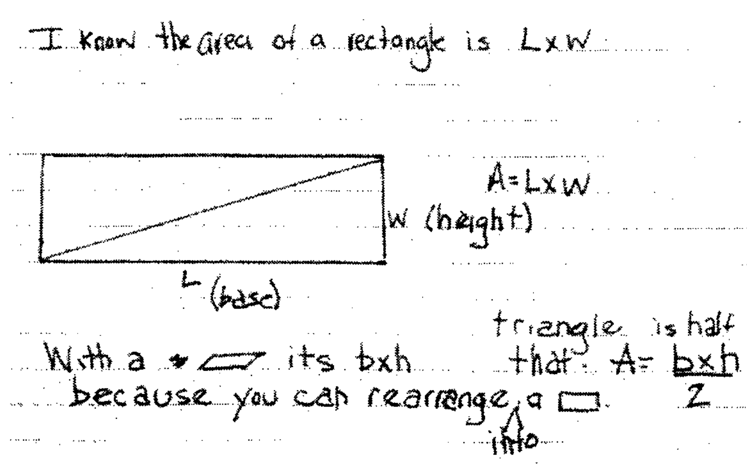
Figure 18. Demonstrating an Understanding of the Static Perspective of Area (Tara)

14. Draw a straight line on the figure (b) to show where you would make a straight cut to have exactly the same amount of space as in figure (a). (Adapted from Kamii & Kysh, 2006, p. 108)



Difficulty B: Static perspective of area - classroom observations. In their day-to-day work, students showed that they understood the static perspective of area. They understood that area was a region around which there is a boundary, and that the region can be quantified. At the beginning of the unit, students demonstrated this by counting squares within a figure (Video, Lesson 2, January 13, 2010). As the unit progressed, they devised more efficient strategies to determine area. Students became more comfortable with the use of the formulas they had developed (Figure 19).

Figure 19. Increased Comfort Level with Area Formulas (Sophie & Kirk, In-Class Notes, January 19, 2010)



Difficulty C: Dynamic perspective of area – test results. Eighteen of the 27 of students were successful on both of the post-test items that related to the understanding of the dynamic perspective of area (Figure 20). On the pre-test, 10 students answered both questions correctly, while 12 answered one question correctly. All students answered at least one of the post-test questions correctly. Of the 27 students in the class, 12 demonstrated an improved understanding, 14 demonstrated the same understanding, and one demonstrated a decline in his understanding. The student whose understanding apparently declined confused area with perimeter (Figure 21), a confusion that was not apparent in any other test questions, nor in any of the in-class work. I believe that the confusion was a result of the student having misread the question. Of the students who demonstrated consistent understanding, only five students had room for improvement. The other nine students answered both questions correctly on the pre-test and post-test. Three of these students did not understand that a parallelogram and rectangle, each with the same base and height, always have the same area. One student did not demonstrate that he understood that the area of a triangle is always half the area of a rectangle, provided each has the same base and height. The last student who did not demonstrate improvement in this area of difficulty confused the word area with perimeter in question 13 on the post-test – something she had not done in a comparable question on the pre-test. On the post-test, students developed a variety of shapes that had a perimeter of 12 cm (Figure 22). Diagrams in question number six on the post-test were not drawn to scale, as is typically the practice in geometry problems. This could have caused problems for students who attempted to compare the areas of the figures using estimation. In the future, I would indicate that diagrams were not to scale to prevent confusion.

Although the test questions touched on the dynamic perspective of area, further review has led me to believe that the items themselves did not allow students the opportunity to demonstrate that they were able to generalize the concept. A question such as: *A farmer has 48 metres of fencing and would like to use it to create a rectangular garden that has the largest possible area. What are the dimensions of the largest and the smallest possible gardens? How do you know?* would have garnered answers that demonstrated whether students had a complete understanding of the dynamic perspective. Nonetheless, class discussions led me to believe that many students did have a generalized understanding of this perspective. For example, when asked about what the best shape for a swimming pool would be if one wanted to have the greatest area in which to play, students were able to explain that a long, skinny pool would have a small area, but that a square pool would provide the largest area.

Figure 20. Pre-Test and Post-Test Results for Difficulty C

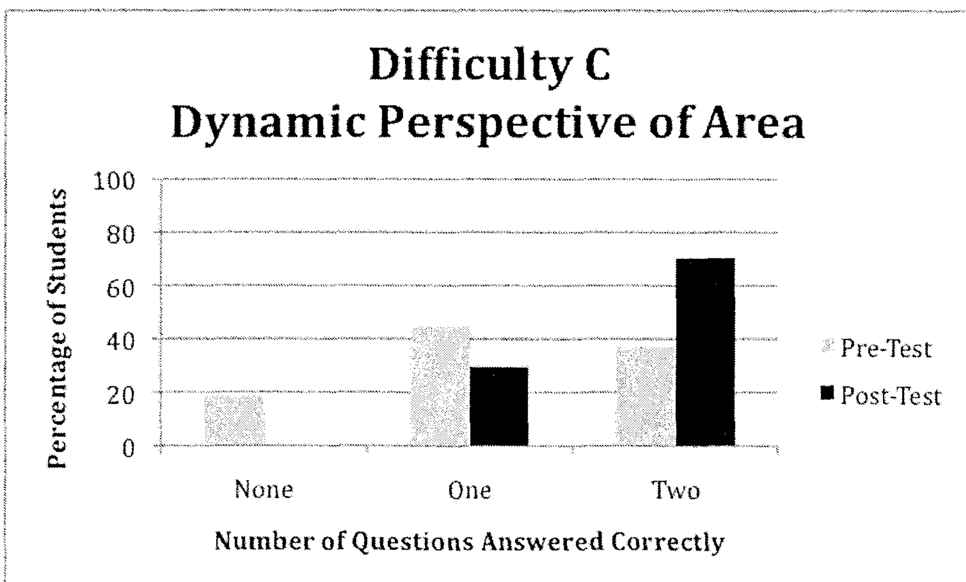


Figure 21. Confusing Area With Perimeter (Chris)

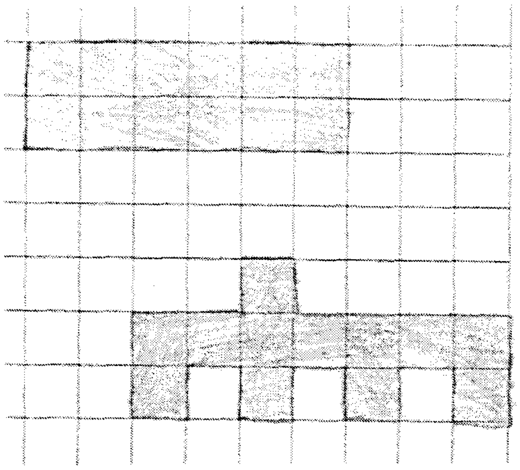
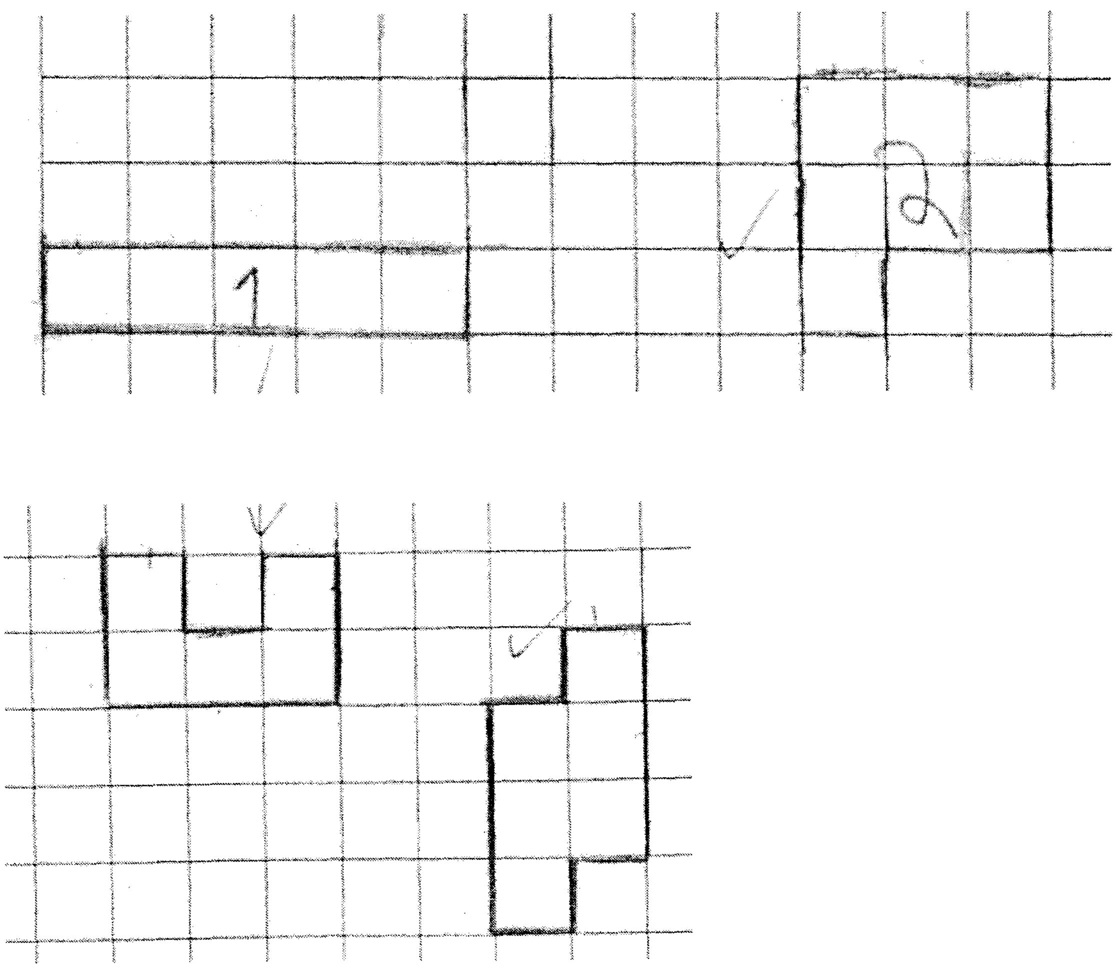


Figure 22. Student Examples of Shapes With a Perimeter of 12 cm (Stacey & Mason)



Difficulty C: Dynamic perspective of area - classroom observations. Two lessons focused specifically on the dynamic perspective of area and students showed progress in understanding this concept. When asked to use their geoboards to draw as many possible polygons with an area of 12 square units, students began to notice that although the area of their figures remained the same, the shape changed, as did the perimeter of the polygons they were creating (Video, Lesson 3, January 13, 2010). In the math congress at the end of the period, students were able to provide real-world examples where this concept would be important. The shape of a swimming pool was mentioned: "You could have a swimming pool that was 12 m by 2 m, but that wouldn't be very good for games and stuff, you would be better to have a pool that was 6 m by 4 m. Then you could play more games in it, but still have the same area" (Alison, Video, Lesson 3, January 13, 2010). Conversely, students began to recognize that shapes with the same perimeter could have different areas (Figure 23). Students further gained an understanding of this concept when they worked on the foot problem in the subsequent class. During the math congress, students enjoyed the opportunity to see that when the string that was equal to the perimeter of their foot was fashioned into a square, it created a larger area (Video, Lesson 4, January 19, 2010. This led some students to correctly conclude that a square always creates the largest area for a given perimeter (Gina & Andrew, Video, Lesson 4, January 19, 2010).

Figure 23. Demonstrating an Understanding of the Dynamic Perspective of Area (Ella & Meredith, Lesson 5, January 19, 2010)

Draw three different shapes on centimeter squared paper following these three rules:

a. Each shape must have a perimeter of 30 cm
b. Stay on the lines when you draw (no diagonals)
c. You must be able to cut your shape out and have it all in one piece.
d. What do you notice about the area of the shapes?

* answers

What I noticed about the area of the shapes, was that even if the perimeter is the same as another shape, the area could be larger or smaller.

example: figure 1 = perimeter = 30 cm, area = 14 cm
figure 2 = perimeter = 30cm, area = 36 cm2
figure 3 = perimeter = 30cm, area = 33 cm2

* perimeters are the same - 30 cm, 30cm, 30cm

* areas are different - 14 cm2, 36 cm2, 33 cm2

Difficulty D: Spatial structuring and covering – test results. Only Mark demonstrated an understanding of spatial structuring on the pre-test. However, on the post-test, 14 students had sufficient understanding of spatial structuring and covering to determine that a hand with fingers together would cover the same area as a hand with fingers apart (Figure 24). Pam verified her answer by tracing her hand in each of these positions on graph paper, and determining that the area was the same (Figure 25). Of the 13 students who struggled with this concept, seven of them indicated that a tracing of a hand with fingers apart has a larger area than a tracing of a hand with the fingers together. The remaining six thought the opposite.

Figure 24. Pre-Test and Post-Test Results for Difficulty D

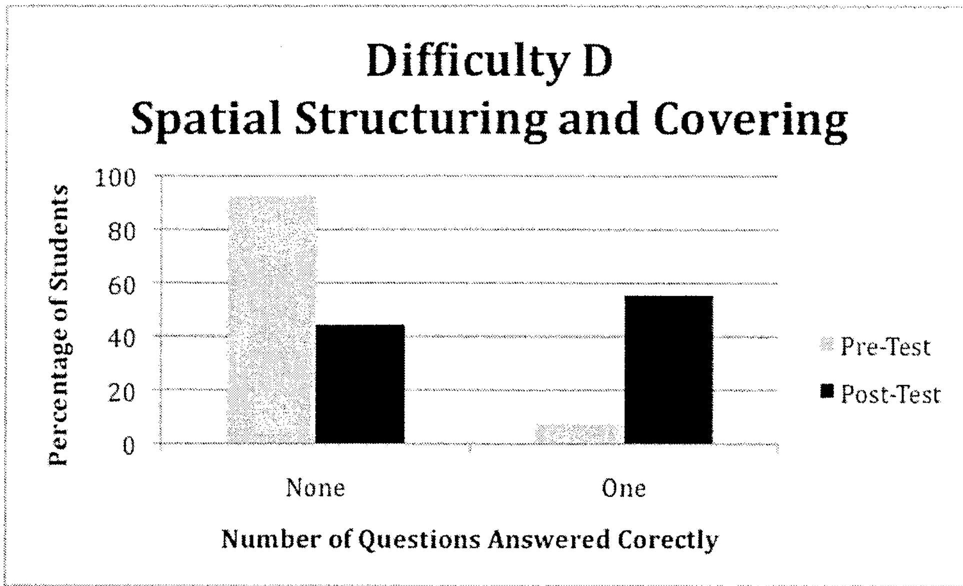
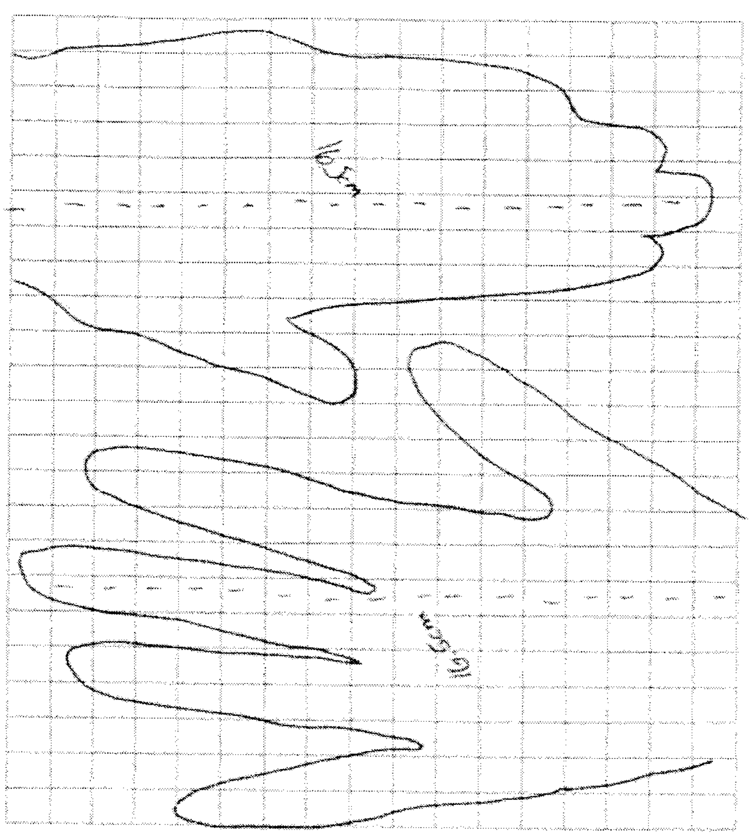
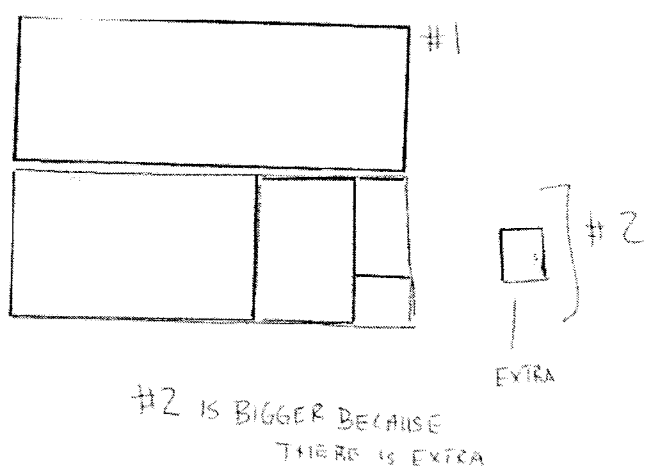


Figure 25. Tracing of Hands to Determine Area with Fingers Together and Fingers Apart (Pam)



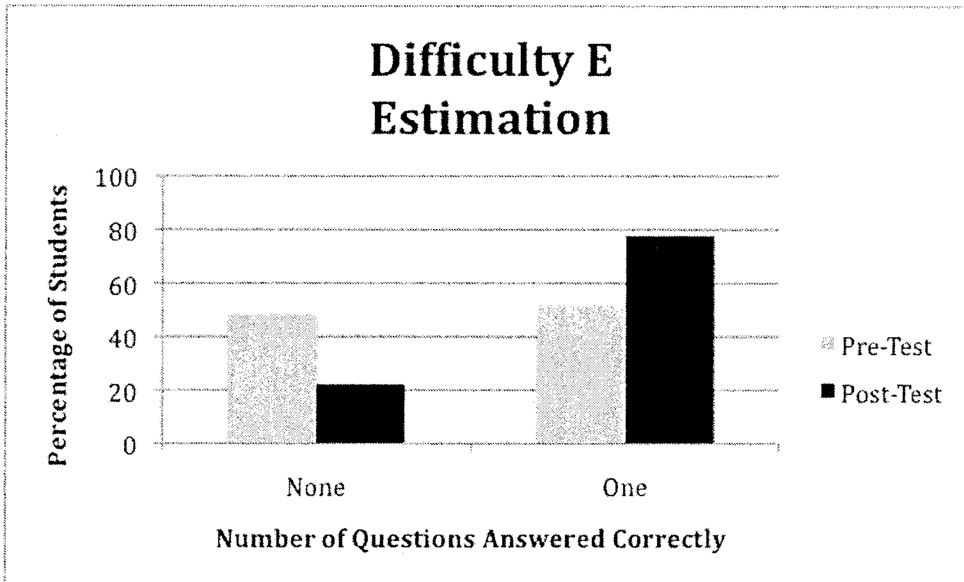
Difficulty D: Spatial structuring and covering – classroom observations. In class, students investigated the concept of covering when asked to compare areas of various rectangles. Students were instructed not to use any known formulas to compare the areas, so they needed to determine alternative methods to compare the rectangles. Eight groups of students counted squares on each rectangle and indicated that the figure with the larger number of squares had the larger area. Another pair of students cut one figure into pieces that could fit on top of the other figure and determined that since there was one leftover piece from the original figure, that it must have had a larger area (Figure 26).

Figure 26. Determining Which Figure has a Larger Area Using a Covering Strategy (Amy & Pam, Lesson 2, January 13, 2010)



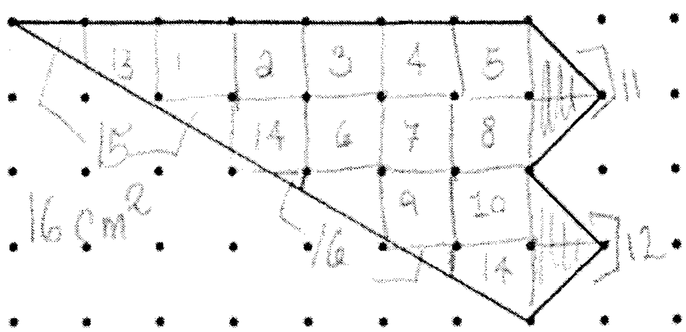
Difficulty E: Estimation – test results. On the pre-test, 14 of the 27 students demonstrated an understanding of how to estimate the area of a figure (Figure 27). An additional 10 students succeeded in correctly estimating the area of a figure on the post-test. Of the 14 students who obtained the correct answer on the pre-test, three obtained incorrect answers on the post-test. Two of the students who indicated that the area of the rolled up rectangle would be less than the area of the rectangle itself mentioned after the test that they had not carefully read the words "without overlap" in the question. The student who indicated that the surface area of the cylinder would be more than the area of the rectangle included the area of the top of the cylinder.

Figure 27. Pre-Test and Post-Test Results for Difficulty E



Difficulty E: Estimation – classroom observations. Lesson 3 was the first opportunity students had to estimate the area of a figure. Initially, one of the most common methods students used was to draw the figure on graph paper and count the squares. This method was used by 12 of the pairs on January 13, 2010. Partial squares were combined with other partial squares to create what appeared to the students to be whole squares (Figure 28). Another method involved obtaining an upper bound of the figure, then subtracting parts contained in the upper bound that were not part of the figure. During the math congress, one pair of students presented the idea of an area of an object being "less than" its upper bound (Chris & Tara, Video, Lesson 6, January 20, 2010). This led another student to comment that the same object could have an area that is "greater than" its lower bound (Alison, Video, Lesson 6, January 20, 2010). Another student exclaimed: "So that means that the area of the object would have to be somewhere in between the upper and lower bounds" (Bianca, Video, Lesson 6, January 20, 2010). Gina agreed with her, and went even further, explaining that: "If you take the average of the upper and lower bounds you would get a more accurate estimation of the figure's area" (Gina, Video, Lesson 6, January 20, 2010).

Figure 28. Combining Partial Squares (Isaac & Elizabeth, Lesson 3, January 13, 2010)



Difficulty F: Area of an amorphous figure – test results. On the pre-test, 10 students were able to provide a method that could be used to calculate the area of an amorphous figure (Figure 29). The method described drawing a box around the figure, measuring the sides of the box, and then calculating the area (Figure 30). This produced a rough estimate of the area of the amorphous figure. Fourteen additional students were able to provide a method to determine the area of an amorphous figure on the post-test. Of the three students who did not answer this question correctly on the post-test, one of the students chose not to provide answer, and the other two students indicated that they would find the upper bound of the area of the figure by drawing a box around it, and multiplying the length by the width, however, proceeded to incorrectly explain the method of averaging the upper bound and lower bound to determine a more precise estimate. Many of these solutions provided a more precise approximation of the area than the strategies that were used on the pre-test. One method used was to find the area of a rectangle that enclosed the entire figure, and find the area of the rectangle that lay completely within the figure. Students knew that these represented the upper and lower bound of the area of the figure, respectively, and realized that the area of the amorphous figure must lie between these two areas (Figure 31). Other strategies involved the use of graph paper and counting squares (Figure 32), and determining the area of a rectangular figure surrounding the amorphous shape, and subtracting unnecessary areas (Figure 33).

Figure 29. Pre-Test and Post-Test Results for Difficulty F

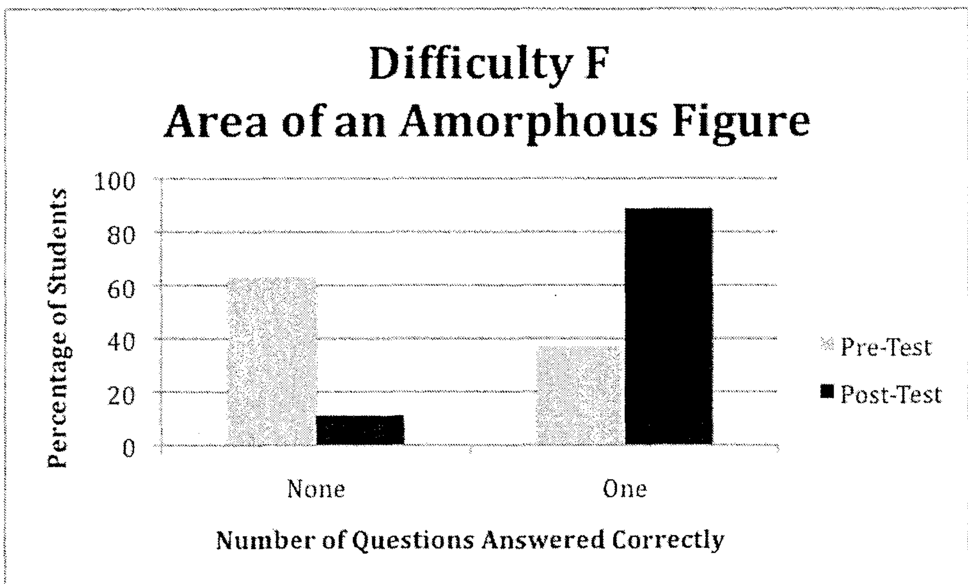


Figure 30. Calculating the Area of an Amorphous Figure – Pre-Test (Andrew)

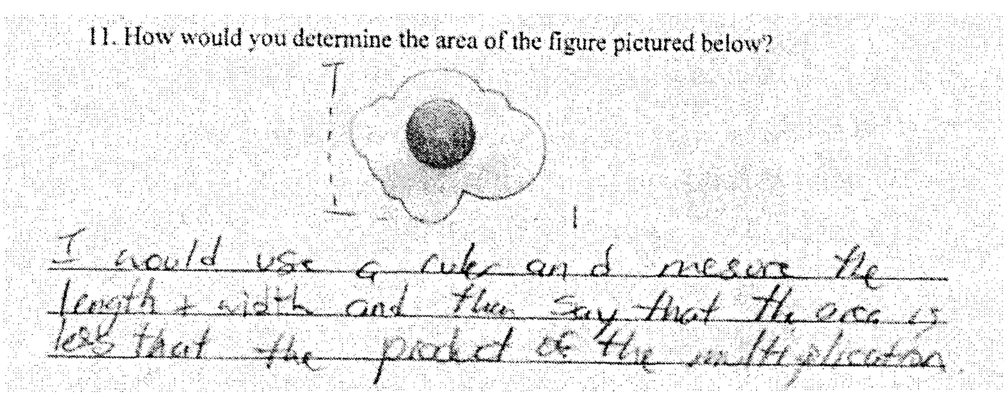


Figure 31. Calculating the Area of an Amorphous Figure – Post-Test (Billy)

10. How would you determine the area of the amorphous shape pictured below?

I would draw an outer box to contain it and then an inner box that doesn't leave it and then the area must be in between those two numbers

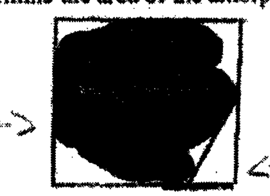
Figure 32. Calculating the Area of an Amorphous Figure – Post-Test (Andrew)

10. How would you determine the area of the amorphous shape pictured below?

I would put a piece of 1 cm square paper over it then count the squares. This method both works and is easy

Figure 33. Calculating the Area of an Amorphous Figure – Post-Test (Ella)

10. How would you determine the area of the amorphous shape pictured below?



To determine the area of this shape, I would draw a square around it and find the area of the square by doing L×W=A. Then, I would find the area of the spaces leftover by making a triangle out of each one of them. Then add them all together, then subtract it from the area of the square

Difficulty F: Area of an amorphous figure - classroom observations. When calculating the area the foot from Lesson 4, six groups chose to count all of the squares within the perimeter of the foot, regardless of whether or not they were whole squares (Figure 34). Another group chose to find the area of a rectangle that fit outside of their foot diagram, then subtracted the parts within that rectangle that were not included in the area of the foot (Figure 35). This lesson demonstrated to students that it was possible to find the area of an amorphous figure. Sophie who, on her pre-test, wrote: "I don't think you can figure out the area of a round [egg] shape", effectively calculated the area of an amorphous figure by drawing a rectangle around the shape, finding its area, and determining that: "the area of the egg has to be less than the area of the box, because the box fits around it" (Sophie, Journal, January 20, 2010). Ashley and Julia worked in class to understand the concept of averaging the upper and lower bound of an amorphous figure to determine a more precise approximation of the area (Figure 36).

Figure 34. Counting Squares (Chris & Tara, Lesson 4, January 19, 2010)

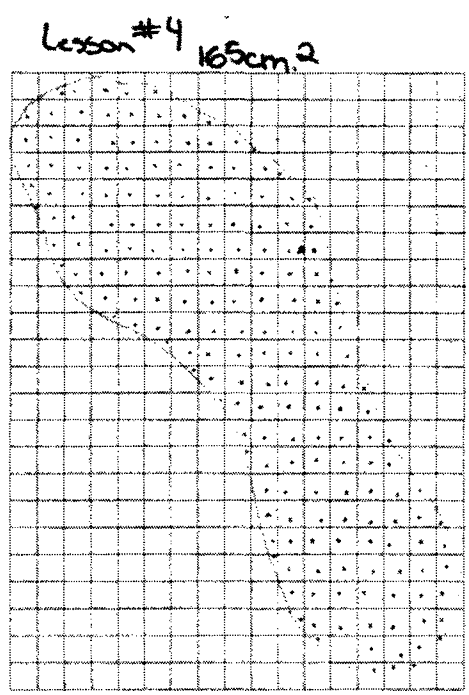


Figure 35. Calculating the Area Around the Foot, then Subtracting Unnecessary Parts (Ella & Meredith, Lesson 4, January 19, 2010)

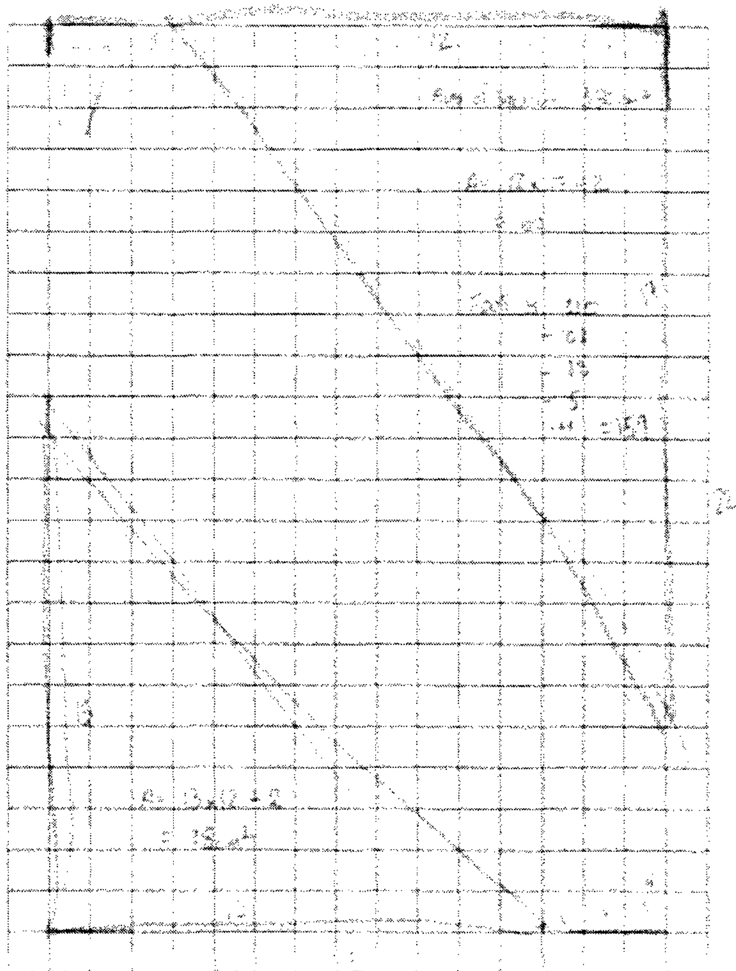
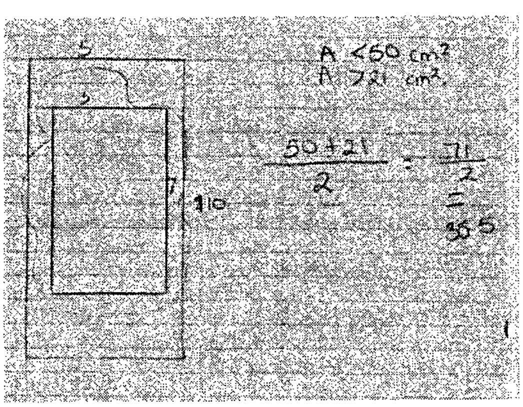


Figure 36. Determining the Area of an Amorphous Figure (Ashley & Julia, In-Class Notes, January 20, 2010).



Video Case Study

Throughout the unit, a video camera was used to capture the lessons. During lessons 4, 6, 7 and 8, the camera was focused on three students who had been chosen based on their results on the pre-test. Students who achieved the median mark of 36% on the pre-test were considered in the selection process. Of the six students who achieved this mark, three students had correctly answered exactly two multiple-choice questions and two short-answer questions. These students were chosen for this portion of the study, and were Amy, Pam, and Alison. Throughout the study, Amy and Pam worked together as partners, and Alison was paired with another student.

Amy, Pam and Alison had all achieved average mathematics marks in the 70's on their first term report card. Alison's was the lowest at 70%, followed by Pam at 76% and Amy with 78%. Initially, none of the three students was very excited about math - a statement that could be used to describe the feelings of the majority of the students in the class (Amy, Pam & Alison, Journal, January 13, 2010). They were all quiet and shy, and, during the first two lessons, did not readily volunteer answers in class, as their confidence level was low. As the unit progressed, the three students became more comfortable contributing to the class discussions. During the math congress for Lesson 8, Pam and Amy were eager to actively participate and successfully led a discussion with their peers about their solution to one of the trapezoid problems. By the end of the unit, the students' journal entries expressed enthusiasm towards mathematics, and conveyed a sense of increased confidence with respect to their understanding of the concept of area. One student wrote: "I get it now! I didn't before, but understanding it makes me feel like I really know what I'm doing" (Bianca, Journal, January 27, 2010).

Evidence was exhibited throughout the unit indicating that these students became more comfortable with the mathematics that was being presented. Initially, conversations between partners were limited. The students appeared to prefer drawing and pointing rather than actually using words to describe to each other what they were thinking. Mid-way through the unit, each student began to converse more with her partner and use mathematical terms related to area. When this increase in discussion was pointed out to them, Pam said "Well, I guess it's 'cause we know the words now. And we're not afraid to be wrong anymore" (Pam, Class Discussion, January 19, 2010).

The coding of the video was a process in which the actions of these three students were monitored. Examples that depicted students solving problems that focused on one or more of the six areas of difficulty, as defined in the research, were noted. Below is a description of recorded events relating to these areas of difficulty.

Difficulty A: Measuring areas of various objects – video record. When initially attempting to calculate the area of an isosceles trapezoid, all three students appeared frustrated. Video data showed Alison drawing the trapezoid, labeling the sides, and then realizing that the top and bottom were different lengths. This led her to the conclusion that you could not multiply the base and height, as there were two different base lengths (Video, Lesson 7, In-Class, January 20, 2010). Pam and Amy also struggled with the fact that the trapezoid had base lengths that were different. They attempted to multiply each length together, but as Pam typed *20 x 12* into her calculator, she appeared confused as the number was much larger than what she had estimated the area to be (Video, Lesson 7, Class Discussion, January 20, 2010).

As the class progressed, Alison began to understand that she could rearrange the isosceles trapezoid by cutting a triangle off the left side, flipping it, and placing it on the right side, so that the top and bottom were the same length. She is shown explaining this to her partner (Video, Lesson 7, In-Class, January 20, 2010). Once she obtained this rectangle, she was able to multiply the base and height to obtain the area of the trapezoid. During the math-congress at the end of the lesson, Pam and Amy also demonstrated that they had determined a method for calculating the area of an isosceles trapezoid. They very eloquently explained that they cut their trapezoid up into two triangles and a rectangle, calculated the area of the rectangle using the formula *base x height,* determined that the base of each triangle was found by subtracting the shorter length of the trapezoid from the longer length and dividing by two ("because there are two triangles"), and then calculating those areas using the formula *(base x height)/2.* To find the area of the trapezoid, they found the sum of the areas of the three figures (Video, Lesson 7, Math Congress, January 22, 2010).

Difficulty B: Static perspective of area – video record. When students demonstrated that a closed perimeter created a figure with area, they were demonstrating an understanding of the static perspective. Pam and Amy were able to show this by counting squares to determine the area of their foot after it had been drawn on graph paper (Video, Lesson 4, In-Class, January 19, 2010).

During the math congress for Lesson 6, Amy was able to discuss ways to calculate areas of various objects. In previous lessons she had developed formulas to calculate the area of a rectangle, triangle and parallelogram. On the video, she reminded the class of the formulas for the area of a rectangle and triangle, and the reason they work

(Video, Lesson 6, Math-Congress, January 20, 2010). Pam was able to calculate the area of a parallelogram using the formula *base x height* and Alison drew the explanation on graph paper (Video, Lesson 6, Math-Congress, January 20, 2010).

Difficulty C: Dynamic perspective of area – video record. After the students calculated the area of their foot, they were asked to determine the perimeter of the foot with a piece of string, rearrange the string into a square on graph paper, and then calculate its area. Pam and Amy came to the conclusion that the area of the square, although it had the same perimeter as the foot, was larger (Video, Lesson 4, In-Class, January 19, 2010). During the math congress, Alison concluded that: "If two objects have the same perimeter, they don't necessarily have the same area" (Video, Lesson 4, Math-Congress, January 19, 2010).

Difficulty D: Spatial structuring and covering – video record. Any instance where a student demonstrated that a figure's area, when rearranged, did not change, or that area was a measure of covering, was demonstrating an understanding of spatial structuring. In each of the lessons that were specifically video-taped, all three girls appeared confident with this concept. In the video that was analyzed, there were 20 instances where the three students demonstrated an understanding of this. Specifically, Pam and Amy demonstrated this understanding when they dissected their trapezoid into three pieces, calculated the separate area of each of the pieces, and then combined the areas to produce a total area (Video, Lesson 7, Math-Congress, January 22, 2010). Alison showed that she understood the concept of covering when determining the area of the foot. She knew that each cm x cm square that covered the foot cut-out was part of the total area of the foot (Video, Lesson 4, In-Class, February 19, 2010).

Difficulty E: Estimation – video record. When Pam and Amy were calculating the area of the foot that was drawn on graph paper, they counted each whole square first. After they had counted each whole square, they dealt with the partial squares by finding two squares that appeared to create a whole square when put together. On the video, Pam can be seen keeping her finger on one partial square while searching for another partial square with which to pair the first one (Video, Lesson 4, In-Class, January 19, 2009).

Difficulty F: Area of an amorphous figure – video record. Successfully calculating the area of the foot in Lesson 4 meant that students were able to calculate the area of an amorphous figure. All three students achieved this. Each of them counted whole, and mostly-whole squares, then proceeded to find partial squares that could be added together to create a whole square. They added these pieced-together whole squares to the number of whole squares originally counted, to obtain an approximate area of the foot (Video, Lesson 4, In-Class, January 19, 2009).

Students' Attitudes

Throughout the unit, students' attitudes were observed. Journals were kept both by the students and me. I wrote about my in-class observations of student attitudes, and students were asked to write reflections at the end of each lesson that included comments about their attitudes towards the math that was being taught, and the method of teaching that was used.

Attitudinal baseline. In the initial class discussion, students stated that their previous mathematics teachers had taught area by presenting them with formulas, asking them to memorize the formula, and then apply it to various practice questions (Video, Lesson 2, January 13, 2010). Elizabeth expressed having very little hands-on experience in previous mathematics classes and recalled that a lot of homework, involving many repetitive practice problems, was assigned (Video, Lesson 2, January 13, 2010). One student commented that math was a "lonely" subject because in previous years he had never been allowed to work with a partner on problems (Mason, Video, Lesson 2, January 13, 2010). They all expressed excitement when they were told this traditional method was not how they would be taught (Video, Lesson 2, January 13, 2010).

Solving area questions. Initially, students were reticent to solve problems involving area. Informal discussions with students allowed me to conclude that their prior knowledge of the topic was limited. One student mentioned that he did not like area problems because "there are too many formulas involved and I'm not good at memorizing" (Matt, Class Discussion, January 13, 2010). However, when the lessons began and students realized that memorizing formulas would not be the focus of the lesson, their attitudes changed.

After the first day on which the test was written, I observed students beginning the task as soon as it was presented. Students remained on task throughout the lesson and were shocked when the period came to an end. One student was overheard saying: "it's over already?" (Mason, Video, Lesson 3, January 13, 2010). After the first lesson, Adrian and Mark approached me and asked if we could spend the entire afternoon the next day working on "this area stuff" because they liked it so much.

In addition to in class discussions, and my informal observations, students described their attitudes towards solving area problems in their journals. One student wrote: "I think that the 'foot' activity was fun and educational even though we were still learning about area! I think that if math was like this all the time I would get better grades" (Bianca, Journal, January 18, 2010). This positive feeling about studying area was echoed by another student who commented that: "Every lesson we got taught was fun and it helped me learn more about area" (Julia, Journal, January 27, 2010). As the unit progressed, students became more excited to come to math class to solve area problems.

In an attempt to overtly determine students' feelings about solving area problems, two questions on the pre-test and post-test were posed that provided students with two word problems, and asked them to indicate which problem they would prefer to solve. One of the problems was algebraic, and the other related to area. On the pre-test, more than 60% of the students indicated a predisposition to solving algebraic problems. The post-test results allowed me to conclude that 60% of students had a desire to solve area related problems, thus indicating that, throughout the course of the unit, students' attitudes had changed for the better with respect to solving area questions.

Method of teaching. A significant change for the students was the method through which the unit was taught. When discussing their previous mathematics classes, students mentioned textbook use, individual work, teacher directed lessons, and pages of practice problems that focused on the correct use of formulas. The reform teaching method focused on partner work, student-directed learning through math congresses, discourse, accountability, hands-on activities, and problem solving. The students enjoyed most aspects of the reform teaching method.

Pairing the students homogeneously caused some concerns at the beginning. One student noted in his journal that: "at first I was appalled that you wanted ME to work with HER, but as the saying goes if you just stuff two nerds in a corner they'll make it work"

(Andrew, Journal, January 13, 2010). After the initial complaints about partners, students worked well together. The ability groupings appeared to provide an opportunity for each partner to contribute equally to the pair. A student wrote: "I like [working in partners] because two heads are better than one" (Mark, Journal, January 18, 2010). Another student indicated that: "working with a partner helped to build my brain power" (Ashley, Journal, January 18, 2010).

Students' journal entries expressed feelings of excitement, enthusiasm, and curiosity towards the student-directed learning and math congresses held in class. One student's comment after the first few lessons was: "Math is really fun! I am really learning a lot through a thoroughly thought-out and creative way" (Andrew, Journal, January 27, 2010). Others commented on the benefits of the reform teaching method: "I like presenting theories in front of the class, and I like that she [Ms. Garrett] teaches while giving us freedom to be creative and do it in our own way. I can't wait for math class these days!" (Gina, Journal, January 27, 2010).

By providing the students with the opportunity to be creative and solve problems in their own ways, I was making them accountable for their learning. Students rose to this challenge and demonstrated a concerted effort to develop their own ideas and strategies to determine area. Their pride in their accomplishments was obvious during the math congress. Students were eager to present and discuss their ideas with the class. One student wrote in his journal: "I'm really glad me and my partner got to talk about how we did the problem to the class. They liked our idea and that made me feel good and smart" (Isaac, Journal, January 19, 2010). However, it was still necessary for me support their learning through daily interactions. For example, when the class began the study of trapezoids, not one student was able to explain what a trapezoid was (Video, Lesson 7, January 22, 2010). At this point, I presented a mini-lesson on trapezoids, so that students would have the general idea that a trapezoid is a four-sided figure with at least one pair of parallel sides, and then I allowed students to further investigate properties of various types of trapezoids.

Student-student and student-teacher discourse was extremely important within the classroom. The majority of the pairs remained on topic at all times. Mathematical language, although not prevalent at the beginning of the unit, became commonplace by the end. Students were using the terms: quadrilateral, rectangle, isosceles, acute, obtuse, parallelogram, and perpendicular, among others, to describe objects with which they were working (Video, January 25, 2010). I modeled appropriate mathematical terminology to assist students in improving their mathematical discourse.

Although the majority of students enjoyed the hands-on aspect, a few students felt that the reform method of teaching did not appeal to their specific learning style as described by Howard Gardner. One pupil wrote: "This wasn't my favourite way to teach because I am an auditory learner and doing hands on stuff isn't my thing" (Karen, Journal, January 27, 2010). Another student wrote: "I think that the math with Ms. Garrett is very hands-on and is designed for kinesthetic learners. I am a kinesthetic learner, and enjoy most of the activities, but I'm not sure whether other types of learners would still like it" (Tara, Journal, January 27, 2010).

As we progressed throughout the unit, students began to see a pattern in the teaching method: a problem was given, and they were asked to solve it. This problem solving approach was a significant change from the text-book based approach students had previously experienced. One student wrote: "I will really miss you teaching us math because I loved how we didn't need to use a text book. I also really enjoyed how you taught the lessons" (Katherine, Journal, January 27, 2010). Students enjoyed the opportunity to solve the problem with their partners because they could work at their own pace and devise a method that they understood, as opposed to being told to use a prescribed solution method. In class, a student was overheard saying to his partner: "I like how she doesn't make us do it her way; I get it way better when I can figure it out myself" (Kirk, Class Discussion, January 15, 2010).

Chapter Five: Discussion and Conclusions

Summary of the Major Findings

This study was conducted to determine if the reform method of teaching helps Grade 7 students move beyond a procedural understanding, to a conceptual understanding, of the topic of area. I found generally that there was a positive response to the reform method of teaching, that this teaching method led to the students' adopting a wider range of strategies to solve problems, that students were able to communicate and justify their ideas to others, and that improvement in the specific problem areas, with respect to the understanding of area concepts as outlined in the literature, was observed.

Positive response to reform method of teaching. Students had previously been taught using a traditional method of instruction. When discussing this method of instruction, students indicated that it did not motivate them to study mathematics. According to the NCTM document *Principles and Standards for School Mathematics* (2000): "Middle-grades students should see mathematics as an exciting, useful, and creative field of study" (p. 210). The reform teaching method elicited a positive response from all students involved in the study. This was evidenced in the video record that shows students who are on task, talking excitedly about the topic at hand, and working together to solve the problems. Math congresses often led to discussions of the real-world application of the problems students were solving. It was evident that most students understood the applicability of the problems, and that they could see a use for the knowledge outside of the classroom, further enhancing their interest in the topic.

Teaching through problem solving, a fundamental aspect of reform instruction as mandated by the Ministry of Education (2005, p. 11), allowed students to explore a variety of problems in a way that promoted conceptual understanding. Cobb and Merkel (1989), in their study of teaching arithmetic through problem solving to second grade students, found that while students were solving problems "they developed an interest in, and curiosity about, mathematics" (p. 72). Students in this study exhibited the same characteristics.

Used a wider range of strategies to solve problems. One of the purported benefits of the reform method of teaching is that students will gain a conceptual understanding of the topic being studied. One way students demonstrated this understanding was in the wide range of solution methods they presented when solving problems. At the very beginning of the unit, students attempted to determine the area of a figure using formulae they had been told to memorize in their previous, traditionally taught, math classes. When they were encouraged to use other methods to solve the problem, most students began by counting unit squares enclosed within the figure to determine the area. However, by the end of the unit, not only were students using a variety of strategies to solve problems, they were picking the most appropriate strategies for the problems that were presented to them. They had become flexible problem solvers demonstrating the ability to construct knowledge from previous experience. This progression, that was evident throughout the unit, is consistent with findings by Cobb and Merkel (1989). They noticed that less conceptually advanced pupils initially used unsophisticated methods to solve problems, but, progressed to more sophisticated strategies near the end of their study.

Communicated and justified ideas to others. Students' understanding was demonstrated by their ability to explain and justify their solutions to others. David

Hilbert (1900), in his lecture on Mathematical Problems delivered before the International Congress of Mathematicians in Paris, quoted a French mathematician who said: "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street" (p. 1). In general, mathematicians who are trying to gain a deep understanding of a problem work through the problem in a variety of ways. Hilbert also espoused his idea that mathematics has a requirement of rigour that "corresponds to a universal philosophical necessity of our understanding" (p. 3). He expressed that mathematicians who are able to prove theories in a number of ways achieve this requirement of rigour. In daily discussions between partners, and especially throughout the math congresses, students demonstrated the ability to explain solutions in a variety of ways. Furthermore, students themselves began to verify the conjectures of other groups, rather than looking to the teacher to validate the solution. This ability to communicate, justify and verify answers is strong evidence of the effectiveness of the reform method of teaching.

Experienced success in areas that are considered cause for concern. The reform mathematics lessons, including the pre-test and post-test segments, focused on the areas of difficulty that had been identified in the literature as causing problems for students. As the unit evolved, students demonstrated success in these areas by progressing from a procedural understanding of the topic to a conceptual understanding. The fact that students attempted more questions and answered more questions correctly on the post-test than on the pre-test was a clear indication that they had a deeper conceptual understanding of the topic.

The reform teaching method was initially frustrating to students because they were asked to be responsible for developing their own approaches to solving the proposed problems. Their previous experience was that they were given a formula and told how to substitute numbers in to get the answer. They realized that this traditional method of teaching did not give them a deep understanding of the subject, or the skills necessary to solve multi-step, atypical problems. As the unit progressed, students' confidence increased because they succeeded at solving the problems in the various lessons, using a variety of methods. Eventually, students were able to reflect on their learning, and thus they became active learners within the classroom as described by Sherin, Mendez and Louis (2004). They proved that they were able to calculate the area of a variety of figures. They demonstrated an understanding of both the static and dynamic perspectives of area, as well as the concepts of spatial structuring and covering. They successfully determined the area of amorphous figures, and applied the concept of estimation in a variety of situations. Additionally, some students were able to demonstrate that they understood the concept of conservation of area, although this problem area was not a particular focus in this research.

While achieving these successes, students showed increasing levels of confidence in their mathematical discourse. Franke, Kazemi and Battey (2007) found that higher levels of discourse result in greater student learning, which was supported by this study.

Throughout the unit, most students improved with respect to the pre-defined areas of difficulty. Nevertheless, not every student achieved the level of understanding that is demanded by the expectations listed in the Grade 7 Ontario Curriculum for the measurement strand. This lack of achievement could be attributed to the broad variations in the backgrounds of the students with respect to the topic of measurement. The results of the pre-test were indicative of the fact that prior knowledge of the topic of area was not consistent within the group and that most students' competence was at a low level. Students experienced difficulty on the question that involved calculating the area of a triangle because of the lack of understanding of the language used in the problem.

An expectation in the curriculum documents at the Grade 7 level is for students to be able to determine the area of a trapezoid. Not all students achieved this expectation. To determine the area of a trapezoid using a decomposition strategy, students must have a firm grasp of spatial structuring, which, according to Outhred and Mitchelmore (2000), is one of the areas of understanding with which students have difficulty. Spatial structuring is not intuitive. It must be taught, and practiced. This unit was an initial opportunity for students to investigate spatial structuring through the reform teaching method. Time constraints prevented the necessary practice I believe these students would require to further their understanding of spatial structuring. In particular, students required more practice manipulating objects to create more familiar objects, as was explored using the Geoboard in Lesson 3. This technique would have led to a simple way to calculate the area of a trapezoid, as one half the area of the parallelogram created by flipping the given the trapezoid and attaching it to the original. Not every student achieved the level of understanding I had hoped they would achieve. Nevertheless, all students made progress toward a complete understanding of the topic of area. All students showed considerable improvement with respect to the areas of concern as defined in the literature.

Conclusion

Using the reform method of teaching is an effective way to teach the concept of area to Grade 7 students. Reform teaching is not the usual method of instruction in Canadian schools, and Robitaille, Taylor and Orpwood (1996) found that Canadian students did not perform well on measurement questions included in the Third International Mathematics and Science Study. The reform method of instruction, used in my study, proved to be successful in teaching this topic. Results of this research project suggest that students who are taught using this method develop an improved conceptual understanding of the topic, and experience success in the areas identified in the literature that are typically a cause for concern when studying this topic. In this study, students were able to develop their own methods to solve the problems, as well as justify the use of familiar formulas to calculate area. They were able to apply the understanding they had developed to a variety of situations and problem types.

Van de Walle's (1999) discussion relating to the theory of constructivism was supported in this study. Students were able to build on previous knowledge, and construct new ideas throughout the unit. Supporting this theory of constructivism is Fosnot and Perry's (2005) research that advocates for students to raise their own questions, develop hypotheses and models, test for viability, and defend their findings – all of which are aspects of the reform teaching method that was put into practice in this study.

Considerations for Future Research

The concept of area is one small part of the mathematics curriculum. It would be interesting to use the reform method of teaching to see how it benefits students studying other strands of the Ontario Mathematics Curriculum.

This study was conducted in one classroom. The control in this case was the literature supporting the claim that students have difficulty with the concept of area, and that they develop a deeper conceptual understanding of topics if taught using the reform method of teaching. Comparative studies could provide additional information about the benefits of the reform teaching method. Contrasting two classes, one taught traditionally, and the other taught using the reform method, could allow for a more in depth discussion regarding the traditional teaching method. A second type of comparative study could be done in one classroom where reform teaching could be used to teach one topic in the measurement strand and the traditional method could be used to teach another topic in the same strand.

An extension of this research would be to see how much of the information the students retained when they enter Grade 8 and begin a further study of the concept of area. This type of study would work if students who took part in the study this year were able to move to Grade 8 as a group. If a pre-test, intervention, post-test model were used again, it would be expected that students correctly answer questions on the pre-test relating to material studied in Grade 7. If this were the case, the conclusion that the reform teaching method provides for increased retention of understanding, might be drawn.

A cross-curricular study of reform teaching methods could be done to ascertain the benefits of this teaching method in other areas of the curriculum. This would benefit all educational stakeholders as results would guide teachers to program more effectively, thus resulting in students gaining a deeper conceptual understanding of all concepts.

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Appendix A – Parent Letter

January 4, 2010

Dear Parent/Guardian of Potential Participants:

My name is Sarah Garrett, and I am a teacher at Robertson Public School. I have a Bachelor of Science degree in Mathematical Science, and a Bachelor of Education degree with a specialization in Mathematics. In addition to teaching full time, I am currently pursuing my Master of Education degree from Lakehead University. One of my goals for working on my Master's Degree is to investigate a concept in mathematics that is difficult for children to understand, and to find ways to improve the teaching of this topic. The topic I have chosen to investigate is the area of plane figures, such as rectangles, squares, triangles, parallelograms, and trapezoids. The title of my study is: "Reform Mathematics teaching and how it helps students understand the concert of area."

The concept of area is used on a daily basis throughout an individual's life. Most likely, you will use it in your job, at home when doing renovations, or even when cooking. Educational research has shown that to work competently in the study of area, children need to have a strong conceptual understanding of the topic. Traditionally, students are provided with formulas, they are shown how to use the formula, and asked to complete practice problems. We find, for many children, that this rote memorization of formulas does not contribute to the development of a deep understanding. Instead we have learned through research that students need to have hands on experience, gain a strong understanding of arrays (grids), and develop formulas through their own experimentation. I would like to determine if these methods, often known as *reform instruction*, assist students in gaining a more thorough understanding of area.

In order to determine the effectiveness of these reform strategies I will be working with Mlle Cartier and teaching a unit on area to your child's class. The unit will be taught over a period of two weeks in January. The students will take a pre-test to assess their initial understanding. Lessons will then be taught following the prescribed Ontario Curriculum, using the methods of reform mathematics, and a post-test will be administered to determine how much the students have gleaned from the teaching method. During classroom instruction, students will be video-taped in order to enable me to listen to how they are solving the problems. The answers will be transcribed and possibly quoted anonymously in my final project to exemplify understanding or lack of understanding. The video tapes will be edited and viewed to further understand and document a student's thinking.

Your child will not be identified in the final thesis or any resulting publications. The edited tapes and tests will be shared only with Mlle Cartier and my Supervisor. During the study, the data that is collected will be stored in a locked cabinet in my home, and electronic data will be stored on a secure hard-drive. Following the completion of the project, data that is collected will be stored at Lakehead University for five years, and your child's results will be kept confidential and then destroyed after five years. Upon the completion of my research, you may contact me at Robertson Public School should you wish to obtain a summary of the research. Participation in this study is voluntary and you may withdraw the use of your child's information at any time.

The research project has been approved by the Lakehead Senate Research Ethics Board, the School Board, and Mr. Smith, Principal of Robertson Public School.

Please note that this research does not affect the classroom instruction time that would be usually devoted to this topic. This research will not take away from the normal learning environment in the classroom, and there is no risk to your child. The research is simply being conducted to study alternative approaches to teaching the concept of area. If you choose not to have your child participate, he or she will still be engaged in the math lessons, the only difference is that his or her data will not be used.

Should you have any questions or concerns regarding this research project, please do not hesitate to contact me at the school at (phone number inserted).

Thank you for considering your child's participation in the research.

Ms. S. Garrett
Master's Student, Lakehead University
Teacher, Robertson PS
(Phone number inserted)

Mr. A. Smith
Principal, Robertson PS
(Phone number inserted)

Dr. A. Lawson PhD
Thesis Supervisor
Lakehead University
807-343-8720

Lakehead University Research
Ethics Board
807-343-8283

Appendix B – Potential Participant Letter

January 4, 2010

Dear Potential Participant:

I will be working with your class during the month of January 2010 to teach the concept of the area of plane figures to you. A plane figure includes such shapes as: rectangles, squares, triangles, parallelograms, and trapezoids. During the class, I will also be conducting research. The reason for this is that I am currently attending university to pursue what is called a Master of Education degree. One of my goals for working on my Master's Degree is to investigate a concept in mathematics that is difficult for children to understand, and to find ways to improve the teaching of this topic. The title of my study is: "Reform Mathematics teaching and how it helps students understand the concept of area."

People who have done a lot of research on the subject have found out that in order for students to really understand area, they have to do more than simply memorize a formula such as *A = length x width*. My goal is to teach this concept so that you develop a deeper understanding.

The way the lessons will be approached may be slightly different from what you are used to. We will do hands on work, you will work in partners, and then, as a whole class, we will discuss what you have figured out. YOU will be the mathematicians. Other differences are that there will be a video camera in the classroom, and a microphone on your work table. These tools will help me in my research by recording what you say and do while working through the problems. My thesis committee from Lakehead University and I will be the only ones watching the videos. Also, I will not use any real names in my project, so your identity will be protected.

We will start the unit with a pre-test so that I can determine what your current understanding of area is. I will then teach a series of lessons, and end the unit with a post-test to see what you have learned from the lessons. We will still be covering all of the material that you would cover if the subject were being taught traditionally.

I would encourage you to ask me any questions you may have about the study. Thank you for considering participating in my research project.

Ms. S. Garrett
Master's Student, Lakehead University

Appendix C – Lakehead University Research Consent

Lakehead University
Research Consent

January 4, 2010

My signature on this form indicates that my son or daughter, ____________________ has my permission to participate in a study by Ms. Garrett on *Reform mathematics teaching and how it helps students understand the concept of area.* I have received an explanation about the nature of the study and its purpose, and have read and understood this explanation.

I understand the following:

1. There is no apparent danger of physical or psychological harm.
2. My child is a volunteer and can withdraw from the study at any time.
3. All information collected will be strictly confidential and the students will not be identified individually in the ensuing thesis or publication.
4. I will receive a summary of the project, upon request, following the completion of the project.
5. In accordance with Lakehead University policy, all information collected during the project will be securely stored at Lakehead University for five years.
6. All participants will remain anonymous in any publication or public presentation of research findings.

______ Please initial here if you give permission for your child to be recorded on video during the research project.

Please keep the introductory letter on file should you have any further questions.

Please complete this page and have your son or daughter return the entire form to me.

Parent Name: ____________________ Parent Signature: ____________________

Date: ______________

Appendix D –School Board Research Consent

School Board
Research Consent

January 4, 2010

1. The Research Liaison Committee of the School Board has given permission for this study to be carried out at Robertson PS.
2. All information collected will be strictly confidential and the students will not be identified individually.
3. Your son or daughter's participation is completely voluntary.
4. The information is collected under the authority of Board Policy #204 and the Municipal Freedom of Information and Protection of Privacy Act. Users of this information will be the members of the Board's Research Liaison Committee. (The contact person for inquiries concerning this information is the Superintendent responsible for this policy.)

Please complete the following permission section of this letter and have your son or daughter return the entire page to me.

I hereby give permission for my son/daughter to participate in the research project being conducted in the School Board.

Name of Child: ___________________________
Age of child at time of research: Years: ______ Months: ____

Signature of Parent/Guardian: ___________________ Date: ______________

Appendix E – Pre-Test

Number: ______

AREA - Gr. 7 Pre-Test

1. Area is a topic in geometry. What does the word "area" mean to you?

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For questions 2-10 please circle your desired answer.

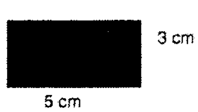
2. Which problem would you prefer to solve?
   a. Determine the total cost of 20 apples, if each apple costs 50 cents.
   b. Determine the area of a rectangle that is 30 cm long by 40 cm wide.

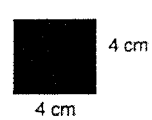
3. Which problem would you prefer to solve?
   a. What is the height of a triangular lot that has a base of 20 m and an area of 342 m^2.
   b. How many 40 passenger busses are required to transport 263 students?

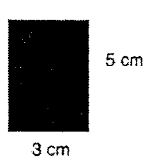
4. Which of the following best describes your understanding of area?
   a. Length times width
   b. Measure of covering
   c. Size of a figure

5. What is the area of a rectangle with length 4 cm, diagonal 5 cm, and height 3 cm?
   a. 12 cm^2
   b. 15 cm^2
   c. 20 cm^2
   d. I don't know

6. Which has the biggest area if the perimeter of each figure is 16 cm?

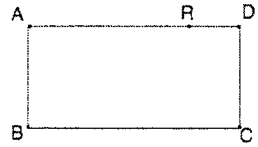
a.


b.


c.


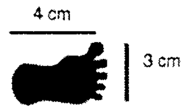
d. I don't know

7. Given that the area of a rectangle ABCD is 12 cm^2, what is the area of the triangle formed by connecting points R, B and C?



a. 4 cm^2 b. 6 cm^2 c. not enough information d. I don't know

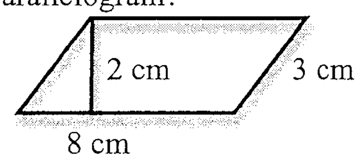
8. What is the best estimate of the area of this foot?



a. greater than 12 cm^2 b. less than 12 cm^2 c. 12 cm^2 d. I don't know

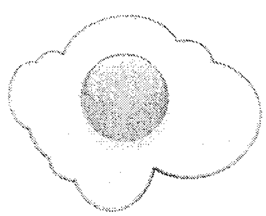
9. John traced his hand with his fingers together, and then did a separate tracing with his fingers apart. Which of the following is true?
   a. The tracing with the fingers apart has a larger area
   b. The tracing with the fingers together has a larger area
   c. The tracings have the same area
   d. I don't know

10. What is the area of this parallelogram?



a. 6 cm^2 b. 16 cm^2 c. 24 cm^2 d. I don't know

11. How would you determine the area of the figure pictured below?



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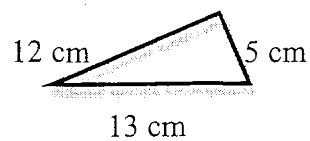
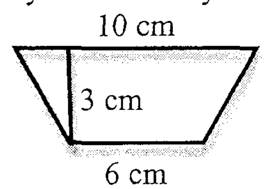
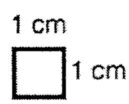
12. Determine the area of a right triangle with legs 5 cm and 12 cm, and hypotenuse 13 cm. Explain how you obtained your answer.



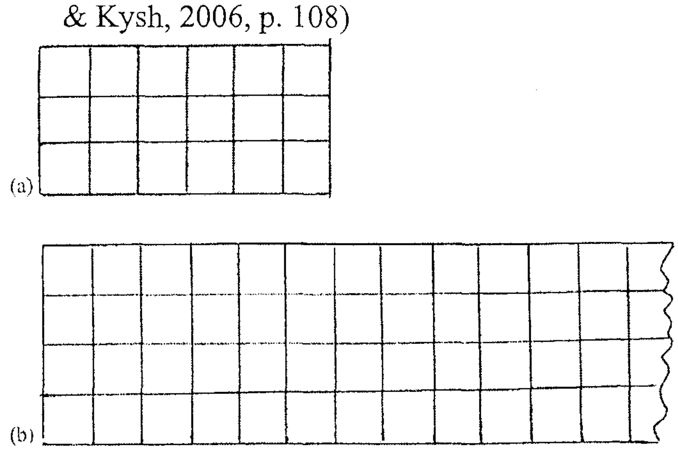
13. Determine the area of the following trapezoid with base 6 cm, top 10 cm, and height 3 cm. Explain how you obtained your answer.



14. Use squares of the size shown below to create two different pictures with a perimeter of 10 cm. Show drawings of your creations.



15. Draw a straight line on figure (b) to show where you would make a straight cut to create a piece that has exactly the same area as in figure (a). (Adapted from Kamii & Kysh, 2006, p. 108)



Appendix F – Post-Test

Number: ______

AREA - Gr. 7 Post-Test

1. Area is a topic in geometry. What does the word "area" mean to you?

____

____

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____

For questions 2-9 please circle your desired answer.

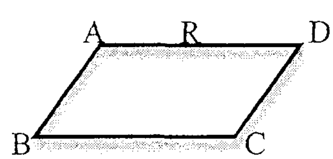
2. Which problem would you prefer to solve?
   a. Determine the total cost of 5 theatre tickets priced at $3.50 each.
   b. Determine the area of a parallelogram with base 8 cm and height 2.5 cm.

3. Which problem would you prefer to solve?
   a. What is the length of a rectangle that has a width of 30 cm and an area of 630 cm^2?
   b. How many shirts costing $20 each can be purchased with $360?

4. What is the area of a rectangle with length 12 cm, diagonal 13 cm, and height 5 cm?
   a. 60 cm^2 b. 65 cm^2 c. 156 cm^2 d. I don't know

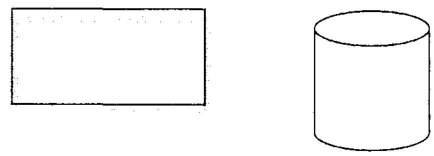
5. Which of the following statements is not true?
   a. Rectangles with the same perimeter always have the same area.
   b. If a square has the same perimeter as a rectangle with unequal sides the square always has a bigger area.
   c. A parallelogram and a rectangle, each with the same base and height, always have the same area.
   d. A triangle with the same base and height as a rectangle always has half the area of the rectangle.

6. Given that the area of a parallelogram ABCD is 24 cm^2, what is the area of the triangle formed by connecting points R, B and C?



   a. 8 cm^2 b. 12 cm^2 c. not enough information d. I don't know

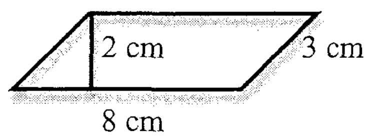
7. The rectangular piece of tin pictured below has an area of 12 cm^2 and is rolled up without overlap to form the pipe shown below. The best estimate of the outside surface of the pipe is:



a. greater than 12 cm^2 b. less than 12 cm^2 c. 12 cm^2 d. I don't know

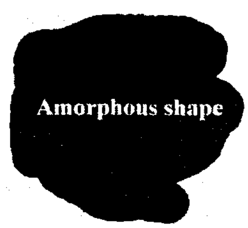
8. John traced his hand with his fingers together, and then did a separate tracing with his fingers apart. Which of the following is true?
   a. The tracing with the fingers apart has a larger area
   b. The tracing with the fingers together has a larger area
   c. The tracings have the same area
   d. I don't know

9. What is the area of this parallelogram?

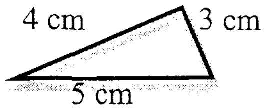


a. 6 cm^2 b. 16 cm^2 c. 24 cm^2 d. I don't know

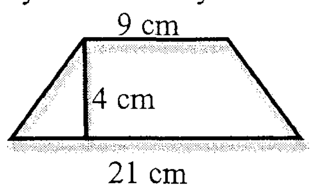
10. How would you determine the area of the amorphous shape pictured below?



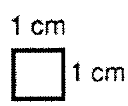
11. Determine the area of a right triangle with legs 3 cm and 4 cm and hypotenuse 5 cm. Explain how you obtained your answer.



12. Determine the area of the following trapezoid with base 21 cm, top 9 cm, and height 4 cm. Explain how you obtained your answer.



13. Use squares of the size shown below to create two different pictures with a perimeter of 12 cm. Show drawings of your creations.



14. Draw a straight line on the figure (b) to show where you would make a straight cut to have exactly the same amount of space as in figure (a). (Adapted from Kamii & Kysh, 2006, p. 108)

Figure (a)

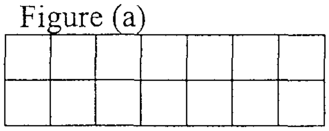
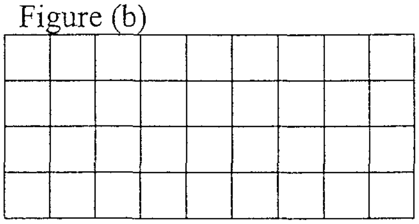


Figure (b)



Appendix G – Problems Posed to Begin Lessons

Lesson 2

1. Without calculating the area of the three given rectangles, determine which has the biggest area and which has the smallest area. *(Provide students with one of the following sets of rectangles: 4 x 4, 3 x 5, 2 x 9 OR 3 x 6, 2 x 10, 5 x 5 OR 8 x 3, 5 x 5, 6 x 4. Rectangles should be drawn to scale, but no side measurements should be provided).*
2. These two rectangles have the same area. How could you prove this without calculating the area? *(Provide students with a pair of rectangles in the following dimensions: 2 x 6, 3 x 4 OR 4 x 6, 3 x 8 OR 4 x 4, 2 x 8)*
3. Given these rectangles, determine their area in square centimeters. Refrain from using any known formulas. *(Provide each group with rectangles that are 8 cm x 4 cm and 3 ½ cm x 6 cm).*

Lesson 3

Using the geoboard, find as many polygons as possible that have an area of twelve square units. Record each new polygon on geoboard dot paper. Be ready to justify why each of your polygons is twelve square units.

Lesson 4

Draw three different shapes on centimeter squared paper following these three rules:

a. Each shape must have a perimeter of 30 cm
b. Stay on the lines when you draw (no diagonals)
c. You must be able to cut your shape out and have it all in one piece.
d. What do you notice about the area of the shapes?

Lesson 5

Provide students with a 10 cm x 10 cm square of paper.

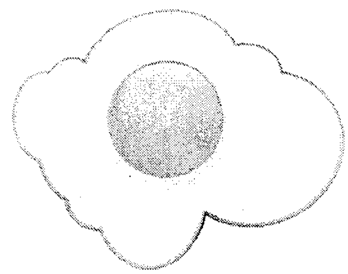
1. Cut your paper in half. Determine the area of each rectangle, and answer the following questions on chart paper:
   a. What relationship, if any, do the rectangles have with each other?
   b. What relationship, if any, do the rectangles have with the original square?
   a. Do you notice any other relationships between the rectangles and the original square.

2. Cut one rectangle along the diagonal. Determine the area of each of the resulting triangles, and answer the following questions on chart paper:
   c. What relationship, if any, do the triangles have with each other?
   d. What type of triangles are they? How do you know?
   e. What relationship do the triangles have with the original rectangle?

3. Take the two triangles and rearrange them into a parallelogram. Determine the area of the parallelogram, and answer the following question on chart paper:
   f. What do you notice about the area of the parallelogram and the area of the rectangle? The area of the triangles and the area of the parallelogram?

Lesson 6

1. You're making fried eggs for breakfast. Your rectangular frying pan has an area of 500 cm^2 and measures 20 cm wide by 25 cm long. Each egg, when cracked, forms a shape like the one below. Determine the largest number of eggs you can cook in this frying pan, without there being any overlap.



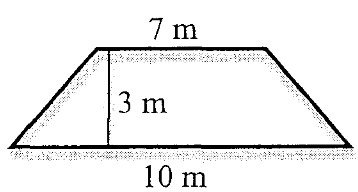
2. There is a circle with a diameter of 7 m painted on a gym floor. You have mats that are 2 m x 2 m.
    a. Estimate the number of mats that can be placed so that they fit entirely within the circle. M. Georges will not let you cut the mats!
    b. Estimate the number of mats that would be necessary to cover the circle completely. Again, you cannot cut the mats.
    c. What is the approximate area of the circle?

Lesson 7

Provide students with a cut-out of a trapezoid. Use a variety of trapezoids.

1. How would you describe your trapezoid? Think of size, shape, angles, as well as relationships to other figures we have looked at during this unit. Record the properties on your chart paper.

2. You are helping your Dad paint a section of a wall in your attic. It has the following shape: base 10 m, top 7 m, height 3 m. If paint comes in cans that will cover 38 m^2, how many cans of paint will you need to paint the wall? What is the area of a wall you could cover with the leftover paint?



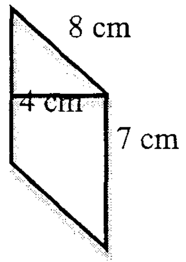
Lesson 8

1. You have a trapezoid with unknown dimensions. Develop a formula to calculate the area of this trapezoid.

Lesson 9

1. A square has an area of 36 square centimetres. What is the area of a square whose sides are twice as long?

2. What is the area of the parallelogram pictured below? (Explain your thinking!)



3. A trapezoid ABCD, with AD parallel to BC, has a height of 4 m. The length of DC is equal to 5 metres, the length of BC is equal to 6 m and the area of the trapezoid is equal to 40 m^2. Calculate the length of AD. Show your work.