Adaptive Almost Disturbance Decoupling for a Class of Nonlinear Differential-Algebraic Equation Systems

by

Hamid Abareghi

Thesis for MSc in Control Engineering

Department of Electrical and Computer Engineering
Lakehead University

Thunder Bay, Ontario, Canada
December, 2015

Lakehead

Abstract

A class of engineering systems is modeled by Differential Algebraic Equations (DAEs), which are also known as singular, descriptor, semistate and generalized systems. In the chemical engineering processes, the differential equations are constituted by the dynamic balances of mass and energy, while the thermodynamic equilibrium relations, empirical correlations, and pseudo-steady-state conditions build the algebraic equations. The robotic systems with kinematic constraints are also modeled by DAE systems.

Physical and complex plants are exposed to extraneous noises and signals such as sensor measurement noise, structural vibration and environmental disturbances. For example, the external disturbance of a wind gust on an aircraft affects its control system. The challenge of almost disturbance decoupling is to design a controller to attenuate the effect of disturbances on the output to an arbitrary degree of accuracy in the L_2 gain sense.

It is worth noting that some parameters of the real plants are naturally unknown due to the difficulty of measurement. For example, the damping, stiffness and friction coefficients in the dynamic equations of a constrained robotic system are difficult to measure.

In this work, the problem of adaptive almost disturbance decoupling for a class of nonlinear DAE systems is investigated. The DAE system is converted to equivalent lower triangular structure by regularization and standardization algorithms and an adaptive almost disturbance decoupling controller is constructed based on adaptive backstepping technique. At the end, an application of the design procedure to a physical model is shown and the results are discussed.

Acknowledgements

I would like to express my deep thanks to Professor Xiaoping Liu who accepted me as his student and suggested the topic and for his guidance during thesis writing. I gratefully thank Professor A. Tayebi for his professional teaching skills and advice on my presentation on Control Engineering Concept and Mechatronics. I also would like to thank Professor M. Uddin and Professor Natarajan for reviewing this work. Finally, I wish to thank my wife and my daughter. Without their support, patience and love, this work would not have been completed.

Table of Contents

List of Figures

List of Tables

List of Symbols

Symbol	Meaning
H_∞	H operator infinity-norm
γ	$L_2\ Gain$
g	Gravity constant m/s^2

List of Abbreviations

Abbreviation	Meaning
ODE	Ordinary Differential Equation
DAE	Differential Algebraic Equation
SISO	Single-Input Single-Output
MIMO	Multi-Input, Multi-Output
2DOF	Two Degree of Freedom

Chapter 1
Introduction

The main focus of this chapter is to review the background of adaptive almost disturbance decoupling for nonlinear differential algebraic equation systems. A brief introduction to singular systems that emphasises the nonlinear differential algebraic equations is given in Section 1.1. The recent research on the problem is reviewed in Section 1.2, focusing on disturbance decoupling problem and adaptive backstepping technique. Section 1.3 provides the motivation for this work, the contribution of this work is explained in Section 1.4, and the outline of the thesis is given in Section 1.5.

1.1 Singular Systems

The implicit differential equations of the form (1-1) describe a wide range of applications [32], [31].

$$f(x(t), \dot{x}(t), t) = 0 \tag{1-1}$$

with $f(0, 0, 0) = 0$.

By representing Jacobian as $E \triangleq \frac{\partial f}{\partial \dot{x}}$, $A \triangleq -\frac{\partial f}{\partial x}$, the equation (1-1) can be linearized to the following form

$$E\dot{x} = Ax(t) + Bu(t) \tag{1-2}$$

If $|E| \neq 0$, then the equation (1-2) can be converted into a state variable equation in $\dot{x}$. If $|E| = 0$, the equation (1-2) represents linear singular system. The linear time-varying singular system, by considering $|EE(t)| = 0$, is written in the following form

$$E(t)\dot{x} = A(t)x(t) + B(t)u(t) \tag{1-3}$$

In the equation (1-3), $x \in \mathbb{R}^n$ is the vector of state variables and $u(t) \in \mathbb{R}^m$ is the vector of input variables. The functions $A(t)$ *and* $B(t)$ are smooth nonlinear time dependent functions with proper dimensions.

A singularly perturbed system is a special case of the equation (1-1) and it is represented as the following form

$$\dot{x} = f\big(x(t), z(t), u(t)\big) \tag{1-4}$$

$$\varepsilon\dot{z} = g\big(x(t), z(t), u(t)\big) \tag{1-5}$$

The situation of $\varepsilon = 0$ is called the singular perturbation because it completely changes the nature of the equation (1-5) from a differential equation to an algebraic equation. Indeed the singular perturbations arise when a high order nonlinear system is approximated with lower order system. In simple words, the objective of singular perturbation is to examine the simplified system $\dot{x} = f\big(x(t), u(t)\big)$ and from this result draw conclusion about original system (1-4) and (1-5) when $\varepsilon \neq 0$

The equation (1-5) becomes an algebraic equation if $\varepsilon = 0$ and as a result, the system (1-4) and (1-5) is a differential algebraic equation system with the form

$$\dot{x} = f\big(x(t), z(t), u(t)\big) \tag{1-6}$$

$$0 = g\big(x(t), z(t), u(t)\big) \tag{1-7}$$

This work focuses on nonlinear differential algebraic systems. The general formats of a semi-explicit nonlinear differential algebraic system include a set of nonlinear differential, algebraic and output equations where there is a distinct separation of differential and algebraic equations.

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)u \tag{1-8}$$

$$0 = f_2(x) + p_2(x)z + g_2(x)u \tag{1-9}$$

$$y = h(x) \tag{1-10}$$

Some of engineering applications [20], [21], [22], [23], [24] are modeled by differential algebraic equations (DAE). The DAE systems are also described as being singular, descriptor, semistate, and generalized systems. In the chemical engineering processes [20], [21], [22] the differential equations are constituted by the dynamic balances of mass and energy, while the algebraic equations come from the thermodynamic equilibrium relations, empirical correlations, and pseudo-steady-state conditions. A holonomic constrained robotic system with kinematic constraints [21], [23], [24] are modeled by singular systems too.

1.2 Literature Review

Physical systems are subjected to some type of extraneous signals or noise during operation. External disturbance such as a wind gust acting on aircraft affects the control system. In complex plants there are not only nonlinear models, uncertain dynamics, time delay and other un-modeled errors, but also measurement noise, control error and structural vibration as well as environmental disturbance. In a robotic system, the source of noise and disturbance is a reaction torque or force vector torque from the object/environment acting on the links.

Disturbance decoupling or disturbance attenuation is a well-known control problem in the field of control engineering. Like most cases, an output disturbance cannot be exactly decoupled but only asymptotically or "almost." It is worth noting that the solution to almost disturbance decoupling problems leads to the design of high gain feedback control. However, introducing large gains in a control loop potentially implies severe robustness problems, as they require very good confidence in the model.

By using a static feedback and introducing the key concept of controlled invariant subspaces, the exact disturbance decoupling problem for linear systems was solved in [1] and [2] in terms of geometric conditions. The geometric conditions were generalized for nonlinear system in [3] and [4] by introducing the controlled invariant distributions, a differential geometric generalization of controlled invariant subspaces. Further achievements were made by using the notion of characteristic indices in [5] to solve the problem of exact disturbance decoupling.

By introducing almost invariant subspaces in [6], the approximately disturbance decoupling with an arbitrary degree of accuracy for linear systems was addressed for the first time and was solved. It showed how subspaces may be viewed as ordinary controlled subspaces when one of them allows distributional inputs, and that others can be approximated by controlled subspaces. The results were applied to the disturbance decoupling problem.

The disturbance decoupling problem is related to high-gain feedback design since in cases where the problem cannot be exactly solved; increasing the accuracy of the decoupling requires increased gains of the linear state feedback control. The singular perturbation techniques can be used in the analysis of high-gain feedback systems.

The almost disturbance decoupling concept for nonlinear systems was addressed for the first time in [7] using singular perturbations and high gain feedback without using almost invariant subspaces. With the introduction of a backstepping technique in [8], a significant improvement has been made in this direction. The controller is explicitly constructed using a Lyapunov-based recursive scheme. The sufficient conditions for the solvability of the H_∞ almost disturbance decoupling problem and the explicit construction of the controller are given for a more restrictive class of nonlinear systems. In [9], it was examined in what extent the hypotheses and assumptions given in [8] can be relaxed and weakened.

In [10], an H_∞ state feedback controller with internal stability is constructed for the affine nonlinear systems in lower triangular form so that the L_2 gain from the disturbance to the tracking error is made arbitrarily small. In [11], the problem of the stability of system was made related to dissipativity. It provides a unique control law which is neither over-parametrized nor dependent on the unknown parameters such that the time dependent disturbances are attenuated with an arbitrary degree of accuracy. Furthermore, the closed loop system is globally asymptotically stable under regularity conditions.

A new set of results dealing with nonlinear single-input single-output (SISO) systems with possibly unstable zero dynamics were presented in [12]. The zero dynamics of a control system are the dynamics describing the internal behavior of the system when input and initial conditions have been chosen in such a way as to constrain the output to remain identically zero.

By deriving the geometric conditions in which those nonlinear systems with vector relative degree can be put globally into a lower triangular form [13], the almost disturbance decoupling problems for multi-input, multi-output (MIMO) nonlinear system was solved by backstepping technique. The H_∞ controller is designed by repeatedly using the backstepping technique so that the L_2 gain from the disturbance input to the closed-loop system output is bounded, and the closed-loop system is internally stable.

In [57], an adaptive controller in its minimum-order property was explicitly constructed without imposing any extra growth condition. The completion of square and the parameter separation technique was used to design the adaptive controller and attenuate the effect of disturbance on the output with an arbitrary degree of accuracy. In this design procedure the order of the dynamic compensator was equal to one.

The property of Young inequality was a useful tool in [17] to design the H_∞ controller for almost disturbance decoupling of MIMO nonlinear systems in nested lower triangular form, the form which is more general than the existing lower triangular form.

1.2.1 Unknown Parameters and Adaptive Control

It is worth noting that some parameters of the real plants are naturally unknown due to the difficulty of measurement or varying with time. For example [16], the damping, stiffness and friction coefficients in the dynamic equations of a constrained robotic system are difficult to measure. Or as an aircraft flies, its mass is slowly decreasing as a result of fuel consumption, so a control law is required to adapt itself to such changing conditions.

By introducing a set of techniques, adaptive control [32], [33], [34] provides a systematic procedure for automatic adjustment of controllers in real time to achieve a desired level of control system performance when some parameters of the plant dynamic model are unknown and/or change in time.

Consider a dynamic model of the plant which is going to be controlled; it includes unknown but constant parameters or at least in a certain region of operation [33]. In such cases, generally the design of the controller does not depend on the particular values of the plant model parameters, but the right tuning of the controller parameters is not possible without knowledge of their nominal values. The adaptive control techniques provide a self-tuning procedure in closed loop for the controller parameters [33]. As a result, the effect of the adaptation vanishes as time elapses.

Now consider the dynamic model of a plant with time varying parameters. The parameters may vary either due to changing environmental conditions [33], or the simplified linearization of nonlinear systems in which a change in operating conditions will lead to a different linearized model. These conditions may also happen because of the parameters of the system which are slowly time-varying [33]. An adaptive control approach should be considered to achieve a reasonable level of control system performance when unknown changes in model parameters occur. The adaptation law fully characterizes the non-vanishing adaptation of operation.

The parameter estimation is the foundation of adaptive control techniques. It provides update laws which are used to modify estimates in real time. Lyapunov stability is typically used to derive control adaptation laws and show convergence.

The adaptive regularization for nonlinear affine differential-algebraic equation (DAE) systems was investigated in [16]. It is assumed that the unknown parameters appear linearly in both differential and algebraic equations. The proposed methodology transforms the DAE system into an equivalent Ordinary Differential Equation (ODE) system with lower triangular form. By defining the change of coordinates globally, the adaptive feedback controller guarantees global asymptotic stability.

The adaptive control of a constrained robots modeled by singular system with parameters uncertainty was proposed in [47] by the theoretical approach presented in [47], [48][47] . Two dynamic equations are considered in the reduced form. One equation characterizes the motion of the robot in the constraint manifold. The second equation is an algebraic equation in the constraint force. Computed torque method is used to design the adaptive control law by introducing the parameter estimates and an additional compensation.

In [42] and [43] the reduced model of 2DOF parallel robot are used to design an adaptive controller and adaptive tracking controller via backstepping design. In general, a parallel robot is modeled by highly nonlinear singular equations. A precise knowledge of its parameters is not easily available and the adaptive backstepping seems to be a convenient control design methodology for the control of a robot.

1.3 Research Motivation

A wide range of practical applications is modeled by singular systems. The most theoretical research and practical works on the singular systems are focused on their solvability and numerical solutions [49], [50]. There is some work on feedback linearization [51], observer design [52], input-output decoupling [14], regulation [19], [53], output tracking [54], [55], stabilization [15], [48] and robust stabilization [56].

On other hand, the problem of almost disturbance decoupling for nonlinear ODE systems with lower triangular structure was significantly investigated by almost invariant subspaces [6], singular perturbations [7] and backstepping technique [8], [12], [13] and [17]. There exists little research on the problem of almost disturbance decoupling for linear singular systems [59], [60] and [61].

Due to the difficulty of measurement or varying with time, there are unknown parameters in the real plants. Therefore, investigating the adaptive control problems of DAE systems becomes natural and significant.

Furthermore, a little research has been done on the problem of adaptive almost disturbance decoupling for SISO nonlinear ODE systems [11], [57], [58] and [60], but from this author's literature search, there is no extensive study on the following problem:

Problem Definition:

Consider the following DAE system with presence of disturbance and unknown parameters:

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)u + \alpha_1(x)\theta + q_1(x)w \tag{1-11}$$

$$0 = f_2(x) + p_2(x)z + g_2(x)u + \alpha_2(x)\theta + q_2(x)w \tag{1-12}$$

$$y = h(x) \tag{1-13}$$

The problem is to design an adaptive controller for the system (1-11)-(1-13) such that the closed-loop system is globally stable and the state converges to zero when the disturbance vanishes; and the effect of the disturbance on the output is not greater than a specific level.

1.4 Research Contribution

To study the problem of adaptive backstepping almost disturbance decoupling for nonlinear singular systems, a systematic design procedure is followed, which can be expanded for large scale singular systems. First, the singular system is regularized with a development on the regularization algorithm in [14], [15] and [16]. The regularization algorithm is proposed to find a static regularization feedback law and make the nonlinear singular system impulse free.

Second, by introducing a standardization algorithm, the regularized system is converted into the ODE system with lower triangular structure.

Third, an adaptive backstepping controller is designed to reduce the effect of the disturbance on the output less than a specific level.

Finally, the algorithmic procedure is applied to a well-known and a high index singular model in order to illustrate the entire design procedure. The simulation results are compared with an adaptive controller without disturbance decoupling to verify the performance of the proposed controllers.

1.5 Organization of Thesis

This thesis work is organized as follows: In Chapter 2, the regularization and standardization algorithms are proposed and necessary condition and assumption are introduced to convert a high index nonlinear singular system into an ODE system with lower triangular structure for designing an adaptive controller based on backstepping technique. In Chapter 3, the basics of adaptive backstepping techniques are illustrated by examples and a systematic design procedure based on the adaptive backstepping technique is proposed and the adaptive almost disturbance decoupling controller is designed to guarantee a finite L_2 gain. Chapter 4 consists of simulation for a practical example to demonstrate the design procedure and show the efficiency of the proposed method. In Chapter 5, the simulation results are reviewed, the advantage and disadvantages of the proposed method are presented and finally, the possible future research on this work is discussed.

Chapter 2
Feedback Regularization

2.1 Introduction

This chapter focuses on the regularization and standardization algorithms to constitute an equivalent ODE system in lower triangular form. After an introduction in Section 2-1, the required mathematical background is reviewed in Section 2.1.1. The concept of index and regularity are introduced in Sections 2.1.2 and 2.1.3 respectively. Chapter 2 covers the proposed regularization algorithm for singular systems. It consists of two algorithms: the first algorithm is used to determine the generalized characteristic number [14], [15], [16] and is introduced in Section 2.2.1. The second algorithm in Section 2.2.2 reveals the hidden constraints behind the algebraic equation. Finally, a standardization algorithm by introducing a regularization feedback in Section 2.2 is developed to convert the nonlinear singular system to the equivalent ODE in lower triangular form.

2.1.1 Mathematical Preliminaries

In the differential-geometric setting [14], [35], the notation of dh for smooth function $h(x)$ is called the differential of h and is defined by $dh(x) = \left[\frac{\partial h(x)}{\partial x_1}, \dots, \frac{\partial h(x)}{\partial x_n}\right]$. For vector-valued smooth functions $f(x) = [f_1(x), \dots, f_n(x)]^T$ and $g(x) = [g_1(x), \dots, g_n(x)]^T$ and a matrix-valued smooth function $p(x) = [p^1(x), \dots, p^r(x)]$ with $p^i(x) = [p_1^i(x), \dots, p_n^i(x)]^T$, the following notations are used in this work:

$$L_f^0 h(x) = h(x)$$

$$L_f h(x) = dh(x) f(x)$$

$$L_f^{k+1} h(x) = L_f(L_f^k h(x))$$

$$L_g L_f h(x) = L_g(L_f h(x))$$

$$L_p L_f^k h(x) = \left[L_{p^1} L_f^k h(x), \dots, L_{p^r} L_f^k h(x)\right], k = 0,1, \dots$$

The expression of $L_f h$ is the Lie derivative of h along f.

2.1.2 Index

Singular systems are characteristically different than ordinary differential equation systems [31]. A difference is the possible impulsive solutions which are caused by arbitrary initial conditions or non-smooth inputs on the singular systems. The difference between singular system and ODE can be measured by a concept called differential index [19], [29]. The index is a nonnegative integer that provides comprehensive information about the complications of analysis and mathematical structure of a DAE system. In simple word, [25], [26], [27], [28], [30], [20] the higher the index of a DAE system, the more difficult numerical solution can be expected. The most general definition of index [19], [29] is the number of differentiations of algebraic equations which is required to obtain the equivalent ODE system.

The above definition for index is shown by the following example. Consider a DAE system the form of (2-1)-(2-3) with three differential equations and one algebraic equation

$$\dot{x_1} = 3x_2 + 2x_3 + z_1 \quad (2\text{-}1)$$

$$\dot{x_2} = u_1 + x_3$$

$$\dot{x_3} = u_2 + 2z_1$$

$$0 = x_1 - x_3 \quad (2\text{-}2)$$

$$y_1 = x_2 \quad (2\text{-}3)$$

where $x_1,\ x_2,\ x_3$are differential variables, $z_1$is algebraic variable and $u_1$is control input. By differentiating the algebraic equation (2-2) with respect to time once and plugging $\dot{x_1}$, $\dot{x}_3$ from (2-1), the algebraic variable z is calculated by the following expressions

$$3x_2 + 2x_3 + z_1 - u_2 - 2z_1 = 0$$

$$z_1 = 3x_2 + 2x_3 - u_2 \quad (2\text{-}4)$$

Differentiating of z_1 in (2-4) with respect to time gives a differential equation the form of $\dot{z}_1 = 3u_1 + 2u_2 + 3x_3 + 4z_1 - \dot{u}_2$. So, by differentiating (2-4) twice, the equivalent ODE is obtained, which means the system (2-1)-(2-3) has an index two.

2.1.3 Regularity

Consider the system in the following form

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)u \quad (2\text{-}5)$$

$$0 = f_2(x) + p_2(x)z + g_2(x)u \quad (2\text{-}6)$$

$$y = h(x) \quad (2\text{-}7)$$

Definition 1.2: DAE system (2-5), (2-6) and (2-7) is regular [19] if

1. It has a finite index.
2. The set where the differential variables x are constrained to evolve is invariant under any control law for manipulated input u.

The system (2-5)-(2-7) is said to be regularizable if there is a smooth regular feedback of the form $u = \beta(x)z + v$ such that the corresponding closed loop system of the form

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)[\beta(x)z + v]$$

$$0 = f_2(x) + p_2(x)z + g_2(x)[\beta(x)z + v]$$

is regular [19], [14] at the neighborhood $U$of x_0.

If $[p_2(x) \quad g_2(x)]$ has a full row rank, then there exists $\beta(x)$ so that $p_2(x) + g_2(x)\beta(x)$ is invertible. As a result, the algebraic variable can be calculated by the following equation:

$$z = -[p_2(x) + g_2(x)\beta(x)]^{-1}[f_2(x) + g_2(x)v]$$

Once we design v, the regular feedback can be calculated in the form of

$$u = -\beta(x)[p_2(x) + g_2(x)\beta(x)]^{-1}[f_2(x) + g_2(x)v] + v$$

2.2 Regularization

A semi-explicit multi-input multi-output (MIMO) differential algebraic system with the presence of unknown parameters and disturbances is in the form

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)u + \alpha_1(x)\theta + q_1(x)w \tag{2-8}$$

$$0 = f_2(x) + p_2(x)z + g_2(x)u + \alpha_2(x)\theta + q_2(x)w \tag{2-9}$$

$$y = h(x) \tag{2-10}$$

The different elements in the system (2-8) - (2-10) are defined as the following

$x \in \mathbb{R}^n: vector\ of\ differential\ variables$

$z \in \mathbb{R}^s: vector\ of\ algebraic\ variables$

$u \in \mathbb{R}^m: vector\ of\ input$

$\theta \in \mathbb{R}^l: vector\ of\ unknown\ parameters$

$w \in \mathbb{R}^p: vector\ of\ bounded\ disturbance$

$h \in \mathbb{R}^m: vector\ of\ output$

$f_1(x): vector - valued\ smooth\ function\ with\ dimension\ n \times 1$

$p_1(x): matrix - valued\ smooth\ function\ with\ dimension\ n \times s$

$g_1(x): matrix - valued\ smooth\ function\ with\ dimension\ n \times m$

$\alpha_1(x): matrix - valued\ smooth\ function\ with\ dimension\ n \times l$

$q_1(x): matrix - valued\ smooth\ function\ with\ dimension\ n \times p$

$f_2(x): vector - valued\ smooth\ function\ with\ dimension\ s \times 1$

$p_2(x): matrix - valued\ smooth\ function\ with\ dimension\ s \times s$

$g_2(x): matrix - valued\ smooth\ function\ with\ dimension\ s \times m$

$\alpha_2(x): matrix - valued\ smooth\ function\ with\ dimension\ s \times l$

$q_2(x)$: $matrix - valued\ smooth\ function\ with\ dimension\ s \times p$

$h(x)$: $vector - valued\ smooth\ function\ with\ dimension\ m \times 1$

In the equations (2-8)-(2-9), the manipulated inputs u and algebraic variables z appear in affine and separable form which covers a wide range of practical applications [14], [19].

If $[p_2(x) \quad g_2(x)]$ has no full row rank, a regularizing feedback of the form $u = \beta(x)z + v$ cannot make the system regular. The regularization algorithm is used to identify the hidden constraints behind algebraic equation (2-9). The algorithm involves a concept of generalized characteristic number [14], [15], [16].

2.2.1 Calculation of Generalized Characteristic Number (Algorithm 1)

Calculation of generalized characteristic number (r) under constraint (2-9) is given in the following algorithm [14] with the assumption as $[p_2(x) \quad g_2(x)]$ has a full row rank of s_0:

Step 1.

Assign $\phi_0(x) = \phi(x)$. Set $k = 0$ and calculate the Lie derivatives

$$L_{f_1}\phi_0(x)\,,\;\; L_{p_1}\phi_0(x),\;\; L_{g_1}\phi_0(x),\;\; L_{\alpha_1}\phi_0(x),\;\; L_{q_1}\phi_0(x)$$

If the matrix $\begin{bmatrix} p_2(x) & g_2(x) \\ L_{p_1}\phi_0(x) & L_{g_1}\phi_0(x) \end{bmatrix}$ has constant rank s_0, then there exists a unique vector-valued smooth function $e_0(x)$ of dimension s_0 such that:

$$[L_{p_1}\phi_0(x) \quad L_{g_1}\phi_0(x)] = e_0(x)[p_2(x) \quad g_2(x)]$$

Define

$$\phi_1(x) = L_{f_1}\phi_0(x) - e_0(x)f_2(x)$$

$$\omega_1(x) = L_{\alpha_1}\phi_0(x) - e_0(x)\alpha_2(x)$$

$$\upsilon_1(x) = L_{q_1}\phi_0(x) - e_0(x)q_2(x)$$

Otherwise set $r = 1$ and terminate the algorithm.

Step $k+1$.

The sequences of $\phi_0(x),\ \phi_1(x),\ \phi_2(x), \dots, \phi_k(x)$ have been defined and now calculate the Lie derivatives

$$L_{f_1}\phi_k(x)\,,\ \ L_{p_1}\phi_k(x),\ \ L_{g_1}\phi_k(x),\ \ L_{\alpha_1}\phi_k(x),\ \ L_{q_1}\phi_k(x)$$

If the matrix $\begin{bmatrix} p_2(x) & g_2(x) \\ L_{p_1}\phi_k(x) & L_{g_1}\phi_k(x) \end{bmatrix}$ has constant rank s_0, then there exists a unique vector-valued smooth function $e_k(x)$ of dimension s_0 such that

$$[L_{p_1}\phi_k(x) \quad L_{g_1}\phi_k(x)] = e_k(x)[p_2(x) \quad g_2(x)]$$

Define

$$\phi_{k+1}(x) = L_{f_1}\phi_k(x) - e_k(x)f_2(x)$$

$$\omega_{k+1}(x) = L_{\alpha_1}\phi_k(x) - e_k(x)\alpha_2(x)$$

$$\upsilon_{k+1}(x) = L_{q_1}\phi_k(x) - e_k(x)q_2(x)$$

Otherwise set $r = k+1$ and terminate the algorithm.

The algorithm terminates at step r. Such an integer is defined to be generalized characteristic number of function $\phi(x)$ under the constraint (2-9).

Now differentiate $\phi_k(x)$ with respect to time and substitute $\dot{x}$ from (2-8) for $k = 0, 1, \dots, r - 2$:

$$\frac{d\phi_k(x)}{dt} = \frac{\partial\phi_k(x)}{\partial x}\dot{x} = \frac{\partial\phi_k(x)}{\partial x}(f_1(x) + p_1(x)z + g_1(x)u + \alpha_1(x)\theta + q_1(x)w)$$

$$= \frac{\partial\phi_k(x)}{\partial x}f_1(x) + \frac{\partial\phi_k(x)}{\partial x}p_1(x)z + \frac{\partial\phi_k(x)}{\partial x}g_1(x)u + \frac{\partial\phi_k(x)}{\partial x}\alpha_1(x)\theta + \frac{\partial\phi_k(x)}{\partial x}q_1(x)w$$

$$= L_{f_1}\phi_k(x) + L_{p_1}\phi_k(x)z + L_{g_1}\phi_k(x)u + L_{\alpha_1}\phi_k(x)\theta + L_{q_1}\phi_k(x)w$$

$$= \phi_{k+1}(x) + \omega_{k+1}(x)\theta + v_{k+1}(x)w + e_k(x)[f_2(x) + p_2(x)z + g_2(x)u + \alpha_2(x)\theta + q_2(x)w]$$

For $k = r - 1$:

$$\frac{d\phi_{r-1}(x)}{dt} = \frac{\partial\phi_{r-1}(x)}{\partial x}\dot{x} = \frac{\partial\phi_{r-1}(x)}{\partial x}(f_1(x) + p_1(x)z + g_1(x)u + \alpha_1(x)\theta + q_1(x)w)$$

$$= \frac{\partial\phi_{r-1}(x)}{\partial x}f_1(x) + \frac{\partial\phi_{r-1}(x)}{\partial x}p_1(x)z + \frac{\partial\phi_{r-1}(x)}{\partial x}g_1(x)u + \frac{\partial\phi_{r-1}(x)}{\partial x}\alpha_1(x)\theta + \frac{\partial\phi_{r-1}(x)}{\partial x}q_1(x)w$$

$$= L_{f_1}\phi_{r-1}(x) + L_{p_1}\phi_{r-1}(x)z + L_{g_1}\phi_{r-1}(x)u + L_{\alpha_1}\phi_{r-1}(x)\theta + L_{q_1}\phi_{r-1}(x)w$$

2.2.2 Regularization Algorithm (Algorithm 2)

The regularization algorithm [14], [15], [16] is used to identify the hidden constraint behind the algebraic equation (2-9). Each step of this algorithm involves calculating the generalized characteristic number.

Algorithm 2:

Step 0.

Consider constraint (2-10) and suppose that the matrix $[p_2(x) \quad g_2(x)]$ has constant rank s_0.

Assumption 1: suppose that $rank\ [p_2(x) \quad g_2(x) \quad \alpha_2(x) \quad q_2(x)] = rank[p_2(x) \quad g_2(x)]$

Take the s_0 rows that produce the matrix with full row rank of s_0 and denote it with $[b_0(x) \quad c_0(x)]$. The matrix $[p_2(x) \quad g_2(x) \quad \alpha_2(x) \quad q_2(x)]$ takes the form

$$[p_2(x) \quad g_2(x) \quad \alpha_2(x) \quad q_2(x)] = \begin{bmatrix} b(x) & c_0(x) & d_0(x) & k_0(x) \\ p_2^{s_0+i}(x) & g_2^{s_0+i}(x) & \alpha_2^{s_0+i}(x) & q_2^{s_0+i}(x) \end{bmatrix},$$

$$i = 1, 2, \dots, t\ (t = s - s_0)$$

There should exist a unique vector $S^i(x)$ such that

$$\left[p_2^{s_0+i}(x) \quad g_2^{s_0+i}(x) \quad \alpha_2^{s_0+i}(x) \quad q_2^{s_0+i}(x)\right] = S^i(x)[p_2(x) \quad g_2(x) \quad \alpha_2(x) \quad q_2(x)]$$

Set:

$$\phi^i(x) = f_2^{s_0+i}(x) - S^i(x)a_0(x) \tag{2-11}$$

$$\omega^i(x) = \alpha_2^{s_0+i}(x) - S^i(x)d_0(x) \tag{2-12}$$

$$\upsilon^i(x) = q_2^{s_0+i}(x) - S^i(x)k_0(x) \tag{2-13}$$

Where $a_0(x), d_0(x), and\ k_0(x)$ are first s_0 row of $f_1(x),\ \alpha_1(x),\ and q_1(x)$ respectively.

The algebraic equation (2-9) becomes:

$$0 = \begin{bmatrix} a_0(x) \\ f_2^{s_0+i}(x) \end{bmatrix} + \begin{bmatrix} b_0(x) \\ p_2^{s_0+i}(x) \end{bmatrix} z + \begin{bmatrix} c_0(x) \\ g_2^{s_0+i}(x) \end{bmatrix} u + \begin{bmatrix} d_0(x) \\ \alpha_2^{s_0+i}(x) \end{bmatrix} \theta + \begin{bmatrix} k_0(x) \\ q_2^{s_0+i}(x) \end{bmatrix} w$$

Decomposing the above matrices gives the following equations

$$0 = a_0(x) + b_0(x)z + c_0(x)u + d_0(x)\theta + k_0(x)w \tag{2-14}$$

$$0 = f_2^{s_0+i}(x) + p_2^{s_0+i}(x)z + g_2^{s_0+i}(x)u + \alpha_2^{s_0+i}(x)\theta + q_2^{s_0+i}(x)w \tag{2-15}$$

If one substitutes $f_2^{s_0+i}(x), p_2^{s_0+i}(x), g_2^{s_0+i}(x), \alpha_2^{s_0+i}(x), q_2^{s_0+i}(x)$ from (2-11), (2-12) and (2-13), respectively, to (2-15), the results are expressed in (2-16)

$$0 = \phi^i(x) + S^i(x)a_0(x) + S^i(x)b_0(x)z + S^i(x)c_0(x)u + \omega^i(x)\theta \ + S^i(x)d_0(x)\theta \\ + \upsilon^i(x)q + S^i(x)k_0(x)q$$

By simplification and appropriate grouping, the equation takes the form

$$0 = \phi^i(x) + \omega^i(x)\theta + \upsilon^i(x)d + S^i(x)[a_0(x) + b_0(x)z + c_0(x)u + d_0(x)\theta \\ + k_0(x)w] \tag{2-16}$$

Now substitute (2-15) in (2-16) to form the following equation

$$0 = \phi^i(x) + \omega^i(x)\theta + \upsilon^i(x)d \tag{2-17}$$

By Assumption 1, the hidden constraint is independent of the unknown parameters and disturbances, which means $\omega^i(x) = 0$ and $\upsilon^i(x) = 0$ and as a result the equation (2-9) takes the form

$$0 = \phi^i(x) \tag{2-18}$$

Step 1.

Assign $\phi(x) = \phi^1(x)$ and perform Algorithm 1 to calculate the generalized characteristic number of $\phi(x)$ under constraint (2-14) and produce r^1, $\phi_0^1(x), \phi_1^1(x), \dots, \phi_{r-1}^1(x)$, $\omega_1^1(x), \dots ,\ \omega_{r-1}^1(x),\ \upsilon_1^1(x), \dots, \upsilon_{r-1}^1(x),\ e_1^1(x), \dots, e_{r-1}^1(x)$.

Now differentiate $\phi_i^1(x)$with respect to time

for $i = 0, 1, \dots ,\ r^1 - 2$

$$\frac{d\phi_i^1(x)}{dt} = \phi_{i+1}^1(x) + \omega_{i+1}^1(x)\theta + \upsilon_{i+1}^1(x)w + e_{i+1}^1(x)[a_0(x) + \ b_0(x)z + \ c_0(x)u + d_0(x)\theta + k_0(x)w] \tag{2-19}$$

The expression (2-18) confirms that $\frac{d\phi_0^1(x)}{dt} = 0$, when put together with (2-14) and (2-19), the hidden constraint becomes $0 = \phi_i^1(x) + w_i^1(x) + \upsilon_i^1(x)$. Since the hidden constraint is independent of θ *and* w, we have $\omega_i^1(x) \equiv 0$ *and* $\upsilon_i^1(x) \equiv 0$ and the hidden constraint takes the form

$$\phi_i^1(x) = 0 \text{ For } i = 0, 1, \dots, \ r^1 - 1 \tag{2-20}$$

and for $i = \ r^1 - 1$:

$$\frac{d\phi_{i-1}^1(x)}{dt} = L_{f_1}\phi_{i-1}^1(x) + L_{p_1}\phi_{i-1}^1(x) + L_{g_1}\phi_{i-1}^1(x)\theta + L_{\alpha_1}\phi_{i-1}^1(x) + L_{q_1}\phi_{i-1}^1(x)w \tag{2-21}$$

Now let

$$a_1(x) = L_{f_1}\phi_{i-1}^1(x),$$

$$b_1(x) = L_{p_1}\phi_{i-1}^1(x),$$

$$c_1(x) = L_{g_1}\phi_{i-1}^1(x),$$

$$d_1(x) = L_{\alpha_1}\phi_{i-1}^1(x),$$

$$k_1(x) = L_{q_1}\phi_{i-1}^1(x)$$

and substitute $a_1(x), b_1(x), c_1(x), d_1(x)$ *and* $k_1(x)$ in (2-21), to form the following

$$\frac{d\phi_{i-1}^1(x)}{dt} = a_1(x) + b_1(x) + c_1(x) + d_1(x)\theta + k_1(x)w \tag{2-22}$$

From (2-22) and (2-20) with $i = r^1 - 1$ it is clear that

$$0 = a_1(x) + b_1(x) + c_1(x) + d_1(x)\theta + k_1(x)w \tag{2-23}$$

Combining (2-14) and (2-23) gives the following algebraic equation

$$0 = a^1(x) + b^1(x)z + c^1(x)u + d^1(x)\theta + k^1(x)w \tag{2-24}$$

with:

$$a^1(x) = \begin{bmatrix} a_0(x) \\ a_1(x) \end{bmatrix},\ b^1(x) = \begin{bmatrix} b_0(x) \\ b_1(x) \end{bmatrix}, c^1(x) = \begin{bmatrix} c_0(x) \\ c_1(x) \end{bmatrix}, d^1(x) = \begin{bmatrix} d_0(x) \\ d_1(x) \end{bmatrix}, k^1(x) = \begin{bmatrix} k_0(x) \\ k_1(x) \end{bmatrix}$$

If the matrix $[b^1(x) \quad c^1(x)]$ has full row rank $s_0 + 1$, then set $k = 2$ and go to the next step. Otherwise terminate the algorithm.

Step 2.

Assign $\phi(x) = \phi^2(x)$ and perform Algorithm 1 to determine the generalized characteristic number of $\phi^2(x)$ under constraint (2-24) and produce $r^2, \phi_0^2(x),\ \phi_1^2(x), \dots, \phi_{r-1}^2(x),\ \omega_1^2(x), \dots, \omega_{r-1}^2(x)$, $\upsilon_1^2(x), \dots, \upsilon_{r-1}^2(x),\ e_1^2(x),\ \dots, e_{r-1}^2(x)$. Now differentiate $\phi_i^2(x)$ with respect to time.

for $i = 0, 1, \dots ,\ r^2 - 2$

$$\frac{d\phi_i^2(x)}{dt} = \phi_{i+1}^2(x) + \omega_{i+1}^2(x)\theta + \upsilon_{i+1}^2(x)w + e_{i+1}^2(x)[a^1(x) + \ b^1(x)z + \ c^1(x)u + d^1(x)\theta + k^1(x)w] \tag{2-25}$$

It comes from (2-18) that $\frac{d\phi_0^2(x)}{dt} = 0$, and together with (2-24) and (2-25) gives $0 = \phi_i^2(x) + \omega_i^2(x) + \upsilon_i^2(x)$. Assumption 1 implies $\omega_i^2(x) \equiv 0$ *and* $\upsilon_i^2(x) \equiv 0$. So the hidden constraint takes the form

$$\phi_i^2(x) = 0 \text{ For } i = 0, 1, \dots ,\ r^2 - 1 \tag{2-26}$$

and for $i = \ r^2 - 1$:

$$\frac{d\phi_{i-1}^2(x)}{dt} = L_{f_1}\phi_{i-1}^2(x) + L_{p_1}\phi_{i-1}^2(x) + L_{g_1}\phi_{i-1}^2(x) + L_{\alpha_1}\phi_{i-1}^2(x) + L_{q_1}\phi_{i-1}^2(x) \tag{2-27}$$

Now let:

$$a_2(x) = L_{f_1}\phi_{i-1}^2(x)$$

$$b_2(x) = L_{p_1}\phi_{i-1}^2(x)$$

$$c_2(x) = L_{g_1}\phi_{i-1}^2(x)$$

$$d_2(x) = L_{\alpha_1}\phi_{i-1}^2(x)$$

$$k_2(x) = L_{q_1}\phi_{i-1}^2(x)$$

By substituting $a_2(x), b_2(x), c_2(x), d_2(x)\ and\ k_2(x)$ in (2-28), it takes the form

$$\frac{d\phi_{i-1}^2(x)}{dt} = a_2(x) + b_2(x) + c_2(x) + d_2(x) + k_2(x) \tag{2-28}$$

It follows from (2-26) with $i = r^2 - 1$ and (2-28) that

$$0 = a_2(x) + b_2(x)z + c_2(x)u + d_2(x)\theta + k_2(x)w \tag{2-29}$$

Combining (2-24) and (2-29) forms the following algebraic equation

$$0 = a^2(x) + b^2(x)z + c^2(x)u + d^2(x)\theta + k^2(x)w \tag{2-30}$$

with

$$a^2(x) = \begin{bmatrix} a^1(x) \\ a_2(x) \end{bmatrix}, b^2(x) = \begin{bmatrix} b^1(x) \\ b_2(x) \end{bmatrix} c^2(x) = \begin{bmatrix} c^1(x) \\ c_2(x) \end{bmatrix}, d^2(x) = \begin{bmatrix} d^1(x) \\ d_2(x) \end{bmatrix}, k^2(x) = \begin{bmatrix} k^1(x) \\ k_2(x) \end{bmatrix}$$

If the matrix $[b^2(x) \quad c^2(x)]$ has full row rank $s_0 + 2$, then set $k = 3$ and go to the next step. Otherwise terminate the algorithm.

Step k.

Suppose Step k-1 produces the algebraic equation of the form

$$0 = a^{k-1}(x) + b^{k-1}(x)z + c^{k-1}(x)u + d^{k-1}(x)\theta + k^{k-1}(x)w \tag{2-31}$$

Assign $\phi(x) = \phi^k(x)$ and perform Algorithm 1 to calculate the generalized characteristic number of $\phi^k(x)$ under constraint (2-31) and produce $r^k, \phi_0^k(x),\ \phi_1^k(x), \dots, \phi_{r-1}^k(x)$, $\omega_1^k(x), \dots, \omega_{r-1}^k(x), v_1^k(x), \dots, v_{r-1}^k(x), e_1^k(x), \dots, e_{r-1}^k(x)$.

With respect to time, differentiating $\phi_i^k(x)$ gives

for $i = 0, 1, \dots ,\ r^k - 2$

$$\begin{aligned} \frac{d\phi_i^k(x)}{dt} = \phi_{i+1}^k(x) + \omega_{i+1}^k(x)\theta + v_{i+1}^k(x)w + e_{i+1}^k(x)[a^{k-1}(x) + \ b^{k-1}(x)z \\ + \ c^{k-1}(x)u + d^{k-1}(x)\theta + k^{k-1}(x)w] \end{aligned} \tag{2-32}$$

The expression (2-18) implies $\frac{d\phi_0^k(x)}{dt} = 0$, when combined with (2-31) and (2-32), the hidden constraint takes the form $0 = \phi_i^k(x) + w_i^k(x) + v_i^k(x)$. Since the hidden constraint is independent of θ *and* w, it is derived that $\omega_i^k(x) \equiv 0$ *and* $v_i^k(x) \equiv 0$. So the hidden constraint becomes

$$\phi_i^k(x) = 0 \text{ For } i = 0, 1, \dots ,\ r^k - 1 \tag{2-33}$$

and for $i = \ r^k - 1$:

$$\frac{d\phi_{i-1}^{k}(x)}{dt} = L_{f_1}\phi_{i-1}^{k}(x) + L_{p_1}\phi_{i-1}^{k}(x) + L_{g_1}\phi_{i-1}^{k}(x) + L_{\alpha_1}\phi_{i-1}^{k}(x) + L_{q_1}\phi_{i-1}^{k}(x) \tag{2-34}$$

Now let $a_k(x) = L_{f_1}\phi_{i-1}^{k}(x)$, $b_k(x) = L_{p_1}\phi_{i-1}^{k}(x)$, $c_k(x) = L_{g_1}\phi_{i-1}^{k}(x)$, $d_k(x) = L_{\alpha_1}\phi_{i-1}^{k}(x)$, $k_k(x) = L_{q_1}\phi_{i-1}^{k}(x)$ and substitute them into (2-35), it take the form

$$\frac{d\phi_{i-1}^{k}(x)}{dt} = a_k(x) + b_k(x) + c_k(x) + d_k(x) + k_k(x) \tag{2-35}$$

With $i = r^k - 1$ in (2-33) it is concluded from (2-35) that

$$0 = a_k(x) + b_k(x)z + c_k(x)u + d_k(x)\theta + k_k(x)w \tag{2-36}$$

Combining (2-32) and (2-36) gives the following algebraic equation

$$0 = a^k(x) + b^k(x)z + c^k(x)u + d^k(x)\theta + k^k(x)w \tag{2-37}$$

with

$a^k(x) = \begin{bmatrix} a^{k-1}(x) \\ a_k(x) \end{bmatrix}$, $b^k(x) = \begin{bmatrix} b^{k-1}(x) \\ b_k(x) \end{bmatrix}$, $c^k(x) = \begin{bmatrix} c^{k-1}(x) \\ c_k(x) \end{bmatrix}$, $d^k(x) = \begin{bmatrix} d^{k-1}(x) \\ d_k(x) \end{bmatrix}$,

$k^k(x) = \begin{bmatrix} k^{k-1}(x) \\ k_k(x) \end{bmatrix}$

If the matrix$[b^k(x) \quad c^k(x)]$ has full row rank $s_0 + k$, then set $k = k + 1$ and go to the next step; otherwise, terminate the algorithm.

The algorithm is considered feasible if it terminates at Step $k = p$ and matrix $[b^p(x) \quad c^p(x)]$ has full row rank of $s = s_0 + p$. If the Algorithm 2 is feasible, the DAE system (2-9)-(2-11) is equivalent to following DAE system:

$$\dot{x} = f_1(x) + p_1(x)z + g_1(x)u + \alpha_1(x)\theta + q_1(x)w \tag{2-38}$$

$$0 = a(x) + b(x)z + c(x)u + d(x)\theta + k(x)w \tag{2-39}$$

with $a(x) = a^p(x), b(x) = b^p(x), c(x) = c^p(x), d(x) = d^p(x), k(x) = k^p(x)$, where $x \in M$ and $[b(x) \quad c(x)]$ has full row rank of s. Then we can change the equivalent DAE system of the form (2-38)-(2-39) to lower triangular form with using a feedback controller of the form $u = \beta(x)z + v$ by the following algorithm.

The initial condition $x(0)$ must satisfy $x(0) \in M$ in order to have impulse free solution for equivalent DAE system, where

$$M = \left\{ x \in R^n \middle| \begin{array}{c} \phi_0^i(0) = 0, \phi_j^i(0) = 0 \\ for\ j = 1, 2, \dots, r^{i-1}, and\ i = 0, 1, 2, \dots, p \end{array} \right\} \tag{2-40}$$

2.3 Standardization Algorithms (Algorithm 3)

Once a regular DAE system has been constructed by the regularizing algorithm, a standardization algorithm [14], [15], [16] converts the regular DAE system into equivalent ODE system with lower triangular structure. This algorithm constitutes the proper change of coordinates, and together with a regular feedback controller of the form $u = \beta(x)z + v$, transforms it into ODE with lower triangular form. The standardization algorithm uses the Algorithm 1 repeatedly.

<u>Algorithm 3:</u>

Step 1.

Consider output equation (2-10) and assign $\psi^1(x) = h^1(x)$ and perform Algorithm 1 to determine the generalized characteristic number of $\psi^1(x)$ under constraint (2-39) to produce $q^1, \psi_0^1(x),\ \psi_1^1(x), \dots, \psi_{r-1}^1(x), \varphi_1^1(x), \dots, \varphi_{r-1}^1(x), \varsigma_1^1(x), \dots, \varsigma_{r-1}^1(x)$, $E_1^1(x), \dots, E_{r-1}^1(x)$. The first order time derivative of $\psi_i^1(x)$ gives

for $i = 0, 1, \dots,\ q^1 - 2$

$$\frac{d\psi_i^1(x)}{dt} = \psi_{i+1}^1(x) + \varphi_{i+1}^1(x)\theta + \varsigma_{i+1}^1(x)w + E_{i+1}^1(x)[a(x) + b(x)z + c(x)u + d(x)\theta + k(x)w]$$

and for $i = q^1 - 1$

$$\frac{d\psi_{i-1}^1(x)}{dt} = L_{f_1}\psi_{i-1}^1(x) + L_{p_1}\psi_{i-1}^1(x) + L_{g_1}\psi_{i-1}^1(x) + L_{\alpha_1}\psi_{i-1}^1(x) + L_{q_1}\psi_{i-1}^1(x)$$

Now let $A_1(x) = L_{f_1}\psi_{i-1}^1(x), B_1(x) = L_{p_1}\psi_{i-1}^1(x), C_1(x) = L_{g_1}\psi_{i-1}^1(x),$

$D_1(x) = L_{\alpha_1}\psi_{i-1}^1(x), K_1(x) = L_{q_1}\psi_{i-1}^1(x)$ and form

$$\frac{d\psi_{i-1}^1(x)}{dt} = A_1(x) + B_1(x) + C_1(x) + D_1(x) + K_1(x)$$

Step k.

Assign $\psi^k(x) = h^k(x)$ and perform Algorithm 1 to determine the generalized characteristic number under of $\psi^k(x)$ constraint (2-39) to produce $q^k, \psi_0^k(x),\ \psi_1^k(x), \dots, \psi_{r-1}^k(x)$, $\varphi_1^k(x), \dots, \varphi_{r-1}^k(x),\ \varsigma_1^k(x), \dots, \varsigma_{r-1}^k(x),\ E_1^k(x), \dots, E_{r-1}^k(x)$. The first order time derivative of $\psi_i^k(x)$ with respect to time gives:

for $i = 0, 1, \dots,\ q^k - 2$

$$\frac{d\psi_i^k(x)}{dt} = \psi_{i+1}^k(x) + \varphi_{i+1}^k(x)\theta + \varsigma_{i+1}^k(x)w + E_{i+1}^k(x)[a(x) + b(x)z + c(x)u + d(x)\theta + k(x)w]$$

and for $i = q^k - 1$:

$$\frac{d\psi_{i-1}^k(x)}{dt} = L_{f_1}\psi_{i-1}^k(x) + L_{p_1}\psi_{i-1}^k(x) + L_{g_1}\psi_{i-1}^k(x) + L_{\alpha_1}\psi_{i-1}^k(x) + L_{q_1}\psi_{i-1}^k(x)$$

Let $A_k(x) = L_{f_1}\psi_{i-1}^k(x)$, $B_k(x) = L_{p_1}\psi_{i-1}^k(x)$, $C_k(x) = L_{g_1}\psi_{i-1}^k(x)$, $D_k(x) = L_{\alpha_1}\psi_{i-1}^k(x)$, $K_k(x) = L_{q_1}\psi_{i-1}^k(x)$ to form $\frac{d\psi_{i-1}^k(x)}{dt} = A_k(x) + B_k(x) + C_k(x) + D_k(x) + K_k(x)$

Algorithm 3 is terminated at $k = m$. The following assumptions are required.

Assumption 1: The matrix $\begin{bmatrix} b(x) & c(x) \\ B_1(x) & C_1(x) \\ \vdots & \vdots \\ B_m(x) & C_m(x) \end{bmatrix}$ is nonsingular.

Assumption 2: $n = r + q$ with $r = r^0 + r^1 + \cdots + r^p$ and $q = q^0 + q^1 + \cdots + q^m$.

The functions $\phi_j^i(x)$ for $j = 0, 1, \ldots, r^i - 1$ and $i = 0, 1, \ldots, p$, and $\psi_j^i(x)$ for $j = 0, 1, \ldots, q^i - 1$ and $= 0, 1, \ldots, m$, form a new set of coordinates which is guaranteed with Lemma 1 [15].

Lemma 1: assume that Algorithms 1-3 are feasible and Assumption 1 is satisfied. Then, the below vectors are linearly independent in U.

$$d\phi_0^1(x), d\phi_1^1(x), \ldots, d\phi_{r-1}^1(x)$$

$$\vdots$$

$$d\phi_0^p(x), d\phi_1^p(x), \ldots, d\phi_{r-1}^p(x)$$

$$d\psi_0^1(x), d\psi_1^1(x), \ldots, d\psi_{r-1}^1(x)$$

$$\vdots$$

$$d\psi_0^m(x), d\psi_1^m(x), \dots, d\psi_{r-1}^m(x)$$

where

$$U = \begin{bmatrix} \frac{\partial \phi(x)}{\partial \varepsilon} & \frac{\partial \psi_0^1(x)}{\partial \varepsilon} & \cdots & \frac{\partial \psi_{q^1-1}^1(x)}{\partial \varepsilon} & \cdots & \frac{\partial \psi_0^k(x)}{\partial \varepsilon} & \cdots & \frac{\partial \psi_i^k(x)}{\partial \varepsilon} & \frac{\partial \phi_i^k(x)}{\partial \varepsilon} \\ \frac{\partial \phi(x)}{\partial \xi} & \frac{\partial \psi_0^1(x)}{\partial \xi} & \cdots & \frac{\partial \psi_{q^1-1}^1(x)}{\partial \xi} & \cdots & \frac{\partial \psi_0^k(x)}{\partial \xi} & \cdots & \frac{\partial \psi_i^k(x)}{\partial \xi} & \frac{\partial \psi_i^k(x)}{\partial \xi} \end{bmatrix}^T$$

Lemma 1 with Assumption 2 implies that the function $\Phi(x) = \begin{bmatrix} \phi(x) \\ \psi(x) \end{bmatrix}$ constitutes a change of coordinates.

$$\phi(x) = [\phi^1(x)^T \quad \phi^2(x)^T \quad \dots \quad \phi^p(x)^T]^T \text{with } \phi^i(x) = \left[\phi_0^i(x) \quad \dots \quad \phi_{r^i-1}^i(x)\right]^T \text{for } i = 1, \dots, p$$

$$\psi(x) = [\psi^1(x)^T \quad \psi^2(x)^T \quad \dots \quad \psi^m(x)^T]^T \text{ with } \psi^i(x) = \left[\psi_0^i(x) \quad \dots \quad \psi_{q^i-1}^i(x)\right]^T \text{for } i = 1, \dots, m$$

Set:

$$\varepsilon_i^k = \phi_i^k \text{ for } i = 1, \dots, r^k - 1 \text{ and } k = 1, \dots, p$$

$$\xi_i^k = \psi_i^k \text{ for } i = 1, \dots, q^k - 1 \text{ and } k = 1, \dots, m$$

Let:

$$\varepsilon = \left[\varepsilon_0^1 \quad \dots \quad \varepsilon_{r^1-1}^1 \quad \dots \quad \varepsilon_0^p \quad \dots \quad \varepsilon_{r^p-1}^p\right]^T$$

$$\xi = \left[\xi_0^1 \quad \dots \quad \xi_{q^1-1}^1 \quad \dots \quad \xi_0^m \quad \dots \quad \xi_{q^m-1}^m\right]^T$$

By differentiating ε_i^k and ξ_i^k with respect to time, the DAE system can be expressed in the new coordinates by:

$$
\begin{gathered}
\varepsilon = 0 \\
y^i = \xi_0^i \\
\dot{\xi}_0^i = \xi_1^i + \varphi_1^i(x)\theta + \varsigma_1^i(x)w \\
\vdots \\
\dot{\xi}_{q^i-2}^i = \xi_{q^i-1}^i + \varphi_{q^i-1}^i(x)\theta + \varsigma_{q^i-1}^i(x)w \\
\dot{\xi}_{q^i-1}^i = A_i(x) + B_i(x)z + C_i(x)u + D_i(x)\theta + K_i(x)w
\end{gathered}
\tag{2-41}
$$

where $i = 1, \ldots, m$

Assumption 3: The matrix W of the following form has a constant row rank $r + \sum_{j=1}^{k-1} q^j + i + 1$ for $i = 0, 1, \ldots, q^k - 1$and $k = 0, 1, \ldots, m$.

$$
W = \left[\frac{\partial\phi(x)}{\partial x} \quad \frac{\partial\psi_0^1(x)}{\partial x} \quad \cdots \quad \frac{\partial\psi_{q^1-1}^1(x)}{\partial x} \quad \cdots \quad \frac{\partial\psi_0^k(x)}{\partial x} \quad \cdots \quad \frac{\partial\psi_i^k(x)}{\partial x} \quad \frac{\partial\phi_i^k(x)}{\partial x}\right]^T
$$

Lemma 2: Suppose Algorithms 1-3 are feasible and Assumptions 1-3 are satisfied. Then, in the ξ coordinate, the system (2-41) takes the form

$$
\begin{gathered}
\varepsilon = 0 \\
y^i = \xi_0^i \\
\dot{\xi}_0^i = \xi_1^i + \varphi_1^i(\varepsilon, \xi_0^i)\theta + \varsigma_1^i(\varepsilon, \xi_0^i)w \\
\vdots \\
\dot{\xi}_{q^i-2}^i = \xi_{q^i-1}^i + \varphi_{q^i-1}^i\left(\varepsilon, \xi_0^i, \xi_1^i, \ldots, \xi_{q^i-2}^i\right)\theta + \varsigma_{q^i-1}^i\left(, \xi_0^i, \xi_1^i, \ldots, \xi_{q^i-2}^i\right)w \\
\dot{\xi}_{q^i-1}^i = A_i(x) + B_i(x)z + C_i(x)u + D_i(x)\theta + K_i(x)w
\end{gathered}
\tag{2-42}
$$

where $i = 1, \ldots, m$.

Lemma 1 has been proved in [15].

2.3.1 Design Regular Feedback

The matrix $[b(x) \quad c(x)]$ in (2-39) has full row rank from Algorithm 2, so a smooth matrix-valued function $\beta(x)$ exists such that the matrix $b(x) + c(x)\beta(x)$ is nonsingular. By introducing a feedback $u = \beta(x)z + v$, the algebraic equation (2-39) takes the form

$$0 = a(x) + [b(x) + c(x)\beta(x)]z + c(x)v + d(x)\theta + k(x)w \tag{2-43}$$

Solving the equation (2-43) for the algebraic variable z and z in $u = \beta(x)z + v$ gives

$$z = -[b(x) + c(x)\beta(x)]^{-1}[a(x) + c(x)v + d(x)\theta + k(x)w] \tag{2-44}$$

$$u = -\beta(x)[b(x) + c(x)\beta(x)]^{-1}[a(x) + c(x)v + d(x)\theta + k(x)w] + v] \tag{2-45}$$

Now substitute z and u from (2-44) and (2-45) into expression for $\dot{\xi}^i_{q^i-1}$ in (2-43) and perform the simplification.

$$\begin{aligned}\dot{\xi}^i_{q^i-1} = {} & A_i(x) - B_i(x)[b(x) + c(x)\beta(x)]^{-1}[a(x) + c(x)v + d(x)\theta + k(x)w] \\ & - C_i(x)\{\beta(x)[b(x) + c(x)\beta(x)]^{-1}[a(x) + c(x)v + d(x)\theta + k(x)w] + v\} \\ & + D_i(x)\theta + K_i(x)w\end{aligned}$$

$$\begin{aligned} = {} & A_i(x) - B_i(x)[b(x) + c(x)\beta(x)]^{-1}\, a(x) - B_i(x)[b(x) + c(x)\beta(x)]^{-1}c(x)v \\ & -B_i(x)[b(x) + c(x)\beta(x)]^{-1}d(x)\theta - B_i(x)[b(x) + c(x)\beta(x)]^{-1}k(x)w \\ & -C_i(x)\gamma(x)[b(x) + c(x)\beta(x)]^{-1}a(x) - C_i(x)\gamma(x)[b(x) + c(x)\beta(x)]^{-1}c(x)v \\ & -C_i(x)\beta(x)[b(x) + c(x)\beta(x)]^{-1}d(x)\theta - C_i(x)\beta(x)[b(x) + c(x)\beta(x)]^{-1}k(x)w \\ & - C_i(x)v + D_i(x)\theta + K_i(x)w\end{aligned}$$

$$= A_i(x) - [B_i(x) + C_i(x)\beta(x)]\,[b(x) + c(x)\beta(x)]^{-1}\, a(x)$$

$$-\{C_i(x) + [B_i(x) + C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}c(x)\}v$$

$$-[B_i(x)C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}d(x)\theta + D_i(x)\theta$$

$$-[B_i(x) + C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}\, k(x)w + K_i(x)w$$

Let:

$$\hat{V}_i = A_i(x) - [B_i(x) + C_i(x)\beta(x)]\, [b(x) + c(x)\beta(x)]^{-1}\, a(x) - \{C_i(x) + [B_i(x) + C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}c(x)\}v$$

$$\varphi^i_{q^i} = -[B_i(x)C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}d(x)\theta + D_i(x)\theta$$

$$\varsigma^i_{q^i} = -[B_i(x) + C_i(x)\beta(x)][b(x) + c(x)\beta(x)]^{-1}\, k(x)w + K_i(x)w$$

The equation (2-42) take the following lower triangular form

$$\varepsilon = 0$$

$$y^i = \xi^i_0$$

$$\dot{\xi}^i_0 = \xi^i_1 + \varphi^i_1(0, \xi^i_0)\theta + \varsigma^i_1(0, \xi^i_0)w \tag{2-46}$$

$$\vdots$$

$$\dot{\xi}^i_{q^i-2} = \xi^i_{q^i-1} + \varphi^i_{q^i-1}\left(0, \xi^i_0, \xi^i_1, \dots, \xi^i_{q^i-2}\right)\theta + \varsigma^i_{q^i-1}\left(0, \xi^i_0, \xi^i_1, \dots, \xi^i_{q^i-2}\right)w$$

$$\dot{\xi}^i_{q^i-1} = \hat{V}_i + \varphi^i_{q^i}(0, \xi^1, \dots, \xi^{m-1}, \xi^m)\theta + \varsigma^i_{q^i}(0, \xi^1, \dots, \xi^{m-1}, \xi^m)w$$

where $i = 1, \dots, m$

Chapter 3
Adaptive Algorithm

3.1 Introduction

The recursive adaptive backstepping [32], [33], [34] design methodology is a powerful tool in adaptive control theory to constitute a feedback control law, the parameter estimation law and adaptation law by employing Lyapunov stability theorem for nonlinear ODE systems with lower triangular structure. The meaning of finite L_2 gain is given in section 3.1.1. The adaptive backstepping design procedure is illustrated by an example of SISO system in Section 3.2. The algorithm is expanded for two input two output systems in Section 3.3. The systematic design procedure for multi-input multi-output and multi-state system with lower triangular structure is also described in Section 3.3.

3.1.1 Finite L_2 Gain

Let consider a system described by the following equations [11], [18]:

$$\dot{\hat{x}} = F(\hat{x}) + G(\hat{x})w \tag{3-1}$$

$$y = H(\hat{x}) \tag{3-2}$$

In the equations (3-1)-(3-2), $\hat{x} \in \mathbb{R}^{\hat{n}}$ is the state vector, $w \in \mathbb{R}^{\hat{m}}$ is a time dependent disturbance, $y \in R^{\hat{p}}$ is the output, and the functions $F(.), G(.)$ and $H(.)$ are smooth functions globally on$\mathbb{R}^{\hat{n}}$. It is assumed there exists an equilibrium point $\hat{x} = \hat{x}^e$ such that $F(\hat{x}^e) = 0$, $H(\hat{x}^e) = 0$ and $G(\hat{x}^e) = 0$.

It is assumed that there exists a solution $\hat{x}(T) = \varphi(T, 0, \hat{x}_0, w)$ to the system (3-1)-(2-2) for all $T > 0$, for all $w \in W$ and $\hat{x}_0 \in \mathbb{R}^{\hat{n}}$, in which $w \in W$ at time $t = T$, and such that $\hat{x}(0) = \hat{x}_0$. The following definition is held

Definition 3.1: Let $\gamma > 0$. The system (3-1)-(3-2) has a L_2 gain less than or equal to γ if:

$$\int_0^T \|y(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt$$

for all $T \geq 0$ and all $w \in W$, and initial condition of $\hat{x}(0) = \hat{x}^e$ with $y(t) = H(\varphi(T, 0, \hat{x}_0, w))$.

3.2 Adaptive Backstepping Technique for SISO System

The recursive backstepping design methodology [62], [63] was originally introduced in adaptive control theory to construct the feedback control law, the parameter adaptation law and the associated Lyapunov function systematically for a class of nonlinear systems satisfying certain structured properties. The backstepping method is Consider the following single-input single-output (SISO) and single state system

$$\begin{aligned} \dot{\xi}_0^1 &= v_1 + \varphi_0^1\theta + \varsigma_0^1 w \\ y^1 &= \xi_0^1 \end{aligned} \tag{3-3}$$

Thanks to Lyapunov stability theorem, we introduce a Lyapunov candidate function V_0^1 of the form

$$V_0^1(\xi_0^1, \hat{\theta}_1) = \frac{1}{2}{\xi_0^1}^2 + \frac{1}{2}\left\|\theta - \hat{\theta}_1\right\|^2$$

First order derivative of $V_0^1(\zeta_0^1, \hat{\theta}_1)$ with respect to time and substituting $\dot{\xi}_0^1$ from (3-3) gives the following expressions

$$\begin{aligned} \dot{V}_0^1 &= \xi_0^1\dot{\xi}_0^1 - (\theta - \hat{\theta}_1)\dot{\hat{\theta}}_1 \\ \dot{V}_0^1 &= \left(\xi_0^1\varphi_0^1 - \dot{\hat{\theta}}_1\right)\left(\theta - \hat{\theta}_1\right) + \xi_0^1\left(v_1 + \varphi_0^1\hat{\theta}_1\right) + \xi_0^1\varsigma_0^1 w \end{aligned} \tag{3-4}$$

The Young inequality and the procedure introduced in [17] is recalled. Assume that X and Y are vector-valued of $n \times 1$. For any $\gamma > 0$, the following inequality is true:

$$X^T Y \leq \frac{1}{4\gamma^2} X^T X + \gamma^2 Y^T Y \tag{3-5}$$

Let $X^T = \zeta_0^1 \varsigma_0^1$ and $Y = w$. By considering the fact that ξ_0^1, φ_0^1 *and* w are scalar in (3-3), the following inequality holds

$$\xi_0^1 \varsigma_0^1 w \leq \frac{1}{4\gamma^2} {\xi_0^1}^2 {\varsigma_0^1}^2 + \gamma^2 w^2 \tag{3-6}$$

By adding and subtracting the necessary terms to both side of inequality (3-6) and grouping terms and simple calculations, it becomes the form

$$\begin{aligned} \dot{V}_0^1 \leq \gamma^2 w^2 + \xi_0^1 \left(v_1 + \varphi_0^1 \hat{\theta}_1 + \frac{1}{4\gamma^2} \xi_0^1 {\varsigma_0^1}^2 \right) \\ + \left(\xi_0^1 \varphi_0^1 - \dot{\hat{\theta}}_1 \right) \left(\theta - \hat{\theta}_1 \right) \end{aligned} \tag{3-7}$$

Introduce parameter estimation law and control input the form

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \xi_0^1 \varphi_0^1 \\ v_1 = \alpha_0^1 &= -\left(\varphi_0^1 \hat{\theta}_1 + \frac{1}{4\gamma^2} \xi_0^1 {\varsigma_0^1}^2 + d_0^1 \xi_0^1 \right) \end{aligned} \tag{3-8}$$

Then substituting (3-8) into (3-7), leads us to the following

$$\dot{V}_0^1 \leq \gamma^2 w^2 - d_0^1 {\xi_0^1}^2 \tag{3-9}$$

By integrating both side of (3-7) from 0 to t we have:

$$V_0^1\left(\xi_0^1(t)\right) - V_0^1\left(\xi_0^1(0)\right) \leq \int_0^t (\gamma^2 w^2 - d_0^1 {\xi_0^1}^2) dt$$

$$0 \leq \int_0^t (\gamma^2 w^2 - d_0^1 {\zeta_0^1}^2) dt$$

$$d_0^1 \int_0^t {\zeta_0^1}^2\, dt \le \int_0^t \gamma^2 w^2 dt$$

and if $d_0^1 \ge 1$

$$\int_0^t {y^1}^2\, dt \le \gamma^2 \int_0^t w^2 dt$$

By Definition 3.1, the zero initial condition is considered for derivation of L_2 gain [18]. It implies $V_0^1\left(\xi_0^1(0)\right) = 0$ where $\xi_0^1(0) = {\xi_0^1}_0$ and $V_0^1\left({\xi_0^1}_0\right) = 0$.

3.3 Adaptive Backstepping Technique for MIMO System

The above design procedure [17] [62], [63][17] can be easily expanded for a multi-input, multi-output (MIMO) system. The procedure is illustrated by the following example for two inputs two outputs and two states system. Then we expand the procedure for large scale system (2-46). Consider a system described by two sub systems

$$\begin{aligned} \dot{\xi}_0^1 &= v_1 + \varphi_0^1\theta + \varsigma_0^1 w \\ y^1 &= \xi_0^1 \\ \dot{\xi}_0^2 &= v_2 + \varphi_0^2\theta + \varsigma_0^2 w \\ y^2 &= \xi_0^2 \end{aligned} \tag{3-10}$$

The control input for the first subsystem has been designed in Section 3.2 by Lyapunov candidate function of the form $V_0^1\left(\zeta_0^1, \hat{\theta}_1\right) = \frac{1}{2}{\xi_0^1}^2 + \frac{1}{2}\left\|\theta - \hat{\theta}_1\right\|^2$. Let introduce a Lyapunov candidate function of the form $V_0^2\left(\zeta_0^1, \zeta_0^2, \hat{\theta}_2\right) = V_0^1 + \frac{1}{2}{\xi_0^2}^2 + \frac{1}{2}\left\|\theta - \hat{\theta}_2\right\|^2$ for second subsystem.

Differentiate $V_0^2(\zeta_0^1, \zeta_0^2, \hat{\theta}_2)$ with respect to time and substitute $\dot{\xi}_0^2$ from (3-10). Grouping terms gives the following expression:

$$V_0^2\left(\xi_0^1, \xi_0^2, \hat{\theta}_2\right) = V_0^1 + \frac{1}{2}{\xi_0^2}^2 + \frac{1}{2}\left\|\theta - \hat{\theta}_2\right\|^2$$

$$\dot{V}_0^2 = \dot{V}_0^1 + \xi_0^2 \dot{\xi}_0^2 - (\theta - \hat{\theta}_1)\dot{\hat{\theta}}_1$$

$$\dot{V}_0^2 = \dot{V}_0^1 + \xi_0^2(v_2 + \varphi_0^2\theta + \varsigma_0^2 w) - (\theta - \hat{\theta}_1)\dot{\hat{\theta}}_1$$

$$\dot{V}_0^2 = \dot{V}_0^1 + \xi_0^2 v_2 + 2\varphi_0^2\theta + \xi_0^2\varphi_0^2\hat{\theta} - \xi_0^2\varphi_0^2\hat{\theta} + \xi_0^1\varsigma_0^2 w - (\theta - \hat{\theta}_1)\dot{\hat{\theta}}_1$$

$$\dot{V}_0^2 = \dot{V}_0^1 + \xi_0^2 v_2 + \xi_0^2\varphi_0^2(\theta - \hat{\theta}_2) + \xi_0^2\varphi_0^2\hat{\theta} + \xi_0^2\varsigma_0^2 w - (\theta - \hat{\theta}_2)\dot{\hat{\theta}}_1$$

$$\dot{V}_0^2 = \dot{V}_0^1 + \left(\zeta_0^2\varphi_0^2 - \dot{\hat{\theta}}_2\right)\left(\theta - \hat{\theta}_2\right) + \zeta_0^2\left(v_2 + \varphi_0^2\hat{\theta}_2\right) + \zeta_0^2\varsigma_0^2 w$$

Now recall the Young inequality by letting $X^T = \zeta_0^2\varsigma_0^2$ and $Y = w$. By considering that ξ_0^2, φ_0^2 and w are scalar in (3-10), then the following inequality is true

$$\xi_0^2\varsigma_0^2 w \leq \frac{1}{4\gamma^2}{\xi_0^2}^2{\varsigma_0^2}^2 + \gamma^2 w^2 \tag{3-11}$$

By adding both side of inequalities (3-9) and (3-11) together and adding and subtracting necessary terms, we have the following expression:

$$\begin{aligned}\dot{V}_0^2 \leq 2\gamma^2 w^2 - d_0^1{\xi_0^1}^2 + \xi_0^2\left(v_2 + \varphi_0^2\hat{\theta}_2 + \frac{1}{4\gamma^2}\xi_0^2{\varsigma_0^2}^2\right) \\ + \left(\zeta_0^2\varphi_0^2 - \dot{\hat{\theta}}_2\right)\left(\theta - \hat{\theta}_2\right)\end{aligned} \tag{3-12}$$

The parameter estimation law and control law is defined the form

$$\begin{aligned}\dot{\hat{\theta}}_2 = \zeta_0^2\varphi_0^2 \\ v_2 = \alpha_0^2 = -\left(\varphi_0^2\hat{\theta}_2 + \frac{1}{4\gamma^2}\xi_0^2{\varsigma_0^2}^2 + d_0^2\xi_0^2\right)\end{aligned} \tag{3-13}$$

Then substituting (3-13) into (3-12) leads to

$$\dot{V}_0^2 \leq 2\gamma^2 w^2 - d_0^1 {\xi_0^1}^2 - d_0^2 {\xi_0^2}^2 \tag{3-14}$$

By the same approach in Section 3.2 and integrating both sides of inequality (3-14) and considering the zero initial condition, the following is derived

$$\int_0^t (d_0^1 {\zeta_0^1}^2 + d_0^2 {\zeta_0^2}^2)\, dt \leq \int_0^t 2\gamma^2 w^2 dt$$

if $d_0^1 \geq 2$ and $d_0^1 \geq 2$

$$\int_0^t ({y^1}^2 + {y^2}^2)\, dt \leq \gamma^2 \int_0^t w^2 dt$$

And it is nothing than definition of finite L_2 Gain.

Now the adaptive backstepping design procedure is expanded to large scale system. Consider an ODE system with lower triangular structure

$$\begin{gathered} y^i = \xi_0^i \\ \dot{\xi}_0^i = \xi_1^i + \varphi_1^i(\xi_0^i)\theta + \varsigma_1^i(\xi_0^i)w \\ \vdots \\ \dot{\xi}_{r-1}^i = \xi_r^i + \varphi_{r-1}^i(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i)\theta + \varsigma_{r-1}^i(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i)w \\ \dot{\xi}_r^i = \hat{V}_i + \varphi_r^i(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i, \xi_r^i)\theta + \varsigma_r^i(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i, \xi_r^i)w \end{gathered} \tag{3-15}$$

where $i = 1, \dots, m$

The target is to design an adaptive back stepping almost disturbance decoupling controller and the parameter estimator of the form $\hat{V}_i = \alpha_{q^i-1}^1(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i, \xi_r^i, \hat{\theta})$ and $\hat{\theta} = \alpha_{q^1-1}^1(\xi_0^i, \xi_1^i, \dots, \xi_{r-1}^i, \xi_r^i, \xi^m)$ to achieve the finite L_1 gain from disturbance to output.

Let consider the first subsystem.

$$y^1 = \xi_0^1$$

$$\dot{\xi}_0^1 = \xi_1^1 + \varphi_0^1(\xi_0^1)\theta + \varsigma_0^1(\xi_0^1)w$$

$$\vdots \tag{3-16}$$

$$\dot{\xi}_{r-1}^1 = \xi_r^1 + \varphi_{r-1}^1(\xi_0^1, \xi_1^1, \dots, \xi_{r-1}^1)\theta + \varsigma_{r-1}^1(\xi_0^1, \xi_1^1, \dots, \xi_{r-1}^1)w$$

$$\dot{\xi}_r^1 = \hat{V}_1 + \varphi_r^1(\xi_0^1, \xi_1^1, \dots, \xi_{r-1}^1, \xi_r^1)\theta + \varsigma_r^1(\xi_0^1, \xi_1^1, \dots, \xi_{r-1}^1, \xi_r^1)w$$

Step 1.1.

Now the adaptive controller is designed for the first equation $\dot{\xi}_0^1 = \xi_1^1 + \varphi_1^1(\xi_0^1)\theta + \varsigma_1^1(\xi_0^1)w$.

The Lyapunov candidate function is selected the form of $V_0^1\left(\zeta_0^1, \hat{\theta}\right) = \frac{1}{2}{\xi_0^1}^2 + \frac{1}{2}\left\|\theta - \hat{\theta}\right\|^2$and apply the design procedure:

$$\dot{V}_0^1 = \xi_0^1\dot{\xi}_0^1 - \dot{\hat{\theta}}^T\left(\theta - \hat{\theta}\right)$$

$$\dot{V}_0^1 = \xi_0^1(\xi_1^1 + \varphi_0^1\theta + \varsigma_0^1 w) - \dot{\hat{\theta}}^T\left(\theta - \hat{\theta}\right)$$

$$\dot{V}_0^1 = \left(\xi_0^1\varphi_0^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) + \xi_0^1(\xi_1^1 - \alpha_0^1 + \alpha_0^1 + F_0^1) + \xi_0^1 G_0^1 w$$

with $G_0^1 = \varsigma_0^1$ and $F_0^1 = \varphi_0^1\hat{\theta}$. By recalling Young inequality and letting $X^T = \zeta_0^1\varsigma_0^1$ and $Y = w$ we have:

$$\xi_0^1 G_0^1 w \leq \frac{1}{4\gamma^2}{\xi_0^1}^2 G_0^1[G_0^1]^T + \gamma^2 w^T w \tag{3-17}$$

By adding and subtracting the terms to both side of inequality (3-15) and grouping terms and simple calculations, it becomes:

$$\dot{V}_0^1 \le \gamma^2 w^T w + \xi_0^1 \left((\xi_1^1 - \alpha_0^1) + \alpha_0^1 + F_0^1 + \frac{1}{4\gamma^2} \xi_0^1 G_0^1 [G_0^1]^T \right) + \left(\xi_0^1 \varphi_0^1 - \dot{\hat{\theta}}^T \right) \left(\theta - \hat{\theta} \right) \tag{3-18}$$

Introduce the tuning function τ_0^1 and input law α_0^1 of the form

$$\begin{aligned} \tau_0^1 &= \xi_0^1 \varphi_0^1 \\ \alpha_0^1 &= -\left(d_0^1 \xi_0^1 + F_0^1 + \tfrac{1}{4\gamma^2} \xi_0^1 G_0^1 [G_0^1]^T \right) \end{aligned} \tag{3-19}$$

By applying (3-19), the inequality (3-18) becomes

$$\dot{V}_0^1 \le \gamma^2 w^T w - d_0^1 {\xi_0^1}^2 + \xi_0^1 (\xi_1^1 - \alpha_0^1) + \left(\tau_0^1 - \dot{\hat{\theta}}^T \right) \left(\theta - \hat{\theta} \right) \tag{3-20}$$

Step 1.2.

$$V_1^1 \left(\zeta_0^1, \zeta_0^2, \hat{\theta} \right) = V_0^1 + \frac{1}{2} (\xi_2^1 - \alpha_0^1)^2$$

$$\dot{V}_1^1 = \dot{V}_0^1 + (\xi_1^1 - \alpha_0^1)(\dot{\xi}_1^1 - \dot{\alpha}_0^1)$$

$$\dot{V}_1^1 = \dot{V}_0^1 + (\xi_1^1 - \alpha_0^1)\dot{\xi}_1^1 - (\xi_1^1 - \alpha_0^1)\dot{\alpha}_0^1$$

$$\dot{V}_1^1 = \dot{V}_0^1 + (\xi_1^1 - \alpha_0^1)\dot{\xi}_1^1 - (\xi_1^1 - \alpha_0^1)(\frac{\partial \alpha_0^1}{\partial \xi_0^1} \dot{\xi}_0^1 + \frac{\partial \alpha_0^1}{\partial \theta} \dot{\hat{\theta}})$$

$$\begin{aligned} \dot{V}_1^1 = \dot{V}_0^1 &+ (\xi_1^1 - \alpha_0^1)(\xi_2^1 - \alpha_0^1 + \alpha_0^1 + \varphi_1^1 \theta + \varsigma_1^1 w) \\ &- (\xi_1^1 - \alpha_0^1) \left(\frac{\partial \alpha_0^1}{\partial \xi_0^1} (\xi_1^1 + \varphi_0^1 \theta + \varsigma_0^1 w) + \frac{\partial \alpha_0^1}{\partial \xi_0^1} \dot{\hat{\theta}} \right) \end{aligned}$$

$$\dot{V}_1^1 = \dot{V}_0^1 + (\xi_1^1 - \alpha_0^1)\left((\xi_2^1 - \alpha_1^1) + \alpha_1^1 + \varphi_1^1\hat{\theta}\right) - (\xi_1^1 - \alpha_0^1)\left(\frac{\partial \alpha_0^1}{\partial \xi_0^1}\left(\xi_1^1 + \varphi_0^1\hat{\theta}\right) + \frac{\partial \alpha_0^1}{\partial \xi_0^1}\dot{\hat{\theta}}\right)$$
$$+ \left((\xi_1^1 - \alpha_0^1)(\varphi_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varphi_0^1\right)\left(\theta - \hat{\theta}\right) + (\xi_1^1 - \alpha_0^1)(\varsigma_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varsigma_0^1)w$$

$$\dot{V}_1^1 = \dot{V}_0^1 + (\xi_1^1 - \alpha_0^1)\left((\xi_2^1 - \alpha_1^1) + \alpha_1^1 + F_1^1\right) + \left((\xi_1^1 - \alpha_0^1)(\varphi_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varphi_0^1)\right)\left(\theta - \hat{\theta}\right)$$
$$+ (\xi_1^1 - \alpha_0^1)G_1^1 w$$

with $G_1^1 = \varsigma_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varsigma_0^1$ and $F_1^1 = \varphi_1^1\hat{\theta} - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\left(\xi_1^1 + \varphi_0^1\hat{\theta}\right) - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\dot{\hat{\theta}}$. Letting $X^T = (\xi_1^1 - \alpha_0^1)G_1^1$ and $Y = w$, we have:

$$(\xi_1^1 - \alpha_0^1)G_1^1 w \le \frac{1}{4\gamma^2}(\xi_1^1 - \alpha_0^1)^2 G_1^1 [G_1^1]^T + \gamma^2 w^T w \tag{3-21}$$

The same approach by adding both side of inequalities (3-20) and (3-21) together and by adding necessary terms, the following inequality is formed

$$\begin{aligned}\dot{V}_1^1 \le -d_0^1{\xi_0^1}^2 + (\xi_1^1 - \alpha_0^1)\Bigg((\xi_2^1 - \alpha_1^1) + \alpha_1^1 + \xi_0^1 + F_1^1 \\ + \frac{1}{4\gamma^2}(\xi_1^1 - \alpha_0^1)(\xi_1^1 - \alpha_0^1)^2 G_1^1[G_1^1]^T\Bigg) \\ + \left(\tau_0^1 + (\xi_1^1 - \alpha_0^1)(\varphi_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varphi_0^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) + 2\gamma^2 w^T w\end{aligned} \tag{3-22}$$

The tuning function and input law are introduced in of the form

$$\tau_1^1 = \tau_0^1 + (\xi_1^1 - \alpha_0^1)\left(\varphi_1^1 - \frac{\partial \alpha_0^1}{\partial \xi_0^1}\varphi_0^1\right) \tag{3-23}$$

$$\alpha_0^1 = -\left(d_1^1(\xi_1^1 - \alpha_0^1) + \xi_0^1 + F_1^1 + \frac{1}{4\gamma^2}(\xi_1^1 - \alpha_0^1)(\xi_1^1 - \alpha_0^1)^2 G_1^1 [G_1^1]^T\right)$$

By applying (3-23), the inequality (3-22) becomes

$$\dot{V}_1^1 \leq 2\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - d_1^1(\xi_1^1 - \alpha_0^1)^2 + (\xi_1^1 - \alpha_0^1)(\xi_2^1 - \alpha_1^1) + \left(\tau_1^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) \tag{3-24}$$

Step $1.k+1$.

Suppose that we have found $\alpha_k^1\left(\xi_0^1, \xi_1^1, \ldots, \xi_k^1, \hat{\theta}\right)$, so that $V_k^1 = V_0^1 + (\xi_k^1 - \alpha_{k-1}^1)^2$ satisfies the following

$$\dot{V}_k^1 \leq (k+1)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{k} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 + (\xi_k^1 - \alpha_{k-1}^1)(\xi_{k+1}^1 - \alpha_k^1) + \left(\tau_k^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) \tag{3-25}$$

with $\tau_k^1 = \tau_{k-1}^1 + (\xi_k^1 - \alpha_{k-1}^1)\left(\varphi_k^1 - \sum_{i=0}^{j-1} \frac{\partial \alpha_{k-1}^1}{\partial \xi_i^1} \varphi_i^1\right)$.

Now choose $V_{k+1}^1 = V_k^1 + \frac{1}{2}(\xi_{k+1}^1 - \alpha_k^1)^2$. Differentiating V_{k+1}^1 with respect to time gives:

$$\dot{V}_{k+1}^1 = \dot{V}_k^1 + (\xi_{k+1}^1 - \alpha_k^1)\left(\dot{\xi}_{k+1}^1 - \dot{\alpha}_k^1\right)$$

$$\dot{V}_{k+1}^1 = \dot{V}_k^1 + (\xi_{k+1}^1 - \alpha_k^1)\dot{\xi}_{k+1}^1 - (\xi_{k+1}^1 - \alpha_k^1)\sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1}(\xi_{i+1}^1 + \varphi_i^1 \theta + \varsigma_i^1 w) - (\xi_{k+1}^1 - \alpha_k^1)\frac{\partial \alpha_k^1}{\partial \hat{\theta}}\dot{\hat{\theta}}$$

$$\dot{V}_{k+1}^1 = \dot{V}_k^1 + (\xi_{k+1}^1 - \alpha_k^1)(\xi_{k+2}^1 - \alpha_{k+1}^1 + \alpha_{k+1}^1 + F_{k+1}^1)$$
$$+ (\xi_{k+1}^1 - \alpha_k^1)\left(\varphi_{k+1}^1 - \sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1}\varphi_i^1\right)(\theta - \hat{\theta}) + (\xi_{k+1}^1 - \alpha_k^1)G_{k+1}^1 w$$

with $G_{k+1}^1 = \varsigma_{k+1}^1 - \sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1}\varsigma_i^1$ and $F_{k+1}^1 = \varphi_{k+1}^1\hat{\theta} - \sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1}\left(\xi_{i+1}^1 + \varphi_i^1\hat{\theta}\right) - \frac{\partial \alpha_k^1}{\partial \hat{\theta}}\dot{\hat{\theta}}$. Now Letting $X^T = (\xi_{k+1}^1 - \alpha_k^1)G_{k+1}^1$ and $Y = w$ and recall Young inequality

$$(\xi_{k+1}^1 - \alpha_k^1)G_{k+1}^1 w \leq \frac{1}{4\gamma^2}(\xi_{k+1}^1 - \alpha_k^1)^2 G_{k+1}^1 [G_{k+1}^1]^T + \gamma^2 w^T w \tag{3-26}$$

Adding both side of inequalities (3-25) and (3-26) together and by adding necessary terms the following inequality is obtained

$$\begin{aligned}\dot{V}_{k+1}^1 \leq {} & (k+2)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{k} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 \\ & + (\xi_{k+1}^1 - \alpha_k^1)\Big((\xi_{k+2}^1 - \alpha_{k+1}^1) + \alpha_{k+1}^1 + (\xi_k^1 - \alpha_{k-1}^1) + F_{k+1}^1 \\ & + \frac{1}{4\gamma^2}(\xi_{k+1}^1 - \alpha_k^1)G_{k+1}^1[G_{k+1}^1]^T\Big) \\ & + \left(\tau_k^1 + (\xi_{k+1}^1 - \alpha_k^1)\left(\varphi_{k+1}^1 - \sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1}\varphi_i^1\right) - \dot{\hat{\theta}}^T\right)(\theta - \hat{\theta})\end{aligned} \tag{3-27}$$

Let us introduce tuning function and input law of the form

$$\begin{aligned}\alpha_{k+1}^1 = -\Big(& d_{k+1}^1(\xi_{k+1}^1 - \alpha_k^1) + (\xi_k^1 - \alpha_{k-1}^1) + F_{k+1}^1 \\ & + \frac{1}{4\gamma^2}(\xi_{k+1}^1 - \alpha_k^1)G_{k+1}^1[G_{k+1}^1]^T\Big)\end{aligned} \tag{3-28}$$

$$\tau_{k+1}^1 = \tau_k^1 + (\xi_{k+1}^1 - \alpha_k^1)\left(\varphi_{k+1}^1 - \sum_{i=0}^{k} \frac{\partial \alpha_k^1}{\partial \xi_i^1} \varphi_i^1\right)$$

By applying (3-28) the inequality (3-27) becomes

$$\dot{V}_{k+1}^1 \leq (k+2)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{k+1} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 + (\xi_{k+1}^1 - \alpha_k^1)(\xi_{k+2}^1 - \alpha_{k+1}^1) + \left(\tau_{k+1}^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) \tag{3-29}$$

Step $1.r$.

By the same reasoning to step $k+1$ and selecting $V_r^1 = V_{r-1}^1 + \frac{1}{2}(\xi_r^1 - \alpha_{r-1}^1)^2$, the parameter adaptation and input laws is derived in the form

$$\begin{aligned} \dot{V}_r^1 \leq{} & (r+1)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{r-1} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 \\ & + (\xi_r^1 - \alpha_{r-1}^1)\Big(V_r^1 + (\xi_{r-1}^1 - \alpha_{r-2}^1) + F_r^1 \\ & + \frac{1}{4\gamma^2}(\xi_r^1 - \alpha_{r-1}^1) G_r^1 [G_r^1]^T\Big) \\ & + \left(\tau_{r-1}^1 + (\xi_r^1 - \alpha_{r-1}^1)\left(\varphi_r^1 - \sum_{i=0}^{r-1} \frac{\partial \alpha_{r-1}^1}{\partial \xi_i^1} \varphi_i^1\right) - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) \end{aligned} \tag{3-30}$$

So let introduce tuning function and input law in the following form

$$\hat{V}_1 = \alpha_r^1 = -\Big(d_r^1(\xi_r^1 - \alpha_{r-1}^1) + (\xi_{r-1}^1 - \alpha_{r-2}^1) + F_r^1 + \frac{1}{4\gamma^2}(\xi_r^1 - \alpha_{r-1}^1)G_r^1[G_r^1]^T\Big) \tag{3-31}$$

$$\tau_r^1 = \tau_{r-1}^1 + (\xi_r^1 - \alpha_{r-1}^1)\left(\varphi_r^1 - \sum_{i=0}^{r-1}\frac{\partial \alpha_{r-1}^1}{\partial \xi_i^1}\varphi_i^1\right)$$

By applying (3-31) the inequality (3-30) becomes

$$\dot{V}_r^1 \le (r+1)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{r} d_i^1(\xi_i^1 - \alpha_{i-1}^1)^2 + \left(\tau_r^1 - \dot{\hat{\theta}}^T\right)\left(\theta - \hat{\theta}\right) \tag{3-32}$$

Now consider the second subsystem:

$$\begin{aligned} y^2 &= \xi_0^2 \\ \dot{\xi}_0^2 &= \xi_1^2 + \varphi_0^2(\xi_0^2)\theta + \varsigma_0^2(\xi_0^2)w \\ &\vdots \\ \dot{\xi}_{r-1}^2 &= \xi_r^2 + \varphi_{r-1}^2(\xi_0^2, \xi_1^2, \dots, \xi_{r-1}^2)\theta + \varsigma_{r-1}^2(\xi_0^2, \xi_1^2, \dots, \xi_{r-1}^2)w \\ \dot{\xi}_r^2 &= \hat{V}_2 + \varphi_r^2(\xi_0^2, \xi_1^2, \dots, \xi_{r-1}^2, \xi_r^2)\theta + \varsigma_r^2(\xi_0^2, \xi_1^2, \dots, \xi_{r-1}^2, \xi_r^2)w \end{aligned} \tag{3-33}$$

Applying the same design procedure $\hat{V}_2$ is derived. The following steps show the procedure.

Step 2.1.

The Lyapunov candidate function for the first equation in (3-33) is selected of the form $V_0^2(\zeta_0^2, \hat{\theta}) = V_r^1 + \frac{1}{2}{\xi_0^2}^2$. Now differentiate V_0^2 with respect to time and substitute $\dot{\xi}_0^2$. The expression for $\dot{V}_0^2$ take the form

$$\dot{V}_0^2 = \dot{V}_r^1 + \xi_0^2\dot{\xi}_0^2 = \dot{V}_r^1 + \xi_0^2\left((\xi_1^2 - \alpha_0^2) + \alpha_0^2 + \varphi_0^2\hat{\theta}\right) + \xi_0^2\varsigma_0^2 w + \xi_0^2\varphi_0^2(\theta - \hat{\theta})$$

with $G_0^2 = \varsigma_0^2$ and $F_0^2 = \varphi_0^2\hat{\theta}$. Let $X^T = \zeta_0^2\varsigma_0^2$ and $Y = w$ and perform Young inequality

$$\xi_0^2 G_0^2 w \leq \frac{1}{4\gamma^2} {\xi_0^2}^2 G_0^2 [G_0^2]^T + \gamma^2 w^T w \tag{3-34}$$

Add both side of inequalities (3-32) and (3-34) together. Then add and subtract the required terms to both side of it and group terms and perform simple calculations get the following inequality

$$\dot{V}_0^2 \leq (r+2)\gamma^2 w^T w + \xi_0^2 \left((\xi_1^2 - \alpha_0^2) + \alpha_0^2 + F_0^2 + \frac{1}{4\gamma^2} \xi_0^2 G_0^2 [G_0^2]^T \right) \\ -d_0^1 {\xi_0^1}^2 - \sum_{i=1}^r d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 + \left(\tau_r^1 + \xi_0^2 \varphi_0^2 - \dot{\hat{\theta}}^T \right) \left(\theta - \hat{\theta} \right) \tag{3-35}$$

Introduce a tuning function of the form $\tau_0^2 = \tau_r^1 + \xi_0^2 \varphi_0^2$ and input law of the form $\alpha_0^1 = -\left(d_0^1 \xi_0^1 + F_0^2 + \frac{1}{4\gamma^2} \xi_0^2 G_0^2 [G_0^2]^T \right)$ and substitute them in (3-35). It is not difficult to verify that $\dot{V}_0^2$ take the form

$$\dot{V}_0^2 \leq (r+2)\gamma^2 w^T w - d_0^2 {\xi_0^2}^2 + \xi_0^2 (\xi_1^2 - \alpha_0^2) - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^r d_i^1 (\xi_i^1 - \\ \alpha_{i-1}^1)^2 + \left(\tau_0^2 - \dot{\hat{\theta}}^T \right) \left(\theta - \hat{\theta} \right) \tag{3-36}$$

Step $2.r$.

By the same procedure and reasoning in Step $1.r$, select $V_r^2 = V_{r-1}^2 + \frac{2}{2}(\xi_r^2 - \alpha_{r-1}^2)^2$ and differentiate it with respect to time to form the following inequality

$$\dot{V}_r^1 \leq 2(r+1)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{r} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 - d_0^2 {\xi_0^2}^2 - \sum_{i=1}^{r-1} d_i^2 (\xi_i^2 - \alpha_{i-1}^2)^2 + (\xi_r^2 - \alpha_{r-1}^2)\left(V_r^2 + (\xi_{r-1}^2 - \alpha_{r-2}^2) + F_r^2 + \frac{1}{4\gamma^2}(\xi_r^2 - \alpha_{r-1}^2) G_r^2 [G_r^2]^T\right) + \left(\tau_{r-1}^2 + (\xi_r^2 - \alpha_{r-1}^2)\left(\varphi_r^2 - \sum_{i=0}^{r-1} \frac{\partial \alpha_{r-1}^2}{\partial \xi_i^2} \varphi_i^2\right) - \dot{\hat{\theta}}^T\right)(\theta - \hat{\theta}) \quad \text{(3-37)}$$

The tuning function and control input law take the form

$$\hat{V}_2 = \alpha_r^2 = -\left(d_r^2(\xi_r^2 - \alpha_{r-1}^2) + (\xi_{r-1}^2 - \alpha_{r-2}^2) + F_r^2 + \frac{1}{4\gamma^2}(\xi_r^2 - \alpha_{r-1}^2) G_r^2 [G_r^2]^T\right) \quad \text{(3-38)}$$

$$\tau_r^2 = \tau_{r-1}^2 + (\xi_r^2 - \alpha_{r-1}^2)\left(\varphi_r^2 - \sum_{i=0}^{r-1} \frac{\partial \alpha_{r-1}^2}{\partial \xi_i^2} \varphi_i^2\right)$$

By applying (3-39), the inequality (3-39) becomes the form

$$\dot{V}_r^2 \leq 2(r+1)\gamma^2 w^T w - d_0^1 {\xi_0^1}^2 - \sum_{i=1}^{r} d_i^1 (\xi_i^1 - \alpha_{i-1}^1)^2 - d_0^2 {\xi_0^2}^2 - \sum_{i=1}^{r} d_i^2 (\xi_i^2 - \alpha_{i-1}^2)^2 + \left(\tau_r^2 - \dot{\hat{\theta}}^T\right)(\theta - \hat{\theta}) \quad \text{(3-39)}$$

Now the same procedure is applied for $i = 1, 2, \ldots, m$ in (3-15) to derive the following results:

$$\hat{V}_i = \alpha_r^i = -\Big(d_r^i\big(\xi_r^i - \alpha_{r-1}^i\big) + \big(\xi_{r-1}^i - \alpha_{r-2}^i\big) + F_r^i + \frac{1}{4\gamma^2}\big(\xi_r^i - \alpha_{r-1}^i\big)G_r^i\big[G_r^i\big]^T\Big) \tag{3-40}$$

$$\tau_r^i = \tau_{r-1}^i + \big(\xi_r^i - \alpha_{r-1}^i\big)\left(\varphi_r^i - \sum_{j=0}^{r-1}\frac{\partial\alpha_{r-1}^i}{\partial\xi_j^i}\varphi_j^i\right)$$

By applying (3-41) the inequality (3-28) in general form becomes

$$\dot{V}_r^i \le i(r+1)\gamma^2 w^T w - \sum_{l=1}^{i}\left\{d_0^l {\xi_0^l}^2 + \sum_{j=1}^{r} d_j^l\big(\xi_j^l - \alpha_{j-1}^l\big)^2\right\} + \Big(\tau_r^l - \dot{\hat{\theta}}^T\Big)\big(\theta - \hat{\theta}\big) \tag{3-41}$$

Reviewing (3-41) for $i = m$ and letting $\dot{\hat{\theta}}^T = \tau_r^m$, gives us:

$$0 \le \dot{V}_r^m \le m(r+1)\gamma^2 w^T w - \sum_{l=1}^{m}\left\{d_0^l {\xi_0^l}^2 + \sum_{j=1}^{r} d_j^l\big(\xi_j^l - \alpha_{j-1}^l\big)^2\right\} \tag{3-42}$$

By choosing $d_0^l \ge m(r+1), (l = 1,2, \dots, m)$, it follows from (3-43) that

$$0 \le m(r+1)\gamma^2 w^T w - m(r+1)\sum_{l=1}^{m} {\xi_0^l}^2$$

And it implies $\int_0^\infty \sum_{l=1}^m {\xi_0^l}^2\, dt \le \gamma^2 \int_0^\infty w^T w dt$. Therefor the L_2 Gain from disturbance w to $(\xi_0^1, \dots, \xi_0^m)$ does not exceed γ.

Chapter 4
Simulation Example to Physical Model

4.1 Introduction

Recently there has been extensive attraction to parallel robot and with great studies have been conducted on different control aspects of the parallel robots [17], [36], [37], [38], [39], [40] and [41]. The attraction comes from advantages of the parallel robots over serial robots, such as high stiffness, high speeds, low inertia and large payload capacity [42], [43], [44].

In this Chapter, the design procedure will be applied to a two degree of freedom (2DOF) planar parallel. The stabilization problem will be demonstrated in this simulation work based on methodology introduced in Chapter 3.

In Section 4.2, the dynamic model for 2DOF planar parallel robot is reviewed. The regularization and standardization algorithms are performed in Sections 4.3 and 4.4 respectively. In Section 4.5, the adaptive controller is constructed and Section 4.6 shows the simulation results.

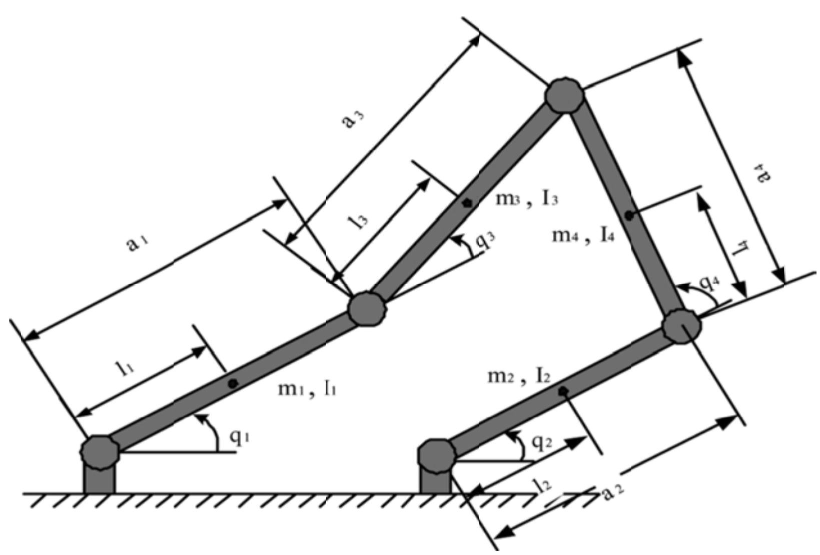

Figure 4-1: 2DOF Planar Parallel Robot.

4.2 Dynamic Model for 2DOF Parallel Robot

The schematic of a 2DOF planar parallel robot [42], [43], [44], [45], [46] is shown in Figure 4-1: 2DOF Planar Parallel Robot. In this schematic, m_i is mass of link i, a_i is length of link i, l_i is distance to the centre of mass from the lower joint of link i and I_i represents the mass moment of inertia of link i. There are four joints from which, the joints q_1 and q_2 are driven by motors mounted at the base of each link while the joints q_3 and q_4 are passive.

The dynamic model of robot [15] is written by the equations of the form

$$D'(q')\ddot{q}' + C'(q',\dot{q}')\dot{q}' + G'(q') = u' + \omega' \tag{4-1}$$

$$\phi(q') = 0 \tag{4-2}$$

In the equations (4-1) and (4-2) the vector $q' = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ is the vector of dependent generalized coordinates.

The selected physical model involves the unknown parameters and disturbances. The unknown parameters [44] are mass of link i, distance to the centre of mass from the lower joint of link i, and the mass moment of inertia of link i, while the disturbance is a reaction torque or force vector torque from the object/environment acting on the links. The vector of unknown parameters [44] is defined as $\Theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}]^T$ with the following elements

$$\theta_1 = m_1 {l_1}^2 + m_3 {a_1}^2 + I_1,\ \theta_2 = m_2 {l_2}^2 + m_4 {a_2}^2 + I_2,\ \theta_3 = m_3 {l_3}^2 + I_3,\ \theta_4 = m_4 {l_4}^2 + I_4,$$
$$\theta_5 = m_3 a_1 l_3,\ \theta_6 = m_4 a_2 l_4,\ \theta_7 = (m_1 l_1 + m_3 a_1)g,\ \theta_8 = (m_2 l_2 + m_4 a_2)g,\ \theta_9 = m_3 l_3 g,$$
$$\theta_{10} = m_4 l_4 g.$$

Different terms in equation (4-1) are defined as follows [44]

$D'(q') = \begin{bmatrix} d'_{11} & 0 & d'_{13} & 0 \\ 0 & d'_{22} & 0 & d'_{24} \\ d'_{31} & 0 & d'_{33} & 0 \\ 0 & d'_{42} & 0 & d'_{44} \end{bmatrix}$: 4×4 positive definite inertia matrix with elements below:

$$d'_{11} = \theta_1 + \theta_3 + 2\theta_5 \cos(q_3),\ d'_{13} = d'_{31} = \theta_3 + \theta_5 \cos(q_3),\ d'_{22} = \theta_2 + \theta_4 + 2\theta_6 \cos(q_4),$$
$$d'_{24} = d'_{42} = \theta_4 + \theta_6 \cos(q_3),\ d'_{33} = \theta_3,\ d'_{44} = \theta_4$$

$C'(q', \dot{q}') = \begin{bmatrix} c'_{11} & 0 & c'_{13} & 0 \\ 0 & c'_{22} & 0 & c'_{24} \\ c'_{31} & 0 & 0 & 0 \\ 0 & c'_{42} & 0 & 0 \end{bmatrix}$: 4×4 Coriolis or centrifugal torque matrix with the following elements

$$c'_{11} = -\theta_5 \sin(q_3)\dot{q}_3,\ c'_{13} = -\theta_5 \sin(q_3)(\dot{q}_1 + \dot{q}_3),\ c'_{22} = -\theta_6 \sin(q_4)\dot{q}_4,$$

$$c'_{24} = -\theta_6 \sin(q_4)(\dot{q}_2 + \dot{q}_4),\ c'_{31} = \theta_5 \sin(q_3)\dot{q}_1,\ c'_{42} = \theta_6 \sin(q_4)\dot{q}_2,$$

$G'(q) = [g'_{11} \quad g'_{21} \quad g'_{31} \quad g'_{41}]^T$: 4×1 gravity vector with the following elements

$$g'_{11} = \theta_7 \cos(q_1) + \theta_9 \cos(q_1 + q_3),\ g'_{21} = \theta_8 \cos(q_1) + \theta_{10} \cos(q_1 + q_3)$$

$$g'_{31} = \theta_9 \cos(q_1 + q_3),\ g'_{41} = \theta_{10} \cos(q_2 + q_4)$$

$u' = [u_1 \quad u_2 \quad 0 \quad 0]^T$: 4×1 input torque vector

$\phi(q') = \begin{bmatrix} \phi_1(q') \\ \phi_2(q') \end{bmatrix}$: is constraint. The expressions for constraint are

$$\phi_1(q') = a_1 \cos(q_1) + a_3 \cos(q_1 + q_3) - c - a_2 \cos(q_2) - a_4 \cos(q_2 + q_4) \tag{4-3}$$

$$\phi_2(q') = a_1 \sin(q_1) + a_3 \sin(q_1 + q_3) - a_2 \sin(q_2) - a_4 \sin(q_2 + q_4) \tag{4-4}$$

$\omega' = [\omega_1 \quad \omega_2 \quad 0 \quad 0]^T$: is disturbance which is a reaction torque or force vector from the object acting on the links.

By simple row operations on the equations (4-1), the following differential equations are obtained

$$d'_{11}\ddot{q}_1 + d'_{42}\ddot{q}_2 + d'_{13}\ddot{q}_3 + d'_{44}\ddot{q}_4 + c'_{11}\dot{q}_1 + c'_{42}\dot{q}_2 + c'_{13}\dot{q}_3 + g'_{11} = u_1 + \omega_1 \tag{4-5}$$

$$d'_{31}\ddot{q}_1 + d'_{22}\ddot{q}_2 + d'_{33}\ddot{q}_3 + d'_{24}\ddot{q}_4 + c'_{22}\dot{q}_2 + c'_{31}\dot{q}_1 + c'_{24}\dot{q}_4 + g'_{21} = u_2 + \omega_2 \tag{4-6}$$

4.3 Regularization

The dynamic model described by equations (4-5), (4-6) and (4-2) is not in a semi-explicit form of (2-9) and (2-10). But, the regularization algorithm introduced in Section 2.2 can be used with some modifications such as defining generalized characteristic number in form of vector. Performing the regularization algorithm on the algebraic equation (4-2), is equivalent to differentiating (4-2) twice, which gives $r^1 = 2$ and $r^2 = 2$ respectively.

Differentiating (4-2) with respect to time, yields the equations of the form

$$\dot{\phi}(q', \dot{q}') = 0 \tag{4-7}$$

where $\dot{\phi}(q', \dot{q}') = \begin{bmatrix} \dot{\phi}_1(q', \dot{q}') \\ \dot{\phi}_2(q', \dot{q}') \end{bmatrix}$ with

$$\dot{\phi}_1(q',\dot{q}') = -a_1\dot{q}_1\sin(q_1) - a_3(\dot{q}_1+\dot{q}_3)\sin(q_1+q_3) + a_2\dot{q}_2\sin(q_2) + a_4(\dot{q}_2+\dot{q}_4)\sin(q_2+q_4) \tag{4-8}$$

$$\dot{\phi}_2(q',\dot{q}') = a_1\dot{q}_1\,cos(q_1) + a_3(\dot{q}_1+\dot{q}_3)\,cos(q_1+q_3) - a_2\dot{q}_2\,cos(q_2) - a_4(\dot{q}_2+\dot{q}_4)\,cos(q_2+q_4) \tag{4-9}$$

Differentiating (4-7) produces

$$\ddot{\phi}(q',\dot{q}') = 0 \tag{4-10}$$

where $\ddot{\phi}(q',\dot{q}') = \begin{bmatrix}\ddot{\phi}_1(q',\dot{q}')\\ \ddot{\phi}_2(q',\dot{q}')\end{bmatrix}$ with

$$\begin{aligned}\ddot{\phi}_1(q',\dot{q}') = &-a_1\ddot{q}_1\sin(q_1) - a_1\dot{q}_1^{\,2}\cos q_1\\ &- a_3(\ddot{q}_1+\ddot{q}_3)\sin(q_1+q_3) - a_3(\dot{q}_1+\dot{q}_3)^2\cos(q_1+q_3)\\ &+ a_2\ddot{q}_2\sin(q_2) + a_2\dot{q}_2^{\,2}\cos(q_2)\\ &+ a_4(\ddot{q}_2+\ddot{q}_4)\sin(q_2+q_4) + a_4(\dot{q}_2+\dot{q}_4)^2\sin(q_2+q_4)\end{aligned} \tag{4-11}$$

$$\begin{aligned}\ddot{\phi}_2(q',\dot{q}') = &\,a_1\ddot{q}_1\,cos(q_1) - a_1\dot{q}_1^{\,2}\,sin(q_1)\\ &+ a_3(\ddot{q}_1+\ddot{q}_3)\,cos(q_1+q_3)\\ &- a_3(\dot{q}_1+\dot{q}_3)^2\sin(q_1+q_3) - a_2\ddot{q}_2\,cos(q_2) + a_2\dot{q}_2^{\,2}\,sin(q_2)\\ &- a_4(\ddot{q}_2+\ddot{q}_4)\,cos(q_2+q_4) + a_4(\dot{q}_2+\dot{q}_4)^2\,sin(q_2+q_4)\end{aligned} \tag{4-12}$$

Solving (4-2) for q_3 and q_4 gives the following results

$$q_3 = \tan^{-1}\left(\frac{\mu + a_4\sin(q_2+q_4)}{\lambda + a_4\cos(q_2+q_4)}\right) - q_1 \tag{4-13}$$

$$q_4 = \tan^{-1}\left(\frac{\sqrt{A^2 + B^2 - C^2}}{C}\right) + \tan^{-1}\left(\frac{B}{A}\right) - q_2 \tag{4-14}$$

with $A = 2a_4\lambda, B = 2a_4\mu, C = {a_3}^2 - {a_4}^2 - \lambda^2 - \mu^2, \lambda = a_2\cos(q_2) - a_1\cos(q_1) + c$

$and\ \ \mu = a_2\sin(q_2) - a_1\sin(q_1)$

Solving (4-7) for $\dot{q}_3$ and $\dot{q}_4$, we have [40]

$$\dot{q}_3 = \rho_{31}\dot{q}_1 + \rho_{32}\dot{q}_2 \tag{4-15}$$

$$\dot{q}_4 = \rho_{41}\dot{q}_1 + \rho_{42}\dot{q}_2 \tag{4-16}$$

In order to get $\ddot{q}_3$ and $\ddot{q}_4$ differentiating (4-15) and (4-16) instead of solving (4-10) results in

$$\ddot{q}_3 = \dot{\rho}_{31}\dot{q}_1 + \rho_{31}\ddot{q}_1 + \dot{\rho}_{32}\dot{q}_2 + \rho_{32}\ddot{q}_2 \tag{4-17}$$

$$\ddot{q}_4 = \dot{\rho}_{41}\dot{q}_1 + \rho_{41}\ddot{q}_1 + \dot{\rho}_{42}\dot{q}_2 + \rho_{42}\ddot{q}_2 \tag{4-18}$$

The expressions for ρ_{31}, ρ_{32}, ρ_{41}, ρ_{42}, $\dot{\rho}_{31}$, $\dot{\rho}_{32}$, $\dot{\rho}_{41}$ and $\dot{\rho}_{42}$ are introduced later. Plug (4-15), (4-16), (4-17) and (4-18) in (4-5) and (4-6) and sort the terms to produce the following equations

$$\begin{aligned}(d'_{11} + d'_{13}\rho_{31} + d'_{44}\rho_{41})\ddot{q}_1 + (d'_{42} + d'_{13}\rho_{32} + d'_{44}\rho_{42})\ddot{q}_2 \\ + (c'_{11} + c'_{13}\rho_{31} + d_{13}\dot{\rho}_{31} + d'_{44}\dot{\rho}_{41})\dot{q}_1 \\ + (c'_{42} + c'_{13}\rho_{32} + d'_{13}\dot{\rho}_{32} + d'_{44}\dot{\rho}_{42})\dot{q}_2 + g'_{11} + g'_{41} = u_1 + \omega_1\end{aligned} \tag{4-19}$$

$$(d'_{31} + d'_{33}\rho_{31} + d'_{24}\rho_{41})\ddot{q}_1 + (d'_{22} + d'_{33}\rho_{32} + d'_{24}\rho_{42})\ddot{q}_2 \\ + (c'_{31} + c'_{24}\rho_{41} + d'_{33}\dot{\rho}_{31} + d'_{24}\dot{\rho}_{41})\dot{q}_1 \\ + (c'_{22} + c'_{24}\rho_{42} + d_{33}\dot{\rho}_{32} + d'_{24}\dot{\rho}_{42})\dot{q}_2 + g'_{21} + g'_{31} = u_2 + \omega_2 \tag{4-20}$$

The compact form of differential equations (4-19) and (4-20) takes the form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = u + \omega \tag{4-21}$$

and the constraint takes the form:

$$\phi_1(q',\dot{q}') = 0 \tag{4-22}$$

$$\phi_2(q',\dot{q}') = 0$$

$$\dot{\phi}_1(q',\dot{q}') = 0$$

$$\dot{\phi}_2(q',\dot{q}') = 0$$

with $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, D(q) = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}, C(q,\dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, G(q) = \begin{bmatrix} g_{11} \\ g_{21} \end{bmatrix}, \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$ and $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$.

The elements of $D(q)$, $C(q,\dot{q})$ and $G(q)$ [42], [43], [44] can be expressed as $D_{ojk}\Theta$,$C_{ojk}\Theta$ $g_{ojk}\Theta$ for $j,k = 1,2$ where

$$D_{ojk} = [D_{ojk1} \quad D_{ojk2} \quad \dots \quad D_{ojk10}]$$

$D_{o111} = 1$, $D_{o112} = 0$, $D_{o113} = (1 + \rho_{31})^2$, $D_{o114} = {\rho_{41}}^2$, $D_{o115} = 2(1 + \rho_{31})cos(q_3)$, $D_{o116} = 0$, $D_{o117} = 0$, $D_{o118} = 0$, $D_{o119} = 0$, $D_{o1110} = 0$

$D_{o121}=0,\ D_{o122}=0,\ D_{o123}=(1+\rho_{31})\rho_{32},\ D_{o124}=(1+\rho_{42})\rho_{41},\ D_{o125}=\rho_{32}cos(q_3),$
$D_{o126}=\rho_{41}cos(q_4), D_{o127}=0, D_{o128}=0, D_{o129}=0, D_{o1210}=0$

$D_{o211}=1,\ D_{o212}=0,\ D_{o213}=(1+\rho_{31})\rho_{32},\ D_{o214}=(1+\rho_{42})\rho_{41},\ D_{o215}=\rho_{32}cos(q_3),$
$D_{o216}=0\ , D_{o217}=0\ ,\ D_{o218}=0, D_{o219}=0, D_{o2110}=0$

$D_{o221}=0, D_{o222}=1, D_{o223}={\rho_{32}}^2, D_{o224}=(1+\rho_{42})^2, D_{o225}=0,$
$D_{o226}=2(1+\rho_{41})cos(q_4), D_{o227}=0, D_{o228}=0, D_{o229}=0, D_{o2210}=0$

$$C_{ojk}=[C_{ojk1}\quad C_{ojk2}\quad \cdots \quad C_{ojk10}]$$

$C_{o111}=0, C_{o112}=0, C_{o113}=(1+\rho_{31})\dot{\rho}_{31}, C_{o114}=\rho_{41}\dot{\rho}_{41},$
$C_{o115}=\dot{\rho}_{31}cos(q_3)-(1+\rho_{31})\dot{q}_3\sin q_3,\quad C_{o116}=0,\quad C_{o117}=0,\quad C_{o118}=0,\quad C_{o119}=0,$
$C_{o1110}=0$

$C_{o121}=0, C_{o122}=0, C_{o123}=(1+\rho_{31})\dot{\rho}_{32}, C_{o124}=\rho_{41}\dot{\rho}_{42},$
$C_{o125}=\dot{\rho}_{32}cos(q_3)-(\dot{q}_1+\dot{q}_3)\rho_{32}\sin q_3,\quad C_{o126}=\rho_{41}\dot{q}_2\sin q_4,\quad C_{o127}=0,\quad C_{o128}=0,$
$C_{o129}=0, C_{o1210}=0$

$C_{o211}=0,\quad C_{o212}=0,\quad C_{o213}=\rho_{32}\dot{\rho}_{31},\quad C_{o214}=(1+\rho_{42})\dot{\rho}_{41},\quad C_{o215}=\rho_{32}\dot{q}_1\sin q_3,$
$C_{o216}=\dot{\rho}_{41}cos(q_4)-(\dot{q}_2+\dot{q}_4)\rho_{41}\sin q_4, C_{o217}=0, C_{o218}=0, C_{o219}=0, C_{o2110}=0$

$C_{o221}=0, C_{o222}=0, C_{o223}=\rho_{32}\dot{\rho}_{32}, C_{o224}=(1+\rho_{42})\dot{\rho}_{42}, C_{o225}=0,$
$C_{o226}=\dot{\rho}_{42}cos(q_4)-(1+\rho_{32})\rho_{42}\dot{q}_4\sin q_4, C_{o227}=0, C_{o228}=0, C_{o229}=0, C_{o2210}=0$

$$g_{ojk}=[g_{ojk1}\quad g_{ojk2}\quad \cdots \quad g_{ojk10}]$$

$g_{o11}=0,\ g_{o12}=0,\ g_{o13}=0,\ g_{o14}=0,\ g_{o15}=0,\ g_{o16}=0,\ g_{o17}=\cos q_1,\ g_{o18}=0,$
$g_{o19}=(1+\rho_{31})\cos(q_1+q_3), g_{o110}=\rho_{41}\cos(q_2+q_4)$

$g_{o21}=0,\ g_{o22}=0,\ g_{o23}=0,\ g_{o24}=0,\ g_{o25}=0,\ g_{o26}=0,\ g_{o27}=0,\ g_{o28}=\cos q_2,$
$g_{o29}=\rho_{32}\cos(q_1+q_3), g_{o210}=(1+\rho_{42})\cos(q_2+q_4)$

The expressions for ρ_{31}, ρ_{32}, ρ_{41}, ρ_{42}, $\dot{\rho}_{31}$, $\dot{\rho}_{32}$, $\dot{\rho}_{41}$ and $\dot{\rho}_{42}$ are derived from the following equations

$$\rho(q') = {\Psi_{q'}}^{-1}(q')\begin{bmatrix}0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\\ \rho_{31} & \rho_{32}\\ \rho_{41} & \rho_{42}\end{bmatrix} \tag{4-23}$$

$$\dot{\rho}(q') = \begin{bmatrix}0 & 0\\ 0 & 1\\ \dot{\rho}_{31} & \dot{\rho}_{32}\\ \dot{\rho}_{41} & \dot{\rho}_{42}\end{bmatrix} = -{\Psi_{q'}}^{-1}(q')\dot{\Psi}_{q'}(q', \dot{q}')\rho(q') \tag{4-24}$$

$$\Psi_{q'}(q') = \begin{bmatrix}\Psi_{q'11} & \Psi_{q'12} & \Psi_{q'13} & \Psi_{q'14}\\ \Psi_{q'21} & \Psi_{q'22} & \Psi_{q'23} & \Psi_{q'24}\\ 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \tag{4-25}$$

The elements of $\Psi_{q'}(q')$ are

$$\Psi_{q'11} = -a_1\sin(q_1) - a_3\sin(q_1+q_3),$$

$$\Psi_{q'12} = a_2\sin(q_2) + a_4\sin(q_2+q_4)$$

$$\Psi_{q'13} = -a_3\sin(q_1+q_3),$$

$$\Psi_{q'14} = a_4\sin(q_2+q_4)\,, \psi_{q'21} = a_1\cos(q_1) + a_3\cos(q_1+q_3)$$

$$\Psi_{q'22} = -a_2\cos(q_2) - a_4\cos(q_2+q_4),$$

$$\Psi_{q'23} = a_3\cos(q_1+q_3), \Psi_{q'24} = a_4\cos(q_2+q_4)$$

The matrix of $\dot{\Psi}_{q'}(q',\dot{q}')$ and its elements take the form

$$\dot{\Psi}_{q'}(q',\dot{q}') = \begin{bmatrix} \dot{\psi}_{q'11} & \dot{\psi}_{q'12} & \dot{\psi}_{q'13} & \dot{\psi}_{q'14} \\ \dot{\psi}_{q'21} & \dot{\psi}_{q'22} & \dot{\psi}_{q'23} & \dot{\psi}_{q'24} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-26}$$

$$\dot{\Psi}_{q'11} = -a_1\cos(q_1)\,\dot{q}_1 - a_3\cos(q_1+q_3)(\dot{q}_1+\dot{q}_3),$$

$$\dot{\Psi}_{q'12} = a_2\cos(q_2)\,\dot{q}_2 + a_4\cos(q_2+q_4)(\dot{q}_2+\dot{q}_4),$$

$$\dot{\Psi}_{q'13} = -a_3\cos(q_1+q_3)\,(\dot{q}_1+\dot{q}_3).$$

$$\dot{\Psi}_{q'14} = a_4\cos(q_2+q_4)\,(\dot{q}_2+\dot{q}_4),$$

$$\dot{\Psi}_{q'21} = -a_1\sin(q_1)\,\dot{q}_1 - a_3\cos(q_1+q_3)(\dot{q}_1+\dot{q}_3)$$

$$\dot{\Psi}_{q'22} = a_2\sin(q_2)\dot{q}_2 + a_4\sin(q_2+q_4)(\dot{q}_2+\dot{q}_4),$$

$$\dot{\Psi}_{q'23} = -a_3\sin(q_1+q_3)(\dot{q}_1+\dot{q}_3)$$

$$\dot{\Psi}_{q'24} = -a_4\sin(q_2+q_4)(\dot{q}_2+\dot{q}_4)$$

4.4 Standardization

Suppose $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} q_{1d} - q_1 \\ q_{2d} - q_2 \end{bmatrix}$ is the output equation, where q_{1d} *and* q_{2d} are desired set points. Let $\psi^1 = q_1 - q_{1d}$ and $\psi^2 = q_1 - q_{1d}$. The resulted system from Section 4.3 is not exactly the same as the outcome from Section 2.2.2, but by using the concept of standardization algorithm, the following results are obtained

$$\psi_1^1 = q_{1d} - q_1, \quad \psi_2^1 = q_{2d} - q_2$$

$$\psi_1^2 = \dot{q}_1, \quad \psi_2^2 = \dot{q}_2$$

And the new coordinate take the form

$$\varepsilon = [\phi_1 \quad \phi_2 \quad \dot{\phi}_1 \quad \dot{\phi}_2]^T$$

$$\xi = \left[\psi_1^1 \quad \psi_2^1 \quad \psi_1^2 \quad \psi_2^2\right]^T$$

By differentiating ε_i^k and ξ_i^k with respect to time, the system (4-5), (4-6) and (4-2) can be expressed in the new coordinates of the form

$$\varepsilon = 0$$

$$\dot{\xi}_0^1 = \xi_1^1 \tag{4-27}$$

$$\dot{\xi}_0^2 = \xi_1^2$$

$$[\dot{\xi}_1^1 \quad \dot{\xi}_1^2]^T = D(q)^{-1}[u - C(q,\dot{q})\dot{q} - G(q) + \omega] \tag{4-28}$$

4.5 Design Adaptive Controller

The adaptive backstepping controller is designed for the system described by (4-27)-(4-28). The Lyapunov candidate function for equations (4-27) takes the form

$$V_1 = \frac{1}{2}{\xi_0^1}^2 + \frac{1}{2}{\xi_0^2}^2 \tag{4-29}$$

By differentiation (4-29) with respect to time and substituting $\dot{\xi}_0^1$ and $\dot{\xi}_0^2$ we have

$$\dot{V}_1 = \xi_0^1\dot{\xi}_0^1 + \xi_0^2\dot{\xi}_0^2 = \xi_0^1\xi_1^1 + \xi_0^2\xi_1^2 = \xi_0^1(\xi_1^1 - \alpha_0^1 + \alpha_0^1) + \xi_0^2(\xi_1^2 - \alpha_0^2 + \alpha_0^2) \tag{4-30}$$

Now by introducing virtual input $\alpha_0^1 = -d_0^1\xi_0^1 and\ \alpha_0^2 = -d_0^2\xi_0^2$ and substituting in (4-30), $\dot{V}_1$ becomes

$$\dot{V}_1 = -d_0^1 {\xi_0^1}^2 - d_0^2 {\xi_0^2}^2 + \xi_0^1(\xi_1^1 - \alpha_0^1) + \xi_0^2(\xi_1^2 - \alpha_0^2) \tag{4-31}$$

The Lyapunov candidate function for (4-28) takes the form

$$V_2 = V_1 + \frac{1}{2}\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T D(q)\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix} + \frac{1}{2}(\Theta - \widehat{\Theta})^T \Gamma(\Theta - \widehat{\Theta}) \tag{4-32}$$

Differentiating V_2 with respect to time gives

$$\dot{V}_2 = \dot{V}_1 + \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T D(q)\begin{bmatrix}\dot{\xi}_1^1 - \dot{\alpha}_0^1 \\ \dot{\xi}_1^2 - \dot{\alpha}_0^2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \dot{D}(q)\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix} - \dot{\widehat{\Theta}}^T \Gamma(\Theta - \widehat{\Theta})$$

Now $[\dot{\xi}_1^1 \quad \dot{\xi}_1^2]^T$ can be substituted from (4-28) and $\dot{V}_1$ from (4-31). As a result, $\dot{V}_2$ becomes

$$\dot{V}_2 = -d_0^1 {\xi_0^1}^2 - d_0^2 {\xi_0^2}^2 + \xi_0^1(\xi_1^1 - \alpha_0^1) + \xi_0^2(\xi_1^2 - \alpha_0^2) + \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T D(q)\begin{bmatrix}\dot{\xi}_1^1 - \dot{\alpha}_0^1 \\ \dot{\xi}_1^2 - \dot{\alpha}_0^2\end{bmatrix}$$
$$+ \frac{1}{2}\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \dot{D}(q)\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix} - \dot{\widehat{\Theta}}^T \Gamma(\Theta - \widehat{\Theta})$$

$$\dot{V}_2 = -d_0^1 {\xi_0^1}^2 - d_0^2 {\xi_0^2}^2 + \xi_0^1(\xi_1^1 - \alpha_0^1) + \xi_0^2(\xi_1^2 - \alpha_0^2)$$
$$+ \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T D(q)\{D(q)^{-1}[u - C(q,\dot{q})\dot{q} - G(q) + \omega]\}$$
$$- \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T D(q)\begin{bmatrix}\dot{\alpha}_0^1 \\ \dot{\alpha}_0^2\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \dot{D}(q)\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix} - \dot{\widehat{\Theta}}^T \Gamma(\Theta - \widehat{\Theta})$$

$$\dot{V}_2 = -d_0^1 {\xi_0^1}^2 - d_0^2 {\xi_0^2}^2 + \xi_0^1(\xi_1^1 - \alpha_0^1) + \xi_0^2(\xi_1^2 - \alpha_0^2) + \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \left(u - D(q)\begin{bmatrix}\dot{\alpha}_0^1 \\ \dot{\alpha}_0^2\end{bmatrix}\right.$$
$$\left. - C(q,\dot{q})\dot{q} - G(q)\right) + \frac{1}{2}\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \dot{D}(q)\begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix} + \begin{bmatrix}\xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2\end{bmatrix}^T \omega$$
$$- \dot{\widehat{\Theta}}^T \Gamma(\Theta - \widehat{\Theta})$$

According to [44], the matrix $\dot{D}-2C$ is skew symmetric and $\frac{1}{2}\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\left(\dot{D}-2C\right)\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}=0$. By using this property, the simplification can be continued

$$\dot{V}_2=-d_0^1{\xi_0^1}^2-d_0^2{\xi_0^2}^2+\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\left(u+\begin{bmatrix}\xi_0^1\\ \xi_0^2\end{bmatrix}+\Lambda\right)+\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\omega-\dot{\widehat{\Theta}}^T\Gamma(\Theta-\widehat{\Theta})$$

with $\Lambda=-G(q)-D(q)\begin{bmatrix}\dot{\alpha}_1\\ \dot{\alpha}_2\end{bmatrix}-C(q,\dot{q})\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}=\Lambda_\circ\Theta$

$$\Lambda_\circ=-\begin{bmatrix}\dot{\alpha}_1{D'}_{\circ 11}+\dot{\alpha}_2{D'}_{\circ 12}+\alpha_1{C'}_{\circ 11}+\alpha_2{C'}_{\circ 12}+{g'}_{\circ 1}\\ \dot{\alpha}_1{D'}_{\circ 21}+\dot{\alpha}_2{D'}_{\circ 22}+\alpha_1{C'}_{\circ 21}+\alpha_2{C'}_{\circ 22}+{g'}_{\circ 2}\end{bmatrix}$$

$$\dot{V}_2=-d_0^1{\xi_0^1}^2-d_0^2{\xi_0^2}^2+\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\left(u+\begin{bmatrix}\xi_0^1\\ \xi_0^2\end{bmatrix}+\Lambda_\circ\widehat{\Theta}\right)+\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\omega \\ -\left(\dot{\widehat{\Theta}}^T\Gamma-\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\Lambda_\circ\right)(\Theta-\widehat{\Theta}) \tag{4-33}$$

Recall Young inequality and let $\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T=X^T$ and $Y=\omega$

$$X^TY\le\frac{1}{4\gamma^2}X^TX+\gamma^2Y^TY$$

$$\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\omega\le\frac{1}{4\gamma^2}\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}+\gamma^2\omega^T\omega \tag{4-34}$$

By adding necessary terms to (4-34), the new inequality takes the form

$$\dot{V}_2\le-d_0^1{\xi_0^1}^2-d_0^2{\xi_0^2}^2+\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\left(u+\begin{bmatrix}\xi_0^1\\ \xi_0^2\end{bmatrix}+\Lambda_\circ\widehat{\Theta}+\frac{1}{4\gamma^2}\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}\right) \\ +\gamma^2\omega^T\omega-\left(\dot{\widehat{\Theta}}^T\Gamma-\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}^T\Lambda_\circ\right)(\Theta-\widehat{\Theta}) \tag{4-35}$$

By selecting the control input u and parameter estimation law of the form

$$u = -\left(\begin{bmatrix}\xi_0^1\\ \xi_0^2\end{bmatrix} + \Lambda_\circ\widehat{\Theta} + \frac{1}{4\gamma^2}\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix} + \begin{bmatrix}d_1^1(\xi_1^1-\alpha_0^1)\\ d_1^2(\xi_1^2-\alpha_0^2)\end{bmatrix}\right) \tag{4-36}$$

$$\dot{\widehat{\Theta}} = \Gamma^{-1}\Lambda_\circ{}^T\begin{bmatrix}\xi_1^1-\alpha_0^1\\ \xi_1^2-\alpha_0^2\end{bmatrix}$$

with $d_0^1 \geq 1$ and $d_1^2 \geq 0$. By applying (4-36) in (4-35), the inequality simplified as

$$\dot{V} \leq \gamma^2\omega^T\omega - d_0^1{\xi_0^1}^2 - d_0^2{\xi_0^2}^2 - d_1^1(\xi_1^1-\alpha_0^1)^2 - d_1^2(\xi_1^2-\alpha_0^2)^2 \tag{4-37}$$

If $d_1^1 \geq 0, d_0^2 \geq 1$, it follows from (4-34) that $\dot{V} \leq \gamma^2\omega^T\omega - d_0^1{\xi_0^1}^2 - d_0^2{\xi_0^2}^2$ which implies

$$\int_0^\infty [\xi_0^1]^2dt + \int_0^\infty [\xi_0^2]^2dt \leq \gamma^2\int_0^\infty w^Twdt$$

Therefore, the L_2 Gain from disturbance w to (ξ_0^1, ξ_0^2) does not exceed γ.

4.6 Simulation Results

The value and parameters for this simulation work are given in Table 4-1. These values were selected from a practical system in [42], [43].

The gravity constant is $g = 9.81m/s^2$and the distance between shafts of motors is measured as $c = 0.4240\ m$.

The initial angles for joint variables are $q_1(0) = 0^\circ C$, $q_2(0) = 0^\circ C$, $q_3(0) = 62.55^\circ C$, $q_4(0) = 117.44^\circ C$ while the desired angles are set to be $q_{1d} = 90^\circ C, q_{2d} = 90^\circ C, q_{3d} = -27.44^\circ C$ and $q_{4d} = 27.44^\circ C$. The initial values for unknown parameters are

$$\Theta(0) = [0.08, 0.08, 0.02, 0.02, 0.03, 0.03, 1.8, 1.8, 0.6, 0.6]^T$$

which are the values calculated based on the nominal values of the parameters.

In order to verify the results, two sets of simulations were performed in MATLAB 2015b with the tuning parameters:

$$d_0^1 = 5,\, d_0^2 = 5,\, d_1^1 = 15,\, d_1^2 = 15,$$

$$\Gamma = diag([30, 30, 30, 30, 30, 30, 30, 60, 150, 150]),$$

In the first simulation, an adaptive backstepping controller without disturbance decoupling was designed while the model includes disturbance. The control input and parameter estimation law are

$$u = -\left(\begin{bmatrix} \xi_0^1 \\ \xi_0^2 \end{bmatrix} + \Lambda_\circ \widehat{\Theta} + \begin{bmatrix} d_1^1(\xi_1^1 - \alpha_0^1) \\ d_1^2(\xi_1^2 - \alpha_0^2) \end{bmatrix}\right) \tag{4-38}$$

$$\dot{\widehat{\Theta}} = \Gamma^{-1} {\Lambda_\circ}^T \begin{bmatrix} \xi_1^1 - \alpha_0^1 \\ \xi_1^2 - \alpha_0^2 \end{bmatrix}$$

The second simulation was conducted based on adaptive backstepping controller with almost disturbance decoupling in the form of (4-36) with $L_2\ gain\ (\gamma) = 0.1$.

The disturbances as shown in Figure 4-2 and Figure 4-3 are used for both simulations to demonstrate the effect of the disturbances.

Table 4-1: Link Parameters.

Link i	$m_i\ (kg)$	$a_i(m)$	$l_i\ (m)$	$I_i\ (kg.m^2)$
1	0.1950	0.4600	0.3367	4.5667×10^{-3}
2	0.1950	0.4600	0.3367	4.5667×10^{-3}
3	0.2538	0.4600	0.2400	8.626×10^{-3}
4	0.2538	0.4600	0.2400	8.626×10^{-3}

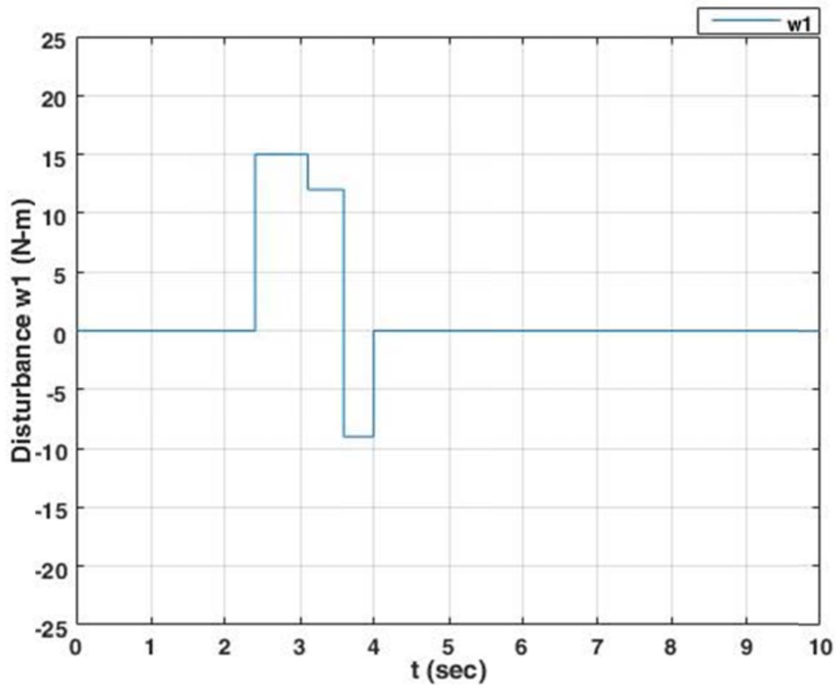

Figure 4-2: Disturbance w_1

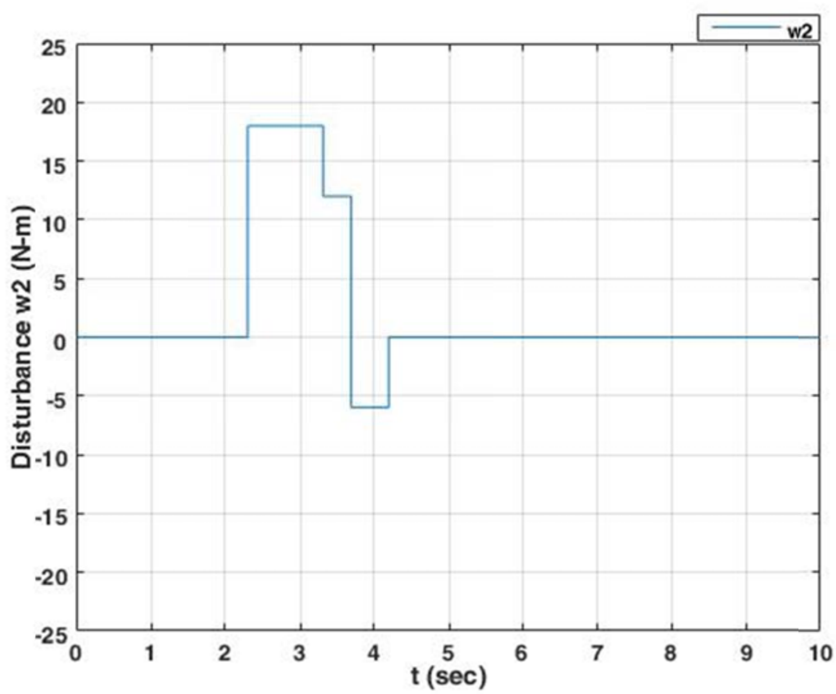

Figure 4-3: Disturbance w_2

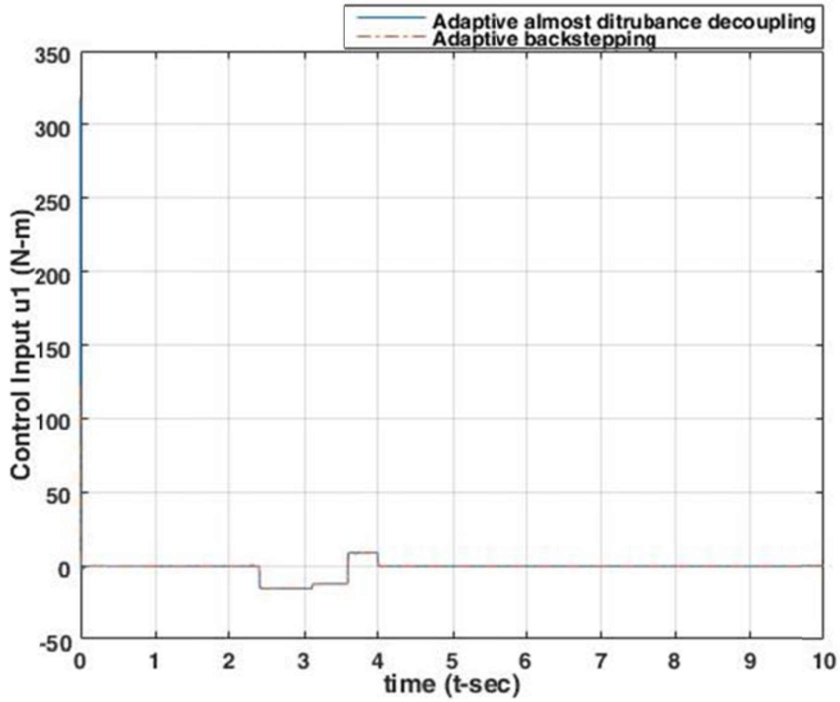

Figure 4-4: Control Input u_1

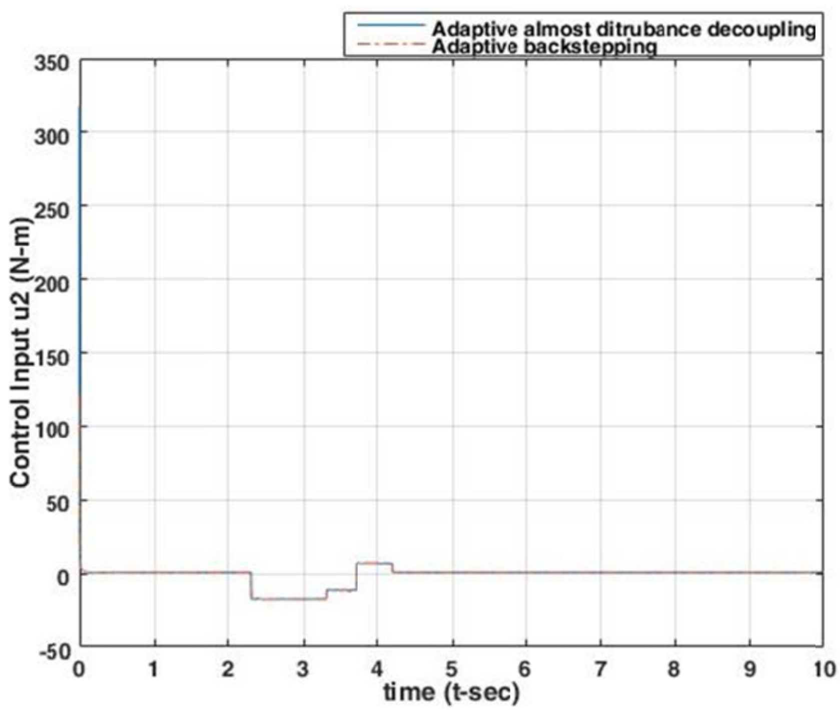

Figure 4-5: Control Input u_2

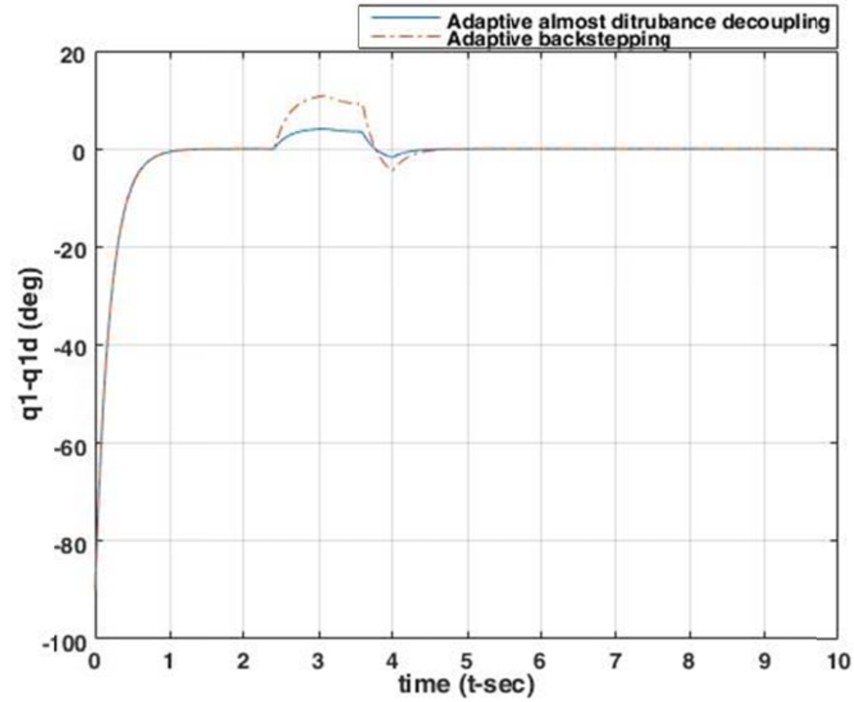

Figure 4-6: Error $q_1 - q_{1d}$

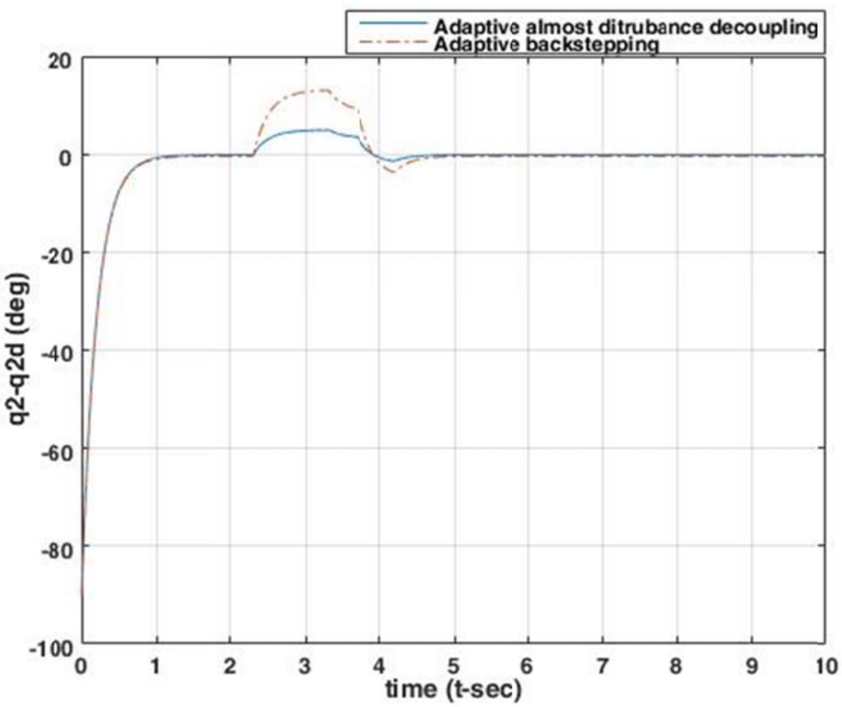

Figure 4-7: Error $q_2 - q_{2d}$

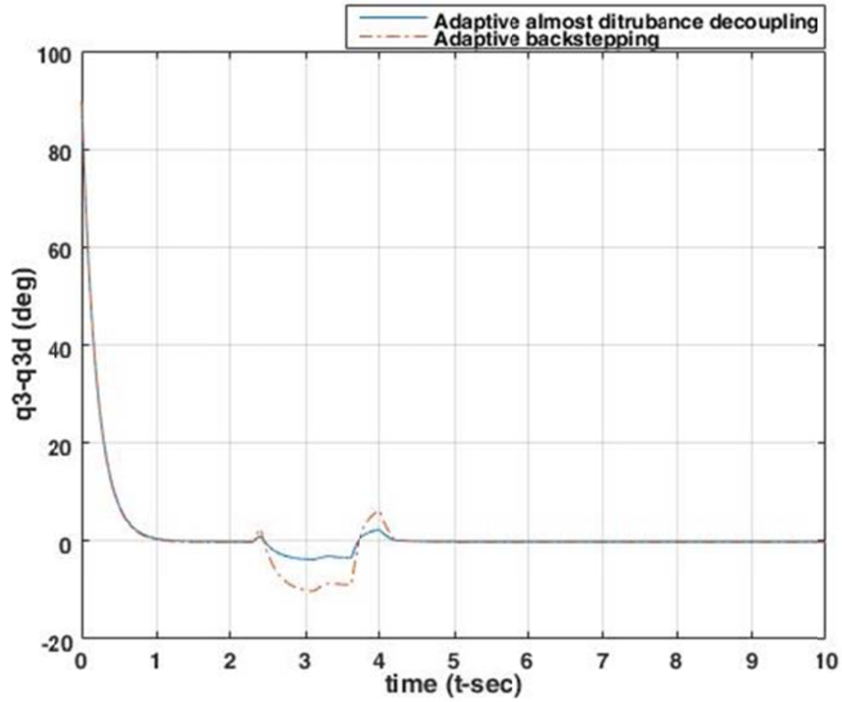

Figure 4-8: Error $q_3 - q_{3d}$

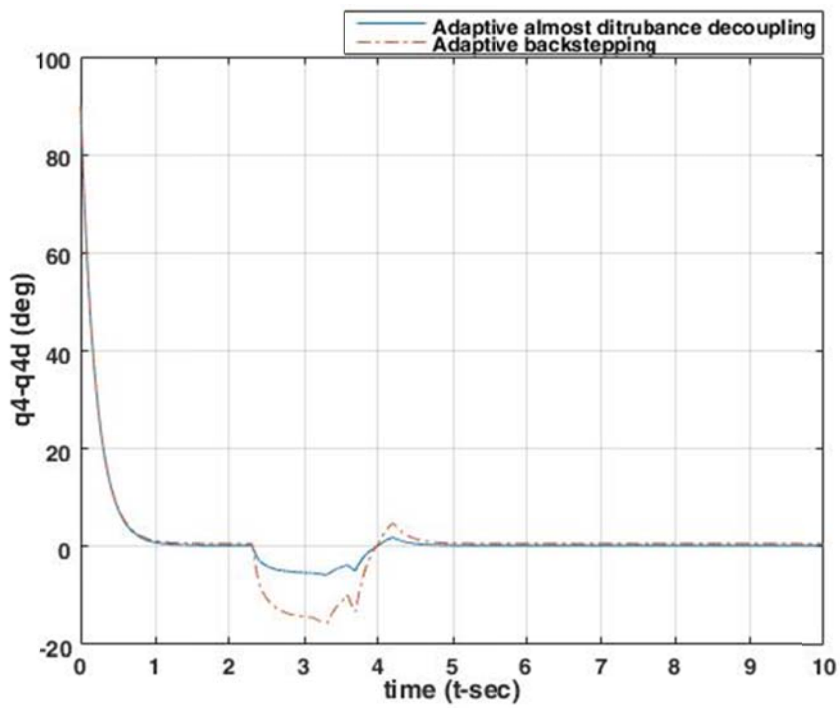

Figure 4-9: Error $q_4 - q_{4d}$

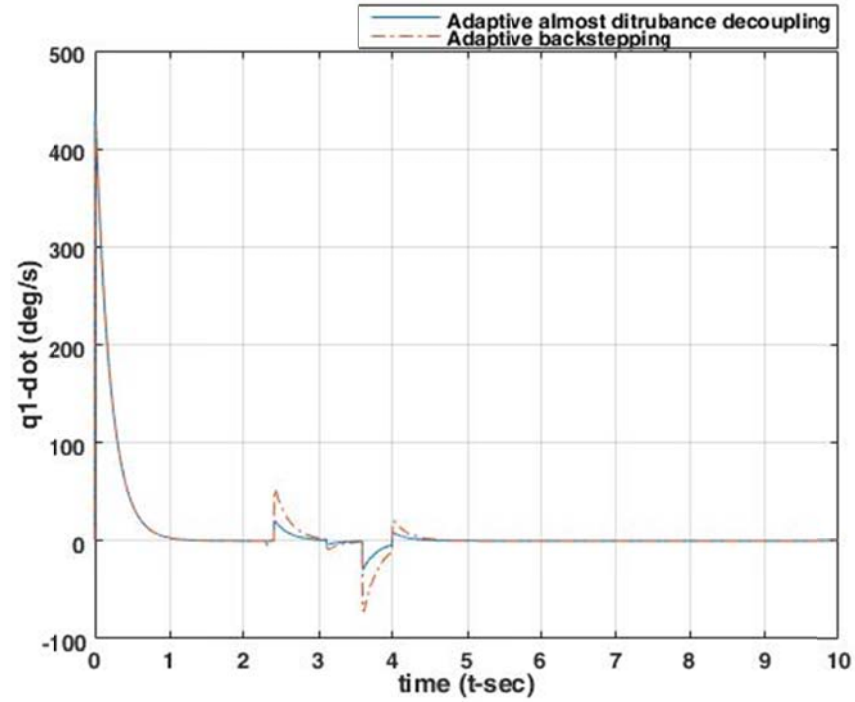

Figure 4-10: Joint Velocity $\dot{q}_1$

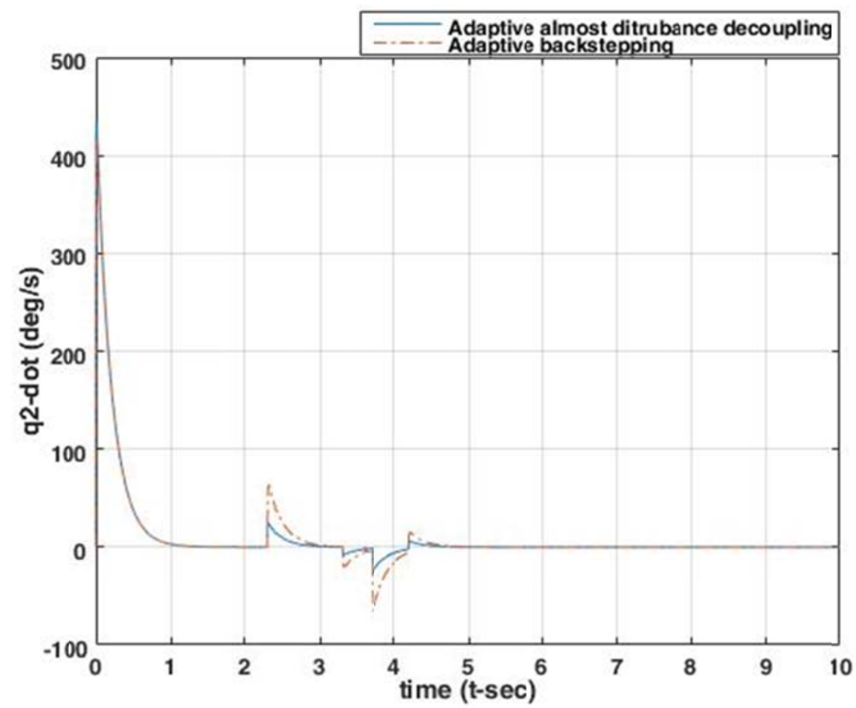

Figure 4-11: Joint Velocity $\dot{q}_2$

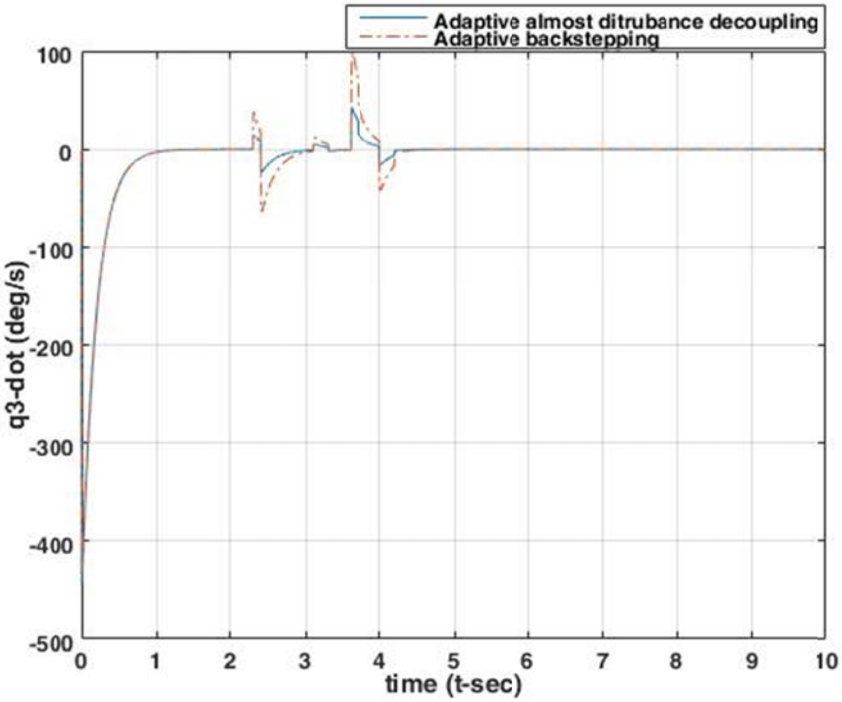

Figure 4-12: Joint Velocity $\dot{q}_3$

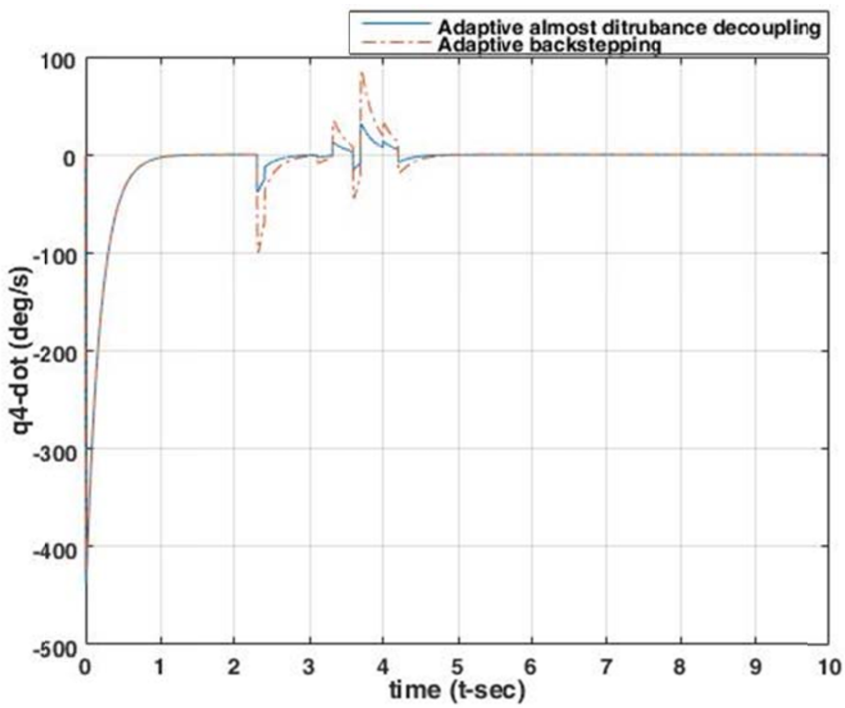

Figure 4-13: Joint Velocity $\dot{q}_4$

4.7 Result Discussion

The simulations were run in the presence of disturbance torque of the form in Figure 4-2 and Figure 4-3. The simulation results are shown in Figure 4-4 to Figure 14-16. The errors for link angles with respect to the set-points are shown in Figure 4-6 to Figure 4-9. In the exposure of such disturbances, the errors converge to a smaller bounded value for adaptive almost disturbance decoupling controller, which is quite satisfactory.

On other hand, Figure 4-10 to Figure 4-13 show the comparison of joint velocities for both adaptive almost disturbance decoupling and adaptive backstepping controllers. It is clear that the disturbance has less effect on the joint velocities with adaptive almost disturbance decoupling controller.

As shown in Figure 4-4 and Figure 4-5, the control inputs for both adaptive almost disturbance controller and adaptive backstepping controllers have the same performance in the area where disturbance is activated. The proposed adaptive almost disturbance controller yields excellent performances vs adaptive backstepping controller. These simulations verify the reasonable performance of the proposed controller.

Chapter 5
Conclusion and Future Work

5.1 Conclusion

In this work, the adaptive almost disturbance decoupling for nonlinear singular systems has been discussed. Below is a summary of the main results and achievements:

1. The singular system has been regularized to find a static regularization feedback law which renders the nonlinear singular system impulse free. An extension has been made to the regularization algorithm introduced in [16].

2. The regularized system has been converted into the ODE system in lower triangular form by extending the standardization algorithm in [16].

3. The proposed regularization and standardization algorithms include one more terms in each step compared to the algorithms in [16].

4. An adaptive backstepping controller has been designed to reduce the effect of the disturbance on the output less than the specific level. The control and parameter adaptation laws have been formulated for large scale systems. The systematic design procedure can be applied to any system in lower triangular form.

5. The proposed regularization, standardization and adaptive almost disturbance decoupling algorithmic procedures have been applied to a 2DOF planar parallel robot, which is a high index singular model. The simulation results show the performance of the proposed method.

The almost disturbance decoupling problem is related to high-gain feedback design. In cases where the problem cannot be exactly solved, increasing the accuracy of the decoupling requires increasing the gains of the linear state feedback control. In terms of almost disturbance decoupling, the performance of controller is acceptable and it can attenuate the disturbance arbitrarily. To achieve a lower effect of disturbance on the output requires higher gain. This problem seems to be a disadvantage of the backstepping technique, which is a high gain feedback controller.

5.2 Future Work

The following four open problems can provide direction for future research:

1. To relax the assumptions in regularization and standardization algorithms; for example, to include zero dynamics in the algorithms.

2. To extend the adaptive almost disturbance decoupling controller to tracking control problem.

3. To perform this simulation on the proposed adaptive almost disturbance decoupling controller in different applications, such as chemical and electrical systems, in order to verify the performance of the controller.

4. To conduct an experimental test, verify the performance of the proposed controller and compare with simulation results.

References

[1] W. M. Wonham and A. S. Morse, “Decoupling and pole assignment in linear multivariable systems: A geometric approach,” SIAM Journal on Control, vol. 8, no. 1, pp. 1-18, 1970.

[2] G. Basile and G. Mano, “Controlled and conditioned invariant subspaces in linear systems theory,” Journal of Optimization Theory and Applications, vol. 3, no. 5, pp. 306-315, 1969.

[3] A. Isidori, A.J. Krener, C. Gori-Giorgi, and S. Monaco "Nonlinear decoupling via feedback: A differential geometric approach", IEEE Transactions on Automatic Control, vol. 26, no.2, pp. 331-345, 1981.

[4] R. M. Hirschom, “(A, B)-invariant distributions and disturbance decoupling of nonlinear systems,” SIAM Journal on Control and Optimization, vol. 19, pp. 1-19, 1981.

[5] H. Nimeijer and K. Tchon, "An input-output characterization of nonlinear disturbance decoupling", Department of Mathematics, Twente University, Memo 502, Feb. 1985.

[6] J. C. Willems, “Almost invariant subspaces: an approach to high-gain feedback design-Part I: Almost controlled invariant subspaces,” IEEE Transactions on Automatic Control,” vol. 26, pp. 235-252, 1982.

[7] R. Marino, W. Respondek, A. J. Van Der Schaft, ‘Almost disturbance decoupling for single-input single-output nonlinear systems,” IEEE Transactions on Automatic Control, vol. 34, no. 9, 1989.

[8] Marino, R., Respondek, W., Van der Schaft, A. J., & Tomei, P., “Nonlinear H∞ almost disturbance decoupling,” Systems and Control Letters, vol. 23, pp. 159–168, 1994.

[9] A. Isidori, “A note on almost disturbance decoupling for nonlinear minimum phase systems,” Systems and Control Letters, vol.27, pp. 191-194, 1996.

[10] A. Isidori and A. Asolfi, “Disturbance attenuation and H_∞ control via measurement feedback in nonlinear systems,” IEEE Transactions on Automatic Control, vol. 37, pp. 1283– 1293, 1992

[11] G. L. Santosuosso, "Nonlinear H∞ almost disturbance decoupling and adaptive stabilization without overparametrization", Proceedings of the 34th Conference on Decision and Control, vol. 3, pp. 2970-2975, 1995.

[12] A. Isidori, "Global almost disturbance decoupling with stability for non-minimum-phase single-input single-output nonlinear systems," Systems and Control Letters, vol. 28, pp.115-122, 1996.

[13] Schwartz, B., Isidori, A., & Tarn, T. J. "Global normal forms for MIMO nonlinear systems with application to stabilization and disturbance attenuation," Proceedings of the 35th Conference on Decision and Control, vol. 12, pp. 1041–1046, 1996.

[14] X. P. Liu, C. S. Celikovsky, "Feedback control of affine nonlinear singular control systems", International Journal of Control, vol. 68, pp. 753–774, 1997.

[15] X. P. Liu, D. W. C. Ho, "Stabilization of non-linear differential-algebraic equation systems," International Journal of Control, vol. 77, no. 7, pp. 671–684, 2004.

[16] Z. Lu and X. P. Liu, "Adaptive regularization for a class of nonlinear affine differential-algebraic equation systems," in American Control Conference, pp. 517-522, 2008

[17] X. P. Liu, A. Jutan, S. Rohani, "Almost disturbance decoupling of MIMO nonlinear systems and application to chemical processes'' Automatica, vol. 40, pp. 465 – 471, 2004.

[18] A. J. van der Schaft, "L_2-gain analysis of nonlinear systems and nonlinear state-feedback H_∞ control" IEEE Transactions on Automatic Control, vol. 37, no. 6, pp. 770-8784, 1992.

[19] A. Kumar, P. Daoutidis, "Feedback regularization and control of nonlinear differential-algebraic-equation systems", AIChE Journal, vol. 42, no. 48, pp. 2175-98, 1996.

[20] A. Kumar and P. Daoutidis, "High-index DAE systems in modeling and control of chemical processes," Preprints of IFAC Conference on Control of Industrial Systems, vol. 1, pp. 518-523, 1997.

[21] G.D. Byrne and P. R. Ponzi, "Differential-algebraic systems, their applications and solutions," Computer Chemical Engineering, vol.12, pp. 377-382, 1988.

[22] R. Gani, I. T. Cameron, "Modelling for dynamic simulation of chemical processes: the index problem," Chemical Engineering Science, vol. 47, no. 5, pp. 1311-1315, 1992.

[23] H. Hemami, B. F. Wyman, "Modeling and control of constrained dynamic systems with application to biped locomotion in the frontal plane," IEEE Transactions on Automatic Control, vol. 24, no. 4, pp. 526-535, 1979.

[24] N. H. McClamroch, "Singular systems of differential equations as dynamic models for constrained robot systems," in Proceeding of the IEEE International Conference on Robotics and Automation, vol. 3, pp. 21-28, 1986.

[25] D. Cobb, "Controllability, observability, and duality in singular systems," IEEE Transactions on Automatic Control, vol. 29, pp. 1076-1082, 1984.

[26] D. G. Luenberger, "Nonlinear descriptor systems," Journal of Economic Dynamics and Control, vol. 1, pp. 212-242, 1979.

[27] E. Griepentrog and R. Marz, "Differential algebraic equations and their numerical treatment", Teubner, Leipzig, 1986.

[28] N. Karcanias, "Regular state-space realizations of singular system control problems," Proceedings IEEE Conference Decision and Control, vol. 26, pp. 1144-1146, 1987.

[29] A. Kumar, P. Daoutidis, "Control of nonlinear differential-algebraic-equation systems with disturbance," Industrial & Engineering Chemistry Research, vol. 34, pp. 2060-2076, 1995.

[30] S. L. Campbell, E. Griepentrog, "Solvability of general differential algebraic equations," SIAM Journal on Scientific Computing, vol. 16, pp.257-270, 1995.

[31] L. R. Petzold, "Differential-algebraic equations are not ODE's," SIAM Journal on Scientific and Statistical Computing, vol.3, pp. 367-384, 1982.

[32] A. Isidori, "Nonlinear control systems," third edition, Berlin, Germany: Springer-Verlag, 1995.

[33] I.D. Landau, R. Lozano, M. M'Saad, A. Karimi, "Adaptive control: algorithms, analysis and applications," London, New York, Springer, 2011.

[34] H. Nijmeijer, A. J. V. D Shaft, "Nonlinear dynamical control systems," New York: Springer-Verlag, 1990.

[35] F. R. Gantmacher, "The theory of matrices," Chelsea Publication, New York, vol.2, 1959.

[36] D. Stewart, "A platform with six degrees of freedom," Proceedings of the Institution of Mechanical Engineers, vol. 180, pp.371-386, 1965.

[37] K.H. Hunt, "Kinematic geometry of mechanisms," Clarendon Press, Oxford, 1978.

[38] E., Fichter, "A Stewart platform-based manipulator: general theory and practical construction," International Journal Robotics Research, vol. 5, no. 2, pp.157–182, 1986.

[39] C.M. Gosselin, "Kinematic analysis, optimization and programming of parallel robotic Manipulators," PhD Thesis, McGill University, 1988.

[40] C. R. Boer, L. Molinari-Tosatti and K.S. Smith, "Parallel kinematic machines," Springer-Verlag, Berlin. 1999.

[41] Zhang, D., "Kinetostatic analysis and optimization of parallel and hybrid architectures for machine tools," PhD Thesis, Laval University, Quebec, Canada, 2000.

[42] L. Wang, Z. Lu, X. Liu, K. Liu, D. Zhang, "Adaptive control of a parallel robot via backstepping technique," International Journal Systems, Control and Communication, vol. 1, no. 3, 2009.

[43] L. Wang, Z. Lu, X. Liu, K. Liu, D. Zhang, "Adaptive tracking control of a parallel robot: backstepping and schemes," International Journal of Control Theory and Applications vol. 2, no. 1. pp. 39-59, 2009.

[44] F.H. Ghorbel, O. Chetelar, R. Guawardana, and R. Longchamp, "Modelling and set point control of closed-chain mechanisms: theory and experiment," IEEE Transactions on Control Systems Technology, vol. 8, no. 5, pp.801–815, 2000.

[45] M. W. Spong and M. Vidysagor, "Robot dynamics and control," John Wiley & Sons, 1989.

[46] N.H. McClamroch and H.P Huang, "Dynamics of closed chain manipulator," Proceedings American Control Conference, Boston, pp. 50-54, 1985.

[47] R. Carelli and R. Kelly, "Adaptive control of constrained robots modeled by singular systems," Proceedings IEEE Conference Decision and Control, vol.3, pp. 2635 – 2640, 1989.

[48] N. H. McClamroch, and D. Wang, "Feedback Stabilization and Tracking of Constrained Robots," IEEE Transaction Automatic Control, vol. 33, pp. 419-426, 1988.

[49] K.E. Brenan, S.L. Campbell, and L.R. Petzold, "Numerical solution of initial-value problems in differential-algebraic equations," New York, 1996.

[50] Campbell, S L, Griepentrog, E. "Solvability of general differential algebraic equations," SIAM Journal on Scientific Computing, vol. 16, pp. 257-270, 1995.

[51] S. Kawaji, E.Z. Taha, "Feedback linearization of a class of nonlinear descriptor systems," Proceedings of the IEEE Conference on Decision and Control, vol. 4, pp. 4035-4042, 1994.

[52] G. Zimmer, J. Meier, "On observing nonlinear descriptor systems," Systems & Control Letters, vol. 32, pp 43-48, 1997.

[53] X.P. Liu, D. W. C. Ho, "Global regularization of nonlinear differential-algebraic equation systems," Proceedings of the 6th World Congress on Intelligent Control and Automation, vol. 1, pp. 863-867, 2006.

[54] H. Krishnan, N.H. McClamroch, "Tracking in nonlinear differential-algebraic control systems with applications to constrained robot systems," Automatica, vol. 30, pp 885-897, 1994.

[55] X. P. Liu. "Asymptotic output tracking of nonlinear differential-algebraic control systems," Automatica, vol. 34, pp. 393-397, 1998.

[56] X.P. Liu. "Robust stabilization of nonlinear singular systems," Proceedings of IEEE Conference on Decision and Control", vol. 4, pp. 2375-2376, 1995.

[57] Y. M. Fu, B. Li, A. G. Wu, G. R. Duan "Adaptive regulation with almost disturbance decoupling for nonlinearly parameterized systems with control coefficients," Journal of Computational Analysis and Applications., vol. 15, pp. 936-946, 2013.

[58] Y. M. Fu, B. Li, A. G. Wu, G. R. Duan "Adaptive regulation with almost disturbance decoupling for integrator triangular systems with nonlinear parametrization," International Journal of Innovative Computing, Information and Control, vol. 7, no. 4, pp. 1699-1711, 2011.

[59] X. Liu, “Local disturbance decoupling of nonlinear differential-algebraic control systems,” International Journal of Control, vol. 70, pp 685-702, 1998.

[60] A. Asolfi, “Singular H_∞ control,” Proceedings of the 33rd Conference on Design and Control, Lake Buena Vista, 1994.

[61] W. Wentao, K. Zhi “Disturbance decoupling of affine nonlinear singular systems,” IEEE Proceeding of International Conference on Control and Automation Guangzhou, China, pp 2198-2201, 2007.

[62] P. Kokotovic, " The joy of feedback: nonlinear and adaptive", Control Systems, IEEE, vol. 2, pp. 7-17, 1992.

[63] I. Kanellakopoulos, P. Kokotovic, A. Morse, "Systematic design of adaptive controllers for feedback linearizable systems", IEEE Transactions on Automatic Control, vol.36, no. 11, pp. 1241-1253, 1991