AN EXPLORATION OF MATHEMATICS DISCOURSE IN A REFORM-ORIENTED CLASSROOM AND ITS ROLE IN THE DEVELOPMENT OF STRATEGIES AND BIG IDEAS IN EARLY ADDITION AND SUBTRACTION

by

Margaret Allen

A Thesis Submitted In Partial Fulfillment of the Requirements for

the Degree of Master of Education

FACULTY OF EDUCATION

LAKEHEAD UNIVERSITY

THUNDER BAY, ONTARIO

Abstract

The focus of this video case study was to analyze the role of social norms (discourse in a learning community), sociomathematical norms (math discourse in a learning community), and my teaching in facilitating these norms on the development of strategies and big ideas in early addition and subtraction in a Grade 1 classroom. Three students from the class were purposely selected to be the focus of this study because of their high level of participation in discussions and their different levels of conceptual understanding. An independent task was given to all the students in the class pre- and poststudy that paralleled the addition and subtraction problems used during the study. The model of the arithmetic rack was used in the context of the double-decker bus, on which the students applied their strategies directly and indirectly. The teaching unit and numeracy continuum *(The Landscape of Learning)* used, supported students in their development of the big mathematical ideas surrounding early addition and subtraction. The frequency of talk, the direction of talk, the type of talk, and the teacher's talk in relation to the students under study were analyzed. Although students' movement along the landscape was not dramatic, it was evident that the discussions helped to deepen their understanding. Recommendations are discussed.

Acknowledgements

Many people helped to make the completion of this thesis possible. First, I want to thank all the students in my Grade 1 class who participated in this study as well as all of my students over the years who have been on this mathematical learning journey with me. They are my partners in teaching and learning.

I sincerely thank Bonnie Jaakkimainen, my first mentor and principal, who trusted me to teach and encouraged me to try. I thank Diana Mason, who helped me stay organized, Dr. Ruth Beatty and Dr. Joan Chambers who provided advice and guidance. I am truly grateful to Dr. Alex Lawson, my supervisor, who coached me expertly and patiently for a very long time.

Finally, I want to thank members of my family: my mom and dad, for always believing in me; my son, Oliver, who shows me each day how to climb over life's hurdles; my daughter, Scarlett, who knows the value of a giggle and a hug; and my husband, Vincent, my lifelong partner in everything that matters. I could not have done this without your ongoing support.

Table of Contents

List of Tables

List of Figures

CHAPTER 1: INTRODUCTION

Context

In response to poor achievement levels and generally weak mathematical understanding found in many American elementary classrooms at the time, the National Council of Teachers of Mathematics (NCTM, 1989) called for a profound shift in instructional practices. In particular they outlined new communication standards they recommended teachers use in order to deepen children's mathematical understanding and proficiency. They reiterated and refined this call in 2000 (NCTM, 2000). Ontario followed suit putting out new mathematics curricula in 1997 and again in 2005 (OME, 1997; 2005).

As a teacher and a researcher I wanted to know whether I was meeting the call, and whether my instructional changes in mathematical communication were deepening my students' understanding and proficiency. Consider, for example, the following independently-initiated mathematical discussion that I witnessed and recorded in my classroom one morning.

As students were filing into my Grade 1 classroom after the morning bell, they slowly made their way to the group meeting area on the carpet to read the morning question printed on chart paper. The question on this day asked, "Would you like to take a math game home over the March break?" Drawn under the question was a t-chart for students to record their choice of "yes" or "no." After they included their data, another question asked, "What do you notice and wonder about these data?" Also on the chart paper was a sketch of two students sitting and facing each other. One of the students in the drawing had a speech bubble that read, “I notice….”

As students were answering the above question, I observed the following conversation and wrote about it at the end of the math lesson with help from the students involved in the discussion.

Al (talking to Joshua M., who was standing at the flip chart wonderingly): “Hey, this is just like the Flat Stanley question we did yesterday…just put your line here (pointing to the “yes” side of the chart) if you want a math game, or here (pointing to the “no” side) if you don’t want one.” “We don’t have a section to put another answer, like, ‘I don’t know’ or ‘maybe.’ ”

Yibin (approaches Al and Joshua after listening to what Al was saying): “Al, this line is called a tally mark. Remember that Rainforest book we read?”

Al: “Yeah.”

Yibin: “The authors called it a tally mark, see look, I’ll show you.” Yibin looks over at me and notices he is being observed and continues to walk over to the class library to pick up a big book. He brings it over to the meeting area, sits down with it on the floor and begins flipping through the pages, then stops and points at one page in particular.

Al (looking at the big book): “Oh yeah. Thanks, Yibin!” (continues to follow Joshua’s actions, who is now putting a tally mark on the “Yes” side of the chart): “That’s another one for the ‘Yes’ side.” “I wonder how many games Mrs. Allen will have to make?”

Yibin: "It doesn't look like it's as many as the kids who wanted a Flat Stanley to take home."

Joshua (begins to point to each tally and counts in a whisper): "7 and mine makes 8. That isn't as many as the kids who wanted a Flat Stanley!"

Yibin (talking to Joshua): "There's a faster way to count. You can look at this and say, "5" because there's 5 here (and points to the tallies) see, this one across makes it an easy group of 5 to count, and just count on what's left."

Joshua: "Yeah, I was just checking, I know we'll have to get Mrs. Roberts's [data] so we know how many kids want games from both classes. I think it'll be less than Flat Stanley from yesterday because ours is less, so I bet theirs will be less, too."

Does this exchange demonstrate the goals for mathematics outlined by the NCTM in its 1989 *Curriculum and Evaluation Standards for School Mathematics* that students should communicate mathematically? If so, are the students constructing mathematical strategies and big ideas through their talk? What were the previous mathematical learning experiences in this classroom for these students? What role did the teacher play so that these students were able to participate in a social and mathematical learning community and contribute in a mathematically productive way? Does mathematical talk of this type engender mathematical understanding and proficiency in all students?

Although a focus on math talk has now achieved widespread acceptance as an essential aspect of good instructional practice (Sherin, 2002), it has been a relatively recent phenomenon, with the first large-scale call for talk appearing in the NCTM's 1989 standards document and then again in its 2000 document. Although its value has been recognized, Hufferd-Ackles, Fuson, and Sherin (2004) found that the prospect of creating a math-talk community is daunting for many teachers because they often do not know where to begin to create the discourse practices described by the NCTM (1989, 2000).

1.1 Research Questions

I investigated the facilitation of math talk and attempted to document its mathematical impact by examining whether students achieved a deeper and better understanding of discussed math concepts through the social construction of ideas. In conducting this study, I hoped to answer the following research questions:

1. What is the role of social norms, (as defined by Sfard, 2000), in my classroom math community, that contribute to the construction of strategies and big ideas in early addition and subtraction?
2. What is the role of sociomathematical norms, (as defined by Sfard, 2000), in my classroom math community, that contribute to the construction of these strategies and big ideas?
3. What pedagogical and mathematical content knowledge do I as the teacher draw on to facilitate the development of strategies and big ideas?

1.2 Significance of the Study

This study will add to extant literature on the impact of classroom talk on student learning. This study also will add to research suggesting that teachers need to do more with the talk generated by students so that accountability among learners improves. Accountability in this case refers to students participating in a math discussion in a purposeful way, which contributes to the learning of the whole group. This study might add to literature documenting the impact of teachers' responsive listening on children's deeper understanding of mathematical big ideas and effective use of models and strategies.

1.3 Contribution to the Community

Teachers often use their classrooms as laboratories, trying out a variety of practices and discussing their findings with their students and other teachers. As a teacher, I also look for ways to improve my professional practice. My school board offers many opportunities for teacher leaders to make presentations about their areas of specialization. My principals have asked me to initiate several lesson study groups and make presentations at staff meetings and professional development days on mathematical literacy. These sessions comprise the development of number concepts through reform-oriented mathematics methodology and specific planning to facilitate accountable, math talk in the classroom setting. In addition, use of the lesson videotapes gathered as data in the study should contribute to teachers' professional development and might lead to increased student success as a result.

1.4 Limitations of the Study

Some limitations of the study were considered. As a video case study, this research allowed me to provide a detailed account of the learning and teaching that occurred in one classroom at one point in time. Case studies usually are conducted to understand a particular case and might not be generalizable to other cases (Stake, 1995). This specific Grade 1 classroom was not representative of all Grade 1 students. I did not design this case study to compare two teaching methods in order to determine which one was more effective. Instead, the purpose of the study was to determine whether student talk had an impact on the construction of mathematical strategies, big ideas, and the development of a model. Another limitation was that when video cameras were introduced into the classroom environment, the dynamic could have changed, resulting in surveillance of student and teacher behaviours that were less than authentic (Stigler & Hiebert, 1999). Each participant could have been behaving for the camera.

Researcher bias was another limitation. Because I was the teacher and the researcher, my review and analysis of the material could have been skewed or interpreted in a way that could have influenced the results. I attempted to reduce the potential of such an outcome by working with my supervisor to co-code part of my data, thereby establishing a reasonable interrater reliability. Also, daily conversations with my supervisor and teaching partner went a long way to mitigating researcher bias.

CHAPTER 2: LITERATURE REVIEW

Introduction: Call for Reform

Mathematics education in North America (i.e., Canada and the United States) has changed over the last 20 years. This ongoing evolution in mathematics education instruction has been termed the *reform movement* (Van De Walle, 2001). In his research, Kilpatrick (1997) envisioned mathematical reform as a movement that energizes teachers to teach mathematics in better ways and motivates them to do so. Reform instruction of mathematics was an initiative originally driven in part by the NCTM (Lampert & Cobb, 2003), which tabled a document on changing instruction after it found that many students were not achieving in school mathematics (as cited in Battista, 1999); many students had poor test results in mathematics, and they were not learning in more than a superficial manner.

The failure of traditional methods of teaching mathematics also has been documented. Carpenter, Fennema, Franke, Levi, and Empson (1999) argued that this failure was partly the result of instruction in early mathematics not being based upon the informal knowledge that children had gained through their own experiences. They argued that the mathematics being taught in schools was disconnected from the way children naturally thought about number concepts. They felt that this disconnection resulted in a lack of deep understanding by students and an inability to communicate and connect thinking to new learning situations. Other researchers have argued that direct instruction of step-by-step solutions and memorization of algorithms were not working (Hiebert, 1999). Therefore, the NCTM (1989, 2000) subsequently called for a reformed vision of mathematics instruction.

Franke, Kazemi, and Battey (2007) described the instruction in a reform mathematics classroom as being very different from traditional instruction. The teacher's role in particular is different. They summarized that teachers in reform mathematics classrooms are expected to pose meaningful problems, but not provide solutions; manage the flow of discussion by stopping or slowing it down to make it accessible to all students; model academic discourse; and probe for comments and the elaboration of student rather than teacher ideas. Moreover, the teachers also need to question student reasoning in order to foster certain habits of mind, defined by Ball, Hoyles, Jahnke, and Movshovitz-Hadar (2003) as giving proof, organizing arguments, and working toward a collective mathematical theory. Teachers also are responsible for the students' learning of math content and for nurturing a math-talk community that supports students as accountable contributors of mathematics (Ball, 1993).

2.1 Theoretical Underpinnings of Reform: Constructivism

Reform mathematics is inquiry based, and the theoretical framework of this study is based upon the learning theory of constructivism. Current approaches to mathematics educational reform in North America, Europe, Japan, and elsewhere in the world tend to emphasize deep conceptual understanding, complex problem solving, and communication more than procedural speed or factual accuracy (Forman & Ansell, 2002). This approach is characterized by instruction that allows the learners to construct their own knowledge, formulate ideas based upon prior experience, and apply old understanding to new situations and then expanding upon them.

Vygotsky (1930/trans. 1978) discussed the idea of imagination in the development of thought. He suggested that as students are asked a question, they should be given time to create imaginings surrounding that question. They connect to prior experience and build more pathways to that knowledge, and then through discussion with their peers, they allow their imaginings to connect with each other and become more solid and substantial. Vygotsky, in his discussion of social constructivism, also included the theory of the zone of proximal development (ZPD). In her talk about ongoing assessment, Shepard (2000) referred to Vygotsky's ZPD as dynamic assessment and stated that it is integral to providing teachers with insight about how students' understanding might be extended. Shepard (2000) explains that dynamic assessment is finding out what a student is able to do independently as well as what can be done with teacher guidance. It is interactive because teachers provide assistance as part of assessment, creating perfect, targeted occasions to teach and scaffold. Teachers encourage students to move forward within the students' ability levels but at levels that they could not achieve independently. This theory informed the first set of NCTM (1989) standards.

2.2 Reform Standards

As stated earlier, the NCTM initiated this movement in 1989. Although the NCTM addressed a number of areas of mathematics instruction needing improvement in its new goals, researchers such as Hufferd-Ackles et al. (2004) felt that math talk in the classroom stood out for particular criticism. Researchers of the Third International Mathematics and Science Study (TIMSS), which was conducted in 1995, for example, used video evidence to report that most of the discussion in middle school math classes in the United States were between students and teachers and comprised only short-answer questions and answers with little or no explanation or elaboration (as cited in Stigler &

Hiebert, 1999). They described that the discussions were comprised of low-level thinking. For example, when taking into consideration all the problems presented in the U.S. lessons, many of the problems per lesson were posed with the apparent intent of using a procedure, whereas in many of the countries where scores were higher, an emphasis was placed on having students make connections between the problems posed. The researchers also noted that when an answer was wrong, the teacher would move on to another student who was willing to try giving an answer rather than use the error as a stepping stone, as was done in Japanese classes (as cited in Stigler & Hiebert, 1999). Geist (2010) contended that this type of instruction promoted math anxiety and the notion that participation should be made up of only right answers. This type of math classroom did not promote development of a math community or support effective mathematics discussion. As a result, the NCTM began to envision a different standard of communication.

2.2.1. The communication standard. The NCTM (1989) document included new goals for students: (a) learn to value mathematics, (b) become confident in their ability to do mathematics, (c) become problem solvers, and (d) learn to reason mathematically. The NCTM also expected children to communicate like young mathematicians, asserting, "the communication process allows students the opportunity to explain and defend one's ideas orally and in writing" (p. 27). What educators and researchers have found in the classroom since the publication of the standards is discussed next.

2.3 Observation of Classroom Talk: Two Waves

As teachers began implementing the new standards, researchers found that the shift in math talk in some classrooms was typically just a show-and-tell communication by students rather than a true mathematical discussion, as envisioned by the reformers (Spillane, 1999). Franke et al. (2007), for example, found that even in classrooms where teachers were attempting to teach for understanding, they still used the initiation, response, and evaluation (IRE) pattern: The teacher asks a question, waits for a response, and then qualifies the answer. Sherin (2002) observed that teachers attempting this reform-oriented talk in the classroom were successful in getting their students to offer different solutions to a problem, but not in engaging them in a true mathematical discussion (i.e., questioning and defence of ideas) with other students.

van Oers (1996) explained that what was missing was building upon the theory of Vygotsky's (1930/trans. 1978) ZPD, where mathematizing is an activity that children can accomplish only to a certain level because it is a sociocultural activity that is mimicked. Although students might use math terms in the correct context, they might not yet have an understanding of the meaning of the terms. He also contended that a scaffold orchestrated by the teacher is needed to help children to move along in their mathematical development. van Oers noted that teachers were reluctant to comment on students' ideas under the mistaken belief that everyone should have a say, thereby avoiding the feedback that would have generated the scaffold. As a result, this first wave of math talk lead to mathematical activity with less rigor than expected (Sfard, 2000). Math talk was discussed much more often and in depth in the last decade. For example, Hufferd-Ackles et al. (2004) and Sherin (2002) specifically mentioned how difficult it is to talk well and for teachers to know what to do with the talk (i.e., knowing what questions to ask the students to clarify placement on a numeracy continuum).

2.3.1. Changing theoretical notions of math talk. A challenge that mathematics teachers face when implementing instructional reform is to orchestrate whole-class discussions that use students' responses to math problems in ways that advance the mathematical learning of the whole class (Ball, 1993; Lampert & Cobb, 2003). Teachers often are faced with a wide array of student responses to problems and must find ways to use them to guide the class toward a deeper understanding of significant mathematics (i.e., strategies and big ideas). To address this issue, some mathematical theorists have revisited the ideas espoused by Freudenthal (1973), among others, who stated that mathematics discussion should be about developing mathematical models and contexts. These new models and contexts give students time to construct mathematics with purpose and meaning. What is needed is not only better math talk but also better math.

Freudenthal (1973), who initiated the realistic mathematics education movement, tried to articulate this assertion. He believed that mathematics is an activity of many levels and that teachers often present the lowest level of mathematics to students. To take mathematics instruction to a higher level, he recommended supporting children's thinking by helping them to construct mathematical models that reflect their reality.

In other words, the problems being posed should reflect specific situations so that children can learn to connect real-life situations to mathematical abstract concepts (Freudenthal, 1973). Moreover, mathematics' classrooms should no longer be filled with rows of desks and silent students. Instead, students should be expected to engage in the inquiry process. Together, Freudenthal (1973) contended that students need to form mathematical arguments and produce mathematical evidence; show their learning in ways that expose their reasoning to one another and to their teachers (Lampert et al., 2003); and participate in math talk to develop new ideas that they can use to construct understanding.

In looking deeper into the evolution of talk in reform mathematics classroom, van Oers (1996) studied classroom teachers' instruction and noted that math talk was a sociocultural activity that required more than just the participation of students. He observed that math talk required the students to participate in meaningful ways by offering ideas that could contribute to strategies to solve problems. van Oers asserted that students need to acquire a certain level of understanding of the difference between participating (just being present and talking) and participating about the math (being present, talking, and adding to the whole-group discussion in ways that help to move the students forward in their thinking). For the discussion to move toward a deep mathematical understanding, it has to go further. van Oers contended that teachers need to participate in the assessment of the students' ideas.

Creating a classroom where students feel comfortable talking (social norms) is necessary but insufficient. To establish sociomathematical norms, Fosnot and Dolk (2001) recommend that students need to work to prove mathematically why ideas make sense and to solve problems. For example, in the opening vignette, Yibin needed to refer back to the book to prove to Al that the term he used (*tallies*) was correct. In addition, when Yibin told Joshua that there was a quicker way to count the tallies, he showed Joshua how to do it. Joshua acknowledged Yibin's strategy by saying, "I know, I just wanted to check," and relying on counting by ones to do so. Each comment and question should be loaded with several mathematical notions or little theories that the students develop. Again, in the vignette, Joshua theorized that because the tallies were fewer in our class, they would be fewer in the other class and would result in a total that was less than the total from the previous day. Sharing his theory with his peers would have engaged them actively in the upcoming addition problem.

Fosnot and Dolk (2001) built on the work of von Glasersfeld (2005) to support their contention that turning a classroom into a real mathematics community is not easy and is quite different from the traditional classroom, which is where most current teachers learned. In a strong community of discourse, the participants speak to one another and do not just direct their answers to the teachers. Fosnot and Dolk, like Yackel (2001), termed this type of talk *discourse,* where students ask questions, comment on one another's ideas, defend their ideas to the community, and together decide whether it is a sound argument to be applied to the mathematical situation being discussed. They contended that in a strong math-talk community, students are using math terms, noticing and discussing relationships between and among numbers, making connections between and among strategies, and developing mathematical generalizations.

Sfard (2007) referred to these types of sociomathematical norms as components of the activity theory, which originated in the work of Vygotsky (1930/trans. 1978) . This theory states that there is a transition between acquisitionism (i.e., taking the information without question, like filling a bucket) and participationism (i.e., conceptualizing developmental transformations as changes in what and how students are doing or discussing something). Many teachers initiating this shift have found that students coming from traditional settings enter into discussions ready to be told the solutions

(Brown, 2001). Once the social norms of a reform math classroom are put into place and practiced, students can transition from one type of learning, namely, participation and talk, to a more intensive form involving defence and debate.

Cobb and Jackson (2010) believed that ambitious mathematics education and talk also could promote equity. They agreed that before real mathematical learning can happen, an established community of norms needs to be implemented because only in this way can strategies and big ideas begin to emerge collectively. Students who are employing social norms appropriately can focus on the mathematical content to the extent that they can explore a math conjecture thoroughly. Social norms mean that students understand the importance of listening to their peers, whereas sociomathematical norms indicate that they are interested in the mathematics that will be shared. The members can generalize once the community members are convinced by evidence that originally was merely mathematical conjecture.

Bauersfeld (1994) concluded that students arrive at what they know about mathematics mainly by participating in social practice in the classroom. If social practice means staying mathematically focused and using their teachers' questions as a guide right from the beginning, then students will eventually enter into the discussion knowing what is expected of them from the community. In order to foster math talk of this type that researchers such as Hill and Ball (2009) and Cobb and Jackson (2010) have suggested, certain conditions must be established. These more nuanced talk conditions constitute a second generation of math reform.

2.3.2. What teachers are doing to make math talk work: The second generation of reform implementation. The hallmark of second-generation math reform is its focus on using student-developed work as the starting point of whole-class discussions that allow teachers to actively shape the ideas that students produce to lead them toward more powerful, efficient, and accurate mathematical thinking (Ball, 1993; Gravemeijer, 2004; Lampert, 2001). Sherin and van Es (2003) studied teachers who were working on improving student talk in the classroom by analyzing the talk already happening in their classrooms. They saw these teachers moving along the path from just attentive listening to what Callahan (2011) later identified as more responsive listening. These teachers were learning to notice.

Sherin and van Es (2011) theorized that learning to notice required the teachers to analyze the talk in three ways: (a) determining what was important in the situation, (b) interpreting the interaction, and (c) asking what this was a case of. They studied three teachers who were committed to learning to notice. One teacher took photographs of students working in groups to solve a problem. This teacher tried to magnify the situation and identify the action. The teacher then categorized the pictures in hopes of identifying the case. Another teacher wrote in a journal at the end of the school day to review and reflect on the student discourse that happened in his math class that day. The third teacher videotaped parts of a math discussion so that she could analyze herself and her students after the lesson and locate important features of instruction. Sherin and van Es argued that learning to notice, as these teachers were, would allow them to move toward more meaningful math talk and prepare them to follow through with the NCTM's (2000) recommendation to make decisions in the midst of instruction. Although these teachers were going further in the development of effective math talk, researchers continued to refine their ideas of the necessary conditions for effective talk.

2.3.3. Necessary conditions of effective math talk. Yackel (2001) agreed that student math talk should include the components of explanation, justification, and argumentation. Like Fosnot and Dolk (2001), Yackel asserted that two constructs are particularly relevant to explanation, justification, and argumentation: social norms and sociomathematical norms. She defined these constructs as the ways in which students interact with each other, with the teacher, and with the mathematics as learners in the classroom. According to Yackel, these two constructs have been helpful in clarifying the functions and the means by which they can be fostered in the math classroom.

These constructs also have been used to make sense of math discourse. As Voigt (1996) pointed out, when studying students' learning in reform-based classrooms, researchers have to take into consideration that different processes are going on, namely, the individual's social processes supported by social norms and the mathematical sense-making process supported by sociomathematical norms. Teachers need to consider that more than just math learning is occurring.

2.3.4. Necessary conditions of math talk: Practical advice. In thinking about creating the social norms of a math-talk community, mathematics educators Chapin, O'Connor, and Anderson (2003) suggested that creating a community of learners takes time and modeling by teachers. Before the students can contribute in responsible and meaningful ways, several instructional strategies need to be used in what they conceptualized as talk moves. These talk moves need to be modeled and then practiced during math games and early year community-building activities.

When students play games, Yackel's (2001) ideas of explanation, justification, and argumentation might occur naturally. When they do, teachers need to make this communication explicit for all students. In the opening vignette, Al noticed that Joshua was taking his time answering the morning question, so he decided to explain how the question was to be interpreted. Joshua was open to that explanation. This interaction occurred without the teacher being present.

Chapin et al. (2003) stated that teachers need to encourage students to use talk moves such as paraphrasing and revoicing their own ideas, ask students to rephrase or repeat another's ideas, and give students time to think and talk with their peers before addressing the whole group. Students and teachers should use moves such as these to participate productively in class. Chapin et al. also contended that in classrooms where specific math-talk moves are followed, quality talk is the focus and will be the most mathematically productive. Is there evidence that this is the case?

2.4 Back to the Classroom: The Impact of Effective Math Talk on Learning

Researchers have suggested that this vision of math talk results in improved learning for students, even for marginalized students. Lipka and Andrew-Ihrke (2009) conducted a study in a remote Aboriginal community in Alaska. Their focus was to have students not only work through meaningful problems but also encourage lots of talk. They used talk moves to encourage the students to share ideas with each other and the teacher. The problems were referenced culturally, and dialogue between and among peers was encouraged in their native tongue. An IRE format of discussion was purposefully avoided. The researchers found that more mathematical ideas were presented in a variety of strategies than was the norm, resulting in students gaining a greater understanding of the mathematical concepts and having the confidence to tackle more realistic mathematical problems.

Some researchers have found that students in classes where math talk is promoted make effective use of it, even when the teacher is not present. Steinberg and Cazden (2001), for example, analyzed videotapes of students acting as tutors for one another and pointed out that the children displayed surprising competence in dealing with educational tasks outside the teacher's span of direct control once they had been taught how to manage themselves as independent learners. By modeling responsible and mathematically focused talk for the students, the teacher allowed them to mimic it and then own it. It might be that the teacher's efforts pushed the classroom community of social norms to sociomathematical norms because the accountability to participate productively was driven by the students.

Webb et al. (2008) found this same result in their study of three classrooms where teachers were attempting to implement reform instruction. The students in one class began mathematically assessing their own strategies and the strategies of their peers. If evidence has shown that creating a community of students whose sociomathematical norms include the argument and defence of ideas and accountability to each other supports improved student learning, then what must teachers know in order to achieve these elements?

2.4.1. Necessary teacher knowledge in creating effective classroom discourse. Beyond creating a community of learners, teachers need to be prepared for the mathematics being investigated. They must understand the mathematical content as well as the possible thinking and mathematical development of their students in order to orchestrate the discussion (Hill & Ball, 2009). Fosnot and Dolk (2001) recommended that teachers replay their teaching day by reflecting on the goal of the lesson; remembering the successes; evaluating the inquiries; and thinking about the insights with a sense of the landscape of learning, a students' developmental numeracy continuum that is progressive rather than linear that can help teachers to track students' mathematical development in number sense (see Figure 2.1).

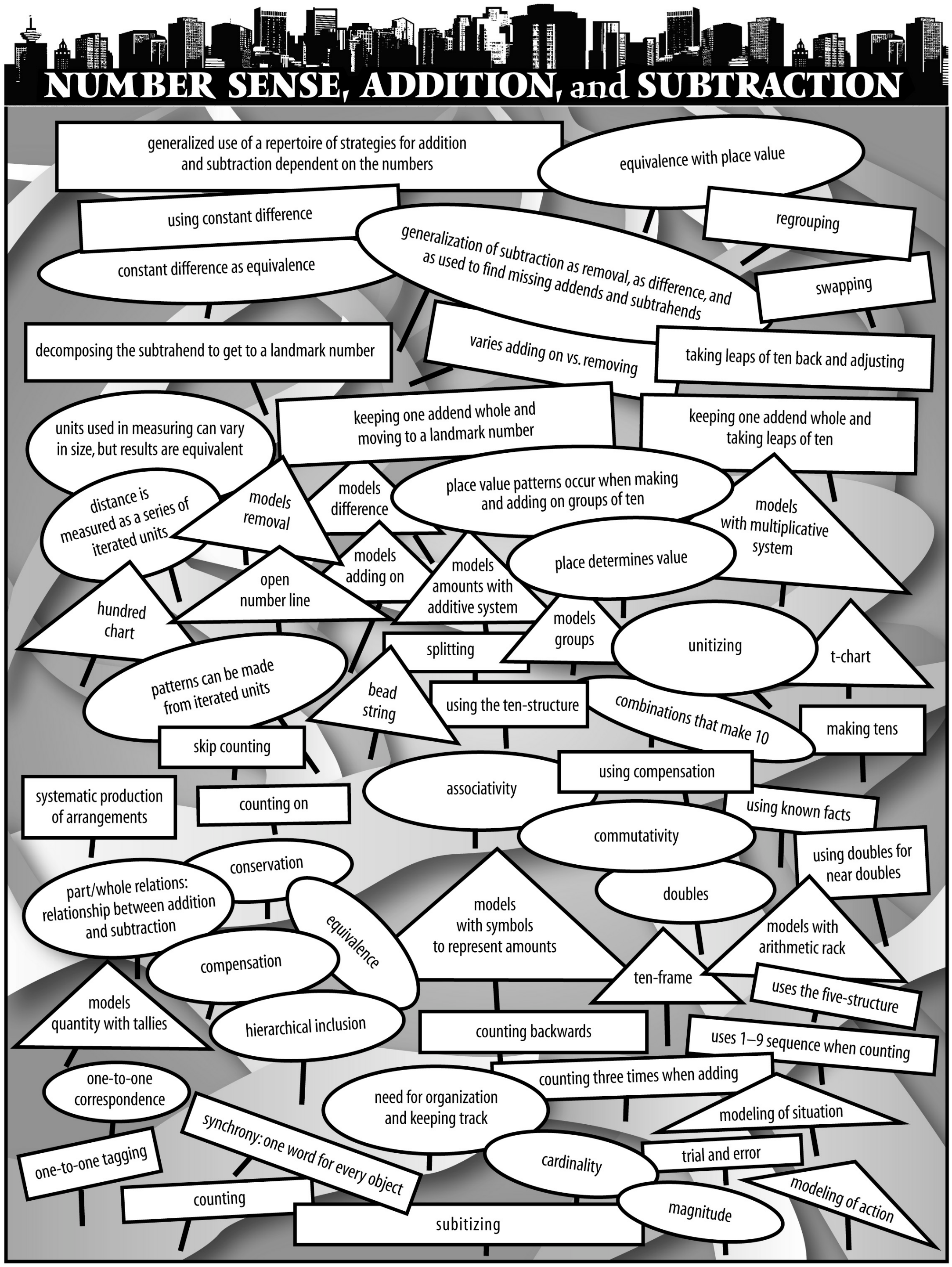

Figure 2.1. Landscape of Learning Number sense, Addition and Subtraction (Dolk et al., 2007).

The metaphor of the landscape consists of mathematical strategies (i.e., inventions that children use to solve problems); big ideas (i.e., central organizing ideas of mathematics, or the principles that define mathematical order), (Schifter & Fosnot, 1992); and models (i.e., concrete or abstract representation of students' thinking). It documents children's development over time in effective reform-oriented classrooms, beginning with the earliest strategies and big ideas of mathematics theory upward through and to their later, more efficient strategies and more sophisticated big ideas or mathematical principles. Without knowing adequately where students are in the mathematical landscape, it is difficult to anticipate the mathematical moves necessary in either a planned or an impromptu math discussion. The landscape offers this support because it is a marker of children's development as well as a foundation for teachers, supporting their further growth.

2.4.2. Using the landscape to facilitate math talk. If the landscape is an accurate reflection and conception of children's addition and subtraction development in an effective, reform-oriented classroom, then what role does a teacher's ability to facilitate productive mathematical talk play in children's progress? Although great strides have been made in the facilitation of student math talk, more needs to be known about the role of math talk in helping students to construct mathematical strategies and big ideas. What types of social norms and sociomathematical norms in the classroom contribute to children's evolving strategies and big ideas in early mathematics? Furthermore, what role does a teacher's ability to facilitate productive mathematical talk play in students' progress? The landscape offers a framework to assess the impact of math talk on students' mathematical development.

CHAPTER 3: METHOD

Research Questions

By conducting this study, I hope to answer the following questions:

1. What is the role of social norms (as defined by Sfard, 2000) in my classroom math community that contribute to the construction of strategies and big ideas in early addition and subtraction?
2. What is the role of sociomathematical norms (as defined by Sfard, 2000) in my classroom math community that contribute to the construction of these strategies and big ideas?
3. What pedagogical and mathematical content knowledge do I as the teacher draw on to facilitate the development of strategies and big ideas?

3.1 Study Context

In 2005, my school in Mississauga, Ontario, was selected as the research site for an ethics board-approved longitudinal study completed in 2012 by Lawson. The school was selected because of the willingness of the principal and the teachers to learn about and implement reform-oriented mathematics instruction. An additional consideration was the school population's transiency, diversity, and lower socioeconomic status. My study was conducted as one part of this larger study.

3.2 Research Design

Lawson's study, which was completed in 2012, focused on the strategies invented by students to solve a series of number sense problems. She followed the students in my Grade 1 class through to their completion of Grade 5. The parents or guardians of the students who took part in her study gave permission for the students to be videotaped for the purposes of my research, which focused on the development of math strategies and big ideas through math talk in a whole-group setting instead of through individual interviews.

I conducted my part of the study using a qualitative case study design, which was an effective way to study the role of mathematical discourse in student learning for several reasons. First, although mathematical discourse is not a new area of research, detailed research of mathematically productive talk has been sparse (Hiebert, 1992). A case study offered on-the-ground details about what effective math discourse looks and sounds like. Moreover, a case study based upon field data from the participants “focuses on connecting categories…not on simply describing categories” (Creswell, 2005, p. 402) and had the potential to make connections between the specific math discourse of students and the students’ constructions of strategies and big ideas in addition and subtraction.

Second, using this design also made sense because I focused on an activity (i.e., the social construction of math knowledge) while implementing a bounded instructional program (i.e., development of a mathematical model). Creswell (2005) asserted that the “case” might represent a process consisting of a series of steps (e.g., a curriculum process) that form a sequence of activities. As math discourse was studied, understanding what the process of math talk comprises, as well as what effective math discourse looks and sounds like, was captured in the experiences of the participants (Dolk, Liu, & Fosnot, 2007). It was a design that could increase current understanding of ineffective and effective mathematics instruction with implications for teaching practice.

I recognize the subjective nature of my research and include my biases while possibly generalizing to other cases (e.g., Hufferd-Ackles et al., 2004). Bias was an inevitable part of my study because I was the teacher and the researcher. I was familiar with the students participating in the case study, so I might have interpreted what they were trying to say in a way that favored the results of the impact of the case study. The case lessons that I taught (i.e., the case that was applied) were designed by Dolk et al. (2007) and were intended to elicit a certain level of math talk, as described by Hufferd-Ackles et al. (2004). My findings, although not generalizable, might offer a rich description of what math talk should look like and sound like in the second wave of math reform.

3.3 Research Sample: Participants

The project was carried out with a convenience sample (Creswell, 2005) comprising all 18 students in my Grade 1 classroom. I purposely selected the three students who were the focus of the case study based upon the two criteria of level of achievement and frequency of discourse. I looked for three students from across the full range of below-grade level to above-grade level who also spoke sufficiently well to facilitate following their thinking over time.

3.4 Procedure

Although the parents or guardians of all students in the class had already given their permission for their children to participate in the larger study, Lawson sent home an additional permission form specific to this study. All parents agreed to have their children take part in my study. The videotaping for the study was carried out by Lawson during the third term of school and lasted approximately three weeks. I taught all of the lessons.

My teaching partner and I met before and after each lesson to discuss the mathematical teaching and learning. I reviewed the minilesson and the problem and anticipated what I thought would happen in terms of discourse and the types of student strategies. A potential goal for the final math meeting (which Dolk et al. [2007] referred to as a congress) also was reviewed. My preparation for the lesson involved looking at the landscape of learning (Dolk et al. 2007) and deciding, based upon the students' work, which strategy to discuss and which big idea to make explicit, providing that the circumstance allowed it. After the lesson, I revisited the students' participation and discussed whether my predictions had been correct. Using the landscape of learning, I tried to identify the strategies that the students had used to solve the minilesson and problem. A discussion ensued on ways to build on the strategies used by the students for the lesson the next day.

3.4.1. The teaching unit. To meet a specific expectation in the Ontario Ministry of Education's (OME) *Revised Ontario Curriculum Grades 1-8* (2005), students at the Grade 1 level need to be able to "solve a variety of problems involving addition and subtraction, of whole numbers, using concrete materials and drawings" (OME, 2005). The unit implemented during the study was designed to fulfill these expectations.

I implemented one of eight research-based mathematics curriculum unit supplements. The unit, *The Double Decker Bus: Early Addition and Subtraction in the Contexts for Learning Mathematics, Investigating Number Sense, Addition, and Subtraction* (Dolk et al., 2007), was intended to support the development of addition and subtraction to 20. The unit introduced the arithmetic rack as a mathematical model through the context of a double-decker bus. The bus eventually became the model, a calculating frame with two rows of 10 beads, with two sets of five (one red and one white) in each row (see Figure 3.1).

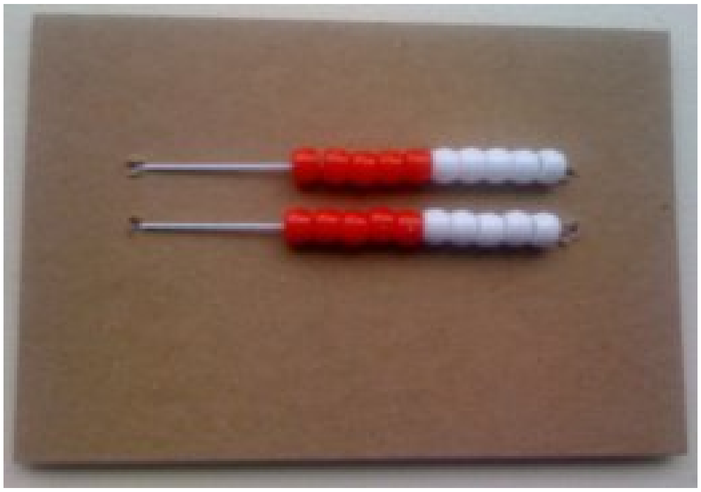

Figure 3.1. Calculating frame.

The students in the study had already had the opportunity to invent and develop mathematical strategies, big ideas, and models through other contexts for learning developed by Dolk's et al. (2007) supplementary units of mathematical study (e.g. *The Sleepover*, and *Grandma's Necklaces*). The students also participated in a math community for the 7 months preceding the study.

3.4.2. Data collection. Three weeks of video data (approximately twelve hours) were captured. Samples of student work (i.e., paperwork or board work) were collected each day. The work collected from the students was work that they had completed in pairs. The students had been paired homogeneously (based upon achievement) and had much experience working together. The final work sample was an independent assessment task.

I also kept notes of events that happened. After reviewing the video footage and my notes, I organized the notes into three categories. The first category dealt with student talk and instances that stood out as a learning community. The second category dealt with talk that would prompt mathematical discussion or deepen mathematical understanding in areas of the landscape of learning. Some of this talk might not have been the focus of the math content, but it could be used later in other investigations. The third category dealt with my reflections about students' reactions to the context and problem (e.g., body language, participation and willingness to contribute, behaviour), which helped me to determine the students' levels of interest. The notes were supported by discussions with my teaching partner, Donna, who was presenting the same lessons at the same time.

I also took anecdotal notes and recorded them directly on the students' work. Table 3.1 shows the types of data collected during the approximately three weeks of the case study. The collection of data was simple and relevant to the continuation of the case. It allowed me to engage in immediate and constant reflection. The frequency by which each type of data was collected depended on time available and ease of collection.

Table 3.1

Data Collection

Day of lesson	Classroom footage	Work on paper/the board	Reflection journal	Pretest	Posttest	Anecdotal notes
1	√		√	√		√
2	√	√	√			√
8	√	√				√
9	√	√	√			√
15		√	√		√	

3.5 Video Data Only

Although data was collected for all 10 lessons and for all students, I decided to code Days 1 and 2, the beginning of the unit, and Days 8 and 9, the end of the unit, only. Each lesson was approximately two hours in length, which I thought would give me enough talk data to analyze. The analysis, therefore, was the result of comparing what the students demonstrated at the beginning of the video study to what they demonstrated at the end of the unit.

3.6 Data Analysis

I developed three main types of a priori codes to identify the types of math talk, the emerging strategies and big ideas on the landscape, and the types of pedagogical content knowledge that I needed to facilitate the math talk.

3.6.1. Math talk codes. To analyze the mathematical talk, I established a modified coding system (see Table 3.2) based upon a framework developed by Hufferd-Ackles et al. (2004). They identified four distinct but related components to capture the growth of the math talk learning community in the classes that they observed. They analyzed the development of mathematical discourse of the teachers and the students in the data using four categories: Questioning, Explanation of Math Thinking, Source of Mathematical Ideas, and Responsibility for Learning. They coded the changes in the actions of the teachers and students based upon these categories on four levels that ranged from Level 0 to Level 3, with Level 3 characterizing the teachers and students as co-teachers and co-learners in math talk in a strong math community. The teachers monitored all that occurred in the classroom and were still fully engaged, albeit in more of a monitoring role and on the periphery. In particular, I used their descriptions of what was happening at Levels 2 and 3 of a mathematics community to develop the codes because they also were the levels that aligned the best with the type of questioning Dolk et al. (2007) wrote about in their supplementary units.

Table 3.2

Modified Coding System

Broad-themed category talk codes *(Pertains to research questions, 1, 2, & 3 –see p. 22)	Subcodes (layering themes) Broad categories broken down into specific items to look for.				Social norm (SN) or Socio-mathematical norm (SMN)	Independent (I) Or teacher prompt (TP)	Researcher/s who discussed this type of talk assert that such discussions are thought to support student learning of mathematics in part by:
1. How often the students talk to each other by…*1	Asking each other a question.	Commenting on each other's ideas.	Participating in paired talk time (on topic)	Addressing the whole group with an "I notice or wonder…" (spontaneous)			Lampert et al. (2003) discuss how classroom discussions evolve through back and forth dialogue between students.
2. How often students defend their ideas by…*1, 2	Referring to a previous problem.	Referring to another students' work.	Rephrasing their explanation.	Using a model of the situation.			Making students' thinking public so it can be guided in mathematically sound directions, by their peers and teachers (Forman, McCormick, & Donato, 1997).
3. How often students make decisions by… *1, 2, 3	Deciding if an argument is sound.		Deciding if an argument can be applied to the mathematical situation being discussed.	Making a mathematical conjecture, generalization or rule.			Encouraging students to construct and evaluate their own and each others' mathematical ideas (Forman et al., 1998)

4. Using math terms…*2	Labeling for the first time.	Correctly.	Incorrectly.	
5. Making connections between strategies by… *1, 2	Building upon what another student did.	Building upon own strategy in current or past problem.	Noticing a similarity and/or difference between one or more strategies.	Freudenthal (1973) contended that allowing students to analyze their own thinking and work helps them to develop more sophisticated ways of solving a problem.
6. Mathematical *Big Idea* can be highlighted on the student's personal learning trajectory (landscape) because…*2, 3	The student used and/or discussed and/or explained most of the strategies surrounding the big idea on the landscape.	The student answered teacher directed questions to communicate understanding of strategies surrounding the big idea even if not obvious on students' work.	Student differentiated between the efficiencies of certain strategies discussed by peers.	The role of the teacher during whole-class discussions is to develop and then build on the personal and collective sense making of students, rather than to simply sanction particular approaches as being correct or demonstrate procedures for solving predictable tasks (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989).

7. Teacher demonstrates pedagogical and/or mathematical content knowledge by…*3	Asking a question to determine the strength and depth of a students' ideas and to place the idea on the landscape	Using talk moves to reinforce a community of learners	Stringing together a discussion and linking strategies	Making content explicit to encompass a math concept		Having student presentations build on each other to develop important mathematical ideas (Hill et al., 2005).

In order for Dolk's et al. (2007) general descriptions to be used for coding, I changed them into observable instances. For example, where they described questioning in a Level 3 classroom as "Students ask questions and listen to responses" (p. 90), I changed this to the first row of codes beginning with the stem Number 1, "How often students talk to each other by… asking a question; commenting on each other's ideas; participating in paired talk on topic; addressing the whole group with an 'I notice' or an 'I wonder.' " Drawing on the work of Franke et al. (2007), I added a layer describing the direction of the talk (e.g., student-to-student, student-to-teacher, etc.) as codes. Following Sfard's (2000) example, I then added a column to identify whether the talk observed could be classified as a social norm or a sociomathematical norm. Finally, just as Hufferd-Ackles et al. (2004) noted the decrease in teacher direction and the increase in teachers and students working as co-learners, I added the last column to note whether the talk was teacher prompted or independent.

3.6.2. Developing the emerging strategy and big idea codes. In row 5 in Table 3.2, I used Dolk et al.'s (2007) landscape of learning to code any instances of strategies being discussed. I referred to the descriptions and definitions of strategies listed in the *Double Decker Bus* resource to substantiate these coding instances. In row 6, again, I referred to the landscape of learning for instances when I could identify the construction of a big idea.

3.6.3. Developing codes to analyze teacher pedagogical knowledge. I continued to draw on the work of Hufferd-Ackles et al. (2004) as well as Hill, Rowan, and Ball (2005) to develop the codes to identify and describe my pedagogical knowledge and mathematical content knowledge, as laid out in row 7 of Table 3.2, necessary to teach the unit.

In Levels 2 and 3 of their table, Hufferd-Ackles et al. (2004) described the talk moves that teachers make to facilitate discussion and learning. They explained that the role of the teacher is that of an instigator rather than a director who asks open-ended questions that target a mathematical concept that might lead to mathematical generalizations. Hill et al. (2005) mentioned that teachers' knowledge of mathematics is not the only variable that predicts student achievement. Other factors such as time spent preparing the lesson as well as time spent analyzing and discussing students' solutions also can be included in the measurement of content knowledge.

3.6.4. Coding. I entered the 4 days of videotapes into Atlas.ti qualitative software for coding. The software allowed me to use tools to examine (i.e., locate, code, and annotate) the multimedia data. I entered the a priori codes outlined in Table 3.2.

Creswell (2005) asserted that the first step in qualitative data analysis is to perform a preliminary exploratory analysis, which involved previewing the videotape segments. I conducted this preview to obtain a general sense of the data and inform any additions or deletions to my codes. As I previewed the data, I also wrote memoes in the margins of my field notes or in Atlas.ti as they occurred to me.

I coded all instances when one of the three chosen students spoke or I spoke in relation to one of the students. I used the work samples and the students' pre- and postassessments as well as my daily notes to substantiate my codes. As I watched the videotapes, I added to the codes iteratively. Lawson (2012) also watched some of the video and examined the coding.

Once I coded all of the data, I looked at the strategy development over time to find specific details of strategies and identify any differences and similarities among them. I also paired the students' talk with their work completed the same day to determine whether what was said and what was written or drawn matched. I then thought about the sophistication of the strategies used by the students. In their research on collective inquiry among teachers analyzing students' work, Kazemi and Franke (2004) mentioned that they had the facilitator of the teachers in the study discuss the relative sophistication of the strategies in order to learn more about what and how their students understood the math being investigated. I discussed my observations with my advisor in an attempt to clarify, add detail to, and modify the coded strategies. I reviewed the next day's footage of selected strategies to identify any changes.

3.6.5. An analysis framework for the codes. To analyze my data, I also drew upon the data analysis cycle described by Jacobs, Tawanaka, and Stigler (1999). They applied this cycle to a wide-scale video study of the TIMSS (1995). They discussed how their cycle was a useful tool in looking at the many components of a math lesson on video. Although Jacobs et al. talked about the benefits of how video analysis aids in conducting a thorough mixed methods study, including qualitative and quantitative data, and even though their cycle promoted an iterative research process that strengthened the qualitative and quantitative findings, I found it to be just as useful a framework in helping me to analyze my video for a qualitative case study.

The cycle that Jacobs et al. (1999) described outlined interconnected steps to analyze the video data and use other pieces of data collected in conjunction with the videos. In my case, the other data included students' work samples, pre- and post independent tasks, and a journal of my reflections after each lesson was taught. This cycle facilitated the emergence of new discoveries because the video data were observed more than once. These new discoveries were applied to existing codes to more clearly define or change the codes.

Jacobs et al. (1999) explained that the data analysis cycle involves watching, coding, and analyzing video data with the goal of transforming the video images into verifiable information. The cycle begins by watching the videos and discussing what is observed. The second step in the cycle is to link the discussions back to the video and make clear (by relabeling), or create new items or passages for coding while generating a hypothesis about what is being observed. The third step is to analyze and interpret what is being observed, followed by developing and applying the codes (a priori, refined, and new). Once I had completed all of the coding in the current study, I tabulated the frequencies.

CHAPTER 4. FINDINGS AND DATA ANALYSIS

The Students

All three students whose talk and work samples I selected to analyze exhibited a willingness to talk during group math discussions. I thought that they talked enough throughout the lessons to offer an adequate amount of data to analyze. These three students also had been selected to participate in this study because they were able to answer questions about their work during conferences with me. One of the students I selected was an English language learner (ELL), and despite the challenges facing ELLs to participate in a discussion-driven learning environment, she still managed to communicate enough during the study to allow for a thorough analysis. The level of conceptual understanding also varied among the three students selected. I thought that this last variable was important in order to assess the success of progress in developing the specific strategies and big ideas that this particular unit supported. Finally, each one of the three students was from low-, middle-, and high-achievement groupings.

Shanzey, an ELL, was the first of the three students. Her performance in Grade 1 at the beginning of the study was considered below the provincial standard. I anticipated that I would see growth throughout the 3 weeks of lessons.

Damien was the second student. He consistently talked a lot. His talk also seemed to be varied, in that he blurted out opinions, connections, and responses to the questions. He asked many questions and commented on what his peers were saying. His work samples left me puzzled because they did not often match what he was saying in terms of sophistication of mathematical strategies. In addition, his performance in Grade 1 at the beginning of the study was considered almost at the provincial standard, and I thought that I would see growth throughout the 3 weeks of lessons.

Endrias was the third student. His performance in Grade 1 at the beginning of the study was above the provincial standard, and I wondered whether his use of strategies would change and develop throughout the 3 weeks of lessons. In particular, I wondered whether he would vary his strategy use to reflect the types of problems that were posed.

4.1 Video Data

I attempted to understand the depth of their understanding in mathematics as well as explore their levels of collaboration based upon how they talked and what they talked about. According to assessment reform initiatives, the processes involved in teaching and learning should evolve as a group or a learning community moves together to achieve a common goal (Marzano, Pickering, & McTighe, 1993). In this case, it was to develop a model that would support decomposition strategies for addition and subtraction rather than counting by ones while consolidating appropriate discourse moves as the students demonstrated movement up the landscape of learning. I hoped that the talk in my classroom engaged in by the students would reflect those goals.

4.1.1. Coding frequency of talk. As I watched the videos, I noticed and wondered about (i.e., began forming and revising our hypothesis) the frequency by which each of the three students participated in the whole-group conversation. I had reason to believe that the frequency of participation through talk was an important aspect in the analysis of a math lesson because it influenced the individual as well as the collective understanding of the mathematical concepts being investigated.

Sfard (2008) asserted that participation means more than just answering a question correctly or incorrectly. In his review of Sfard's book, Stahl (2009) summarized what Sfard had written about math discourse. He summarized that math discourse among children is a social routine. He reiterated that students individualize the social language pertaining to math in their own personal math thinking. Stahl noted that according to Sfard, a discursive social process is not acquisition knowledge, but participation in the co-construction of realizations. Therefore, the development of personal math thinking comes with, and is revealed in, the unfolding of discourse over time. The more involved in the process students become, the more time students have to develop their personal math thinking.

After thinking through this assertion, I hypothesized that the students would participate more (i.e., increase in the frequency of mathematical talking) as the lessons unfolded over the 3 weeks of the study. I hypothesize that as time elapsed, personal math thinking would develop and the frequency of talk would increase. I also thought that this would happen because Chapin et al. (2003) asserted that the more time that students are given to talk and mull over a good question, the more they talk and the more mathematically productive is their talk.

Table 4.1 shows how often the three students participated in the discussion at the beginning of the 3 weeks of lessons and at the end of the 3 weeks of lessons. Participation included talking to the whole group and being captured on film while talking with a math partner during the whole-group phase of the lessons.

Table 4.1

Frequency of Talk for Each Student for Days 1 and 2 and Days 8 and 9

	Shanzey		Damien		Endrias	
Instances of talk	Days 1-2	Days 8-9	Days 1-2	Days 8-9	Days 1-2	Days 8-9
1-10	√	√	√	√	√	√
10-20		√	√	√	√	√
20-30			√	√		√
30 +			√	√		

I coded talk if it was obvious and part of the discussion that I was listening to. The talk coded here does not include muffled utterances with a partner, even if on topic, and in the background that I was not present for. At the beginning of the video study, Shanzey was coded for talk up to 10 times and, by the end of the video study, she was coded for talk up to 20 times. Endrias's instances of coded talk also increased. At the beginning of the study, Endrias was coded for talk up to 20 times, but at the end of the study, he was coded for talk up to 30 times. My hypothesis was correct for Endrias and Shanzey, whose frequency of talk increased. The instances in which Damien was coded for talk remained the same. He was coded as talking more than 30 times at the beginning of the study and at the end of the study. What still needs to be discussed is the type of talk that they were participating in and how long their participation lasted. Another question requiring exploration is whether their talk was in response to a question or was self -initiated.

4.1.2. Direction of talk. As I watched the videos of the students, I noticed the direction of the talk, that is, to whom were the students speaking? I hypothesized that the direction of talk would change. I expected to see the direction of talk move student to teacher toward student to student. By the end of the video study, I predicted that there would be more student-to-student talk rather than student–to-teacher talk based upon Hufferd-Ackles et al.'s (2004) analysis of talk data that as time passed and the mathematical discussions continued, students would become more persistent in challenging peers' ideas as well as persistently pursuing clarification of peers' strategies. They noticed that as students took on more responsibility for the direction, the conversation changed because the students entertained more questions and comments from their peers. They concluded that the conversation was being managed by the students and with minimal intervention from the teacher. I was expecting to see the same outcome in this video study, and this subsequently was the case. My results were similar in that the conversations between the students were lasting longer.

In the following excerpt from Day 2, students were talking about what they noticed about the arithmetic rack. I directed the discussion by prompting the students to explain their thinking and rephrase their strategies in order to make the math explicit to them and check for my own understanding. This excerpt was mainly a student-to-teacher discussion.

Me: "Endrias, thank you. Henry, make sure you're listening to see if he gets it. If you were a bit unsure, this is a chance to think through it again. Talk to us, Endrias."

Endrias: "He said he knew what this was because it's two groups of 5, and 5 plus 5 equals 10."

Me: "And Endrias, how many groups of 5 are on the arithmetic rack?"

Endrias: "Two. I mean."

Henry: "Two on each row."

Endrias: "So that's 2 on each row, so it's 4."

Me: "And how many groups of 5 were on the double-decker bus? -2 – 4."

Damien: "No it's 4 because 10 seats on the top, and 10 seats on the bottom. So 5 and 5 and 5 and 5."

Endrias: "So the white seats are 5, and the red seats is 5."

Me: "Show me with your thumbs [directed at the class] – thumbs up – if you think you understand what Henry said about seeing two groups of 5 on the bottom deck."

The next excerpt demonstrates a mainly student-to-student discussion with limited teacher involvement. The students were explaining their work during a congress on the second day.

Shanzey: "I was gonna say that this one (pointing to a peer's strategy on the board)."

Me: "Wait until you're being respected, Shanzey."

Shanzey: "This strategy is like what I did yesterday on the sheet, with 2 left."

Damien: "That's exactly the same. Both."

Shanzey: "Because there are 2 away, the two here. Yesterday, there were 2 left, and now there are 2 left."

Moses: "And they counted backwards."

Damien: "Yah, it's 17 here and there because there are 2 left. That means it's 19 and 18, which leaves 17."

Ozair: "Actually, it's 20 and 19, so it leaves 18."

Endrias: "So it's kind of the same."

Me: "How is it kind of the same?"

Shanzey, Damien (at the same time): "Because there are two not here [absent from class]."

As I continued to watch the video for the direction of talk, I also noticed that the lengths of the conversations that the students were having with each other increased, along with the length of time that the students took to explain a strategy. Because of this, I also hypothesized that not only would students talk to each other more but also that the discussions among the students would last longer as the math investigation continued.

I believe that the students became more accountable. My reasons are based upon the explanation provided by Michaels, O'Connor, and Resnick (2008) in their review of literature on the development of accountability. They summarized that the critical features of academically productive classroom talk fall under three broad dimensions of accountability: accountability to the community, accountability to the knowledge, and accountability to accepted standards of reasoning. They explained that students who learn curriculum content guided by accountable talk standards are socialized into communities of practice (Lave & Wenger, 1991) in which respectful and grounded discussion, not "noisy assertions" or "uncritical acceptance of the voice of authority" (p. 4), are the norm.

Michaels et al. (2008) continued to explain that although forms of discussion that are accountable in all three dimensions are heavily discipline dependent, they also create environments in which students have the time and social safety to formulate ideas, challenge others, accept feedback, and develop shared solutions. I think that the students in this video study participated in this kind of classroom and, therefore, talked to each other more about the math and for longer periods of time as the study progressed.

Table 4.2 shows the frequency of the direction of talk for each student for Days 1 and 2 and Days 8 and 9. The frequency added up to more than the times the students were coded as participating because in one discussion, the student was coded once for participating and then within that discussion, the number of features of talk was also coded. For example, in the earlier excerpt, Shanzey was coded once for participating in a discussion and again for talking to another student in relation to building on a peer's strategy. She was coded yet again for talking to the whole group and making a connection.

Table 4.2

Direction of Talk

	Shanzey		Endrias		Damien	
Direction of talk	Days 1-2	Days 8-9	Days 1-2	Days 8-9	Days 1-2	Days 8-9
To teacher	21	7	14	19	19	10
To peers	19	15	29	27	78	55

The discussion in the previous excerpt exemplifies how Shanzey's participation (coded once) evolved into a multicoded discussion. When Shanzey referred to her past work and compared it to the situation at hand, it was coded again. When she took comments from peers, it was coded again. One of Shanzey's conversations might have had many codes to it, and that is why the total number of times of student-to-student talk was more than the actual number of times that students were coded to participate in a discussion.

On Days 1 and 2, Shanzey directed most of her talk toward me, but the difference in direction was not large: She talked to me 21 times and to her peers 19 times. On Days 8 and 9, however, the direction of talk toward her peers was almost double that of her talk toward me. What is interesting is that there were fewer coded instances on Days 8 and 9 than on Days 1 and 2. When I discussed these observations with my supervisor, I wondered whether there were fewer coded instances because the conversations lasted longer.

Table 4.2 showed similar results for Endrias and Damien. Both of them were coded as talking far more to their peers than to me on Days 1 and 2 and also on Days 8 and 9. What is interesting is that although the instances of talk did not change much for Shanzey and Endrias, there was a drastic difference in how often Damien was coded for directing his talk to his peers between Days 1 and 2 and Days 8 and 9. He was coded almost 20 fewer times on Days 8 and 9. When I discussed this outcome with my supervisor, I thought that instances might have been coded less often because the length of the conversations increased or there was more participation by the other students than on Day 1. Instances of talk directed toward peers over the course of the lesson study was not consistent for the three students and was not entirely consistent with my hypothesis.

4.1.3. Types of talk. I also delved into the different types of discourse. Good discourse has the elements of explanation, justification, and argumentation (Yackel, 2001). This type of discourse gives important feedback to students while allowing teachers to formatively assess students' understanding of the mathematical concepts being investigated. Often, their talk might be in relation to the feedback given by peers. Black and Wiliam (1998) argued that peer assessment is one way to enhance formative assessment because peers are honest with each other and help to make each other's thinking more explicit.

I coded two types of talk: social norm talk and sociomathematical norm talk. My hypothesis was that as the case study progressed, the sociomathematical norm talk would increase. Cobb, Stephan, McClain, and Gravemeijer (2011) contended that social norm talk becomes sociomathematical norm talk when students are not only presenting a mathematical argument but also judging whether it is an appropriate mathematical contribution to make and whether it constitutes an acceptable contribution. This process requires that students judge what counts as a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, or an acceptable mathematical explanation. Cobb et al. said that all of these requirements are negotiated when establishing mathematical norms, making them reflexive in nature. Cobb et al.'s ideas of sociomathematical norms aligned with Black and Wood's (1998) notion of the importance of self-assessment and peer assessment.

An example of talk that I analyzed is in the following excerpt from Day 1. I thought that Damien was demonstrating a sociomathematical norm because he was blending a talk move, such as rephrasing what a peer said, to help the whole group to understand a math idea. What made this social norm a sociomathematical one was that he explained his peer's math strategy in his own way and connected it to a math fact that he already knew. He did it at a time when there was a class pause. It seemed as if everyone was pondering what a student said.

Me: "Who can explain what she did?"

Damien: "What she did is she had 5 here, then she moved 5 over here (moving beads on the rack), and 5 and 5, I know this was 5 and this was 5, and I knew that 5 and 5 equals 10."

The next excerpt was taken from a sociomathematical norm discussion. The students were referring to a past mathematical context and discussing the similarities and differences of the mathematical concepts involved. Without any prompting from me about the connection, the students saw the big idea of unitizing in both contexts.

Damien: “I can explain. Tyler, do you remember when you and Storm did the blocks? It’s like that. You put them in groups.”

Tyler: “I remember that, but when were we talking about it?”

Me: “That was Collecting and Organizing [the unit].”

Damien: “Yeah, it’s just like that, in groups…Yeah, like you put so many in the little basket, then there, you, like, it’s almost the same thing.”

Tyler: “But we didn’t have enough blocks to make another group of 10.”

Me: “You didn’t have enough blocks to make another group of 10.”

Damien: “But still, it’s still like that, Tyler.”

Me: “Damien’s saying it’s still the same idea. You had groups of 10, but these are groups of 5.”

Damien: “Yeah, but there’s enough to equal 10. There is groups of 10 there.”

Table 4.3 shows the type of talk in which the students participated. In the instances that were coded, I found that all three students used sociomathematical norm type talk more often than social norm talk on Days 1 and 2 and again on Days 8 and 9. I was not surprised to see this because I believed that the participating students were already speaking in mathematically productive ways when the study began. Instances of sociomathematical norm talk decreased slightly for Shanzey and Damien by the end of the video study but stayed the same for Endrias. Again, I thought that this outcome was the result of conversations lasting longer or more participation from other students.

Table 4.3

Type of Talk

	Shanzey		Endrias		Damien	
Type of talk	Days 1-2	Days 8-9	Days 1-2	Days 8-9	Days 1-2	Days 8-9
Social norm	5	2	1	3	7	3
Sociomathematical norm	10	11	47	39	15	15

After analyzing the frequency of talk (Table 4.1), I found that the talk for Shanzey and Endrias increased from Days 1 and 2 to Days 8 and 9; the frequency of Damien's talk stayed the same. Damien consistently talked a lot on Days 1 and 2 and on Days 8 and 9. I believe that the overall frequency of talk increased because the students became more invested in discussing and defending their strategies in an attempt to solve the problems.

After analyzing the direction of talk (Table 4.2), I found that on Days 1 and 2, the students talked more with me and less with their peers. By Days 8 and 9, the students were coded as talking to me less than on Days 1 and 2 and more with their peers. Their total frequency of talk, however, decreased. I believe that the conversations that the students had with each other lasted longer on Days 8 and 9 than on Days 1 and 2 and that other members of the class were also engaged in the dialogue.

After analyzing the type of talk (Table 4.3), I found that there was already a high level of sociomathematical norm talk on Days 1 and 2, perhaps because I engaged in a lot of repetition and questioning to get the conversation going or because the level of mathematics was easier than the math on Days 8 and 9. Frequency of this type of talk by the students decreased by Days 8 and 9, perhaps because there was more participation from other members of the class.

4.2 Using Landscape of Learning to Track Students' Development of Strategies and Big Ideas

I also coded using Dolk et al.'s (2007) landscape of learning, namely, number sense, addition, and subtraction on the horizon showing landmark strategies (rectangles), big ideas (ovals), and models (triangles), to track students' development of strategies and big ideas. I used the landscape for analysis before, during, and after the case study. As one of the three students demonstrated a strategy through written work, conferencing, or on video, I check-marked it on the landscape. A checkmark with a tag (pre, post, 1, 2, 8, or 9) was added to show when the strategy was used or the big idea was developed. As strategies surrounding a big idea were checked, I also checked the big idea. Table 4.4 lists those big ideas and strategies and also illustrates the students' achievement of those big ideas and strategies during Days 1 and 2 and Days 8 and 9.

Table 4.4

Overview of Big Ideas and Strategies and Students' Achievement

Big ideas and strategies	Shanzey	Damien	Endrias
Cardinality – subitizing, trial and error	Achieved prior to study	Achieved prior to study	Achieved prior to study
One-to-one correspondence – counting, tagging, synchrony	Achieved prior to study	Achieved prior to study	Achieved prior to study
Hierarchical inclusion – counting on and counting back	Demonstrated throughout the unit	Achieved prior to study	Achieved prior to study
Compensation and equivalence – using the 5 and 10 structures, using doubles and near doubles	Emerging	Demonstrated throughout the unit	Achieved prior to study
Unitizing – making 10s	Emerging	Emerging	Demonstrated throughout the unit
Commutativity and associativity – using compensation, using known facts	Emerging	Emerging	Demonstrated throughout the unit
Relationship between addition and subtraction	Emerging	Emerging	Demonstrated throughout the unit

I found that all three students began the unit with an understanding of the big ideas of cardinality, one-to-one correspondence, and hierarchical inclusion, as well as the strategies to achieve those big ideas. The big ideas of compensation and equivalence, unitizing, commutativity, and the relationship between addition and subtraction, however, were just emerging at the beginning of the unit, despite some evidence prior to the unit of some of the strategies being used by all three students, albeit inconsistently. By the end of the unit, all three students had talked enough for me to assess their understanding of all the strategies; however, only Endrias showed it in his work samples. We hypothesized that there would not be too much movement up the landscape of learning because the students had entered the case study with a high level of conceptual understanding for Grade 1. This assessment was based upon their work samples collected prior to the case study.

Before and after the video study, the students worked on addition and subtraction problems. They worked independently to solve the problems. My teaching partner and I designed all of the problems so that they would align with the big ideas and strategies being supported by the Double Decker Bus unit and fell within the expectations of the curriculum. We believed that the problems posed prior to the Double Decker Bus unit and video study, as well as after the unit (as a postindependent task assessment) would support the students in their development of the strategies and big ideas found on Dolk et al.'s (2007) landscape of learning (see Table 4.5).

Table 4.5

Pre- and Postassessment Big Ideas and Strategies

Strategy	Big idea
subitizing, trial and error	magnitude, cardinality
synchrony, counting, one-to-one tagging, counting three times when adding, counting backwards	one-to-one correspondence, need for organization and keeping track, hierarchical inclusion, compensation, conservation
counting on, skip counting	Part-whole relations
using known facts, using compensation, using doubles for near doubles, combinations that make 10	doubles, commutativity, unitizing

Following are the two problems that I gave to the students prior to the study:

1. We are going for a walking trip to the fire station. We need to know how many Grade 1 students in total will be going. There are 18 students from our class and 19 students from Mrs. R' s class going. How many students are going on the trip? Explain how you know.
2. We are working on bringing litterless lunches. Today, in Mrs. Allen's class, 12 students brought a completely litterless lunch, and 4 students from Mrs. R's class brought one. How many students from our pod, in total, brought a litterless lunch to school today? Show how you solved the problem, and explain your thinking.

After the video study, the students worked on more addition and subtraction problems independently. I hoped that these problems would serve as an independent task assessment and would show me whether the students had progressed through the landscape. Following are the two problems given to the students poststudy:

1. Mrs. Allen only had 8 book order forms. Mrs. R gave Mrs. Allen 12 more. How many book order forms does Mrs. A have now? Do you think there are enough for our class? Show how you solved the problem, and explain your thinking.
2. Oh, no! Mrs. A had 17 cat treats. Now, there are only 8 left in the treat can. How many did Piglet sneakily take? How do you know? Show your work, and explain your thinking.

4.2.1. Shanzey. I noticed that Shanzey made many gains throughout the 3 weeks of lessons. She seemed to feel more comfortable contributing to the discussions because as the days progressed, she talked more. Her talk also became more focused: She asked some very specific questions and referred back to how past problems were solved to add to the discussions. She referred to how she had solved past problems but also made some connections to what some of her peers thought and shared. In both of these primary documents captured on video (8: 0806_08_moMA and P 9: 0806_moMA), Shanzey answered a question to help to connect strategies and entered into a conversation about the relationship between addition and subtraction without prompting. She also showed more confidence because she often had to repeat what she was saying to make it clearer for her peers.

4.2.1.1. ***Pre-unit assessments.*** Figure 4.1 is a photograph of Shanzey's work on the first addition problem given prior to the study.

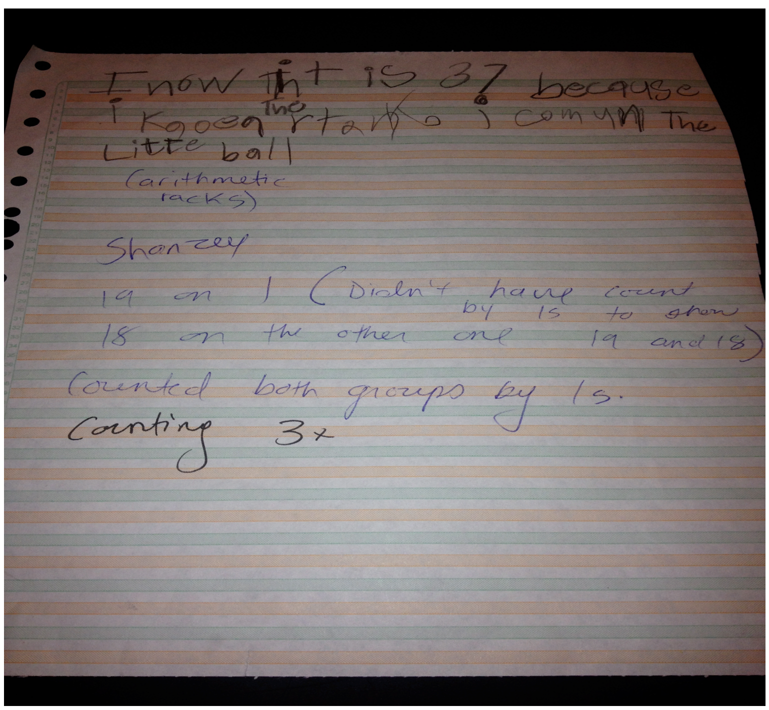

Figure 4.1. Illustration of Shanzey's work prior to first addition problem in study.

I decided that a conference with Shanzey was necessary in order to see how she counted "the little balls" she referred to in her explanation. After I conferred with Shanzey, she showed me how she got 37 in total. She used two arithmetic racks. She showed 19 on one rack and 18 on the other rack. She showed these amounts quickly by removing one bead from 20 to show 19 on one rack and two beads from the 20 to show 18 on the other rack. Shanzey knew what 18 and 19 beads looked like on the rack from previous experience using the racks. Then, once she had the two groups, she counted them all by ones until she got to 37. From this conference, I was able to check off on her personal landscape of learning the strategy *counting 3 times* and the big idea *need for organizing and keeping track*. I think it is important to note here that she exhibited confidence in her strategy. I made a note in my reflective journal that "she asked to share her strategy." Figure 4.2 shows Shanzey's work from the second addition problem.

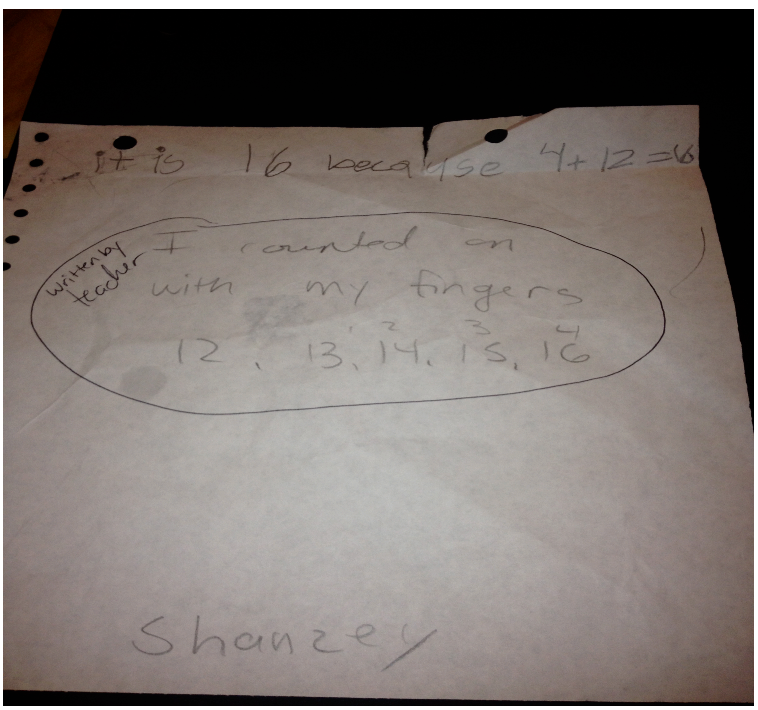

Figure 4.2. Illustration of Shanzey's work on the second addition problem.

She wrote, “It is 16 because 4+12 = 16.” When I conferred with her, she told me that she had used the *counting on* strategy by using her fingers and counting four more times from 12. After talking with her, I checked the *counting on* strategy on her personal landscape of learning. I also made a note in my journal as I previewed some work before the study that “written work not matching oral explanation. Take note of other students who may be doing the same. Have a class discussion about matching our thoughts with our posters or adding on to the picture and the words when done.”

By tracking Shanzey’s progress on the landscape, the following excerpt from Day 1 showed that Shanzey might have demonstrated an understanding of Dolk et al.’s (2007) big idea of hierarchical inclusion, the idea that numbers grow by one and exactly one each time. Based upon the following excerpt, I checked hierarchical inclusion on the landscape.

Shanzey: “I know it’s 6 because … this is 5, and this, add 1 more, and this is 6.”

Me: “5 and 1 more is 6?”

In the next excerpt from Day 2, Shanzey used counting on and counting back and showed an initial understanding of the big idea of the relationship between addition and subtraction. Dolk et al. (2007), in their definitions of big ideas, asserted that as students gain flexibility in composing and decomposing numbers, they begin to generalize about the way in which the parts are related to the whole.

Me: “That was hard. That was hard. It was, I think, too quick. Let’s try again.”

Damien: “I can do it, actually.”

Me: “You can do it? Okay, do you remember what it is?”

Shanzey: “Ahh.”

Me: " 'Cause you need to remember what it is."

Shanzey: "Yeah."

Me: "Okay, show him what you saw. … So what did you see?"

Damien: "14."

Me: "Is it 14?"

Shanzey: "No, it was 11."

Me: "It was 11."

Shanzey: "Yeah, it was 11. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11."

Me: "That's what you had?"

Shanzey: "Um…"

Me: "You need to remember what you have, so maybe, maybe in the beginning, make them a bit easier by including less so that you can keep track of what you have. Close your eyes, Shanzey."

Damien: "Okay."

Me: "How many?"

Damien: "No, not actually ready. 18."

Me: "Alright, okay."

Damien: "Okay, Shanzey."

Me: "And move them then."

Shanzey: "Okay, I saw this."

Me: "And how many is that, Shanzey?"

Shanzey: "I got 18."

Me: "And how do you know that's 18? We saw you started counting by ones and then you just knew it was 18?"

Shanzey: "Because [Damien interrupts]."

Me: "Let her finish; let her explain."

Shanzey: "Because that's 19, and that's 20." (indicating one more bead than 18 and then 2 more)

Me: "Then you know that 18 is left. She's – do you know what she's doing there?"

Damien: "She knew that – she's counting backwards.

Me: "She is counting backwards, that's right. So this is what you can write. You can say 18 people on our bus. What we saw – what did you see?"

Shanzey: "I sawed – I sawed 10 – actually, 1, 2, 3 –"

Me: "But you didn't even count by ones, you knew it without counting by ones."

Shanzey: "I sawed two left."

Me: "You saw two left."

Shanzey: "Then there was 8. So I sawed 18, and that's how I knowed it was 18, because 2 left means that it's 18."

Me: "You saw 2 left. You saw 20, and 19, and knew 18 were left. Okay. If you're having trouble writing your ideas, come and get Mrs. Allen, and I'll help you write them, okay?"

Shanzey: "Okay."

Damien: "Your turn!"

Shanzey: "Close eyes!" (P 2: 0806_02_moMA)

In this next excerpt from Day 8, Shanzey showed her understanding of the big idea of compensation.

Me: “Shanzey, did you want to say something? Make sure you are being respected. Wait until you are being respected and listened to.”

Shanzey: (moves beads on the rack 8 on the top and 2 on the bottom) “8 and 2 is 10.”

Me: “You just know that 8 plus 2 equals 10? Is it a fact you know?”

Shanzey: “Because I’m using the 1 plus 9 to do the 8 plus 2.”

Me: “Can you show us that 9 plus 1 you’re talking about?”

Shanzey: [moves 1 away from the bottom and adds 1 to the top]

Damien: “Ohhh! Like up there.” (pointing to a board)

Shanzey: “That’s 9 and then I did this.”

Damien: “You can keep going back and forth adding 1 and taking 1.”

Me: (addressing the whole class) “Shanzey really knows her 9 and 1, and she used that to help her figure out the 8 plus 2.”

Me: “Did anyone else see 9 and 1?”

Me: “Shanzey, please record your strategy over there on the flip chart. Thanks Shanzey, good for you. I noticed you edited your strategy.”

On Day 9, during a group discussion on why both addition and subtraction can work to solve a problem, Shanzey showed again (which consolidated for me) her understanding of commutativity and the part/whole relations, the relationship of addition and subtraction, when she spoke out during a whole-class discussion on why 4 plus 6 and 6 plus 4 and 10 minus 4 can help to solve the same problem.

4.2.1.2. Shanzey's postassessment. Figure 4.3 is a photograph of Shanzey's work on the first poststudy problem.

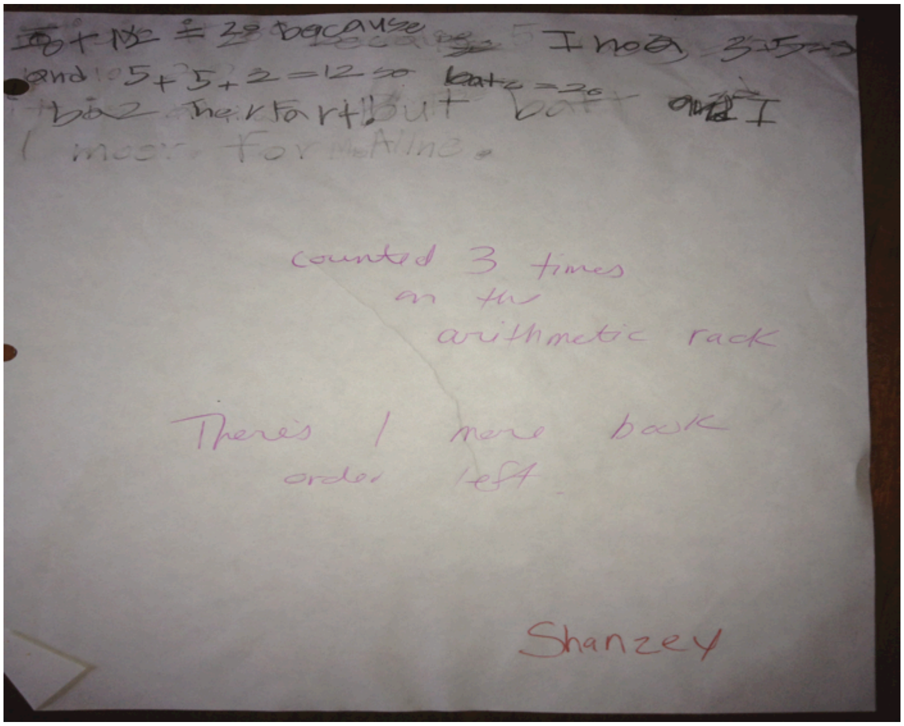

Figure 4.3. Illustration of Shanzey's work on first poststudy problem.

When I looked at the work that Shanzey did for her postassessment, I noticed that to answer 8 + 12, what she wrote and what she told me in a conference were different. When I asked her to explain her work, she said that she used the arithmetic rack, which she had used in a previous set of problems during the school year, and showed 5 red and 3 white on the top and counted to 8 by ones, then 5 red, 5 white on the bottom and counted those, then added 2 more to the top, separate from the 8. Then she counted all of them again by ones until she got to 20. On the landscape of learning, I checked that she demonstrated the strategy of counting three times when adding and the big idea of a need for organization and keeping track. Figure 4.4 is a photograph of Shanzey's work on the second poststudy problem.

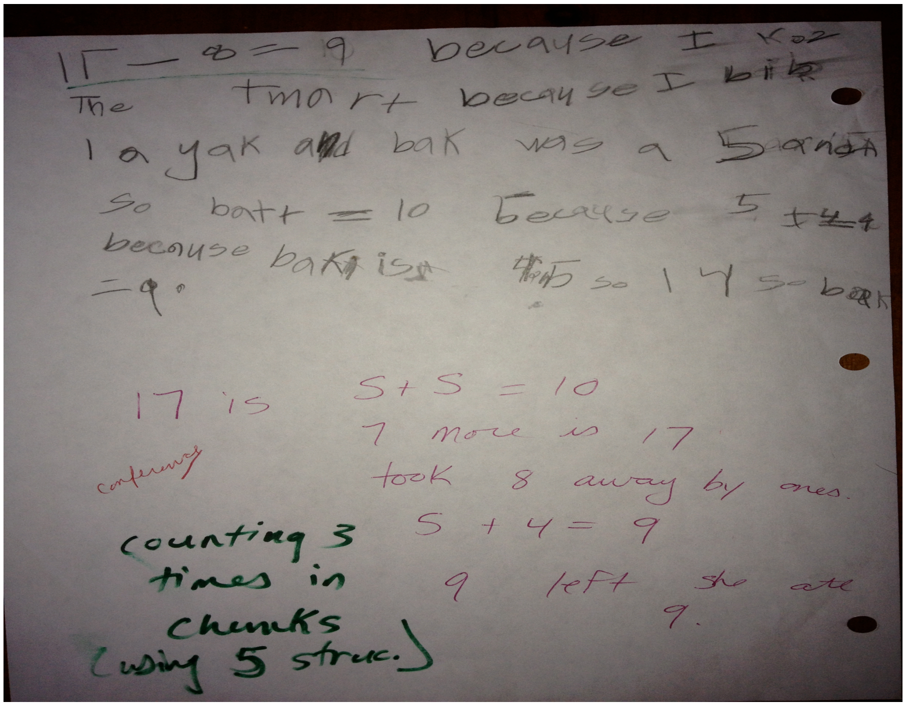

Figure 4.4. Illustration of Shanzey's work on the second poststudy problem.

It is difficult to understand what Shanzey wrote on her work, so a conference was necessary not only for her to read to me what she wrote but also because I noticed with Shanzey, her work often did not match her explanation. She showed me her strategy using the arithmetic rack and explained aloud that "5 + 5 = 10 and 7 more equals 17." Then she took away 8 by ones and said that "5 + 4 = 9. With 9 left, that is 17 - 8 so she ate 9." I also added my notes on the side of her paper indicating that she kept counting the total over again but maintained the groups (I wrote chunks) of 5 beads.

Figure 4.5 is a photograph of the landscape of learning that I used solely for the purpose of this data analysis. A checkmark indicates what Shanzey demonstrated through her pre-video work samples, through oral conferences and anecdotal notes on her work, throughout analysis of the videos, and from her post-video work samples. The numbers indicate which days of the lesson it was demonstrated; if it was demonstrated in her work before the video, then I wrote the word “pre-”; if it was demonstrated after the videos, than I wrote “post.” More than one checkmark meant that it was demonstrated more than once.

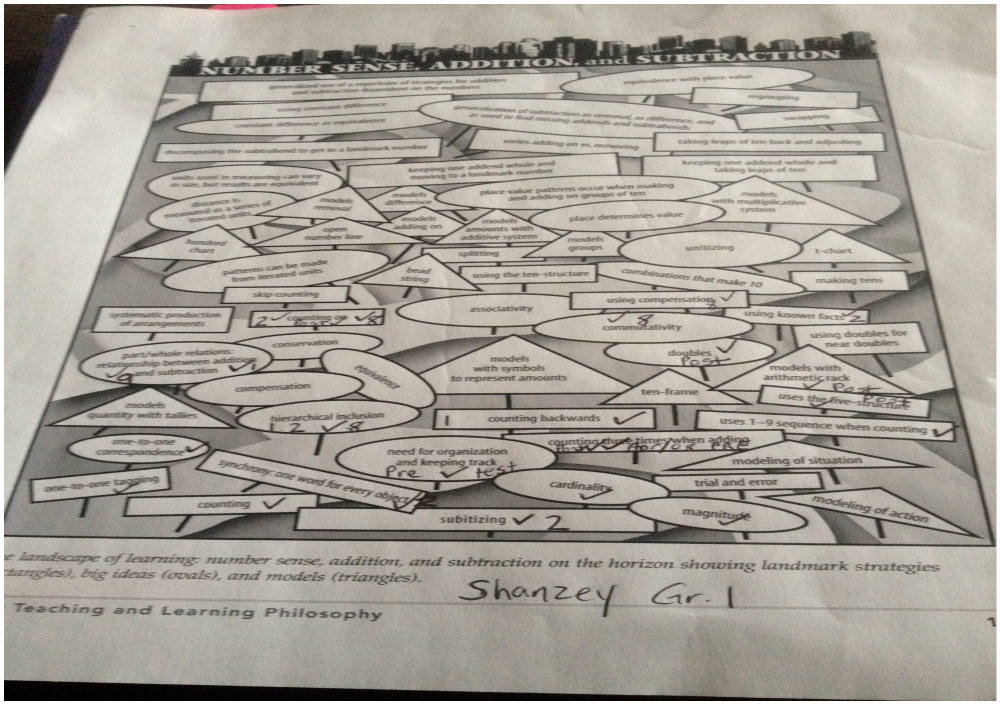

Figure 4.5. Shanzey’s landscape of learning.

Overall, Shanzey could solve addition and subtraction problems to 20. When she explained her strategy orally during a group discussion, she demonstrated an understanding of the relationship between addition and subtraction using the concrete model (the arithmetic rack). When posed with an independent paper-and-pencil task that required her to record her thinking, she reverted back to counting three times. Shanzey’s personal landscape of learning shows partial achievement of the big idea of commutativity (because it was checked only once) and evidence of the use of compensation (on Day 8). A viable next step for Shanzey would be for her to listen to her peers, discuss their strategies of compensating, and then try what she heard and understood on the arithmetic rack.

4.2.2. Damien. Damien's progress throughout the 3 weeks was difficult to track because he talked a lot and the content of his talk varied significantly. Every day, his talk throughout the 3 weeks comprised both social norms and math norms. The composition of both did not change much. His math talk became more mathematical as the 3 weeks progressed, but that was in relation to the math getting more complicated. What remained consistent for Damien was that his talk was more sophisticated than what he demonstrated in his work samples. What is interesting about Damien's talk is how it impacted the rest of the group. He pushed everyone else by talking more, and as a result, he moved all of them up the landscape toward constructing the big ideas of unitizing, compensation, and commutativity.

4.2.2.1. Pre-unit assessment. Figure 4.6 shows how Damien solved the "going to the fire station" problem, one of the pre-video study problems given to the students.

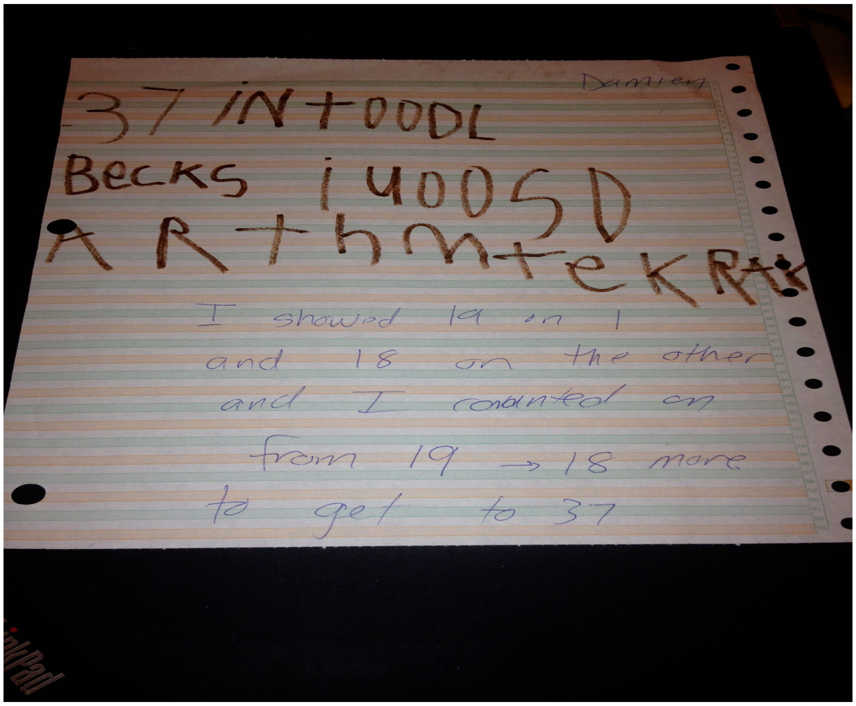

Figure 4.6. Illustration of Damien’s solution to the fire station problem.

Damien told me in his conference that he knew 37 students were going on the trip because he used two arithmetic racks. On one, he showed 18 students by removing 2 from the set of 20, and on the other, he showed 19 by removing one. He then counted on from 19 by ones the remaining 18. In my reflective journal, I made a note to “encourage Damien and others, to use the counting on strategy to get to a landmark number and then to skip count what’s left.” From his work, I was able to put a checkmark on the strategies *subitizing* (because I saw him move the beads on the rack without counting) and *counting on* and I checked the big idea *need for organization and keeping track*. I also checked that he modeled with the arithmetic rack because I mentioned in my notes that he specifically said that “he used the beads to show the kids going on the trip.” Figure 4.7 is a photograph of Damien’s work on the second problem completed prior to the study.

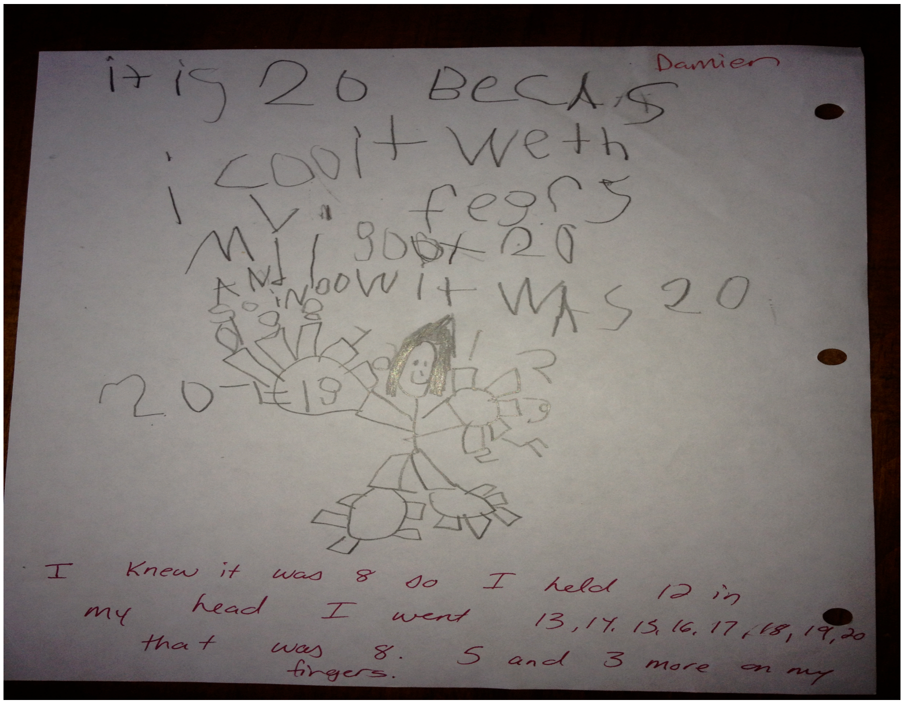

Figure 4.7. Illustration of Damien's work on the second problem.

After studying Damien's work, I found that his explanation and what he showed in his picture and words matched. His picture shows him using the *counting on* strategy on his fingers. He held the bigger number in his head, which was 12, and counted on 8 more to get to a total of 20. From this piece of work, I checked *counting on* as a strategy and *hierarchical inclusion* as a big idea because he said in conference that he knew in the total 8 there was a 5 and a 3.

In a transcription from Day 1 of the video study, Damien showed an understanding of the big idea of commutativity, (a + b = b + a; Dolk et al. 2007). Later, he nicknamed it the "Switcheroo" strategy. It was a strong example of his taking ownership of the mathematics that he was constructing.

Damien: “4 plus 5 equals 9. It’s not actually 4 plus… it’s not changed…. So we just switched it…and then 5 plus 4, 4 plus 5.

Me: “Lots of thinking. Thank you, Damien.”

Damien: “ ’Cause if this 5 is up here, there would be 5 here and 5 here, and if this were on the bottom, there’d be 4 here and 4 here.”

In this next excerpt from Day 1, Damien showed me that he could subitize and unitize. This was a key strategy that later directed the entire class toward seeing the beads on the arithmetic racks as 4 groups of 5 rather than 20 single beads.

Damien: “Because I saw you move this many, and I knew that was, that, there was 10.”

Me: “Mmm.”

Damien: “ ’Cause if you had moved this many, it would have been 20.”

Me: “You know that that’s 20, Damien? Do you – does everyone know that that’s 20?”

Whole group: “Yes.”

On Day 2, Damien demonstrated the *counting on* strategy again.

Damien: “I know it’s 7, because there’s 5 here, 6, 7 (counting on).”

On Day 2 Damien also demonstrated the big idea of unitizing. Dolk et al. (2007) explained that being able to unitize means that students have made a shift in their perception of numbers from counting single objects using numbers to using numbers to count groups.

Damien: “He said that, he knew that each group—He said that each group of 5, that it can equal 10. It keeps equaling.”

Me: “And he said that’s one group of 5, 2 groups.”

Damien: “And if it gets in the 2, so it’s 10, and then another group – 20.”

Me: “So how many groups of 5 are on that arithmetic rack?”

Shanzey : “2.”

Me: “2 groups of 5?”

Damien: “4.”

Me: “How do you know it’s 4?”

Damien: “Because 1, 2, 3, 4.” (indicating groups in the rack)

The following excerpt from Day 8 showed Damien demonstrating his understanding of the big idea of commutativity.

Damien: “It’s the switcheroo.”

Me: “It’s the switcheroo?”

Damien: “Ya.”

Others: “Yes.”

Me: “How do you know?”

Damien: “Because it’s 5 and 3 and 3 and 5.”

Me: “Yaaaa?”

Damien: “4 plus 4 is also 8.”

On Day 9, Damien varied adding versus subtracting when figuring out how many seats were available on the bus.

Damien: “I almost did the same, except I did 10 straightly and added 5. I holded 10 in my head and counted 5 more.”

Tyler: “Using 5 plus 10.”

Other: “Instead of 20 minus.”

Damien: “That’s not exactly what I meant.”

On Day 9, in the following excerpt, the students discussed how 6 plus 4 and 10 minus 6 helped to solve the same problem. Damien entered the conversation and showed that he could discuss the relationship between addition and subtraction.

Me: “Tatiana did 10 minus 6.”

Damien: “It’s the same thing, but it’s just switching it. They still both work.”

After studying Damien, I realized that he showed more conceptual understanding in discussion rather than through his written work.

4.2.2.2. Posttest. Figure 4.8 is a photograph of how Damien solved the second poststudy problem.

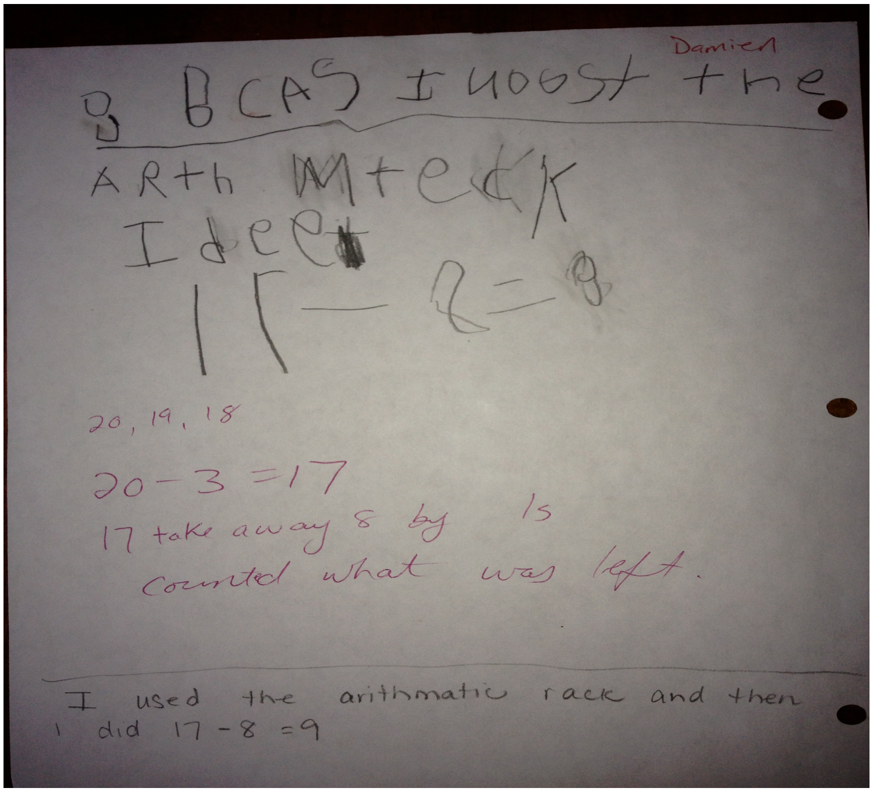

Figure 4.8. Illustration of Damien's work on the second poststudy problem.

It was not possible to know how Damien solved 17 minus 8 based upon what he showed on the paper, so a conference was necessary. He showed 17 on the rack by removing 3, and then he removed 8 more by ones and counted what was left. I was hoping that he would notice that 8 was close to 10 and then add 2 more or use the 5 structure of the rack to subtract 8. Here he used counting three times. Figure 4.9 is a photograph of Damien's landscape of learning.

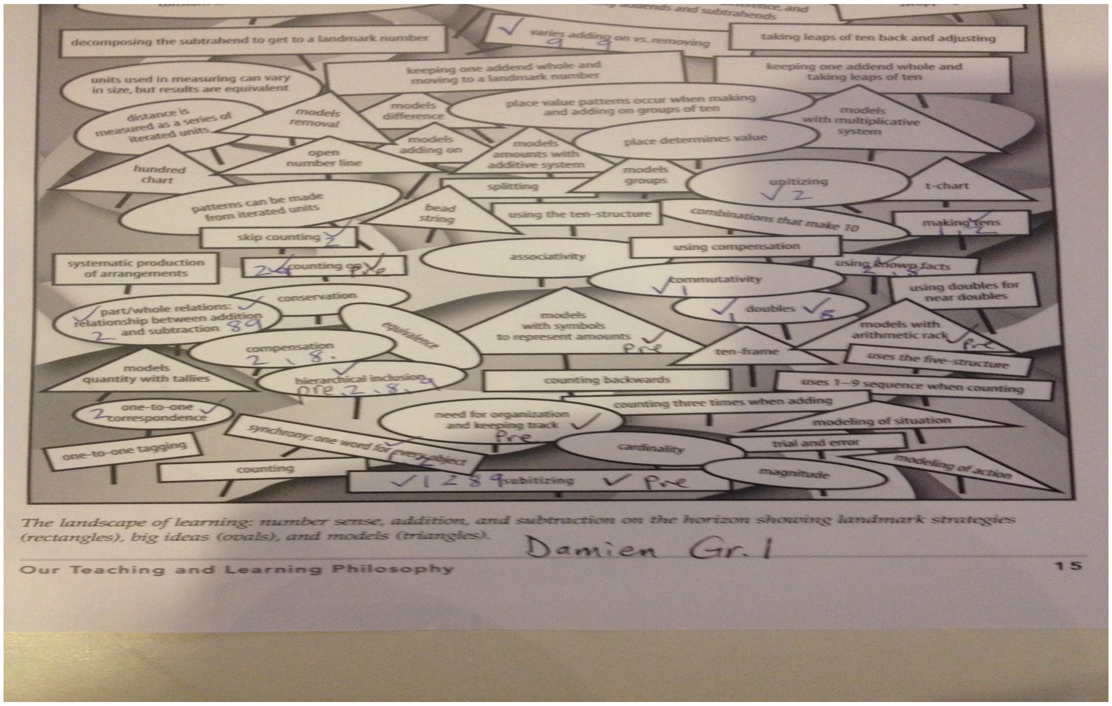

Figure 4.9. Damien’s landscape of learning.

Overall, Damien can solve addition and subtraction problems to 20. He demonstrated an understanding of the big ideas of *commutativity* and *unitizimg* through oral discussions during large-group meetings. This understanding did not transfer to his assessment tasks. In his written tasks, Damien showed subtraction in groups of 5s and 10s to get to a landmark number. A viable goal for Damien would be for him to continue to solve “how many” for the quick images and then have him annotate his strategy in pictorial or written form so that his use of grouping would become visible in his work.

4.2.3. Endrias. Endrias demonstrated growth throughout the 3 weeks of lessons. Although he was more of a teacher than a learner because much of his talk was coded as explanation (he often had to make his thinking explicit for the rest of his peers), I think that it helped him to demonstrate his understanding better on his work samples. What was the most notable was how flexibly he used addition and subtraction and was able to explain the connection between them.

4.2.3.1. Pretest. Figure 4.10 is a photograph of Endrias's work from the first problem before the study began. Here he answered the question 19 + 18.

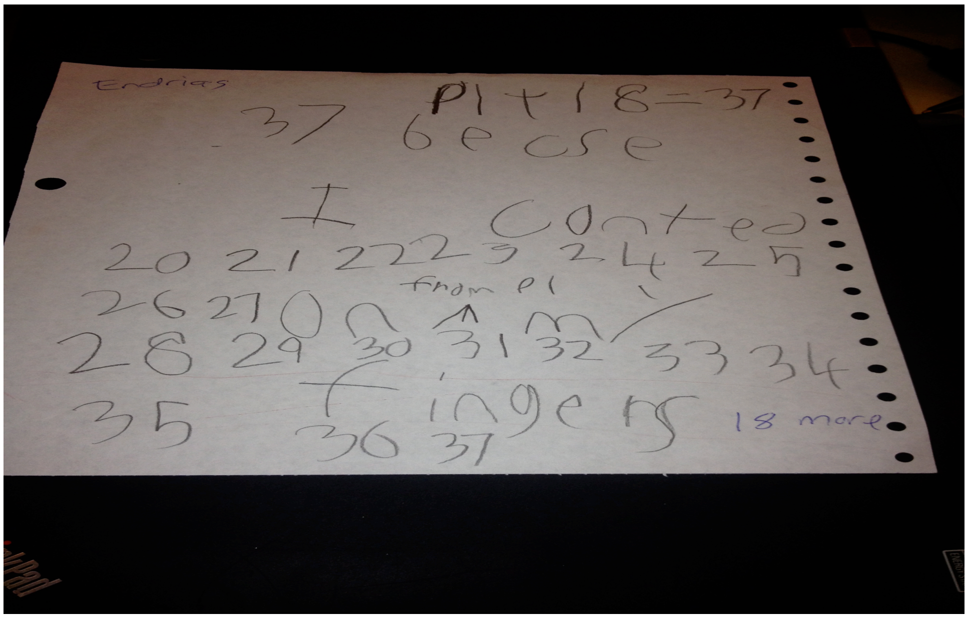

Figure 4.10. Illustration of Endrias's work on the first pre-study problem.

In his work, Endrias clearly explained how he arrived at his solution. He used the *counting on* strategy and kept track of how many to count on by using his fingers. On his landscape of learning, I checked *counting on* and *keeping track*. On the second problem prior to the study, I thought that his method was much more sophisticated than what he did on the first task because he manipulated the numbers so that he could work with 10, a benchmark number (see Figure 4.11). On his landscape, I checked the strategy *using the 10 structure*. I also wondered how his use of this strategy would play out in with the Double Decker Bus lesson.

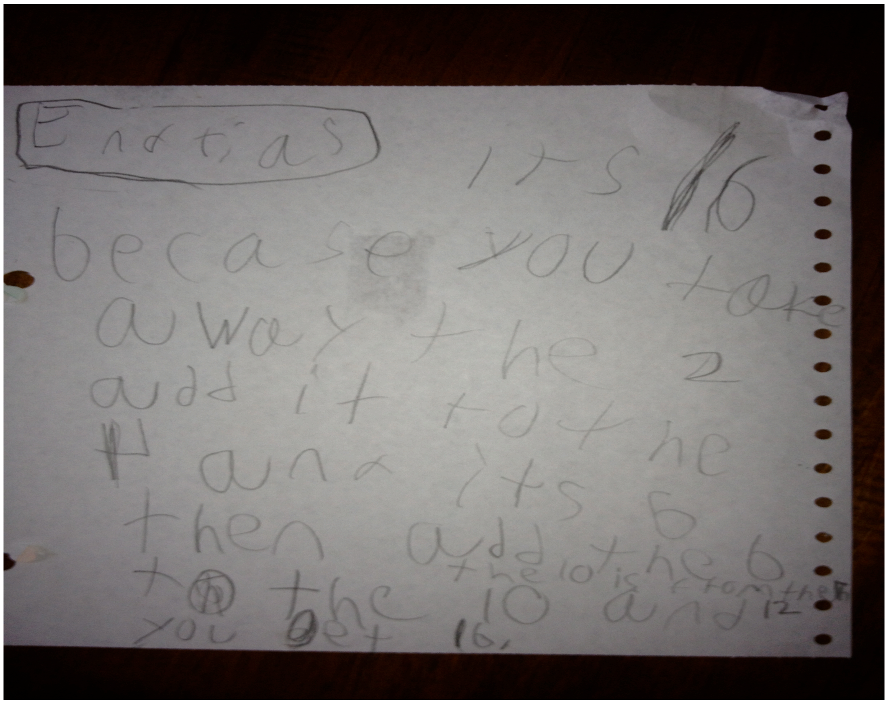

Figure 4.11. Illustration of Endrias's work on the second pre-study problem.

In an excerpt from Day 1 of the video study, Endrias demonstrated an understanding of the big idea of *hierarchical inclusion* and was able to make a mathematical generalization for the entire group. He also demonstrated once again his comfort using the 5 and 10 structures.

Me: "Make 6. Okay, Endrias, come on up, Endrias. Make sure you're being respected."

Endrias: "This is 5. If you add one more to 5, this is 6. It's just like plus 1 to 10, you get 11."

Me: "Just like 10 plus 1."

Endrias: "Is 11."

Me: "Is 11—Just like 10 plus 1 equals 11… and then what?"

Endrias: "Just plus 1, like, then 1 is just 1 more."

Me: “Oh, it’s just 1 more?”

Endrias: “And then, you count 1 more and you see what it is.”

Me: “Just count 1 more, okay.”

In this next excerpt from Day 2, Endrias subitized, and he used the *counting on* strategy. He also demonstrated an understanding of the big ideas of *cardinality* and *hierarchical inclusion*. My questioning was done to get Endrias to talk more. I tend to do that type of open-ended questioning more with students whom I feel have a higher level of conceptual understanding, or rather, what I think they do understand is solid and can handle elaboration of their explanation.

Endrias: “I saw 5, and I saw 8, and I counted on from 8, 5 more, and then I got 13.”

Me: “Can you show us?”

Endrias: “9, 10, 11, 12, 13.”

Me: “So what did you see? How did you know that was 8 down at the bottom there, Endrias?”

Endrias: “Because this is 5, and 3 more from 5 is 8.”

Following is a quote taken from my field notes after Day 4 of the video study (which is Lesson 3). In the following excerpt, Endrias demonstrated the commutative property.

On that day, the morning question was, “Do you think what Henry and Namrah did yesterday [referring to 13 passengers – commutative property and compensation (moving 1 bead up and 1 down)] can be applied to a different number of passengers?” Endrias tried it with 12. I ensured he knew what the goal was because I asked him what his first idea was, and he said that it was “10+2 = 12,” and to be sure he would be able to explain this to the class, I asked him where he was going to go next. He said after thinking “9+3.”

On Day 9 of the video study, the morning message asked students to help the bus driver by thinking about the following problem: “He (the bus driver) knows there are 13 on the bus, then 7 get off. None get on. He wants to know how many are still on the bus.” After the students talked with their math partners and determined that there were 6 passengers left, using various *counting on and back* strategies, Endrias added some sophistication to the repertoire of more basic counting strategies. He explained how he knew there were 6 passengers left in the following in a clip from day 9.

Endrias: “7 plus 7 is 14 so 7 plus 6 must be 13. Take away 7 from the 13, and then you have the 6 left because you have the 6 that you added from the 13 because the 6 is left there.”

Me: “What strategy do you think you used there, Endrias?”

Endrias: “I was adding and taking away.”

Others: “He used doubles.”

Based upon this conversation, I checked off on Endrias’s personal landscape of learning that he demonstrated the big ideas of *part/whole relations* and *compensation* as well as the strategies of *using doubles for near doubles* and *varies adding versus removing*. I also coded that he was approaching an understanding of constant difference because he showed in his demonstration that taking 7 away from 14 needed to be shifted to one less on an imaginary number line to 13 in total with 6 passengers remaining.

4.2.3.2. Posttest. Figure 4.12 shows Endrias's work on the cat treats problem done after the video study. Again, he used his understanding of the 10 structure and compensated to help him solve the problem. He also used a second strategy to prove that his answer was correct. I think that he used a second strategy that was much less sophisticated, for two reasons: (a) to check his answer to make sure that it was correct, and (b) he was getting accustomed to being asked to explain his thinking over and over again.

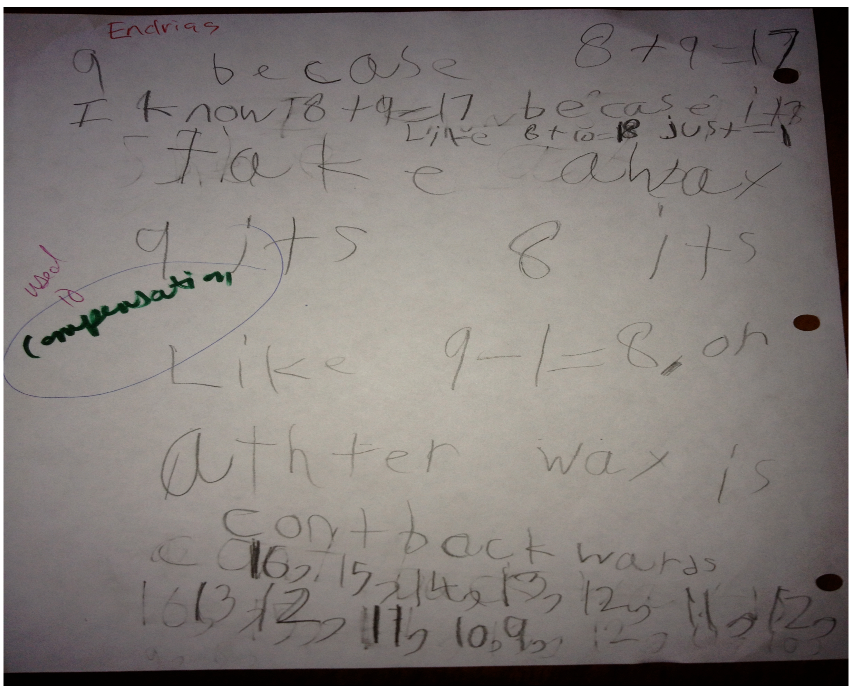

Figure 4.12. Illustration of Endria's work done after the video study.

Figure 4.13 is a photograph of Endrias's personal landscape of learning. Overall, Endrias continued to use the less sophisticated strategies of *counting by ones* and *counting on* throughout the unit while talking in pairs in a more casual problem-solving setting. However, he also made leaps up the landscape of learning by building on his use of doubles and 10 as a benchmark to solve more complicated double-digit addition and subtraction problems when asked to think about a different way to solve a morning message problem. On the last day, he began to describe a subtraction of a set as a shift along an open number line. It would take further conferencing with him and more group discussions with the entire class to determine whether this was what he actually visualized and whether the majority of students would be ready to pursue those types of constant difference problems.

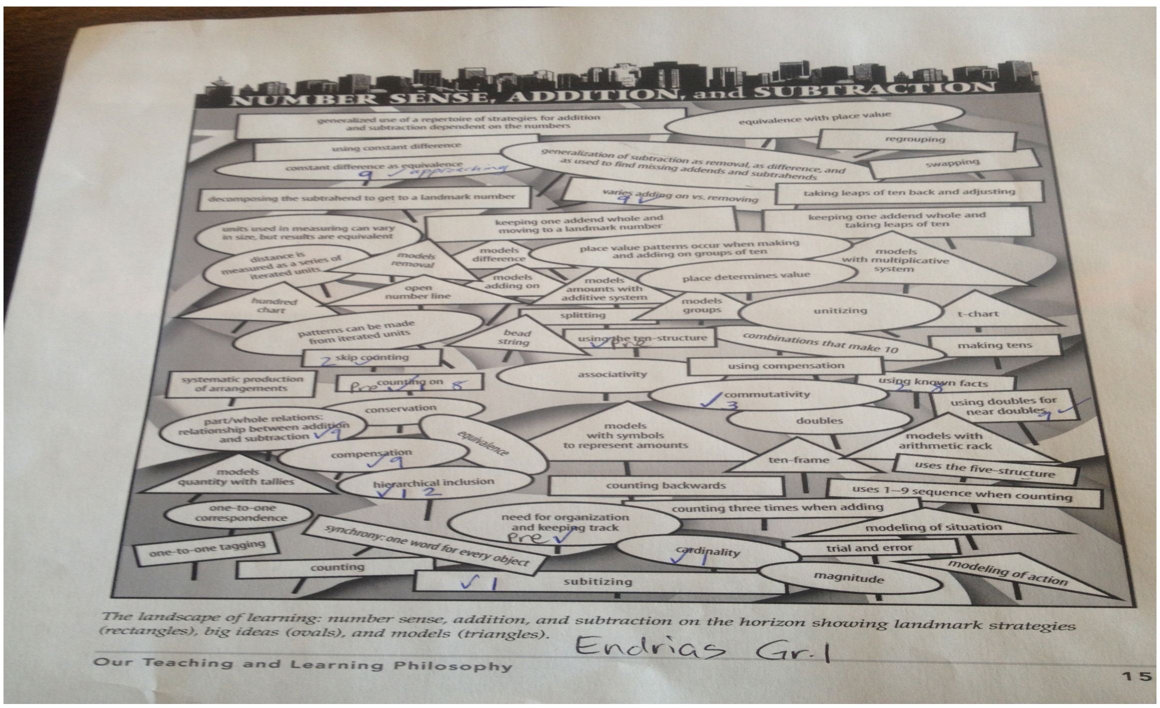

Figure 4.13. Endrias's landscape of learning.

Overall, the three students in the study moved up the landscape of learning and consolidated their current levels of understanding of addition and subtraction. They all used more sophisticated strategies to solve problems during large-group discussions in comparison to what they demonstrated on their pretests. Shanzey was able to apply compensation to new mathematical situations. Damien was able to work through the big idea of *unitizing* and imagined how one group can mean 5 things or 10 things, too. Endrias made explicit the reciprocal relationship between addition and subtraction, and he solidified his idea of part/whole relations. More importantly, listening to these students talk throughout the unit allowed me to assess their understanding and craft questions for strategy development and future investigations meeting their ZPD (Vygotsky, 1978).

4.3 Teacher's Knowledge

In their review of literature, Lampert et al. (2010) summarized that in a dialogue between students and teacher, the work of the teacher is to maintain coherence and focus on ensuring that mathematical concepts are explained in a way that is co-constructed, not produced by the teacher alone. What is the teacher's role in getting the students to talk and to talk mathematically?

Two questions or themes emerged as I watched the videos. The first theme was pedagogy, and I asked about the kind of knowledge of pedagogy (instructional strategies) that I needed to understand and use in order to get the students to talk and to talk accountably (social norm)? The second theme was mathematical content knowledge, and we asked about the kind of mathematical content knowledge that I needed to understand in order to get the students to participate in socio mathematical norm discussions.

I hypothesized that as the lessons progressed, my talk at first would demonstrate a reflection of knowledge of pedagogy and then would move into talk reflecting my knowledge of math content. In relation to the students being observed, I thought that my questions and prompts would become more mathematically specific and direct so that I could identify the strategy being used and understand the big idea being developed. I needed to know this for two reasons, namely, (a) so that I could highlight progress on their personal landscape of learning, and (b) because as the time for a congress (math meeting or time for consolidation) approached, I needed to be certain about what strategies were to be discussed as viable options for solving the problems so that the intended goal of the lesson would be summarized by the students with my help. Therefore, we thought that there would be more evidence of pedagogy at the beginning of the 2 weeks to get the students talking and then more evidence of math content knowledge near the end to get the students talking more mathematically.

This excerpt from Day 1 is an example of my use of an instructional pedagogy. I allowed the students to struggle within their ZPD (Vygotsky, 1930/ trans.1978). Even though the students appeared to be complaining, I was comfortable with their unease and encouraged them to work with their partner to help them get through it. Dolk, et al. (2007) asserted that a real math community invites the learners to take risks and gives them time to collaborate.

Me: "Whoa, I'm hearing some people say they didn't get enough time! Probably the same problem the girl had when she's looking out her window. Talk to your partner about what you saw, (pause), talk to your partner about what you saw."

In the next excerpt from Day 2, I took the time to have a dialogue with the students in order to make clear that the strategies that they were using were not the same. This was an example of using pedagogy and math content knowledge.

Me: "Okay, so Endrias and Jayden got 13 in two different ways."

Other: "So did I."

Me: "What was Endrias's way? What did Endrias see? Jaylen, you think you can explain that? Thanks, Jaylen. Let's see – Endrias, listen to see if Jaylen understood what you said."

Jaylen: "He saw 8 on the bottom, and then he added 5 more, 'cause I think he saw what 5 here, and then 5 on the top, and then 3. I think, he took away one 5 and then added 3, then that would be 8, then added the 5 back, and that would be 13."

Me: "Is that what you saw, Endrias?"

Endrias: "Not exactly."

Me: "Not exactly. Do you want to explain again what you saw? Talk to your friends."

Endrias: "I saw 5 here, and I saw 8 there."

Me: "We can't see because you're standing in front of it."

Endrias: "I saw 5 here, I saw 8 here. I counted on the 5 because I didn't have time to count by ones, like this, so then I just counted on the 5, and then I got 13."

Me: "Okay, so he counted on the 5."

Damien: "I actually had time to count by ones."

Me: "Okay, so Endrias, you said how did you know this was 8 again?"

Endrias: "Because 5 plus 3 is 8."

Me: "So you saw 5 plus 3 equals 8, and then you knew you had 8 in your head."

Endrias: "And I counted on 5."

Me: "Then you counted on 5 more, so 8, 9, 10, 11, 12, 13. -And he's right."

In this final excerpt from Day 2, the students demonstrated how they applied the model to a new situation and became flexible with the total of the numbers. Again, this was an example of using pedagogy and math content knowledge to work with the students. It was an example of pedagogy because asking the students to apply a mathematical situation to a new context allowed them to dig deeper into the context and make it more meaningful. This also might give more reluctant mathematicians more time to understand the problem being investigated. This also was an example of math content knowledge because we were seeing how far the students could stretch their use of the strategies of *counting on* and *counting back* or whether they would take the leap to using a benchmark number.

Alex: “Mrs. Allen, I just wanted to ask the group, I wondered if all of you were sitting on the bus, if there’d be any seats left over.”

Students: “Yes. – One more – One (multiple times).”

Students: “No, if Ms. Allen was sitting on the bus.”

Damien: “But she would be the driver!”

Me: “So would there be enough seats on the double-decker bus for you?”

Students: “Yes.”

Damien: “There’d be one left.”

Me: “How do you know?”

Damien: “Because if we have 19 kids.”

Endrias: “Because there’s 10 in each row, and 10 plus 10 is 20, so each row is like this, it’s a group of 5.”

Damien: “So you’d be one passenger.”

Me: “I’d be one passenger. I’d want a window seat.”

Alex: “Mrs. Allen, I’m not sure I agree. I wondered if you could show me ’cause I’m not sure there would be enough free space.”

Shanzey: “This is the 1 left 1, and this is 19, so if another people comes, then it’d be 20.”

Other: “Like Ms. Allen. She could still get the last seat.”

Shanzey: “And then if there’s – so there’s 19 kids in our class, so if one more peoples comes, then it’s only – it’s gonna be 20. And it’s gonna be enough for us, but there’s just one more, so another people can come if they want.”

Damien: “But it’ll be enough for 21, cause the driver’s seat. It would be 2.”

Shanzey: “Yeah, but the driver’s not a passenger, he’s the driver.”

That last excerpt also demonstrated that the students felt comfortable enough changing the context slightly and creating a very real situation for the numbers. It also showed how long a discussion the students were able to have without too much teacher intervention.

In the excerpt from Day 8, which we read earlier, where Shanzey showed her understanding of the big idea of *compensation*, we saw a combination of both types of teacher knowledge.

Me: “Shanzey, did you want to say something? Make sure you are being respected. Wait until you are being respected and listened to.” (pedagogical move)

Shanzey: (moves beads on the rack 8 on the top and 2 on the bottom) “8 and 2 is 10.”

Me: “You just know that 8 plus 2 equals 10? Is it a fact you know?”

Shanzey: “Because I’m using the 1 plus 9 to do the 8 plus 2.”

Me: “Can you show us that 9 plus 1 you’re talking about?”

Shanzey: (moves one away from the bottom and adds one to the top)

Damien: “Ohhh! Like up there.” (pointing to a board)

Shanzey: “That’s 9, and then I did this.”

Damien: “You can keep going back and forth adding 1 and taking 1.”

Me: “Shanzey really knows her 9 and 1, and she used that to help her figure out the 8 plus 2.”

Me: “Did anyone else see 9 and 1?”

Me: “Shanzey, please record your strategy over there on the flip chart. Thanks Shanzey, good for you. I noticed you edited your strategy.”

I wanted to see how much talk I did in order to get the students talking and what type of talking I did. Table 4.6 lists the codes that I developed for the talking that I did throughout the video study and the frequency of that type of talking. The talking I did was coded only if it was in relation to the students being studied.

Table 4.6

Teacher Talk Codes

Codes for pedagogical knowledge		Codes for math content knowledge	
• Teacher prompting students in the study to talk • Using a talk move		• Stringing together a discussion to link strategies • Making math content explicit • Asking a question to determine placement on the landscape of learning	
86 instances Days 1 and 2	95 instances Days 8 and 9	111 instances Days 1 and 2	186 instances Days 8 and 9

Originally, I were going to compare how often I participated in each type of talk on Days 1 and 2 to Days 8 and 9, but after revisiting the video clips, I realized that both types of talk were needed throughout the unit to complete the lessons adequately. For instance, the resource helped me to direct my questioning during the congress in order to elicit certain strategies. During that time, my talk would obviously be more mathematical than when I am developing a different context every day at the beginning of a lesson.

Table 4.6 showed that the instances of my talk did not fluctuate throughout the 3 weeks; in fact, they actually increased in both types of talk from Days 1 and 2 to Days 8 and 9. After I discussed this with my supervisor, I hypothesized that it might have been because there were longer instances of talk coded. Conversations were lasting longer in relation to the students being studied. The lessons were lasting longer from beginning to end. For instance, before I could develop a new context or pose a new problem for the day, we revisited the previous day's conjectures or questions. A longer lesson elicited more talk from everyone. In addition, the frequency of math content knowledge codes increased more than the pedagogy codes on Days 1 and 2 and on Days 8 and 9. This might have been because the students were truly invested in the context and did not need too much prompting to participate. Therefore, my talk moves were geared more toward assessing for understanding rather than focusing on classroom management.

Overall, it was evident that I drew on math content knowledge and pedagogy when facilitating the development of strategies and big ideas in my math classroom. The talk moves that I used to support both norms coexisted throughout and were made to keep the discussion going and to work with the students on co-constructing emerging strategies. The students did lead more of the discussion toward the end of the unit, and their talk with each other lasted longer and without my intervention on Days 8 and 9. My participation was still active rather than passive, however, because I was present and in close proximity to the students during their conversations.

An interesting aspect of my role that was evident in the video was my activity during peer-to-peer talk. I used a lot of gestures (nodding, thumbs up, hooray, shrugging); I silently redirected students by tapping their shoulders and making eye contact with them; I quietly repeated or whispered to myself, but loud enough for the students to hear, important terms and ideas; I annotated students' strategies on the board or flip chart, wrote down words or phrases, and dramatized what thinking or pondering a situation looks like; and I acted excited during "wow" moments. I did this to encourage students to keep talking, especially when an important idea that I thought would impact the development of an emerging strategy or a big idea was being debated by the students or to redirect them so that their discussion was purposeful and continued along the path of our math community's learning goal. I believe that I was using what Stein, Engle, Smith & Hughes (2008) referred to as the five practices for orchestrating productive math discussions in an inquiry-based classroom. These researchers asserted that to move from the first generation of reform to the second, teachers need to use these five practices: anticipating likely student responses, monitoring these responses, selecting particular students, sequencing the students' responses, and helping the class to make mathematical connections. In my video analysis, while my students were engaged in discussions, I believe that I orchestrated some of these practices to keep the discussion productive.

CHAPTER 5: CONCLUSION

Summary of Major Findings

I conducted this study to answer the following questions:

1. What is the role of social norms (as defined by Sfard, 2000) in my classroom math community that contribute to the construction of strategies and big ideas in early addition and subtraction?
2. What is the role of sociomathematical (as defined by Sfard, 2000) norms in my classroom math community that contribute to the construction of these strategies and big ideas?
3. What pedagogical and mathematical content knowledge do I as the teacher draw on to facilitate the development of strategies and big ideas?

I conducted a video case study specifically to analyze talk in my Grade 1 math classroom to determine the role of social norms, sociomathematical norms, and the teacher's role in facilitating these norms on the development of strategies and big ideas for addition and subtraction. I taught and videotaped the 9-day unit (i.e., nine lessons videotaped over 3 weeks), developed by Dolk et al. (2007). The model of the arithmetic rack was introduced in the context of the double-decker bus, in which the students applied their strategy directly (moved the beads or passengers on the rack) or indirectly (described the rack when explaining a strategy). Dolk et al.'s (2007) landscape of learning was the trajectory used to track students' strategy use and conceptual understanding of the mathematical big ideas for addition and subtraction.

I selected three students, Shanzey, Damien, and Endrias, to analyze their talk. They were purposefully selected based upon their frequency of talk and their levels of conceptual understanding, which varied. I analyzed their talk on the first 2 days of the unit (Days 1 and 2) and again on the last 2 days of the unit (Days 8 and 9). I also analyzed my talk on these days in relation to the students. The students completed two independent tasks before and after the unit was taught that paralleled the addition and subtraction investigations in the unit. Otherwise, the students worked in homogeneous pairs or with the whole group during the study. There were five main findings in this study.

5.1 First Finding: Change in Frequency and Length of Student Talk

The frequencies of both types of talk increased for Shanzey and Endrias; Damien's talk stayed the same. Looking more closely at Damien's conversations, I think it is important to note that on Days 1 and 2, Damien's instances of talk lasted less than 1 minute. On Days 8 and 9, discussions led by Damien typically lasted more than 5 minutes. I think that this is the reason that Damien's instances did not increase: He simply was talking longer.

5.2 Second Finding: Change in Direction of Student Talk

All students directed their talk increasingly toward their peers rather than to me on all days. The instances of peer-to-peer talk were less frequent on Days 8 and 9. Again, I think that this was the result of the conversations lasting longer on Days 8 and 9 (>5 minutes) than on Days 1 and 2 (< 1 minute).

5.3 Third Finding: Change in Students' Type of Talk

The two types of talk that were measured were social norms and sociomathematical norms. The data showed a shift in the type of talk for Shanzey and Damien. For both students, the proportion of talk shifted from greater social norms to greater sociomathematical norms. The type of talk did not shift for Endrias. This outcome was what I had expected. I thought that Shanzey would talk more and that her talk would become more mathematical. Damien talked a lot, and I expected his type of talk to become more mathematically productive. Endrias's talk was always mathematically sophisticated, and he often took the role of explainer.

Although there was an increase in mathematical norms, the social norms and the sociomathematical norms were present consistently over time. It is true that before sociomathematical norms can be established, social norms need to be in place and must be consistently reinforced (Chapin et al., 2003). What I can conclude from my video study is that sociomathematical norms do not replace social norms; instead, they work in tandem while maintaining their own unique roles in moving the whole group forward, or in this case, up the landscape of learning.

5.4. Fourth Finding: Some Growth in Students' Strategies and Deepening of Big Ideas

Although there was not much movement up the landscape of learning for Shanzey, Damien, and Endrias, they did make gains. I believe that there is enough evidence to suggest that all three students deepened their comfort levels with the strategies that they were already using sufficiently to not only try a more sophisticated strategy with the support of a partner and in the context of an investigation but also use a different strategy to suit a specific type of question. For example, when playing the passenger pairs game with Damien, Shanzey explained that she used a removal strategy to get to 18 passengers instead of counting three times, a strategy that she had used in her pretest, to get the correct number of passengers. Another example of using a more sophisticated strategy was when Endrias explained that he was adding as well as taking away to solve 7 + 6 is 13 because 7 + 7 is 14, instead of counting on, which he had done on his pretest.

Shanzey consolidated her understanding of the big idea of the relationship between addition and subtraction. I believe that this happened for her because there was so much talk surrounding this big idea and through the use of the model, she was able to reimagine it over and over again when we discussed why 4 + 6 and 10 – 4 were used to solve one problem. She also developed an understanding of compensation when she played around with the idea that 8 + 2 and 9 + 1 both equal 10. She showed trust in her peers when she questioned them over again and listened to their explanations. Shanzey also had the opportunity to investigate the big idea of *unitizing* when her peers talked about the double-decker bus having two groups of seats on one level and two groups of seats on another.

Damien also did not show much movement up the landscape, but he did demonstrate a deepening of understanding. Damien was coded for most of the talk in the video. Much of his talk was coded as sociomathematical norm talk. Throughout his talk time, he was able to make many connections (i.e., to other peers' comments, to past investigations), which led us as a group toward investigating the big idea of *unitizing*, the big idea that one refers to a group. What I also think was an important step for Damien throughout the 3 weeks was having his peers and me model for him how to record his thinking on paper. There were many instances when I asked the students to write down their thoughts or to record their thinking on the flip chart. I was hopeful that these explicit experiences would eventually translate to Damien's independent tasks, which were not apparent in his posttest but would be, I expect, in the near future.

Endrias consistently demonstrated a very high level of conceptual understanding throughout the 3 weeks. In his pretest, he demonstrated an understanding of equivalence when he showed two ways to solve for 19. When faced with a similar problem using the model of the double-decker bus, his reasoning became very flexible when trying to show the similarities and differences between his strategy and a peer's strategy for 8 + 5. I was interested to see this understanding of equivalence translate to subtraction when he helped a peer to explain why when 7 people got off the bus from a total of 13 passengers, he used 14 as an initial total to help.

5.5 Fifth Finding: What Pedagogical and Mathematical Content Knowledge Did I Draw On?

To develop a community of learners who participate in discussions and talk productively, I drew on the work of Chapin et al. (2003). They outlined five talk moves for teachers to use: revoicing, asking students to restate each other's reasoning, asking students to apply their own reasoning to someone else's, prompting students for further participation, and using wait time. In the video, I used all of these moves consistently, with 86 instances coded on Days 1 and 2 and 95 instances coded on Days 8 and 9. Using these moves helped me to assess my students' understanding enough to be able to plot them on the landscape of learning.

It also was evident in the video that I consistently practiced what was summarized in Lampert et al. (2010) and maintained my coherence and focus of the mathematical concepts introduced in the context and being discussed by the students. The 111 instances coded on Days 1 and 2 and the 186 instances coded on Days 8 and 9 reflected my understanding of the landmark strategies and big ideas for early addition and subtraction shown on Dolk's et al. (2007) landscape of learning. I used this trajectory for early number sense development to track the students' development. I spent time discussing the meanings of the strategies and big ideas with my teaching partner and my supervisor. We looked at the students' work examples and matched them with the definitions supplied by Dolk et al., as well as collected student samples to compare strategies across various types of addition and subtraction questions and their responses. Studying and discussing the students' responses and written work helped me to label their strategies and assess their conceptual understanding.

Final Conclusions

It has been emphasized in recent education reform curricula (NCTM, 1989, 2000) that learning in a mathematics community fosters the communication of students' ideas and deepens their conceptual understanding. I believe that I was able to foster this type of community in my classroom with these students. I facilitated the development of social norms and sociomathematical norms.

I provided opportunities and expectations for the students to think and talk with each other and then prove and defend and test their conjectures. This practice supports a more rigorous mathematical experience (Gravemeijer, 1973). To do this properly, Sfard (2008) asserted that we need to move away from a show-and-tell type of mathematics discussion (Phase 1 of reform implementation) and toward a more accountable form of participation (Phase 2 of reform implementation), where students can practice sharing their thoughts at appropriate points in a discussion and their thinking will contribute to the learning of the whole group.

I found that the students' frequency of talk, direction of talk, and type of talk became more mathematically focused from the first 2 days to the last 2 days of the unit. This change over time co-occurred with a shift toward greater sophistication of mathematical strategies for two of the three students and a deepening of understanding and the construction of some big ideas in early addition and subtraction.

Considerations for Future Research

I think that students and teachers would benefit from more research in the area of math talk. Specifically, research should focus on analyzing effective math talk and revealing student outcomes over the course of several years. Because many teachers are concerned about the amount of time that this type of instruction takes, they are not certain that it will elicit the learning required to cover the required curriculum content. Another concern is the amount of time necessary to implement this type of learning community, meaning that students might not be prepared for standardized tests in middle and high school. More research showing how students in talk-heavy classrooms fare in the higher grades would help to convince more teachers to support talk-driven math classes.

REFERENCES

Ball, D. L. (1993). With an eye on mathematical horizon: Dilemmas of teaching elementary mathematics. *Elementary Journal, 93*(4), 373-397.

Ball, D. L., Hoyles, C., Jahnke, H. N., & Movshovitz-Hadar, N. (2002). The teaching of proof. *International Congress of Mathematics, 3*(1-3), 907-920.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Battista, M. (1999). The mathematical miseducation of America's youth. *Phi Delta Kappan, 80*(6), 425-433.

Bauersfeld, H. (1994). Theoretical perspectives on interaction in the mathematics classroom. In P Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 133-146). Hillsdale, NJ: Erlbaum.

Black, P., & Wiliam, D. (1998). Inside the black box: Raising standards through classroom assessment. *Phi Delta Kappan, 80*(2), 139-148.

Brown, T. (2001). *Mathematics education and language: Interpreting hermeneutics and post-structuralism* (2^{nd} ed., rev.). Dordrecht, Netherlands: Kluwer Academic.

Callahan, K. M. (2011). Listening responsively. *Teaching Children Mathematics, 8*(15), 296-305

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics.* Portsmouth, NH: Heinemann.

Carpenter, T., Fennema, E., Peterson, P., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26,* 499-531.

Chapin, S. H., O'Connor, C. & Anderson, N. C. (2003). *Classroom discussions: Using math talk to help students learn, grades 1-6.* Sausalito, CA: Math Solutions.

Cobb, P., & Jackson, K. (2010, February). *Reconceptualizing policies as designs for supporting learning.* Paper presented at the symposium on supporting instructional improvement, Haifa, Israel.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. P. E. (2011). Participating in classroom mathematical practices. In A. Sfard, K. Gravemeijer, & E. Yackel (Eds.), *A journey in mathematics education research: Insights from the work of Paul Cobb* (pp. 117-163). Berlin, Germany: Springer.

Creswell, J. W. (2005). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (2nd ed.). Columbus, OH: Pearson.

Dolk, M., Liu, L., & Fosnot, C. T. (2007). *The Double-Decker Bus early addition and subtraction.* Orlando, FL: Harcourt School.

Forman, E. A., & Ansell, E. (2002). The multiple voices of a mathematics classroom community. *Education Studies in Mathematics, 46,* 115-142.

Forman, E. A., McCormick, D. E., & Donato, R. (1997). Learning what counts as a mathematical explanation. *Linguistics and Education, 9*(4), 313-339.

Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing number sense, addition, and subtraction.* Portsmouth, NH: Heinemann.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Charlotte, NC: Information Age.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, The Netherlands: Reidel.

Geist, E. (2010). The anti-anxiety curriculum: Combating math anxiety in the classroom. *Journal of Instructional Psychology, 37*(1), 24-31

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105-128.

Hiebert, J. (1992). Reflection and communication: Cognitive considerations in school mathematics reform. *International Journal of Educational Research*, *17*(5), 439-456.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30*(1), 3-19.

Hill, H. C., & Ball, D. L. (2009). The curious-and crucial-case of mathematical knowledge for teaching. *Phi Delta Kappan, 91*(2), 68-71.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371- 406.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Jacobs, J. K., Tawanaka, T. M., & Stigler, J. W. (1999). Integrating qualitative and quantitative approaches to the analysis of video data on classroom teaching. *International Journal of Educational Research, 31*(8), 771-724.

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203-235.

Kilpatrick, J. (1997). Confronting reform. *American Mathematical Monthly, 104,* 955-962.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

Lampert, M. K., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kuea (Eds.), *Instructional explanations in the disciplines* (pp. 129-141). New York, NY: Springer.

Lampert, M., & Cobb, P. (2003). Communication and language. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 237-249). Reston, VA: National Council of Teachers of Mathematics.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

Lawson, A. C. (2012). *Data filmed for longitudinal study on student generated alternative algorithms.* Thunder Bay, ON, Canada: Lakehead University.

Lipka, J., & Andrew-Ihrke, D. (2009). Ethnomathematics applied to classrooms in Alaska: Math in a cultural context. *NASGEm Newsletter, 3*(1), 8-10.

Marzano, R. J., Pickering, D., & McTighe, J. (1993). *Assessing student outcomes: Performance assessment using the dimensions of learning model*. Alexandria, VA: ASCD.

Michaels, S., O'Connor, C., & Resnick, L. B. (2008). Deliberative discourse idealized and realized: Accountable talk in the classroom and in civic life. *Studies in Philosophy and Education, 27*(4), 283-297.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation of standards for school mathematics* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Ontario Ministry of Education. (1997). *Ontario mathematics curriculum grades 1-8.* Toronto, ON, Canada: Author.

Ontario Ministry of Education. (2005). *Revised Ontario mathematics curriculum grades 1-8.* Toronto, ON, Canada: Author.

Schifter, D., & Fosnot, C. (1992). *Reconstructing mathematics education: Stories of teachers meeting the challenge of reform.* New York, NY: Teachers College Press.

Sfard, A. (2000). Steering (dis)course between metaphors and rigor: Using focal analysis to investigate an emergence of mathematical objects. *Journal for Research in Mathematics Education, 31*(3), 296-329.

Sfard, A. (2007). When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint. *Journal of the Learning Sciences, 16*(4), 565-613.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses and mathematizing*. Cambridge, UK: Cambridge University Press.

Shepard, L. A. (2000). The role of assessment in a learning culture. *Educational Researcher, 29*(7), 4-14.

Sherin, M. G. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education, 5,* 205-233.

Sherin, M. G., & van Es, E. A. (2003). A new lens on teaching: Learning to notice. *Mathematics Teaching in the Middle School, 9*(2), 92-95.

Spillane, J. P. (1999). External reform initiatives and teachers' efforts to reconstruct their practice: The mediating role of teachers' zone of enactment. *Journal of Curriculum Studies, 31*(2), 143-175.

Stahl, G. (2009). Mathematical discourse as group cognition. In G. Stahl (Ed.), *Studying virtual math teams* (pp. 31-40). New York, NY: Springer.

Stake, R. (1995). *The art of case study research.* Thousand Oaks, CA: Sage.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313-340.

Steinberg, Z. D., & Cazden, C. B. (2001). Children as teachers – of peers and ourselves. *Theory Into Practice, XVIII*(4), 258-266.

Stigler, J., & Hiebert, J. (1999). *The teacher gap: Best ideas from the world's teachers for improving education in the classroom.* New York, NY: Free Press.

Van De Walle, J. (2001). *Elementary and middle school mathematics teaching developmentally* (4th ed.). New York, NY: Longman.

van Oers, B. (1996). Learning math as a meaningful activity. In P. Nesher, L. P. Steffe, P. Cobb, G. Goldin, & B. Green (Eds.), *Theories of mathematical learning* (pp. 91-113). Mahwah, NJ: Erlbaum.

Voigt, J. (1996). Negotiation of mathematical meaning in classroom processes: Social interaction and learning mathematics. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 21-50). Mahwah, NJ: Erlbaum.

von Glasersfeld, E. (2005). Introduction: Aspects of constructivism. In C. T. Fosnot (Ed.), *Constructivism: Theory, perspectives, and practice* (2nd ed., pp. 3-7). New York, NY: Teachers College Press.

Vygotsky, L. S. (1930/trans. 1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Webb, N. M., Franke, M. L., Ing, M., Chan, A., De, T., Freund, D., & Battey, D. (2008). The role of teacher instructional practices in student collaboration. *Contemporary Educational Psychology, 33*(3), 360-381.

Yackel, E. (2001, July 12-17). Explanation, justification and argumentation in mathematics classrooms. *Proceedings of the conference of the International Group for the Psychology of Mathematics Education,* Utrecht, Netherlands. Vol. 1-4.